Standard Model electroweak baryogenesis
A new hope?

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Outline

- Introduction
  - Matter–antimatter asymmetry
  - CP-violation in the Standard Model
  - (Cold) electroweak baryogenesis
- Calculational framework
  - Derivative expansion
  - Method of (covariant) symbols
- Results
- Summary and outlook

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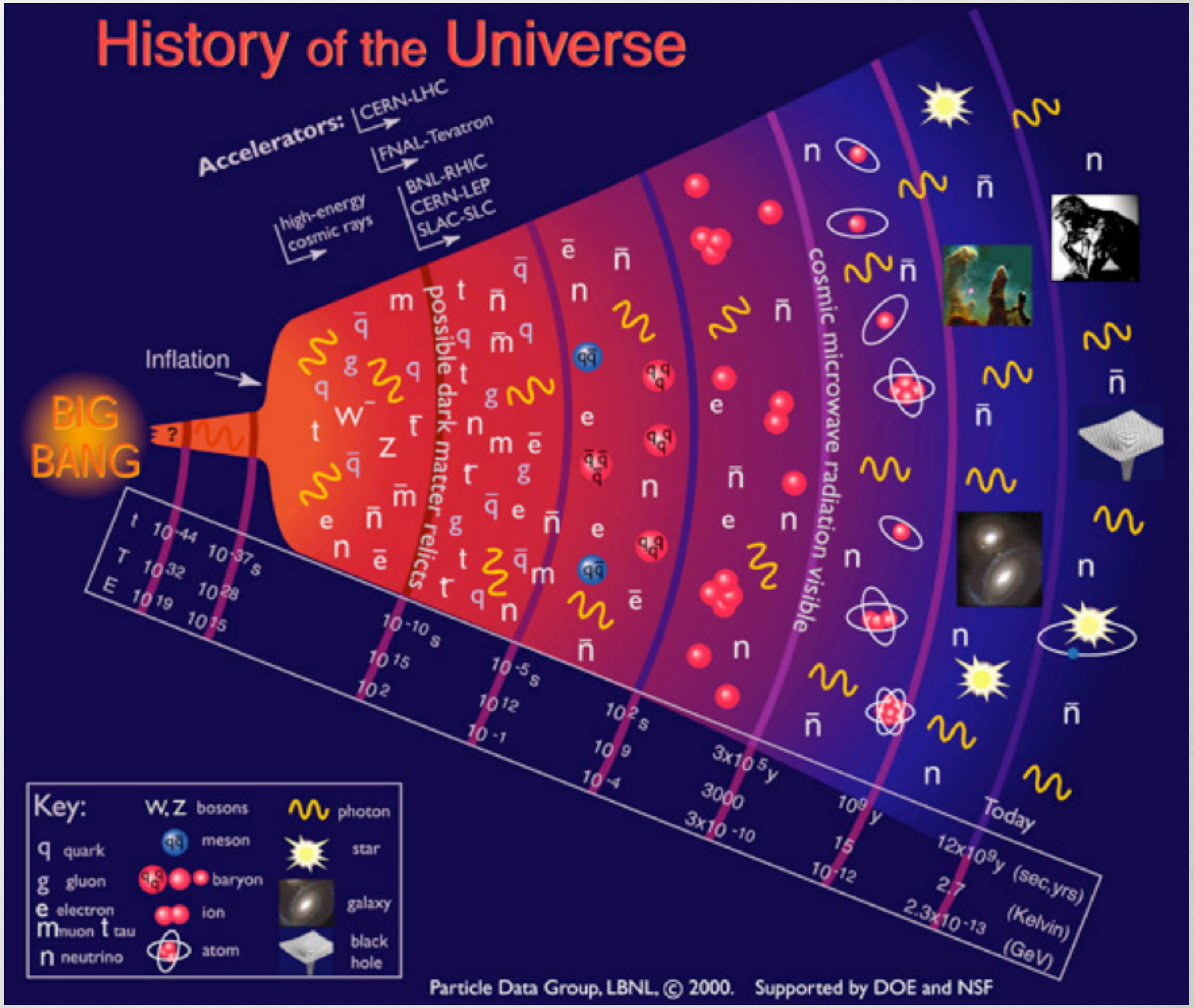
Matter–antimatter asymmetry

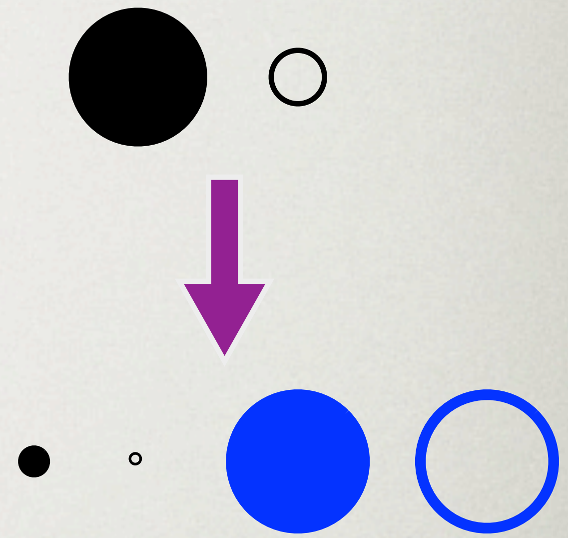
$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 6 \times 10^{-10}$$



