Spatially covariant gravity with dynamical lapse function

Xian Gao

Department of Physics and Astronomy

Sun Yat-sen University

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ICTS, USTC

Based on:

- **XG,** Phys.Rev. D **90** (2014) 081501(R), [arXiv: 1406.0822]
- **XG**, Phys.Rev. D **90** (2014) 104033, [arXiv: 1409.6708]
- **XG** and Zhi-bang Yao (姚志邦), to appear

Why modified gravity

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Phenomenological:

To explain the early and later accelerated expansion of our universe.

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To explain the early and later accelerated expansion of our universe.

Theoretical:

To understand why GR is unique.

How to modify gravity (GR)

Einstein equation is quite unique.

$$\alpha G_{\mu\nu} + \lambda g_{\mu\nu} = 0$$

[Lovelock, 1971]

Any metric theory of gravity alternative to GR must satisfy (at least):

- extra degrees of freedom **(with exotic couplings with gravity)**,
- extra dimensions (brane world),
- higher derivative terms ($f(R)$),
- extension of (pseudo-)Riemannian geometry ($f(T)$),
- giving up locality.

Covariant scalar-tensor theories

From *k*-essence to Horndeski

1915 • GR

$$\mathcal{L} = \frac{1}{16\pi G} R$$

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$$\mathcal{L} = g(\phi)\, R + F(\phi, \partial\phi)$$

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k-essence:
The most general theory for a scalar field coupled to gravity, of which the Lagrangian involves up to the first derivative of the scalar field.

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Introducing second derivatives?

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Ghost?

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2011 — Generalized galileon

*[Deffayet, **XG**, Steer & Zahariade*
***Phys.Rev. D** 84 (2011) 064039]*

In *D*=4:

$$\begin{aligned}\mathcal{L} = {} & G_0(\phi, X) + G_1(\phi, X)\Box\phi \\ & + G_2(\phi, X)\, R + \frac{\partial G_2}{\partial X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ & + G_3(\phi, X)\, G^{\mu\nu}\nabla_\mu\nabla_\nu\phi \\ & - \frac{1}{6}\frac{\partial G_3}{\partial X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]\end{aligned}$$

with $X \equiv -\frac{1}{2}(\partial\phi)^2$

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$$\ldots \nabla_\nu\phi)^3]$$

Generalized galileon/Horndeski theory:

- The most general theory for a scalar field coupled to gravity, of which the Lagrangian/EoMs involve up to the second derivatives of the scalar field and the metric.
- Propagates 1 scalar + 2 tensor dofs.

Beyond Horndeski?

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*Even **beyond galileon/Horndeski** theory?*

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In D=4:

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$+G_3(\phi, X$ …

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We need some alternative approach.

Spatially covariant gravity

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Key question： how to introduce a scalar degree of freedom?

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Breaking U(1) ➡ Massive vector (2 vector + 1 scalar)

Spatially covariant gravity

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Reduction of symmetries	⇒	Extra mode(s)
Breaking U(1)	⇒	Massive vector (2 vector + 1 scalar)
Breaking whole spacetime diff	⇒	Massive gravity (2 tensor + 2 vector + 1 scalar)

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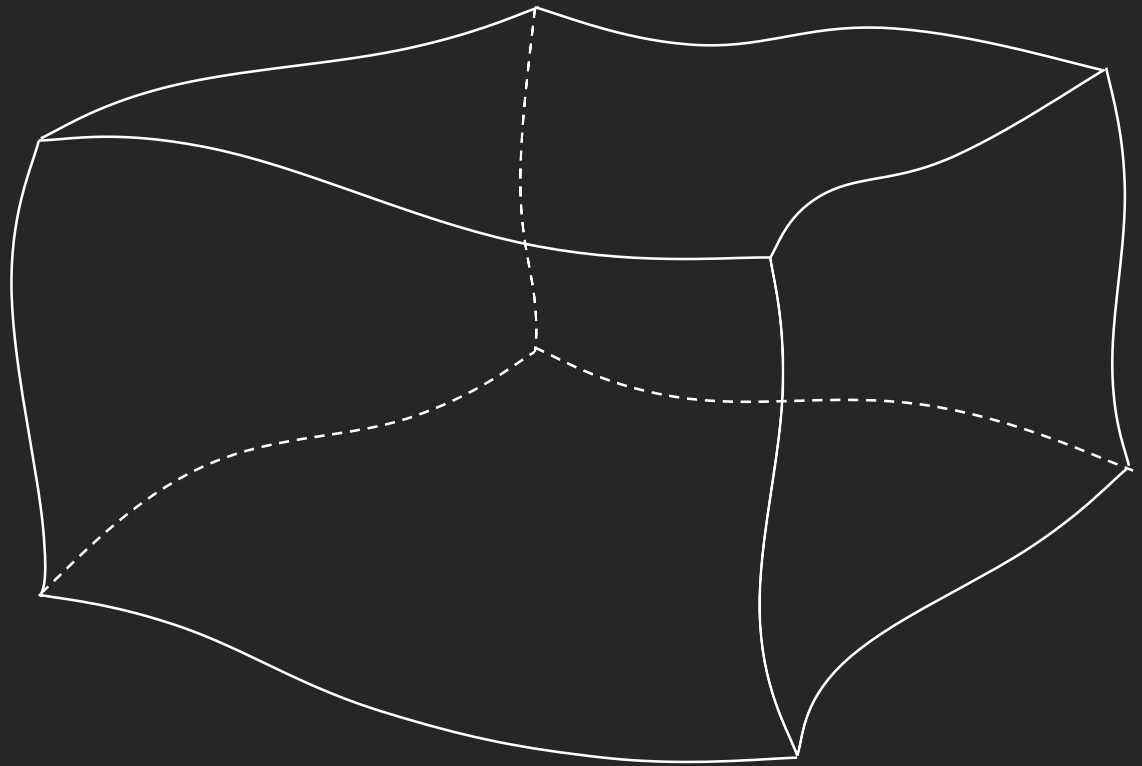
Breaking U(1)	➡	Massive vector (2 vector + 1 scalar)
Breaking whole spacetime diff	➡	Massive gravity (2 tensor + 2 vector + 1 scalar)
Breaking time diff, respecting only spatial symmetries	➡	2 tensor + 1 scalar

