Time-dependent BPS configurations in String and M-theory

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1 Motivations

- **Singularity theorem: (Penrose, Hawking ... 1960s) Singularities could be ill defined in General Relativity. Especially, spacelike singularities are BAD: Black holes, big-bang (crunch) singularities;**

- **Very near singularities: quantum effects should be taken into account. The concept of geometry must be broken down. So what? Quantum foam, spin foam, ... It's a question for every kind of quantum gravity theory!**

- **The resolution of cosmic singularity? One of the central issues in String theory. Singularity should be resolved in string theory due to the extensive nature of string.**
  - **Orbifold singularity: resolved by perturbative D.O.F.**

    Dixon et.al., ...
  - **Conifold singularity: resolved by nonperturbative D.O.F**

    A. Strominger

- **What are the right degrees of freedom?**
  - **Perturbative description? Null orbifolds (suffer from the large blueshift)H. Liu et.al., Horowitz et.al.,... Milne universe Berkooz et.al., Nekrasov, ...**
  - **Closed string Tachyon phase? (poorly understood) E. Silverstein, ...**
  - **Pre-big-bang scenario? Veneziano et.al.**
  - **Nonperturbative description? Matrix cosmology!**

- **Matrix cosmology:** **C. Craps, Sethi and Verlinde (2005)**
  - **IIA flat spacetime with a linear null dilaton background;**
  - **1/2 BPS;**
  - **Perturbatively solvable;**
  - **Geodesics incomplete;**
  - **String coupling diverges near the big-bang singularity;**
  - **A dual Matrix string description (2-dim SYM on Milne orbifold);**
  - **1-loop effective potential?** **Miao Li and Wei Song, Craps et.al.**

- **Questions: other time-dependent configurations with SUSY**
  - **1/2 BPS configurations in M-theory?**

    Miao Li, Bin Chen, N. Ohta ...
  - **Other solvable BPS background in String theory?**

    S. Das et.al, B. Chen et.al.

– **Configurations with holographic description?**

Chu and Ho, S. Das et.al., F.L. Lin et.al.

Holographic description of BH singularity via AdS/CFT

Shenker et.al....

Emergent time?

2 Time-dependent BPS configurations in M-theory and Matrix models

Start from 11-dimensional supergravity

$$
\begin{aligned}
R_{MN} &= \frac{1}{12}(F_{MPQR}F_N{}^{PQR} - \frac{1}{12}g_{MN}F^2) \\
d * F &= \frac{1}{2}F \wedge F
\end{aligned}
$$

and the Killing spinor equations

$$\tilde{D}_M\epsilon = (D_M - \Omega_M)\epsilon$$

where D_M is the spin connection defined by

$$D_M = \partial_M + \frac{1}{4}\omega^{ab}\Gamma_{ab}$$

and

$$\Omega_M = \frac{1}{288} F_{PQRS}(\Gamma^{PQRS}{}_M + 8\Gamma^{PQR}\delta^S_M).$$

We would like to find the solutions to the equations of motions, which have at least sixteen standard supersymmetries, corresponding to the Killing spinors ϵ **satisfying** $\Gamma^+\epsilon = 0$**.**

- **Vacuum configurations: we make the following ansatz:**

$$ds^2 = 2A_0(u)dudv + A_i(u)(dx^i)^2,$$

where the $A_0(u), A_i(u), i = 1, \cdots 9$ **are the functions of** u**.**

An orthogonal frame

$$\theta^+ = \sqrt{A_0(u)}du, \quad \theta^- = \sqrt{A_0(u)}dv \quad \theta^I = \sqrt{A_i(u)}dx^i\delta_i^I.$$

The nonvanishing spin connections are

$$\omega^{-i} = -\frac{\partial_u\sqrt{A_i}}{\sqrt{A_0}}dx^i, \quad \omega^{-+} = -\frac{\partial_u\sqrt{A_0}}{\sqrt{A_0}}du$$

and the only non-zero Ricci tensor is

$$R_{uu} = \sum_i \frac{\sqrt{A_0}}{\sqrt{A_i}}\left(-\partial_u(\frac{\partial_u\sqrt{A_i}}{\sqrt{A_0}}) + \frac{\partial_u\sqrt{A_i}\partial_u\sqrt{A_0}}{A_0}\right).$$

