BCS instabilities of electron stars to holographic superconductors

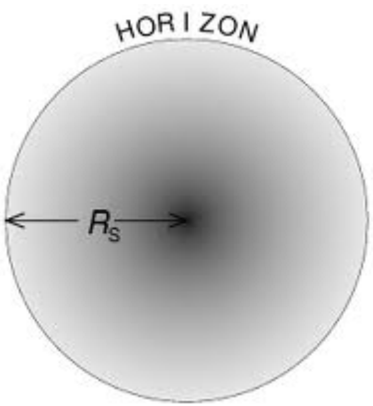
Yan Liu
IFT-UAM/CSIC

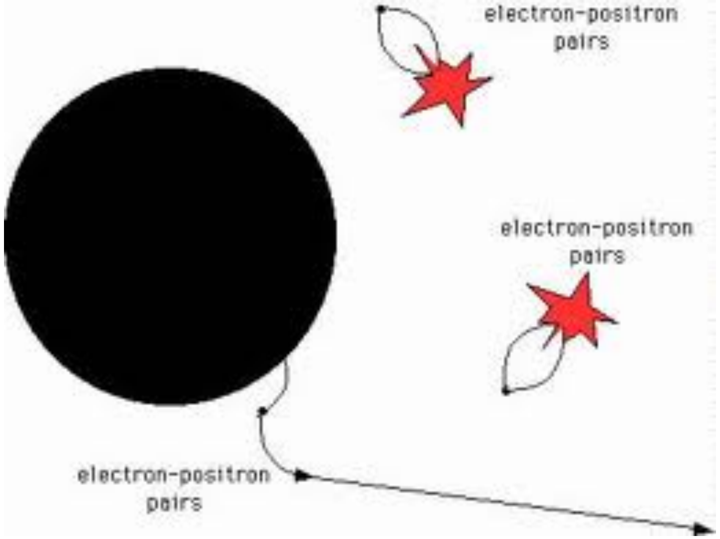
talk at ICTS-USTC, June 13, 2014

Based on: YL, Schalm, Sun, Zaanen, 1307.4572, 1404.0571

AdS/CMT: a correspondence between condensed matter theory and gravity

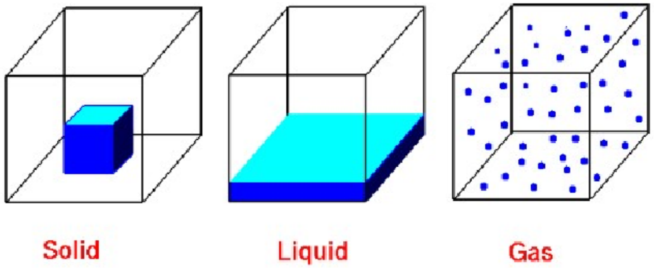
- Gravity: black holes

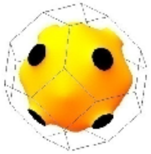


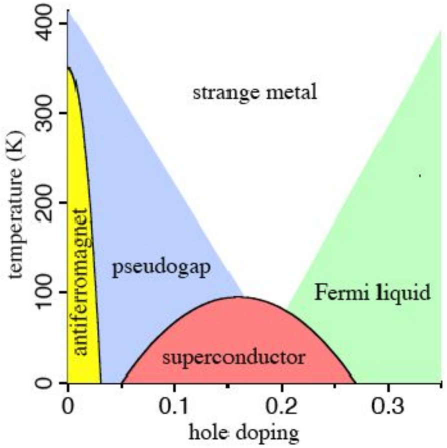


AdS/CMT: a correspondence between condensed matter theory and gravity

- Condensed matter physics: condensed phases of matter





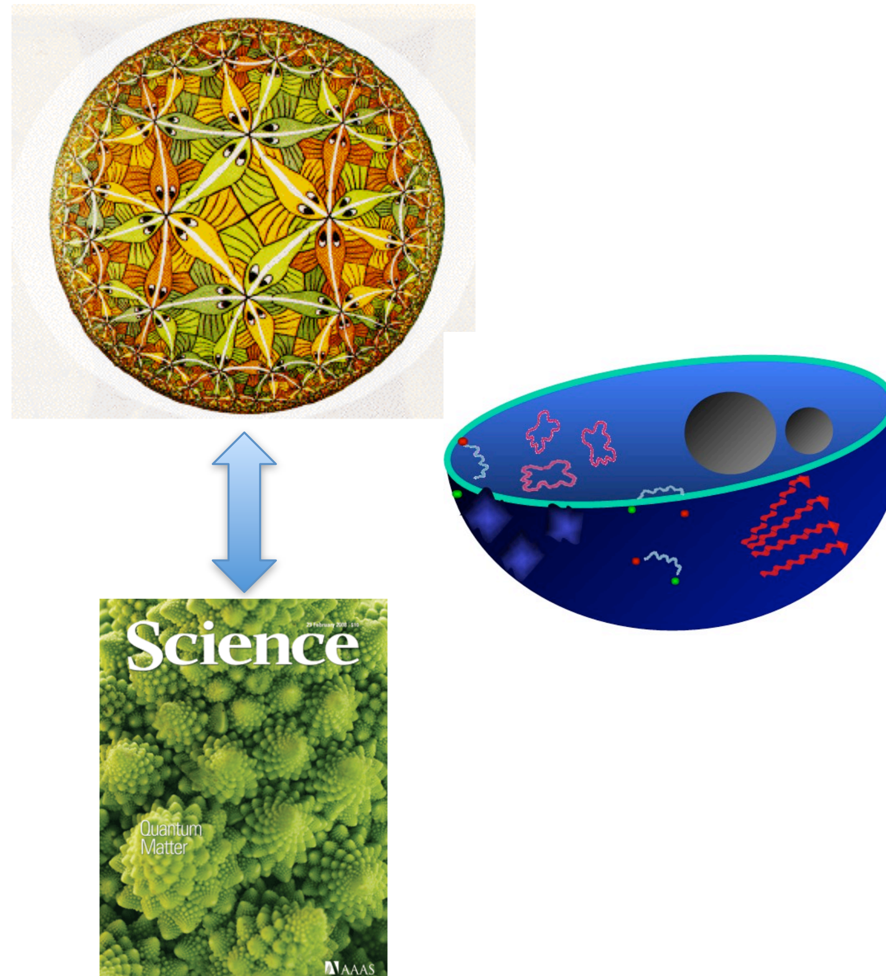


AdS/CFT correspondence

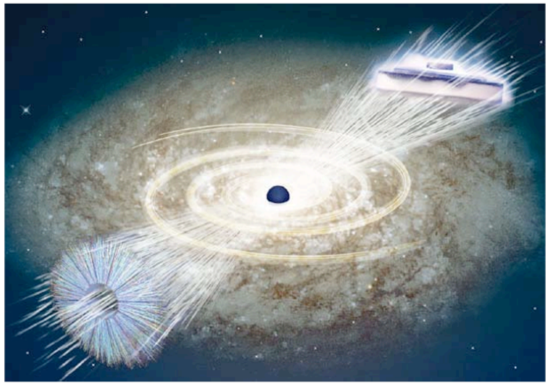
(Maldacena, 1997; Gubser, Klebanov, Polyakov; Witten, 1998)

- *Gravity in d+1 dimensional anti-de Sitter spacetime*

- *Conformal field theory in d dimensions*



AdS/CMT



CMT: systems at finite density

AdS/CMT*:* application of AdS/CFT correspondence to condensed matter physics

Developments:

First contact (2007, S. Sachdev, D. T. Son et al.)
Holographic superconductor (2008, HHH)
Holographic (non-) Fermi liquid (2009, MIT, Leiden)
Holographic lattice (2012, Horowitz, Santos, D. Tong)
……

State of art: it is suggestive but not decisive

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Motivation: towards a more realistic holographic superconductor

- Superconductivity*: zero electrical resistance below Tc*

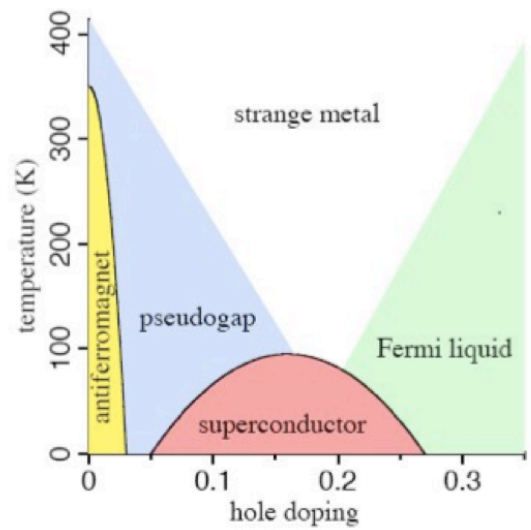
Normal state: Fermi liquid / strange metal (marginal FL)
Ordered state: superconductor

- Conventional superconductors:

low Tc, BCS theory, Cooper pair

- Unconventional superconductors:

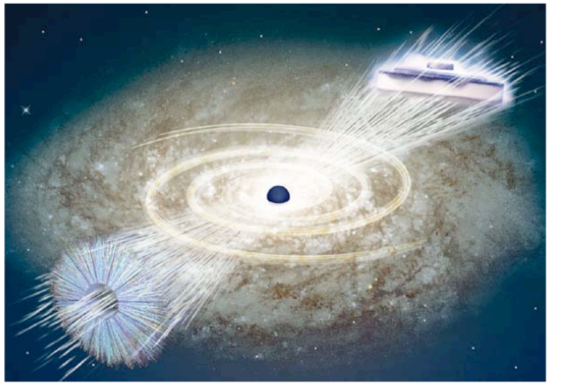
high Tc, Pairing mechanism unsolved



High T_c superconductors

- **Ingredients in common:**
  FS in normal state; Pairing of fermions in ordered state

Motivation: towards a more realistic holographic superconductor



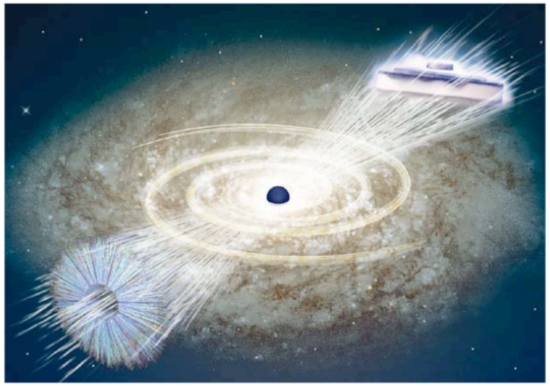
AdS/CMT: application of AdS/CFT correspondence to condensed matter physics

CMT: systems at finite density. Strongly coupled finite density system is difficult from QFT approach.

Dictionary for finite density system

CMT_d		**AdS_d+1**
Many body system	⟺	Classical gravity
Finite temperature	⟺	Black hole
Finite density	⟺	gauge field

Motivation: towards a more realistic holographic superconductor



CMT: systems at finite density

Simplest holographic finite density system

$$\mathcal{L} = \frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4e^2} F_{\mu\nu}F^{\mu\nu} + \text{charged matter}$$

$$A_t(r \to \infty) = \mu$$

Achievements: holographic superconductor,
holographic (non-)Fermi liquid, holographic lattice etc...

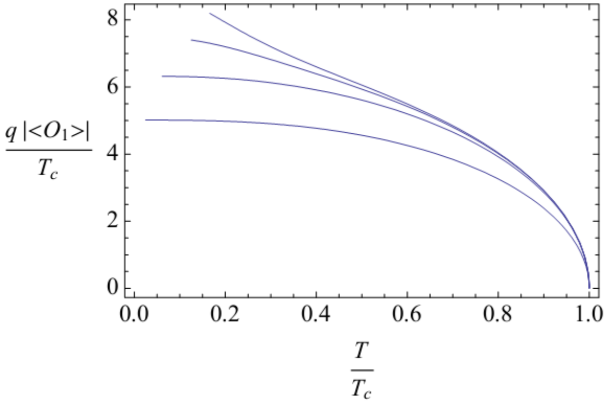
Motivation: towards a more realistic holographic superconductor

Holographic superconductor: (Hartnoll, Herzog, Horowitz, 2008)

$$\mathcal{L} = \frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} - |(\partial_\mu - iqA_\mu)\phi|^2 - V(\phi)$$

a charged scalar field ⟺ order parameter

At finite temperature, 2nd order mean-field phase transition



Normal state AdS RN black hole

Ordered state Hairy black hole

Motivation: towards a more realistic holographic superconductor

Holographic superconductor: (Hartnoll, Herzog, Horowitz, 2008)

$$\mathcal{L} = \frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} - |(\partial_\mu - iqA_\mu)\phi|^2 - V(\phi)$$

Ordered phase: hairy black hole

- Bulk: all the charges carried by the order parameter and the horizon
- Boundary: a superconducting ground state with no fermionic charge density

The ordered state in BCS: Cooper pairing near the Fermi surface

Holographic superconductor: No clear relation between condensate and paired fermions.

Aim I: Holographic superconductor with paring of fermions

Motivation: towards a more realistic holographic superconductor

Holographic superconductor: (Hartnoll, Herzog, Horowitz, 2008)

$$\mathcal{L} = \frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} - |(\partial_\mu - iqA_\mu)\phi|^2 - V(\phi)$$

Normal state: AdS RN black hole

Probe fermions in AdS RN background --- holographic (non-) Fermi liquids (MIT, Leiden, 2009)

Problems with this RN black hole as a model for holographic (non-) Fermi liquids

- Charge carried by the horizon at T=0;
- Properties of Fermi surface relying on the probe while not the background
- Nonzero entropy at T=0, indicating a possible new ground state

New ground states at zero temperature, corresponding to the normal state.

Motivation: towards a more realistic holographic superconductor

A better normal state:

Charge totally carried by fermions, no finite size horizon at zero temperture

- Backreactions of fermions

Electron star: Thomas-Fermi approximation

(S. Hartnoll et al., 2010; de Boer et al. 2009)

Quantum electron star: fermions being treated quantum mechanically (S. Sachdev, 2011, J. McGreevy et al., 2013)

Aim II: Consider a holographic superconductor whose normal state is an electron star

Motivation: towards a more realistic holographic superconductor

Construct a holographic superconductor combining “fermion paring” and “ordering”

Normal state: electron star
Ordered state: with pairing of fermions

BCS star: BCS interaction in the bulk. A superconducting instability of an electron star induced by Cooper paring.

