Distributed quantum information and the structure of spacetime

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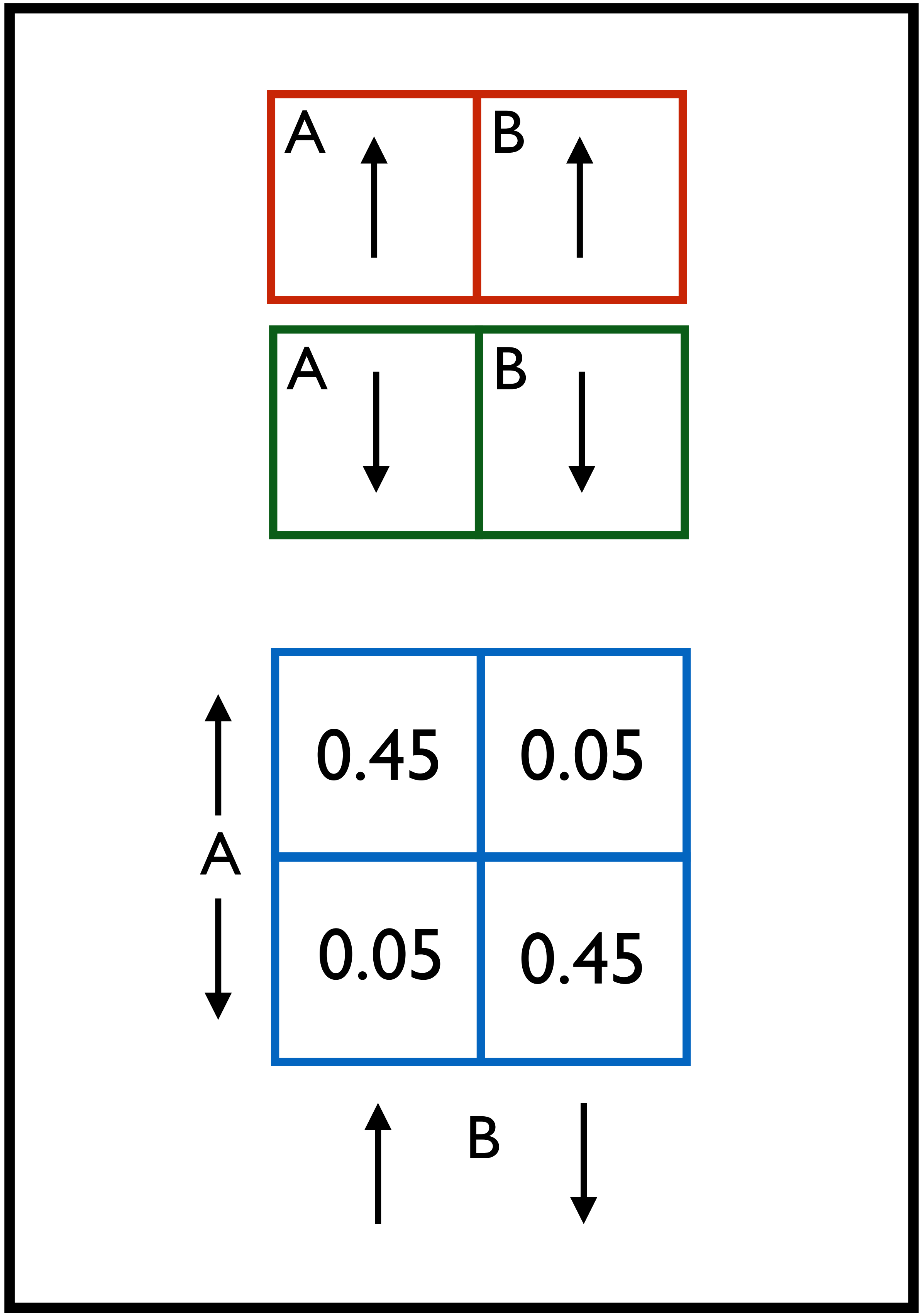
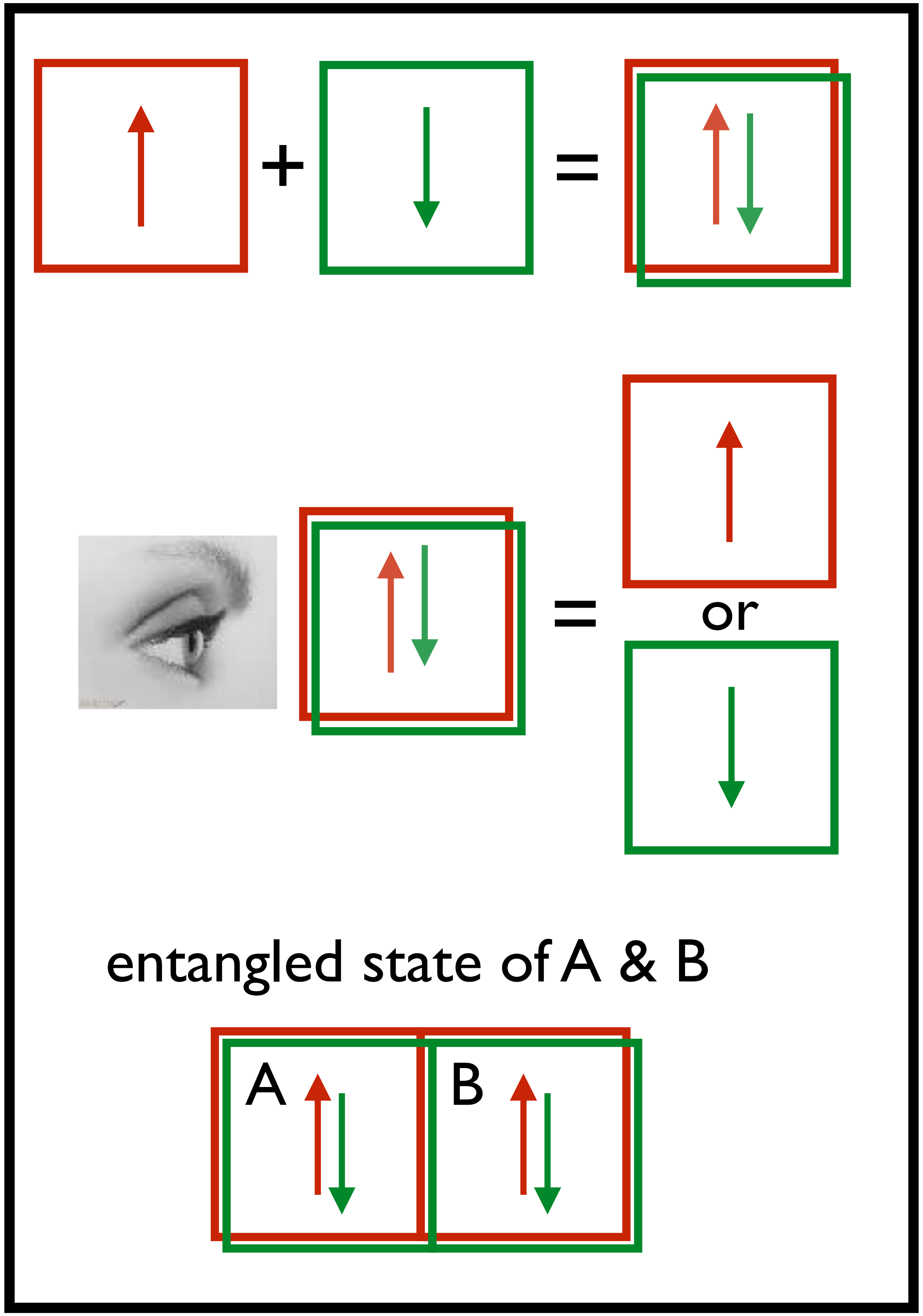
Classical Information



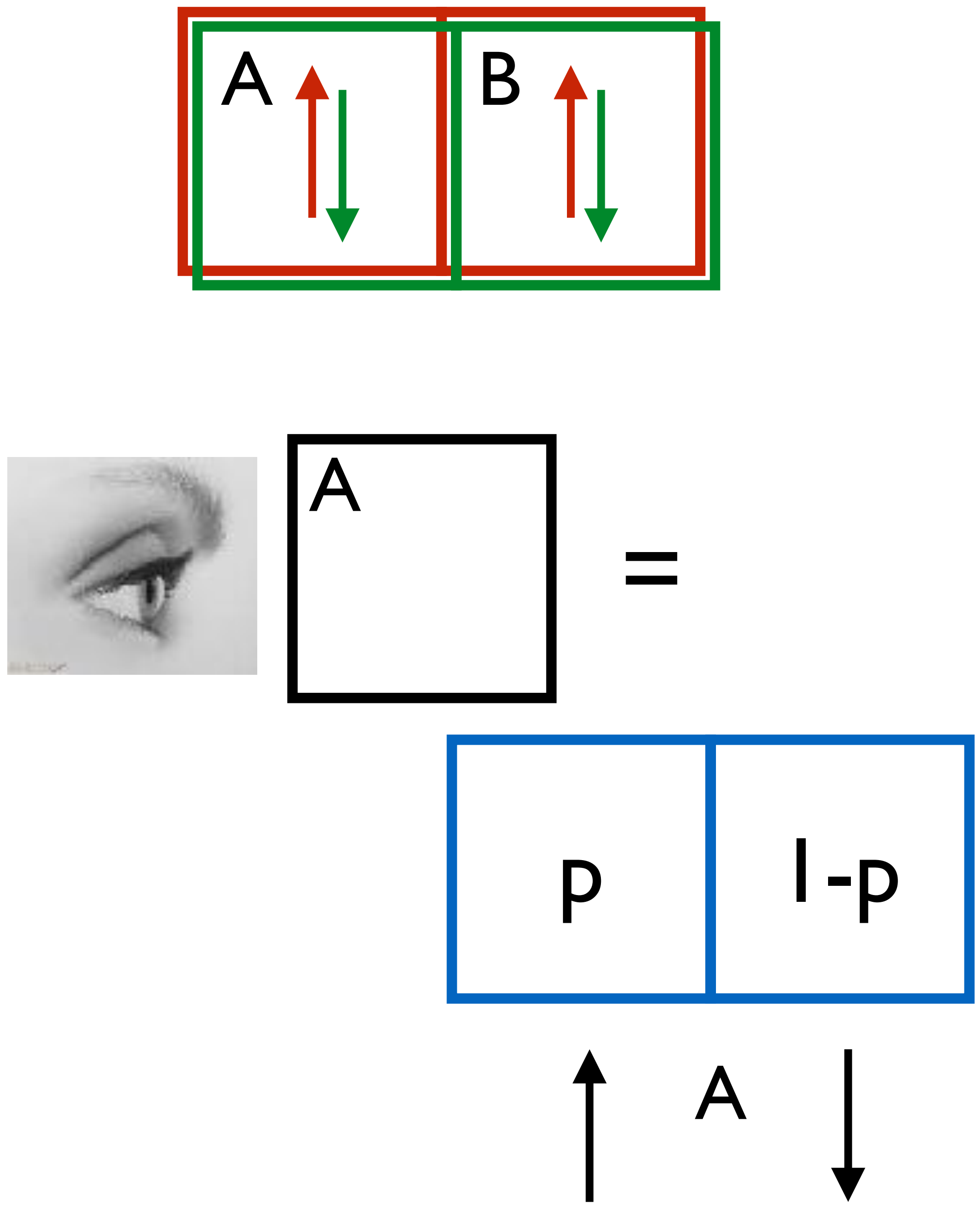
- System A contains information about System B = measuring A tells you about B
- Equivalently, A and B are correlated
- “Information” is a way of quantifying this correlation
- “A contains N bits of information about B” = measuring A allows us to distinguish between 2^N possible configurations of B.