Next let us check the remaining supersymmetries. In the vacuum case, $\Omega_M = 0$ and the Killing spinor equations are

$$\begin{aligned} \partial_u \epsilon &= -\frac{1}{2}(\omega_u^{+-} + \omega_u^{-+}\Gamma_{-+} + \omega_u^{-i}\Gamma_{-i})\epsilon \\ \partial_i \epsilon &= -\frac{1}{2}\omega_i^{-k}\Gamma_{-k}\epsilon. \end{aligned}$$

Choosing a constant spinor ϵ_0 with $\Gamma^+\epsilon_0 = 0$, then the Killing spinor satisfying the above equations is

$$\epsilon(u) = \exp(-\frac{1}{2}\int \omega_u^{-+} du)\epsilon_0.$$

- **Plane-wave like configurations**

Let us introduce a constant 4-form field strength

$$F_{u123} = f_0(u),$$

and make the following ansatz on the metric

$$ds^2 = 2A_0(u)dudv + B_0(u,x)du^2 + A_i(u)(dx^i)^2 + B_i(u,x)dx^i du,$$

where the B_0, B_i's are the functions of u and x. Very recently, N. Ohta et.al. generalized the above metric and found more general supersymmetric configurations.

The metric allows an orthogonal frame

$$\begin{aligned}
\theta^{+} &= \sqrt{A_0(u)}du \\
\theta^{-} &= \sqrt{A_0(u)}dv + \frac{B_0(u,x)}{2\sqrt{A_0(u)}}du + \frac{B_i(u,x)}{2\sqrt{A_0(u)}}dx^i \\
\theta^{I} &= \sqrt{A_i(u)}dx^i\delta_i^I .
\end{aligned}$$

The corresponding spin connections are

$$\begin{aligned}
\omega^{-+} &= -\frac{\partial_u\sqrt{A_0}}{\sqrt{A_0}}du \\
\omega^{+i} &= 0 \\
\omega^{ij} &= -\frac{\partial_i B_j}{2\sqrt{A_i A_j}}du \\
\omega^{-i} &= \frac{1}{\sqrt{A_i}}\left(\frac{\partial_i B_0}{2\sqrt{A_0}} - \frac{\partial_u B_i}{2\sqrt{A_0}} + \frac{\partial_u\sqrt{A_0}}{A_0}B_i\right)du \\
&\quad -\frac{\partial_u\sqrt{A_i}}{\sqrt{A_0}}dx^i + \sum_{j\neq i}\frac{\partial_i B_j}{2\sqrt{A_0 A_i}}dx^j.
\end{aligned}$$

The existence of Killing spinor with $\Gamma^+\epsilon = 0$ requires that

$$B_i = A_{ij}(u)x^j$$

with $A_{ij} = -A_{ji}$[1]. And the equation of motion asks B_0 to be bilinear in x^i, namely

$$B_0 = B_{ij}(u)x^i x^j.$$

Therefore, we find a class of time-dependent configurations akin to the plane-waves

$$ds^2 = 2A_0(u)dudv + B_{ij}(u)x^i x^j(du)^2 + A_i(u)(dx^i)^2 + A_{ij}(u)x^j dx^i du$$

with at least 16 supersymmetries.

[1] The antisymmetric condition is not necessary

- **Extra supersymmetries?**
  - **Why? The "standard" 16 supersymmetries lead to nonlinearly realized supersymmetries on Matrix model (or on the action of other objects embedded in the background).**
  - **Strategy: check if such configurations have Killing vector with nonvanishing ∂_u component and no dependence on v.**

1. **Consider the Killing vector**

$$K = \bar{\epsilon}\Gamma^{\mu}\epsilon\partial_{\mu},$$

which has the component $K^u = \frac{1}{\sqrt{2}}(\Gamma^+\epsilon)^T(\Gamma^+\epsilon)$**. The extra supersymmetries with** $\Gamma^+\epsilon \neq 0$ **will give us the nonvanishing Killing component** K^u**.**