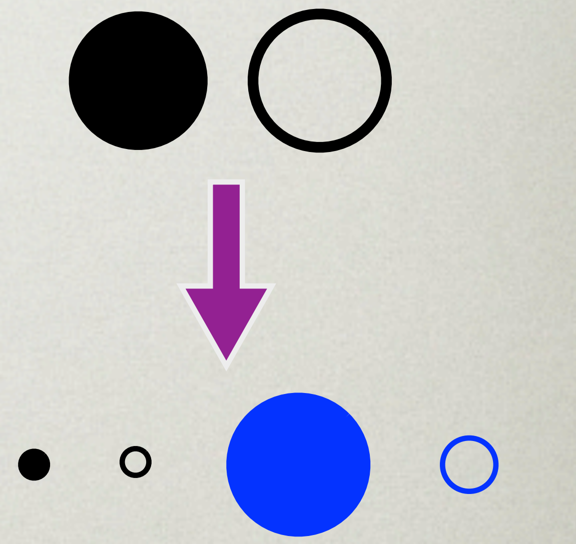
- Chemical composition of the Universe well known in BBN.

Achieving baryon asymmetry

- Asymmetric initial condition?
- Contradicts the inflation paradigm.
  Any preexisting asymmetry washed out.



- Symmetric initial condition.
- Generate asymmetry dynamically!
- Baryo*genesis*.



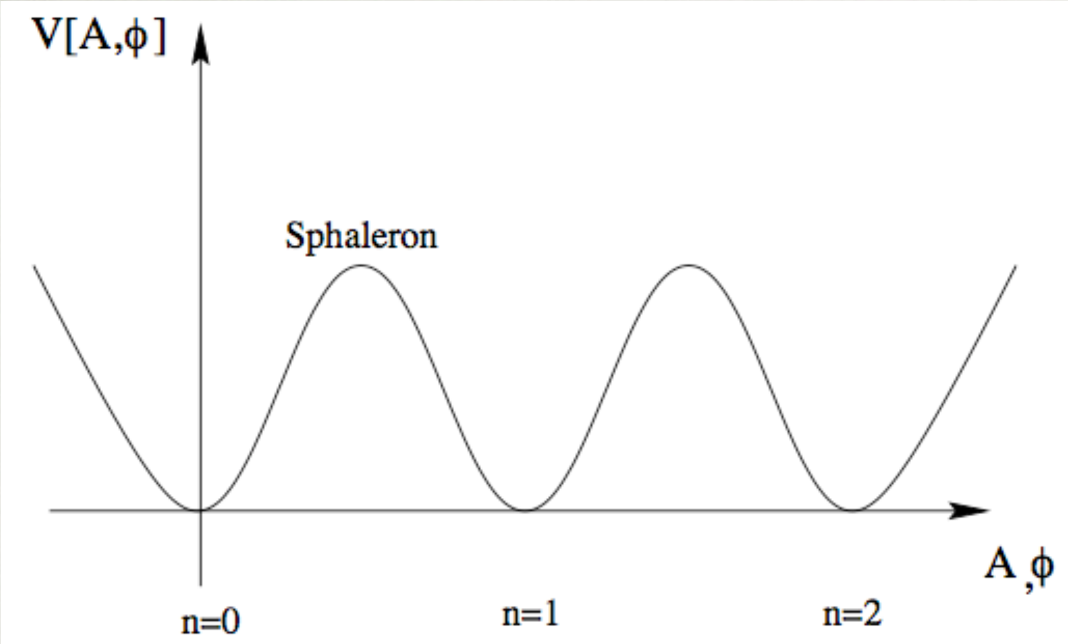
Sakharov's conditions

Any candidate theory of baryogenesis must incorporate:

1) Baryon number violation.
2) C- as well as CP-violation.
3) Departure from chemical equilibrium.

Condition #1

- Satisfied directly in GUT scenarios.
- In Standard Model, B is a classical symmetry.
- Violated on the quantum level by a global anomaly!



$$\partial_\mu j^\mu_B = \frac{n_f}{32\pi^2}\left(g^2 W^a_{\mu\nu}\widetilde{W}^{a\mu\nu} - g'^2 F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)$$

$$\Gamma(T=0) \propto \exp\left(-4\pi\sin^2\theta_W/\alpha_W\right) \simeq 10^{-170}$$

$$\Gamma(T \gg M_W) \propto (\alpha_W T)^4$$

- Baryon number changes in sphaleron processes.
- Active at temperatures higher than the electroweak scale.