Foliation of spacetime



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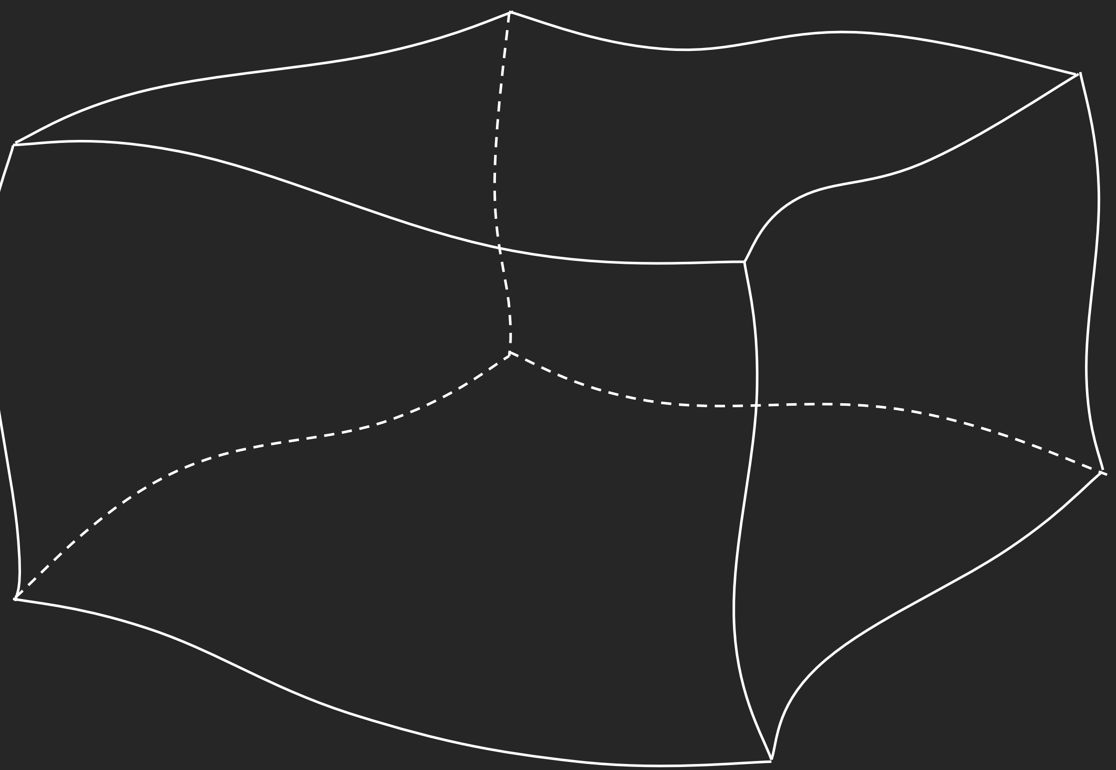

Spacetime covariant

4-D quantities

$\phi, g_{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\mu$

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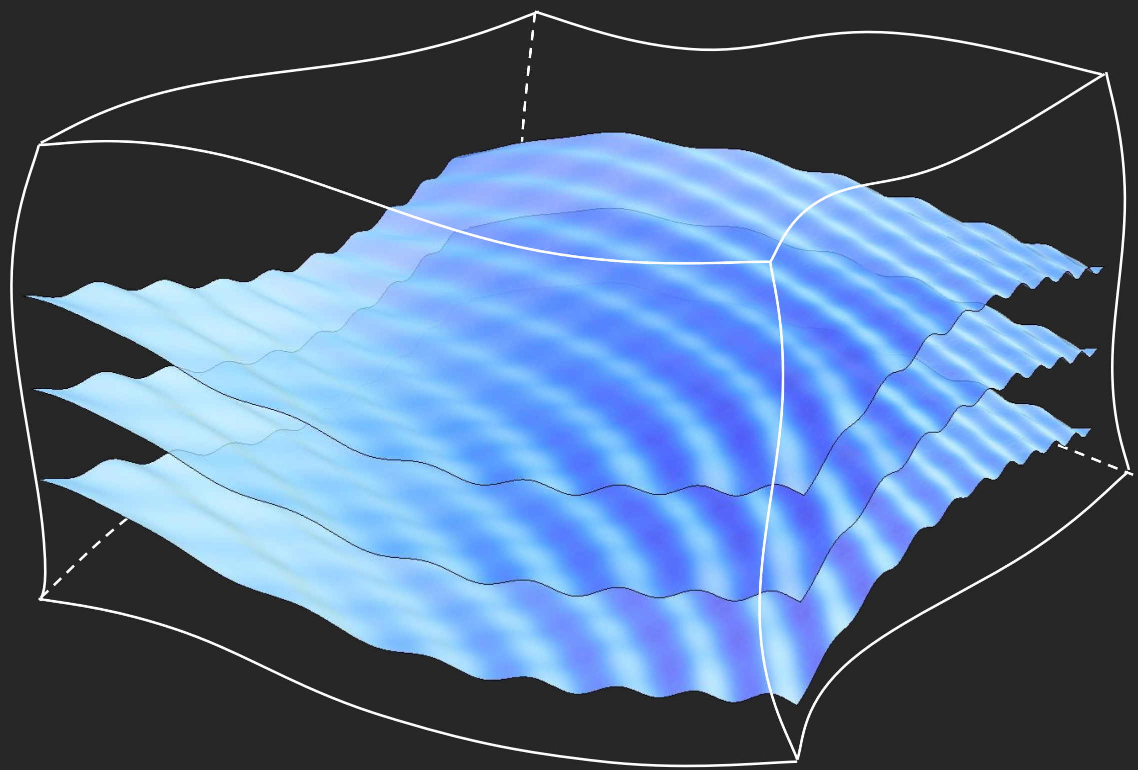

