暗物质与束缚电子散射的有效理论

Effective theory for dark matter-bound electron scattering

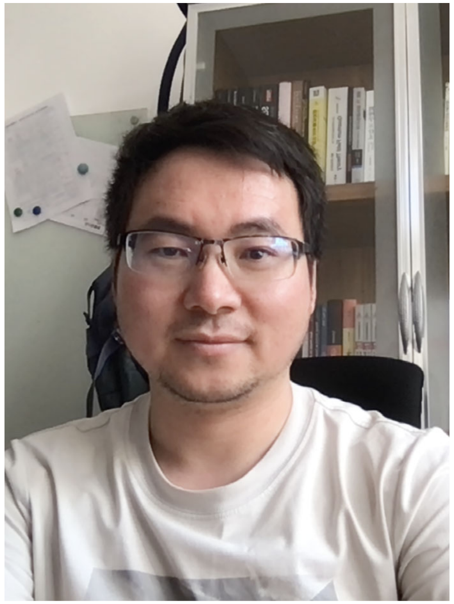
廖益

量子物质研究院，华南师范大学

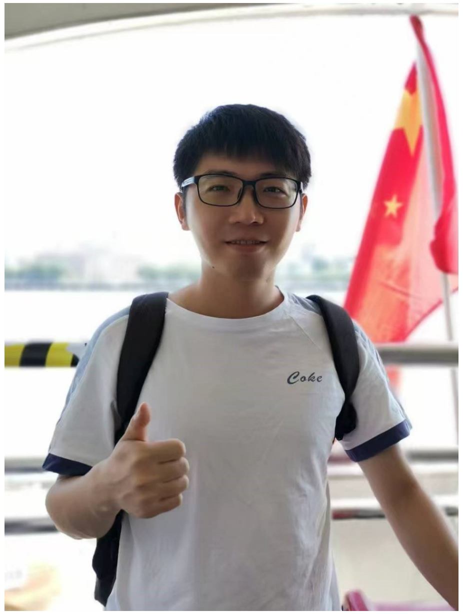
2025年1月9日，中国科学技术大学

Based on work in collaboration with

王昊琳 **Hao-Lin Wang**



马小东 **Xiao-Dong Ma**



梁锦汉 **Jin-Han Liang**

- *Revisiting general dark matter-bound electron interactions*, PRD 110 (2024) L091701 [arXiv:2405.04855]
- *A systematic investigation on dark matter-electron scattering in effective field theories*, JHEP 07 (2024) 279 [arXiv:2406.10912]

ABC about dark matter

- Plenty of evidence for its existence
  but is restricted to gravitational effects

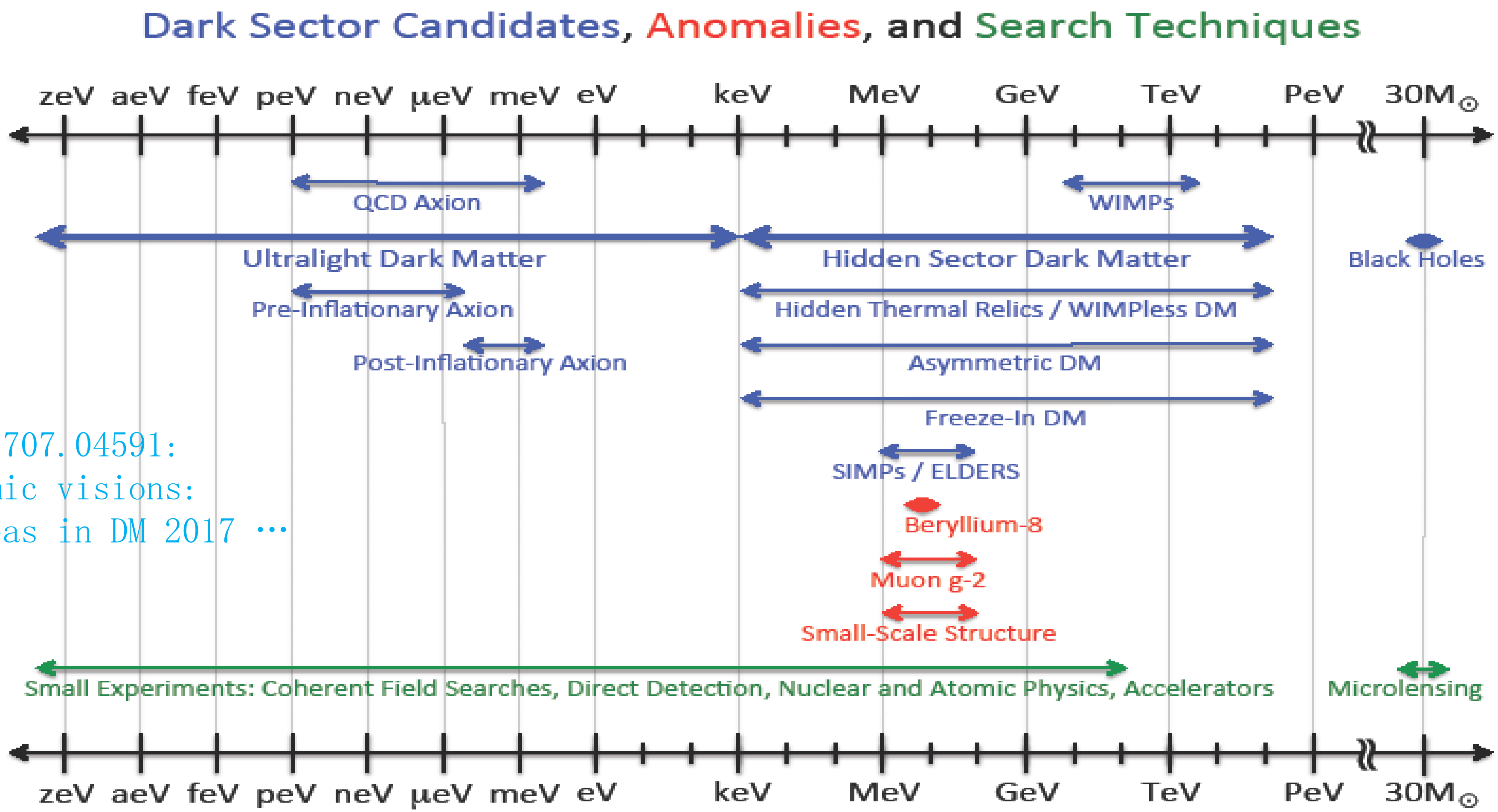
- Constitutes ≈ 25% of total energy budget in the universe,
  ≈ 5 x ordinary/baryonic matter
  energy density ≈ 0.4 GeV/cm^3