Quantum Information



- Quantum systems, unlike classical ones, can be in a "superposition" of states
- Measurement "collapses" the superposition
- Because of this there is a new kind of correlation, *entanglement*
- If System A is entangled with System B, measuring A can collapse B, even though you have have not touched it
- So entanglement also means that measuring A tells you about B

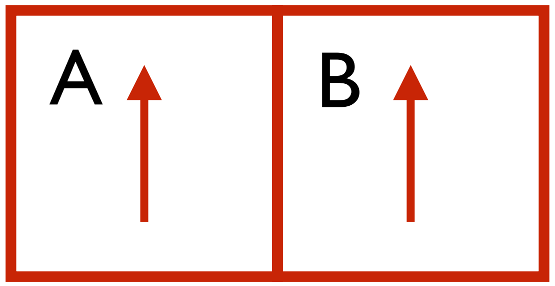
Entanglement Entropy



$$S = -p \log p - (1-p) \log(1-p)$$

- Suppose A and B are entangled and you only measure A
- To predict the result you must sum over all the possibilities for B that are quantumly superposed
- This leaves A in a "mixed state" which is classically uncertain. The uncertainty is quantified by *entanglement entropy* (S)
- Similarly, *mutual information quantifies* how well we can predict B if we only measure A.

Entanglement Entropy



$$|\psi\rangle = |\uparrow_A \uparrow_B\rangle$$

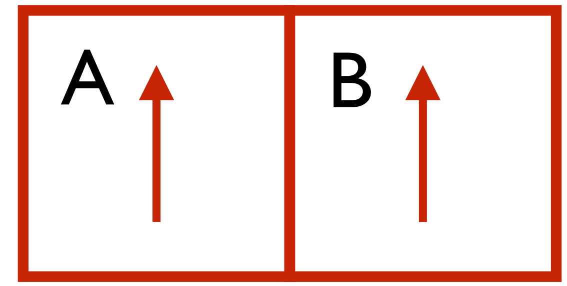
$$\rho = |\psi\rangle\langle\psi|$$

$$= |\uparrow_A \uparrow_B\rangle\langle\uparrow_A \uparrow_B|$$

$$= \begin{array}{c} \begin{array}{cccc} \uparrow\uparrow & \uparrow\downarrow & \downarrow\uparrow & \downarrow\downarrow \end{array} \\ \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{c} \\ \uparrow\uparrow \\ \uparrow\downarrow \\ \downarrow\uparrow \\ \downarrow\downarrow \end{array}$$

$$S = -\mathrm{Tr}[\rho \log \rho] = 0$$

von Neumann entropy of pure states vanishes



$$|\psi\rangle = |\uparrow_A \uparrow_B\rangle$$

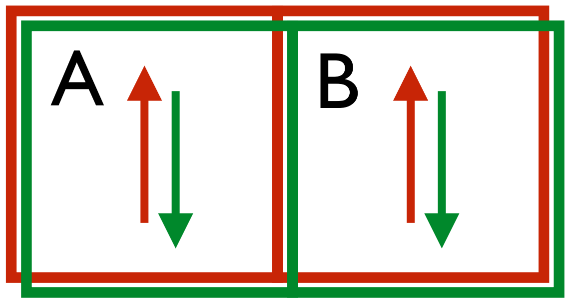
$$\rho = |\psi\rangle\langle\psi| = |\uparrow_A \uparrow_B\rangle\langle\uparrow_A \uparrow_B|$$

$$\rho_A = \mathrm{Tr}(\rho)_B = |\uparrow_A\rangle\langle\uparrow_A| = \begin{array}{c} \left[\begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array}\right] \\ \begin{array}{cc} \uparrow & \downarrow \end{array} \end{array} \begin{array}{c} \uparrow \\ \downarrow \\ \end{array}$$

$$\boxed{S_A = -\mathrm{Tr}(\rho_A \ln \rho_A) = 0}$$

von Neumann entropy of subsystems vanishes in pure product (disentangled) states

Multi-party entanglement



$$|\psi\rangle = \frac{|\uparrow_A \uparrow_B\rangle + |\downarrow_A \downarrow_B\rangle}{\sqrt{2}}$$

$$\rho = |\psi\rangle\langle\psi| = \begin{array}{c|cccc} & \uparrow\uparrow & \uparrow\downarrow & \downarrow\uparrow & \downarrow\downarrow \\ \hline \uparrow\uparrow & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \uparrow\downarrow & 0 & 0 & 0 & 0 \\ \downarrow\uparrow & & & & \\ \downarrow\downarrow & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{array}$$

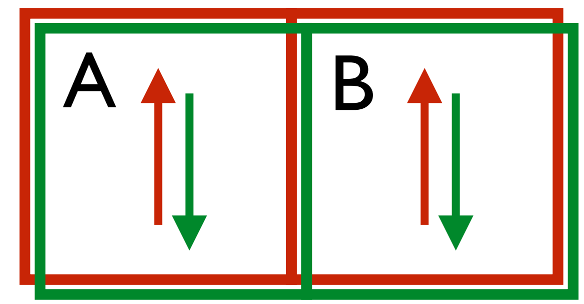
$$\rho_A = Tr(\rho)_B = \sum_{S_B} |S_A S_B\rangle\langle S'_A S_B|$$

$$= \begin{array}{cc|c} \uparrow & \downarrow & \\ \hline \frac{1}{2} & 0 & \uparrow \\ 0 & \frac{1}{2} & \downarrow \end{array}$$

