Spacetime Geometry of Black Holes Wormholes and Time Machines

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Event Horizon

Event Horizon Telescope

Apr. 10, 2019

- Space-like singularity
- Classical Black Hole:
  only seen at infinite future.
- Singularity is unphysical.
- What is a black hole?

General Relativity

Distance

$$ds^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu}(x) dx^\mu dx^\nu$$

Einstein's eqation

$$G_{\mu\nu}[g] = \kappa T_{\mu\nu} \qquad (\kappa = 8\pi G_N)$$

predictive?

Energy-Momentum Tensor

$$T_{\mu\nu} = \begin{bmatrix} \rho & \vec{j} \\ \vec{j} & \overleftrightarrow{p} \end{bmatrix}$$

Classical Energy Condition

Classical Matter $$\sum_{\mu,\nu=0}^{3} T_{\mu\nu}k^{\mu}k^{\nu} \geq 0$$

★ Weak Energy Condition $$k \ni \sum_{\mu,\nu} g_{\mu\nu}k^{\mu}k^{\nu} < 0$$

★ Null Energy Condition $$k \ni \sum_{\mu,\nu} g_{\mu\nu}k^{\mu}k^{\nu} = 0$$

“Energy is non-negative.”

Quantum Energy Conditions

Quantum Fields $\quad T_{\mu\nu} \quad \rightarrow \quad \langle T_{\mu\nu} \rangle$

- ★ Average Weak/Null Energy Condition
  (weak but *violated*)

$$\int_\gamma \sum_{\mu,\nu} \langle T_{\mu\nu} \rangle k^\mu k^\nu d\tau \geq 0$$

- ★ Quantum Inequalities [Ford, Roman, Fewester]

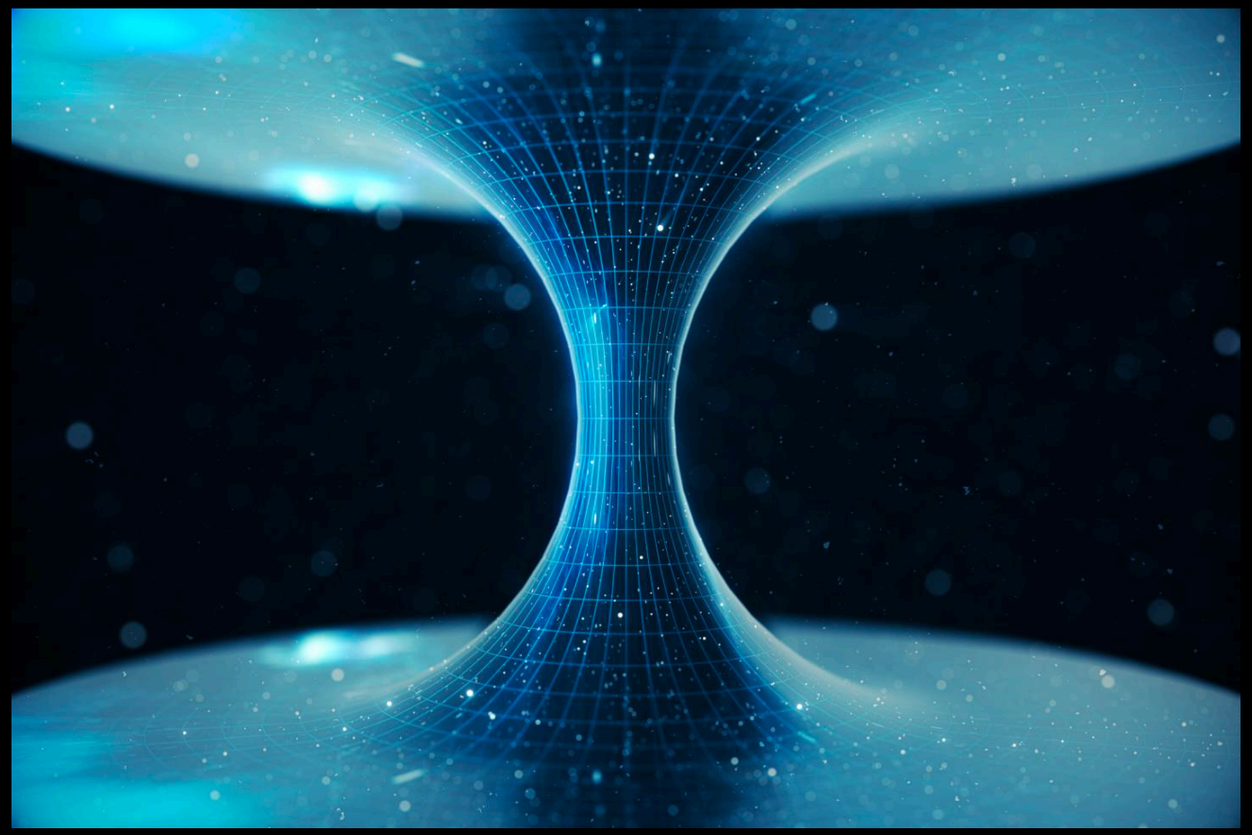
$$\sum_{\mu,\nu} \langle T_{\mu\nu} \rangle k^\mu k^\nu \geq -\frac{C}{\tau^4}$$

Energy Conditions and Geometry

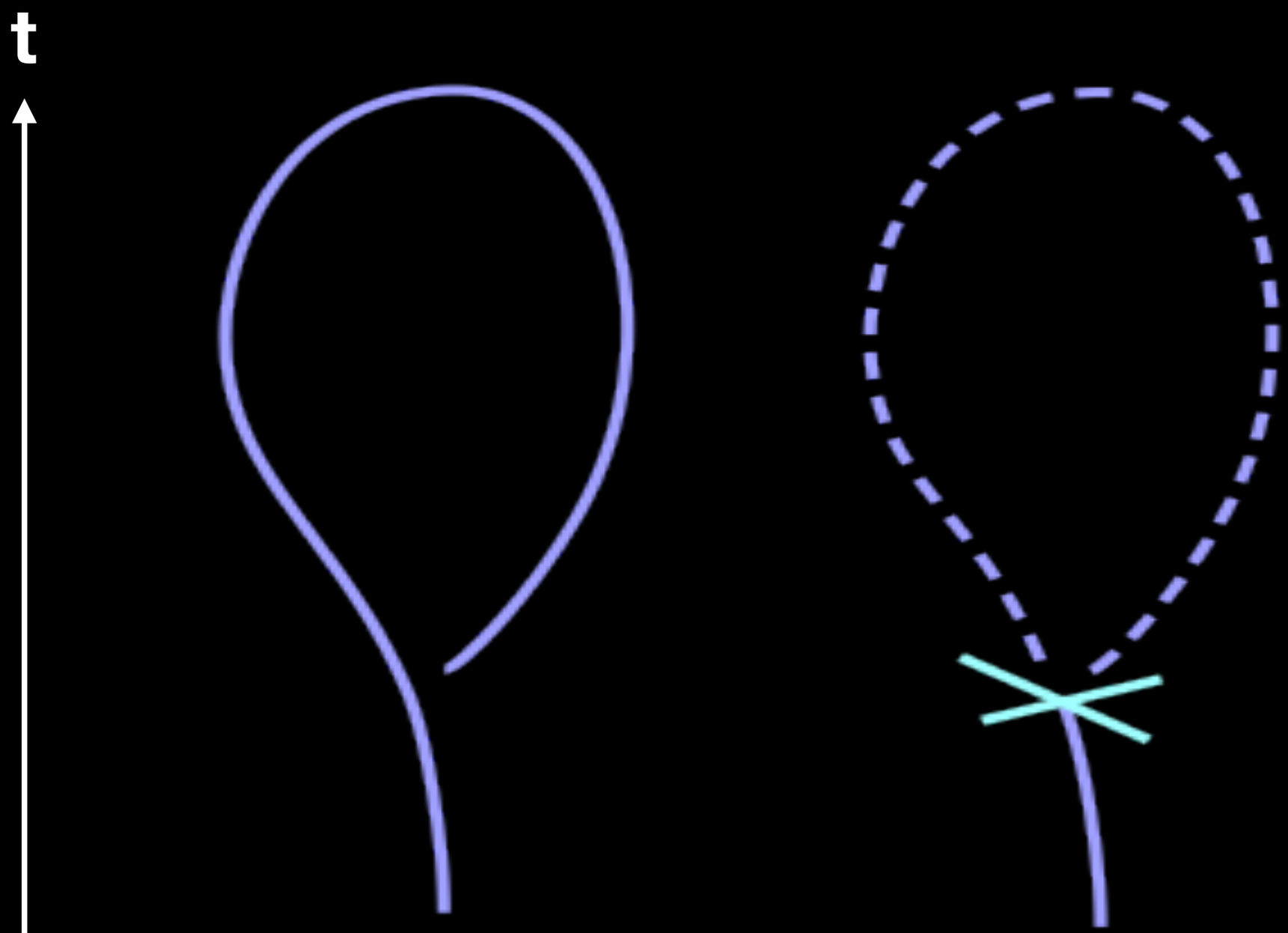
ANEC broken → Traversable Wormholes. [Morris-Thorne 88]

Traversable Wormholes can be used as *time machines*.
[Morris-Thorne-Yurtsever 88]

