The EDGES 21 cm anomaly, and axion dark matter



N. Houston, 中国科学院理论物理研究所

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In collaboration with Chuang Li, Tianjun Li, Qiaoli Yang and Xin Zhang
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中国科学技术大学, March 2019

nhouston@itp.ac.cn

Experiment to Detect the Global EoR Signature

- Epoch of reionisation: the era in which first luminous structures form and reionise matter
- After CMB formation and prior to reionisation: the Dark Ages. Essentially untested by observations



Observational technique: 21 cm cosmology

- Once formed, the first luminous structures will heat, ionise and excite various atomic states of the neutral gas present
- CMB radiation at that time feels these effects, creating a characteristic spectral distortion carrying invaluable information
- Abundance of neutral hydrogen: 21 cm line a great target



CMB supplies background light



Neutral gas filters this, imprinting a signature



Before

After

Proton

Electron

21cm emission

Redshifts to ~ 80 MHz today

Basics of global 21 cm signal evolution

- Spin-flip transition of neutral hydrogen can be used to probe the temperature and distribution of neutral gas in the early universe prior to reionization (z > 6)
- 21cm absorption/emission signal depends on $T_{\rm SPIN}$, a measure # of H in ground vs excited state expected to lie between $T_{\rm GAS}$ and $T_{\rm CMB}$





perature

$$I_{\nu} = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad n_1 \qquad q_1 \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad \tau_{\nu} = \frac{1}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1} \qquad C_{01} \qquad$$



(A. Loeb, Phys. Rept. 2012)

Global of su Flagtatoigns



(Courtesy of Jonathan Pritchard)



- LCDM predicts T₂₁ depth to be at most 0.2 K. 3.8 sigma discrepancy
- Unlike e.g. missing satellites in LCDM, this is a simple observable, derived from wellunderstood atomic physics. Very problematic for the standard cosmology
- 'This may deserve two Nobel prizes', A. Loeb, Chair of Harvard Astrophysics Dept.
- Confirmation could come within a few years (LEDA, PRIZM, SARAS2 etc)

(EDGES collaboration, Nature 2018)

Possible explanations?

Most obviously, we can focus on $\left(1 - \frac{T_{CMB}}{T_{SPIN}}\right)$ factor in T₂₁

- Late time injection of photons? Synchotron emission? (M. Pospelov et al, 1803.07048, T. Moroi et al, 1804.10378 etc)
- Hydrogen cooling: DM-hydrogen interactions? (Hooper et al, 1803.02804 etc)

Or alternatively

- Non-standard thermal history? (Chunlong Li et al 1804.04816 etc, A. Falkowksi et al, 1803.10096).
- Foreground mismodelling or instrument miscalibration? (R. Hills et al 1805.01421, R. Bradley et al, 1810.09015)

(EDGES response recently published in Nature, mismodelling now seems unlikely.)

CMB heating

- We can modify the Rayleigh-Jeans tail of the CMB to account for the EDGES observations
- This can be achieved via a dark photon A' which resonantly mixes with the photon, or via some other process which can inject 21 cm photons

(M. Pospelov et al, 1803.07048)



- Required cross section for DM to sufficiently cool hydrogen is ~ 10^3 barns = 10^{-21} cm²!
- Seems to imply Coulomb-type interactions, which are enhanced at low temperatures: $\sigma(v) \propto 1/v^4$. Millicharged DM.



Can only work if tuned to ~ 1% of total DM, with mass 10 - 80 MeV and charge 10⁻⁴ to 10⁻⁶. A
mechanism to deplete the early overabundance is also required.

(Hooper et al, 1803.02804)

Does this cover every DM possibility? What about axions?





"CP Conservation in the Presence of Instantons," R. Peccei and H. Quinn, Phys. Rev. Lett. 38 (1977) 1440.