- Attracts ordinary matter gravitationally, but is nonluminous
  → very weak interaction with ordinary matter

- Typical velocity ≈ 10^{-3} → nonrelativistic

- We know almost nothing else

ABC about dark matter



arXiv:1707.04591:
US cosmic visions:
New ideas in DM 2017 …

How to detect DM particles

- Detection means via ordinary matter scattering/production/annihilation depend on
  both momentum transfer – kinematics,
  and types of interaction – dynamics
  direct detection: DM + OM → DM + OM, in terrestrial labs
  indirect detection: DM + DM → OM, in cosmic rays
  collider searches: OM + OM → DM + DM

- I focus on direct detection from now on:
  A DM particle *in halo* around us collides on
  a target particle *in lab* a bulk/liquid/gas of matter nucleus electron
  to trigger an observable phenomenon

Direct detection: DM + OM → DM + OM

DM particle must be *energetic enough* to make target particle *recoil visibly*

both nonrelativistic

kinetic energy $\frac{1}{2}m_{\mathrm{DM}}v_{\mathrm{DM}}^2$ gain energy

the heavier, more energetic the heavier, more reluctant to recoil

Only relatively heavy DM can kick a nucleus,

while electron recoils visibly against relatively light DM.

Minimal energy required defines detection threshold,

which translates into detectable lower limit of DM mass.

Different types of recoil result in different signals to observe.

I'll concentrate on electron recoil below.

Effective theory approach: Outline

- From now on, DM-bound electron scattering
- Both DM and atomic electron are nonrelativistic (NR)
  i.e., experiments at low energy, but interest in physical origin at high energy
  How to relate the two?
- Bottom-up approach: from something certain to something less
  Knowns: interactions based on established symmetries and power counting
  Unknowns: interaction strengths parameterized without theoretical biases
- We distinguish clearly between what we know and what we don't.

Effective theory approach: Outline

- The DM-bound electron scattering: NR quantum mechanics for 2 bodies

  Parameterize possible NR interactions at leading order

  Compute event rate using NR interactions

- Above NR interactions are from reduction of relativistic interactions
  - Low energy effective field theory (LEFT)

- Getting closer to new physics
  - Standard model EFT (SMEFT)

  Match LEFT with SMEFT, and further SMEFT with your favorite new phys model

- Employ data to constrain interactions/models at various energy scales

Step 1: DM-bound electron scattering

- Initial state: $|\boldsymbol{p}, 1\rangle$ with atomic electron $|1\rangle = |n, l, m\rangle$

  Final state: $|\boldsymbol{p}', 2\rangle$ with ionized atomic electron $|2\rangle = |k', l', m'\rangle$

- Transition amplitude:

$$\mathcal{M}_{1\to 2} = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \tilde{\psi}_2^*(\boldsymbol{k}+\boldsymbol{q})\mathcal{M}(\boldsymbol{q}, \boldsymbol{v}_{\mathrm{el}}^{\perp})\tilde{\psi}_1(\boldsymbol{k})$$

$$\boldsymbol{q} = \boldsymbol{p} - \boldsymbol{p}'$$

$$\boldsymbol{v}_{\mathrm{el}}^{\perp} = \boldsymbol{v} - \frac{\boldsymbol{q}}{2\mu_{xe}} - \frac{\boldsymbol{k}}{m_e}$$

amplitude for free particles depends only on $\boldsymbol{q}, \boldsymbol{v}_{\mathrm{el}}^{\perp}$ by rotational and Galilean invariance

Electron mass, DM mass m_x, reduced mass DM initial velocity, relative velocity (initial-final averaged, $\perp \boldsymbol{q}$)

- Work at excellent precision to linear order in $\boldsymbol{v}_{\mathrm{el}}^{\perp}$

$$\mathcal{M}(\boldsymbol{q}, \boldsymbol{v}_{\mathrm{el}}^{\perp}) = \mathcal{M}_{\mathrm{S}} + \boldsymbol{v}_{\mathrm{el}}^{\perp} \cdot \boldsymbol{\mathcal{M}}_{\mathrm{V}}$$

$$\rightarrow \quad \mathcal{M}_{1\to 2} = f_{\mathrm{S}}(\boldsymbol{q})\mathcal{M}_{\mathrm{S}} + \boldsymbol{f}_{\mathrm{V}}(\boldsymbol{q}) \cdot \boldsymbol{\mathcal{M}}_{\mathrm{V}}$$

$$f_{\mathrm{S}}(\boldsymbol{q}) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \tilde{\psi}_2^*(\boldsymbol{k}+\boldsymbol{q})\tilde{\psi}_1(\boldsymbol{k})$$

$$\boldsymbol{f}_{\mathrm{V}}(\boldsymbol{q}) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \tilde{\psi}_2^*(\boldsymbol{k}+\boldsymbol{q})\boldsymbol{v}_{\mathrm{el}}^{\perp}\tilde{\psi}_1(\boldsymbol{k})$$

This expansion in $\boldsymbol{v}_{\mathrm{el}}^{\perp}$ makes our formalism more advantageous than previous one in $\boldsymbol{k}$ expansion – see later.

Step 1: DM-bound electron scattering

- DM-spin-averaged and –summed:

$$\overline{|\mathcal{M}_{1\to 2}|^2} = a_0|f_\mathrm{S}|^2 + a_1|\boldsymbol{f}_\mathrm{V}|^2 + \frac{a_2}{x_e}\left|\frac{\boldsymbol{q}}{m_e}\cdot\boldsymbol{f}_\mathrm{V}\right|^2 + ia_3\frac{\boldsymbol{q}}{m_e}\cdot(\boldsymbol{f}_\mathrm{V}\times\boldsymbol{f}_\mathrm{V}^*) + 2\mathrm{Im}\left[a_4 f_\mathrm{S}\boldsymbol{f}_\mathrm{V}^*\cdot\frac{\boldsymbol{q}}{m_e}\right]$$

$a_{0,1,2,3,4}$: DM response functions (DMRF)

$x_e = \boldsymbol{q}^2/m_e^2$

- Summing over initial m and final (k', l', m'):

$$\overline{|\mathcal{M}_{1\to 2}|^2} \quad \to \quad \overline{|\mathcal{M}_\mathrm{ion}^{n\ell}|^2} = a_0\tilde{W}_0 + a_1\tilde{W}_1 + a_2\tilde{W}_2$$

$\widetilde{W}_{0,1,2}$: generalized atomic response functions (ARF)

- Differential ionization rate:

$$\frac{\mathrm{d}\mathcal{R}_\mathrm{ion}^{n\ell}}{\mathrm{d}\ln E_e} = \frac{n_\mathrm{dm}}{128\pi m_\mathrm{dm}^2 m_e^2}\int \mathrm{d}q\,q \int_{v_\mathrm{min}} \frac{\mathrm{d}^3\boldsymbol{v}}{v} f_\mathrm{dm}(\boldsymbol{v})\overline{|\mathcal{M}_\mathrm{ion}^{n\ell}|^2}$$

- This formalism is universal, and does not depend on specific forms of interactions. Single restriction: include NR interactions to linear order in small $\boldsymbol{v}_\mathrm{el}^\perp$.