BCS instabilities of electron stars to “holographic superconductors”

Outline:

- A simple review of BCS in condensed matter

- Construction of the holographic BCS star

- Properties of the dual field theory:
  - Gap in the spectral function of the dual theory
  - Charge density
  - Conductivity

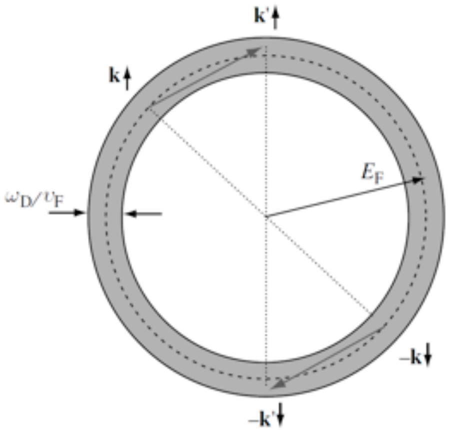
- A more generalized construction

- Summary

BCS theory in Condensed Matter Physics

BCS theory: by Bardeen, Cooper and Schrieffer, 1957

An attractive interaction between electrons is introduced. This interaction induces an instability forming Cooper pairs.



The attractive interaction is induced by phonons, introducing a UV scale: Debye frequency

Effective Hamiltonian for interacting fermion within thin shell:

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - \frac{\lambda}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} c^\dagger_{\mathbf{k}+\mathbf{q}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} c_{-\mathbf{k}'+\mathbf{q}\downarrow} c_{\mathbf{k}'\uparrow}$$

BCS theory in Condensed Matter Physics

Start from the Hamiltonian:

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - \frac{\lambda}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} c^\dagger_{\mathbf{k}+\mathbf{q}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} c_{-\mathbf{k}'+\mathbf{q}\downarrow} c_{\mathbf{k}'\uparrow}$$

Define $\Delta = \frac{\lambda}{V} \sum_{\mathbf{k}} \langle \Omega_{\mathrm{BCS}} | c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} | \Omega_{\mathrm{BCS}} \rangle$

We get the mean field BCS Hamiltonian

$$H - \mu N \simeq \sum_{\mathbf{k}} \left[\xi_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - \left(\bar{\Delta} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} + \Delta c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} \right) \right] + V \frac{\Delta^2}{2\lambda}$$

EoM for Δ gives (Gap Equation)

$$\Delta = \frac{\omega_D}{\sinh(1/\lambda\nu)} \simeq 2\omega_D \exp\left(- \frac{1}{\lambda\nu} \right) \quad \text{when} \quad 1/\lambda\nu \ll 1$$

BCS theory in Condensed Matter Physics

Bogoliubov transformation

$$\begin{pmatrix} \alpha_{\mathbf{k}\uparrow} \\ \alpha^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix} = \begin{pmatrix} \cos\theta_{\mathbf{k}} & \sin\theta_{\mathbf{k}} \\ \sin\theta_{\mathbf{k}} & -\cos\theta_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix}$$

Diagonal form:

$$H - \mu N = \sum_{\mathbf{k}\sigma} E_{\mathbf{k}} \alpha^\dagger_{\mathbf{k}\sigma} \alpha_{\mathbf{k}\sigma} + \sum_{\mathbf{k}} (\xi_{\mathbf{k}} - E_{\mathbf{k}}) + V \frac{\Delta^2}{2\lambda}$$

BCS vacuum state:

$$|\Omega_{\mathrm{BCS}}\rangle = \prod_{\mathbf{k}} \alpha_{\mathbf{k}\uparrow}\alpha_{-\mathbf{k}\downarrow}|\Omega\rangle \sim \prod_{\mathbf{k}} (\cos\theta_{\mathbf{k}} - \sin\theta_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow})|\Omega\rangle$$

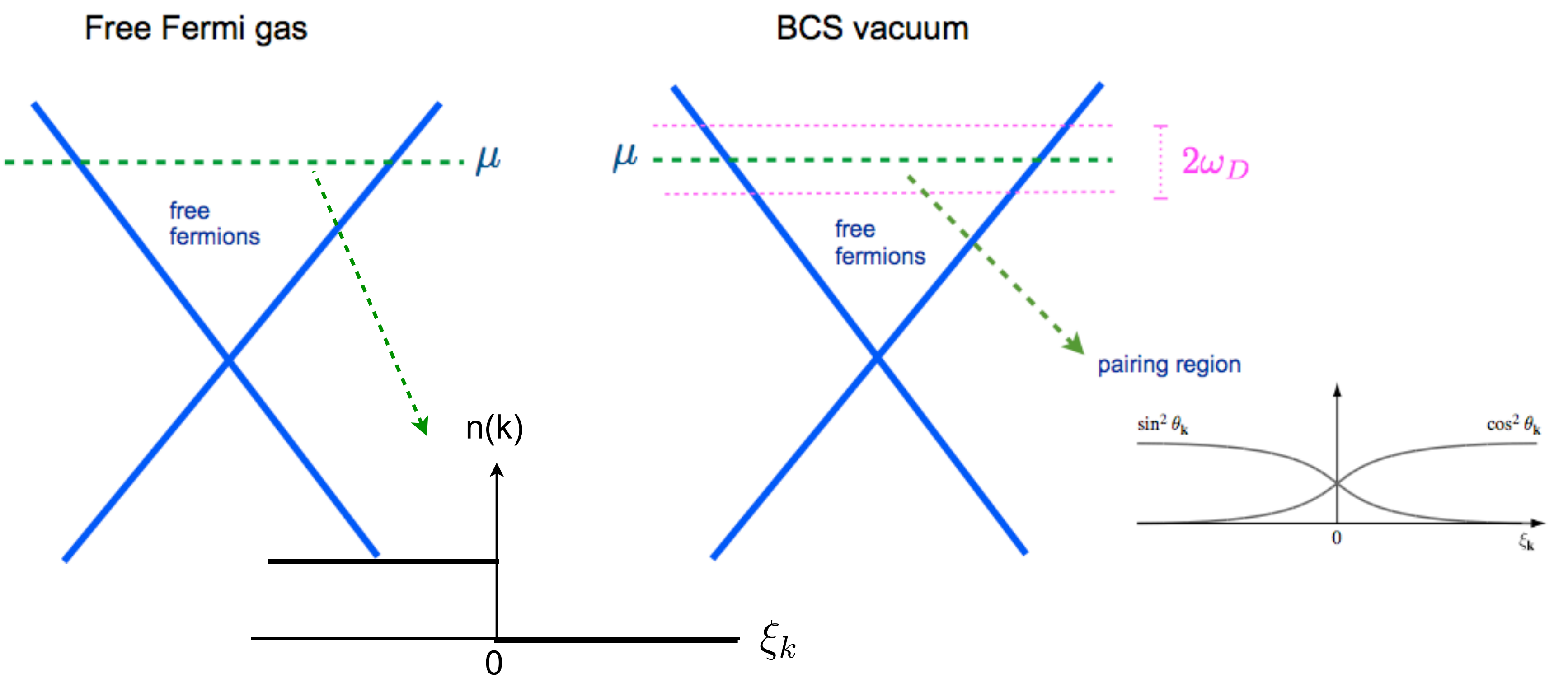
Energy of excitations:

$$E_{\mathbf{k}} = \sqrt{\Delta^2 + \xi^2_{\mathbf{k}}}$$

$$\cos(2\theta_{\mathbf{k}}) = \xi_{\mathbf{k}}/E_{\mathbf{k}}, \quad \sin(2\theta_{\mathbf{k}}) = -\Delta/E_{\mathbf{k}},$$

