Giant Graviton Configurations in String Theory

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一、Giant Graviton 问题的由来：

AdS/CFT 对应：

J. Maldacena, hep-th/9711200

AdS/CFT Conjecture

$\Downarrow$

Type IIB String theory on $AdS_5 \times S^5$

$\Longleftrightarrow$ $N=4$, $d=3+1$ $U(N)$ SYM theory ⟸ CFT

$\Downarrow$

gravity theory in $(D+1)$-dimensional AdS space

$\Longleftrightarrow$ Conformal field theory in D dimensions

e.g. J. Maldacena, A. Strominger, hep-th/9804085

microstate level

Type IIB String theory on $AdS_3 \times S^3 \times M^4$

$\Longleftrightarrow$ CFT whose target space is a deformation of a symmetric product of copies of M^4

K3 or T^4

主要结论: Chiral primaries in this CFT

correspond to multi-particle BRS states

in the string theory carrying S^3 angular momentum

multi-particles $\Longrightarrow$ particle number $2k$

$\uparrow$ GKO level

$\downarrow$

abruptly terminated

$\uparrow$ SCFT

the maximum R-change for chiral ~~par~~ primaries

$$Q_M = 2k$$

ST $\rightarrow$ multi-particle state $\Longrightarrow$ particular mode (Supergraviton mode)

the occupation number

$\Downarrow$ $2k$

limit

effect $\longrightarrow$ Stringy exclusion principle

<u>Remarks</u>: 1. non-perturbative effect

$\not\Updownarrow$

free bosons $\rightarrow$ Type IIB – AdS_3 perturbative theory

2. CFT 观点的自洽性

Gauge symmetry group with a finite rank

$\Downarrow$

A family of chiral primary operators in CFT terminates

AdS/CFT
$\longleftrightarrow$ dual AdS description: these operators are associated with single particle state carrying angular momentum

3. 超引力观点的不可理解性:

appearence of an upper bound on the angular momentum

$\uparrow$??

the point of view of supergravity theory

J. McGreevy, L. Susskind, N. Toumbas, hep-th/0003075

物理解释: The supergraviton states expand into the spherical part of the space-time geometry with a radius proportional to the angular momentum. The radius of these 'giant gravitons' must be smaller than the radius of the sphere, which through the AdS/CFT is related ~~with~~ to the rank of the gauge group in the CFT.

'giant graviton' 的主要性质:

1. Energy of the giant graviton state
   = Energy of the usual 'point-like' graviton
2. Giant graviton is a BPS state.
   $\hookrightarrow$ preserving some of supersymmetries

二. Giant graviton 的发现:

物理对象: test brane configurations in background spacetime of the form $AdS_m \times S^n$

$AdS_4 \times S^7$ $\quad AdS_5 \times S^5$ $\quad AdS_7 \times S^4$

$\Downarrow \leftarrow$ ~~red~~ realize

stable test brane solutions where a $(n-2)$-brane has expanded on S^n

MST, hep-th/0003075

M. Grisaru, R. Myers, Ø. Tafjord, hep-th/0008015

the line element for metric on $AdS_m \times S^n$:

$$ds^2 = ds^2_{AdS} + ds^2_{sphere}$$

$$ds^2_{AdS} = -\left(1 + \frac{r^2}{\tilde{L}^2}\right)dt^2 + \frac{dr^2}{1 + \frac{r^2}{\tilde{L}^2}} + r^2 d\Omega^2_{m-2}$$

$$ds^2_{sphere} = L^2\left(d\theta^2 + \cos^2\theta\, d\phi^2 + \sin^2\theta\, d\Omega^2_{n-2}\right)$$

~~retal~~ $\tilde{L}$: radius of curvature for the AdS_m.