Conditions #2 and #3

- C-violation.
- Maximal in the Standard Model (through P-violation).
- CP-violation.
- Present, in SM enters only through the CKM matrix.
- Departure from chemical equilibrium.
- Several possibilities:
  - Phase transitions.
  - Classical field dynamics (e.g. inflaton).
  - Out-of-equilibrium decay of heavy particles (GUTs).

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CP-violation in Standard Model



- Need at least three fermion families.
- Diagonalization of the complex mass matrices:

$$u_L(d_L) \to U_{u,d} u'_L(d'_L), \quad u_R(d_R) \to V_{u,d} u'_R(d'_R)$$

$$\bar{u}_L M_u u_R + \bar{d}_L M_d d_R \to \sum_f \left(m_{uf} \bar{u}'_{fL} u'_{fR} + m_{df} \bar{d}'_{fL} d'_{fR} \right)$$

$$\bar{u}_L \gamma^\mu W^+_\mu d_L \to \bar{u}'_L \gamma^\mu W^+_\mu C d'_L, \quad C = U^\dagger_u U_d$$

- Similar structure in the lepton sector, less clear due to Dirac/Majorana nature of neutrinos.
- Here consider only quark sector CP-violation.
- CKM matrix source of all CP-violation effects up to now!

CKM and rephasing invariance

- Physics must not be changed by arbitrary rephasing:

$$u_{fL} \to e^{i\alpha_f} u_{fL}, \quad d_{fL} \to e^{i\beta_f} d_{fL}, \quad C_{fg} \to C_{fg}\, e^{-i(\alpha_f - \beta_g)}$$

- CP-violating effects proportional to Jarlskog invariant:

$$J \varepsilon_{fk} \varepsilon_{g\ell} = \mathrm{Im}\left(C_{fg} C^\dagger_{gk} C_{k\ell} C^\dagger_{\ell f} \right), \quad J \simeq 3 \times 10^{-5}$$

- Kobayashi–Maskawa parameterization of CKM matrix:

$$C = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix}, \quad J = s_1^2 s_2 s_3 c_1 c_2 c_3 \sin\delta$$

- Simplest perturbative CP-violating operator corresponds to the Jarlskog determinant:

$$\Delta \equiv \mathrm{Im}\det[M_u M_u^\dagger, M_d M_d^\dagger] = J(m_u^2 - m_c^2)(m_c^2 - m_t^2)(m_t^2 - m_u^2)(m_d^2 - m_s^2)(m_s^2 - m_b^2)(m_b^2 - m_d^2)$$

- No CP-violation in case of “horizontal” mass degeneracy.

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Palette of scenarios

- Many different scenarios for baryogenesis.
  - *Electroweak baryogenesis*: see later.
  - *GUT baryogenesis*: B-violation by new interactions, off-equilibrium by decay of heavy particles.
  - *Leptogenesis*: uses conservation of B−L in SM.
- If CKM is the sole source of CP-violation, baryogenesis must occur during the electroweak phase transition.
  - No B-violation below electroweak scale.
  - Quark masses (if present) degenerate above T_{EW}.

Electroweak baryogenesis

- Generate baryon number during EW phase transition.
- Is SM capable of producing enough baryon asymmetry?
- Problem #1:
- 1st order transition requires m_H<80 GeV; in contradiction with experimental limit. Kajantie *et al.*, PRL **77** (1996)
- Problem #2:
- At T above EW scale, CP-violation suppressed by

$$\Delta/T^{12} \lesssim \Delta/v^{12} \simeq 10^{-24}$$

- Corollary: SM cannot explain baryon asymmetry!?

Cold electroweak baryogenesis

- Both problems of Standard Model EWBG alleviated.
  García-Bellido, Grigoriev, Kusenko, Shaposhnikov, PRD **60** (1999)
- Universe supercooled to T much below EW scale.
- EW transition via tachyonic instability.
- Problem #1 bypassed:
- Off-equilibrium by classical (tachyonic) field dynamics.
- Problem #2 bypassed:
- Nonperturbative infrared enhancement. CP-violation $\propto$ to Jarlskog *invariant*, not Jarlskog *determinant*!

It can work!

- The possibility to explain the observed asymmetry using just know physics is very appealing!
- Result of initial numerical simulations:
  Tranberg, Hernandez, Konstandin, Schmidt, PLB **690** (2010)

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 4 \times 10^{-6}$$

- Four orders of magnitude more than observed!
- There is a lot of space for approximation uncertainties.

