Quantum Anomalies
and
Hawking radiation

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1. Introduction

§1.1 Black holes

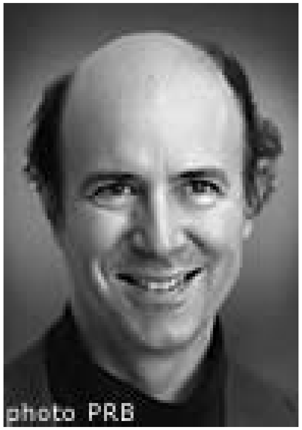
A brief introduction: Concept and Properties

§1.2 Hawking radiation

(a) Radiant Mechanism: quantum tunnelling; vacuum fluctuation

(b) Derivation Method: so many approaches, such as, gravitational collapse, path integral, Euclidean action integral, Euclidean periodic extension, temperature Green function, renormalization EMT, second quantization, density matrix, Bogoliubov transformation, **trace (con-formal) anomaly**, conformal flat, **DRS**, etc.

Two recent attracted ones due to F. Wilczek: **tunnelling picture** and **anomaly cancellation**
A few words about Frank Wilczek



Frank Wilczek, 2004 Nobel Prize Owner

Phys.Rev.Lett. **56** papers

[**1** (2000) +**1** (1000) +**2** (500) +**7** (250) +**12** (100)
+**10** (50) +19(10)+3(1)+1(0)] (**2007.10.23**)

Hawking radiation from charged black holes via gauge and gravitational anomalies
S. Iso, H. Umetsu, **F. Wilczek**, *PRL* **96** (2006) 151302 Cited **37**

A Relationship between Hawking radiation and gravitational anomalies
S.P. Robinson, **F. Wilczek**, *PRL* **95** (2005) 011303 Cited **46**

Hawking radiation as tunneling
M.K. Parikh, **F. Wilczek**, *PRL* **85** (2000) 5042 Cited **203**

Problem of Strong p and t Invariance in the Presence of Instantons
F. Wilczek, *PRL* **40** (1978) 279 Cited **1283**

Ultraviolet Behavior of Nonabelian Gauge Theories
D.J. Gross, **F. Wilczek**, *PRL* **30** (1973) 1343 Cited **2324**

2. Anomaly

§2.1 Definitions:

Classically conserved law is violated in quantum mechanism version.
QFT: quantum field theory
Anomaly must be cancelled by introducing a new mechanism—new physics—related to symmetry: **CFT** ?!

§2.2 Examples:

Gauge anomaly: $\nabla_\mu J^\mu \neq 0$
Gravitational anomaly: $\nabla_\mu T^{\mu\nu} \neq 0$
Conformal (trace) anomaly: $T^\mu_\mu \neq 0$
Chiral anomaly: $\nabla_\mu J^\mu_5 \neq 0$
Axial-vector anomaly, triangle anomaly, etc...

3. Anomaly cancellation

§3.1 Introduction

Dimensional reduction—2-dimensional effective theory—omitting classically irrelevant modes—effective theory becomes chiral in the near-horizon region (Brick-Wall model)—anomalies (appearance and cancellation)—d=2 thermal radiant flux—Hawking radiation reproduction

Static BHs:

S.P. Robinson and F. Wilczek, PRL **95** (2005) 011303, gr-qc/0502074

S. Iso, H. Umetsu and F. Wilczek, PRL **96** (2006) 151302, hep-th/0602146

Rotating BHs:

S. Iso S, H. Umetsu and F. Wilczek, PRD **74** (2006) 044017, hep-th/0606018

K. Murata and J. Soda, PRD **74** (2006) 044018, hep-th/0606069

S. Iso S, H. Morita and H. Umetsu, JHEP **04** (2007) 068, hep-th/0612286

§3.2 Dimensional reduction

A most general, static and spherically symmetric black hole:

$$\begin{aligned} ds^2 &= -f(r)dt^2 + h(r)^{-1}dr^2 + P(r)^2 d\Omega^2 \\ A &= A_t dt = \frac{q}{r} dt \end{aligned}$$