L: 〃 〃 S^n

relations of $\tilde{L}$, L: $\quad L = \frac{n-3}{2}\tilde{L}$

$$\Downarrow$$

$$(m,n) = \begin{cases} (4,7) & L = 2\tilde{L} \\ (5,5) & L = \tilde{L} \\ (7,4) & L = \tilde{L}/2 \end{cases}$$

the spherical components in the metric:

$$d\Omega^2_{m-2} = d\alpha_1^2 + \sin^2\alpha_1\left(d\alpha_2^2 + \sin^2\alpha_2\left(\cdots + \sin^2\alpha_{m-3}\, d\alpha^2_{m-2}\right)\right),$$

$$d\Omega^2_{n-2} = d\chi_1^2 + \sin^2\chi_1\left(d\chi_2^2 + \sin^2\chi_2\left(\cdots + \sin^2\chi_{n-3}\, d\chi^2_{n-2}\right)\right)$$

coupling of brane and potentials:

geometries $\Leftarrow$ maximally supersymmetric solutions $\Longleftrightarrow$ supergravity

11-dimensional supergravity $\Leftarrow$ four-form field strength

$\xleftrightarrow{\text{Hodge-dual}}$ seven-form field strength

Type IIB supergravity $\Leftarrow$ self-dual five-form field strength

$$\Downarrow$$

branes $\underset{\text{coupling}}{\longleftrightarrow}$ the corresponding potentials

(n-1)-form potential on S^n:

$$A^{(n-1)}_{\phi\chi_1\cdots\chi_{n-2}} = \beta_n L^{n-1}\sin^{n-1}\theta\,\sin^{n-3}\chi_1\cdots\sin\chi_{n-3} \equiv \beta_n L^{n-1}\sin^{n-1}\theta\sqrt{g_\chi}$$

$\sqrt{g_\chi}$: volume element on the unit (n-2)-sphere described by $d\Omega^2_{n-2}$

$\beta_4 = \beta_5 = 1$, $\beta_7 = -1$.

$(m-1)$-form potentials on AdS_m:

$$A^{(m-1)}_{t\alpha_1\cdots\alpha_{m-2}} = -\frac{r^{m-1}}{\tilde{L}}\sin^{m-3}\alpha_1 \cdots \sin\alpha_{m-3} \equiv -\frac{r^{m-1}}{\tilde{L}}\sqrt{g_\alpha}$$

$\sqrt{g_\alpha}$: the volume element on the unit $(m-2)$-sphere described by $d\Omega^2_{m-2}$

Giant Gravitons: $\Longleftarrow$ the p-brane action $\Rightarrow$ DBI action + CS term

$$S_p = -T_p\int d^{p+1}\sigma\sqrt{-g} + T_p\int P[A^{(p+1)}]$$

$$= -T_p\int d^{p+1}\sigma\sqrt{-g}\left[1+\frac{1}{(p+1)!}\epsilon^{i_0\cdots i_p}\partial_{i_0}X^{M_0}\cdots\partial_{i_p}X^{M_p}A^{(p+1)}_{M_0\cdots M_p}\right]$$

g_{ij} : the pull-back of the spacetime metric to the world-volume

i.e.

$$g_{ij} = \partial_i X^M \partial_j X^N G_{MN}$$

$P[A^{p+1}]$: the pull-back of the $(p+1)$-form potential

MST giant graviton 图象

$\Downarrow$

GG $\Big\{$ brane configurations where an $(n-2)$-brane has expanded on the S^n: S^{n-2} in S^n, θ fixed, orbits in the ϕ-direction.

static gauge: $\sigma_0 \equiv \tau = t$, $\sigma_1 = \chi_1$, $\cdots$, $\sigma_{n-2} = \chi_{n-2}$

$L\sin\theta \rightarrow$ the radius of $(n-2)$-brane moving around inside the S^n.