Step 1: DM-bound electron scattering

- Advantages over previous formalism Catena et al, PR Res 2 (2020) 033195

(1) 3 generalized ARFs instead of 4.

(2) Our DMRF $a_{0,1,2,3,4}$ are indept. of atomic properties,
and our ARF $\widetilde{W}_{0,1,2}$ are indept. of DM at level better than 1% for $m_x \geq 5\ \mathrm{MeV}$.
VS Their DMRF depend significantly on atomic properties at level up to 40%.

(3) Clear physical significance:
$\widetilde{W}_0$, a_0 ↔ velocity-indept. NR interactions including spin-indept./dept. ones
$\widetilde{W}_{1,2}$, $a_{1,2}$ ↔ velocity-dept. NR interactions, involving axial-vector currents
$a_{3,4}$ contain only interference of different NR interactions,
vanish for real effective couplings (Wilson coefficients)

Step 1: DM-bound electron scattering

- Examples of DMRF $a_{0,1,2}$:

scalar DM: $a_0 = |c_1|^2 + \frac{1}{4}|c_{10}|^2 x_e, \quad a_1 = \frac{1}{4}|c_7|^2 + \frac{1}{4}|c_3|^2 x_e, \;\; a_2 = -\frac{1}{4}|c_3|^2 x_e.$

fermion DM: $a_0 = |c_1|^2 + \frac{3}{16}|c_4|^2 + \left(\frac{1}{8}|c_9|^2 + \cdots\right) x_e + (\cdots) x_e^2,$

$$a_1 = \frac{1}{4}|c_7|^2 + \frac{1}{4}|c_8|^2 + \frac{1}{8}|c_{12}|^2 + \left(\frac{1}{4}|c_3|^2 + \cdots\right) x_e + (\cdots) x_e^2,$$

$$a_2 = 0 + \left(-\frac{1}{4}|c_3|^2 + \cdots\right) x_e + (\cdots) x_e^2.$$

$$x_e = \frac{\boldsymbol{q}^2}{m_e^2}, \; y_e = \frac{\boldsymbol{q} \cdot \boldsymbol{v}_0^\perp}{m_e}$$
$$\boldsymbol{v}_0^\perp = \boldsymbol{v} - \frac{\boldsymbol{q}}{2\mu_{xe}}$$

vector DM: $a_0 = |c_1|^2 + \frac{1}{2}|c_4|^2 + \left(\frac{1}{3}|c_9|^2 + \cdots\right) x_e + (\cdots) x_e^2,$

$$a_1 = \frac{1}{4}|c_7|^2 + \frac{2}{3}|c_8|^2 + \frac{1}{3}|c_{12}|^2 + \frac{5}{36}|c_{21}|^2 + \left(\frac{1}{4}|c_3|^2 + \cdots\right) x_e + (\cdots) x_e^2,$$

$$a_2 = 0 + \left(-\frac{1}{4}|c_3|^2 + \cdots\right) x_e + (\cdots) x_e^2.$$

Catena et al, PR Res 2 (2020) 033195

- Relations between our ARF $\widetilde{W}_{0,1,2}$ and previous ones $W_{1,2,3,4}$

$$\widetilde{W}_0 = W_1, \;\; \widetilde{W}_1 = |\boldsymbol{v}_0^\perp|^2 W_1 - 2\frac{y_e}{x_e} W_2 + W_3, \;\; \widetilde{W}_2 = \frac{y_e^2}{x_e} W_1 - 2\frac{y_e}{x_e} W_2 + \frac{1}{x_e} W_4$$

wrong sign of W_2 corrected

Step 2: NR interactions

- DM-bound electron scattering:

  $v_{DM} \sim 10^{-3}, v_e \sim \alpha \sim 10^{-2}$ $\rightarrow$ well suited for NR quantum mechanics
- Construct basis of complete and indept. interaction operators up to

  $O(q^2)$, $O(\boldsymbol{v}_{\mathrm{el}}^{\perp})$ $\quad q \sim$ few $10 - 10^2$keV, $\boldsymbol{v}_{\mathrm{el}}^{\perp} \sim 10^{-2}$

  for DM of spin 0, ½, and 1.

- Symmetries:

  Rotational invariance, Galilean invariance
- Building blocks:

  coordinate space: $\boldsymbol{q}$, $\boldsymbol{v}_{\mathrm{el}}^{\perp}$

  spin space: $\mathbf{1}_e$, $\boldsymbol{S}_e$; $\mathbf{1}_\chi$, $\boldsymbol{S}_\chi$, and $\tilde{\boldsymbol{S}}_\chi$for rank-2 traceless spin tensor for spin-1 DM

  $$\tilde{S}_x^{ij} = \frac{1}{2}\left(S_x^i S_x^j + S_x^j S_x^i\right) - \frac{2}{3}\delta^{ij}$$