$$\boxed{S_A = -Tr(\rho_A \log_2 \rho_A) = 1 \text{ bit}}$$

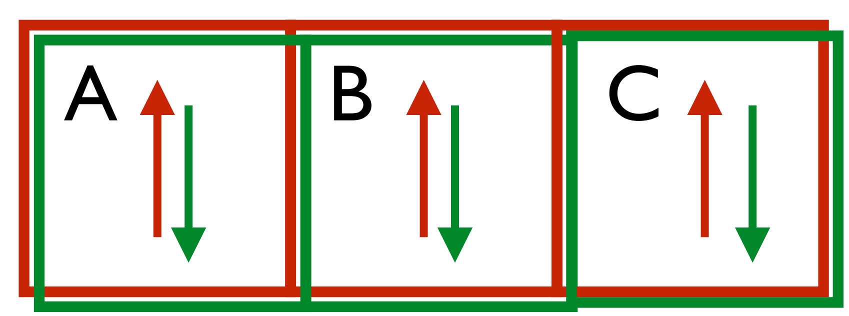
Qubit A is in a "mixed" state

Mutual information quantifies classical and quantum correlation between A and B



$$I(A, B) = S(A) + S(B) - S(A \cup B)$$

Tripartite Information quantifies extensivity of mutual information



$$I_3(A, B, C) = I(A, B) + I(A, C) - I(A, B \cup C)$$

Interactions produce entanglement

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

$$H_0 = H_A + H_B$$

$$\Rightarrow |\psi\rangle_{GND} = |\psi_A\rangle \otimes |\psi_B\rangle$$

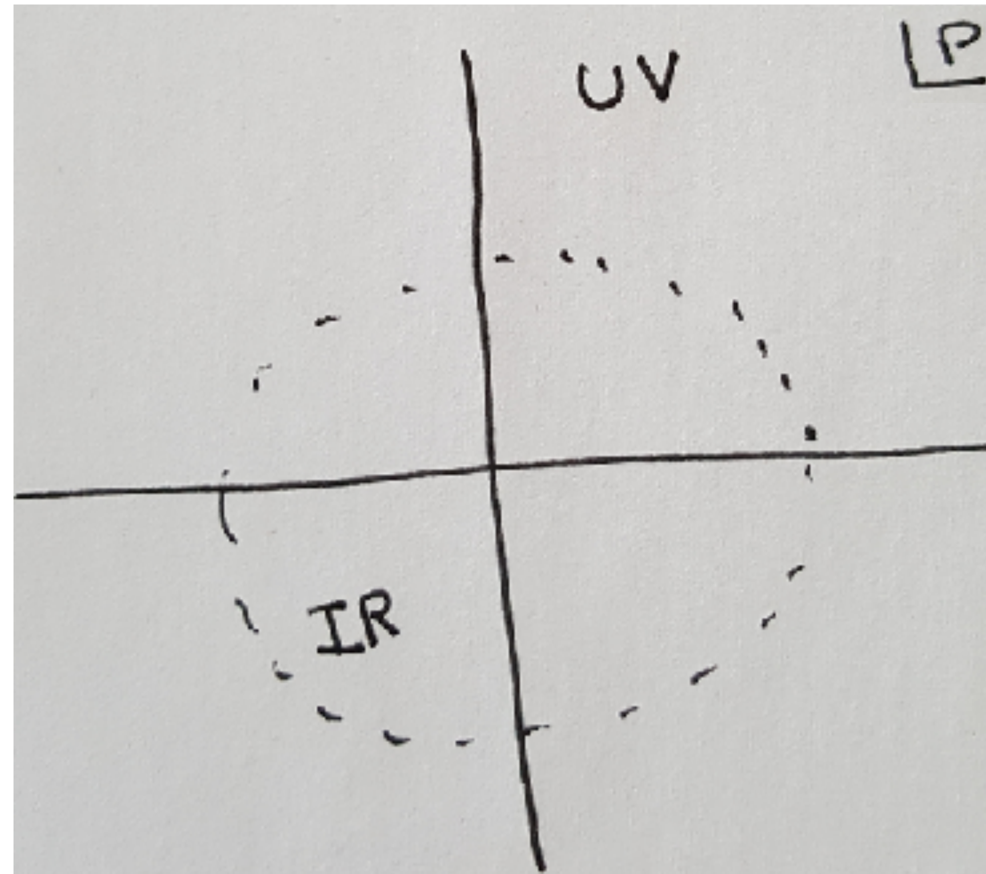
$$H = H_A + H_B + H_{AB}$$

$$\Rightarrow |\psi\rangle_{GND} \neq |\psi_A\rangle \otimes |\psi_B\rangle$$

$$e^{iHt} |\psi_A\rangle \otimes |\psi\rangle_B$$

$\longrightarrow$ ENTANGLED STATE

Interactions between degrees of freedom entangle their wavefunctions

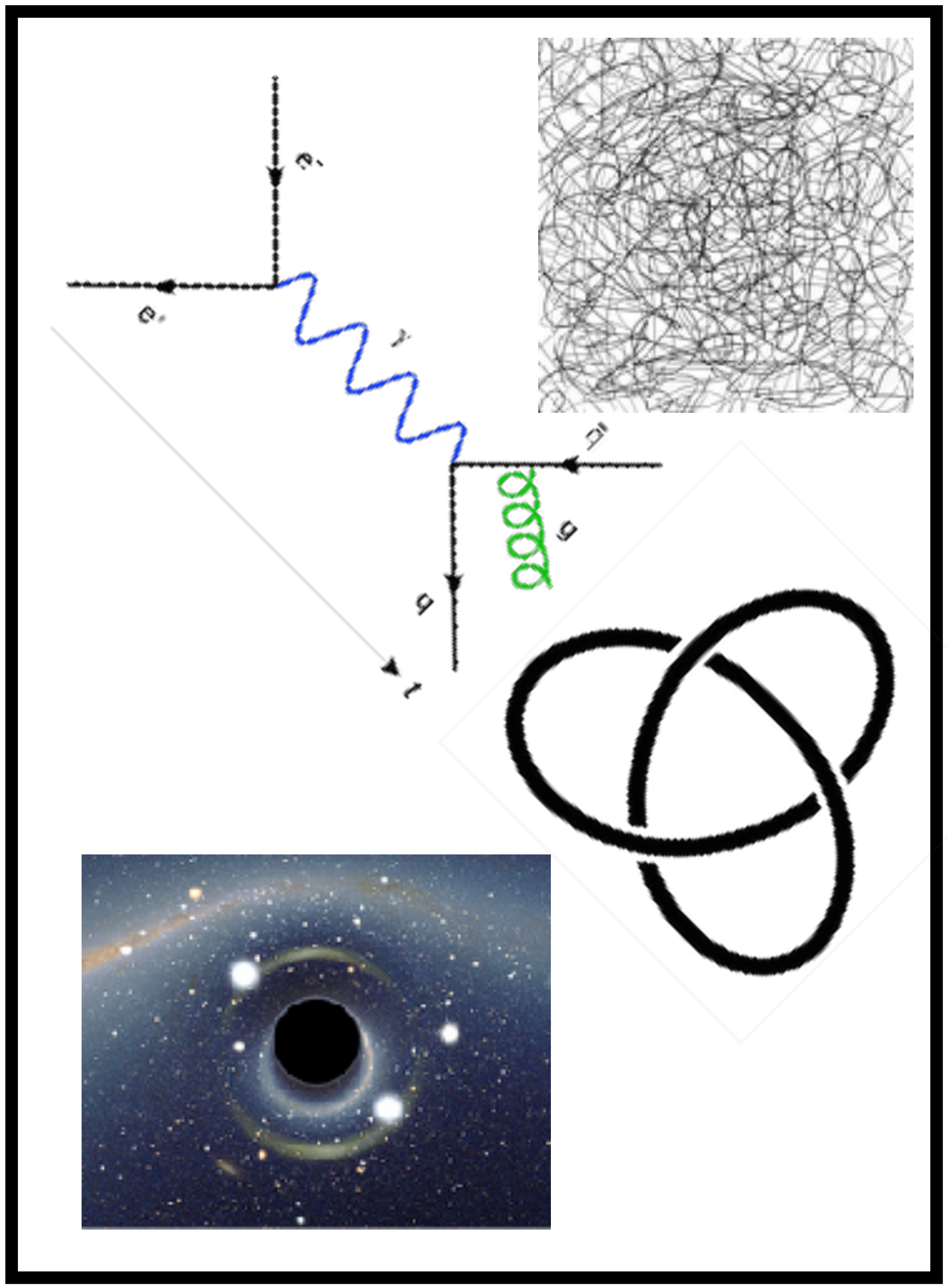


$$\mathcal{L}_0 = \nabla\phi^2 + m^2\phi^2$$

$$\mathcal{L}_1 = \nabla\phi^2 + m^2\phi^2 + \lambda\phi^4$$

Information, matter, spacetime and forces

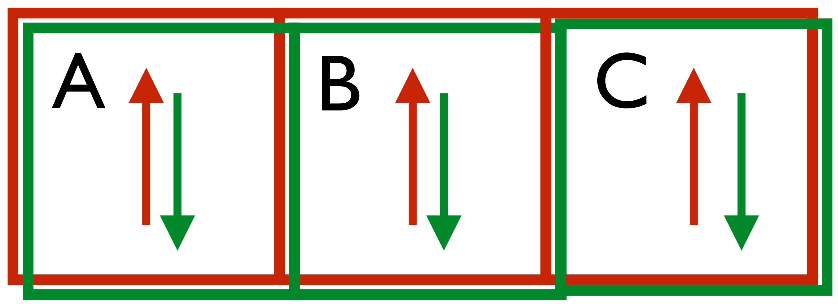
Application to the fundamental theory of matter and forces



- The topology of quantum entanglement
- Quantum information as a probe of microscopic physics
- Thermalization and chaos as quantum entangling processes.
- Information recovery from black holes through inside-outside entanglement.
- Entanglement knitting spacetime

The topology of entanglement

The topology of entanglement



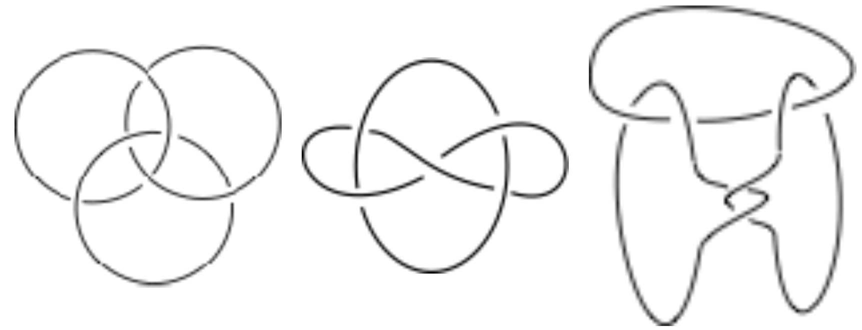
$$|GHZ\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$

$$|W\rangle = \frac{|100\rangle + |010\rangle + |001\rangle}{\sqrt{3}}$$

- Entanglement is a property that implies that a many body system cannot be separated smoothly into pieces
- Thus, it concerns the *topology* of the quantum states
- **GHZ-like states:** partial traces leave a separable state (entanglement is intrinsically multi-party)
- **W-like states**: partial traces leave an entangled state (all parties are robustly entangled)
- What "topological" classes of entanglement arise naturally in quantum field theory?

Information and topology of manifolds

Links



Chern-Simons theory on the sphere with a link drilled out

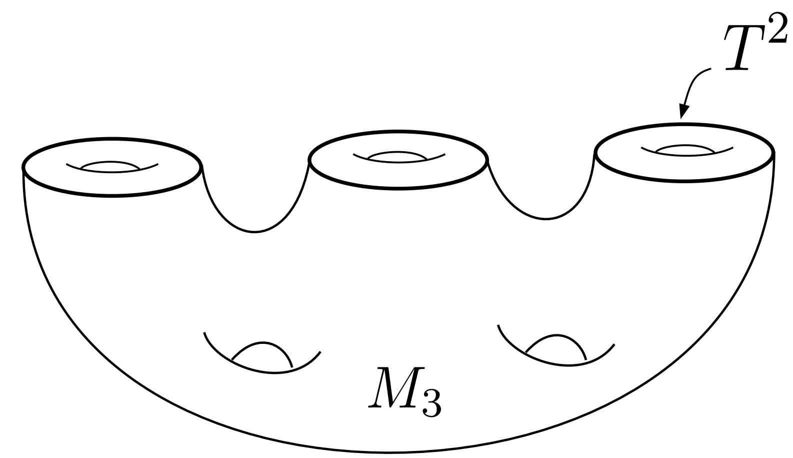
quantum wavefunction ~ colored Jones polynomials

- link topology controls entanglement
- entanglement entropy classifies types of links

- To separate local deformations from global topology, consider a Topological Field Theory
- Example: Chern-Simons theory in 2+1 dimensions.

$$A = A_\mu dx^\mu$$

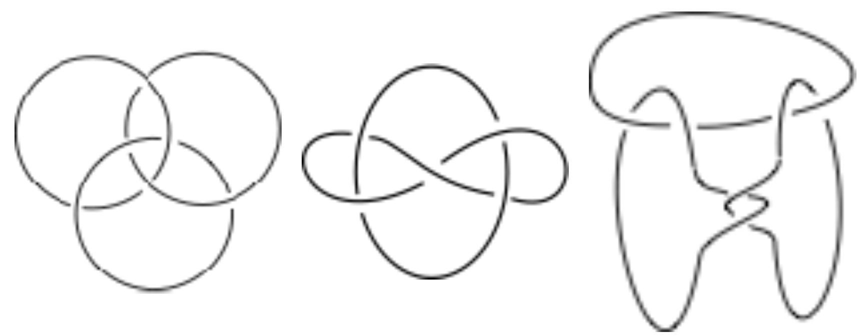
$$S_{CS}[A] = \frac{k}{4\pi}\int_M \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$$



The wavefunction on the equal time surface (multiple copies of a torus) is calculated by the Euclidean path integral on a 3-manfold with this boundary.