special form of solutions:

$$\theta = \text{const}\ , \quad \phi = \phi(\tau)\ , \quad r = 0$$

all expressions $\xrightarrow{\text{into}}$ the p-brane action

area of S^{n-2}

flux

$$A_{n-2}\, T_{n-2} = \frac{N}{L^{n-1}}$$

Lagrangian:

$$L_{n-2} = \frac{N}{L}\left[-\sin^{n-2}\theta \sqrt{1 - L^2 \sin^2\theta\, \dot\phi^2} - L \sin^{n-1}\theta\, \dot\phi \right]$$

angular momentum: $$P_\phi = \frac{\partial L_{n-2}}{\partial \dot\phi} = N\left[\frac{L \sin^{n-2}\theta \cos^2\theta\, \dot\phi}{\sqrt{1 - L^2 \cos^2\theta\, \dot\phi^2}} + \sin^{n-1}\theta \right]$$

$$\Leftrightarrow \quad \dot\phi = \dot\phi\,[\theta, N, L, P_\phi]$$

Hamiltonian:

$$H_n = P_\phi \dot\phi - L_{n-2} = \frac{N}{L}\sqrt{\left(\frac{P_\phi}{N}\right)^2 + \tan^2\theta \left(\frac{P_\phi}{N} - \sin^{n-3}\theta\right)^2}$$

stable solution $\Rightarrow \dfrac{\partial H_n}{\partial \theta} = 0$

$$\Leftrightarrow \quad \sin\theta = 0 \quad ; \quad \frac{P_\phi}{N} = \sin^{n-3}\theta$$

$$\Rightarrow \quad H_n = \frac{P_\phi}{L} = \frac{N}{L}\cdot\frac{P_\phi}{N}$$

giant graviton configuration: $\dfrac{P_\phi}{N} = \sin^{n-3}\theta\ , \quad P_{\phi\,max} = N$

$$\sin\theta = 1\ , \quad \theta = \frac{\pi}{2}$$

'point like' graviton configuration: $\sin\theta = 0$

$\theta = 0$ (= the center of mass position)

$$\Leftrightarrow \quad \dot\phi = 1/L = \text{const}\ , \quad P_\phi = N$$

the form of solutions $\Rightarrow ds^2$

$\Longleftrightarrow \quad ds^2 = -(1 - L^2\cos^2\theta\,\dot{\phi}^2)dt^2$

$\Longleftrightarrow$ giant graviton $\Rightarrow$ time-like trajectory

$\sin\theta = 0,\ \dot{\phi} = \frac{1}{L}$

$\longrightarrow$ 'point-like' graviton $\Rightarrow$ null trajectory

Conclusion:

1. Energy of 'point-like' graviton = Energy of giant graviton.
2. angular momentum of the giant graviton has an upper bound N.
3. giant graviton configurations are described by the $(n-2)$-branes which had expanded into the sphere part of background geometry.

Giant Gravitons in AdS $\Longleftrightarrow$ dual Giant Gravitons:

giant graviton: spherical $(n-2)$-brane expands in S^n

giant graviton configuration $\Leftrightarrow$ 'point-like' graviton

same the physical properties as

dual giant graviton: stable expanded $(m-2)$-brane configuration in the AdS_m space

"dual": supergravity solution $AdS_m \times S^n$

n-form field strength $\longleftrightarrow$ m-form field strength

Hodge dual in $(m+n)$-d spacetime

Similar to the case of giant gravitons,

Static gauge:

$$\sigma_0 = \tau = t\ , \quad \sigma_1 = \alpha_1\ , \ \cdots\ , \quad \sigma_{m-2} = \alpha_{m-2}$$

the form of solution:

$$\theta = 0\ , \quad \phi = \phi(\tau)\ , \quad r = \text{const}$$

the p-brane action

Lagrangian:

$$L_{m-2} = b(m)\, N^{\frac{m-3}{2}} \Big/ L\, \tilde{L}^{m-2} \left[-r^{m-2}\sqrt{1+\frac{r^2}{\tilde{L}^2} - L^2\dot{\phi}^2} + \frac{r^{m-1}}{\tilde{L}} \right]$$

$$\Leftrightarrow\ P_\phi = \frac{\partial L_{m-2}}{\partial \dot{\phi}} \longrightarrow P_\phi = P_\phi[\dot{\phi}] \rightarrow \dot{\phi} = \dot{\phi}[P_\phi]$$