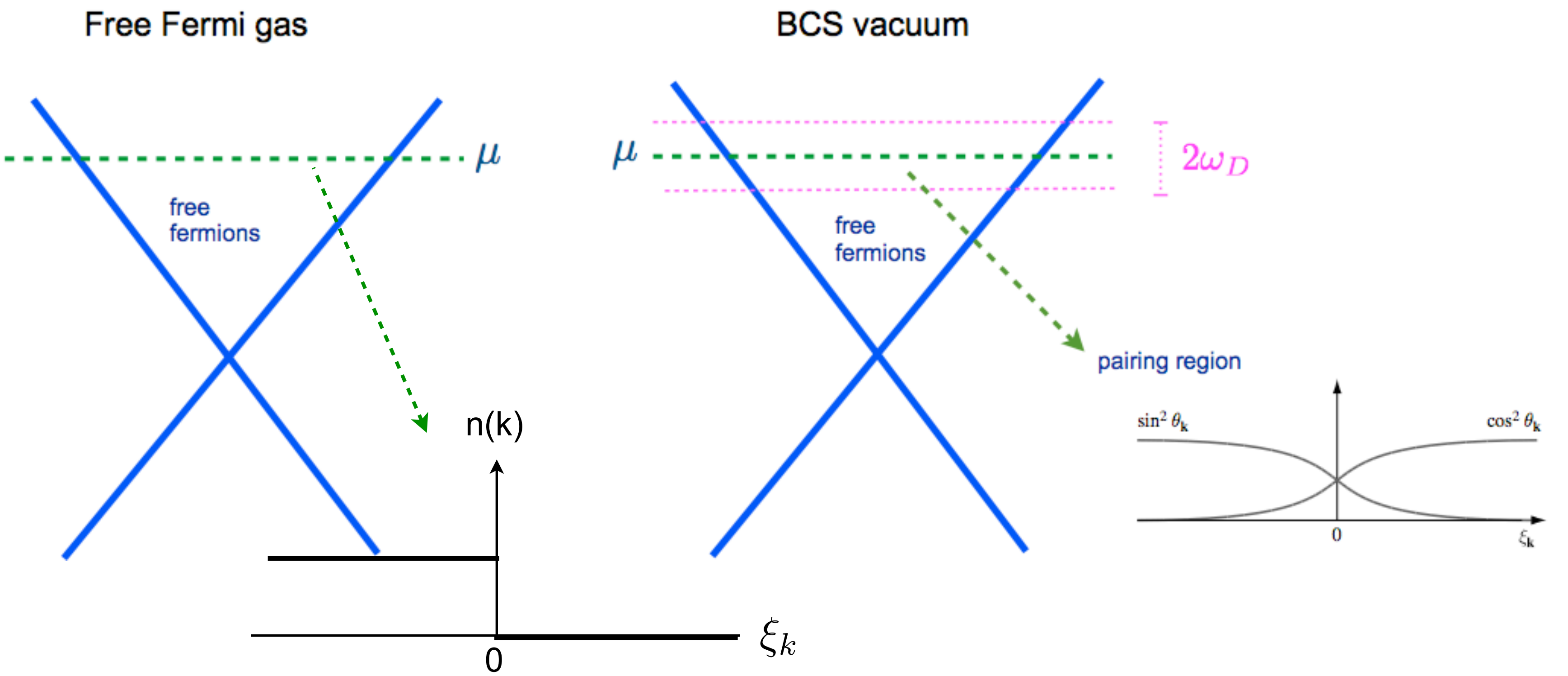
BCS theory in Condensed Matter Physics

An illustration picture



ES and BCS star

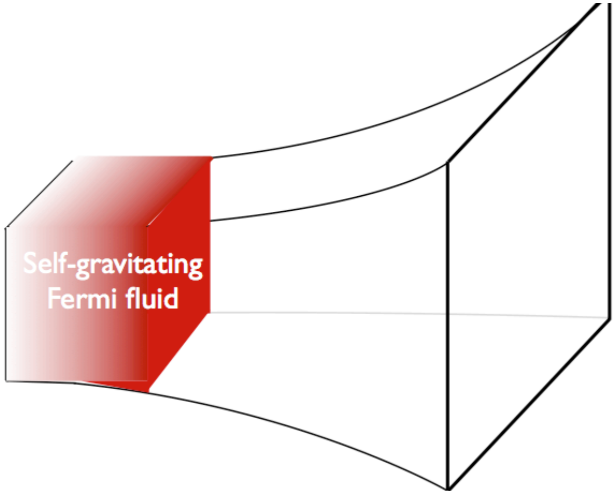
When we couple locally free fermion gas (or BCS system) to Einstein-Maxwell gravity, we get electron star (BCS star).



BCS Star: A holographic superconductor

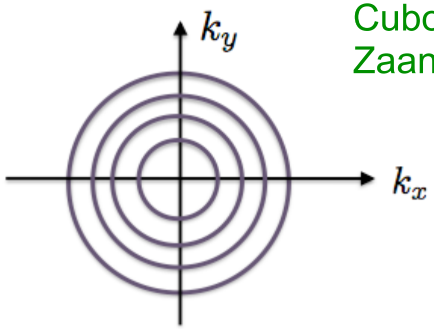
Normal phase: electron star (Hartnoll, Tavanfar, 2010; de Boer et al. 2009)

- Electron star solution:
  - Charged version of neutron star
  - Fermions in *Thomas-Fermi limit*
  - Fermions populating within an edge in the radial direction
  - Near horizon geometry becomes Lifshitz



Hartnoll, Hofman, Vegh, 2011
N.Iqbal, H.Liu, M.Mezei, 2011
Cuborvic, YL, Schalm,Sun, Zaanen, 2011

- Properties of dual field theory:
  - FL with Multiple and closely spaced Fermi surfaces
  - Luttinger's theorem satisfied



BCS Star: A holographic superconductor

Now we focus on the ordered state at zero temperature

Einistein-Maxwell +BCS in the bulk to model the pairing:

$$\mathcal{L} = \frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{\rm BCS}$$

A relativistic BCS interaction (D. Bertrand, 2005)

$$\mathcal{L}_{\rm BCS} = -i\bar{\Psi}(\Gamma^\mu \mathcal{D}_\mu - m_f)\Psi + \frac{\lambda}{2}(\bar{\Psi}_c\Gamma^5\Psi)^\dagger(\bar{\Psi}_c\Gamma^5\Psi)$$

Electron star solution recovered at $\lambda = 0$.

Previous studies: by Hartnoll&Harman, 2010 in the probe limit.
Backreaction is crucial in some limit.

