The Λ_2 Limit of Massive Gravity

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de Rham, Tolley & SYZ, arXiv:1512.06838
de Rham, Tolley & SYZ, arXiv:1602.03721

Outline

- Introduction: dRGT massive gravity
- Non-compact NSM: A new perspective of dRGT
- The Λ_2 limit of massive gravity
- Summary

Motivations for massive gravity

- Theoretically, does a consistent, interacting, massive spin-2 field theory exist?

- Experimentally, there is only an upper bound on the graviton mass.

  de Rham, Deskins, Tolley & SYZ, "Graviton mass bounds", RMP, to appear

- The dark energy problem: Maybe the graviton mass is of the current Hubble scale?

Fierz-Pauli theory

- **Fierz-Pauli theory:**

Fierz & Pauli, 1930s

$$\mathcal{L} = -\frac{M_P^2}{4} h^{\mu\nu}\mathcal{E}_{\mu\nu}^{\rho\sigma} h_{\rho\sigma} - \frac{m^2 M_P^2}{8}(h^\mu_\nu h^\nu_\mu - h^2) + \frac{1}{2} h_{\mu\nu} T^{\mu\nu}$$

linearized GR — mass term — matter coupling

The unique, ghost-free, linear, Lorentz invariant, massive spin-2 theory

- **vDVZ discontinuity:**

van Dam, Veltman & Zakharov, 1970s

Scattering amplitude between $T^{\mu\nu}_{(1)}$ and $T^{\mu\nu}_{(2)}$

$$\mathcal{A} \propto \frac{M_P^{-2}}{(k^2 - m^2 + i\epsilon)} \left(2T^{\mu\nu}_{(1)} T_{(2)\mu\nu} - \frac{2}{3} T_{(1)} T_{(2)} \right)$$

GR value = 1

O(1) deviation!

Vainshtein Mechanism

- **Nonlinear Fierz-Pauli theory:**

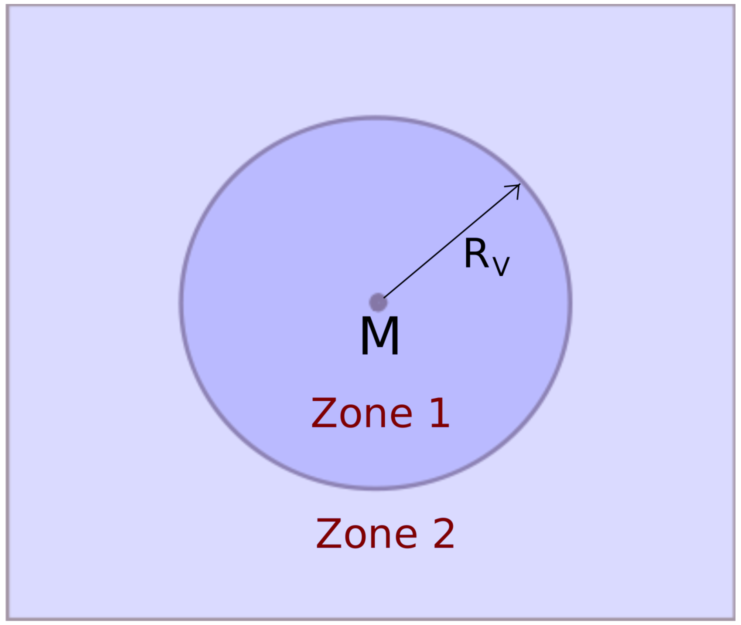
Vainshtein, 1970s

$$\mathcal{L} = M_P^2 \sqrt{-g} \left[\frac{R}{2} - \frac{m^2}{8} \left(h^\mu_\nu h^\nu_\mu - h^2 \right) + \mathcal{L}_m \right]$$

- **Vainshtein mechanism (nonlinear screening):**

Nonlinear effects suppress deviations from GR!