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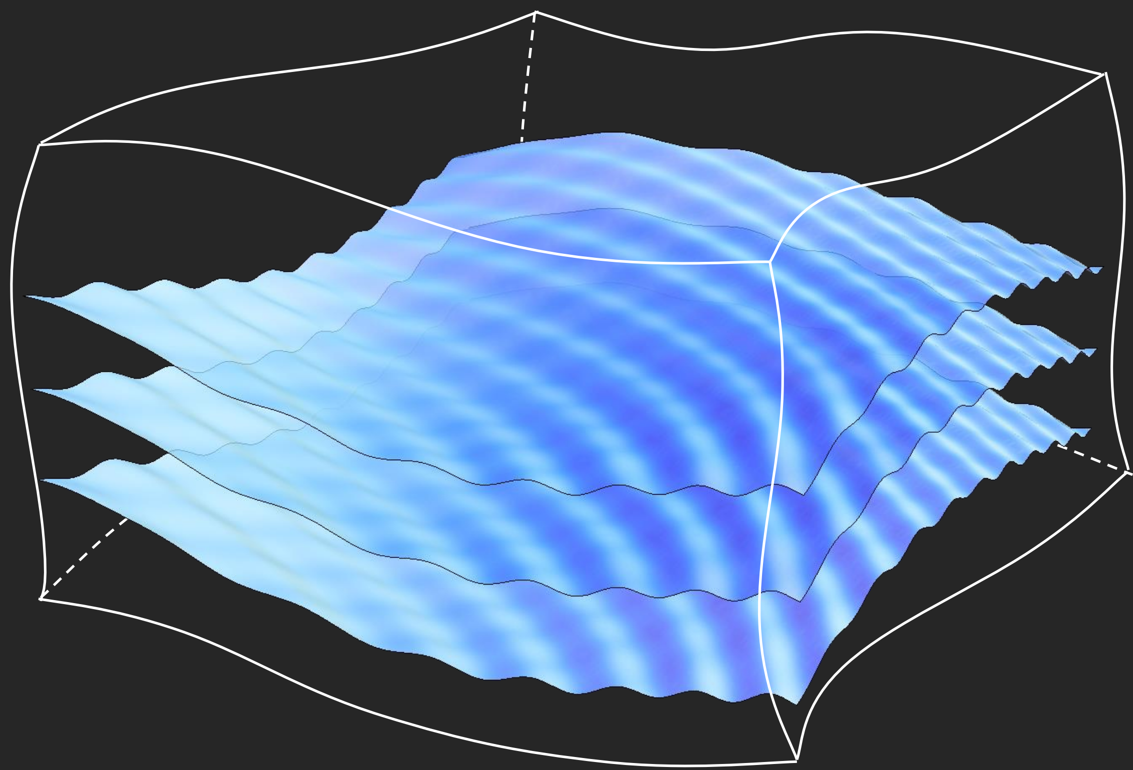

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➡

Spatially covariant

3-D quantities

$t, N, h_{ij}, R_{ij}, \nabla_i, K_{ij}$

Early examples

2004 Ghost condensation

[Arkani-Hamed, Cheng, Luty & Mukohyama]

2007 Effective field theory of inflation

[Cheung, Creminelli, Fitzpatrick, Kaplan & Senatore]

Cosmological background naturally breaks time diff, which has a preferred time direction or set of spatial sclices, on which:

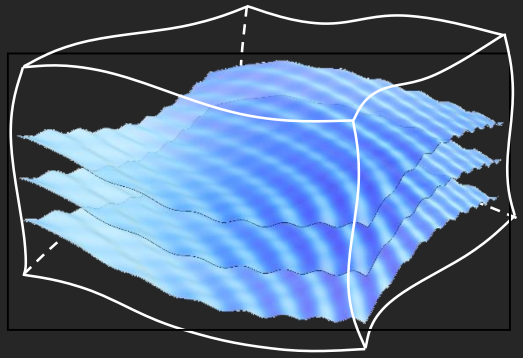
$$\phi(t,\vec{x}) = \bar{\phi}(t)$$

➡ unitary gauge (uniform scalar field gauge)

Instead of perturbatively expanding a "covariant" theory like

$$\mathcal{L}(\bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \bar{\phi} + \delta\phi)$$

we may start directly by constructing Lagrangians respecting only spatial symmetries.



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$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \Lambda(t) + f_1(t)\,\delta N + f_2(t)\,\delta N^2 + \cdots \right.$$
$$\left. + g_1(t)\,\delta K^i_i + g_2(t)\left(\delta K^i_i\right)^2 + g_3(t)\,\delta K_{ij}\delta K^{ij} + \cdots \right]$$

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- The Lagrangians are built of spatial invariants;
- The theories propagates one scalar mode (besides the two tensor modes).

Beyond Horndeski

Two equivalent languages

Spacetime covariant
Scalar-tensor theories

$$\mathcal{L}(\phi, g_{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_{\mu})$$

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Spatially covariant
gravity theories

$$\mathcal{L}(t, N, h_{ij}, R_{ij}, K_{ij}, \nabla_i)$$

Two equivalent languages

Gauge fixing: $\phi(t, \vec{x}) \to t$
(unitary gauge)

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ϕ is just the Goldstone mode that nonlinearly realizes time diff.