Paradoxes → Consistency condition



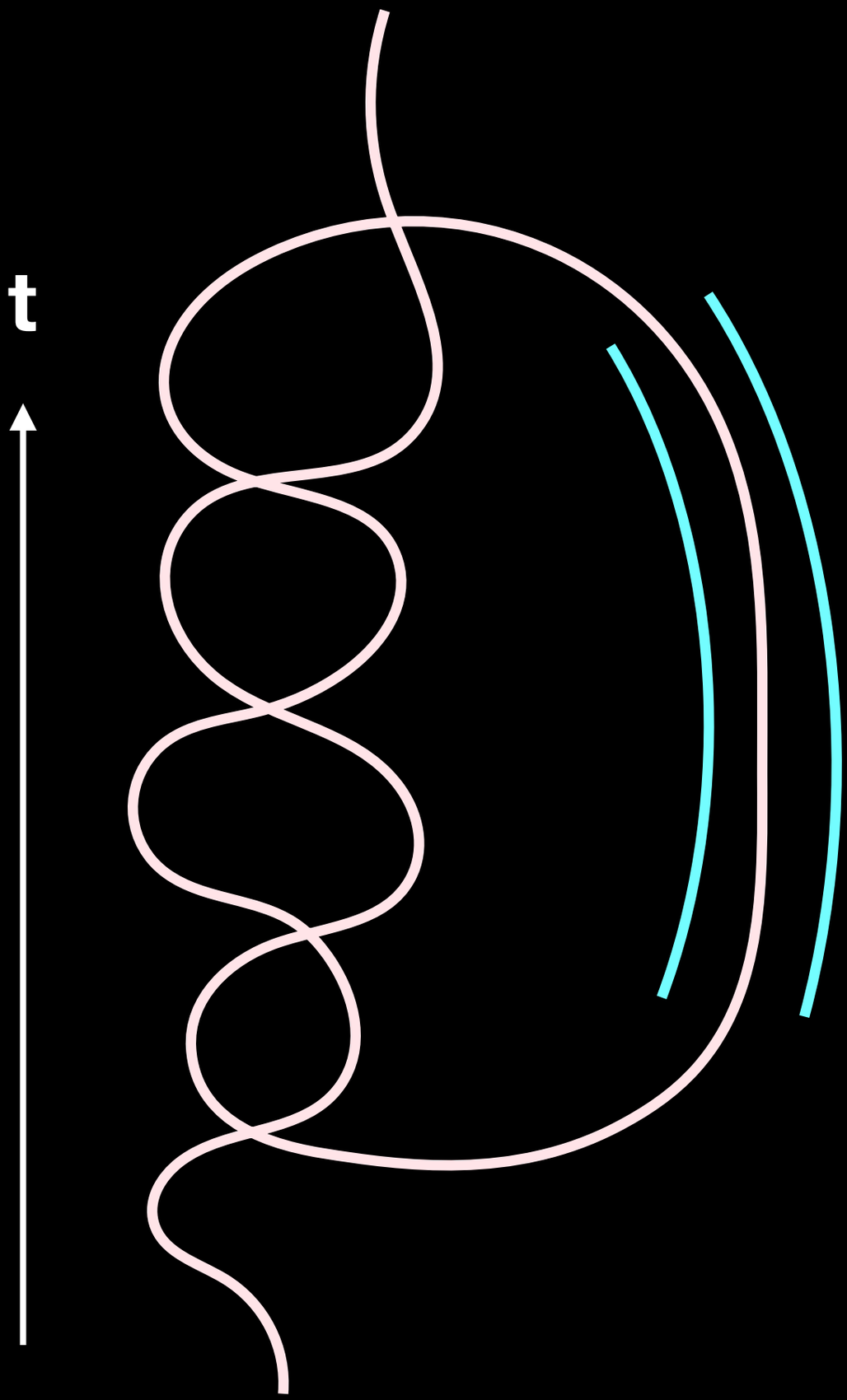
Grandfather Paradox

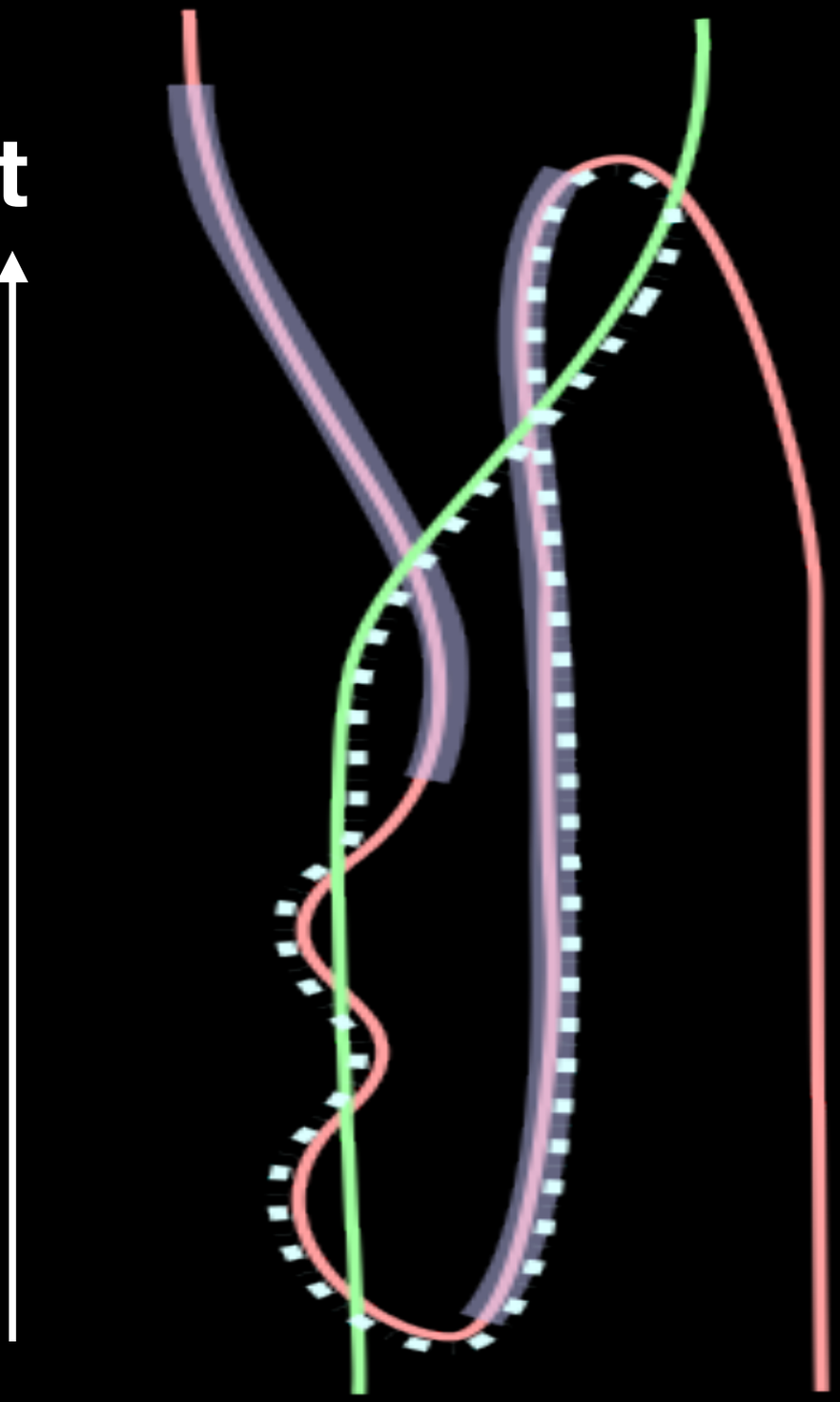


Inconsistency!

Bootstrap Paradox

Information?

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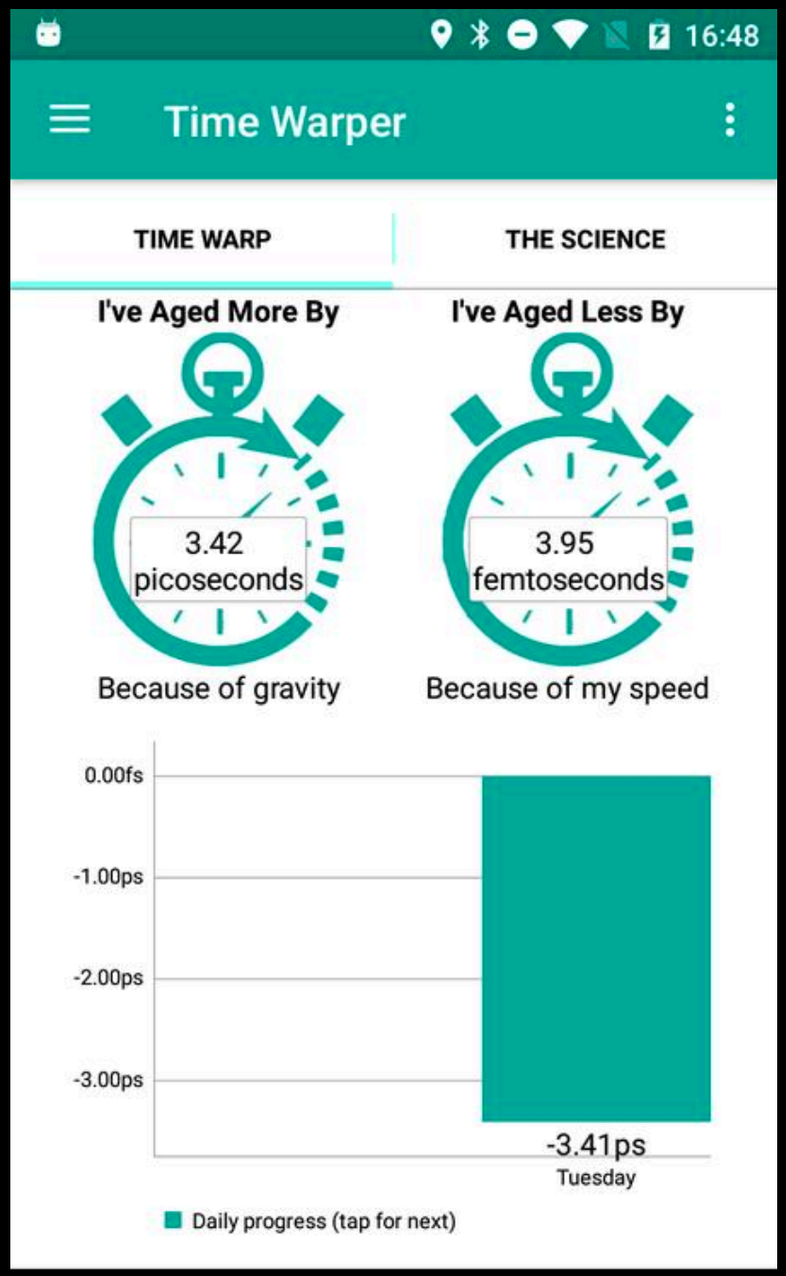
Someday in the past he will find her....
Somewhere in Time
CHRISTOPHER REEVE
"SOMEWHERE IN TIME"
JANE SEYMOUR · CHRISTOPHER PLUMMER
TERESA WRIGHT
RICHARD MATHESON
JOHN BARRY