Step 2: NR interactions

NR operators	Refs. [39, 50]	Power counting	DM type		
			scalar	fermion	vector
$\mathcal{O}_1 = \mathbb{1}_x \mathbb{1}_e$	✓	1	✓	✓	✓
$\mathcal{O}_3 = \mathbb{1}_x \left(\frac{iq}{m_e} \times \boldsymbol{v}_{\rm el}^{\perp}\right) \cdot \boldsymbol{S}_e$	✓	qv	✓	✓	✓
$\mathcal{O}_4 = \boldsymbol{S}_x \cdot \boldsymbol{S}_e$	✓	1	–	✓	✓
$\mathcal{O}_5 = \boldsymbol{S}_x \cdot \left(\frac{iq}{m_e} \times \boldsymbol{v}_{\rm el}^{\perp}\right) \mathbb{1}_e$	✓	qv	–	✓	✓
$\mathcal{O}_6 = \left(\boldsymbol{S}_x \cdot \frac{q}{m_e}\right)\left(\frac{q}{m_e} \cdot \boldsymbol{S}_e\right)$	✓	q^2	–	✓	✓
$\mathcal{O}_7 = \mathbb{1}_x \boldsymbol{v}_{\rm el}^{\perp} \cdot \boldsymbol{S}_e$	✓	v	✓	✓	✓
$\mathcal{O}_8 = \boldsymbol{S}_x \cdot \boldsymbol{v}_{\rm el}^{\perp} \mathbb{1}_e$	✓	v	–	✓	✓
$\mathcal{O}_9 = -\boldsymbol{S}_x \cdot \left(\frac{iq}{m_e} \times \boldsymbol{S}_e\right)$	✓	q	–	✓	✓
$\mathcal{O}_{10} = \mathbb{1}_x \frac{iq}{m_e} \cdot \boldsymbol{S}_e$	✓	q	✓	✓	✓
$\mathcal{O}_{11} = \boldsymbol{S}_x \cdot \frac{iq}{m_e} \mathbb{1}_e$	✓	q	–	✓	✓
$\mathcal{O}_{12} = -\boldsymbol{S}_x \cdot (\boldsymbol{v}_{\rm el}^{\perp} \times \boldsymbol{S}_e)$	✓	v	–	✓	✓
$\mathcal{O}_{13} = (\boldsymbol{S}_x \cdot \boldsymbol{v}_{\rm el}^{\perp})\left(\frac{iq}{m_e} \cdot \boldsymbol{S}_e\right)$	✓	qv	–	✓	✓
$\mathcal{O}_{14} = (\boldsymbol{S}_x \cdot \frac{iq}{m_e})(\boldsymbol{v}_{\rm el}^{\perp} \cdot \boldsymbol{S}_e)$	✓	qv	–	✓	✓
$\mathcal{O}_{15} = \boldsymbol{S}_x \cdot \frac{q}{m_e}\left[\frac{q}{m_e} \cdot (\boldsymbol{v}_{\rm el}^{\perp} \times \boldsymbol{S}_e)\right]$	✓	q^2v	–	✓	✓

✓: known previously

✗: unknown previously

-- : NA

Conventions in [39] Catena et al, PR Res 2 (2020) 033195 and [50] JCAP 03 (2023) 052.

Step 2: NR interactions

NR operators	Refs. [39, 50]	Power counting	DM type		
			scalar	fermion	vector
$\mathcal{O}_{17} = \frac{iq}{m_e} \cdot \tilde{\boldsymbol{S}}_x \cdot \boldsymbol{v}_{\text{el}}^{\perp} \mathbb{1}_e$	$\frac{1}{3}\frac{iq\cdot\boldsymbol{v}_{\text{el}}^{\perp}}{m_e}\mathcal{O}_1 - \mathcal{O}'_{17}$	qv	–	–	✓
$\mathcal{O}_{18} = \frac{iq}{m_e} \cdot \tilde{\boldsymbol{S}}_x \cdot \boldsymbol{S}_e$	$\frac{1}{3}\mathcal{O}_{10} - \mathcal{O}'_{18}$	q	–	–	✓
$\mathcal{O}_{19} = \frac{q}{m_e} \cdot \tilde{\boldsymbol{S}}_x \cdot \frac{q}{m_e} \mathbb{1}_e$	$\frac{1}{3}\frac{q^2}{m_e^2}\mathcal{O}_1 - \mathcal{O}'_{19}$	q^2	–	–	✓
$\mathcal{O}_{20} = -\frac{q}{m_e} \cdot \tilde{\boldsymbol{S}}_x \cdot \left(\frac{q}{m_e} \times \boldsymbol{S}_e\right)$	$-\mathcal{O}'_{20}$	q^2	–	–	✓
$\mathcal{O}_{21} = \boldsymbol{v}_{\text{el}}^{\perp} \cdot \tilde{\boldsymbol{S}}_x \cdot \boldsymbol{S}_e$	✗	v	–	–	✓
$\mathcal{O}_{22} = \left(\frac{iq}{m_e} \times \boldsymbol{v}_{\text{el}}^{\perp}\right) \cdot \tilde{\boldsymbol{S}}_x \cdot \boldsymbol{S}_e + \boldsymbol{v}_{\text{el}}^{\perp} \cdot \tilde{\boldsymbol{S}}_x \cdot \left(\frac{iq}{m_e} \times \boldsymbol{S}_e\right)$	✗	qv	–	–	✓
new → $\mathcal{O}_{23} = -\frac{iq}{m_e} \cdot \tilde{\boldsymbol{S}}_x \cdot (\boldsymbol{v}_{\text{el}}^{\perp} \times \boldsymbol{S}_e)$	✗	qv	–	–	✓
$\mathcal{O}_{24} = \frac{q}{m_e} \cdot \tilde{\boldsymbol{S}}_x \cdot \left(\frac{q}{m_e} \times \boldsymbol{v}_{\text{el}}^{\perp}\right)$	✗	q^2v	–	–	✓
$\mathcal{O}_{25} = \left(\frac{q}{m_e} \cdot \tilde{\boldsymbol{S}}_x \cdot \boldsymbol{v}_{\text{el}}^{\perp}\right)\left(\frac{q}{m_e} \cdot \boldsymbol{S}_e\right)$	✗	q^2v	–	–	✓
$\mathcal{O}_{26} = \left(\frac{q}{m_e} \cdot \tilde{\boldsymbol{S}}_x \cdot \frac{q}{m_e}\right)(\boldsymbol{v}_{\text{el}}^{\perp} \cdot \boldsymbol{S}_e)$	✗	q^2v	–	–	✓

scalar and fermion DM: Del Nobile, PR D98 (2018) 123003; Fitzpatrick et al, JCAP 02 (2013) 004.
vector DM in simplified models: Catena et al, JHEP 08 (2019) 030; Dent et al, PR D92 (2015) 063515.

Step 3: NR interactions from EFT

- NR interactions in QM can be considered as the low-energy limit of relativistic EFT
- At energy scale < electroweak scale Λ_{EW}, this is called low-energy EFT, i.e., LEFT

  (1) symmetries: $SU(3)_{\text{c}} \times U(1)_{\text{em}}$ and Poincare

  (2) dynamical DoFs: SM plus DM fields

  (3) no requirements on other conservation laws or renormalizability,

  relevance of effective interactions assessed by power counting in p/Λ_{EW}

  Again, EFT framework is universal, and new phys models are parameterized by effective couplings (Wilson coefficients).

- LEFT has been widely applied, in particular, in low energy processes involving light DM. extensive literature not cited here
- Here I focus on DM-electron and DM-photon interactions directly related to DM direct detection via electron recoil.