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2014 GLPV theory
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The first explicit example of scalar-tensor theories beyond Horndeski.

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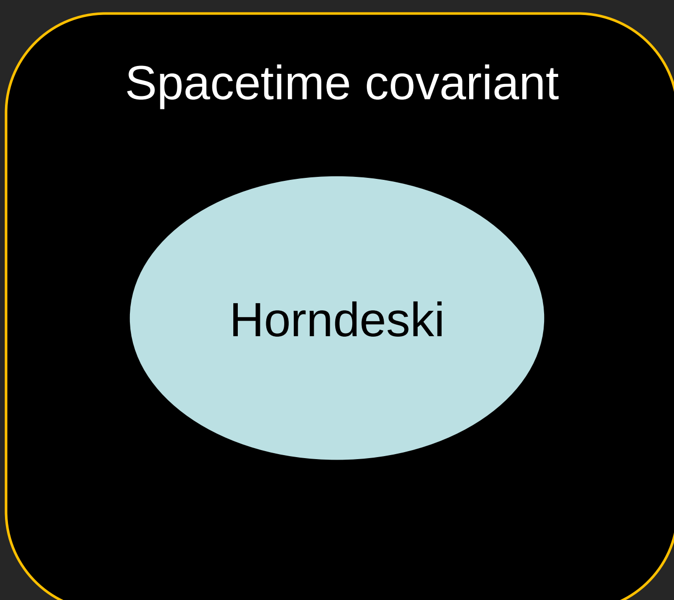

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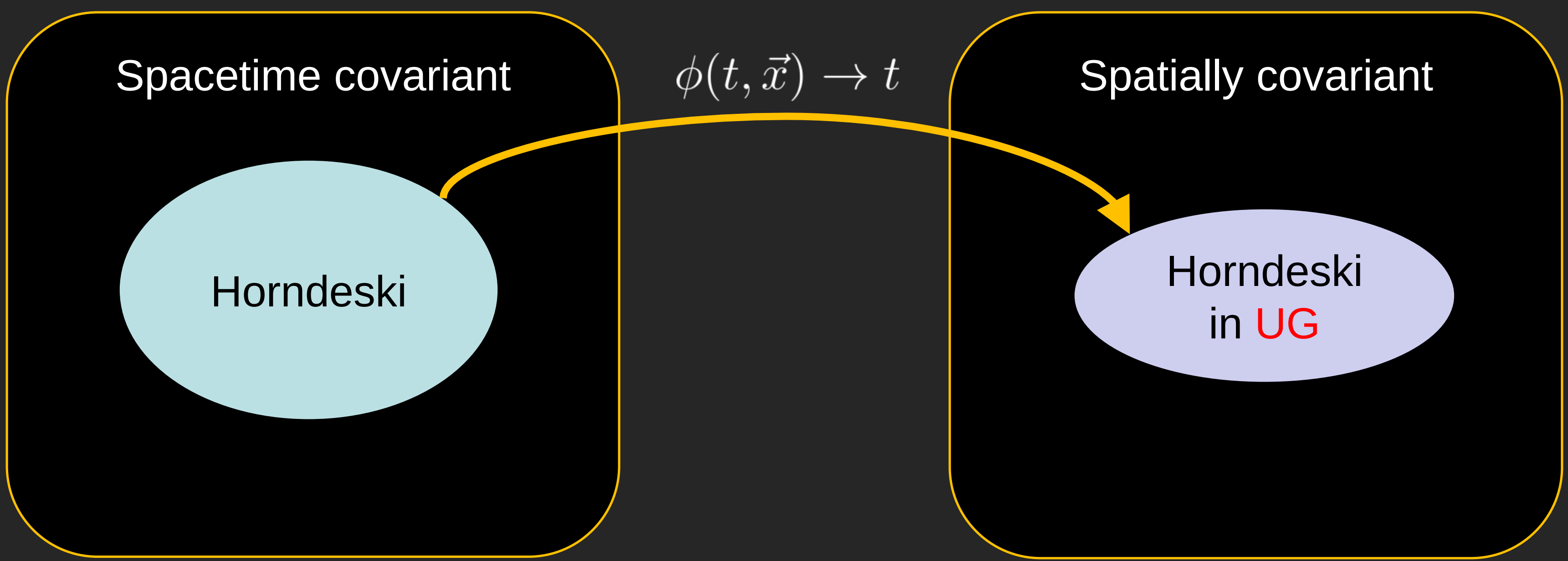