Time Travel

- Simplest way to time travel
- Faster than light (Special Relativity)
- Wormhole (General Relativity)



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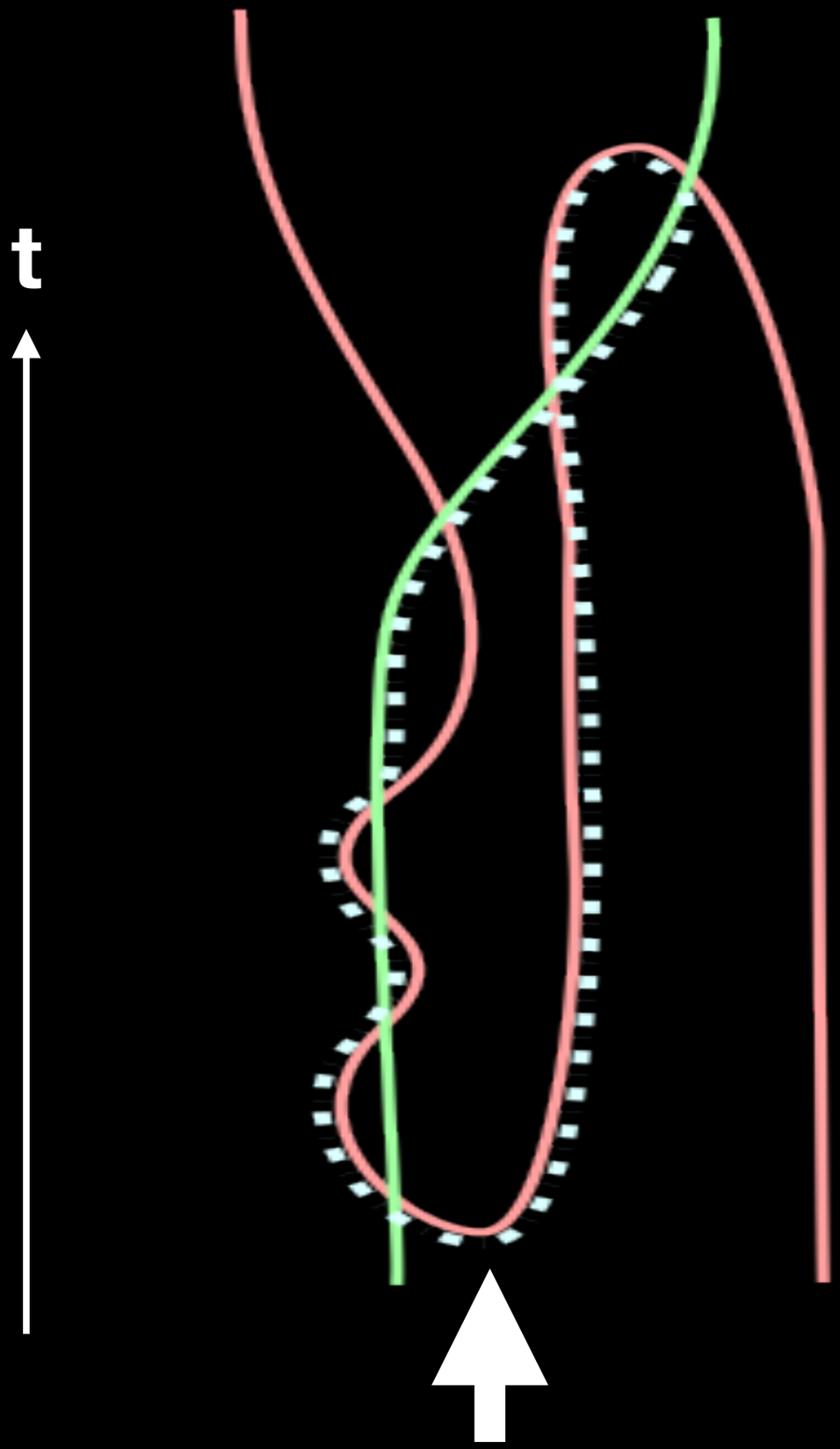
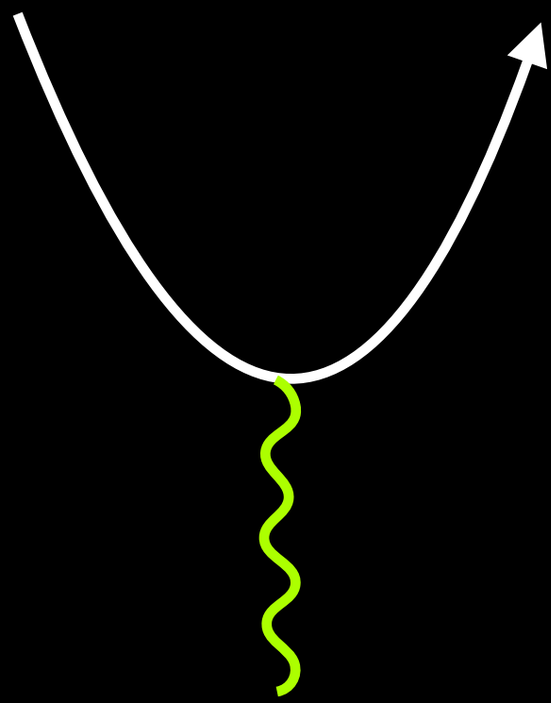
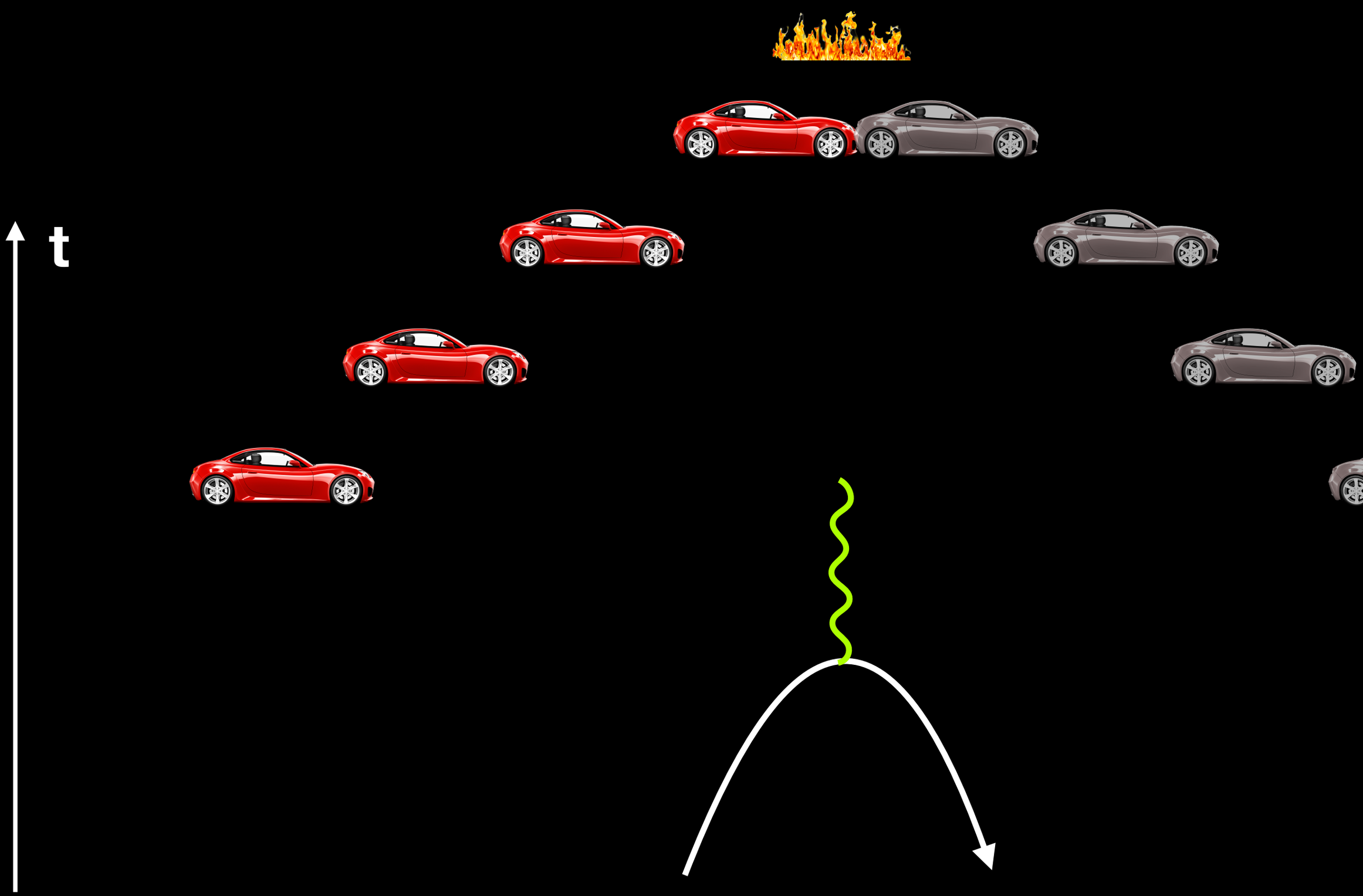