Assume: $f(r_+) = 0$, $h(r_+) = 0$

Surface gravity: $\kappa = \frac{1}{2}\sqrt{f_{,r} h_{,r}}\Big|_{r_+}$

Dimensional reduction—massless scalar field (or massive complex scalar field with a mass term and a minimal electro-magnetic coupling interaction, Dirac, Maxwell fields)—partial wave decomposition: $\phi = \sum_{lm} \phi_{lm}(t, r) Y_{lm}(\theta, \varphi)$

$$\begin{aligned}
S[\phi] &= -\frac{1}{2}\int d^4x\sqrt{-g_{(4)}}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = \frac{1}{2}\int d^4x\sqrt{-g_{(4)}}\phi\Box\phi \\
&= \frac{1}{2}\int dtdrd\theta d\varphi\ P^2\sin\theta\sqrt{-g}\phi\Big\{-\frac{1}{f}\partial_t^2 + h\partial_r^2 \\
&\quad + \Big[\frac{(fh)_{,r}}{2f} + \frac{2h}{P}P_{,r}\Big]\partial_r + \frac{1}{P^2}\Delta_\Omega\Big\}\phi \\
&= \frac{1}{2}\sum_{lm}\int dtdr\ P^2\sqrt{-g}\phi_{lm}\Big\{-\frac{1}{f}\partial_t^2 + h\partial_r^2 \\
&\quad + \Big[\frac{(fh)_{,r}}{2f} + \frac{2h}{P}P_{,r}\Big]\partial_r - \frac{l(l+1)}{P^2}\Big\}\phi_{lm} \\
&\simeq \frac{1}{2}\sum_{lm}\int dtdr\ P^2\sqrt{-g}\phi_{lm}\Big[-\frac{1}{f}\partial_t^2 + h\partial_r^2 + \frac{(fh)_{,r}}{2f}\partial_r\Big]\phi_{lm}
\end{aligned}$$

where $\sqrt{-g} = \sqrt{f/h}$, [Tortoise: $r_* = \int dr/\sqrt{fh}$].

Physics near the horizon can be described by an infinite collection of massless fields in the $(1+1)$-dimensional effective theory, each partial wave propagating in a space-time with a metric given by the "$r-t$" section of the full space-time metric and the dilaton field $P(r)^2$.

Metric ansätz: the $(1+1)$-dimensional effective metric and the gauge potential:

$$
\begin{aligned}
ds^2 &= -f(r)dt^2 + h(r)^{-1}dr^2 \\
A_t &= \frac{q}{r}
\end{aligned}
$$

with the dilaton field $P(r)^2$.

A scalar field in the original (3+1)-dimensional background can be effectively described by an infinite collection of massless fields in the $(1+1)$-dimensional background space-time with the effective metric and the gauge potential, together with the dilaton field $\Psi = P(r)^2$.

When reducing to d=2, a factor $\Psi = P(r)^2$ in the Lagrangian can be interpreted as a dilaton background coupled to the charged fields. The contribution to the anomaly from the dilaton field can be **neglected** due to a static background.

The non-vanishing Christoffel symbols and the Ricci scalar:

$$\Gamma^t_{tr} = \Gamma^t_{rt} = \frac{f_{,r}}{2f}, \quad \Gamma^r_{tt} = \frac{hf_{,r}}{2}, \quad \Gamma^r_{rr} = -\frac{h_{,r}}{2h}$$

$$R = -\frac{hf_{,rr}}{f} - \frac{f_{,r}h_{,r}}{2f} + \frac{hf^2_{,r}}{2f^2}$$

§3.3 Gauge anomaly

Hawking effect—near-horizon region—horizon: one-way membrane—modes interior to the horizon can not affect physics outside the horizon, *classically* $\longrightarrow$ **only** consider the physics outside the horizon, and define the effective theory in the outer region: $[r_+, \ +\infty] \equiv [r_+, \ r_+ + \varepsilon] \cup [r_+ + \varepsilon, \ +\infty]$

$[r_+, \ +\infty]$	effective theory	anomaly	laws
$[r_+, \ r_+ + \varepsilon]$	chiral	Yes	$\times$
$[r_+ + \varepsilon, \ +\infty]$	non-chiral	No	conserved

In the near-horizon region—there are **only** outgoing modes—if neglecting quantum effects of (classically irrelevant) ingoing modes—effective theory becomes **chiral**—gauge and gravitational **anomalies**—invariance of the underlying theory under gauge and diffeomorphism symmetries—these anomalies must be **cancelled** by quantum effects of the classically irrelevant modes—anomaly cancellation at the horizon—Hawking flux of charge and energy momentum—Hawking temperature.