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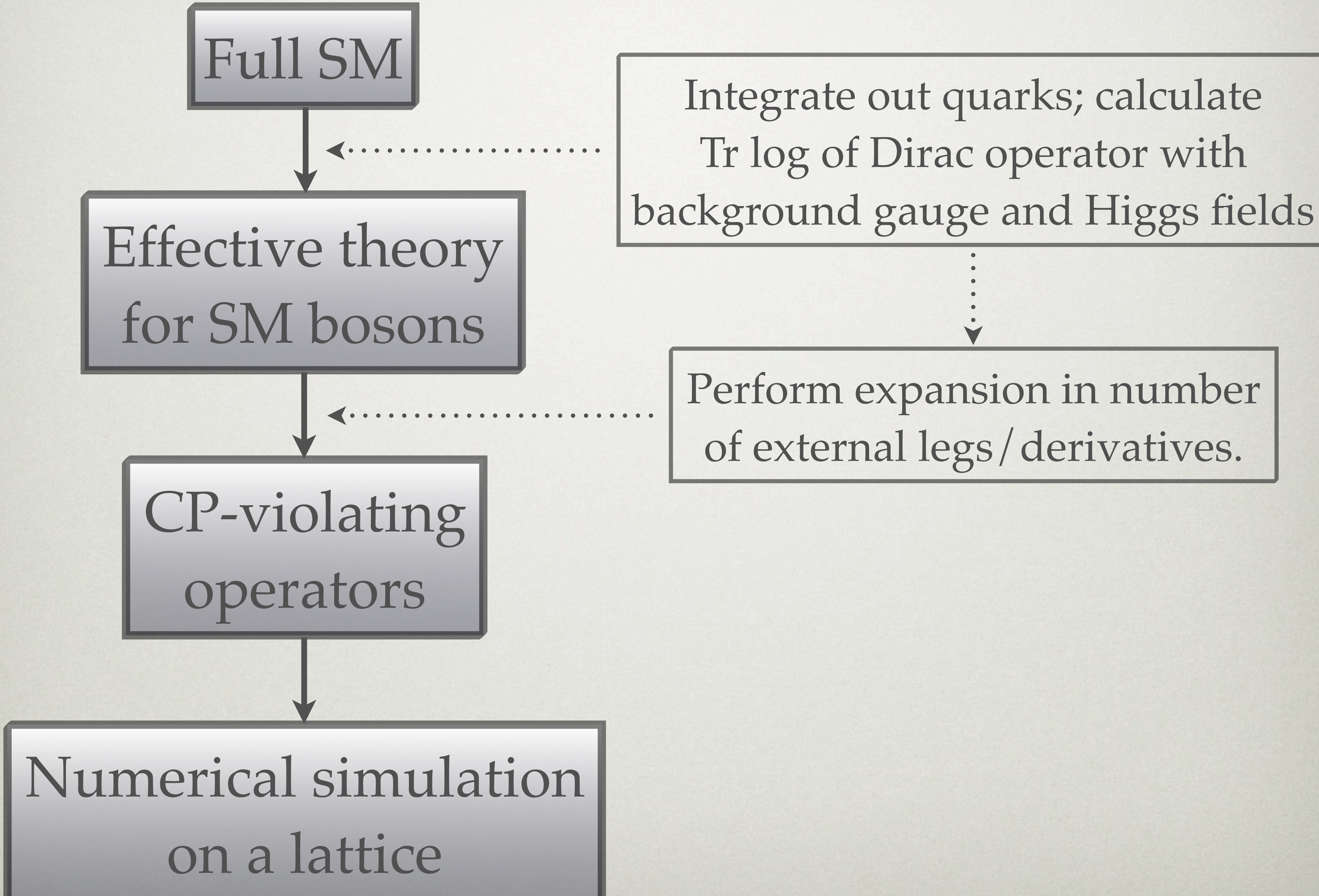
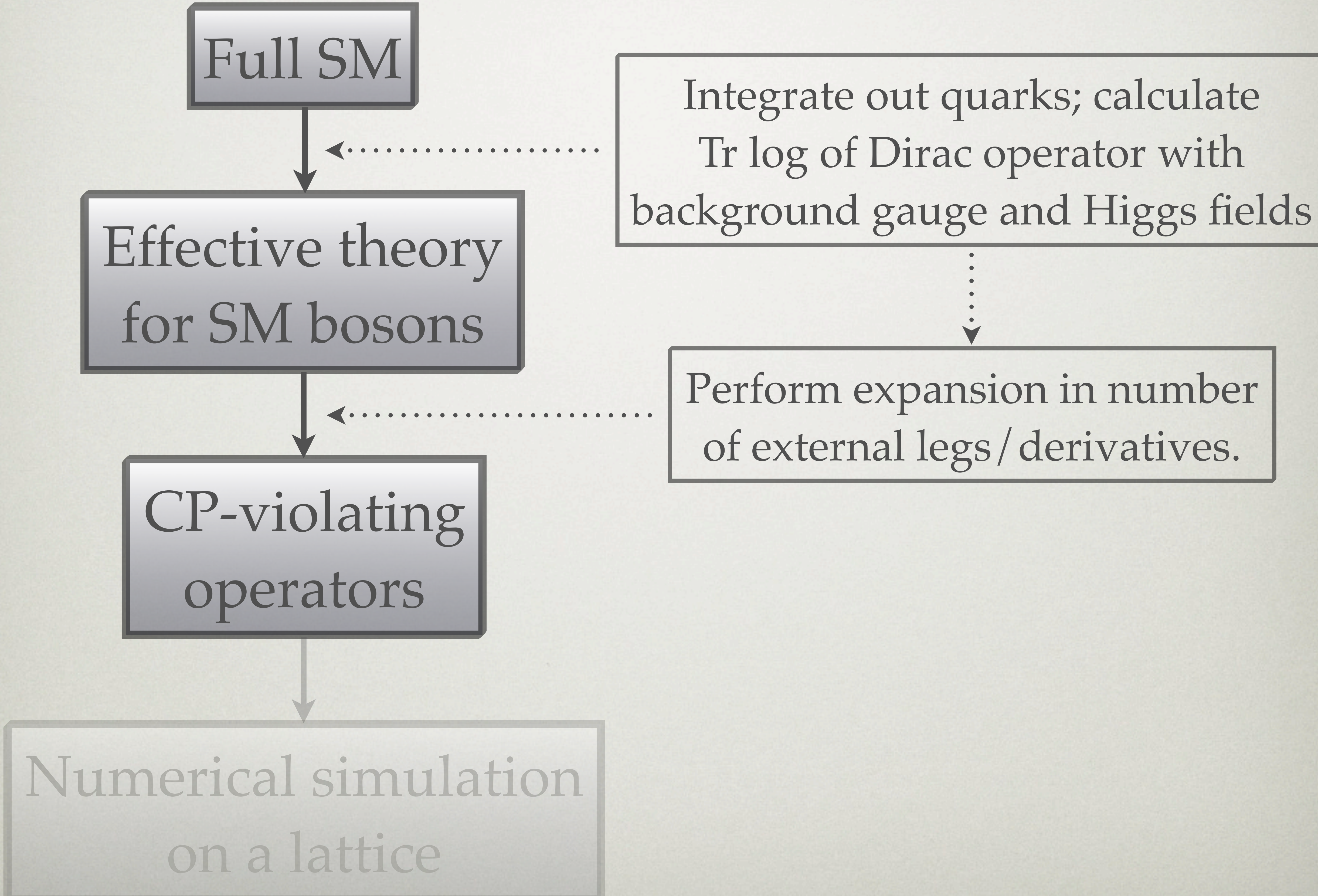
How to obtain this number