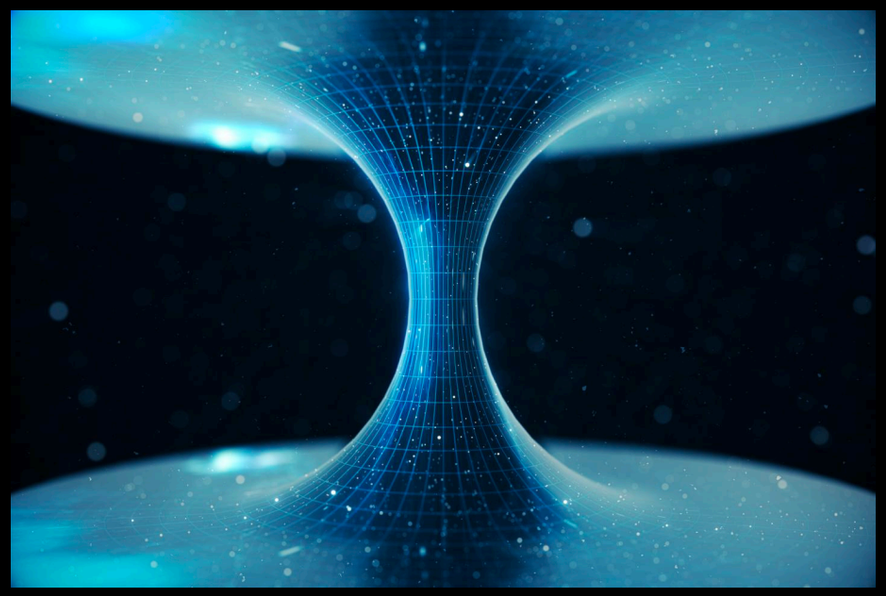

BACK
TO THE FUTURE
OUTATIME

Continuity



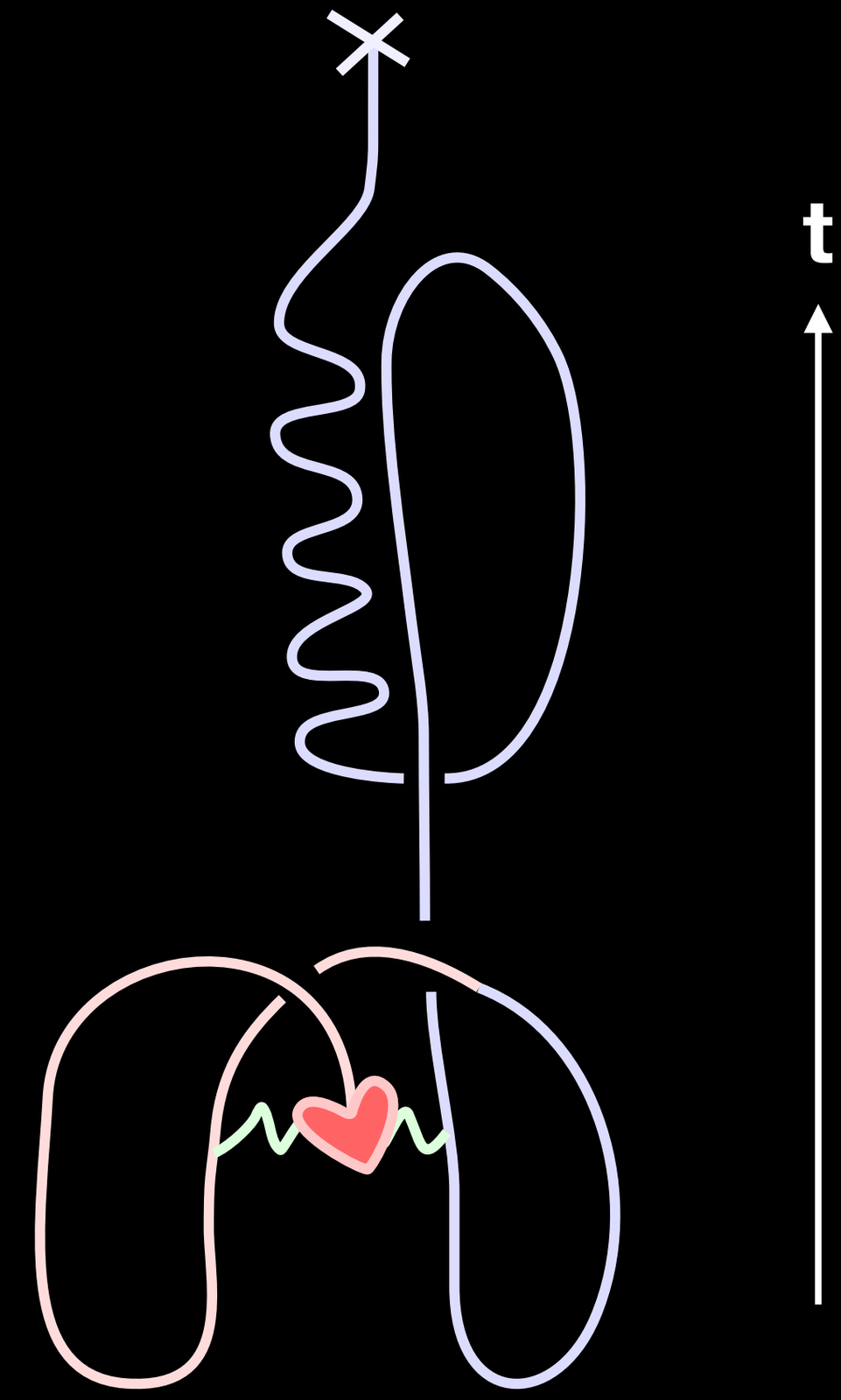

SPECIAL Newsweek EDITION
DOCTOR WHO
THE ULTIMATE WHONIVERSE COMPANION*
Featuring interviews with Sylvester McCoy, Arthur Darvill and more.
*Sorry...we couldn't resist.


POLICE PUBLIC BOX




Doctor Who:
Bigger On The Inside

Consistency?



[Wheeler]