2. **A Killing vector with nonvanishing** K^u **component is not enough to ensure the extra supersymmetries. Since in our discussion the Killing spinor is independent of** v**, the corresponding Killing vector cannot have** v **dependence.**

- **Results: For the metric**

$$ds^2 = 2A_0(u)dudv + B_{ij}(u)x^i x^j (du)^2 + A_i(u)(dx^i)^2$$

the possible time-dependent configurations with supernumerary supersymmetries are

- $B_0 = 0, A_0 = 1$, **and** A_i**'s are exponential functions;**
- $B_0 \neq 0, A_0 = 1$, A_i**'s are exponential functions and** B_{jk} **take the form of**

$$B_{jk} = e^{\gamma_j u}(e^{2uf} B_s e^{-2uf})_{jk}$$

where B_s **is a constant symmetric matrix and** f **is a constant antisymmetric matrix.**

However, after coordinate transformation, the above backgrounds are reduced to the well-studied homogeneous plane-waves. Blau et.al.

We conclude that our time-dependent supersymmetric configurations of the metric form (2.1) have no supernumerary supersymmetry, except the cases with constant A_0, A_i**'s and the appropriately chosen** B_{ij}, A_{ij}**'s.**

3 Matrix models

- **Matrix models in the curved backgrounds is a subtle issue;**
- **Weak field limit following DLCQ prescription;** W. Taylor et.al.

  However, for the above time-dependent configurations, it could be far from flat near the big-bang singularity, and the validity of weak field limit is in doubt;

- **Another way: matrix regularization of the membrane action in the curved background: order by order expansion in fermionic coordinates;** **de Wit et.al.** **People has applied this route to the flat spacetime and maximal plane-wave background and obtained the BFSS and BMN matrix model succesfully.** **de Wit et.al., K. Dasgupta et.al.**

- **In our case, we obtained the membrane action to all orders of fermionic coordinates.** **H. Z. Chen and B. Chen**

The supermembrane action is:

$$S[Z(\xi)] = \int d^3\xi[-\sqrt{-g(Z(\xi))} - \frac{1}{6}\epsilon^{abc}\Pi_a^A\Pi_b^B\Pi_c^C B_{CBA}(Z(\xi))],$$

where $Z^A(\xi) = (x^\mu(\xi), \theta(\xi))$ **is the curved superspace coordinates,** $g_{ab} = \Pi_a^\mu \Pi_b^\nu g_{\mu\nu} = \Pi_a^r \Pi_b^s \eta_{rs}$ **is the induced metric,** $\eta_{rs} = diag(-1, 1, ...1)$ **is the 11-d Lorentz metric, and** $\xi^a = (\xi^0, \xi^1, \xi^2) = (\tau, \xi^\alpha)$**,** $\alpha = 1, 2$ **represent the coordinates on the world volume. Here** Π_a^A **are the supervielbein pullback,** B_{ABC} **are the super three-potential.**

In our case, the gravitino is zero, so the supervielbein pull-back is:

$$\begin{aligned}\Pi_a^r &= \partial_a Z^A E_A^r \\ &= \partial_a x^\mu (e_\mu^r - \frac{1}{4}\bar{\theta}\Gamma^{rst}\theta\omega_{\mu st} + \bar{\theta}\Gamma^r\Omega_\mu\theta) + \bar{\theta}\Gamma^r\partial_a\theta + \mathcal{O}(\theta^3),\end{aligned}$$

where $\omega_{\mu st}$ **is the spin connection, and**

$$\Omega_\mu = \frac{1}{288}F_{\nu\rho\sigma\lambda}(\Gamma_\mu^{\nu\rho\sigma\lambda} + 8\Gamma^{\nu\rho\sigma}\delta_\mu^\lambda).$$

The super three-potential pullback is:

$$
\begin{aligned}
-\quad & \frac{1}{6}\Pi_a^A\Pi_b^B\Pi_c^C B_{CBA} \\
=\quad & \frac{1}{6}\epsilon^{abc}\partial_a x^\mu \partial_b x^\nu \partial_c x^\rho \Big[C_{\mu\nu\rho} + \frac{3}{4}\bar\theta\Gamma_{rs}\Gamma_{\mu\nu}\theta\omega_\rho^{rs} - 3\bar\theta\Gamma_{\mu\nu}\Omega_\rho\theta \Big] \\
-\quad & \epsilon^{abc}\bar\theta\Gamma_{\mu\nu}\partial_c\theta \Big[\frac{1}{2}\partial_a x^\mu (\partial_b x^\nu + \bar\theta\Gamma^\nu\partial_b\theta) + \frac{1}{6}\bar\theta\Gamma^\mu\partial_a\theta\bar\theta\Gamma^\nu\partial_b\theta \Big] + \mathcal{O}(\theta^3),
\end{aligned}
$$

where $C_{\mu\nu\rho}$ is the three-form potential.