Conditions:
(1) anomaly cancellation;
(2) regularity requirement.

Consistent anomaly:

$$\nabla_\mu J^\mu = \frac{-e^2}{4\pi\sqrt{-g}}\epsilon^{\mu\nu}\partial_\mu A_\nu$$

Covariant anomaly:

$$\nabla_\mu \tilde{J}^\mu = \frac{-e^2}{4\pi\sqrt{-g}}\epsilon^{\mu\nu}F_{\mu\nu}$$

minus $(-)$: outgoing (right-handed) fields; $\varepsilon^{tr} = 1$; J^μ: non-covariant; Coefficient: $\tilde{J}^\mu \simeq 2J^\mu$;
Covariant versus **Consistent**:

$$\tilde{J}^\mu = J^\mu + \frac{e^2}{4\pi\sqrt{-g}} A_\lambda \epsilon^{\lambda\mu}$$

Non-vanishing r-component:

$$\tilde{J}^r = J^r + \frac{e^2}{4\pi\sqrt{-g}} A_t(r) H(r)$$

Region $[r_+ + \varepsilon, +\infty]$: $J^\mu_{(O)}$ is conserved;
Region $[r_+, r_+ + \varepsilon]$: $J^\mu_{(H)}$ satisfies the anomaly equation:

$$\begin{aligned} \nabla_\mu J^\mu_{(O)} &= 0 \\ \nabla_\mu J^\mu_{(H)} &= \frac{-e^2}{4\pi\sqrt{-g}} \epsilon^{\mu\nu} \partial_\mu A_\nu \end{aligned}$$

namely

$$\begin{aligned} \partial_\mu \left[\sqrt{-g} J^\mu_{(O)}\right] &= 0\,, \\ \partial_\mu \left[\sqrt{-g} J^\mu_{(H)}\right] &= \frac{-e^2}{4\pi} \epsilon^{\mu\nu} \partial_\mu A_\nu \end{aligned}$$

$$\partial_r\left[\sqrt{-g}J^r_{(O)}\right] = 0$$

$$\partial_r\left[\sqrt{-g}J^r_{(H)}\right] = \frac{e^2}{4\pi}\partial_r A_t$$

Solutions:

$$\sqrt{-g}J^r_{(O)} = c_O$$

$$\sqrt{-g}J^r_{(H)} = c_H + \frac{e^2}{4\pi}\left[A_t(r) - A_t(r_+)\right]$$

$$= c_O + \frac{e^2}{4\pi}A_t(r)$$

c_O, c_H: integration constants.

Total current outside the horizon:

$$J^\mu = J^\mu_{(O)}\Theta(r) + J^\mu_{(H)}H(r)$$

$\Theta(r) = \Theta(r - r_+ - \varepsilon)$: scalar step

$H(r) = 1 - \Theta(r)$: scalar top hat

Variation of the effective action under gauge transformations: (λ: gauge parameter)

$$
\begin{aligned}
-\delta_\lambda W &= \int dtdr\sqrt{-g}\lambda\nabla_\mu J^\mu \\
&= \int dtdr\lambda\Big\{\partial_r\Big(\frac{e^2}{4\pi}A_t H\Big) + \Big[\frac{e^2}{4\pi}A_t \\
&\quad +\sqrt{-g}\Big(J^r_{(O)} - J^r_{(H)}\Big)\Big]\delta(r - r_+ - \epsilon)\Big\}
\end{aligned}
$$

The first term should be cancelled by quantum effects of the classically irrelevant ingoing modes. $[\partial_\mu\Theta(r) = \delta^r_\mu\delta(r)]$

Gauge invariance of the total effective action $\delta_\lambda W = 0$:

$$
\sqrt{-g}\Big[J^r_{(O)} - J^r_{(H)}\Big](r_+) + \frac{e^2}{4\pi}A_t(r_+) = 0
$$

$$
\Longrightarrow c_O = c_H - \frac{e^2}{4\pi}A_t(r_+)
$$

c_H: the value of the consistent current at the horizon.