$\Leftrightarrow$ Hamiltonian:

$$H_m = P_\phi \dot{\phi} - L_{m-2} = \frac{b(m)\, N^{\frac{m-3}{2}}}{L} \left[\sqrt{\left(1+\frac{r^2}{\tilde{L}^2}\right)\left(\frac{P_\phi^2}{b^2(m)\, N^{m-3}} + \frac{r^{2m-4}}{\tilde{L}^{2m-4}}\right)} - \frac{r^{m-1}}{\tilde{L}^{m-1}} \right.$$

$$\Leftrightarrow \quad b(4) = \sqrt{2}\ , \qquad b(5) = 1 = b(7) .$$

Stable configuration:

$$\frac{\partial H_m}{\partial r} = 0 \ \Rightarrow \ \text{minima located at}$$

$$r = 0 \qquad \text{and} \qquad \left(\frac{r}{\tilde{L}}\right)^{m-3} = \frac{P_\phi}{b(m)\, N^{\frac{m-3}{2}}} .$$

At the above minima,

$$H_m = \frac{P_\phi}{L} .$$

Remarks:

1. $r=0 \Longrightarrow$ 'point-like' graviton

$r = \text{const}$

$= \dfrac{\tilde{L}\, P_\phi^{\frac{1}{m-3}}}{N^{\frac{1}{2}}\{b(m)\}^{\frac{1}{m-3}}} \Longrightarrow$ dual giant graviton

Energy $H_m = \dfrac{P_\phi}{L}$

(m-2)-brane has expanded into the AdS_m space to a sphere of constant r while it orbits in the ϕ direction on the S^n.

2. From the expression of P_ϕ, we can see that

→ no upper bound for the angular momentum

The origin of geometry: AdS is not compact.

giant graviton ⟷ String exclusion principle

dual giant graviton ✗

'puzzle'

See below → the spacetime uncertainty relation in M-theory

三、Giant gravitons in type IIA pp-wave background.

Chen, Shao, hep-th/0310062

motivation:

M. Blau et al, hep-th/0110242
hep-th/0201081.

String theory on the maximally SUSY type IIB pp-wave

↑ taking the Penrose limit of the formalism on $AdS_5 \times S^5$

↓

the String theory on the pp-wave → exactly solvable

↑

R. Metsaev et.al
hep-th/0112044, hep-th/0202109

AdS/CFT duality

↑ ← Check ← BMN, hep-th/0202021

correspondence between the gauge theory operators and perturbative closed string states

non-perturbative objects ⇒ D-branes

$\xrightarrow{\text{in the limit}}$ survive

⇓ ?

gauge dual

e.g. giant graviton ⇐ D-3 brane in typeIIB ST

on the pp-wave background

⇓ non-perturbatively understanding

duality between the pp-wave and gauge theory

The reason of pp-wave background into type IIA:

1. Type II theory on the general gravity background

$\Downarrow \leftarrow$ diffcult

problem

$\xrightarrow{\text{restricted}}$ on the pp-wave background $\rightarrow$ solvable

2. type IIA supergravity background $\leftarrow$ 11-d Supergravity backgro

KK compactification

D0-branes $\longrightarrow$ microscopic description of giant graviton

pp-wave and Penrose limit:

M theory $\xrightarrow[\text{low energy limit}]{}$ 11-d supergravity

$\hookrightarrow$ pp-wave solution for the metric: $AdS_4 \times S^7 \leftrightarrows AdS_7 \times S$

$$ds^2 = -2dx^+dx^- - \left(\frac{\mu^2}{9} \sum_{i=1}^{3} (x^i)^2 + \frac{\mu^2}{36} \sum_{i'=4}^{9} (x^{i'})^2 \right)(dx^+)^2 + \sum_{I=1}^{9} (dx^I)^2$$