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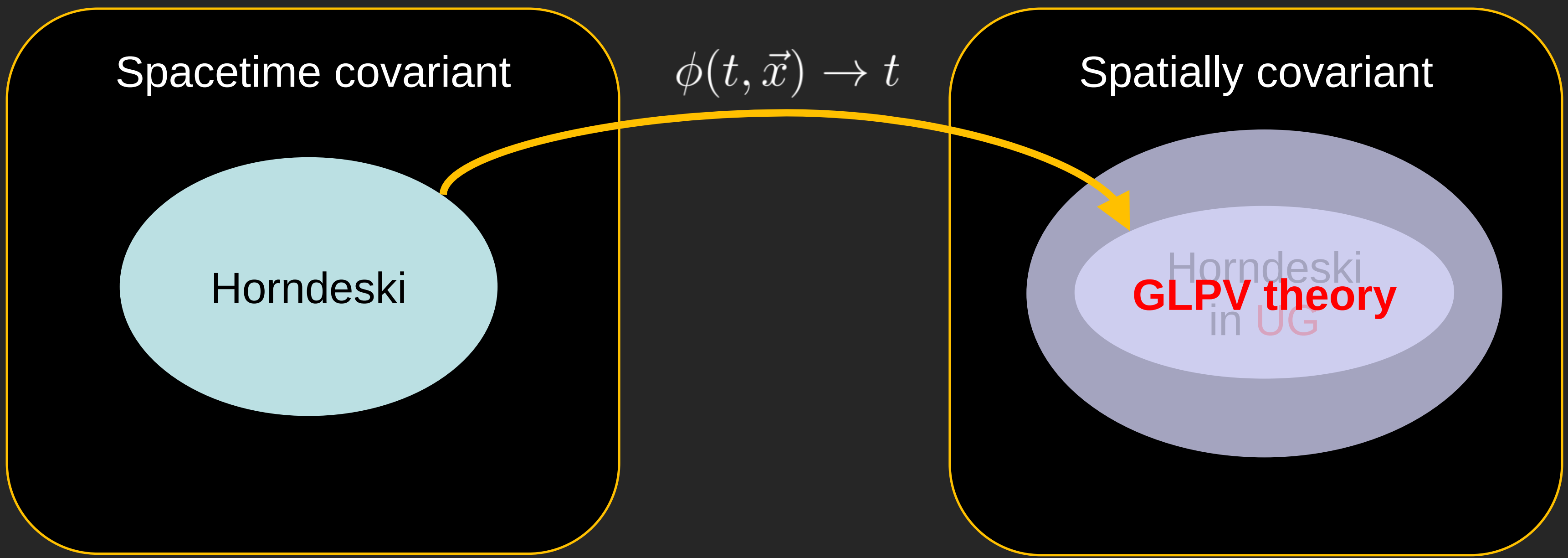

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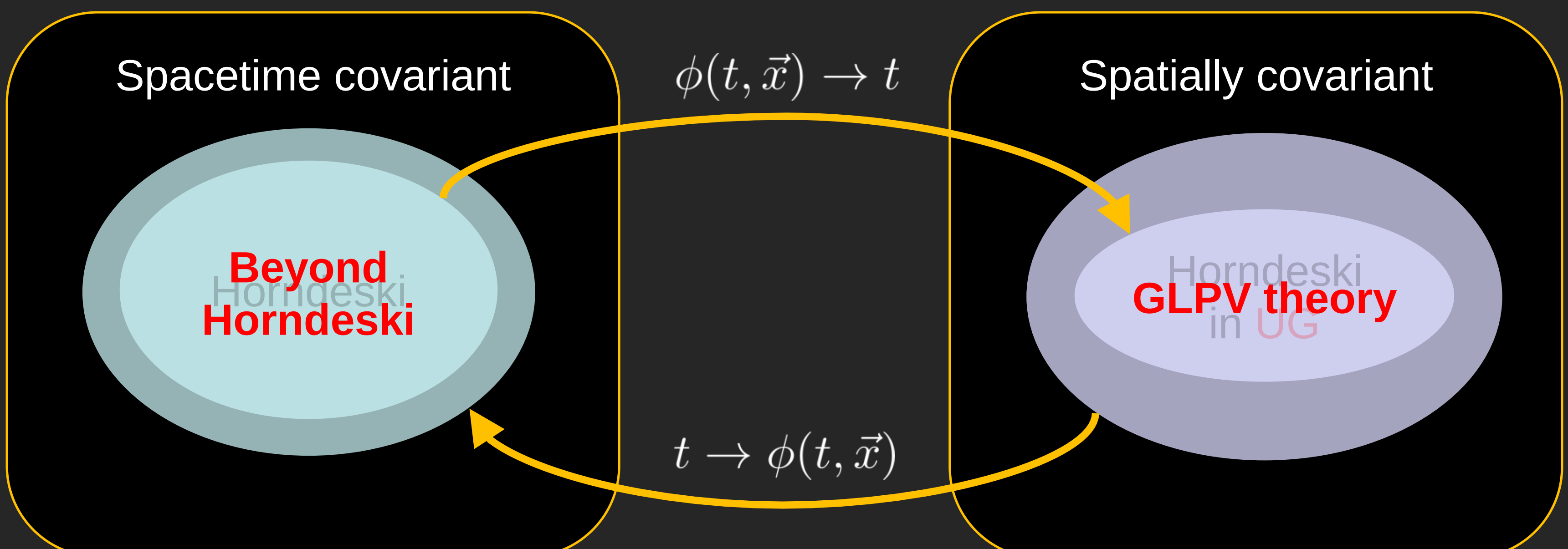

Spatially covariant gravity (ver 1.0)

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*[**X. Gao, Phys.Rev. D** 90 (2014) 081501 (**Rapid Communication**)]*

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2 tensor + 1 scalar DoFs with higher derivative EoMs.
[X.Gao, Phys.Rev. D90 (2014) 104033]

Dynamical lapse function

Geometric picture revisit

The basic picture:

4d spacetime + foliation of spacelike hypersurfaces

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4d spacetime + foliation of spacelike hypersurfaces

Basic geometric quantities:

4d metric $g_{\mu\nu}$ $\left\{ \begin{array}{l} \text{timelike normal vector field: } n_\mu = -N\nabla_\mu \phi \\ \text{Induced metric: } h_{\mu\nu} \end{array} \right.$

Geometric picture revisit

The basic picture:

4d spacetime + foliation of spacelike hypersurfaces

Basic geometric quantities:

4d metric $g_{\mu\nu}$
- timelike normal vector field: $n_\mu = -N\nabla_\mu\phi$
- Induced metric: $h_{\mu\nu}$

Basic building blocks:

$\phi,\ N,\ h_{\mu\nu}$ with derivatives in terms of
- $\pounds_{\boldsymbol{n}}$ time der.
- D_μ space der.