Spacetime theory

⇣

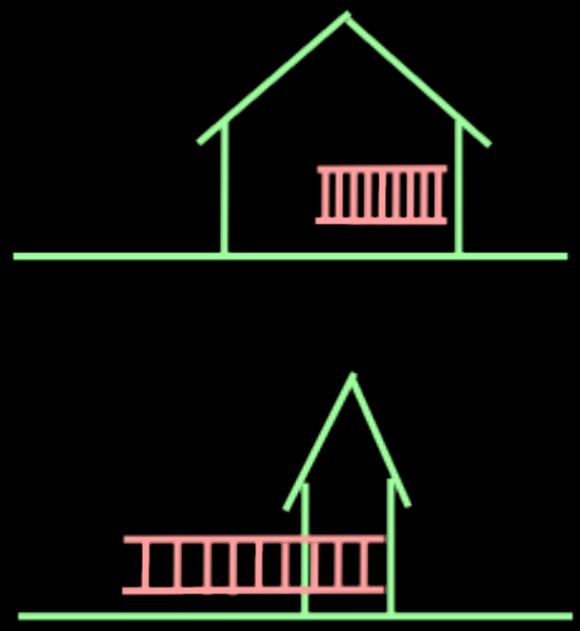
Paradoxes

⇣ ← Not always

Consistency Conditions
on Matter

Paradoxes in Special Relativity

- Twin Paradox
- Barn and Ladder Paradox
- Ehrenfest Paradox

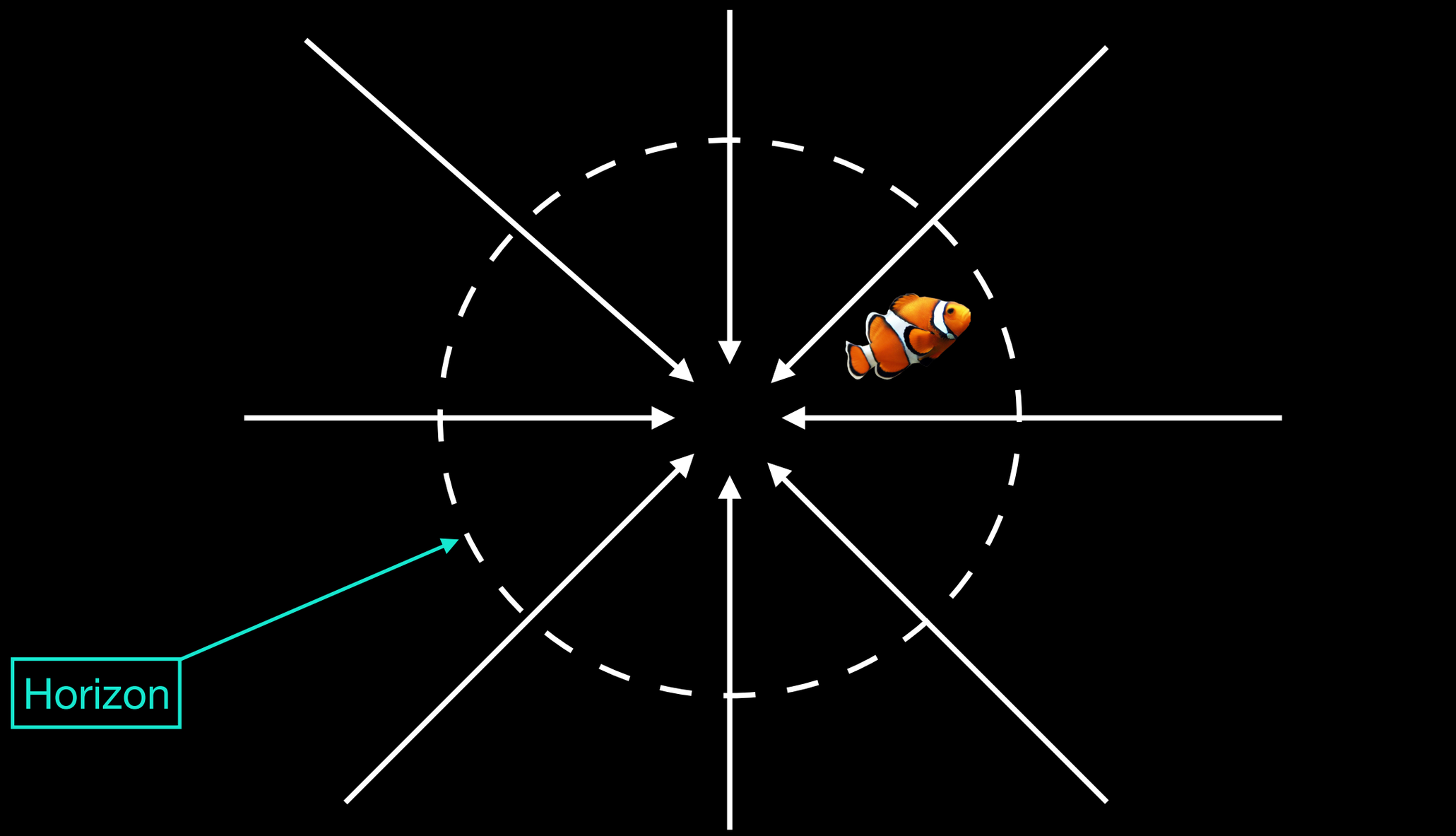


Properties of matter are restricted by consistency in Relativity

Information Loss Paradox

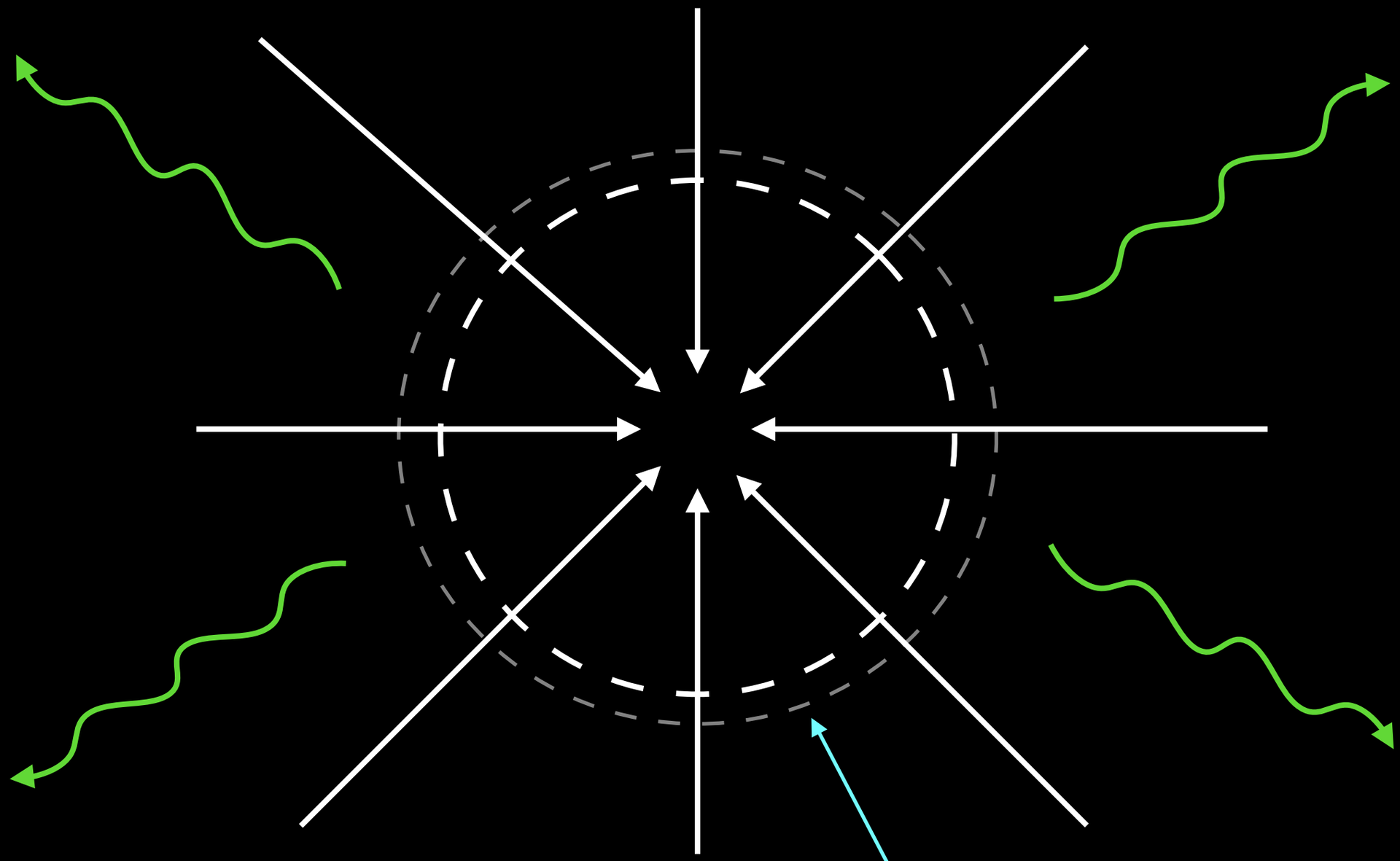
What kind of paradox is this?

Dumb Hole



[Unruh]

Hawking Radiation



Conventional Model

Based on Einstein equations and
quantum field theories in curved spacetime.

- Hawking radiation outside horizon at distance ~ a.
- Negative ingoing energy flux.
- Collapsing matter in free fall. (Nothing happens.)
- Total mass decreases.
- Schwarzschild radius shrinks.
- The “singularity” at the origin is irrelevant.

Accumulation of macroscopic negative energy!

Information Loss Paradox

Where is the information of the matter?

- Hawking radiation?
- Remnant?
- Low-energy effective theory (Einstein equations and quantum field theories) invalid around horizon?

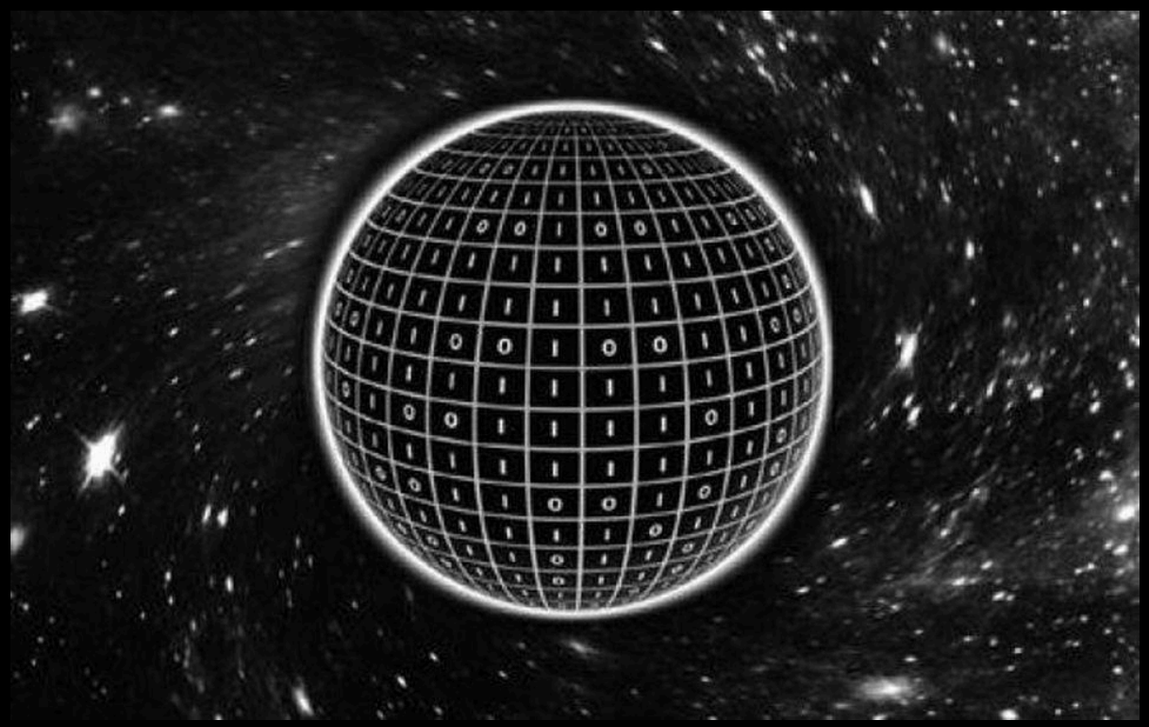
Why Is It Important?