Condensed-phase axion DM



- BEC conditions: a system must comprise a large number of identical bosons, conserved in number, which are sufficiently degenerate and in thermal equilibrium
- CDM axions are then natural candidates to condense (at least temporarily). What will be the coherence length of this state? It should be set by the horizon
- Large occupation numbers and coherence lead to significant enhancement of certain effects. No longer in the ordinary 'particle regime', and previous EDGES-DM constraints do not necessarily apply
- Will this state persist? It has been argued that gravitational interactions allow this state to rethermalise over time, preserving large-scale coherence (Sikivie et al, PRL 2009, PRL 2012, PRD 2012)
- Due to the difficulty of incorporating gravitational effects in QFT there is some debate about this latter point. However, for more general ALP models the same effect is (uncontroversially) possible via axion self-interactions (Guth et al, PRD 2015)

Axion-induced cooling



- Coherence enhances scattering energy losses, & thus hydrogen cooling
- Simple picture: total cooling amplitude comes from a sum over scattering targets (i.e. 'individual' axions), which interfere constructively if coherent

axion mass, number density, coherence length

- Mechanism originally invoked to solve the primordial ⁷Li problem via photon cooling (Sikivie et al, Phys.Rev.Lett. 108 (2012) 061304)
- With cooling rate, boundary conditions & RECFAST we can solve for T_{Hydrogen}

CMB/Hydrogen decoupling

EDGES measurement

• Cooling begins once
$$\Gamma/H \sim 1$$
, when $200 > z > 20$. This sets the required axion CDM density as a function of redshift.

Strategy:

• For the QCD axion:

 $\Gamma \simeq 4\pi G m_a n_a l_a \frac{\omega}{\Delta p}$

$$\Omega_a h^2 \simeq 0.15 X \left(\frac{f}{10^{12} \text{GeV}}\right)^{7/6} \qquad m_a \simeq 6 \text{eV} \left(\frac{10^6 \text{GeV}}{f}\right)$$

- This sets m_a as a function of redshift, or equivalently $T_{Hydrogen}$ when cooling begins
- Solve $\rho_H(T_i) = \rho_H(T_f) + \rho_a^{HOT}(T_f)$ for the change in T_{Hydrogen}, to find T₂₁ as a function of X and m_a

A pleasant surprise



IAXO/CAST primer

- Axions should be copiously produced in our sun via a number of physical processes
- Can be converted into photons in intense B fields via the Primakoff effect, and detected on Earth



- EDGES best fit value translates to a (100, 450) meV QCD axion, in the absence of fine tuning of the misalignment angle
- 'Standard' axions are technically in tension here with SN1987A, but uncertainty exists around this point





(Archidiacono et al, JCAP 2015)

• For generic ALPS, relic density and mass are unrelated EDGES-favoured range extends to (10, 450) meV



Conclusions & discussion

- The EDGES collaboration have reported an anomalously strong 21 cm absorption feature corresponding to the era of first star formation, offering an exciting new window into a previously unexplored epoch
- For condensed-phase axion DM cooling, this favours an ALP mass in the (10, 450) meV range, which can be compressed for the QCD axion to (100, 450) meV in the absence of tuning
- This could be tested in the future at IAXO & EUCLID
- Thanks for your attention!

(full details in 1805.04426, Phys.Rev.Lett 121 111301 (2018))

Extra: axion constraints



Accurate Calculation of Global 21 cm Signal

Extra material courtesy of Ran Ding (丁然), Peking University

1. Fine structure of hydrogen

Energy levels in hydrogen atoms are split due to spin orbit interaction (fine splitting), and the smaller effect of electron-proton spin interaction (hyperfine splitting). Anti-aligned spins lead to lower energy levels. The splittings of the lowest energy levels are: Here we have used spectroscopic notation $n_F L_J$, where n is the principal quantum number, L = 0(S), 1(P) are the electron orbital angular momentum. $S_e(S_p)$ shall denote the electron (proton) spin (not to be confused with the S orbital). Then $J = |L + S_e|$ is the electron total angular momentum, and $F = |L + S_e + S_p|$ is the hydrogen total angular momentum.