The action

The general action:

$$S = \int \mathrm{d}^4x \sqrt{-g}\, \mathcal{L}\left(\phi, N, h_{\mu\nu}, \pounds_{\boldsymbol{n}} N, \pounds_{\boldsymbol{n}} h_{\mu\nu}, \mathrm{D}_\mu\right)$$

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In the unitary gauge ($\phi = t$):

$$S = \int \mathrm{d}t \mathrm{d}^3 x\, N \sqrt{h}\, \mathcal{L}\left(t, N, h_{ij}, F, K_{ij}, \nabla_i\right)$$

with $F = \frac{1}{N}\left(\dot{N} - \pounds_{\vec{N}} N\right)$, $K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - \pounds_{\vec{N}} h_{ij}\right)$

↓ lapse is dynamical

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Generally, such kind of theories have 2 scalar dof's, one of which is an Ostrogradsky ghost.

Known healthy examples

Healthy examples:

[Domènech, Mukohyama, Namba, Naruko, Saitou and Watanabe,
Phys. Rev. D92 (2015), no. 8 084027, [arXiv:1507.05390]

$$g_{\mu\nu} \to \mathcal{A}(\phi, X)\, g_{\mu\nu} + \mathcal{B}(\phi, X)\, \partial_\mu\phi\partial_\nu\phi$$

$$S_{\mathrm{EH}} \to S \supset \int \mathrm{d}t\mathrm{d}^3x\, N\sqrt{h} F(t, N)\left(K^2 - K_{ij}K^{ij} + 2KL + \frac{3}{2}L^2\right)$$

$$L = \frac{\mathcal{A}_{,N}}{\mathcal{A}}\frac{1}{N}\left(\dot{N} - N^i\nabla_i N\right) + \frac{\mathcal{A}_{,t}}{\mathcal{A}N}$$

[Takahashi and Kobayashi, JCAP 1711(2017), no. 11 038]

$$g_{\mu\nu} \to -X g_{\mu\nu}$$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(f_2(\phi, X)\,\mathcal{R} + f_3(\phi, X)\,\mathcal{G}^{\mu\nu}\nabla_\mu\nabla_\nu\phi + F(g_{\mu\nu}, \phi, \nabla\phi, \nabla\nabla\phi)\right)$$

$$\to \cdots$$

Hamiltonian analysis

The action

Unitary gauge action:

$$S = \int \mathrm{d}t\mathrm{d}^3x \, N\sqrt{h}\,\mathcal{L}\left(t, N, h_{ij}, F, K_{ij}, \nabla_i\right)$$

with $\quad F = \frac{1}{N}\left(\dot{N} - \pounds_{\vec{N}}N\right), \qquad K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - \pounds_{\vec{N}}h_{ij}\right)$

An equivalent action:

$$\tilde{S} = S + \int \mathrm{d}t\mathrm{d}^3x \left[\frac{\delta S}{\delta A}\left(F - A\right) + \frac{\delta S}{\delta B_{ij}}\left(K_{ij} - B_{ij}\right)\right]$$

with $\quad S = \int \mathrm{d}t\mathrm{d}^3x \, N\sqrt{h}\mathcal{L}\left(t, N, h_{ij}, A, B_{ij}, \nabla_i\right)$

Primary constraints

17 variables: $\{N^i, A, B_{ij}, N, h_{ij}\}$

17 conjugate momenta:

$$
\begin{aligned}
\pi_i &\approx 0, \\
p &\approx 0, \\
p^{ij} &\approx 0, \\
\pi &= \frac{1}{N}\frac{\delta S}{\delta A}, \qquad \Rightarrow \qquad \tilde{\pi} := \pi - \frac{1}{N}\frac{\delta S}{\delta A} \approx 0, \\
\pi^{ij} &= \frac{1}{2N}\frac{\delta S}{\delta B_{ij}}, \qquad \Rightarrow \qquad \tilde{\pi}^{ij} := \pi^{ij} - \frac{1}{2N}\frac{\delta S}{\delta B_{ij}} \approx 0,
\end{aligned}
$$

In total 17 primary constraints: $\{\pi_i, p, p^{ij}, \tilde{\pi}, \tilde{\pi}^{ij}\}$

Playing with Poisson brackets

Primary

$[\,,\,]_{PB}$	π_k	p	p^{kl}	$\tilde{\pi}$	$\tilde{\pi}^{kl}$
π_i					
p					
p^{ij}					
$\tilde{\pi}$					
$\tilde{\pi}^{ij}$					

Playing with Poisson brackets

Primary

$[\,,\,]_{PB}$	π_k	p	p^{kl}	$\tilde{\pi}$	$\tilde{\pi}^{kl}$
π_i	0	0	0	0	0
p	0				
p^{ij}	0				
$\tilde{\pi}$	0				
$\tilde{\pi}^{ij}$	0				

Playing with Poisson brackets

$[\,,\,]_{PB}$	π_k	p	p^{kl}	$\tilde{\pi}$	$\tilde{\pi}^{kl}$	$\mathcal{C}_k$
π_i	0	0	0	0	0	
p	0					
p^{ij}	0					
$\tilde{\pi}$	0					
$\tilde{\pi}^{ij}$	0					
$\mathcal{C}_i$						

Primary

Secondary

Playing with Poisson brackets

$[\,,\,]_{PB}$	π_k	p	p^{kl}	$\tilde{\pi}$	$\tilde{\pi}^{kl}$	$\mathcal{C}_k$
π_i	0	0	0	0	0	0
p	0					0
p^{ij}	0					0
$\tilde{\pi}$	0					0
$\tilde{\pi}^{ij}$	0					0
$\mathcal{C}_i$	0	0	0	0	0	0