Information and topology of manifolds

Links



Chern-Simons theory on the sphere with a link drilled out

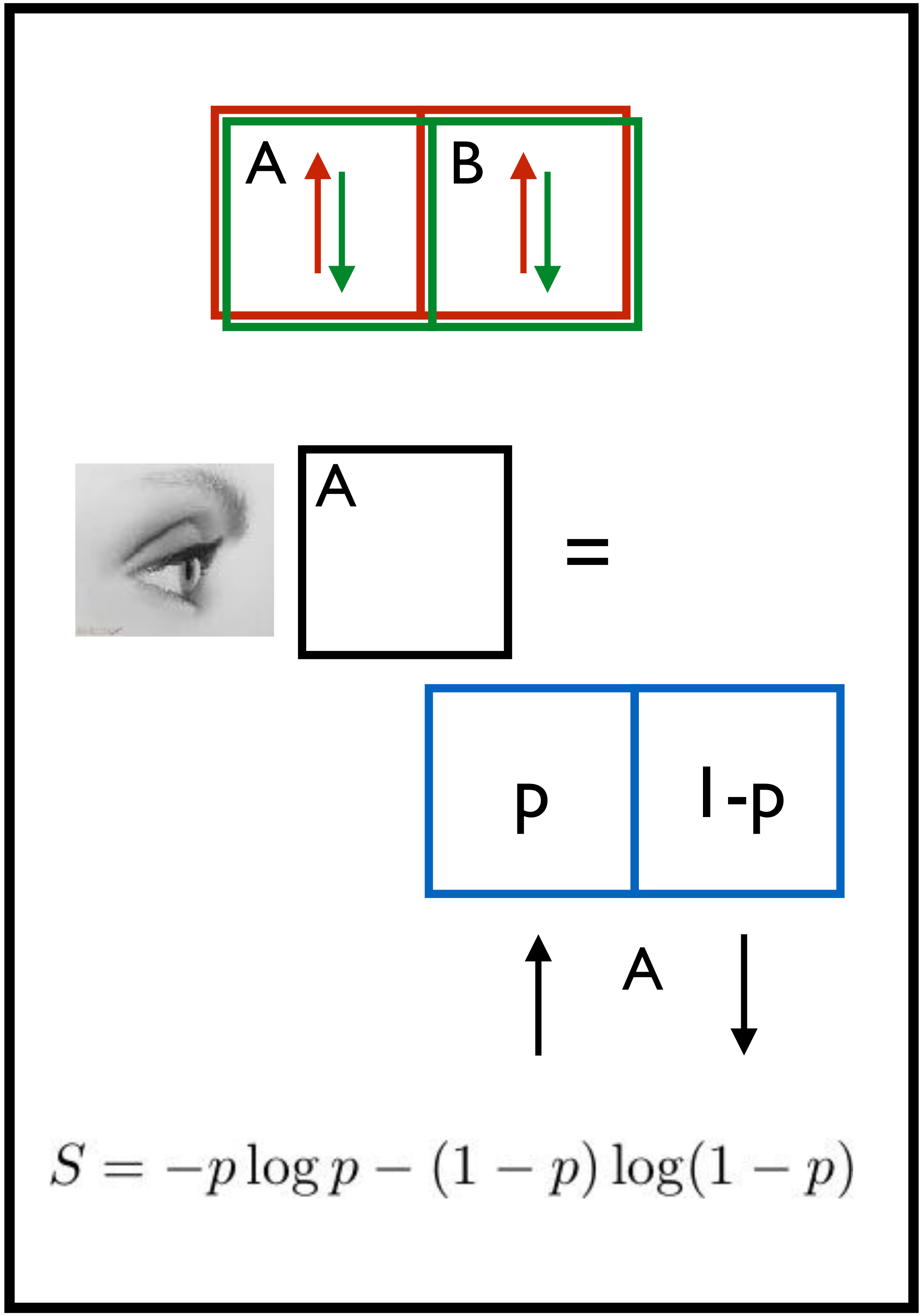
quantum wavefunction = Jones polynomial of link

- link topology controls entanglement
- entanglement entropy classifies types of links

- U(1) Chern-Simons theory: entanglement entropy between sublinks vanishes if and only if they have zero Gauss linking number
- All torus links (links that can be drawn on a torus) have **GHZ-like entanglement.**
- Hyperbolic links (whose link complement admits a hyperbolic structure) have **W-like entanglement**
- A direct connection between topology of manifolds and the topology of quantum entanglement

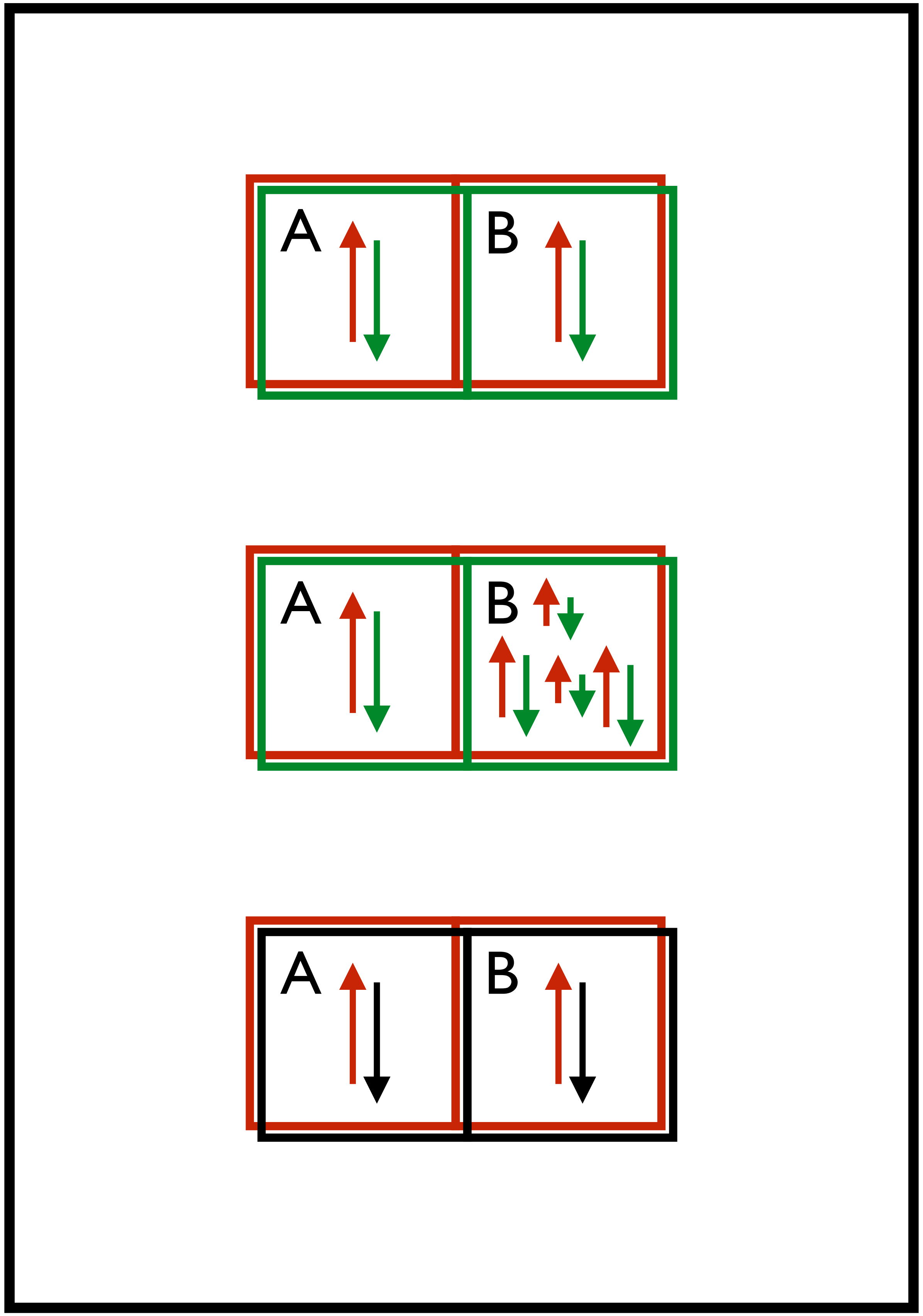
Quantum entanglement as a probe

Cosequences of entanglement



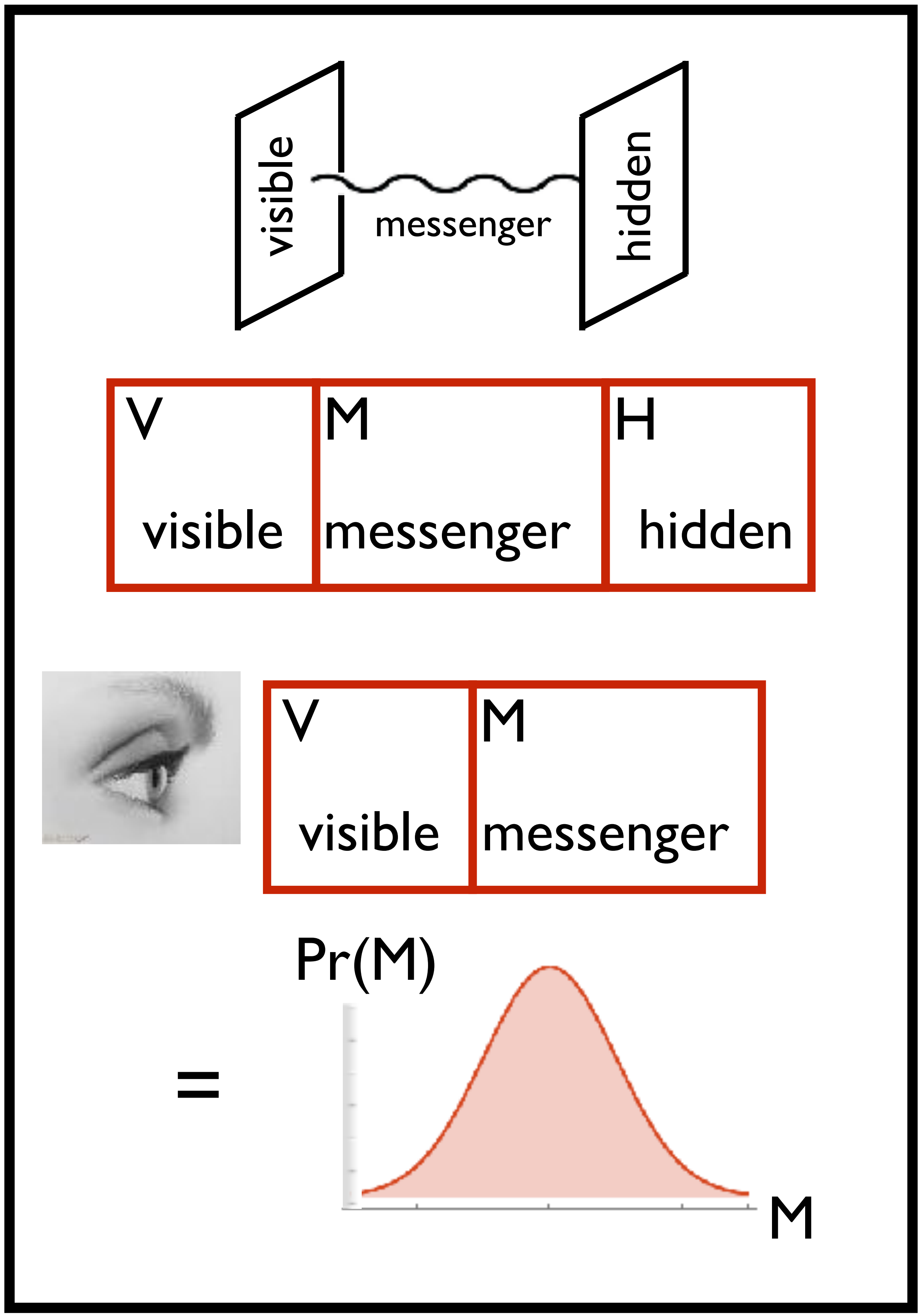
- Suppose A and B are entangled and you only measure A. This measurement can collapse B, even though you have have not touched it
- So measuring A tells you something about B
- Not measuring B leaves A in a classically uncertain "mixed state", quantified by *entanglement entropy* (S)