Step 3: NR interactions from EFT

- Higher-dimension operators are more suppressed by power in $p/\Lambda_{\rm EW}$:

$$\text{effective interaction} = \text{Wilson coefficient} \times \text{effective operator}$$
$$\text{dim } 4 = (4-n) + n$$
$$(\Lambda_{\rm EW})^{4-n}$$

$\rightarrow$ concentrate on first few high-dimension operators

Dim	Relativistic operators	NR reduction
Scalar case		
dim-5	$\mathcal{O}^S_{\ell\phi} = (\bar{\ell}\ell)(\phi^\dagger\phi)$	$2m_e\mathcal{O}_1$
	$\mathcal{O}^P_{\ell\phi} = (\bar{\ell}i\gamma_5\ell)(\phi^\dagger\phi)$	$-2m_e\mathcal{O}_{10}$
dim-6	$\mathcal{O}^V_{\ell\phi} = (\bar{\ell}\gamma^\mu\ell)(\phi^\dagger i\overleftrightarrow{\partial_\mu}\phi)$ (×)	$4m_em_\phi\mathcal{O}_1$
	$\mathcal{O}^A_{\ell\phi} = (\bar{\ell}\gamma^\mu\gamma_5\ell)(\phi^\dagger i\overleftrightarrow{\partial_\mu}\phi)$ (×)	$-8m_em_\phi\mathcal{O}_7$
	$\mathcal{L}^Q_\phi = \lvert(\partial_\mu - iQ_\phi eA_\mu)\phi\rvert^2$ (×)	$-4Q_\phi e^2\frac{m_em_\phi}{q^2}\mathcal{O}_1$
	$\mathcal{L}^{\rm cr}_\phi = b_\phi(\phi^\dagger i\overleftrightarrow{\partial^\mu}\phi)\partial^\nu F_{\mu\nu}$ (×)	$4b_\phi em_em_\phi\mathcal{O}_1$

x: not for real scalar

NR reduction or matching known previously

contribute via photon exchange

Step 3: NR interactions from EFT

Dim	Relativistic operators	NR reduction
Fermion case		
dim-6	$\mathcal{O}^S_{\ell\chi 1} = (\bar{\ell}\ell)(\bar{\chi}\chi)$	$4m_e m_\chi \mathcal{O}_1$
	$\mathcal{O}^S_{\ell\chi 2} = (\bar{\ell}\ell)(\bar{\chi}i\gamma_5\chi)$	$4m_e^2 \mathcal{O}_{11}$
	$\mathcal{O}^P_{\ell\chi 1} = (\bar{\ell}i\gamma_5\ell)(\bar{\chi}\chi)$	$-4m_e m_\chi \mathcal{O}_{10}$
	$\mathcal{O}^P_{\ell\chi 2} = (\bar{\ell}i\gamma_5\ell)(\bar{\chi}i\gamma_5\chi)$	$4m_e^2 \mathcal{O}_6$
	$\mathcal{O}^V_{\ell\chi 1} = (\bar{\ell}\gamma^\mu\ell)(\bar{\chi}\gamma_\mu\chi)\,(\times)$	$4m_e m_\chi \mathcal{O}_1$
	$\mathcal{O}^V_{\ell\chi 2} = (\bar{\ell}\gamma^\mu\ell)(\bar{\chi}\gamma_\mu\gamma_5\chi)$	$8m_e m_\chi(\mathcal{O}_8 - \mathcal{O}_9)$
	$\mathcal{O}^A_{\ell\chi 1} = (\bar{\ell}\gamma^\mu\gamma_5\ell)(\bar{\chi}\gamma_\mu\chi)\,(\times)$	$-8m_e(m_\chi\mathcal{O}_7 + m_e\mathcal{O}_9)$
	$\mathcal{O}^A_{\ell\chi 2} = (\bar{\ell}\gamma^\mu\gamma_5\ell)(\bar{\chi}\gamma_\mu\gamma_5\chi)$	$-16m_e m_\chi \mathcal{O}_4$
	$\mathcal{O}^T_{\ell\chi 1} = (\bar{\ell}\sigma^{\mu\nu}\ell)(\bar{\chi}\sigma_{\mu\nu}\chi)\,(\times)$	$32m_e m_\chi \mathcal{O}_4$
	$\mathcal{O}^T_{\ell\chi 2} = (\bar{\ell}\sigma^{\mu\nu}\ell)(\bar{\chi}i\sigma_{\mu\nu}\gamma_5\chi)\,(\times)$	$8m_e(m_e\mathcal{O}_{10} - m_\chi\mathcal{O}_{11} - 4m_\chi\mathcal{O}_{12})$
	$\mathcal{L}^Q_\chi = \bar{\chi}i\gamma^\mu(\partial_\mu - iQ_\chi e A_\mu)\chi\,(\times)$	$-4Q_\chi e^2 \frac{m_e m_\chi}{q^2}\mathcal{O}_1$
	$\mathcal{L}^{\rm mdm}_\chi = \mu_\chi(\bar{\chi}\sigma^{\mu\nu}\chi)F_{\mu\nu}\,(\times)$	$4\mu_\chi e\left(m_e\mathcal{O}_1 + 4m_\chi\mathcal{O}_4 + \frac{4m_e^2 m_\chi}{q^2}(\mathcal{O}_5 - \mathcal{O}_6)\right)$
	$\mathcal{L}^{\rm edm}_\chi = d_\chi(\bar{\chi}i\sigma^{\mu\nu}\gamma_5\chi)F_{\mu\nu}\,(\times)$	$d_\chi e\frac{16m_e^2 m_\chi}{q^2}\mathcal{O}_{11}$
	$\mathcal{L}^{\rm cr}_\chi = b_\chi(\bar{\chi}\gamma^\mu\chi)\partial^\nu F_{\mu\nu}\,(\times)$	$4b_\chi e\, m_e m_\chi \mathcal{O}_1$
	$\mathcal{L}^{\rm anap.}_\chi = a_\chi(\bar{\chi}\gamma^\mu\gamma_5\chi)\partial^\nu F_{\mu\nu}$	$8a_\chi e\, m_e m_\chi\,(\mathcal{O}_8 - \mathcal{O}_9)$