In our configurations, the supervielbein and super three-potential are exact to all orders of θ since the higher order terms vanish. Why?

- **The supervielbein pullback $\Pi_a^r = \partial_a Z^A E_A^r$ is linear in $\partial_a X^\mu$, while E_A^r is constituted with other quantities. It can be seen from their explicit form that these other quantities, θ, Γ^r, Ricci tensor, Ω_μ, and field strength et.al. have no lower curved spacetime index v, and hence no upper curved spacetime index u.**

- **The nonvanishing bilinear fermionic terms $\bar{\theta}\Gamma^{rst\cdots}\theta$ always have one and only one Γ^- and no Γ^+ due to the gauge condition $\Gamma^+\theta = 0$. The upper tangent space index $r = -$ require an upper curved spacetime index $\mu = u$ coming from other geometrical quantities because the only nonzero vielbein with a lower tangent index $r = -$ is e_{-u}. Such an index cannot be cancelled by the above mentioned quantities except $\partial_a X^u$.**

- **the super three-potential pullback term can only have bilinear θ terms. This is due to the antisymmetric nature of ϵ^{abc} and the fact that bilinear θ term in Π_a^A must be proportional to $\partial_a X^u$.**

To simplify the action, we go to light-cone gauge:

$$x^u = u = \tau.$$

And because of the κ-symmetry of the action, we can also impose an additional gauge

$$\Gamma^+\theta = 0.$$

There still exist the residue gauge D.O.F. We should fix them and write our action in pure physical D.O.F.

Let us focus on the following metric and field strength

$$ds^2 = 2e^{r_0 u} du dv + \sum_i c_i e^{r_i u} (x^i)^2 (du)^2 + \sum_i e^{r_i u} (dx^i)^2 + \sum_{ij} A^0_{ij} e^{(r_i + r_j)u/2} x^j dx^i du,$$

where

$$A^0_{ij} = -A^0_{ji} = const,$$

and r_0, r_i **are all constants,too. We also have a four-form field strength**

$$F_{u123} = e^{(r_1+r_2+r_3)u/2} f^0, \qquad f^0 = const.$$

The Lagrangian of the membrane action in this background is

$$
\begin{aligned}
\mathcal{L} &= \sum_i \frac{P^u}{2} e^{(r_i - r_0)\tau} (D_\tau x^i)^2 + \frac{P^u}{2} \sum_{ij} A^0_{ij} e^{(\frac{r_i + r_j}{2} - r_0)\tau} x^j D_\tau x^i \\
&+ \frac{P^u}{2} \sum_i c_i e^{(r_i - r_0)\tau} (x^i)^2 - \frac{e^{r_0 \tau}}{4P^u} \sum_{ij} e^{(r_i + r_j)\tau} \{x^i, x^j\}^2 \\
&- \frac{1}{2} \sum_{i,j=1,2,k=3} \{x^i, x^j\} x^k \epsilon_{ijk} f^0 e^{(r_i + r_j + r_k)\tau/2} \\
&+ i P^u e^{-r_0 \tau/2} \psi^T D_\tau \psi - \frac{i}{6} P^u e^{-r_0 \tau/2} f^0 \psi^T \gamma^{123} \psi \\
&- \frac{i}{8} P^u \sum_{IJ} e^{-r_0 \tau/2} A^0_{IJ} \psi^T \gamma^{IJ} \psi - i \sum_{I,i} \psi^T \gamma^I \{x^i, \psi\} e^{(r_0 + r_i)\tau/2} \delta^I_i .
\end{aligned}
$$