BCS Star: A holographic superconductor

Following the electron star construction, we work at the Thomas-Fermi approximation with the assumption:

$$\partial_r \mu_l \ll \mu_l^2 \qquad \text{and} \qquad \partial_r \Delta \ll \Delta^2$$

The adiabatic limit allows us to study the fermionic contributions in a locally tangential spacetime

$$\begin{aligned} T_{\mu\nu}^{\mathrm{BCS}} &= \frac{1}{2}\langle i\bar{\Psi}\Gamma_{(\mu}\mathcal{D}_{\nu)}\Psi - i\bar{\Psi}\overleftarrow{\mathcal{D}}_{(\mu}\Gamma_{\nu)}\Psi\rangle + g_{\mu\nu}\langle\mathcal{L}_{\mathrm{BCS}}\rangle, \\ J_{\mu}^{\mathrm{BCS}} &= -q\langle\bar{\Psi}\Gamma_{\mu}\Psi\rangle, \\ \Delta &= \lambda\langle\bar{\Psi}_c\Gamma^5\Psi\rangle \end{aligned}$$

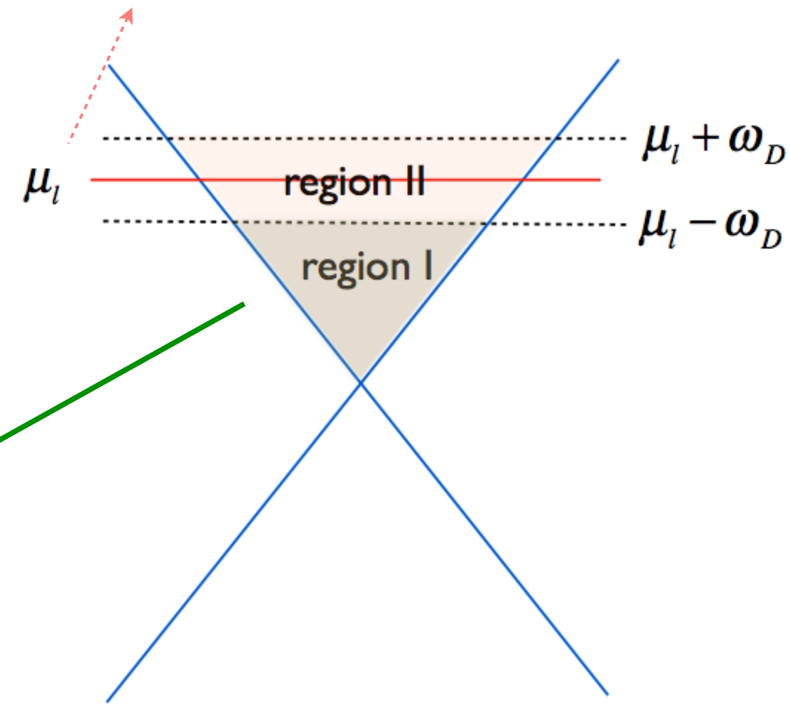
Expectation values are taken on the BCS vacuum state.

BCS Star: A holographic superconductor

Various backreaction parameters:

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu},$$
$$J_\mu = -q n u_\mu$$

local chemical potential



$$\rho = \rho_{\rm I} + \rho_{\rm II}$$
$$p = p_{\rm I} + p_{\rm II}$$
$$n = n_{\rm I} + n_{\rm II}$$

BCS Star: A holographic superconductor

Various backreaction parameters:

$$p_{\rm II} = \langle \Omega_{\rm BCS} | T_{11} | \Omega_{\rm BCS} \rangle$$
$$= \frac{1}{3} \int_{-\omega_D}^{\omega_D} d\xi \nu(\xi) \frac{(\mu_l + \xi)^2 - m_f^2}{\mu_l + \xi} \left(1 - \frac{\xi}{\sqrt{\xi^2 + |\Delta|^2}} \right) - \frac{\Delta^2}{2\lambda}$$

$$n_{\rm II} = \langle \Omega_{\rm BCS} | \hat{n} | \Omega_{\rm BCS} \rangle = \int_{\rm region\ II} \frac{d^3 k}{(2\pi)^3} 2(\sin^2 \theta_{\mathbf{k}})$$
$$= \int_{-\omega_D}^{\omega_D} d\xi \nu(\xi) \left(1 - \frac{\xi}{\sqrt{\xi^2 + \Delta^2}} \right)$$

$$\rho_{\rm II} = \langle \Omega_{\rm BCS} | T_{00} | \Omega_{\rm BCS} \rangle$$
$$= \int_{-\omega_D}^{\omega_D} d\xi \nu(\xi) \left(\xi - \sqrt{\xi^2 + \Delta^2} \right) + \mu_l n_{\rm II} + \frac{\Delta^2}{2\lambda}$$

BCS Star: A holographic superconductor

Rewrite these parameters in the difference between the BCS vacuum and the free fermions:

$$\delta\rho_{\rm total} = \frac{2\mu_l^2 - m_f^2}{(\mu_l^2 - m_f^2)} \frac{\Delta^2}{2\lambda},$$

$$\delta n_{\rm total} = \frac{2\mu_l^2 - m_f^2}{\mu_l(\mu_l^2 - m_f^2)} \frac{\Delta^2}{2\lambda},$$

$$\delta p_{\rm total} = 0.$$

$$\delta(\rho, n, p)_{\rm total} = (\rho, n, p) - (\rho, n, p)_{\rm FL}$$

$$\Delta = 2\omega_D e^{-1/(2\lambda\nu_0)}$$

BCS Star: A holographic superconductor

Solve the system:

Parameter rescaling

$$A = \frac{eL}{\kappa}\hat{A}, \quad (\rho, p) = \frac{1}{\kappa^2 L^2}(\hat{\rho}, \hat{p}), \quad n = \frac{1}{e\kappa L^2}\hat{n}, \quad \lambda = \frac{e^2 L^2}{\beta}\hat{\lambda},$$

$$(m_f, \mu_l) = \frac{e}{\kappa}(\hat{m}_f, \hat{\mu}), \quad (\Delta, \omega_D) = \frac{e}{\kappa}(\hat{\Delta}, \hat{\omega}_D)$$

Leaving $\beta, \hat{m}_f$ and $\hat{\lambda}$

where $\beta = \frac{e^4 L^2}{\pi^2 \kappa^2}$

BCS Star: A holographic superconductor

Solve the system:

$$ds^2 = L^2\big(-f(r)dt^2 + g(r)dr^2 + r^2(dx^2+dy^2)\big), \quad \hat{A}_t = h(r),$$

Near horizon: Lifshitz geometry

$$\begin{aligned} f &= r^{2z}(1+f_1 r^\alpha), \\ g &= \frac{g_0}{r^2}(1+g_1 r^\alpha), \\ h &= h_0 r^z(1+h_1 r^\alpha), \\ \hat{\mu} &= \hat{\mu}_0(1+\mu_1 r^\alpha). \end{aligned}$$