x: not for Majorana

cr: charge radius
anap: anapole

NR reduction or matching known previously

Step 3: NR interactions from EFT

Dim	Relativistic operators	NR reduction
Vector case A		
dim-5	$\mathcal{O}^{\mathsf{S}}_{\ell X} = (\bar\ell\ell)(X^\dagger_\mu X^\mu)$	$-2m_e\mathcal{O}_1$
	$\mathcal{O}^{\mathsf{P}}_{\ell X} = (\bar\ell i\gamma_5\ell)(X^\dagger_\mu X^\mu)$	$2m_e\mathcal{O}_{10}$
	$\mathcal{O}^{\mathsf{T}}_{\ell X1} = \frac{i}{2}(\bar\ell\sigma^{\mu\nu}\ell)(X^\dagger_\mu X_\nu - X^\dagger_\nu X_\mu)$, $(\times)$	$-4m_e\mathcal{O}_4$
	$\mathcal{O}^{\mathsf{T}}_{\ell X2} = \frac{1}{2}(\bar\ell\sigma^{\mu\nu}\gamma_5\ell)(X^\dagger_\mu X_\nu - X^\dagger_\nu X_\mu)$, $(\times)$	$-m_e(\mathcal{O}_{11}+4\mathcal{O}_{12}) + 4\frac{m_e^2}{m_X}\left(\frac{1}{3}\mathcal{O}_{10} - \mathcal{O}_{18}\right)$
dim-6	$\mathcal{O}^{\mathsf{V}}_{\ell X1} = \frac{1}{2}[\bar\ell\gamma_{(\mu}i\overleftrightarrow{D}_{\nu)}\ell](X^{\mu\dagger}X^\nu + X^{\nu\dagger}X^\mu)$	$m_e^2\mathcal{O}_1$
	$\mathcal{O}^{\mathsf{V}}_{\ell X2} = (\bar\ell\gamma_\mu\ell)\partial_\nu(X^{\mu\dagger}X^\nu + X^{\nu\dagger}X^\mu)$	$-4m_e^2(\mathcal{O}_{17}+\mathcal{O}_{20}) + \frac{4}{3}m_e(i\boldsymbol{q}\cdot\boldsymbol{v}_{\rm el}^\perp)\mathcal{O}_1$
	$\mathcal{O}^{\mathsf{V}}_{\ell X3} = (\bar\ell\gamma_\mu\ell)(X^\dagger_\rho\overleftrightarrow{\partial}_\nu X_\sigma)\epsilon^{\mu\nu\rho\sigma}$	$-4m_e m_X(\mathcal{O}_8 - \mathcal{O}_9)$
	$\mathcal{O}^{\mathsf{V}}_{\ell X4} = (\bar\ell\gamma^\mu\ell)(X^\dagger_\nu i\overleftrightarrow{\partial}_\mu X^\nu)$, $(\times)$	$-4m_e m_X\mathcal{O}_1$
	$\mathcal{O}^{\mathsf{V}}_{\ell X5} = (\bar\ell\gamma_\mu\ell)i\partial_\nu(X^{\mu\dagger}X^\nu - X^{\nu\dagger}X^\mu)$, $(\times)$	$2m_e^2\left(\mathcal{O}_5 - \mathcal{O}_6 - \frac{m_e}{m_X}\mathcal{O}_{19}\right) + 2\boldsymbol{q}^2\mathcal{O}_4 + \frac{2}{3}\frac{m_e}{m_X}\boldsymbol{q}^2\mathcal{O}_1$
	$\mathcal{O}^{\mathsf{V}}_{\ell X6} = (\bar\ell\gamma_\mu\ell)i\partial_\nu(X^\dagger_\rho X_\sigma)\epsilon^{\mu\nu\rho\sigma}$, $(\times)$	$-2m_e^2\mathcal{O}_{11}$
	$\mathcal{O}^{\mathsf{A}}_{\ell X1} = \frac{1}{2}[\bar\ell\gamma_{(\mu}\gamma_5 i\overleftrightarrow{D}_{\nu)}\ell](X^{\mu\dagger}X^\nu + X^{\nu\dagger}X^\mu)$	$-2m_e^2\left(\frac{m_e}{m_X}\mathcal{O}_9 - 4\mathcal{O}_{21} + \frac{4}{3}\mathcal{O}_7\right)$
	$\mathcal{O}^{\mathsf{A}}_{\ell X2} = (\bar\ell\gamma_\mu\gamma_5\ell)\partial_\nu(X^{\mu\dagger}X^\nu + X^{\nu\dagger}X^\mu)$	$-8m_e^2\left(\frac{1}{3}\mathcal{O}_{10} - \mathcal{O}_{18}\right)$
	$\mathcal{O}^{\mathsf{A}}_{\ell X3} = (\bar\ell\gamma_\mu\gamma_5\ell)(X^\dagger_\rho\overleftrightarrow{\partial}_\nu X_\sigma)\epsilon^{\mu\nu\rho\sigma}$	$8m_e m_X\mathcal{O}_4$
	$\mathcal{O}^{\mathsf{A}}_{\ell X4} = (\bar\ell\gamma^\mu\gamma_5\ell)(X^\dagger_\nu i\overleftrightarrow{\partial}_\mu X^\nu)$	$8m_e m_X\mathcal{O}_7$
	$\mathcal{O}^{\mathsf{A}}_{\ell X5} = (\bar\ell\gamma_\mu\gamma_5\ell)i\partial_\nu(X^{\mu\dagger}X^\nu - X^{\nu\dagger}X^\mu)$, $(\times)$	$4m_e^2\mathcal{O}_9$
	$\mathcal{O}^{\mathsf{A}}_{\ell X6} = (\bar\ell\gamma_\mu\gamma_5\ell)i\partial_\nu(X^\dagger_\rho X_\sigma)\epsilon^{\mu\nu\rho\sigma}$, $(\times)$	$4m_e^2\left(\mathcal{O}_{14} - \frac{m_e}{m_X}\mathcal{O}_{20}\right)$