The usual matrix regularization:

$$\begin{aligned} x^i &\longrightarrow X^i_{N\times N}, \\ \psi &\longrightarrow \psi_{N\times N}, \\ P^u \int d^2\sigma &\longrightarrow \frac{1}{R} Tr, \\ \{,\} &\longrightarrow -i[,], \end{aligned}$$

Then the matrix model action reads

$$
\begin{aligned}
S &= \int d\tau Tr\Big(\sum_i \frac{1}{2R} e^{(r_i-r_0)\tau}(D_\tau X^i)^2 + \frac{1}{2R}\sum_{ij} A^0_{ij} e^{(\frac{r_i+r_j}{2}-r_0)\tau} X^j D_\tau X \\
&+ \frac{1}{2R}\sum_i c_i e^{(r_i-r_0)\tau}(X^i)^2 + \frac{R}{4} e^{r_0\tau}\sum_{ij} e^{(r_i+r_j)\tau}\left[X^i, X^j\right]^2 \\
&+ \frac{i}{2}\sum_{i,j=1,2,k=3}\left[X^i, X^j\right] X^k \epsilon_{ijk} f^0 e^{(r_i+r_j+r_k)\tau/2} \\
&+ \frac{i}{R} e^{-r_0\tau/2}\psi^T D_\tau \psi - \frac{i}{6R} e^{-r_0\tau/2} f^0 \psi^T \gamma^{123}\psi \\
&- \frac{i}{8R}\sum_{IJ} e^{-r_0\tau/2} A^0_{IJ}\psi^T\gamma^{IJ}\psi - \sum_{I,i}\psi^T\gamma^I\left[X^i, \psi\right] e^{(r_0+r_i)\tau/2}\delta^I_i\Big).
\end{aligned}
$$

- **Fuzzy sphere solutions: consider the matrix model in the sector:**

$$X^4 = X^5 = ... = X^9 = 0, \psi = 0.$$

To simplify the problem, let

$$r_1 = r_2 = r_3 = r, c_1 = c_2 = c_3 = c.$$

We want to find solution of the form:

$$X^a(\tau) = S(\tau) J^a, a = 1, 2, 3,$$

where J^a is N dimensional representation of $SU(2)$.

Use

$$Tr\sum_a (J^a)^2 = \frac{N(N-1)}{4},$$

and

$$[J^a, J^b] = i\epsilon^{abc}J^c.$$

We finally get

$$\frac{d^2S}{d\tau^2} + (r - r_0)\frac{dS}{d\tau} + 2R^2 e^{(2r_0+r)\tau}S^3 + Rf^0 e^{(r_0+r/2)\tau}S^2 - cS = 0.$$

4 IIB time-dependent plane-wave with linear null dilaton

- Linear null dilaton is different from the usual linear (spatial) dilaton;
- Plane-fronted waves are exact solutions of the string theory; Horowitz and Stief
- Plane-wave/SYM correspondence. Metsaev et.al., BMN

Let's consider the IIB GS superstrings in the following backgrounds:

$$ds^2 = -2dx^+dx^- - \lambda(x^+)\, x_I^2\, dx^+dx^+ + dx^I dx^I \,,$$

$$\phi = \phi(x^+)\,, \qquad (F_5)_{+1234} = (F_5)_{+5678} = 2f.$$

The world-sheet conformal invariance requires

$$\lambda = -\frac{1}{4}\phi'' + f^2 e^{2\phi}\,.$$

If we restrict the dilaton to be linear in the light-cone time coordinate x^+, i.e. $\phi = -cx^+$ with c being a constant, then we have

$$\lambda = f^2 e^{-2cx^+}.$$

In general, f could be an arbitrary function of x^+ and there is a large class of the models. These models could be studied in the lightcone gauge.

- **Two special cases:**

  1. **One is that λ is a constant. This happens when $f = f_0 e^{-\phi}$ with f_0 being constant so that**

     $$\lambda = f_0^2 ,$$

     and the metric in the string frame reduces to the form of the maximally supersymmetric plane wave;

     Blau et.al., Mestaev et.al., BMN

  2. **The other special case is when the self-dual RR field strength $f = f_0$ is constant so that**

     $$\lambda = f^2 e^{-2cx^+} .$$

- **Symmetry algebra:**

  1. **A null Killing vector T, corresponding to the translation in the x^-;**
  2. **Two $SO(4)$ rotations + Z_2;**
  3. **Translation along x^I plus the shift along x^-;**

  The the continuous symmetry algebra is $[so(4) \oplus so(4)] \oplus_s h(8)$.