To determine the current flow—to fix the value of the current at the horizon—condition: gauge covariant—impose: the coefficient of the *co-variant* current at the horizon should vanish: $\widetilde{J}^r_{(H)} = 0 \longmapsto$ **Regularity requirement**

$$\widetilde{J}^r_{(H)} = J^r_{(H)} + e^2 A_t(r)/(4\pi\sqrt{-g})$$

$$c_H = -\frac{e^2}{4\pi} A_t(r_+)$$
$$c_O = -\frac{e^2}{2\pi} A_t(r_+) = -\frac{e^2 q}{2\pi r_+}$$

This agrees with the current flow associated with the Hawking thermal (blackbody) radiation including a chemical potential.

§3.3 Gravitational anomaly

Consistent anomaly:

$$\nabla_\mu T^\mu_{\ \nu} = \frac{1}{96\pi\sqrt{-g}}\epsilon^{\beta\delta}\partial_\delta\partial_\alpha\Gamma^\alpha_{\nu\beta} \equiv \mathcal{A}_\nu = \frac{1}{\sqrt{-g}}\partial_\mu N^\mu_{\ \nu}$$

Covariant anomaly:

$$\nabla_\mu \widetilde{T}^\mu_{\ \nu} = \frac{-1}{96\pi\sqrt{-g}}\epsilon_{\mu\nu}\partial^\mu R \equiv \widetilde{\mathcal{A}}_\nu = \frac{1}{\sqrt{-g}}\partial_\mu \widetilde{N}^\mu_{\ \nu}$$

(1) First ignore the electro-magnetic interaction and only concentrate on the pure gravitational anomaly.

Region $[r_+ + \varepsilon, +\infty]$: $T^\mu_{(O)\nu}$ is covariantly conserved; Region $[r_+, r_+ + \varepsilon]$: $T^\mu_{(H)\nu}$ obeys the anomalous equation:

$$\begin{aligned} \nabla_\mu T^\mu_{(O)\nu} &= 0 \\ \nabla_\mu T^\mu_{(H)\nu} &\equiv \mathcal{A}_\nu = \frac{1}{\sqrt{-g}}\partial_\mu N^\mu_{\ \nu} \end{aligned}$$

Region $[r_+ + \varepsilon, +\infty]$: $N^\mu_\nu = \mathcal{A}_\nu = 0$;
Region $[r_+, r_+ + \varepsilon]$: $N^r{}_r = N^t{}_t = 0$

$$
\begin{aligned}
N^r{}_t &= \frac{1}{96\pi}\partial_r \Gamma^r_{tt} = \frac{1}{192\pi}\left(f_{,r}h_{,r} + hf_{,rr}\right) \\
N^t{}_r &= \frac{-1}{96\pi}\partial_r \Gamma^r_{rr} = \frac{-1}{192\pi h^2}\left(h_{,r}{}^2 - hh_{,rr}\right)
\end{aligned}
$$

Consistent and covariant anomalies are purely time-like ($\mathcal{A}_r = \widetilde{\mathcal{A}}_r = 0$)

$$
\begin{aligned}
\sqrt{-g}\mathcal{A}_t &= \partial_r N^r{}_t = \partial_r\left[\sqrt{-g}T^r{}_t\right] = \frac{1}{96\pi}\partial_r^2 \Gamma^r_{tt} \\
&= \frac{1}{192\pi}\partial_r\left(hf_{,rr} + f_{,r}h_{,r}\right) \\
\sqrt{-g}\widetilde{\mathcal{A}}_t &= \partial_r \widetilde{N}^r{}_t = \partial_r\left[\sqrt{-g}\widetilde{T}^r{}_t\right] = \frac{-1}{96\pi} f\partial_r R \\
&= \frac{1}{96\pi}\partial_r\left(hf_{,rr} + \frac{f_{,r}h_{,r}}{2} - \frac{hf_{,r}^2}{f}\right)
\end{aligned}
$$

$\Longrightarrow$

$$
\begin{aligned}
N^r{}_t &= (hf_{,rr} + h_{,r}f_{,r})/(192\pi) \\
\widetilde{N}^r{}_t &= \left[hf_{,rr} + f_{,r}h_{,r}/2 - hf_{,r}^2/f\right]/(96\pi)
\end{aligned}
$$

Covariant versus **Consistent**:

$$\sqrt{-g}\widetilde{T}^r{}_t = \sqrt{-g}T^r{}_t + \frac{h}{192\pi f}\left(f f_{,rr} - 2f_{,r}^2\right)$$

(2) Now include the electro-magnetic interaction.