Entanglement as a probe of microscopic physics



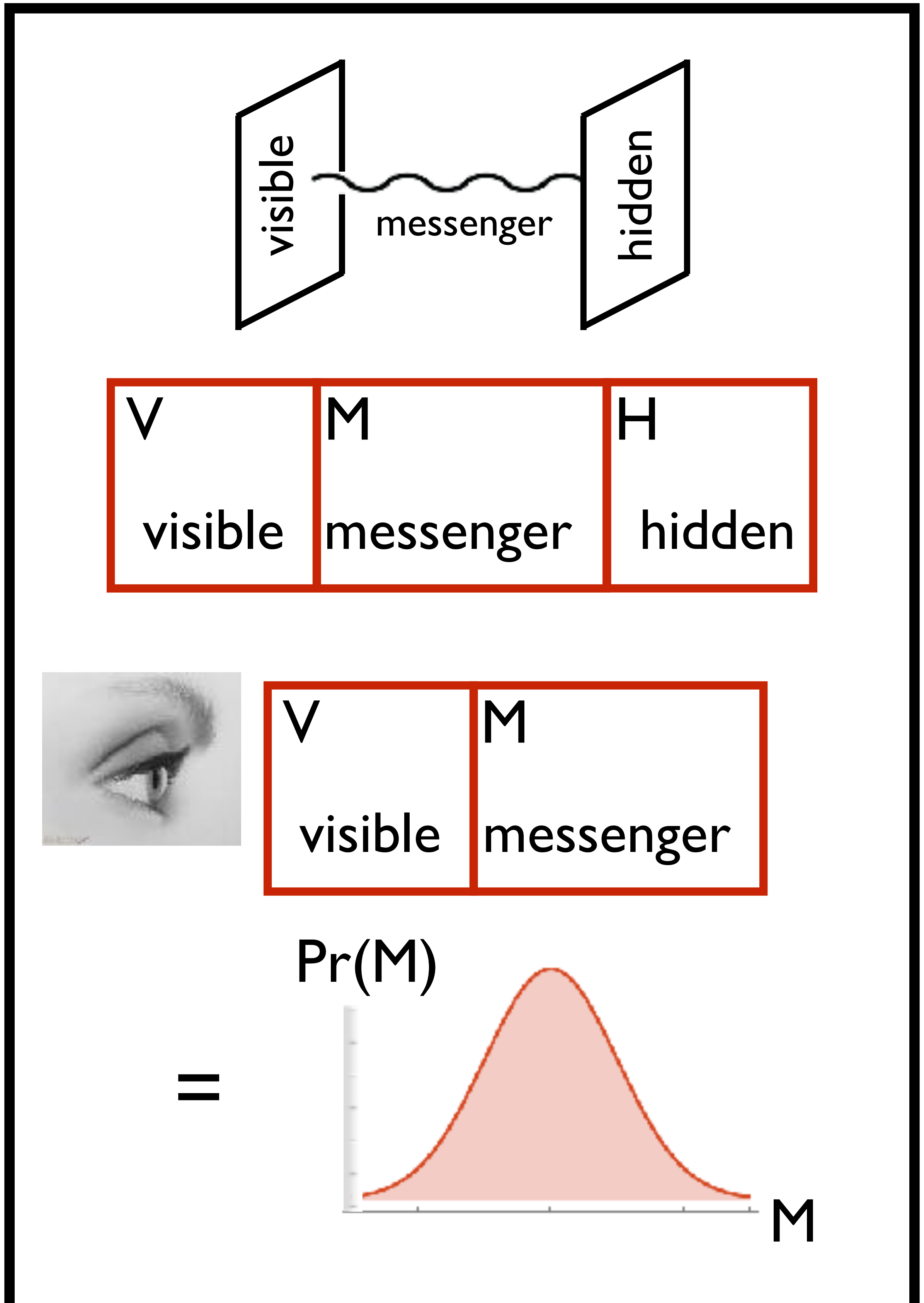
- The entangled systems A & B need not be spatially separated.
- They can be:
  - microscopic vs. macroscopic
  - visible (standard model) vs. hidden (dark matter)
- Can such information be used to probe microscopic physics or dark matter that cannot be directly measured?

Example: a generic model in string theory



- A visible sector (us) interacts with messengers, which interact with hidden (dark?) matter
- The messengers "freeze" and their frozen values M determine the "couplings" of nature, i.e. the strengths of the forces.
- The messengers are entangled with the hidden sector.
- The hidden sector is not measured
- *So*: the messengers should be in a mixed state, giving statistically distributed couplings

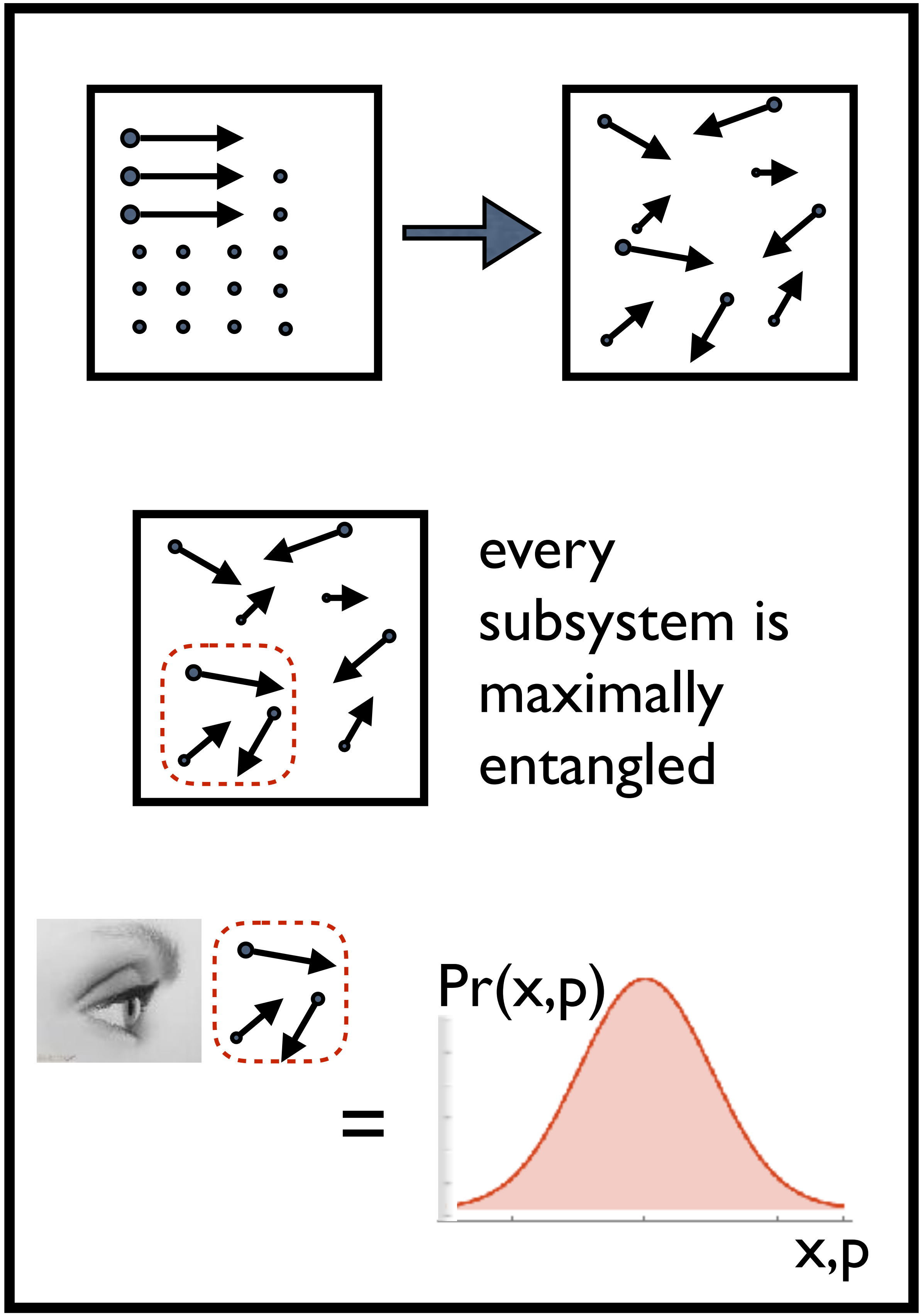
Example: a generic model in string theory



- Can we use such entanglement as a probe to extend the reach of high energy experiments?
- **Idea**: each time an interaction occurs, the coupling is statistically sampled
- **Strategy**: treat the coupling as statistically distributed and fit it with a mean and a variance
- Perhaps this strategy can exploit entanglement to extend the reach of experiments.

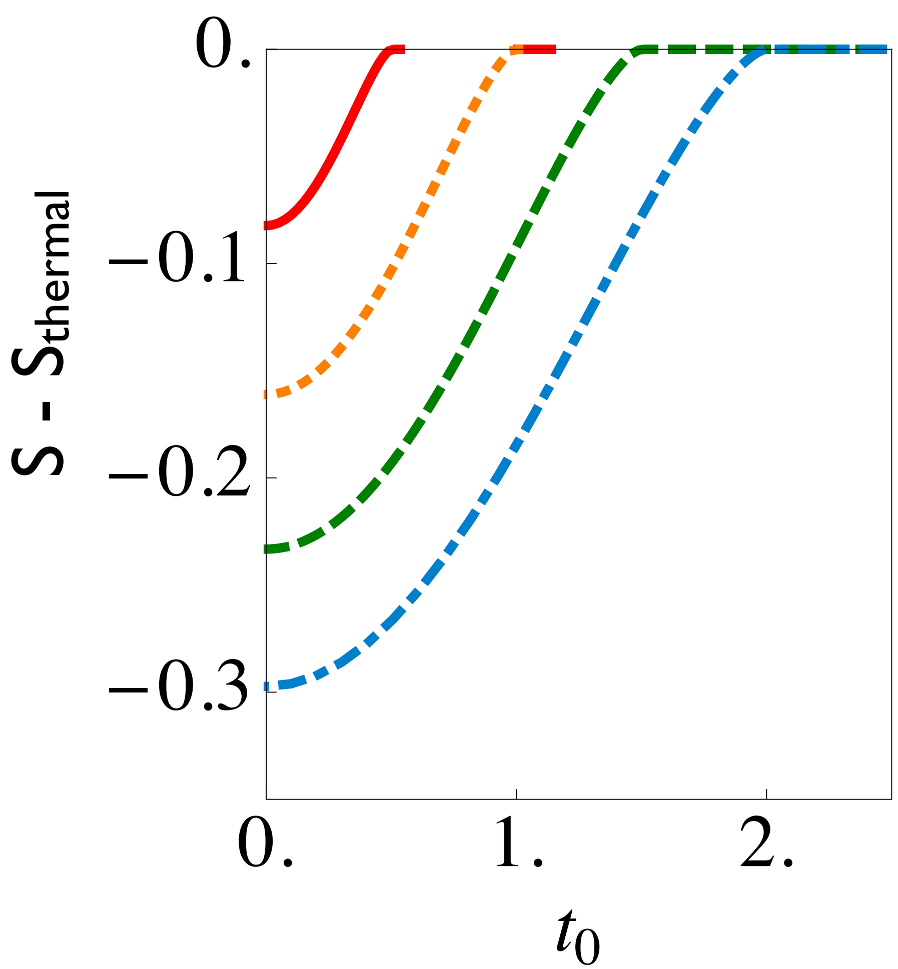
Quantum entanglement, thermalization, and chaos

Information and thermalization



- Colliding heavy ions and black holes seem to *thermalize*, so that any subsystem is randomly organized with maximum entropy.
- How can isolated systems thermalize when physics specifies deterministic evolution?
- *Information perspective*: each sub-system becomes maximally entangled with everything else.
- If we observe only the sub-system it has entanglement entropy and is statistically distributed.