- **Implications: light-cone momentum conserved; no particle or string creation; no supernumerary supersymmetries; Hamiltonian not conserved.**

- **Geodesics incomplete**

Let us focus on $x^I = 0, x^- =$ constant, and consider the geodesic equation for x^+

$$\frac{d^2x^+}{d\sigma^2} + \frac{c}{2}\left(\frac{dx^+}{d\sigma}\right)^2 = 0 \ ,$$

which gives

$$\sigma = e^{\frac{c}{2}x^+}$$

up to an affine transformation. Therefore the singularity $x^+ \to -\infty$ corresponds to $\sigma = 0$ and it has finite affine distance to all points in the interior.

- **Supersymmetries**

The gravitino and dilatino variations should vanish for independent Killing spinors, i.e.

$$\delta_\epsilon \lambda^A \equiv (\tilde{\mathcal{D}})^A_B\, \epsilon^B = 0\,, \qquad \delta_\epsilon \psi^A_\mu \equiv (\hat{\mathcal{D}}_\mu)^A_B\, \epsilon^B = 0\,,$$

where $\mu = +, -, 1, ..., 8$ **and** $A = 1, 2$ **and**

$$\begin{aligned} (\,\tilde{\mathcal{D}})^A_B &= \frac{1}{2}\,\delta^A_B\,\Gamma^\mu\,\partial_\mu \phi\,, \\ (\hat{\mathcal{D}}_\mu)^A_B &= \delta^A_B\,\partial_\mu + (\Omega_\mu)^A_B\,, \end{aligned}$$

with

$$(\Omega_\mu)^A_B = \frac{1}{4}\omega_\mu^{\hat\nu\hat\rho}\Gamma_{\hat\nu\hat\rho}\,\delta^A_B + \frac{i\,e^\phi}{8\cdot 5!}\,\Gamma^{\kappa\nu\rho\sigma\delta}F_{\kappa\nu\rho\sigma\delta}\Gamma_\mu(\sigma_2)^A_B\,,$$

where σ_2 is the Pauli matrix, $\omega_\mu^{\hat\nu\hat\rho}$ is the spin connection and the hatted indices are used for the tangent space.

It's straightforward to check our background keep one-half of the original supersymmetries.

- **Bosonic sector**

The bosonic part of the GS action is

$$S_B = -\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-g}\, g^{ab}G_{\mu\nu}\partial_a x^\mu \partial_b x^\nu$$

In the light-cone gauge the bosonic action takes the form

$$S_B = \frac{1}{4\pi\alpha'}\int d\tau \int_0^{2\pi} d\sigma \, (\partial_\tau x^I \partial_\tau x^I - \partial_\sigma x^I \partial_\sigma x^I - \tilde{f}^2 e^{-2\,\tau} x_I^2) \; ,$$

where for simplicity we have defined $\tilde{f} \equiv \alpha' p^+ f$ **which is dimensionless.**

The equations of motions are

$$(\partial_\tau^2 - \partial_\sigma^2 + \tilde{f}^2 e^{-2\,\tau})\, x^I = 0\,.$$

Expanding in Fourier modes in σ, we get an infinite collection of oscillators with time-dependent frequencies.

The bosonic light-cone Hamiltonian is

$$H_B = H_{B0}(\tau) + \frac{1}{\alpha' p^+} \sum_{n=1}^{\infty} \omega_n(\tau) \left[A^I_{-n}(\tau) A^I_n(\tau) + \tilde{A}^I_{-n}(\tau) \tilde{A}^I_n(\tau) + 1 \right],$$

where

$$\omega_n = \sqrt{n^2 + \tilde{f}^2 e^{-2\tau}}. \tag{4.1}$$

The bosonic Hamiltonian reminisces the 2-d field theory of free scalars with time-dependent masses.