If there were **no** gravitational anomaly, the Ward identity is

$$\nabla_\mu T^\mu{}_\nu = F_{\mu\nu}J^\mu + A_\nu \nabla_\mu J^\mu$$

Adding the gravitational anomaly, the Ward identity becomes

$$\nabla_\mu T^\mu{}_\nu = F_{\mu\nu}J^\mu + A_\nu \nabla_\mu J^\mu + \mathcal{A}_\nu = F_{\mu\nu}\widetilde{J}^\mu + \mathcal{A}_\nu$$

Region $[r_+ + \varepsilon, +\infty]$: $J^\mu_{(O)}$, $T^\mu_{(O)\nu}$ are conserved;
Region $[r_+, r_+ + \varepsilon]$: $J^\mu_{(H)}$, $T^\mu_{(H)\nu}$ obey the (modified) anomaly equations:

$$\begin{aligned}
\nabla_\mu T^\mu_{(O)\nu} &= F_{\mu\nu}J^\mu_{(O)} \\
\nabla_\mu T^\mu_{(H)\nu} &= F_{\mu\nu}J^\mu_{(H)} + A_\nu \nabla_\mu J^\mu_{(H)} + \mathcal{A}_\nu \\
&= F_{\mu\nu}\widetilde{J}^\mu_{(H)} + \mathcal{A}_\nu
\end{aligned}$$

Solve $T^r_{(O)t}$:

$$\partial_r\left[\sqrt{-g}T^r_{(O)t}\right] = \sqrt{-g}F_{rt}J^r_{(O)} = c_O\partial_r A_t$$

$F_{rt} = \partial_r A_t$, $T^r{}_t = -fhT^t{}_r$, $\sqrt{-g}J^r_{(O)} = c_O$

$$\begin{aligned}\sqrt{-g}T^r_{(O)t} &= a_O + c_O A_t(r) \\ &= a_O - \frac{e^2}{2\pi}A_t(r_+)A_t(r)\end{aligned}$$

a_O: an integration constant.

Solve $T^r_{(H)t}$:

$$\begin{aligned}\partial_r\left[\sqrt{-g}T^r_{(H)t}\right] &= \sqrt{-g}\left[F_{rt}J^r_{(H)} + A_t\nabla_\mu J^\mu_{(H)}\right] + \partial_r N^r{}_t \\ &= \sqrt{-g}F_{rt}\tilde{J}^r_{(H)} + \partial_r N^r{}_t\end{aligned}$$

namely: $[\sqrt{-g}\tilde{J}^r_{(H)} = c_O + e^2 A_t(r)/(2\pi)]$

$$\begin{aligned}\partial_r\left[\sqrt{-g}T^r_{(H)t}\right] &= \partial_r N^r{}_t + \sqrt{-g}J^r_{(H)}\partial_r A_t \\ &\quad + A_t\partial_r\left[\sqrt{-g}J^r_{(H)}\right] \\ &= \sqrt{-g}\tilde{J}^r_{(H)}\partial_r A_t + \partial_r N^r{}_t\end{aligned}$$

$$\sqrt{-g}T^r_{(H)t} = a_H + \int_{r_+}^{r} dr\partial_r\left(c_O A_t + \frac{e^2}{4\pi}A_t^2 + N^r{}_t\right)$$

Total energy momentum tensor outside the horizon:

$$T^\mu_\nu = T^\mu_{(O)\nu}\Theta(r) + T^\mu_{(H)\nu}H(r)$$

Under the infinitesimal general coordinate transformation, the effective action varies as

$$\begin{aligned}
-\delta_\xi W &= \int dtdr\sqrt{-g}\ \xi^\nu \nabla_\mu T^\mu_\nu \\
&= \int dtdr\ \xi^t\Big\{c_O\partial_r A_t + \partial_r\Big[\Big(\frac{e^2}{4\pi}A_t^2 + N^r{}_t\Big)H\Big] \\
&+ \Big[\sqrt{-g}\big(T^r_{(O)t} - T^r_{(H)t}\big) + N^r{}_t + \frac{e^2}{4\pi}A_t^2\Big]\delta(r - r_+ - \epsilon)\Big\} \\
&+ \int dtdr\xi^r\sqrt{-g}\big(T^r_{(O)r} - T^r_{(H)r}\big)\delta\big(r - r_+ - \epsilon\big)
\end{aligned}$$

The **1st term**: the classical effect of the background electric field for constant current flow; The **2nd term** should be cancelled by the quantum effect of the classically irrelevant ingoing modes.