How to obtain this number



A bit of history

- External gauge fields count as derivatives.
- Need at least four W's to get the Jarlskog invariant, so CP-violation can only start at order four.
- All calculations were done at zero temperature so far.

order 4	Smit, JHEP **09** (2004)	no CP-odd terms at this order!
order 6	García-Recio, Salcedo, JHEP **07** (2009)	only CP-odd P-even operators
order 6	Hernandez, Konstandin, Schmidt, NPB **812** (2009)	also CP-odd P-odd operators

Our goal

- Resolve the discrepancy of the existing calculations at zero temperature.
- Extend the calculation to nonzero temperature in order to see the extrapolation between the T=0 infrared enhancement and the high-T perturbative suppression.
- Do four orders of magnitude provide enough space for finite-T effects?

Calculation of chiral determinant

- Euclidean Dirac operator in general background field:

$$\boldsymbol{D} = \begin{pmatrix} \not{D}_L & m_{LR} \\ m_{RL} & \not{D}_R \end{pmatrix}, \quad \not{D}_{L,R} = \not{\partial} + \not{V}_{L,R}$$

- Reduce the rank of the Dirac operator: Salcedo, EPJC **58** (2008)

$$\boldsymbol{K} = m_{LR} m_{RL} - \not{D}_L m_{RL}^{-1} \not{D}_R m_{RL}$$

- Parity-even and -odd parts of the Euclidean effective action coincide with its real and imaginary parts.

$$\Gamma = \Gamma^+ + \Gamma^-, \quad \Gamma^+ = -\frac{1}{2} \mathrm{Re}\,\mathrm{Tr}\,(\log \boldsymbol{K})$$

$$\Gamma^- = -\frac{i}{2} \mathrm{Im}\,\mathrm{Tr}\,(\gamma_5 \log \boldsymbol{K}) + \Gamma_{gWZW}$$

- Smit tells us that anomaly does not contribute, only need to calculate traces of log $\boldsymbol{K}$. Smit, JHEP **09** (2004)

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Method of symbols

- Technique to calculate traces of differential operators.
- For a matrix function $M(x)$ and covariant derivative D_x:

$$\mathrm{Tr}\, f(M(x), D_x) = \int \frac{d^d x\, d^d p}{(2\pi)^d}\, \mathrm{tr}\big[f(M(x), D_x + ip)\mathbf{1}\big].$$

- Loses manifest covariance by "free" covariant derivatives.
- Method of covariant symbols makes the expansion manifestly covariant already on the integrand level.
García-Recio, Salcedo, JHEP **07** (2009)

$$\mathrm{Tr}\, f(M(x), D_x) = \int \frac{d^d x\, d^d p}{(2\pi)^d}\, \mathrm{tr}\left[f(\overline{M}(x), \overline{D}_x)\mathbf{1}\right]$$

$$\overline{M} = M + i[D_\alpha, M]\frac{\partial}{\partial p_\alpha} - \frac{1}{2}[D_\alpha, [D_\beta, M]]\frac{\partial^2}{\partial p_\alpha \partial p_\beta} + \cdots$$

$$\overline{D}_\mu = ip_\mu + \frac{i}{2}[D_\alpha, D_\mu]\frac{\partial}{\partial p_\alpha} - \frac{1}{3}[D_\alpha, [D_\beta, D_\mu]]\frac{\partial^2}{\partial p_\alpha \partial p_\beta} + \cdots$$