$$\hat{\omega}_D = c\frac{\hat{\mu}^2 - \hat{m}_f^2}{\hat{\mu}}$$

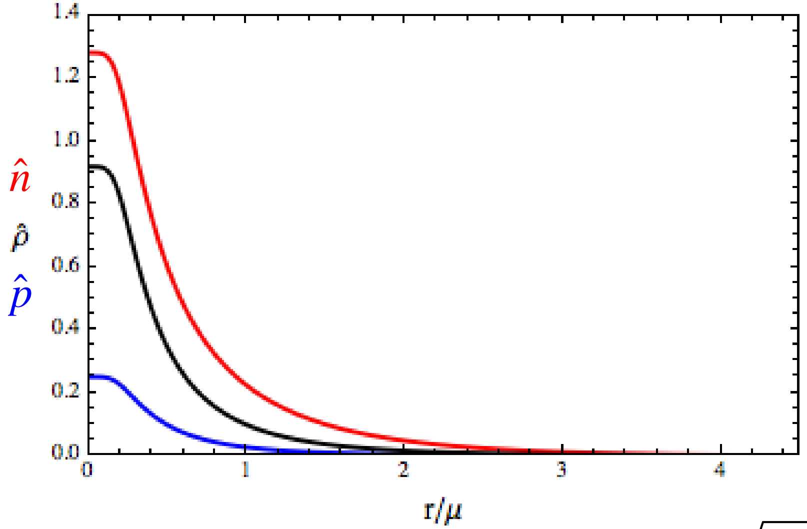
Boundary: AdS

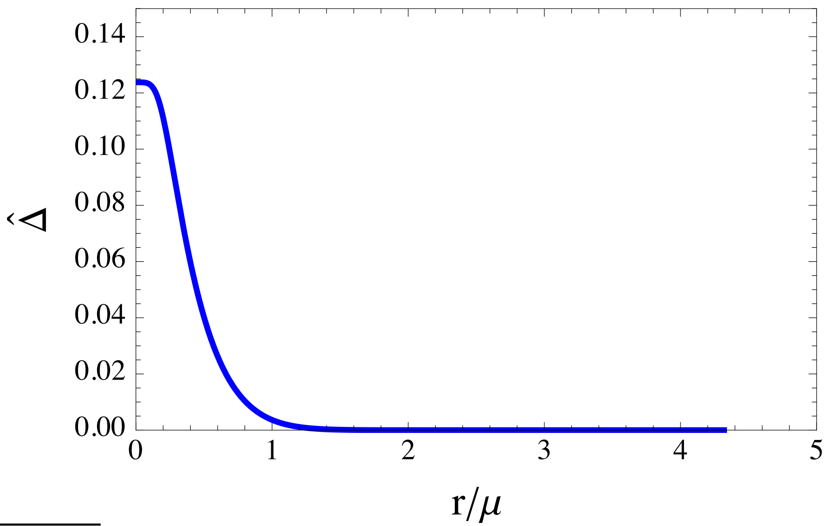
Edge of the BCS star: $\hat{\mu}(r_s) = \hat{m}_f$

Four parameters: (f,g,h,mu) where mu is determined from energy-momentum conservation.

BCS Star: A holographic superconductor

- A typical BCS star:



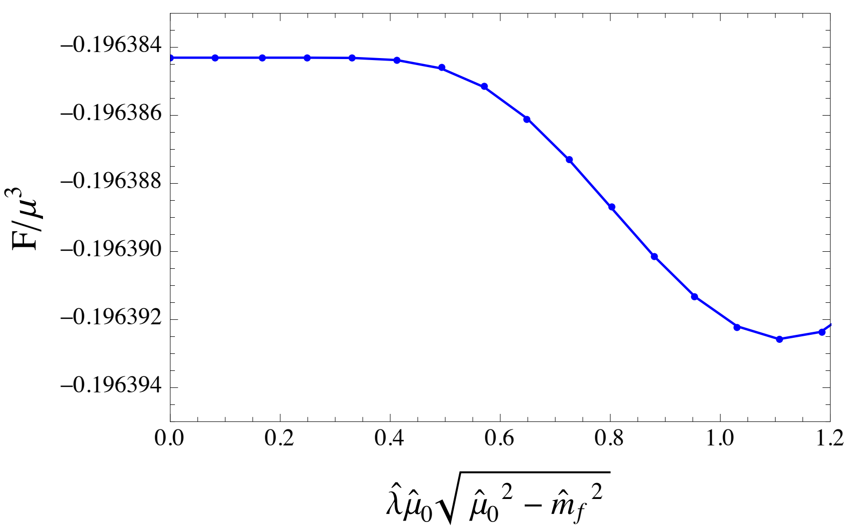


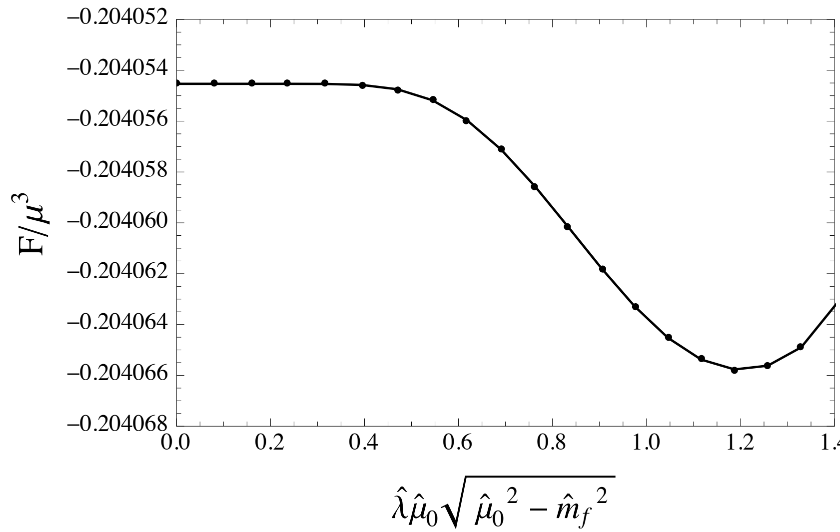
$$\hat{\lambda}\hat{\mu}\sqrt{\hat{\mu}^2 - \hat{m}_f^2} = 0.649$$

$$\hat{m}_f = 0.2, c = 1/3, \beta = 5$$

BCS Star: A holographic superconductor

• Free energy of the stars compared to electron star (lambda=0)





$\hat{m}_f = 0.2, c = 1/3, \beta = 5$

$\hat{m}_f = 0.2, c = 1/3, \beta = 6$

BCS Star: A holographic superconductor

- Remarks:

More stable than electron star

Interaction driven phase transition, (BKT?)

It inherits a lot of properties of electron star: multiple and closely located Fermi surfaces, fields not visible at the boundary

- The properties of the dual field theory

A new ingredient: gap in the dual spectral function

Luttinger theorem

Conductivity

Properties of the dual field theory

- Gap in the dual field theory

Probe with the same fermion that constitute the BCS star
(Faulkner et al. 2009):

$$S_{\mathrm{probe}} = \int d^4x\sqrt{-g}\Big[-i\bar{\Psi}(\Gamma^\mu \mathcal{D}_\mu - m_f)\Psi + \frac{1}{2}\Delta^* \bar{\Psi}_c \Gamma^5 \Psi - \frac{1}{2}\Delta \bar{\Psi}\Gamma^5 \Psi_c\Big]$$

Dirac equation:

$$i(\Gamma^\mu D_\mu - m_f)\Psi + \Delta\Gamma^5\Psi_c = 0.$$

$$\big(-\sqrt{g^{rr}}\sigma^3\partial_r \mp i\sqrt{g^{xx}}\sigma^2 k + (\omega + A_t)\sqrt{g^{tt}}\sigma^1 - m_f\big)\psi_{1,2}(r,k,\omega) \pm i\Delta\sigma^1\psi^*_{2,1}(r,-k,-\omega) = 0$$

$\psi_1(r,k,\omega)$ is coupled to $\psi_2^*(r,-k,-\omega)$ due to BCS interaction and they have the same spectrum at $\omega = 0.$