- Our physics is low-energy effective theory.

  Can we trust physics?

- Holography for QG? ['t Hooft, Susskind]

  AdS/CFT duality, … [Maldacena]



$$S_{BH} = \frac{c^3 k_B A}{4\hbar G_N}$$

To Solve Information Paradox

- Does the low-energy effective theory break down around the horizon?

- ***High-energy event around the horizon?***

- Look for “large” quantum corrections.

Spherically Symmetric Metrics

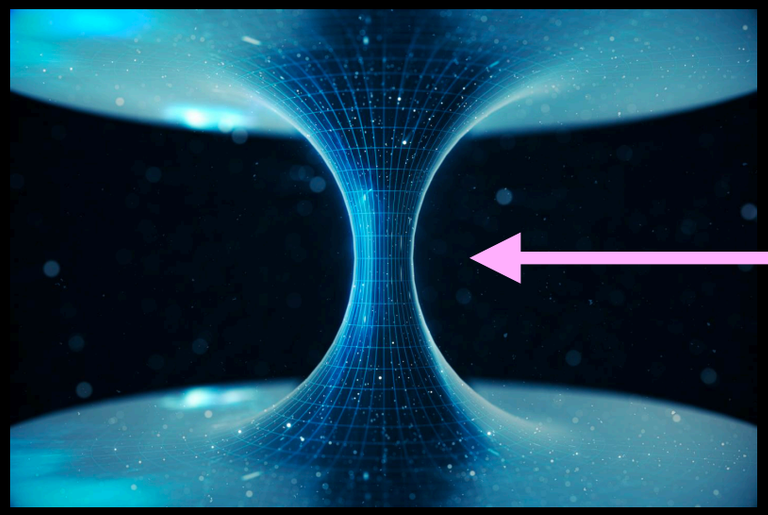
- Generic spherically symmetric metric

$$ds^2 = -f(t,z)dt^2 + dz^2 + r^2(t,z)(d\theta^2 + \sin^2\theta d\phi^2)$$

- Example of a wormhole

Areal radius

$$ds^2 = -dt^2 + dz^2 + r^2(t,z)d\Omega^2, \qquad r^2(t,z) = a^2 + z^2$$



$$\frac{\partial r}{\partial z} = 0$$

Static Black Holes

- The energy-momentum operator $\langle T_{\mu\nu} \rangle$ in curved spacetime is different for different QFTs.
- 2D massless field [Davies-Fulling-Unruh 1976][PMH-Matsuo 17 (1)] [PMH-Matsuo 17 (2)]
- 4D conformal matter [Christensen-Fulling 1977][PMH-Kawai-Matsuo-Yokokura 18]
- Literature [Solodukhin 04, 06; Fabbri-Farese-Navarro-Salas-Olmo-Sanchis-Alepuz 05 (1), 05 (2)]

3 Classes of Geometries

[Ho-Kawai-Matsuo-Yokokura, JHEP1811]

- Wormhole-like neck $\boxed{q < 0}$

  Local minimum in *r* occurs at $r > a$ (*No event horizon.*)

- Event horizon $q = 0$

  Equivalent to a shift of Schwarzschild radius a. (*fine tuning*)

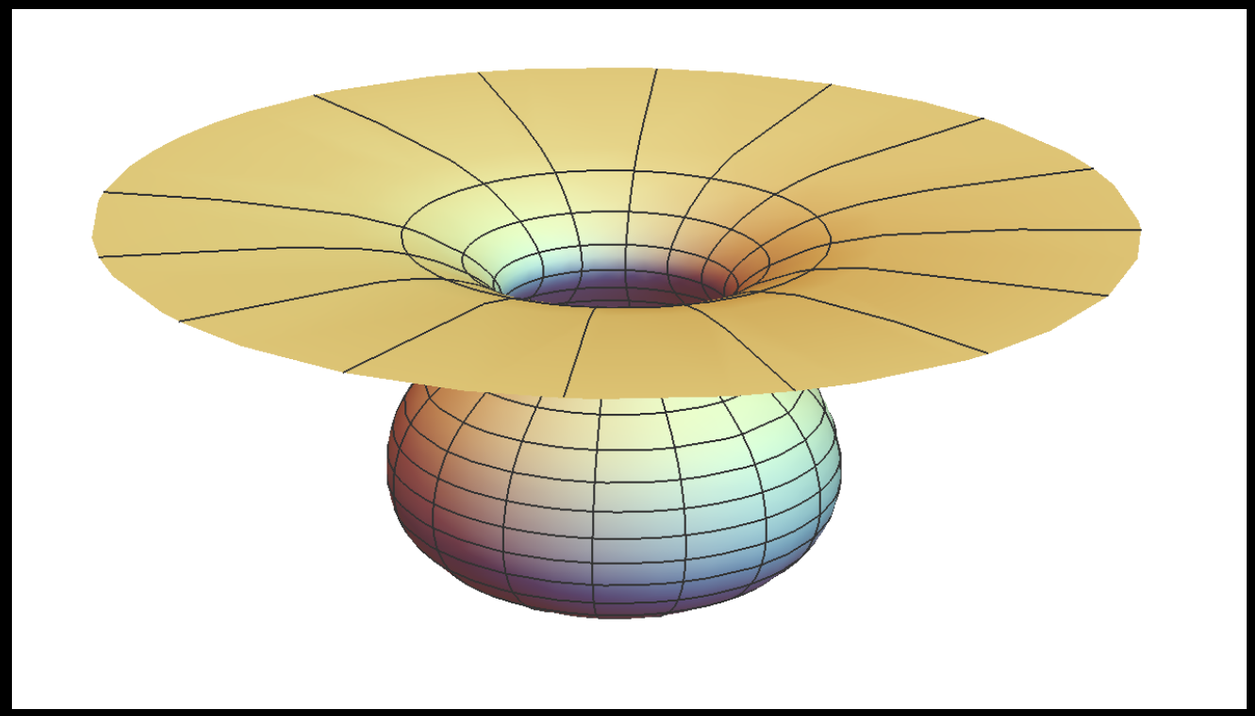
- No neck, no horizon $q > 0$

  Perturbation theory breaks down when $r - a \ll \kappa |q| a$

Negative Energy

“Defocusing”

“Bigger on the inside”



[Ho-Kawai-Matsuo-Yokokura 18]

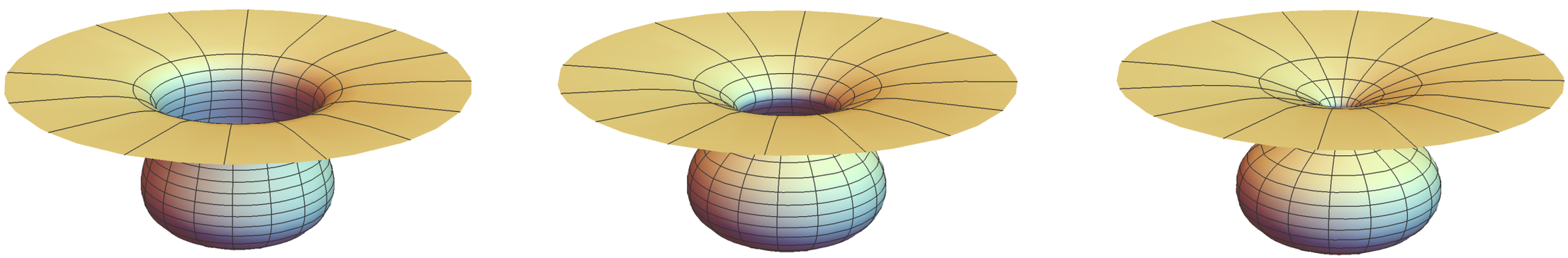
Dynamical Black Holes

- Conventional Model $q < 0$
- Kawai-Matsuo-Yokokura (KMY) Model $q \geq 0$

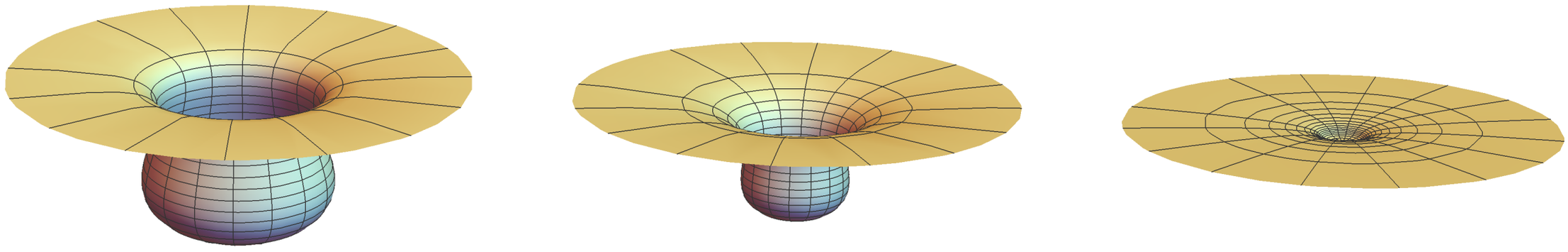
Black-Hole Geometry

[Ho-Matsuo, JHEP 1807]

Wheeler's Bag of Gold as Remnant



Everything Evaporated (need high-energy events)



Firewall

[Almheiri-Marolf-Polchinski-Sully, 13]

- Postulate 1: unitary evolution from infalling matter to Hawking radiation
- Postulate 2: semi-classical field equations
- Postulate 3: $e^{S(M)}$ where $S(M) \propto A$ is the Bekenstein entropy
- Postulate 4: A freely falling observer experiences nothing out of the ordinary when crossing the horizon.