$F_{+123} = \mu$

$\Uparrow \leftarrow$ Penrose limit

$AdS_4 \times S^7$ and $AdS_7 \times S^4$

Steps of the limit:

1. the metric of $AdS_m \times S^n$ $\longrightarrow$ KP coordinates (AdS part ↓)

2. 考虑特定的粒子运动轨道，再考虑靠近这一轨道的几何

$\Longrightarrow$ L (or $\tilde{L}$) $\to \infty$ ↓

3. the form of the metric $\Longrightarrow$ the form of a plane wave metric

$\Longrightarrow$ pp-wave.

Type IIA pp-wave:

the above pp-wave form $\xrightarrow{\text{compactification}}$ ~~the~~ type IIA pp-wave form

procedure:

1.
$$X^- = x^- - \frac{\mu}{6} x^4 x^9$$
$$X^4 = x^4 \cos\left(\frac{\mu}{6} x^+\right) - x^9 \sin\left(\frac{\mu}{6} x^+\right)$$
$$X^9 = x^4 \sin\left(\frac{\mu}{6} x^+\right) + x^9 \cos\left(\frac{\mu}{6} x^+\right)$$

2.
$$dS^2 = -2dx^- dx^+ - \left(\frac{\mu^2}{9}\sum_{i=1}^{4}(x^i)^2 + \frac{\mu^2}{36}\sum_{i'=5}^{8}(x^{i'})^2\right)(dx^+)^2 + \sum_{I=1}^{8}(dx^I)^2$$
$$+ \left(dx^9 + \frac{\mu}{3} x^4 dx^+\right)^2$$

3. the standard dimension reduction along x^9

$$dS^2 = -2dx^- dx^+ - \left(\frac{\mu^2}{9}\sum_{i=1}^{4}(dx^i)^2 + \frac{\mu^2}{36}\sum_{i'=5}^{8}(x^{i'})^2\right)(dx^+)^2 + \sum_{I=1}^{8}(dx^I)^2$$

$$F_{+123} = \mu, \quad F_{+4} = -\frac{\mu}{3}$$

Remarks: 1. potentials:

$$F_{+123} = \mu \quad \Longrightarrow \quad C_{+ij} = -\frac{\mu}{3}\epsilon_{ijk}x^k$$

$$F_{+4} = -\frac{\mu}{3} \quad \Longrightarrow \quad A_+ = \frac{\mu}{3}x^4$$

2. KK-reduction: $N = P_q R_c$ $\quad g_{uc} \to$ source of RR-fie

$$g_{uc} \to A_+ = \frac{\mu}{3}x^4 \qquad \qquad \| \atop N \text{ D0-branes}$$

Giant gravitons on type IIA pp-wave:

The low energy effective action of a D-2brane in general type IIA background = DBI action + Chern-Simons term

$$\Leftrightarrow S_2 = -T_2\int d^3\sigma \sqrt{-\det(P[G+B]_{ab}+\lambda F_{ab})} + \mu_2\int P\left[\sum C^{(n)}e^B\right]e^{\lambda F}$$

$T_2 = \frac{2\pi}{(2\pi l_s)^3 g_s}$, $\quad G_{\mu\nu}$: metric, $\quad B_{\mu\nu}$: Kalb-Ramond

F_{ab}: world-volume gauge field, $\quad \lambda = 2\pi l_s^2$

$P[\cdots]$: pull back to the world volume

$C^{(n)}$: the potentials of RR fields $\to F^{(n+1)} = dC^{(n)}$

supersymmetry $\xrightarrow{\text{requires}}$ $\mu_2 = \pm T_2 \Rightarrow$ brane and anti-b