There is a gap

Properties of the dual field theory

Infalling boundary condition can be chosen independently for $\psi_1(r,k,\omega)$ and $\psi_2^*(r,-k,-\omega)$

$$\begin{pmatrix} B_1^{\mathrm{I}} & B_1^{\mathrm{II}} \\ B_2^{*\mathrm{I}} & B_2^{*\mathrm{II}} \end{pmatrix} = \begin{pmatrix} G_{O_1 O_1^\dagger} & G_{O_1 O_2} \\ G_{O_2^\dagger O_1^\dagger} & G_{O_2^\dagger O_2} \end{pmatrix} \begin{pmatrix} A_1^{\mathrm{I}} & A_1^{\mathrm{II}} \\ -A_2^{*\mathrm{I}} & -A_2^{*\mathrm{II}} \end{pmatrix}.$$

Without BCS coupling: matrices A and B are diagonal and

$$A_1^{\mathrm{I}}(k_F, 0) = 0 \qquad\qquad A_2^{*\mathrm{II}}(-k_F, 0) = 0$$

With BCS coupling, the sources matrix

$$A(k_F, \omega) \sim \begin{pmatrix} a_1^{\mathrm{I}}\omega & a_1^{\mathrm{II}}\Delta \\ -a_2^{*\mathrm{I}}\Delta & -a_2^{*\mathrm{II}}\omega \end{pmatrix} + \mathcal{O}(\Delta^2, \omega^2)$$

Properties of the dual field theory

In a more detailed form: (T. Faulkner, et al. 2009, H. Liu, et al, 2009)

$$G_R^{-1}(k_F,\omega) \sim \begin{pmatrix} \omega P_1 & Q_1 \\ Q_2 & \omega P_2 \end{pmatrix}$$

where

$$P_\alpha = \int dr \sqrt{g_{rr}} \bar{\xi}_\alpha^{(0)} \sqrt{g^{tt}} \xi_\alpha^{(0)} (-1)^\alpha,$$

$$Q_1 = \int dr \sqrt{g_{rr}} \bar{\xi_1}^{(0)} i\Delta \xi_2^{(0)},$$

$$Q_2 = \int dr \sqrt{g_{rr}} \bar{\xi_2}^{(0)} i\Delta \xi_1^{(0)}$$

$$\xi_1^{(0)} = \psi_1^{\mathrm{I}}(k_F, 0)$$

$$\xi_2^{(0)} = \psi_2^{*\mathrm{II}}(-k_F, 0)$$

The gap scale: $\sqrt{Q_1 Q_2 / P_1 P_2}$

The gap is of the order Δ taking value at the horizon.

Properties of the dual field theory

- Luttinger's theorem violated

Electron star: Luttinger's theorem satisfied, a filled Fermi sea

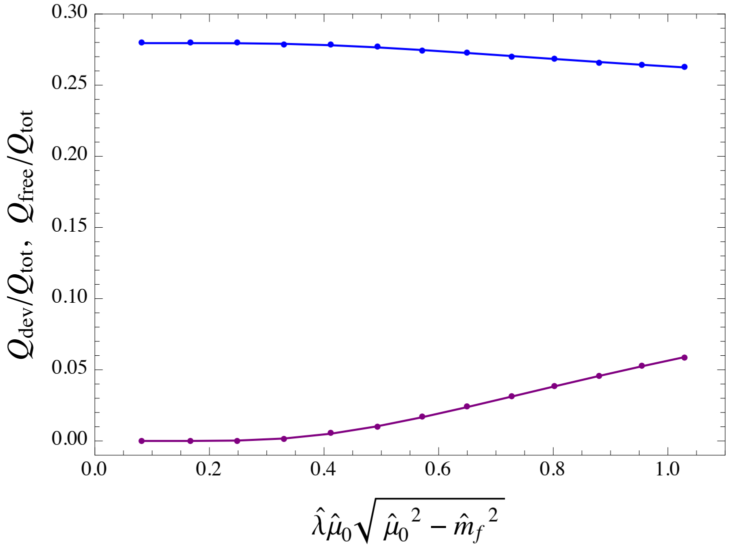
BCS star: $V(k_F) \neq Q_{\rm total}$

$$Q_{\rm free} = \int_0^{r_s} dr r^2 \sqrt{g_{rr}} n_{\rm I}^{\rm FL},$$

$$Q_{\rm total} = \int_0^{r_s} dr r^2 \sqrt{g_{rr}} n,$$

$$Q_{\rm pair} = Q_{\rm total} - Q_{\rm free},$$

$$Q_{\rm dev} = \int_0^{r_s} dr r^2 \sqrt{g_{rr}} \delta n_{\rm total}$$



Properties of the dual field theory

- Conductivity

Consider the time dependent fluctuations:

$$A_x = \frac{eL}{\kappa}\delta a_x(r)e^{-i\omega t}, \quad g_{tx} = L^2\delta g_{tx}(r)e^{-i\omega t}, \quad u_x = L\delta u_x(r)e^{-i\omega t}.$$

The small frequency behavior can be obtained analytically

$$\mathrm{Re}\ \sigma \propto \delta(\omega) + \omega^2$$

No hard gap; The same happens for holographic superconductor.
Same behavior in the normal phase.

Properties of the dual field theory

Evidence that BCS star corresponds to a superconducting state:

- Fermionic spectrum with a gap
- Luttinger's theorem not satisfied
- Conductivity?

A more generalized construction

- We introduce a dynamical scalar field in the bulk which causes the pairing of fermion

$$\mathcal{L} = \frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\lambda m_\phi^2}|(\partial_\mu - 2iqA_\mu)\Delta|^2 - \frac{1}{2\lambda}|\Delta|^2$$
$$- i\bar{\Psi}(\Gamma^\mu \mathcal{D}_\mu - m_f)\Psi + \frac{1}{2}\Delta^*\bar{\Psi}_c\Gamma^5\Psi - \frac{1}{2}\Delta\bar{\Psi}\Gamma^5\Psi_c.$$

- It is no longer the condensate field

$$\Delta - \lambda\bar{\Psi}_c\Gamma^5\Psi = \frac{1}{m_\phi^2}\left(\nabla^\mu - 2iqA^\mu\right)\left(\nabla_\mu - 2iqA_\mu\right)\Delta$$

- Kinetic term of Delta: scalar field could be visible at the boundary.

A more generalized construction

- We solve this system at $\kappa \to 0$.
- All the parameters and fields can be rescaled to eliminate κ, however, there is no scaling limit that can keep all the terms in the Lagrangian.
- An interesting scaling limit:

$$(\rho, p) = \frac{1}{\kappa^2}(\hat{\rho}, \hat{p}), \quad n = \frac{1}{e\kappa}\hat{n}, \quad (A_\mu, \mu_l, m_f, \Delta, \omega_D) = \frac{e}{\kappa}(\hat{A}_\mu, \hat{\mu}, \hat{m}_f, \hat{\Delta}, \hat{\omega}_D), \quad \lambda = \frac{e^2}{\beta}\hat{\lambda}$$

- the gap equation becomes

$$\hat{\Delta} - \hat{\lambda}\langle\bar{\Psi}_c\Gamma^5\Psi\rangle = \frac{1}{m_\phi^2}(\nabla^\mu - 2i\frac{q_{\text{eff}}}{\sqrt{\kappa}}\hat{A}^\mu)(\nabla_\mu - 2i\frac{q_{\text{eff}}}{\sqrt{\kappa}}\hat{A}_\mu)\hat{\Delta}$$

with $q_{\text{eff}} = \sqrt{\pi}\beta^{1/4}$

A more generalized construction

$$\hat{\Delta} - \hat{\lambda}\langle\bar{\Psi}_c\Gamma^5\Psi\rangle = \frac{1}{m_\phi^2}(\nabla^\mu - 2i\frac{q_{\text{eff}}}{\sqrt{\kappa}}\hat{A}^\mu)(\nabla_\mu - 2i\frac{q_{\text{eff}}}{\sqrt{\kappa}}\hat{A}_\mu)\hat{\Delta}$$

Depending on the scaling of m_ϕ^2, we can have three kinds of system

- When $m_\phi^2 = \kappa^{-1-\delta}\hat{m}_\phi^2$ with $\delta > 0$: the bulk BCS Lagrangian.
- When $m_\phi^2 = \kappa^{-1}\hat{m}_\phi^2$: Bulk BCS + charged scalar
- When $m_\phi^2 = \kappa^{-1-\delta}\hat{m}_\phi^2$ with $\delta < 0$: Charged scalar with kinetic term, neutral fermions.