x: not for real vector

First systematic NR reduction or matching

Step 3: NR interactions from EFT

Dim	Relativistic operators	NR reduction
	Vector case A	
	$\mathcal{L}_{\kappa_\Lambda} = i\frac{\kappa_\Lambda}{2}(X_\mu^\dagger X_\nu - X_\nu^\dagger X_\mu)F^{\mu\nu}\,(\times)$	$-2e\kappa_\Lambda\left[\frac{m_e}{m_X}\left(\frac{1}{3}\mathcal{O}_1 - \frac{m_e^2}{\boldsymbol{q}^2}\mathcal{O}_{19}\right) - \mathcal{O}_4 - \frac{m_e^2}{\boldsymbol{q}^2}\left(\mathcal{O}_5 - \mathcal{O}_6\right)\right]$
	$\mathcal{L}_{\tilde\kappa_\Lambda} = i\frac{\tilde\kappa_\Lambda}{2}(X_\mu^\dagger X_\nu - X_\nu^\dagger X_\mu)\tilde F^{\mu\nu}\,(\times)$	$2e\tilde\kappa_\Lambda m_e^2\frac{1}{\boldsymbol{q}^2}\mathcal{O}_{11}$
dim-6	$\mathcal{O}_{X\gamma 1} = \epsilon^{\mu\nu\rho\sigma}\left(X_\rho^\dagger \overleftrightarrow{\partial_\nu} X_\sigma\right)\partial^\lambda F_{\mu\lambda}$	$-4em_e m_X\left(\mathcal{O}_8 - \mathcal{O}_9\right)$
	$\mathcal{O}_{X\gamma 2} = \epsilon^{\mu\nu\rho\sigma} i\partial_\nu\left(X_\rho^\dagger X_\sigma\right)\partial^\lambda F_{\mu\lambda}\,(\times)$	$-2em_e^2\mathcal{O}_{11}$
	$\mathcal{O}_{X\gamma 3} = \left(X_\nu^\dagger i\overleftrightarrow{\partial^\mu} X^\nu\right)\partial^\lambda F_{\mu\lambda}$	$-4em_e m_X\mathcal{O}_1$
	$\mathcal{O}_{X\gamma 4} = \partial_\nu(X^{\mu\dagger}X^\nu + X^{\nu\dagger}X^\mu)\partial^\lambda F_{\mu\lambda}$	$4e\,m_e\left[\frac{1}{3}(i\boldsymbol{q}\cdot\boldsymbol{v}_{\rm el}^\perp)\mathcal{O}_1 - m_e\left(\mathcal{O}_{17} + \mathcal{O}_{20}\right)\right]$
	$\mathcal{O}_{X\gamma 5} = i\partial_\nu(X^{\mu\dagger}X^\nu - X^{\nu\dagger}X^\mu)\partial^\lambda F_{\mu\lambda}\,(\times)$	$e\left[2m_e^2\left(\mathcal{O}_5 - \mathcal{O}_6 - \frac{m_e}{m_X}\mathcal{O}_{19}\right) + 2\boldsymbol{q}^2\mathcal{O}_4 + \frac{2}{3}\frac{m_e}{m_X}\boldsymbol{q}^2\mathcal{O}_1\right]$
	Vector case B	
	$\tilde{\mathcal{O}}_{\ell X1}^{\rm S} = (\bar\ell\ell)X_{\mu\nu}^\dagger X^{\mu\nu}$	$4m_e m_X^2\mathcal{O}_1$
	$\tilde{\mathcal{O}}_{\ell X2}^{\rm S} = (\bar\ell\ell)X_{\mu\nu}^\dagger \tilde X^{\mu\nu}$	$4m_e^2 m_X\mathcal{O}_{11}$
dim-7	$\tilde{\mathcal{O}}_{\ell X1}^{\rm P} = (\bar\ell i\gamma_5\ell)X_{\mu\nu}^\dagger X^{\mu\nu}$	$-4m_e m_X^2\mathcal{O}_{10}$
	$\tilde{\mathcal{O}}_{\ell X2}^{\rm P} = (\bar\ell i\gamma_5\ell)X_{\mu\nu}^\dagger \tilde X^{\mu\nu}$	$4m_e^2 m_X\mathcal{O}_6$
	$\tilde{\mathcal{O}}_{\ell X1}^{\rm T} = \frac{i}{2}(\bar\ell\sigma^{\mu\nu}\ell)(X_{\mu\rho}^\dagger X_\nu^\rho - X_{\nu\rho}^\dagger X_\mu^\rho),\,(\times)$	$4m_e m_X^2\mathcal{O}_4$
	$\tilde{\mathcal{O}}_{\ell X2}^{\rm T} = \frac{1}{2}(\bar\ell\sigma^{\mu\nu}\gamma_5\ell)(X_{\mu\rho}^\dagger X_\nu^\rho - X_{\nu\rho}^\dagger X_\mu^\rho),\,(\times)$	$\frac{1}{3}m_e m_X\left[3m_X(\mathcal{O}_{11} + 4\mathcal{O}_{12}) - 4m_e(2\mathcal{O}_{10} + 3\mathcal{O}_{18})\right]$
dim-6	$\tilde{\mathcal{O}}_{X\gamma 1} = i(X_{\mu\rho}^\dagger X_\nu^\rho - X_{\nu\rho}^\dagger X_\mu^\rho)F^{\mu\nu}\,(\times)$	$2e\left[\frac{2}{3}m_X(m_e + m_X)\mathcal{O}_1 + 2m_X^2\mathcal{O}_4 + \frac{1}{\boldsymbol{q}^2}\left(2m_e^2 m_X^2(\mathcal{O}_5 - \mathcal{O}_6) - 2m_e^2 m_X(m_X - 2m_e)\mathcal{O}_{19}\right)\right]$
	$\tilde{\mathcal{O}}_{X\gamma 2} = i(X_{\mu\rho}^\dagger X_\nu^\rho - X_{\nu\rho}^\dagger X_\mu^\rho)\tilde F^{\mu\nu}\,(\times)$	$-4em_e^2 m_X^2\frac{1}{\boldsymbol{q}^2}\mathcal{O}_{11}$

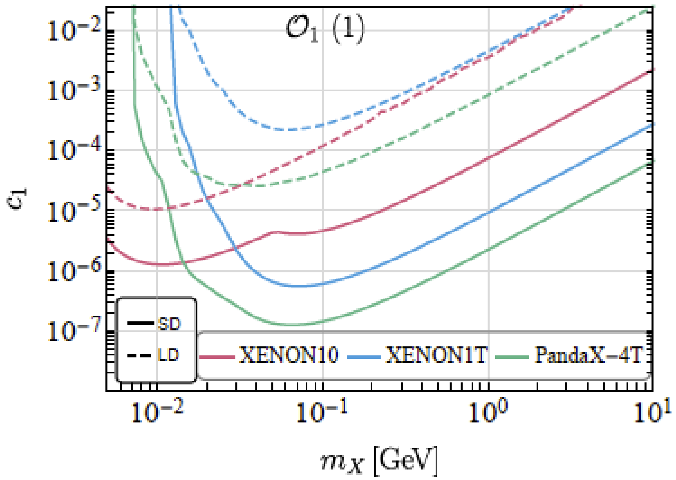
First systematic NR reduction or matching