$\Downarrow$ ← Type IIA pp-wave background
← anti-brane

$$S_2 = -T_2\int d^3\sigma \left(\sqrt{-\det(P[G]_{ab}+\lambda F_{ab})} + \lambda C_0 F_{12} + C_{012}\right)$$

Ansatz of the giant graviton configuration. (static spherical configuration)

$$x^+ = t, \quad x^1 = R\sin\theta\cos\phi, \quad x^2 = R\sin\theta\sin\phi, \quad x^3 = R\cos\theta$$

$\hookrightarrow$ in this ansatz

$$C_{t\theta\phi} = -\frac{\mu}{3}R^3\sin\theta, \quad C_t = \frac{\mu}{3}x^4$$

world-volume gauge field: $F_{\theta\varphi} = \lambda^{-1}\sin\theta$

world-volume magnetic flux: $N = \frac{1}{2\pi}\int d\theta\, d\varphi\, F_{\theta\varphi} = 2\lambda^{-1}q$

$\hookrightarrow$ 2-brane action

Lagrangian:

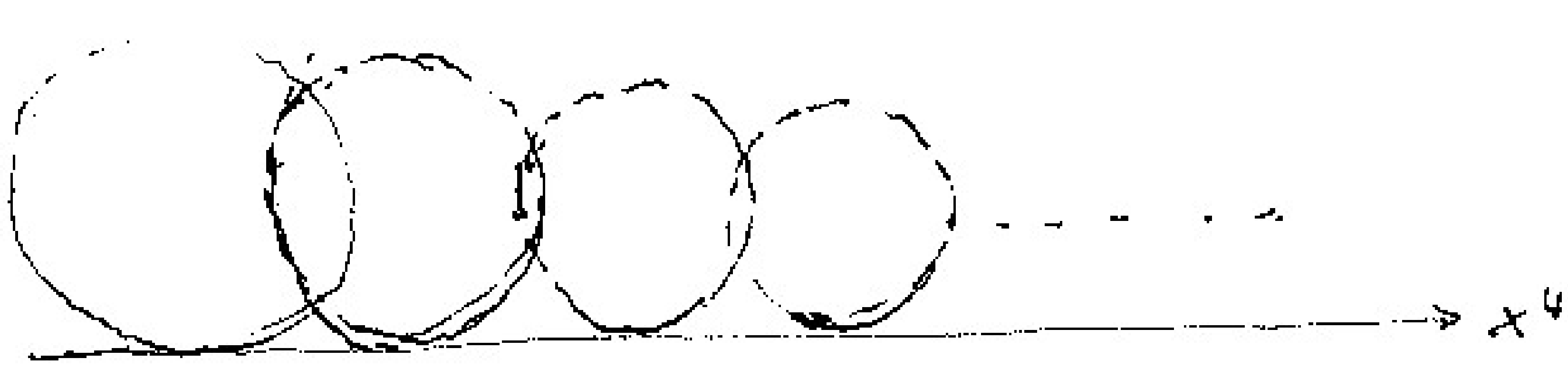
$$L = -\frac{\mu}{3}T_2\left(\sqrt{(R^2+(x^4)^2)(R^4+q^2)} + x^4 q - R^3\right)\sin\theta$$

$$\frac{\partial L}{\partial x^4} = 0 \implies x^4\sqrt{\frac{R^4+q^2}{R^2+(x^4)^2}} + q = 0$$

$$\frac{\partial L}{\partial R} = 0 \implies \frac{2R}{3}\sqrt{\frac{R^2+(x^4)^2}{R^4+q^2}} + \frac{1}{3R}\sqrt{\frac{R^4+q^2}{R^2+(x^4)^2}} - 1 = 0$$

$\hookrightarrow$ $Rx^4 = -q$ $\leftarrow$ giant graviton configuration

Remarks: 1. Configuration $Rx^4 = -q \rightarrow$ giant graviton configuration



Lifting M theory:

$$X^4 = x^4\cos\left(\frac{\mu}{6}x^+\right), \quad X^9 = x^4\sin\left(\frac{\mu}{6}x^+\right)$$