A more generalized construction

- Bulk BCS+ charged scalar in more detail:
- Gap equation: $$\hat{\Delta} - \hat{\lambda}\langle\bar{\Psi}_c\Gamma^5\Psi\rangle = -\frac{s^2}{\sqrt{\beta}}\hat{A}^\mu\hat{A}_\mu\hat{\Delta}$$

  with $$s \equiv \frac{2\pi\sqrt{\beta}}{\hat{m}_\phi^2}$$

- In Thomas-Fermi approximation:

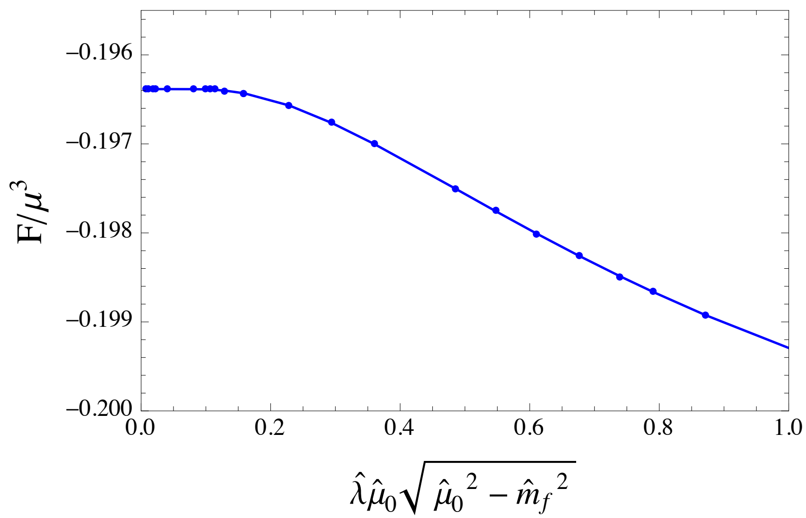
$$\hat{\Delta} = 2\hat{\omega}_D e^{-(1-\frac{2sh^2}{f})/(\hat{\lambda}\hat{\mu}\sqrt{\hat{\mu}^2-\hat{m}_f^2})}$$

- Different from the BCS Lagrangian: scalar no longer meaning condensate.
- The charged scalar enhance the system to condensate.

A more generalized construction

- Bulk geometry: star structure

- Normal phase: ES
  It is much stabler than
  ES.

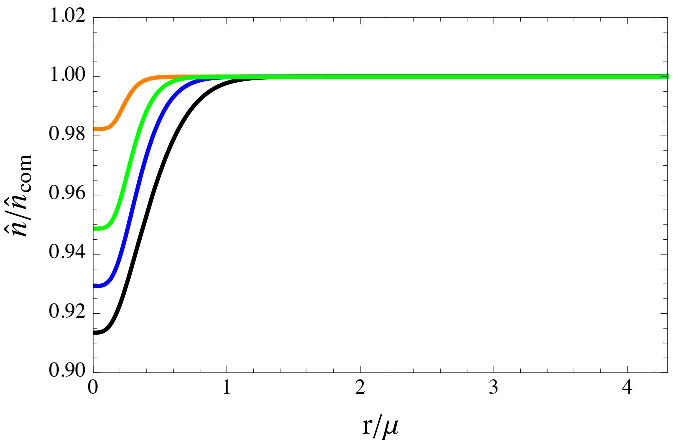


$$\hat{m}_f = 0.2, c = 1/3, \beta = 5, s = 0.4$$

A more generalized construction

- Fermionic spectrum function -- still a gap

- Enhancement of charge density:
  Coupling goes larger, more bosons in the bulk



$$\hat{n}_{\rm com} = \hat{n} + \frac{2s\beta h}{\sqrt{f}}\frac{\hat{\Delta}^2}{\hat{\lambda}}$$

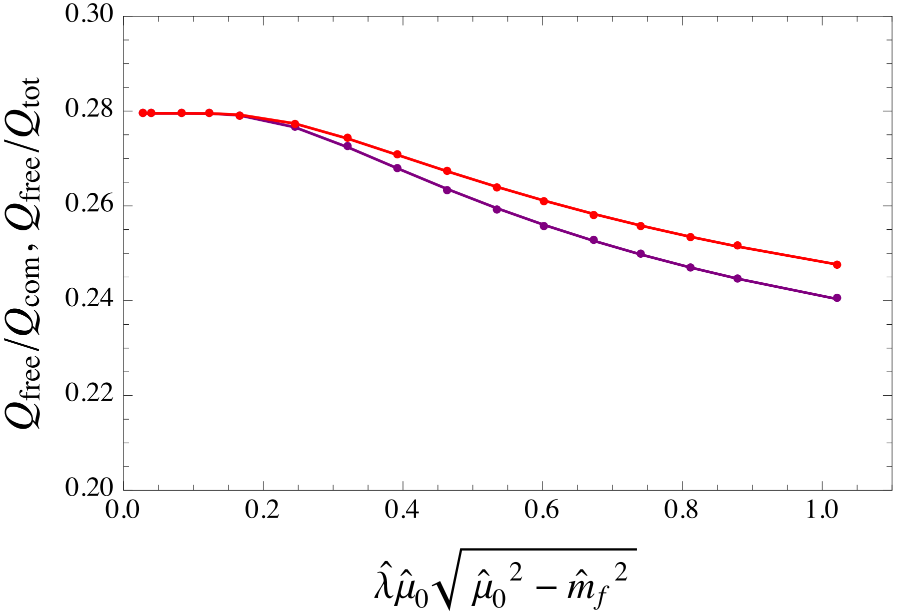
$$\hat{\lambda}\hat{\mu}\sqrt{\mu^2 - \hat{m}_f^2} = 0.245 \text{ (Orange)},$$

$$0.393 \text{ (Green)}, 0.534 \text{ (Blue)}, 0.810 \text{ (Black)}.$$

A more generalized construction

- The resulting system is quite similar to BCS star.

$$Q_{\rm com} = \int_0^{r_s} dr r^2 \sqrt{g_{rr}} n_{\rm com}$$



- Enhancement of the susceptibility of the system towards superconductivity and also the charge density.

Summary

- **What we did**

- BCS star: pairing in the bulk; More stable than electron star
- gap in the fermion spectral function
- Scaling limits of a more generalized construction

- **Drawbacks**

- Star limit: Multiple FS, Fermions and pairing not visible at the boundary
- No hard gap in the conductivity

Future Direction

- Finite temperature
- Lattice, d-wave, to make it more realistic
- Other scaling limit

Thank you!