x: not for real vector

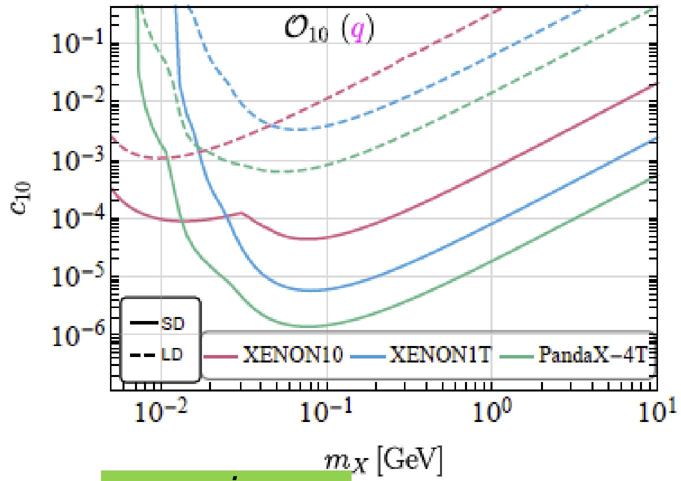
previously missed →

Step 4: constraints on NR interactions

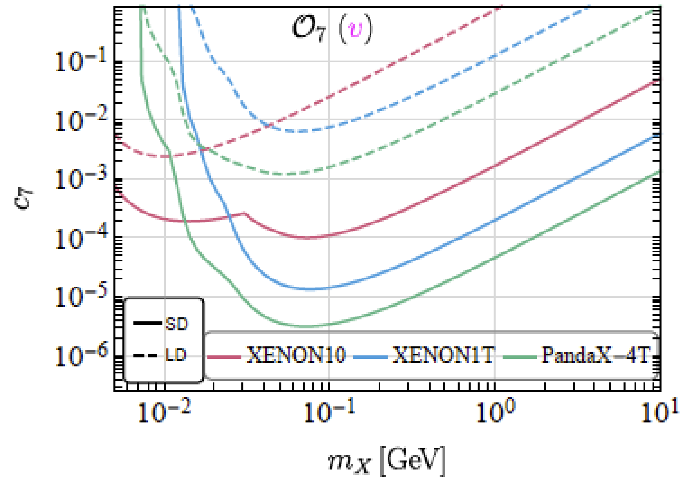
- I skip all details about numerical analysis, but show directly a few results.
- I assume one operator is activated a time.
- All results for scalar, fermion, or vector DM and for all operators can be obtained from results for 12 operators for vector DM by equivalent or scaling relations.
- Here are 3 best constraints among 12:



$c_1 \mathbf{1}_x \mathbf{1}_e$



$c_{10} \mathbf{1}_x \frac{i\boldsymbol{q}}{m_e} \cdot \boldsymbol{S}_e$



$c_7 \mathbf{1}_x \boldsymbol{v}_{\mathrm{el}}^{\perp} \cdot \boldsymbol{S}_e$

Step 5: constraints on LEFT interactions

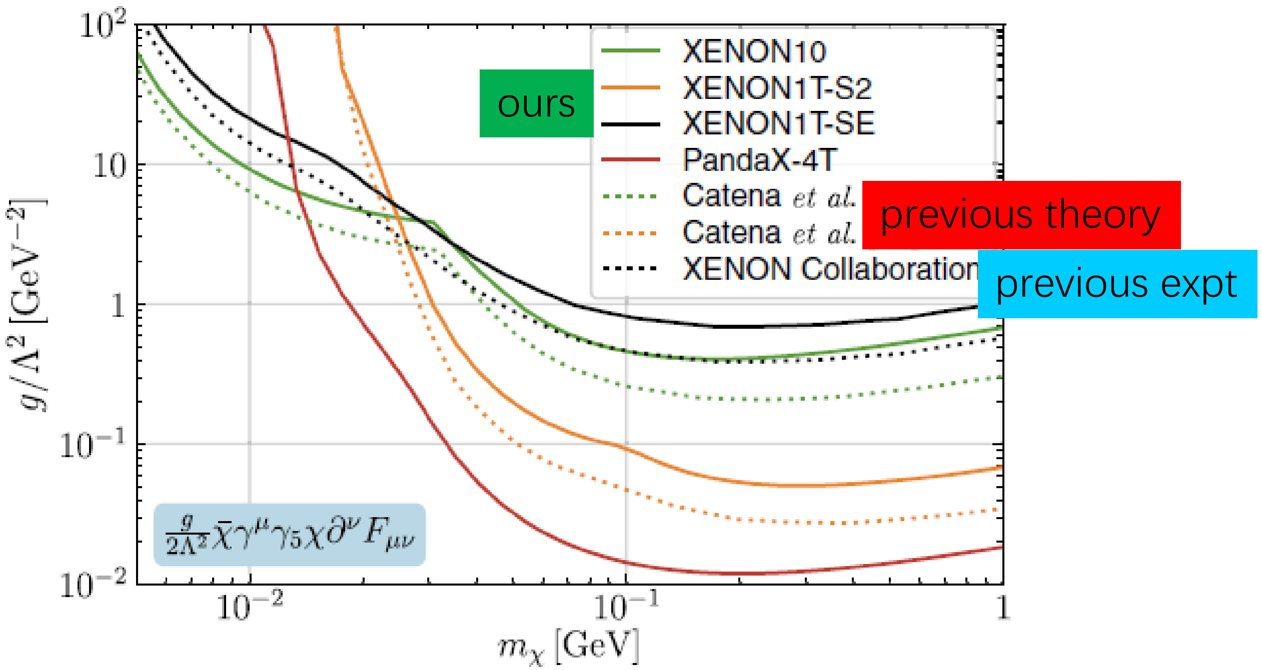
- Again I assume one LEFT operator is activated a time. But it usually reduces to several NR operators, whose interference should be included.
- I show as an example for

$$L_\chi^{\mathrm{anap}} = a_\chi \bar{\chi}\gamma^\mu\gamma^5\chi\partial^\rho F_{\mu\rho}, \ \ a_\chi = \frac{g}{2\Lambda^2}$$

  which reduces in NR limit to

$$L^{\mathrm{NR}} = c_8 \boldsymbol{v}_{\mathrm{el}}^{\perp}\cdot\boldsymbol{S}_\chi \mathbf{1}_e - c_9 \boldsymbol{S}_\chi \cdot \frac{i\boldsymbol{q}}{m_e}\times\boldsymbol{S}_e,$$

$$c_8 = c_9 = 8 e m_e m_\chi \frac{g}{\Lambda^2}$$



- Our constraints are weaker by a factor ~2 than previous theory and experiment results, because they were based on a formalism incurring a wrong sign in one ARF.

Summary

- Established a formalism for DM-bound electron scattering, aiming at direct detection of DM via electron recoil.

➢ universal for general NR and R interactions up to some orders

➢ advantages over previous formalism:

✓ 3 generalized ARFs instead of 4;

✓ ARFs depend only on atomic properties and DMRF only on DM properties, without cross reference;

✓ clear physical significance:

$\widetilde{W}_0$ and a_0 ($\widetilde{W}_{1,2}$ and $a_{1,2}$) associated with velocity-indept (dept) NR interactions.

Summary

- Provided a basis of complete and indept NR operators for spin-1 DM.
- Accomplished first systematic NR reduction/matching of LEFT operators for spin-1 DM.
- Comprehensive constraints on all NR interactions up to $\boldsymbol{q}^2$ and $\boldsymbol{v}_{\rm el}^{\perp}$ for DM of spin 0, ½, and 1.
- Comprehensive constraints on all LEFT interactions up to dim-6 (-7) for DM of spin 0, ½, and 1, with interference among reduced NR operators.
- Corrected a sign mistake in previous calculation of ARF W_2, thus modified constraints significantly.