For example:
M: a star
Zone1: close to GR
Zone2: close to FP



"Strong gravity" regimes of MG are much more common!

Problems of Nonlinear FP

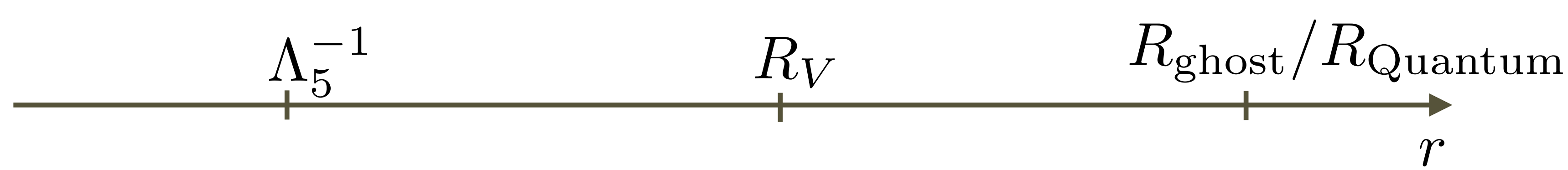
- **Boulware-Deser ghost** — Boulware & Deser, 1970s

6th polarization
Hamiltonian unbounded from below

- **Low strong coupling scale** — Arkani-Hamed, Georgi & Schwarz, 2003

$$\Lambda_5 = (M_P m^4)^{\frac{1}{5}} \sim (10^{16}\mathrm{cm})^{-1} \qquad \Lambda_n \equiv (M_P m^{n-1})^{\frac{1}{n}}$$

$$R_{\mathrm{V}} = \left(\frac{M}{M_P}\right)^{\frac{1}{5}} \Lambda_5^{-1} \qquad R_{\mathrm{ghost}} \sim R_{\mathrm{Quantum}} = \left(\frac{M}{M_P}\right)^{\frac{1}{3}} \Lambda_5^{-1}$$



Nonlinear Fierz-Pauli theory is inconsistent!

Higher order graviton potential

$$\mathcal{L} = -\frac{M_P^2}{4} h^{\mu\nu} \mathcal{E}_{\mu\nu}^{\rho\sigma} h_{\rho\sigma} - \frac{m^2 M_P^2}{8} (h_\nu^\mu h_\mu^\nu - h^2) + \frac{1}{2} h_{\mu\nu} T^{\mu\nu}$$

$$\mathcal{L} = M_P^2 \sqrt{-g} \left[\frac{R}{2} + \mathcal{L}_m - \frac{m^2}{8} (h_\nu^\mu h_\mu^\nu - h^2) + \mathcal{O}(h_{\mu\nu}^3) + \mathcal{O}(h_{\mu\nu}^4) + \cdots \right]$$

dRGT massive gravity

The dRGT model:

de Rham & Gabadadze, 2010
de Rham, Gabadadze & Tolley, 2010

$$\mathcal{L} = M_P^2 \sqrt{-g} \left(\frac{R}{2} + m^2 \left(\mathcal{K}^\mu_{[\mu} \mathcal{K}^\nu_{\nu]} + \alpha_3 \mathcal{K}^\mu_{[\mu} \mathcal{K}^\nu_\nu \mathcal{K}^\rho_{\rho]} + \alpha_4 \mathcal{K}^\mu_{[\mu} \mathcal{K}^\nu_\nu \mathcal{K}^\rho_\rho \mathcal{K}^\sigma_{\sigma]} \right) + \mathcal{L}_m \right)$$

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \mathcal{X}^\mu_\nu, \quad \mathcal{X}^\mu_\rho \mathcal{X}^\rho_\nu = g^{\mu\rho}\eta_{\rho\nu} \quad \text{or} \quad \mathcal{X}^\mu_\nu = \sqrt{g^{\mu\rho}\eta_{\rho\nu}}$$

The unique, Lorentz invariant, ghost-free non-linearization of FP theory!