Primary

Secondary

Playing with Poisson brackets

$[\,,\,]_{PB}$	π_k	p	p^{kl}	$\tilde{\pi}$	$\tilde{\pi}^{kl}$	$\mathcal{C}_k$
π_i	0	0	0	0	0	0
p	0					0
p^{ij}	0					0
$\tilde{\pi}$	0					0
$\tilde{\pi}^{ij}$	0					0
$\mathcal{C}_i$	0	0	0	0	0	0

Primary

Secondary

Spatial symmetry

Playing with Poisson brackets

$[\,,]_{PB}$	π_k	p	p^{kl}	$\tilde{\pi}$	$\tilde{\pi}^{kl}$	$\mathcal{C}_k$
π_i	0	0	0	0	0	0
p	0					0
p^{ij}	0					0
$\tilde{\pi}$	0					0
$\tilde{\pi}^{ij}$	0					0
$\mathcal{C}_i$	0	0	0	0	0	0

Primary

Secondary

Playing with Poisson brackets

$[\,,\,]_{PB}$	π_k	p	p^{kl}	$\tilde{\pi}$	$\tilde{\pi}^{kl}$	$\mathcal{C}_k$
π_i	0	0	0	0	0	0
p	0					0
p^{ij}	0					0
$\tilde{\pi}$	0					0
$\tilde{\pi}^{ij}$	0					0
$\mathcal{C}_i$	0	0	0	0	0	0

Primary

Secondary

If the blue block is not degenerate, no further secondary constraint,

Playing with Poisson brackets

$[\,,]_{PB}$	π_k	p	p^{kl}	$\tilde{\pi}$	$\tilde{\pi}^{kl}$	$\mathcal{C}_k$
π_i	0	0	0	0	0	0
p	0					0
p^{ij}	0					0
$\tilde{\pi}$	0					0
$\tilde{\pi}^{ij}$	0					0
$\mathcal{C}_i$	0	0	0	0	0	0

Primary

Secondary

If the blue block is not degenerate, no further secondary constraint,

$$\begin{aligned}\#_{\rm dof} &= \frac{1}{2}\left(2\times\#_{\rm var} - 2\times\#_{\rm 1st} - \#_{\rm 2nd}\right)\\ &= \frac{1}{2}\left(2\times 17 - 2\times 6 - 14\right)\\ &= 4\end{aligned}$$

Playing with Poisson brackets

	$[\,,]_{PB}$	π_k	p	p^{kl}	$\tilde{\pi}$	$\tilde{\pi}^{kl}$	$\mathcal{C}_k$	$\mathcal{C}$
Primary	π_i	0	0	0	0	0	0	
	p	0					0	
	p^{ij}	0					0	
	$\tilde{\pi}$	0					0	
	$\tilde{\pi}^{ij}$	0					0	
Secondary	$\mathcal{C}_i$	0	0	0	0	0	0	
	$\mathcal{C}$							

The blue block must be degenerate:

→ Putting constraints on the theory,

→ Additional secondary constraint $\mathcal{C}$.

Playing with Poisson brackets

	$[\,,]_{PB}$	π_k	p	p^{kl}	$\tilde{\pi}$	$\tilde{\pi}^{kl}$	$\mathcal{C}_k$	$\mathcal{C}$
Primary	π_i	0	0	0	0	0	0	0
	p	0					0	
	p^{ij}	0					0	
	$\tilde{\pi}$	0					0	
	$\tilde{\pi}^{ij}$	0					0	
Secondary	$\mathcal{C}_i$	0	0	0	0	0	0	0
	$\mathcal{C}$	0					0	

The “extended” blue block is also degenerate.

Nullity = 1 $\rightarrow$ $\#_{\text{dof}} = \frac{1}{2}(2 \times 17 - 2 \times 7 - 14) = 3$

(correct number of dof’s)

Playing with Poisson brackets

	$[\,,]_{PB}$	π_k	p	p^{kl}	$\tilde{\pi}$	$\tilde{\pi}^{kl}$	$\mathcal{C}_k$	$\mathcal{C}$
Primary	π_i	0	0	0	0	0	0	0
	p	0					0	
	p^{ij}	0					0	
	$\tilde{\pi}$	0					0	
	$\tilde{\pi}^{ij}$	0					0	
Secondary	$\mathcal{C}_i$	0	0	0	0	0	0	0
	$\mathcal{C}$	0					0	

The "extended" blue block is also degenerate.

Nullity = 2 → $\#_{\mathrm{dof}} = \frac{1}{2}(2 \times 17 - 2 \times 8 - 13) = 2.5$

Not consistent.

Playing with Poisson brackets

	$[\,,]_{PB}$	π_k	p	p^{kl}	$\tilde{\pi}$	$\tilde{\pi}^{kl}$	$\mathcal{C}_k$	$\mathcal{C}$
Primary	π_i	0	0	0	0	0	0	0
	p	0					0	
	p^{ij}	0					0	
	$\tilde{\pi}$	0					0	
	$\tilde{\pi}^{ij}$	0					0	
Secondary	$\mathcal{C}_i$	0	0	0	0	0	0	0
	$\mathcal{C}$	0					0	

The "extended" blue block is also degenerate.