Application to Standard Model

- Quark Dirac operator in the chiral basis:

$$\boldsymbol{D} = \begin{pmatrix} \not{D}_u + \not{Z} + \not{G} & \not{W}^+ & \frac{\phi}{v} M_u & 0 \\ \not{W}^- & \not{D}_d - \not{Z} + \not{G} & 0 & \frac{\phi}{v} M_d \\ \frac{\phi}{v} M_u^\dagger & 0 & \not{D}_u + \not{G} & 0 \\ 0 & \frac{\phi}{v} M_d^\dagger & 0 & \not{D}_d + \not{G} \end{pmatrix}, \quad \not{D}_{u,d} = \not{\partial} + q_{u,d} \not{B}$$

- Identify the reduced Dirac operator $\boldsymbol{K}=\boldsymbol{K}_D+\boldsymbol{K}_A$:

$$\boldsymbol{K}_D = \begin{pmatrix} (\phi^2/v^2) M_u M_u^\dagger - (\not{D}_u + \not{Z})(\not{D}_u + \not{\varphi}) & 0 \\ 0 & (\phi^2/v^2) M_d M_d^\dagger - (\not{D}_d - \not{Z})(\not{D}_d + \not{\varphi}) \end{pmatrix}$$

$$\boldsymbol{K}_A = \begin{pmatrix} 0 & -\not{W}^+ (\not{D}_d + \not{\varphi}) \\ -\not{W}^- (\not{D}_u + \not{\varphi}) & 0 \end{pmatrix}$$

- Gluons do not contribute at order six.
- Expand the trace in powers of derivatives/gauge fields:

$$\mathrm{Tr}\left[(\gamma_5) \log \boldsymbol{K}\right] = \sum_{n=0}^{\infty} \mathrm{Tr}[(\gamma_5) \log \boldsymbol{K}]_{2n}$$

$$\mathrm{Tr}\left[(\gamma_5) \log \boldsymbol{K}\right]_{2n} = -\frac{1}{n} \mathrm{Tr} \left\{ (\gamma_5) \left[\left(\frac{\phi^2}{v^2} M_u M_u^\dagger - (\not{D}_u + \not{Z})(\not{D}_u + \not{\varphi}) \right)^{-1} \not{W}^+ (\not{D}_d + \not{\varphi}) \times \right.\right.$$
$$\left.\left. \times \left(\frac{\phi^2}{v^2} M_d M_d^\dagger - (\not{D}_d - \not{Z})(\not{D}_d + \not{\varphi}) \right)^{-1} \not{W}^- (\not{D}_u + \not{\varphi}) \right]^n \right\}$$

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Order six (T=0)

- All contributions depend on a single master integral.

$$\kappa_{CP} = \frac{\Delta}{G_F} \int \frac{d^4 p}{(2\pi)^4} (p^2)^3 \prod_{f=1}^{6} \frac{1}{(p^2 + m_f^2)^2} \approx 9 \times 10^{-3}.$$

- Infrared enhancement is clearly visible here.
- Full result for the CP-violating effective action:

$$\Gamma_{\text{eff}} = -\frac{i}{2} N_c G_F \kappa_{CP} \int d^4x \left(\frac{v}{\phi}\right)^2 (\mathcal{O}_0 + \mathcal{O}_1 + \mathcal{O}_2), \quad W^{\pm}_{\mu\nu} = \partial_\mu W^{\pm}_\nu \pm B_\mu W^{\pm}_\nu, \quad \varphi_\mu = (\partial_\mu \phi)/\phi$$

$$\mathcal{O}_0 = -\frac{1}{3}(W^+)^2 W^-_{\mu\mu} W^-_{\nu\nu} + \frac{5}{3}(W^+)^2 W^-_{\mu\nu} W^-_{\mu\nu} - \frac{1}{3}(W^+)^2 W^-_{\mu\nu} W^-_{\nu\mu} + \frac{4}{3} W^+_\mu W^+_\nu W^-_{\mu\alpha} W^-_{\alpha\nu} -$$
$$- \frac{2}{3} W^+_\mu W^+_\nu W^-_{\mu\alpha} W^-_{\nu\alpha} - 2 W^+_\mu W^+_\nu W^-_{\alpha\mu} W^-_{\alpha\nu} + \frac{4}{3} W^+_\mu W^+_\nu W^-_{\mu\nu} W^-_{\alpha\alpha} - \text{c.c.}$$

$$\mathcal{O}_1 = \frac{8}{3}(Z_\mu + \varphi_\mu)\big[(W^+)^2 W^-_\mu W^-_{\nu\nu} - (W^+)^2 W^-_\nu W^-_{\mu\nu} - (W^+)^2 W^-_\nu W^-_{\nu\mu} -$$
$$- (W^+ \cdot W^-) W^+_\mu W^-_{\nu\nu} + (W^+ \cdot W^-) W^+_\nu W^-_{\mu\nu} + W^+_\mu W^+_\nu W^-_\alpha W^-_{\alpha\nu}\big] - \text{c.c.}$$