The BD ghost is projected out by 2 second class constraints.

Strong coupling in dRGT

- **Strong coupling scale and Vainshtein radius**

$$\Lambda_3 = (M_P m^2)^{\frac{1}{3}} \sim (10\text{km})^{-1} \qquad R_V = \left(\frac{M}{M_P}\right)^{\frac{1}{3}} \Lambda_3^{-1}$$

GR analogy:

- **“Vainshtein on top of Vainshtein”**

$$\Lambda_{true} \gg \Lambda_3$$

Non-compact NSM:
A new perspective of dRGT

Massive gravity as NSM

- Massive gravity needs 2 metrics (spaces)

$g_{\mu\nu}$ spacetime

$\eta_{\mu\nu}$ reference space

- Stueckelberg formulation

$\eta_{\mu\nu}$ explicitly breaks diff invariance

Restore diff invariance:

$$\eta_{\mu\nu} \;\to\; \partial_\mu \phi^A \partial_\nu \phi^B \eta_{AB}$$

Massive gravity as a mapping ϕ^A:

from base space $g_{\mu\nu}$ to target space η_{AB}

Compact requirement for NSM

Typical nonlinear sigma model

$$S = \int \mathrm{d}^D x \left(-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi^A\partial_\nu\phi^B f_{AB}(\phi) - V(\phi) \right)$$

The symmetry group should be compact!

$f_{AB}(\phi)$: **Riemannian (+,+,+,...)**

Auxiliary gauge trick

- **p-brane action:**

$$S_{\text{Polyakov}} = -\frac{T_p}{2}\int \mathrm{d}^{p+1}\xi\sqrt{-\gamma}\left(\gamma^{\mu\nu}\partial_\mu X^A(\xi)\partial_\nu X^B(\xi)G_{AB}(X) - p + 1\right)$$

Ghost eliminated by diff invariance

- **Cremmer–Julia NSM**

SU(1,1)/U(1): $$S_{\text{CJ}} = \int \mathrm{d}^D x\left(|\partial_\mu\phi_1 - A_\mu\phi_1|^2 - |\partial_\mu\phi_0 - A_\mu\phi_0|^2\right)$$

$$|\phi_0|^2 - |\phi_1|^2 = 1$$

Ghost eliminated by gauge invariance

Generally: G/H non-compact group/gauge subgroup

All known non-compact NSMs use 'auxiliary gauge trick'.

dRGT massive gravity

- **BD ghost**

  is the non-compact ghost

- **dRGT massive gravity**

$$\mathcal{L}_{\mathrm{dRGT}} = \sqrt{-g}\left(\frac{M_P^2}{2}R + m^2 M_P^2 \sum_{n=0}^{D} \beta_n \mathcal{X}^{\mu_1}_{[\mu_1}\mathcal{X}^{\mu_2}_{\mu_2}\cdots\mathcal{X}^{\mu_n}_{\mu_n]} + \mathcal{L}_{\mathrm{m}}\right)$$

$$\mathcal{X}^{\mu}_{\nu} = \sqrt{g^{\mu\rho}\partial_\rho\phi^A\partial_\nu\phi^B\eta_{AB}}$$

- **Nambu-Goto p-brane as special case of dRGT**

$$\mathcal{L}_{\mathrm{NG}} = \sqrt{-g}\mathcal{X}^{\mu_1}_{[\mu_1}\mathcal{X}^{\mu_2}_{\mu_2}\cdots\mathcal{X}^{\mu_D}_{\mu_D]}$$

The Λ_2 limit of massive gravity

vDVZ and S. C. scale in Stueckelberg

- **Trivial (Minkowski) vacuum:** $g_{\mu\nu} = \eta_{\mu\nu}, \qquad \phi^\alpha = x^\alpha$
- **Helicity decomposition:**

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad \phi^\alpha = x^\alpha + A^\alpha + \partial^\alpha \pi$$