Postulates 1,2, 4 are incompatible.

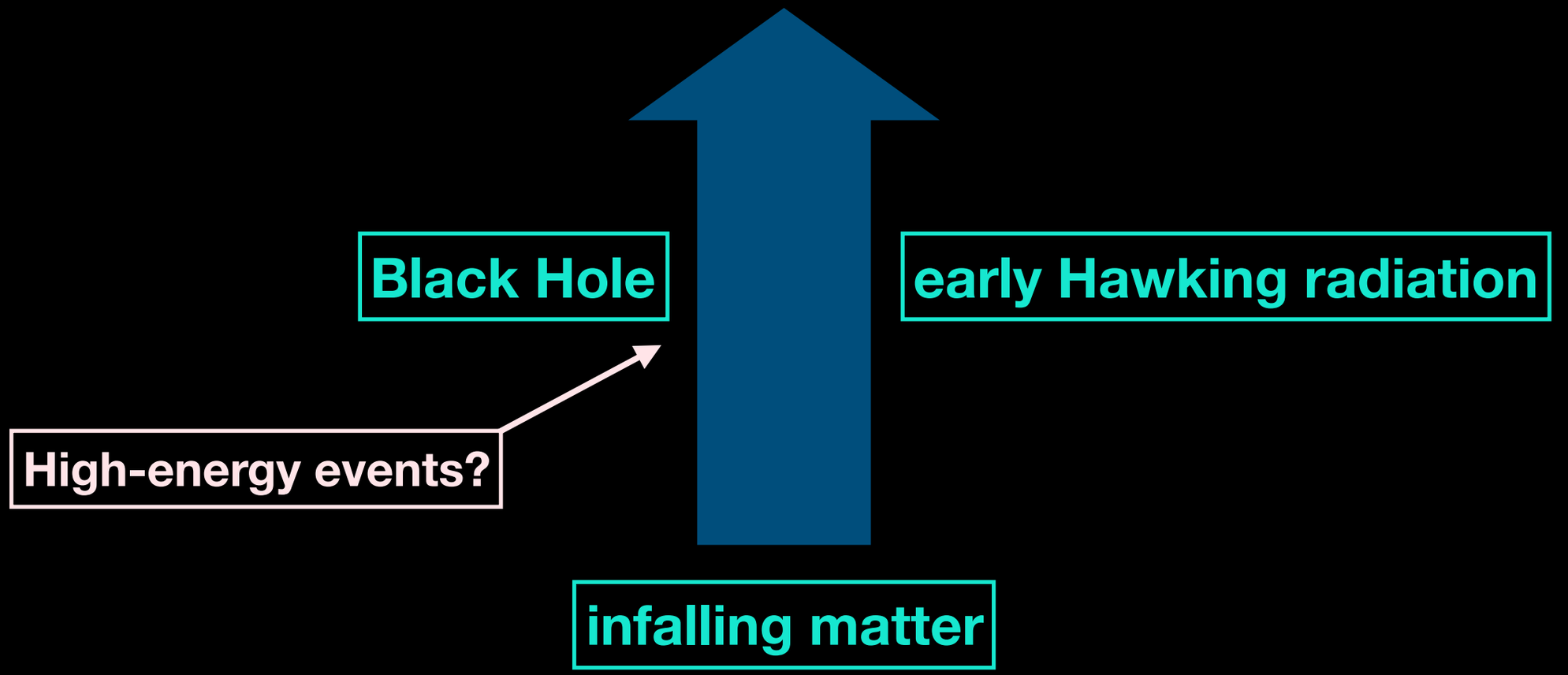
Conventional model violated Postulate 3.

Effect of “Observation”

[Almheiri-Marolf-Polchinski-Sully, 13]

decoherence

early Hawking radiation + late Hawking radiation



What is “Vacuum”?

- For a different vacuum, the notion of particles is different.

- Reason for Hawking radiation:

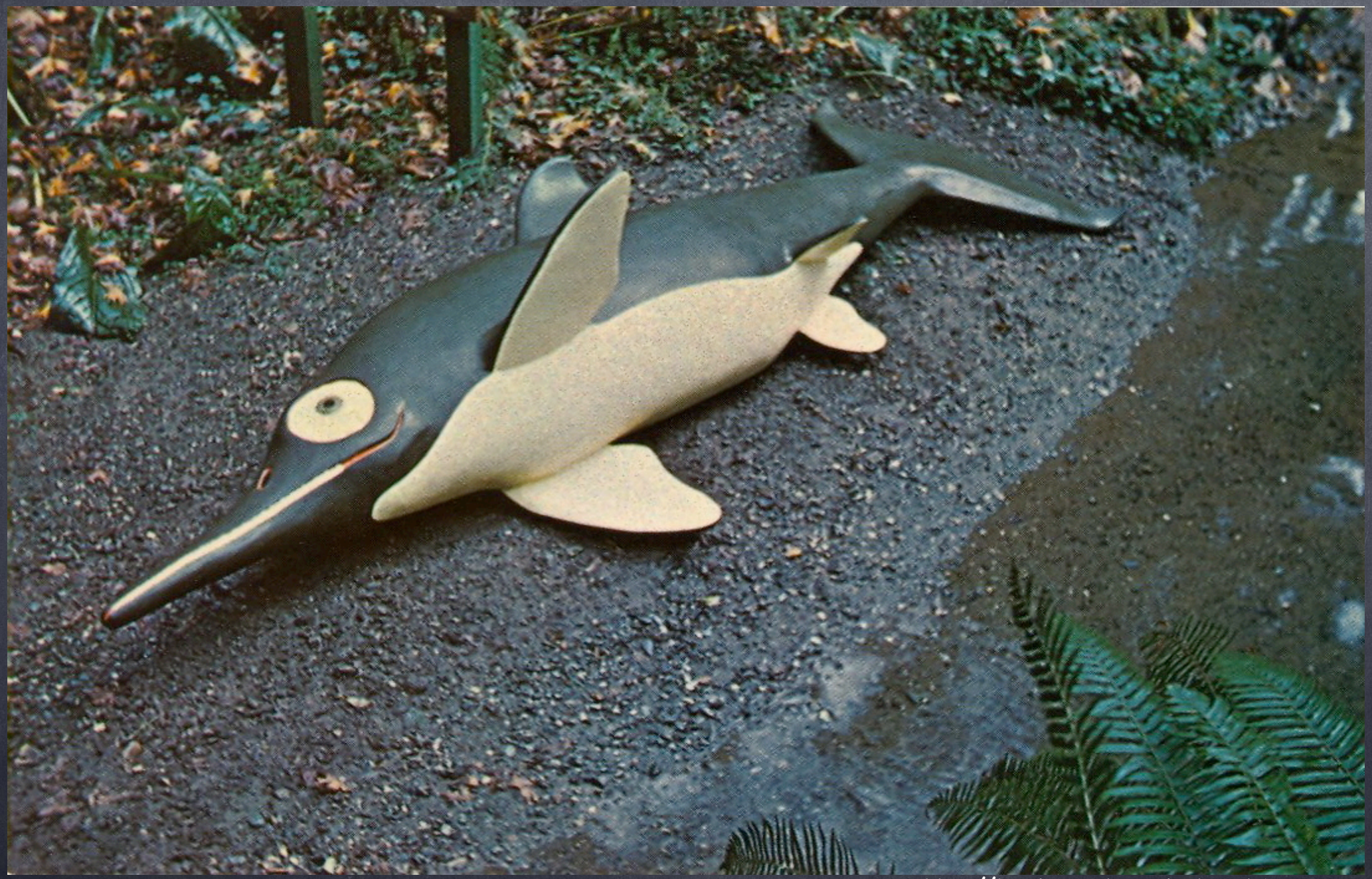
  vacuum for free-falling observers ≠

  vacuum for distant observers.

  - But vacuum for free-falling observers cannot interact with free-falling matter. ⇒ No high-energy event.

Vacuum and Particle for Fish

http://wallpaperstate.og



http://bad-postcards.tumblr.com


Paul

Schrodinger's Cat

Schrodinger's cat

- A cat is put in a box with a bottle of poisonous gas.
- If the radioactive substance decays (50% change), Geiger counter detects it and triggers a hammer to break the bottle.
- $|\text{decay}\rangle + |\text{no decay}\rangle \rightarrow |\text{dead}\rangle + |\text{alive}\rangle ?$
- ***Decoherence*** (Everett interpretation)

$$(|\textbf{dead}\rangle + |\textbf{alive}\rangle) \otimes |\textbf{environ.}\rangle$$

$$\rightarrow \quad |\textbf{dead}\rangle \otimes |\textbf{d}\rangle + |\textbf{alive}\rangle \otimes |\textbf{a}\rangle$$

What we know is not “The cat is always either dead of alive”, but “Whenever we see the cat dead/alive, we do not see anything suggesting that it is alive/dead at the same time”, or that “There is no *interference pattern* between the dead/alive states”.