$\Rightarrow$ metric of maximally SUSY M-theory pp-wave

$\uparrow$

$AdS_4 \times S^7$

$\hookrightarrow$ dual giant graviton : it is blown up in AdS_4 and is circling in S^7

the action of giant graviton on M-theory pp-wave

$$\Rightarrow \; R(x^4)^2 = 2L \qquad L : \text{angular momentum}$$

$$\downarrow \leftarrow p = \frac{L}{x^4}$$

$$Rx^4 \sim p \xleftarrow{\text{Lifting M-theory}} Rx^4 = -\frac{q}{\ }$$

2. the solution of 'puzzle':

KK ~~mod~~ reduction $\longrightarrow$ $N = P_9 R_c \longrightarrow E \sim \frac{N}{R_c}$

uncertainty of the energy and the time

$$\downarrow$$

$$\Delta t \sim \frac{R_c}{N}$$

if $\Delta x^i \sim R$, $\quad Rx^4 = -\underline{q} = \frac{N\lambda}{2} = \pi l_s^2 N$

$$\Leftrightarrow \quad \Delta x^i \Delta x^4 \sim l_s^2 N \qquad T_1 = \frac{R_c}{l_p^3} = \frac{1}{l_s^2}$$

$$\Leftrightarrow \quad \Delta t \Delta x^i \Delta x^4 \sim R_c l_s^2 \sim l_p^3 \leftarrow$$

M. Li and T. Yoneya, hep-th/9806240

$\hookrightarrow$ dual giant graviton $\Rightarrow Rx^4 = -\underline{q}$

$\longrightarrow$ spacetime uncertainty relation in M-theory

M. Li, hep-th/0003173, spacetime uncertainty relation $\Rightarrow$ Stringy exclusion princ

$\uparrow$

giant graviton expanded in S

3. $Rx^4 = -q \Rightarrow$ complementary relation of the spacetime geometry associated with the giant graviton on type IIA pp-wave.

Since, R is the size of the coordinates x^i, $i=1,2,3$

x^i, x^4 are the coordinate configuration of the giant graviton

$Rx^4 = -q \rightarrow$ complementary relation

$\Downarrow$ constrained

the spacetime structure of the giant graviton configuration

4. Supersymmetric property of the giant graviton:

Type IIA pp-wave background has 24 supercharges

$\uparrow$ 32−8

toroidal compactification $\xrightarrow{\text{break}}$ 8 supercharges

24 supersymmetries of type IIA pp-wave

$\Downarrow$

16 kinematical SUSYs + 8 dynamical SUSYs

$\uparrow$ non-linear realized on the string world-sheet

$\uparrow$ Linear ~~rela~~ realization and time independent

the giant graviton brane $\xleftrightarrow{\text{embeding}}$ Kappa symmetry projection

completely broken $\rightarrow$ 16 kinematical SUSYs

~~pr~~ preserving $\rightarrow$ 8 dynamical SUSYs

$$\Downarrow$$

the giant graviton $\Rightarrow$ $\frac{1}{4}$ BPS state

hint: $RX^4 = -q$ $\rightarrow$ geometrical picture of the giant graviton

+ Myers effect: N D0-branes $\underset{\text{blown up}}{\Longrightarrow}$ D2-brane

$\leftarrow$ X^4 into the deformed parameter in fuzzy algebra

fuzzy structure of the giant graviton:

~~Se~~ Setup: magnetic flux $N = 2\lambda^{-1}q$ $\Rightarrow$ N D0-brane coupling

+ the background of 4-form RR field strength

$\overset{\text{Myers effect}}{\Longrightarrow}$ D2-brane to be blown up from N D0-brane