2. Application to 21 cm Global Signal

Back to the 21cm global signal. We define a series of temperatures:

• Spin temperature:

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-\Delta E_{21}/k_B T_s} = \frac{g_1}{g_0} e^{-h\nu_{21}/k_B T_s} = 3e^{-T_*/T_s} \approx 3\left(1 - \frac{T_*}{T_s}\right).$$

• CMB temperature (black-body):

$$I_{\nu} = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/k_B T_{\rm CMB}} - 1}$$

We define the CMB temperature such that the black-body form is retained. It does tell you the distribution and average energy, but it's not exactly a real temperature. Practically speaking though, it's useful to define something as a temperature.

- T_K : the kinetic temperature of the gas. This is a real temperature.
- T_b : brightness temperature. Not a real temperature but defined as the temperature needed to obtain a measured intensity if the source is a black-body:





$$I_{\nu} = \frac{2k_B}{\lambda_{21}^2} T_b$$

• There are various temperatures involved in the evolution of the 21 cm hydrogen signal. The figure below shows the three temperatures we've previously discussed and how they are coupled and adds the color temperature: the characteristic temperature of the Wouthuysen-Field effect.



• From $dI_{\nu}/d\tau = -I_{\nu} + S_{\nu}$, we can approximate S_{ν} as a black-body:

$$S_{\nu} = \frac{2h\nu_{21cm}^3}{c^2} \frac{1}{e^{h\nu_{21}/k_B T_s} - 1} = \frac{2h\nu^3}{c^2} \frac{1}{e^{T_*/T_s} - 1},$$

where $T_* = h\nu_{21}/k_B = 0.068 \text{K} \ll T_s, T_K, T_{\text{CMB}}$.

• In the Rayleigh-Jeans limit, we then have

$$S_{\nu} = \frac{2h\nu^3}{c^2} \frac{T_s}{T_*} = \frac{2k_B T_s}{\lambda_{21}^2}.$$

• Recall $I_{\nu} = \frac{2k_B T_b}{\lambda^2}$, which is exactly the same form as the previous equation. From this, we can rewrite $dI_{\nu}/d\tau = -I_{\nu} + S_{\nu}$ in terms of the temperatures. We have

$$\frac{dT_b}{d\tau} = -T_b + T_s$$

• To solve this, we need an initial condition: $T_b(\tau = 0) = T_{CMB}$. That is, before we have anything else, we have the CMB. We then have as a solution for the differential brightness temperature:

$$\delta T_b = T_b - T_{\rm CMB} = \frac{T_s - T_{CMB}}{1+z} (1 - e^{-\tau}),$$

where the 1 + z is a correction factor, accounting for the expansion of the universe.

- If δT_b is positive, negative, or zero, we have emission, absorption, or no signal, respectively.
- Local thermal equilibrium is reached when $T_K = T_s = T_b$.

• Collisions set the rate of moving from one energy level to the other. For the n = 1 levels, let the rates of transition between the levels be C_{01} (from 0 to 1) and C_{10} (from 1 to 0). Remember that for collisional excitations of the hyperfine transition, in thermal equilibrium, the gas temperature is coupled directly to the spin temperature. In other words, collisions are the dominant effect that sets the hyperfine level population. This means the transition rates C_{01} and C_{10} are given by: $n_0C_{01} = n_1C_{10}$. We can solve for the ratio of the rates

$$\frac{C_{01}}{C_{10}} = \frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-T_*/T_K} \approx 3\left(1 - \frac{T_*}{T_K}\right).$$

• Similarly, the Wouthuysen-Field effect sets the rates of transition are given by the "color temperature" T_{α} :

$$\frac{P_{01}}{P_{10}} = \frac{g_1}{g_0} e^{-T_*/T_\alpha} \approx 3\left(1 - \frac{T_*}{T_\alpha}\right)$$



• To get all the astrophysical processes at play, we must write a balance equation:

$$n_1(A_{10} + B_{10}I_{\rm CMB} + C_{10} + P_{10}) = n_0(B_{01}I_{\rm CMB} + C_{01} + P_{01}),$$

where A_{10} , B_{10} and B_{01} are Einstein coefficients: A_{10} = rate of spontaneous decay

 $B_{10} =$ stimulated emission

 $B_{01} = absorption$

In the absence of collisions and Wouthuysen-Field effect, the above simplifies to

$$n_1(A_{10} + B_{10}I_{\nu}) = n_0(B_{01}I_{\nu}).$$

• If equilibrium is reached, I_{ν} must be a black body: $I_{\nu} = I(A, B) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/k_BT} - 1}$.