Nullity = 3 $\rightarrow$ $\#_{\rm dof} = \frac{1}{2}(2 \times 17 - 2 \times 9 - 12) = 2$

Playing with Poisson brackets

	$[\,,\,]_{PB}$	π_k	p	p^{kl}	$\tilde{\pi}$	$\tilde{\pi}^{kl}$	$\mathcal{C}_k$	$\mathcal{C}$
Primary	π_i	0	0	0	0	0	0	0
	p	0	0	0	0	0	0	0
	p^{ij}	0	0		0		0	0
	$\tilde{\pi}$	0	0	0	0	0	0	0
	$\tilde{\pi}^{ij}$	0	0		0		0	0
Secondary	$\mathcal{C}_i$	0	0	0	0	0	0	0
	$\mathcal{C}$	0	0	0	0	0	0	0

The “extended” blue block is also degenerate.

Nullity = 3 $\rightarrow$ $\#_{\text{dof}} = \frac{1}{2}(2 \times 17 - 2 \times 9 - 12) = 2$

GR as a special example.

Evolution of the theories

Evolution of the theories

1915 GR

1961 Brans-Dicke

1999 *k*-essence

Evolution of the theories

1915 GR

1961 Brans-Dicke

1999 *k*-essence

2011 Galileon/Horndeski

Evolution of the theories

1915 GR

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1999 *k*-essence

2011 Galileon/Horndeski

??

Evolution of the theories

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1961 Brans-Dicke

1999 *k*-essence

2011 Galileon/Horndeski

2004 Ghost condensate

2007 EFT of inflation

2009 Hořava gravity

Evolution of the theories

1915 GR

1961 Brans-Dicke

1999 *k*-essence

2011 Galileon/Horndeski

Beyond Horndeski

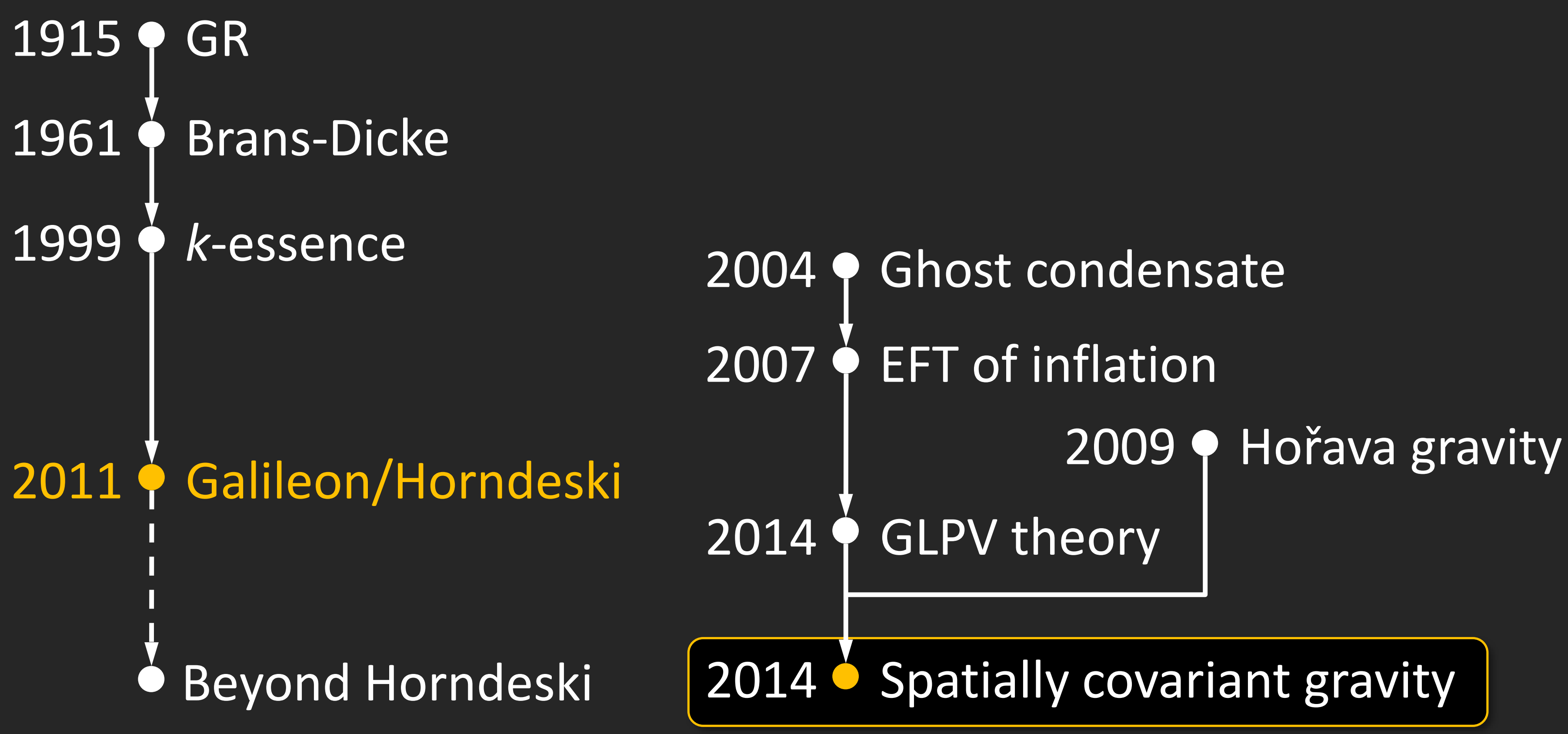
2004 Ghost condensate

2007 EFT of inflation

2009 Hořava gravity

2014 GLPV theory

Evolution of the theories



Evolution of the theories

Evolution of the theories

Evolution of the theories

1915 GR

1961 Brans-Dicke

1999 *k*-essence

ensate

“Almost” the most general scalar-tensor theories beyond Horndeski (at this moment).

2011 Galileon/Ho

Hořava gravity

Beyond H

y covariant gravity

Even beyond Horndeski

2018 SCG with dynamical lapse

Thank you for your attention!