To restore the diffeomorphism invariance, the variation of the effective action should vanish $\delta_\xi W = 0 \Longrightarrow$ constrains:

$$\sqrt{-g}\Big[T^r_{(O)t} - T^r_{(H)t}\Big](r_+) + N^r{}_t(r_+) + \frac{e^2}{4\pi}A_t^2(r_+) = 0$$

$\Longrightarrow$

$$\begin{aligned} a_O &= a_H + \frac{e^2}{4\pi}A_t^2(r_+) - N^r{}_t(r_+) \\ &= \frac{e^2}{4\pi}A_t^2(r_+) + N^r{}_t(r_+) \end{aligned}$$

To determine a_O—to fix the value of the energy momentum tensor at the horizon—impose: a vanishing condition for the *covariant* energy

momentum tensor at the horizon: $\widetilde{T}^r_{(H)t} = 0$
$\longmapsto$ **Additional regularity condition**

$$a_H = 2N^r{}_t(r_+) = \left.\frac{f_{,r}h_{,r}}{96\pi}\right|_{r=r_+} = \frac{\kappa^2}{24\pi}$$

The total flux of the energy momentum tensor:

$$a_O = \frac{e^2q^2}{4\pi r_+^2} + N^r{}_t(r_+) = \frac{e^2q^2}{4\pi r_+^2} + \frac{\kappa^2}{48\pi}$$

§3.3 Blackbody radiation

Uncharged case: $\sqrt{-g}T^r_{(O)t} = N^r{}_t(r_+) \Longleftrightarrow$ the energy momentum flux of Hawking radiation.

A $(1+1)$-dimensional black body radiation at temperature T has a flux of the form: $N^r{}_t(r_+) = (\pi/12)T^2$, accurately giving the Hawking temperature $T = \kappa/(2\pi)$.

Charged case: the blackbody radiation at a temperature $T = \kappa/(2\pi)$ with a chemical potential $\omega_0 = eA_t(r_+) = eq/r_+$—Planck distribution:

$$I^{(\pm)}(\omega) = \frac{1}{e^{2\pi(\omega\pm\omega_0)/\kappa} - 1} \qquad \text{bosons}$$

$$J^{(\pm)}(\omega) = \frac{1}{e^{2\pi(\omega\pm\omega_0)/\kappa} + 1} \qquad \text{fermions}$$

$I^{(-)}$, $J^{(-)}$: particles with charge $-e$.

To keep things simple—only consider the fermion case. The fluxes of charged current and energy momentum:

$$\sqrt{-g}J^r = \int_0^\infty e\frac{d\omega}{2\pi}\left[J^{(-)}(\omega) - J^{(+)}(\omega)\right]$$

$$= -\frac{e^2 q}{2\pi r_+}$$

$$\sqrt{-g}T^r{}_t = \int_0^\infty \omega\frac{d\omega}{2\pi}\left[J^{(-)}(\omega) + J^{(+)}(\omega)\right]$$

$$= \frac{e^2 q^2}{4\pi r_+^2} + \frac{\kappa^2}{48\pi}$$

The results derived from the **anomaly cancellation conditions** coincide with these results, showing that the required thermal flux is capable of cancelling the anomaly.

4. Concluding remarks

Conclusion: The compensating energy momentum flux and charged current flux required to cancel gravitational and gauge anomalies at the horizon are precisely equivalent to thermal fluxes associated with a $(1+1)$-dimensional blackbody radiation emanating from the horizon at the Hawking temperature.