This gives us equilibrium ratios of the Einstein coefficients.

$$B_{01} = \frac{g_1}{g_0} B_{10} = \frac{3c^2}{2h\nu^3} A_{10}.$$

In the Rayleigh-Jeans limit, $I_{\text{CMB}} \approx \frac{2\nu^2}{c^2} k_B T_{\gamma}$, here $T_{\gamma} = 2.725(1+z)K$ is the temperature of the CMB. Radiative coefficients then satisfy

$$B_{01}I_{\rm CMB} = \frac{3k_B T_{\gamma}}{h\nu} A_{10} = \frac{3T_{\gamma}}{T_*} A_{10}$$
$$B_{10}I_{\rm CMB} = \frac{g_0}{g_1} B_{01}I_{\rm CMB} = \frac{T_{\gamma}}{T_*} A_{10}$$

• Plugging the above results into the balance equation, we have the relation of the temperatures:

$$\begin{aligned} \frac{n_1}{n_0} &= \frac{B_{01}I_{\rm CMB} + C_{01} + P_{01}}{A_{10} + B_{10}I_{\rm CMB} + C_{10} + P_{10}} \\ \Rightarrow 3\left(1 - \frac{T_*}{T_s}\right) = 3\frac{\frac{T_{\gamma}}{T_*}A_{10} + \left(1 - \frac{T_*}{T_K}\right)C_{10} + \left(1 - \frac{T_*}{T_{\alpha}}\right)P_{10}}{\left(1 + \frac{T_{\gamma}}{T_*}\right)A_{10} + C_{10} + P_{10}} \\ \Rightarrow T_s^{-1} &= \frac{T_{\gamma}^{-1} + x_c T_K^{-1} + x_{\alpha} T_{\alpha}^{-1}}{1 + x_c + x_{\alpha}}. \end{aligned}$$

where

$$x_c = \frac{C_{10}}{A_{10}} \frac{T_*}{T_{\gamma}}, \quad x_\alpha = \frac{P_{10}}{A_{10}} \frac{T_*}{T_{\gamma}}$$

represents the relative rates. It remains to determine these coefficients.

3. collision coefficients

• The collision coefficients x_c is the sum of three terms:

$$x_c = \frac{0.068 \,\mathrm{K}}{A_{10} T_{\gamma}} \left[n_H \kappa_{\mathrm{HH}}(T_K) + n_e \kappa_{\mathrm{eH}}(T_K) + n_p \kappa_{\mathrm{pH}}(T_K) \right],$$

with $n_H \kappa_{\text{HH}}$, $n_e \kappa_{\text{eH}}$ and $n_p \kappa_{\text{pH}}$ are the de-excitation rates of the triplet due to collisions with neutral atoms, electrons, and protons.

- Free electron and proton fractions small so HH collisions dominates.
- By $z \sim 30$, collisions negligible \Rightarrow gas diffuse and T_K falls below 100 K.



4. Wouthuysen-Field effect

The Wouthuysen-Field (WF) effect is the process for a hydrogen atom to absorb a Ly- α photon, exciting the atom into an n = 2 state. If it then decays back into a different n = 1 hyperfine state, the spin flips in the process, producing a 21 cm photon. The WF effect becomes important during reionization due to the abundance of Ly- α photons. The WF effect is also relatively competitive due to the quantum mechanical selection rules. Laws such as the conservation of parity and angular momentum tell us that only certain transitions are allowed. In particular, the electric dipole mode (strongest) of the direct 21 cm transition is forbidden by parity, thus making it a factor α^2 weaker.