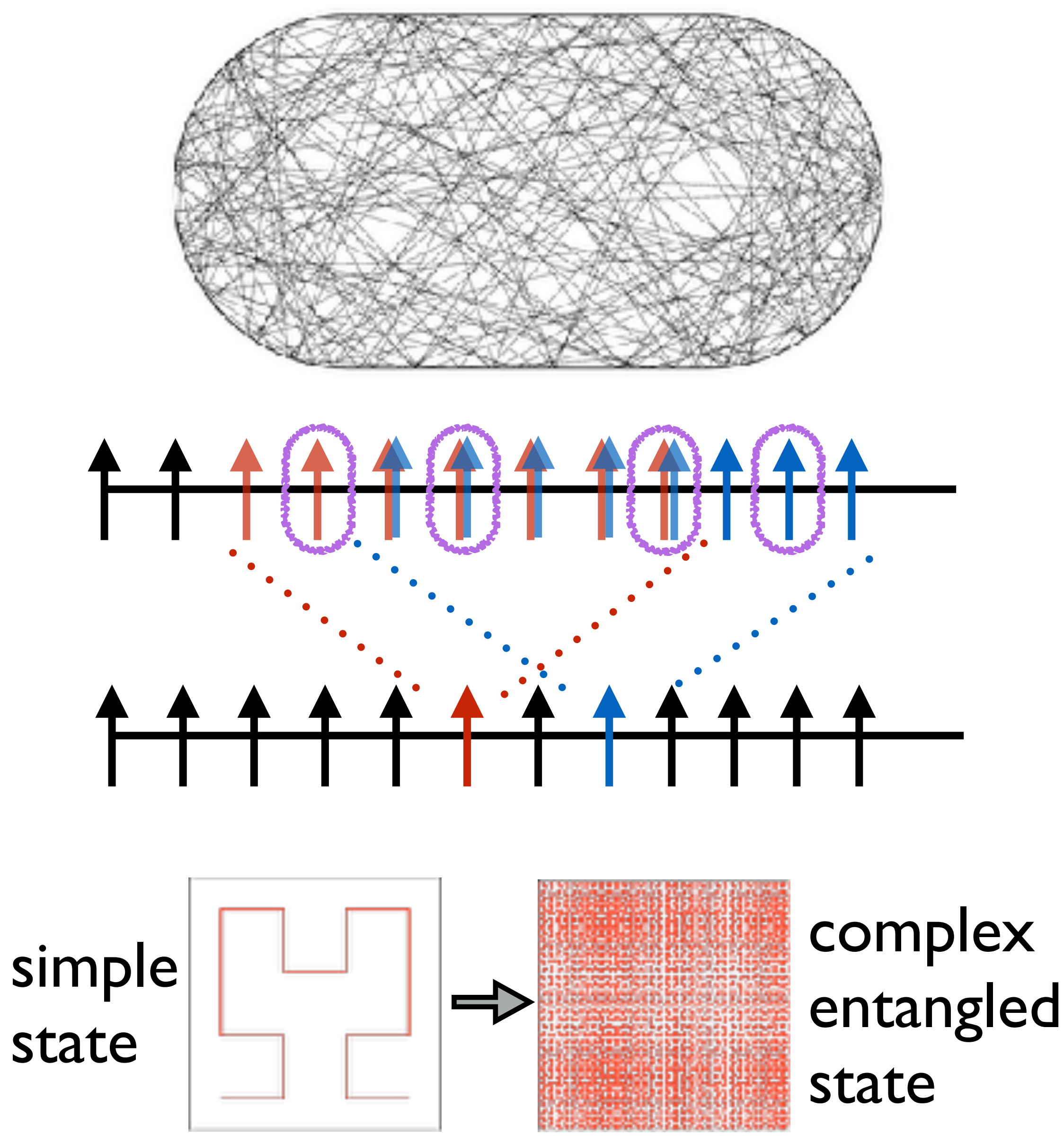
Information spread during thermalization



von Neumann entropy of intervals of different length after a quantum quench in a two-dimensional conformal field theory

- Inject energy uniformly into the ground state of a field theory
- After a while the system thermalizes
- Track the von Neumann (entanglement) entropy of a subsytem
- The entropy grows and sharply reaches the expected thermal value

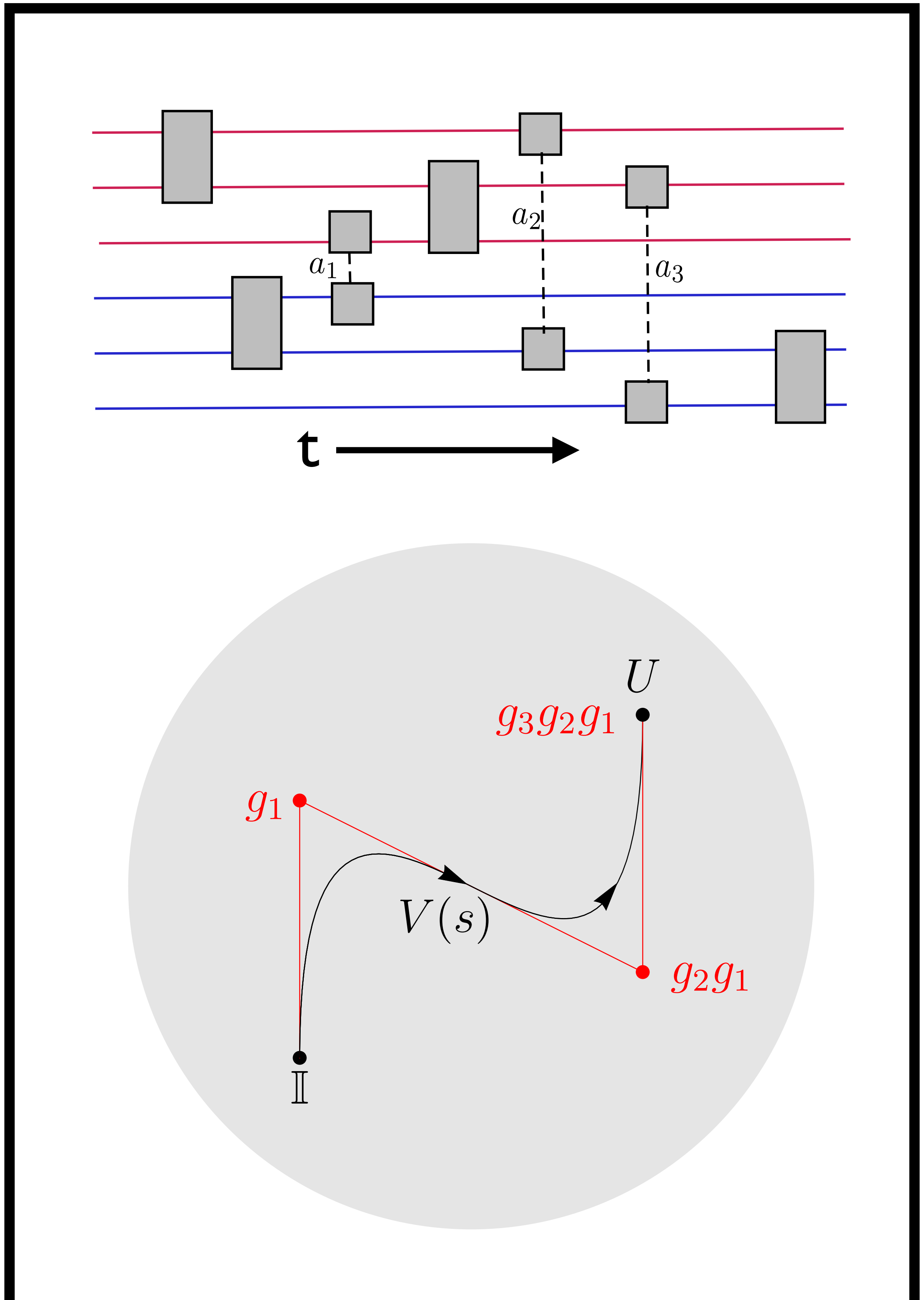
Information, complexity and chaos



measuring complexity:
(a) **number** of operations?
(b) **geometry** of functions?

- Some thermalizing systems (e.g. heavy ion collisions & possibly black holes) thermalize at speeds approaching a physical bound, and may be maximally chaotic.
- Classical chaos = extreme sensitivity to initial conditions in *nonlinear* dynamics
- What is quantum chaos? Quantum mechanics is *linear* in the state! Random energy spectrum.
- An information perspective:
  - multiparty information over time
  - complexity of states

Information, complexity and chaos

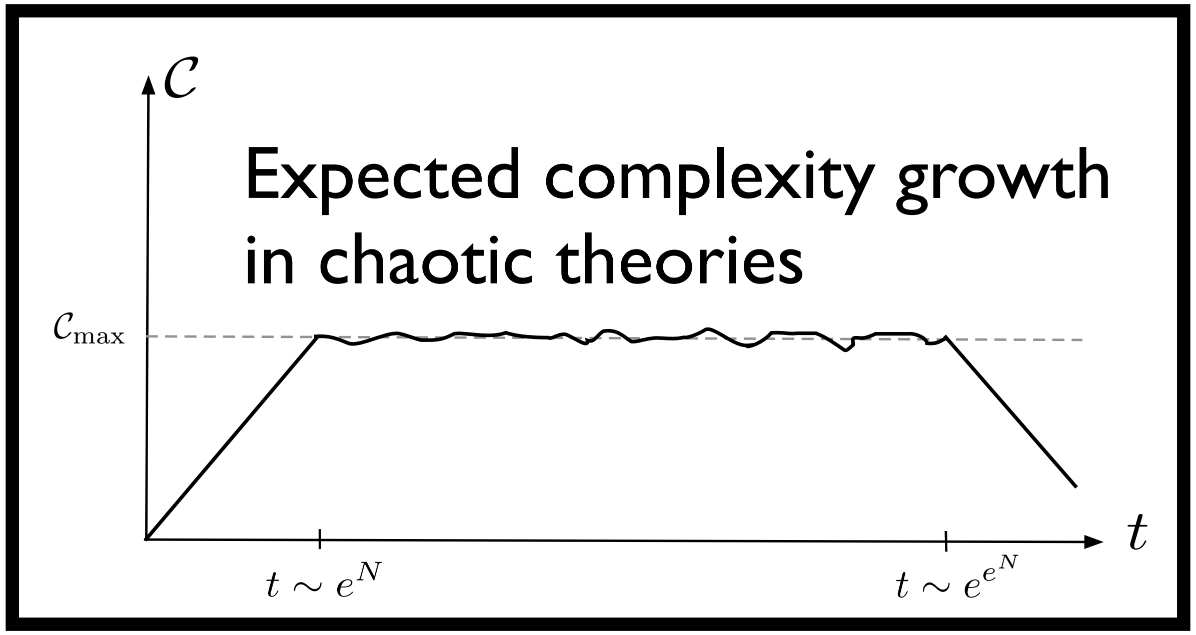


- Quantifying complexity: how "hard" is it to construct the time evolution from "easy" gates

$$U(t) = e^{-i \int dt\, H(t)} = g_1 g_2 g_3$$

- Continuous version: find the length of the shortest geodesic in the unitary group manifold between the identity and U(t), with a metric that is small in the "easy" (local) directions and big in the "hard" (nonlocal) directions.