- **Fermionic sector**

  The fermionic action in the light-cone gauge can be written as

$$
\begin{aligned}
S_F \;&=\; \frac{i\,p^+}{\sqrt{2}\,\pi}\int d\tau \int_0^{2\pi} d\sigma\,\big(\theta^{1T}\partial_\tau\theta^1 + \theta^{2T}\partial_\tau\theta^2 \\
&\quad +\theta^{1T}\partial_\sigma\theta^1 - \theta^{2T}\partial_\sigma\theta^2 + 2\tilde{f}e^{-\tau}\theta^{1T}\Pi\theta^2\big)\,.
\end{aligned}
$$

  Similar to the bosonic case, when $f = f_0 e^{-\phi}$, the action reduces to the one in the usual plane-wave metric.

The equations of motion of the fermionic sector are

$$(\partial_\tau + \partial_\sigma) \theta^1 + \tilde{f} e^{-\tau} \Pi \theta^2 = 0 \ ,$$
$$(\partial_\tau - \partial_\sigma) \theta^2 - \tilde{f} e^{-\tau} \Pi \theta^1 = 0 \, .$$

The fermionic part of the Hamiltonian can be diagonalized as

$$H_F = H_{F0}(\tau) + \frac{1}{\alpha' p^+} \sum_{n=1}^{\infty} \omega_n(\tau) \left[B_n^\dagger(\tau) B_n(\tau) + \tilde{B}_n^\dagger(\tau) \tilde{B}_n(\tau) - 1 \right] .$$

Just like the bosonic case, it also looks like the Hamiltonian of a free 2-d field theory with time-dependent mass.

- **Quantum string mode creation**

  Generically, in a time-dependent background, one may expect the particle or string creation occurs from our knowledge of the quantum field theory in curved spacetime. However, in a plane-fronted background, due to the existence of null Killing vector, this would not happen. Gibbons 1975

  Nevertheless, there does exist the string mode creation. Horowitz 1990

In our case, the total number of created oscillator modes is

$$\bar{N}_T(\tau) = \sum_{n=1}^{\infty} \left[\bar{N}_n^B + \bar{N}_n^F\right] .$$

1. **As** $\tau \to +\infty, \bar{N}_T(\tau) \sim 0$ **;**
2. **As** $\tau \to -\infty$**, the total number of created oscillator modes is**

$$\bar{N}_T(\tau) = 4d \sum_{n=1}^{\infty} \frac{1}{e^{2n\pi} - e^{-2n\pi}}$$
$$\sim 0.06 .$$

Effectively, the problem could be restated as a quantum mechanical problem. A Bosonic mode $T_n(\tau)$ should satisfy the equation

$$\partial_\tau^2 T_n + (\, n^2 + \tilde{f}^2 e^{-2\tau}) \, T_n = 0.$$

Replacing τ by x and T_n by ψ, the above equation takes a form of one-dimensional Schrodinger equation for a particle with energy n^2 in a potential $-\tilde{f}^2 e^{-2\tau}$. The problem of calculating the number of the creating modes reduces to the problem of calculating the reflective amplitude in this one-dimensional system.

- **Remarks**

1. **The symmetric spectrum of the bosonic and fermionic excitations does not come from the spacetime supersymmetries. It is from the special choice of the metric. Consider the following background**

$$ds^2 = -2dx^+dx^- - \sum_I \lambda_I(x^+)\, x_I^2\, dx^+dx^+ + dx^I dx^I\,,$$

$$\phi = \phi(x^+)\,, \qquad (F_5)_{+1234} = (F_5)_{+5678} = 2f.$$

Here λ_I could be different from each other. The transverse bosons in the bosonic action have different masses proportional to λ_I.

2. **Matrix description near the big-bang singularity? There exists the cosmological singularity at $x^+ = -\infty$, where the string coupling is divergent. (Matrix model in IIB ?)**

3. **Membrane creation? S. Das et.al**

- **Other directions?**

  Holographic description of the time-dependent backgrounds

  Chu and Ho, S.Das et.al., F.L. Lin et.al.

  (S)YM with time-dependent coupling, what does it mean? nontrivial RG flow? Exotic QFT!

  Why the correspondence could be true? Short of evidence!

  Stability? 1-loop issue.