Comments:

(1) Hawking radiation can be understood as a compensating flux to cancel anomaly at the horizon;

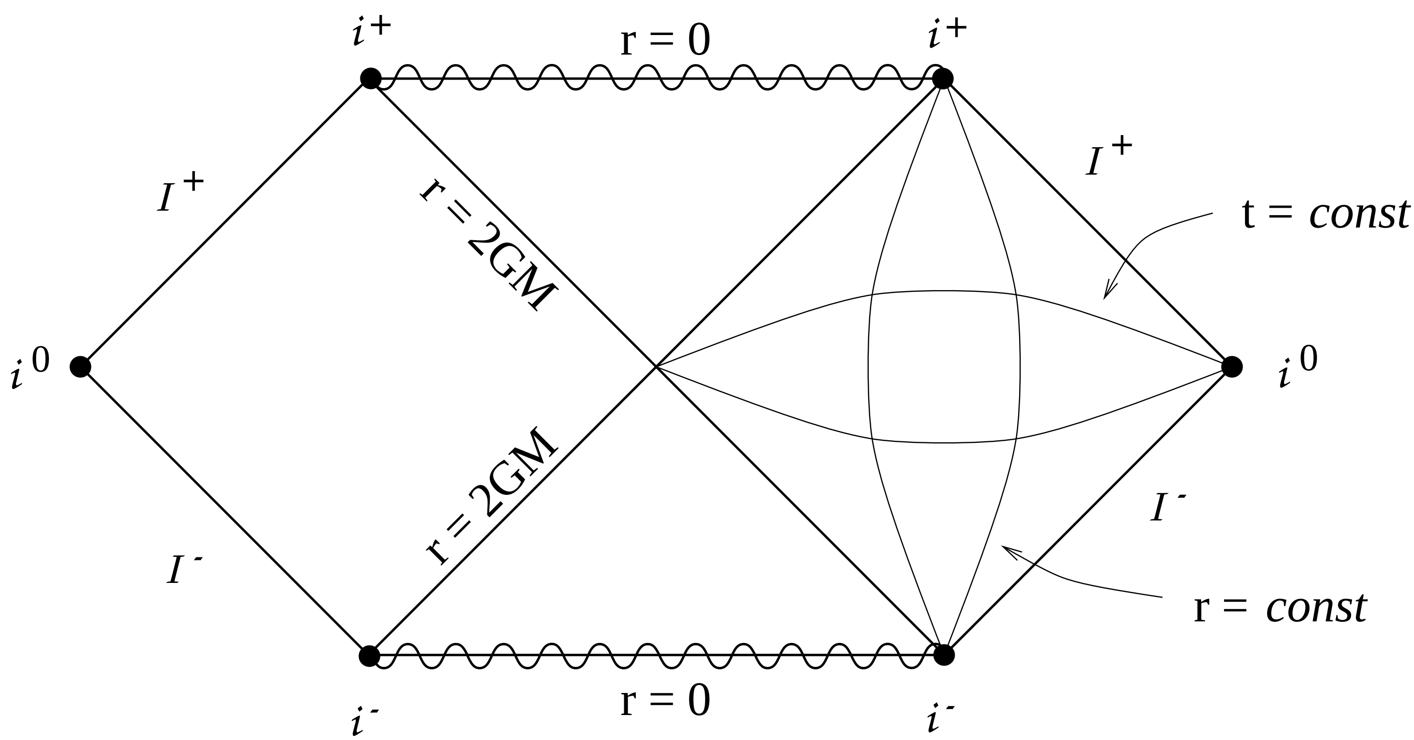
(2) Hawking radiation is a universal quantum phenomenon only related to the horizon;

(3) Anomaly cancellation method is universal, but can **not** determine the Bekenstein-Hawking entropy;

(4) Anomaly cancellation method is closely related to the properties of the horizon: the cancellation of anomaly takes place at the horizon; the regularity condition requires the covariant physical quantities to vanish at the horizon $\Longleftrightarrow$ Unruh vacuum

Three **different** definitions of vacuum: Boulware, Hartle-Hawking, and Unruh—**Penrose diagram**

(5) Symmetry: the near-horizon conformal symmetry (**CFT: horizon = boundary**) — holography ?!



i^+
r = 0
i^+
I^+
r = 2GM
I^+
t = *const*
i^0
i^0
r = 2GM
I^-
I^-
r = *const*
i^-
r = 0
i^-

5. Our related work

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Thank!