Information, complexity and chaos



- **Chaotic theories**: expect linear growth of complexity for exponential time
- **Integrable theories**: expect oscillation of complexity in polynomial time

Eigenstate complexity hypothesis

$$R_{mn} = \frac{\sum_\alpha |\langle m|T_\alpha|n\rangle|^2}{\sum_\alpha |\langle m|T_\alpha|n\rangle|^2 + \sum_{\dot\alpha} |\langle m|T_{\dot\alpha}|n\rangle|^2}$$

T_α local Lie algebra generators

$T_{\dot\alpha}$ nonlocal Lie algebra generators

$|m\rangle$ energy eigenstates

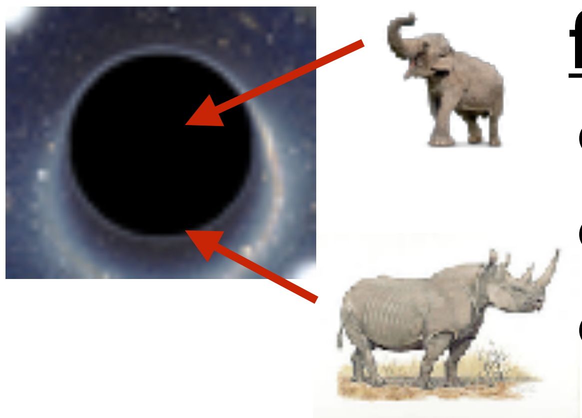
The Hamiltonian and the gate set satisfy the Eigenstate Complexity Hypothesis (ECH) if $R_{mn} = e^{-2S}\mathrm{poly}(S) r_{mn}$ for any m and n with $E_m \neq E_n$ and S = log(dimension of Hilbert space), r_{mn} = O(1)

If a theory satisfies ECH, can prove that complexity grows linearly for exp. time.

Hypothesis: all chaotic theories satisfy ECH because of nonlocal, multiparty entanglement in the energy eigenstates.

Quantum entanglement and spacetime architecture

Black holes



formation

- mass
- charge
- angular mom.

evaporation

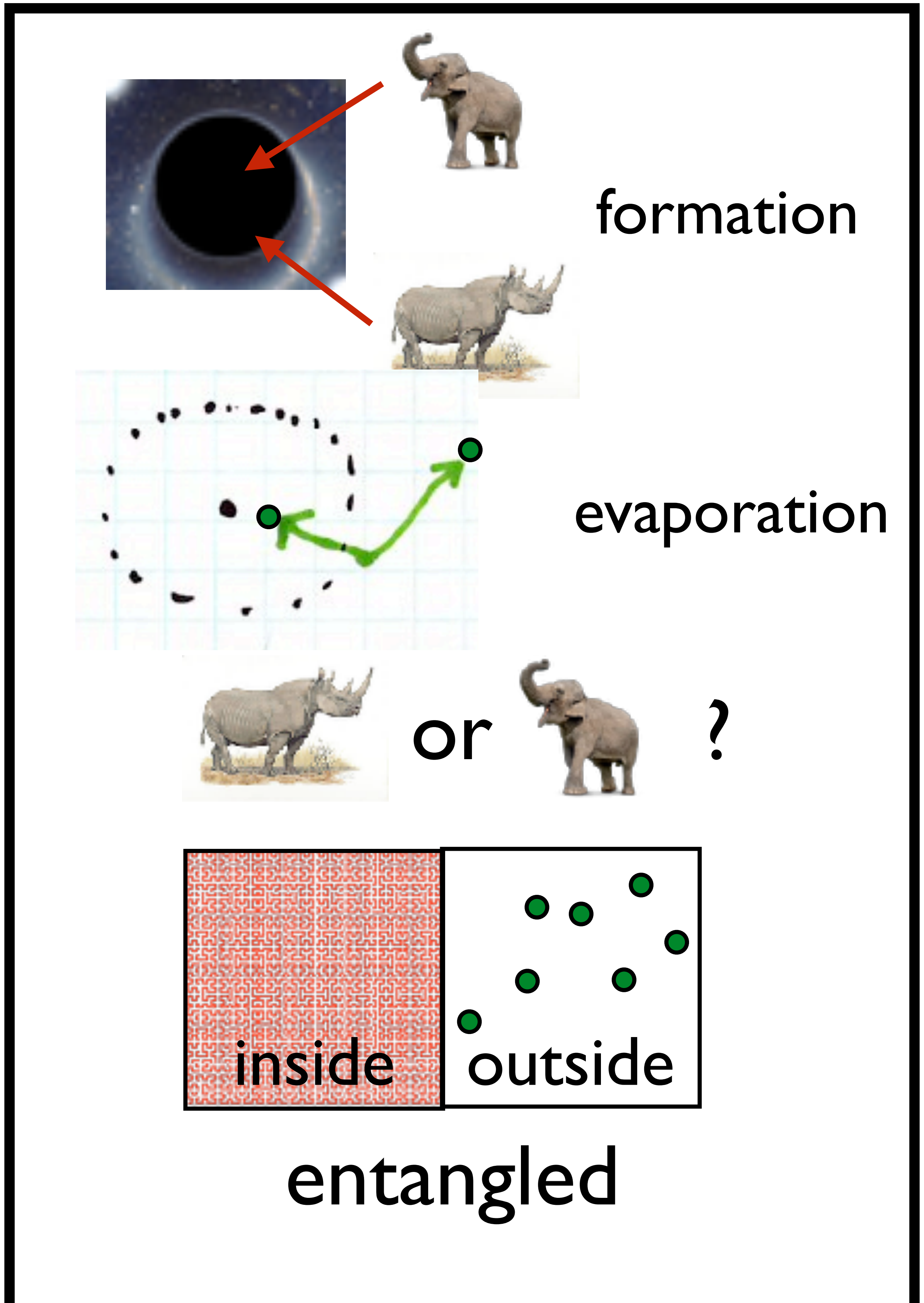
quantum pair production

BLACK HOLES	THERMODYNAMICS
$\Delta M = \kappa \Delta A + \cdots$	$\Delta E = T \Delta S + \cdots$
$\Delta A \geq 0$	$\Delta S \geq 0$
κ = CONST ON HORIZON	T = CONST. IN EQUILIBRIUM
(κ = surface gravity)	

horizon area = entropy
surface gravity = temperature

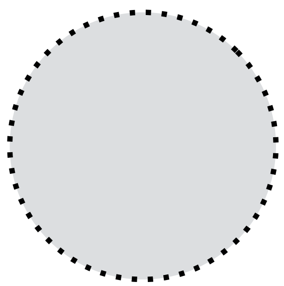
- Classically, things enter black holes horizons and never leave
- But black holes evaporate away due to quantum mechanics.
- The radiation looks thermal (totally random) = destruction of information. PARADOX!
- *What are the quantum microstates that give rise to the entropy?*
- *How do we recover information about the microstate?*

Information recovery from black holes



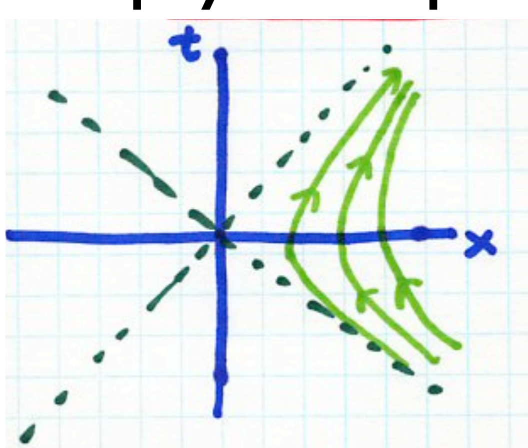
- IDEA: The emitted radiation and the internal microstates are quantum mechanically entangled.
- So, measuring the radiation gives you information about the microstate. The general theory of quantum communication then predicts:
  - the identity of the microstate is concealed until the half-way point of evaporation
  - after that the information is recovered very rapidly

Does gravity geometrize information?



$$S = \frac{A}{4G_N\hbar}$$

Empty flat space



accelerated observers see a horizon and Area ~ Entropy

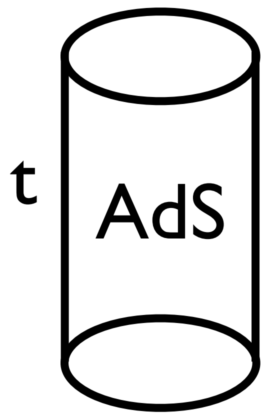
Unruh radiation: the analog of for accelerated observers of Hawking radiation

- Horizon area ~ entropy. Why?
- In many theories, entanglement entropy of a region is proportional to the area of the boundary
- Is horizon area = inside/outside entanglement entropy?
- Need:
  - enough microstates from quantum gravity/string theory
  - a mechanism for entanglement

A dream

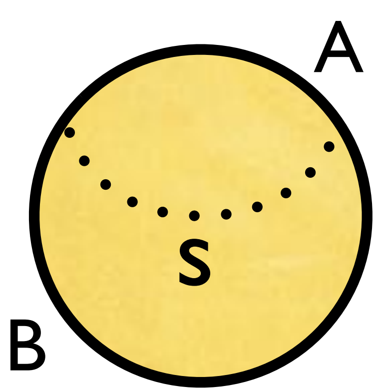
All of geometry & gravity from information?

Geometry = Information?