- **vDVZ discontinuity and strong coupling scale**

$$\mathcal{L}^{\text{lin}}_{\text{dRGT}} \supset -M_P^2 h\mathcal{E}h - m^2 M_P^2 \partial A \partial A + m^2 M_P^2 h \partial\partial\pi + hT$$

$$\xrightarrow{h = \tilde{h} + m^2\pi}$$

$$\mathcal{L}^{\text{lin}}_{\text{dRGT}} \supset -M_P^2 \tilde{h}\mathcal{E}\tilde{h} - m^2 M_P^2 \partial A \partial A - m^4 M_P^2 \partial\pi\partial\pi + \tilde{h}T + m^2 \pi T$$

$$\xrightarrow{\hat{h} \sim M_P \tilde{h}, \quad \hat{A} \sim m M_P A, \quad \hat{\pi} \sim m^2 M_P \pi}$$

$$\mathcal{L}_{\text{dRGT}} \sim -\hat{h}\mathcal{E}\hat{h} - \partial\hat{A}\partial\hat{A} - \partial\hat{\pi}\partial\hat{\pi} + \frac{1}{M_P}\hat{h}T + \frac{1}{M_P}\hat{\pi}T + \cdots$$

Strong coupling scale: $\Lambda_3 = (m^2 M_P)^{1/3}$

Λ_3 decoupling limit of dRGT

To study physics around Λ_3, take the limit:

$$M_P \to \infty, \quad m \to 0, \quad \Lambda_3 \to \text{fixed}$$

$$\begin{aligned}
\mathcal{S}_{\rm D.L.} = &\int \mathrm{d}^4x \, \frac{1}{8} h^{\mu\nu} \hat{\mathcal{L}}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + \frac{1}{2} h^{\mu\nu} \left(X^{(1)}_{\mu\nu} + \frac{1+\alpha_3}{\Lambda_3^3} X^{(2)}_{\mu\nu} + \frac{\alpha_3+\alpha_4}{\Lambda_3^6} X^{(3)}_{\mu\nu} \right) \qquad (3.36)\\
&- \frac{\beta_1}{4} \delta^{\mu\nu\rho\sigma}_{abcd} \left(\frac{1}{2} G^a_\mu \omega^b{}_\nu \delta^c_\rho \delta^d_\sigma + (\delta+\Pi)^a_\mu [\delta^b_\nu \omega^c{}_\rho \omega^d{}_\sigma + \frac{1}{2} \delta^b_\nu \delta^c_\rho \omega^d{}_\alpha \omega^\alpha{}_\sigma] \right)\\
&- \frac{\beta_2}{8} \delta^{\mu\nu\rho\sigma}_{abcd} \left(2 G^a_\mu (\delta+\Pi)^b_\nu \omega^c{}_\rho \delta^d_\sigma + (\delta+\Pi)^a_\mu (\delta+\Pi)^b_\nu [\omega^c{}_\rho \omega^d{}_\sigma + \delta^d_\sigma \omega^c{}_\alpha \omega^\alpha{}_\rho] \right)\\
&- \frac{\beta_3}{24} \delta^{\mu\nu\rho\sigma}_{abcd} \left((\delta+\Pi)^a_\mu (\delta+\Pi)^b_\nu (\delta+\Pi)^c_\rho \omega^d{}_\alpha \omega^\alpha{}_\sigma + 3 \omega^a{}_\mu G^b_\nu (\delta+\Pi)^c_\rho (\delta+\Pi)^d_\sigma \right) ,
\end{aligned}$$

Ondo & Tolley, 2013

N.B.: the limit is established around $g_{\mu\nu} = \eta_{\mu\nu}, \ \phi^\alpha = x^\alpha$

What about Λ_2 decoupling limit?