R. Myers, hep-th/9910053

coincident D-branes $\Rightarrow$ non-Abelian symmetry group

$$\Downarrow$$

Myers' non-Abelian D-brane action

= non-Abelian DBI action + Wess-Zumino term

$$= -T_p \int d^{p+1}\sigma \, \mathrm{STr}\left(\sqrt{\det(Q^i{}_j)} \times \sqrt{-\det\left(P\left[E_{ab} + E_{ai}(Q^{-1}-\delta)^i{}_j E_{jb}\right] + \lambda F_{ab}\right)}\right)$$

$$+ \mu_p \int \mathrm{STr}\left(P\left[e^{i\lambda i_\phi i_\phi}\left(\sum C^{(n)} e^{B}\right)\right] e^{\lambda F}\right)$$

$$E_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}, \quad Q^i{}_j = \delta^i{}_j + i\lambda[\phi^i, \phi^k] E_{kj}$$

$\phi^i \rightarrow$ Matrix $\Rightarrow$ STr : trace is averaged over all possible ordering

N D0-branes : $\phi^i \equiv (2\pi\alpha')^{-1} X^i$ $\quad X^i$ matrix $\Rightarrow x^i$

Static configuration

$$\hookrightarrow \quad S = -T_0 \int dt\, \mathrm{STr} \sqrt{-G_{00} \det(Q^i_j)} + \mu_0 \int dt\, \frac{1}{3} \lambda \mu\, \mathrm{STr}(\phi^4)$$

$$+ \mu_0 \int dt\, \frac{i}{3} \lambda^2 \mu\, \mathrm{STr}(\phi^a \phi^b \phi^c) \epsilon_{abc}$$

$a, b, c = 1, 2, 3$

$$Q^i_j = \delta^i_j + i\lambda [\phi^i, \phi^j] \qquad G_{00} = -\frac{\mu^2}{9} (\phi^i)^2$$

$i = 1, 2, 3, 4.$

$\mu_0 = -T_0$

$$S = -\frac{\mu}{3} T_0 \lambda \int dt\, \mathrm{STr} \left\{ \sqrt{((\phi^4)^2 + (\phi^a)^2)(1 - \frac{\lambda^2}{2} [\phi^i, \phi^j]^2)} + \phi^4 + i\lambda \phi^a \phi^b \phi^c \epsilon_{abc} \right\}$$

$$\phi^4 = x^4 \mathbb{1}, \qquad (\phi^a)^2 << (x^4)^2 \qquad x^4 \text{ is negative}$$

$$V = \frac{\mu}{3} T_0 \lambda\, \mathrm{STr} \left(-\frac{1}{2x^4} (\phi^a)^2 + \frac{\lambda^2}{4} x^4 [\phi^a, \phi^b]^2 + i \phi^a \phi^b \phi^c \epsilon_{abc} \right)$$

$$= -\frac{1}{2x^4} \frac{\mu}{3} T_0 \lambda\, \mathrm{STr} \left(\phi^a - \frac{i}{2} x^4 [\phi^b, \phi^c] \epsilon_{abc} \right)^2$$

$V = 0 \Rightarrow$ stable state

$$\hookrightarrow \qquad [\phi^a, \phi^b] = -\frac{i}{\lambda x^4} \epsilon^{abc} \phi^c \quad \rightarrow \text{fuzzy algebra}$$

x^4 is into the deformed parameter of this algebra

isomorphic $[J^a, J^b] = i\epsilon^{abc} J^c$, but, truncated

Casimier invariant

$$\hookrightarrow \quad R x^4 = -\lambda^{-1} N/2 = -q$$

$\downarrow$

complamentary relation

VI. Conclusions

1. giant graviton configuration on type IIA pp-wave

2. $R x^4 = -q$ complementary relation

$\Downarrow$

geometrical picture of the giant graviton

3. space-time non-commutative property of the giant graviton.

4. giant graviton — non-perturbative object — BPS state

$\Downarrow$

brane configurations in string theory