Anti-de Sitter space

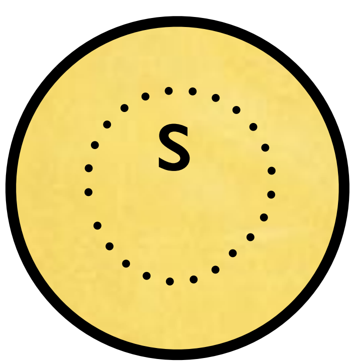
Quantum gravity in AdS
=
Boundary "field theory"



Equal time slice

s = minimal surface

$$S_A = \frac{\text{Area}(s)}{4G_N\hbar}$$

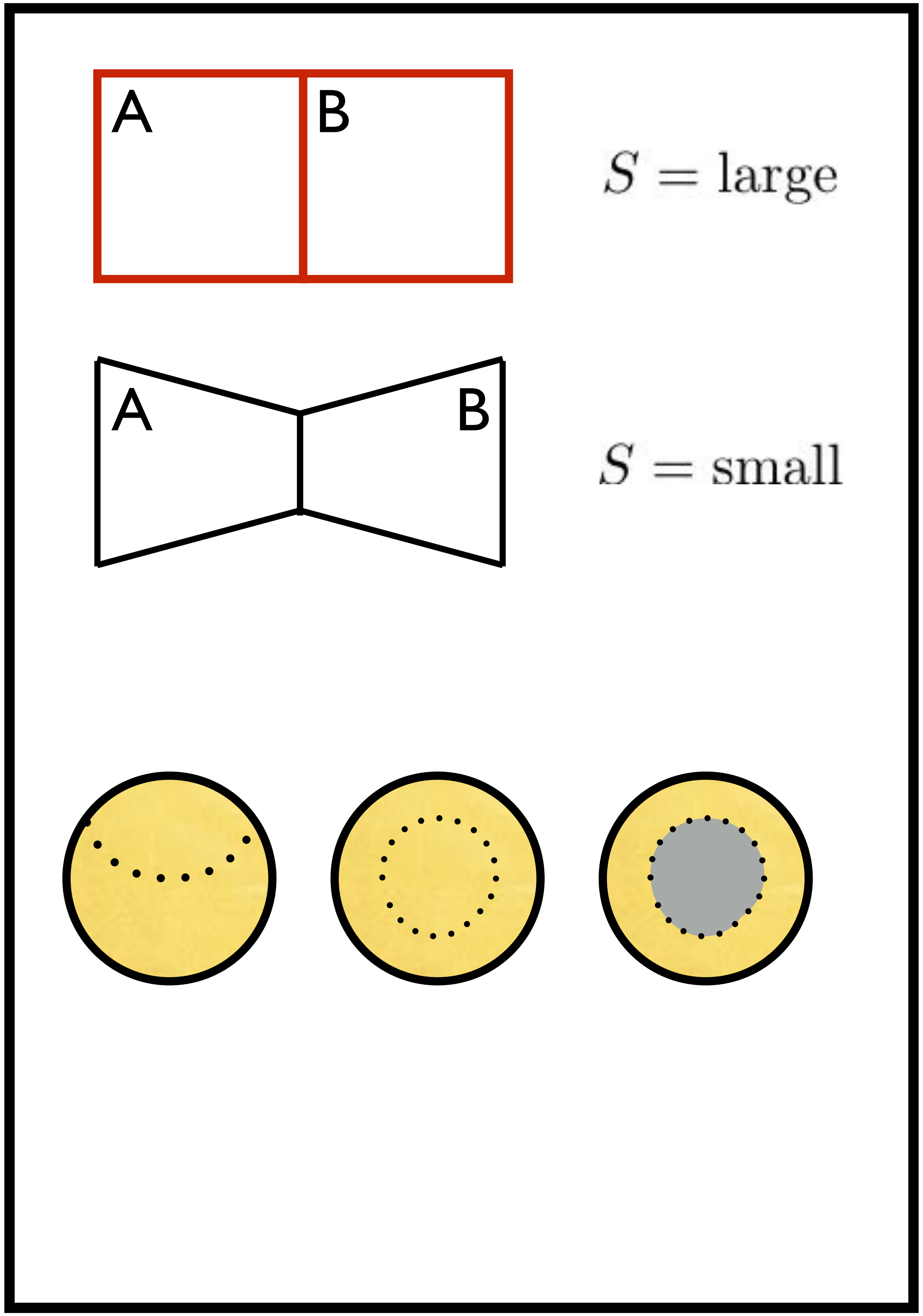


s = general surface

- *Toy model*: gravity in universes with negative curvature (negative dark energy) = "AdS space"
- Area of minimal surfaces in AdS = entanglement entropy of subtended region in the boundary
- Area of general surfaces AdS ~ differential entropy in boundary (macro-micro entanglement)
- First law of entanglement = Einstein's equation in an order by order expansion

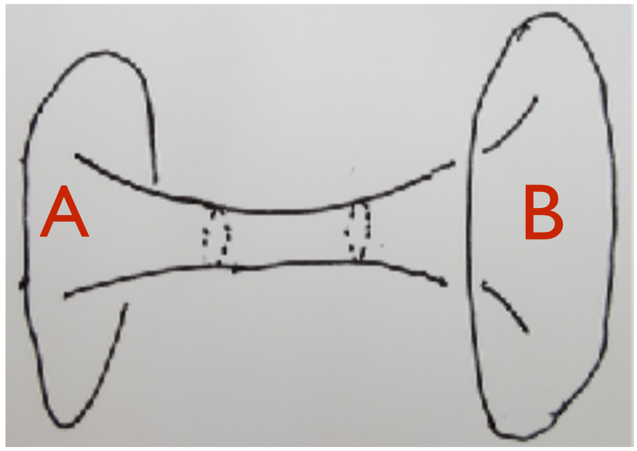
Is spacetime emergent from information?

Information knits spacetime: It from bit?

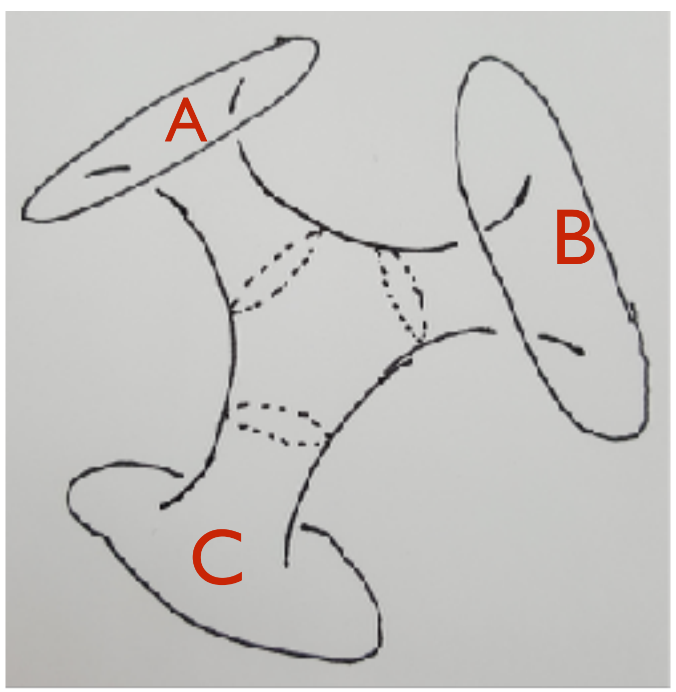


- Two regions of space A and B are connected if they are entangled
- The area of the boundary between A & B is related to their entanglement entropy
- Evidence in AdS space: many examples where increasing/decreasing entanglement between subregions increases/decreases area of the interface

Entanglement and wormholes



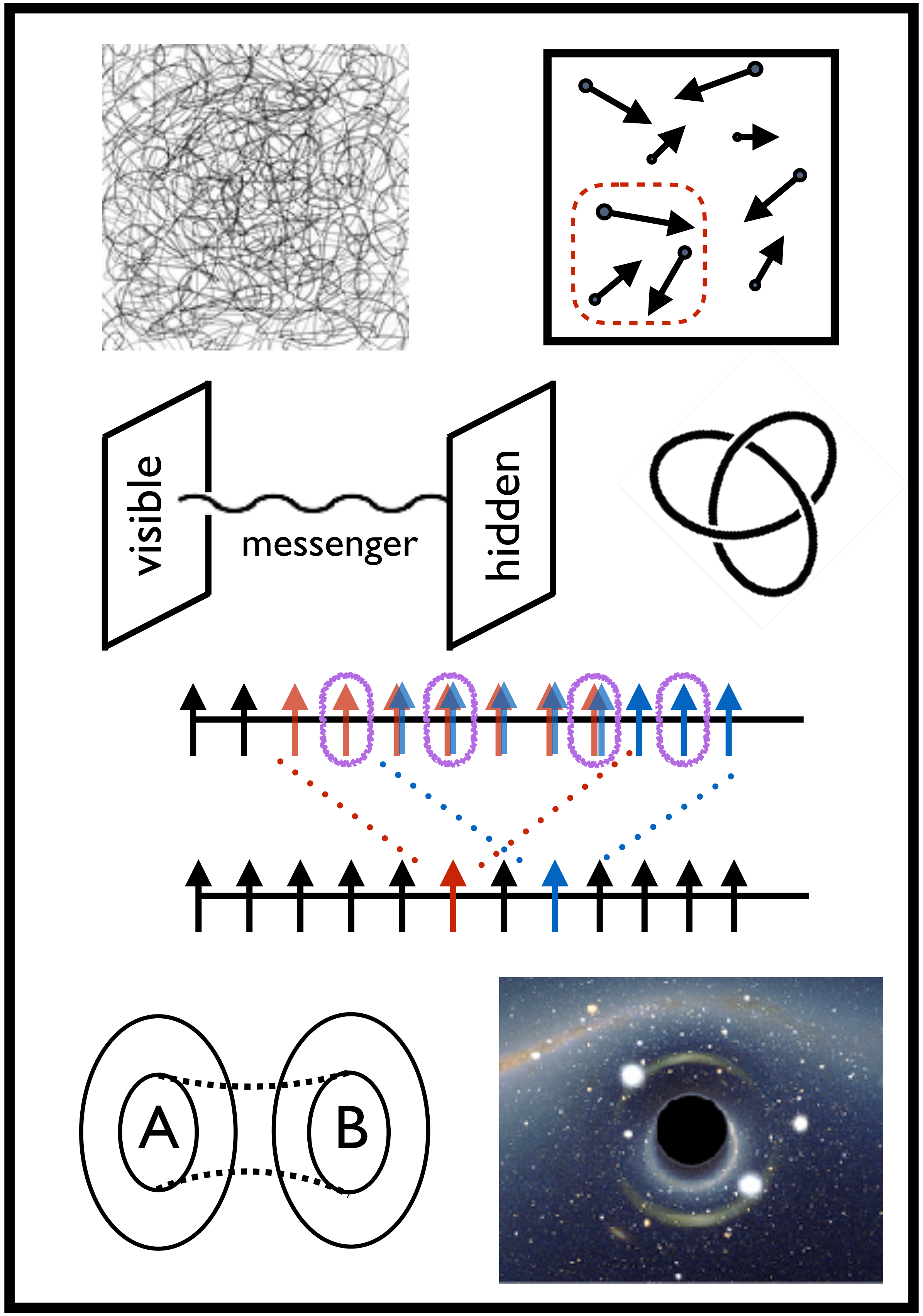
$$|\psi\rangle = \sum_{a,b} c_{ab}|a\rangle \otimes |b\rangle$$



$$|\psi\rangle = \sum_{a,b,c} c_{abc}|a\rangle \otimes |b\rangle \otimes |c\rangle$$

- **Test**: entanglement between distant regions A & B should create a wormhole.
- Examples in the AdS/CFT correspondence: entangling distinct boundary field theories produces wormholes in the corresponding gravity description
- *So maybe spacetime connectedness = entanglement of the underlying quantum "atoms of spacetime"*

Many questions to think about



- How to measure entanglement/ information across time?
- How to characterize information shared by many parties?
- How to measure the complexity of chaotic states?
- Is there a topology of entanglement?
- Can entanglement be used to probe microscopic, hidden physics?
- Does entanglement rescue information from black holes?
- Does entanglement create wormholes/spacetime connection?
- Does It come from Bit?

The End