Decoherence explains why the interference pattern is diminished.

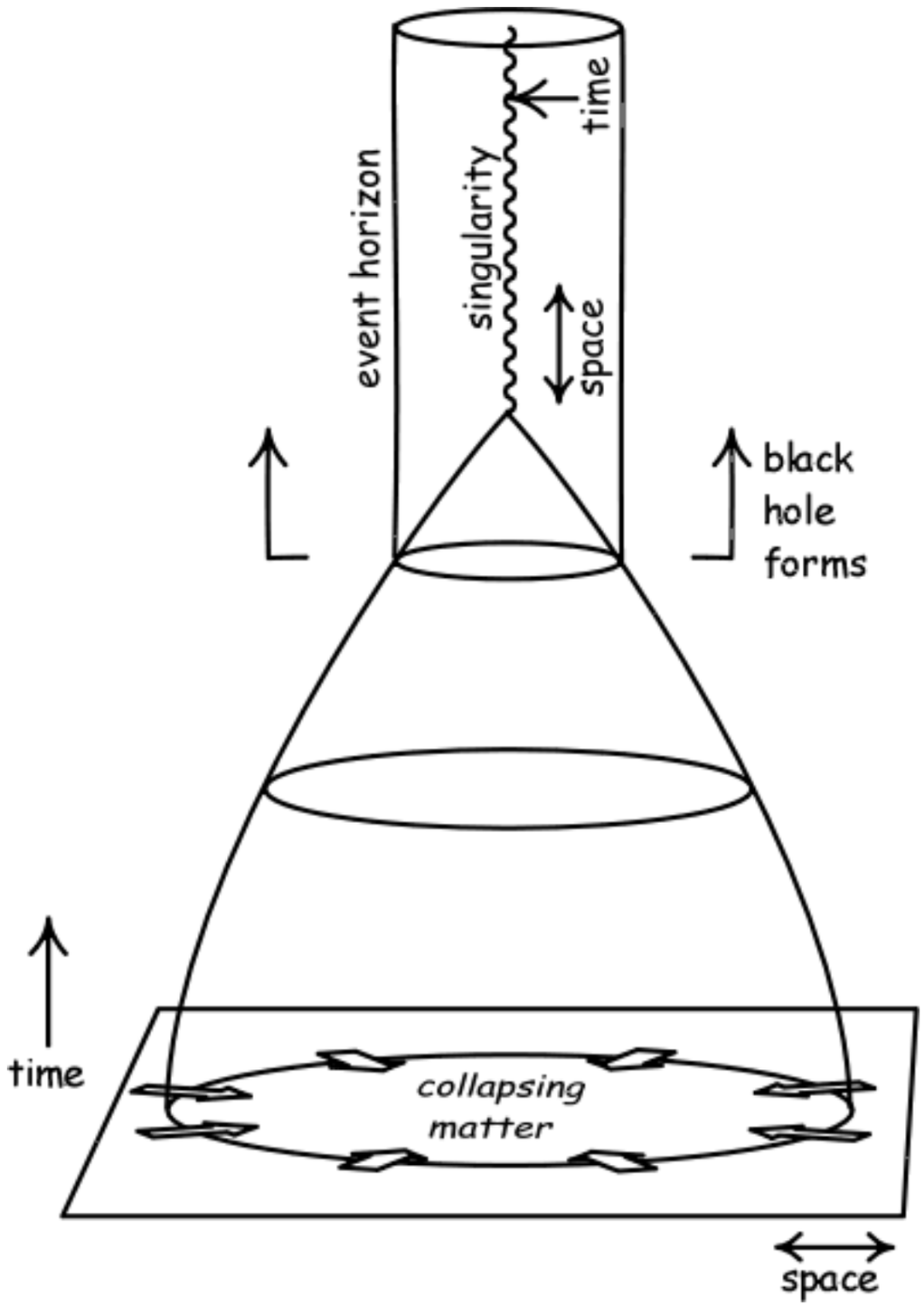
Hawking radiation as Schrodinger's cat

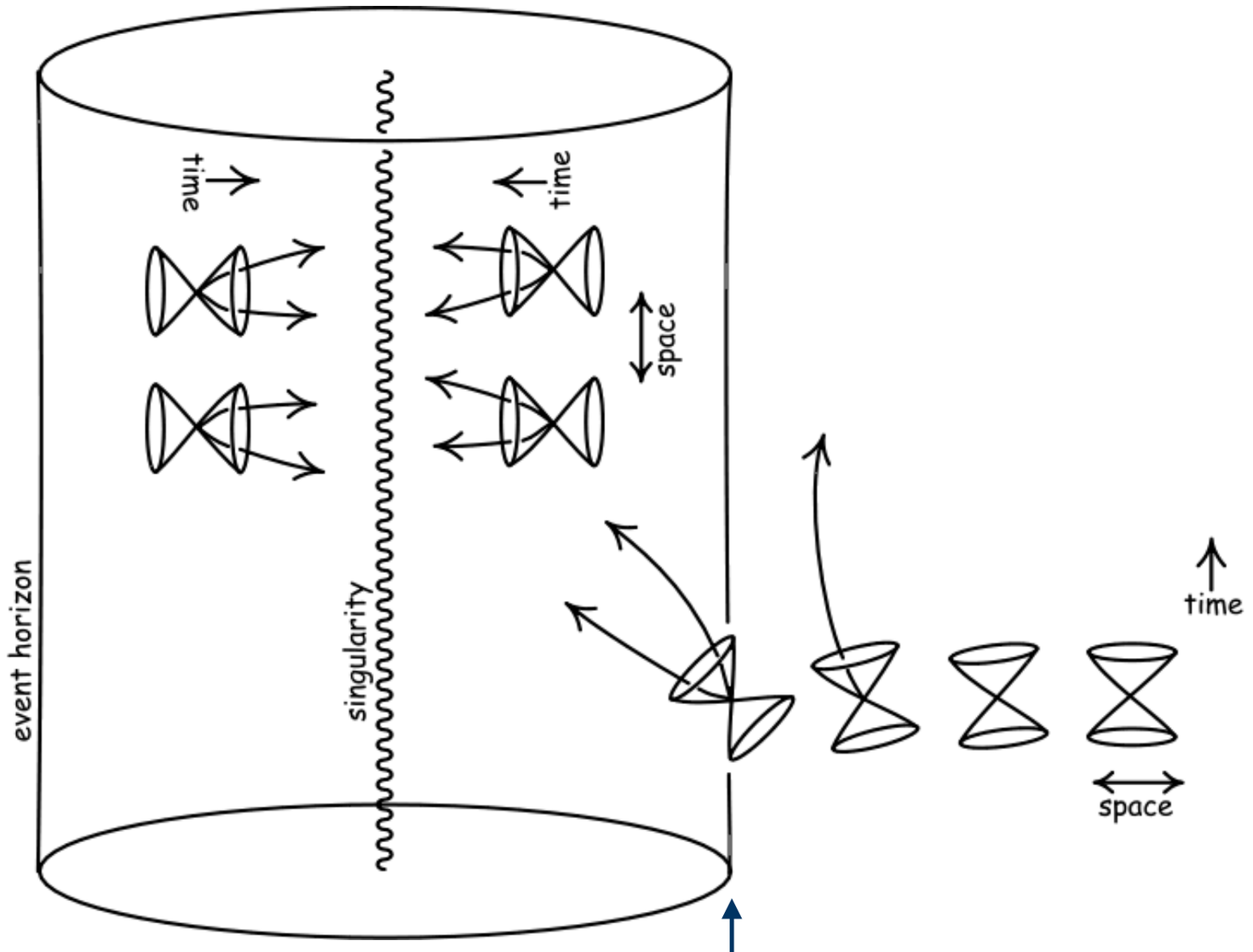
- Why is the cat either dead or alive? Why not $|\text{dead}\rangle + |\text{alive}\rangle$?
- Interactions naturally carries out observations.
- What is the "pointer basis" for Hawking radiation?

We might need a better answer to Schrodinger's cat for a better answer to the black-hole information paradox, which may be related to how to understand the wave fx of the universe.

Thank you!

Classical Black Hole





Schwarzschild radius $a = 2G_N M/c^2$

Classical Black Hole

Different observers have different observations.

For observers outside the horizon:

- Everything falls ever slower as it approaches the horizon.
- Everything takes an infinite time to fall into the horizon.
- It takes an infinite time to see the horizon appear.

For observers in free fall:

- It takes only a finite time to cross the horizon.
- Nothing special at the horizon.
- Everything falls to the singularity within a finite proper time after passing the horizon.

What's wrong with naive perturbation

- Conventional model assumes perturbative expansion around horizon.
- Perturbative expansion in κ is different when $r \sim a + \mathcal{O}(\kappa/a)$

$$G_{\mu\nu} = \kappa \langle \hat{T}_{\mu\nu} \rangle$$

$$ds^2 = -\left(1 - \frac{a}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{a}{r}} + d\Omega^2$$

- Perturbative approx. good only for $r > a + O(\kappa/a)$.