The WF effect involves absorption of an Ly- α photon from the $1_0S_{1/2}$ state and subsequent decay into the $1_1S_{1/2}$ state. The CMB temperature during the dark ages have temperatures $15K < T_{\gamma} < 3000K$, and is much smaller than the Lyman- α transition temperature of $T = 13.6 \text{eV} \sim 1.6 \times 10^5 \text{K}$, and thus we expect WF effect to be most important during the epoch of reionization, when the first stars provide the abundance of Ly- α photons.

In summary, we have the following selection rules for dipole transitions in a neutral hydrogen atom



Thus finally $P_{10} = 4P_{\alpha}/27$ and the coupling coefficient becomes

 $x_{\alpha} = \frac{4P_{\alpha}}{27A_{10}} \frac{T_{*}}{T_{\gamma}}$

$$x_{\alpha} = S_{\alpha} \frac{J_{\alpha}}{J_{\nu}^{c}}, \quad \text{where} \quad J_{\nu}^{c} = 1.165 \times 10^{-10} \left(\frac{1+z}{20}\right) \text{cm}^{-2} \text{s}^{-1} \text{Hz}^{-1} \text{sr}^{-1}.$$

the suppression factor is given by

$$S_{\alpha} \sim \exp\left[-0.803 T_K^{-2/3} (10^{-6} \tau_{\rm GP})^{1/3}\right],$$

where the Gunn-Peterson optical depth can be written for overdensity δ as roughly:

$$\tau_{\rm GP} = 7 \times 10^5 \left(\frac{\Omega_b h^2}{0.03}\right) \left(\frac{\Omega_m h^2}{0.25}\right)^{-1/2} \left(\frac{1+z}{10}\right) (1+\delta).$$

5. brightness temperature

• For gas in local thermal equilibrium, optical depth is:

$$\tau_{\nu} = \int ds [1 - \exp(-E_{10}/k_B T_s)] \sigma_0 \phi(\nu) n_0,$$

- $n_1 = 3n_0 \Rightarrow n_0 = n_H/4$, also can expand $1 e^{-h\nu_{21}/k_B T_s} \approx h\nu_{21}/k_B T_s$.
- Cross section: split into amplitude and line-profile: $\sigma(\nu) = \sigma_0 \phi(\nu)$ with $\sigma_0 \equiv 3c^2 A_{10}/8\pi\nu^2$, We thus have

$$\tau_{\nu} = \int ds \frac{h\nu_{21}}{k_B T_s} \frac{n_H}{4} \sigma_0 \phi(\nu).$$

• Expansion of Universe $ds = -cdt = -\frac{cdz}{(1+z)H(z)}$,

with $H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda} \approx H_0 \Omega_m^{1/2} (1+z)^{3/2}$.

• The number density of neutral hydrogen is related to baryon overdensity:

$$n_H = x_{\rm HI} n_b = x_{\rm HI} (1 + \delta_b) \Omega_b \rho_{c,0} (1 + z)^3 / m_b,$$

with $\rho_{c,0} = 3H_0^2/8\pi G$ is the critical density.

• 21 cm line-width very small $\Rightarrow z, H(z) \sim \text{constant over line-width}$:

$$\tau_{\nu_0} = \frac{3}{128\pi^2 G} \frac{hc^3 A_{10}\nu_{21}}{k_B T_s \nu_0^2} x_{\rm HI} (1+\delta_b) \frac{H_0}{m_b} \Omega_b \Omega_m^{-1/2} (1+z)^{3/2}$$
$$= \frac{300}{128\pi^2 G} \frac{hc^3 A_{10}\nu_{21}}{k_B T_s \nu_0^2} \frac{x_{\rm HI} (1+\delta_b)}{m_b} (\Omega_b h^2) (\Omega_m h^2)^{-1/2} (1+z)^{3/2}.$$