$$\mathcal{L}_{\rm dRGT} = \sqrt{-g}\left(\frac{M_P^2}{2}R + m^2 M_P^2 \sum_{n=0}^{D} \beta_n \mathcal{X}^{\mu_1}_{[\mu_1} \mathcal{X}^{\mu_2}_{\mu_2} \cdots \mathcal{X}^{\mu_n}_{\mu_n]} + \mathcal{L}_{\rm m}\right)$$

Λ_2 decoupling limit?

$$M_{\rm Pl} \to \infty, \quad m \to 0, \quad \Lambda_2 = \sqrt{M_{\rm Pl} m} \to \text{fixed}$$

$$S_{\rm dRGT} \to \int {\rm d}^4 x \left(-\frac{1}{4}\hat{h}^{\mu\nu}\mathcal{E}^{\rho\sigma}_{\mu\nu}\hat{h}_{\rho\sigma} + \frac{\hat{h}_{\mu\nu}}{2M_P}T^{\mu\nu} + \Lambda_2^4 \mathcal{L}_{\rm NSM}\right)$$

$$\mathcal{L}_{\rm NSM} = \sum_{n=1}^{D} \beta_n X^{\mu_1}_{[\mu_1} X^{\mu_2}_{\mu_2} \cdots X^{\mu_n}_{\mu_n]} \qquad X^\mu_\nu = \sqrt{\eta^{\mu\rho}\partial_\rho\phi^\alpha\partial_\nu\phi^\beta\eta_{\alpha\beta}}$$

Obstruction to Λ_2?

Problem:

$$\mathcal{L}_{\rm NSM} = \sum_{n=1}^{D} \beta_n X^{\mu_1}_{[\mu_1} X^{\mu_2}_{\mu_2} \cdots X^{\mu_n}_{\mu_n]} \qquad X^\mu_\nu = \sqrt{\eta^{\mu\rho}\partial_\rho\phi^\alpha\partial_\nu\phi^\beta\eta_{\alpha\beta}}$$

Around $\phi^\alpha = x^\alpha$:

$$\mathcal{L}^{\rm lin}_{\rm NSM} \sim -\partial^\mu \hat{A}^\nu \partial_{[\mu}\hat{A}_{\nu]}$$

1.U(1) gauge symmetry: Λ_2 limit impossible

2. $\hat{\pi}$ strongly coupled: Λ_2 limit possible

Check the number of DoFs

DoFs in dRGT NSM

$$\mathcal{L}_{\rm NSM} = \sum_{n=1}^{D} \beta_n X^{\mu_1}_{[\mu_1} X^{\mu_2}_{\mu_2} \cdots X^{\mu_n}_{\mu_n]}$$

$$X^{\mu}_{\nu} = \sqrt{\eta^{\mu\rho}\partial_\rho\phi^\alpha\partial_\nu\phi^\beta\eta_{\alpha\beta}}$$

- **Nonlinear Hamiltonian analysis**
- **Perturbations on a generic background**
- **Find an exact background and check DoFs**

There are 3 DoFs in 4D!

→ Λ_2 **limit is possible!**

Stable backgrounds in dRGT NSM

$$\mathcal{L}_{\rm NSM} = \sum_{n=1}^{D} \beta_n X^{\mu_1}_{[\mu_1} X^{\mu_2}_{\mu_2} \cdots X^{\mu_n}_{\mu_n]} \qquad\qquad X^{\mu}_{\nu} = \sqrt{\eta^{\mu\rho}\partial_\rho\phi^\alpha\partial_\nu\phi^\beta\eta_{\alpha\beta}}$$

There exist non-trivial vacua $\phi^\alpha = \bar{\phi}^\alpha(x)$

- **activate all 3 DoFs**
- **are free of ghosts**
- **are free of gradient instabilities**