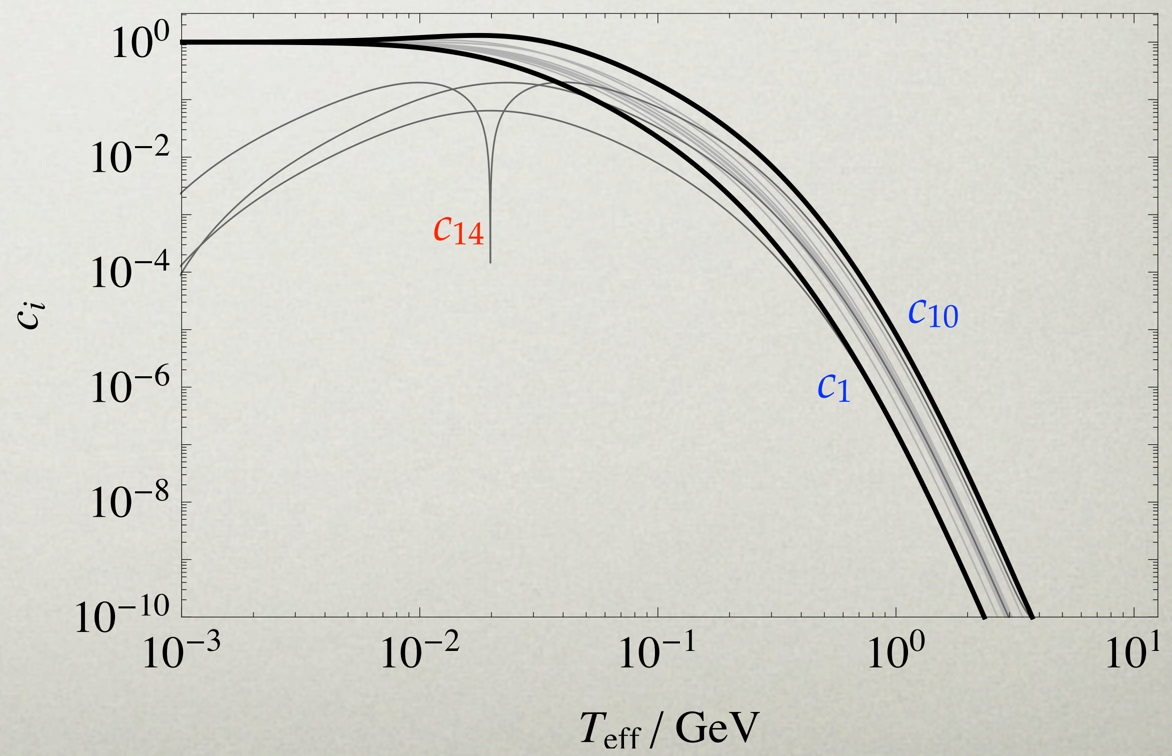
$$\mathcal{O}_2 = 4(Z_\mu Z_\nu + \varphi_\mu \varphi_\nu)\big[(W^+)^2 W^-_\mu W^-_\nu - (W^-)^2 W^+_\mu W^+_\nu\big] - \frac{16}{3}(Z \cdot \varphi)\big[(W^+ \cdot W^-)^2 - 2(W^+)^2 (W^-)^2\big] +$$
$$+ \frac{4}{3}(Z_\mu \varphi_\nu + Z_\nu \varphi_\mu)\big[(W^+)^2 W^-_\mu W^-_\nu + (W^-)^2 W^+_\mu W^+_\nu - 2(W^+ \cdot W^-)(W^+_\mu W^-_\nu + W^+_\nu W^-_\mu)\big]$$

- We fully confirm the result of García-Recio & Salcedo!

Order six (T≠0)