• Final brightness temperature then given by

$$\begin{split} \delta T_b &= \frac{T_s - T_{\rm CMB}}{1+z} (1 - e^{-\tau}) \\ &\approx \frac{T_s - T_{\rm CMB}}{1+z} \tau \\ &= \frac{300}{128\pi^2 G} \frac{hc^3 A_{10}\nu_{21}}{k_B \nu_0^2 m_b} x_{\rm HI} (1 + \delta_b) (\Omega_b h^2) \left(\frac{1+z}{\Omega_m h^2}\right)^{1/2} \times \left(1 - \frac{T_{\rm CMB}}{T_s}\right) \\ &= 27 x_{\rm HI} (1 + \delta_b) \left(\frac{\Omega_b h^2}{0.023}\right) \left(\frac{0.15}{\Omega_m h^2} \frac{1+z}{10}\right)^{1/2} \left(1 - \frac{T_{\rm CMB}}{T_s}\right). \end{split}$$

6. Thermal history and heating processes

• Heating of IGM in expanding Universe:

$$\frac{dT_K}{dt} = -2H(z)T_K + \frac{2}{3}\sum_i \frac{\epsilon_i(z)}{k_B n}.$$

- After recombination, free electron fraction $x_e = 3.1 \times 10^{-4}$ at z = 200.
- Compton scattering initially dominates heating/cooling post-recombination:

$$\frac{2}{3}\frac{\epsilon_{\rm comp}}{k_B n} = \frac{x_e}{1 + f_{\rm He} + x_e} \frac{8\sigma_T u_\gamma (T_\gamma - T_K)}{3m_e c}.$$

• Thermal decoupling when scattering CMB photons can no longer heat gas at

$$1 + z_{\rm dec} \approx 150 \left(\frac{\Omega_b h^2}{0.023}\right)^{2/5},$$

thereafter gas cools adiabatically as non-relativistic particles $T_K \sim (1+z)^2$.

- After first galaxies form, gas is reheated by X-ray and Ly- α photons.
- X-ray heating can be efficient

$$\frac{2}{3} \frac{\epsilon_X(z)}{k_B n H(z)} = 10^3 \mathrm{K} f_X \left(\frac{f_*}{0.1} \frac{f_{X,h}}{0.2} \frac{df_{\mathrm{coll}}/dz}{0.01} \frac{1+z}{10} \right).$$

- Ly- α heating less efficient, unless gas very cold

$$\frac{2}{3} \frac{\epsilon_{\alpha}(z)}{k_B n_H H(z) T_K} \approx \frac{0.80}{T_K^{4/3}} \frac{x_{\alpha}}{S_{\alpha}} \left(\frac{10}{1+z}\right).$$

• X-ray heating dominates for sensible parameters

$$\frac{\epsilon_X(z)}{\epsilon_\alpha(z)} \sim 140 f_X T_K^{1/3} \left(\frac{f_{X,h}}{0.2} \frac{9690}{N_\alpha} \frac{1+z}{10} \right).$$

• The evolution of gas temperature T_K can be numerically solved by package **RecFast** or **HyRec** with following standard evolution equations

$$\begin{aligned} \frac{dT_K}{dt} &= -2H(z)T_K + \frac{8\sigma_T a_R T_\gamma^4 x_e (T_\gamma - T_K)}{3(1 + f_{\text{He}} + x_e)m_e c}, \\ \frac{dx_e}{dt} &= -C \left[n_H \alpha_B (T_K) x_e^2 - (1 - x_e)\beta_B (T_\gamma) e^{-E_{12}/T_\gamma} \right]. \end{aligned}$$

• Taking into account X-ray heating, introducing additional terms in evolution equations

$$\begin{split} \delta\left(\frac{dT_K}{dt}\right) &= \frac{2\epsilon_X(z)}{3k_B} \frac{1+2x_H + f_{\rm He}(1+2x_{\rm He})}{3(1+f_{\rm He})},\\ \delta\left(\frac{dx_H}{dt}\right) &= \frac{\epsilon_X(z)}{13.6{\rm eV}} \frac{1-x_H}{3(1+f_{\rm He})},\\ \delta\left(\frac{dx_{\rm He}}{dt}\right) &= \frac{\epsilon_X(z)}{24.6{\rm eV}} \frac{1-x_{\rm He}}{3(1+f_{\rm He})}, \end{split}$$

with $x_e = x_H + f_{\text{He}} x_{\text{He}}$.



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