Λ_2 decoupling limit

There exists a Λ_2 decoupling limit:

$$M_{\rm Pl} \to \infty, \qquad m \to 0, \qquad \Lambda_2 = \sqrt{M_{\rm Pl} m} \to \text{fixed}$$

$$S_{\rm dRGT} \to \int \mathrm{d}^4 x \left(-\frac{1}{4} \hat{h}^{\mu\nu} \mathcal{E}^{\rho\sigma}_{\mu\nu} \hat{h}_{\rho\sigma} + \frac{\hat{h}_{\mu\nu}}{2M_P} T^{\mu\nu} + \Lambda_2^4 \mathcal{L}_{\rm NSM} \right)$$

$$\mathcal{L}_{\rm NSM} = \sum_{n=1}^{D} \beta_n X^{\mu_1}_{[\mu_1} X^{\mu_2}_{\mu_2} \cdots X^{\mu_n}_{\mu_n]} \qquad X^\mu_\nu = \sqrt{\eta^{\mu\rho} \partial_\rho \phi^\alpha \partial_\nu \phi^\beta \eta_{\alpha\beta}}$$

around a non-trivial vacuum or Λ_2 vacuum:

$$g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(m^2), \quad \phi^A = \bar{\phi}^A(x) \neq x^A$$

unitary gauge: $g_{\mu\nu} = \partial_\mu \tilde{\phi}^A \partial_\nu \tilde{\phi}^B \eta_{AB} + \mathcal{O}(m^2)$

No vDVZ discontinuity

$$S_{\mathrm{dRGT}} \to \int \mathrm{d}^4x \left(-\frac{1}{4}\hat{h}^{\mu\nu}\mathcal{E}^{\rho\sigma}_{\mu\nu}\hat{h}_{\rho\sigma} + \frac{\hat{h}_{\mu\nu}}{2M_P}T^{\mu\nu} + \Lambda_2^4\mathcal{L}_{\mathrm{NSM}} \right)$$

No vDVZ discontinuity around Λ_2 vacua!

Easily pass GR tests!

Phenomenologically more natural!

Λ_2 vacua have Vainshtein mechanism already built-in!

Strong coupling scale

$$S_{\mathrm{dRGT}} \to \int \mathrm{d}^4x \left(-\frac{1}{4}\hat{h}^{\mu\nu}\mathcal{E}^{\rho\sigma}_{\mu\nu}\hat{h}_{\rho\sigma} + \frac{\hat{h}_{\mu\nu}}{2M_P}T^{\mu\nu} + \Lambda_2^4\mathcal{L}_{\mathrm{NSM}} \right)$$

Explicit scale: Λ_2

$$\bar{\phi}^{\alpha} = x^{\rho}\left[H^{\mu}_{\rho} + M^{\alpha}{}_{\rho\sigma}\left(\frac{x^{\sigma}}{L}\right) + \mathcal{O}\left(\left(\frac{x}{L}\right)^2\right)\right]$$

Implicit scale: L^{-1}

Strong coupling scale:

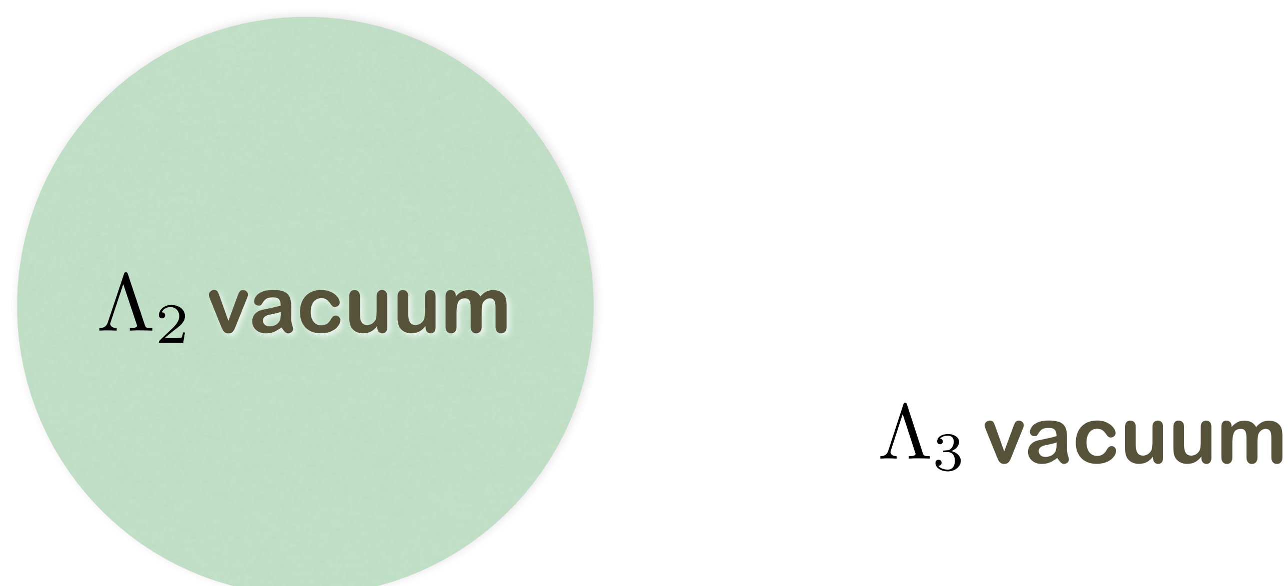
$$(\Lambda_2 L^{-3})^{\frac{1}{4}} < \Lambda_{2*} < (\Lambda_2 L^{-1})^{\frac{1}{2}}$$

$$\Lambda_{2*} > \Lambda_3$$

Relation between Λ_2 and Λ_3 vacua

All DoFs are activated by

$$g_{\mu\nu} = \eta_{\mu\nu}, \quad \phi^\alpha = x^\alpha + \epsilon B^\alpha$$



Summary

- By studying dRGT NSM, we find that dRGT has a Λ_2 decoupling limit

$$M_{\rm Pl} \to \infty, \quad m \to 0, \quad \Lambda_2 = \sqrt{M_{\rm Pl} m} \to \text{fixed}$$

$$S_{\rm dRGT} \to \int {\rm d}^4 x \left(-\frac{1}{4} \hat{h}^{\mu\nu} \mathcal{E}^{\rho\sigma}_{\mu\nu} \hat{h}_{\rho\sigma} + \frac{\hat{h}_{\mu\nu}}{2M_P} T^{\mu\nu} + \Lambda_2^4 \mathcal{L}_{\rm NSM} \right)$$

$$\mathcal{L}_{\rm NSM} = \sum_{n=1}^{D} \beta_n X^{\mu_1}_{[\mu_1} X^{\mu_2}_{\mu_2} \cdots X^{\mu_n}_{\mu_n]} \qquad X^\mu_\nu = \sqrt{\eta^{\mu\rho} \partial_\rho \phi^\alpha \partial_\nu \phi^\beta \eta_{\alpha\beta}}$$

Around a Λ_2 vacuum

$$g_{\mu\nu} = \partial_\mu \bar{\phi}^\alpha \partial_\nu \bar{\phi}^\beta \eta_{\alpha\beta} + \mathcal{O}(m^2)$$

- no vDVZ discontinuity
- strong coupling scale is raised to quasi-Λ_2

Summary (2)

- By product: discovered a "genuine" ghost-free, non-compact NSM

$$\mathcal{L}_{\rm NSM} = \sum_{n=1}^{D} \beta_n X^{\mu_1}_{[\mu_1} X^{\mu_2}_{\mu_2} \cdots X^{\mu_n}_{\mu_n]} \qquad X^\mu_\nu = \sqrt{\eta^{\mu\rho}\partial_\rho\phi^\alpha\partial_\nu\phi^\beta\eta_{\alpha\beta}}$$

Uniqueness of dRGT implies uniqueness of non-compact NSM

Thank you!