$$
\begin{aligned}
\mathcal{O}_0^+ =& -\frac{c_1}{3}(W^+)^2 W^-_{\mu\mu}W^-_{\nu\nu} + \frac{5c_2}{3}(W^+)^2 W^-_{\mu\nu}W^-_{\mu\nu} - \frac{c_1}{3}(W^+)^2 W^-_{\mu\nu}W^-_{\nu\mu} + \frac{4c_3}{3}W^+_\mu W^+_\nu W^-_{\mu\alpha}W^-_{\alpha\nu} - \\
& - \frac{2c_1}{3}W^+_\mu W^+_\nu W^-_{\mu\alpha}W^-_{\nu\alpha} - 2c_4 W^+_\mu W^+_\nu W^-_{\alpha\mu}W^-_{\alpha\nu} + \frac{4c_3}{3}W^+_\mu W^+_\nu W^-_{\mu\nu}W^-_{\alpha\alpha} - \text{c.c.} \\
\mathcal{O}_1^+ =& \frac{8}{3}(Z_\mu + \varphi_\mu)\big[c_5(W^+)^2 W^-_\mu W^-_{\nu\nu} - c_6(W^+)^2 W^-_\nu W^-_{\mu\nu} - c_6(W^+)^2 W^-_\nu W^-_{\nu\mu} - \\
& - c_3(W^+\cdot W^-)W^+_\mu W^-_{\nu\nu} + c_7(W^+\cdot W^-)W^+_\nu W^-_{\mu\nu} + c_7 W^+_\mu W^+_\nu W^-_\alpha W^-_{\alpha\nu} - \\
& - c_{12}(W^+\cdot W^-)W^+_\nu W^-_{\nu\mu} - c_{12}W^+_\mu W^+_\nu W^-_\alpha W^-_{\nu\alpha} + c_{13}W^-_\mu W^+_\nu W^+_\alpha W^-_{\nu\alpha}\big] - \text{c.c.} \\
\mathcal{O}_1^- =& \frac{2}{3}c_{12}W^-_{\mu\nu}W^+_\beta\big[2(W^+\cdot W^-)Z_\alpha\epsilon_{\mu\nu\alpha\beta} - W^+_\nu W^-_\alpha(3Z_\gamma + \varphi_\gamma)\epsilon_{\mu\alpha\beta\gamma}\big] + \text{c.c.} \\
\mathcal{O}_2^+ =& 4(Z_\mu Z_\nu + \varphi_\mu\varphi_\nu)\big[c_8(W^+)^2 W^-_\mu W^-_\nu - c_8(W^-)^2 W^+_\mu W^+_\nu\big] - \frac{16}{3}(Z\cdot\varphi)\big[c_9(W^+\cdot W^-)^2 - 2c_6(W^+)^2(W^-)^2\big] + \\
& + \frac{4}{3}(Z_\mu\varphi_\nu + Z_\nu\varphi_\mu)\big[c_{10}(W^+)^2 W^-_\mu W^-_\nu + c_{10}(W^-)^2 W^+_\mu W^+_\nu - 2c_{11}(W^+\cdot W^-)(W^+_\mu W^-_\nu + W^+_\nu W^-_\mu)\big] \\
\mathcal{O}_2^- =& c_{14}(W^+\cdot W^-)Z_\alpha\varphi_\beta W^-_\mu W^+_\nu\epsilon_{\alpha\beta\mu\nu}
\end{aligned}
$$

- Effective couplings drop very fast with temperature!
- Dependence on $T_{\rm eff}=Tv/\phi$.
- P-odd coupling c_{14} important at higher temperatures!
- Only Lorentz-invariant op-s!



Order eight (T=0) – preliminary

- First P-odd contributions appear at order eight.

  Salcedo, PLB **700** (2011)

- Relevant for the electric dipole moment calculation.
- Full order-eight result useful for analysis of convergence properties of the derivative expansion.
- Two types of terms:
  - $6\times W + 2\times Z,\varphi,\partial$.
  - $4\times W + 4\times Z,\varphi,\partial$.
- 6+2 terms: full list of P-even terms, no P-odd terms.
- 4+4 terms: in progress...

Computational complexity

- Order six at $T=0$:
  - Initially 10–20 pages of manipulations + angular average and Dirac trace with *Mathematica*.
  - Now 2 min with *Mathematica* code (using *Feyncalc*).
- Order eight at $T=0$:
  - 6+2 terms in 50 min using the *Mathematica* code.
  - Most time-consuming part: Dirac trace ($O(10^4)$ terms with up to 14 γ-matrices each).
- Order six at $T\neq 0$:
  - Temperature enters in (3+1)-dim angular averaging.
  - Each $T=0$ operator yields up to 8×4 different terms.
  - Total runtime $\approx$ 1 hour using the *Mathematica* code.

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Summary

- We calculated the leading CP-violating operators for Standard Model bosons in a derivative expansion.
- Result of García-Recio, Salcedo, JHEP **07** (2009) fully confirmed.
- Result of Hernandez, Konstandin, Schmidt, NPB **812** (2009): doubts.
- Generalization of order six to nonzero temperature; critical for the cold EWBG scenario.
  See arXiv in the the following few days for details.
- Order-eight calculation in progress; relevant for estimates on convergence of the derivative expansion.

Outlook

- Finish the order-eight calculation (a few weeks).
- Numerical lattice simulations of baryogenesis:
  - Redo with correct order-six operators.
  - Insert the correct temperature dependence.
  - Parameter space strongly constrained, but generation of sufficient baryon asymmetry still seems possible!
- Possible issues with the derivative expansion:
  - Violates gauge invariance at nonzero temperature (here only the gauge-covariant terms kept).
  - Fully gauge-invariant action is nonlocal in time.