标量场暗能量的一般性质

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暗能量及其基本理论高级研讨班, 2014.4.12

提纲

- 追踪解及其特性 $w_\phi - \Omega_\phi,\ w_\phi - w'_\phi$
- thawing解及其特性
- 参数化及w_0 与 w_a的简并关系
- 一般性质的应用
- 增长因子及增长指数
- 观测限制（宇宙学常数）

参考文献

Gao & Gong, Int. J. Mod. Phys. D 22 (13) 1350035

Gong & Gao, Eur. Phys. J. C 74 (14) 2729

Gong, PLB 731 (14) 342

Gao & Gong, CQG in press

Gong etal., PRD 80 (2009) 023002

动机

- 观测数据越来越多，且越来越精确，动力学暗能量的空间越来越小
- 观测结果支持宇宙学常数模型
- 动力学模型与宇宙学常数模型接近，是否表现出某些共性？
- 动力学模型与宇宙学常数模型的区分不大，是否可以区分？
- 如何区分动力学模型（是否需要加初始条件）
- 是否不需要考虑动力学模型？

动机

- 共性问题
- thawing模型具有普遍的 $w_\phi - \Omega_\phi$ 关系
- 这个普遍 $w_\phi - \Omega_\phi$ 关系可以导出近似的参数化，并且可利用观测数据限制势函数
- thawing模型的 w'_ϕ 有一个上限
- 追踪、freezing模型的 w'_ϕ 也有下限
- 追踪模型具有不太依赖于初始条件的 $w_\phi - \Omega_\phi$ 及 w'_ϕ 普遍关系
- 追踪模型这些关系是否也可以用来限制模型

追踪解

- 追踪场的状态方程参数w_ϕ具有类似吸引子的解，即对于很大范围的初始条件，标量场会很快沿着一个共同的轨迹 $\rho_\phi(t)$, $w_\phi(t)$ 运动

$$\rho_{eq} < \rho_{\phi i} < \rho_{r,mi}$$

PRL82 (99) 896

Steinhardt, Wang and Zlatev, PRD 59 (99) 123504

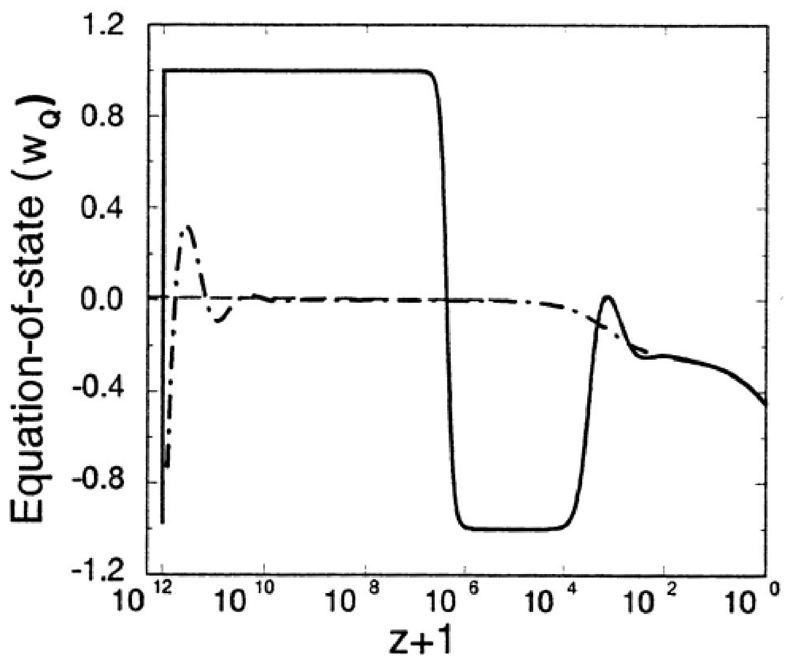
- 特点：$w_\phi \lesssim w_B$ 追踪背景的演化，ρ_ϕ 比背景衰减的慢，最后能够超过物质而占主导，从而可以提供宇宙加速
- 渐近行为：$w_\phi \to -1,\ \Omega_\phi \to 1$

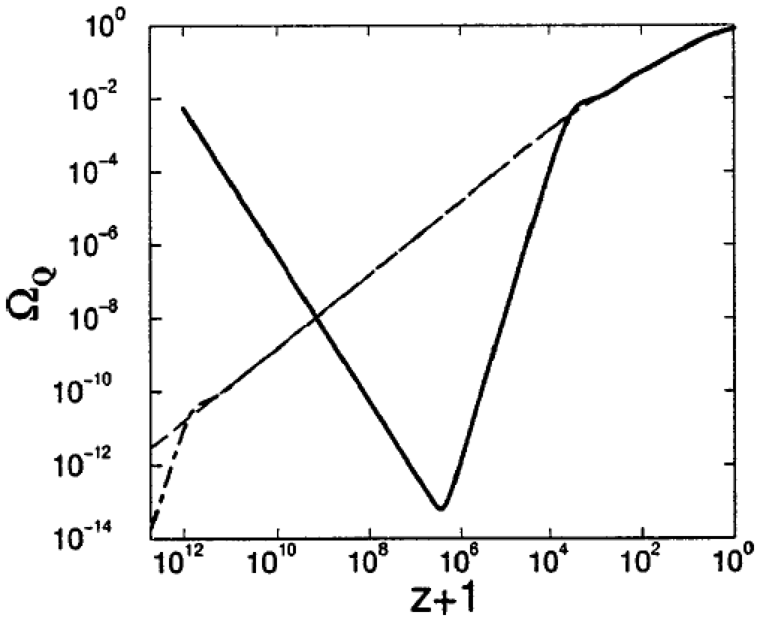
追踪解

- 追踪场的势能函数 $V(\phi) = M^4 f(\phi/M)$
- 追踪解的必要条件 $\Gamma = \frac{VV_{,\phi\phi}}{V_{,\phi}^2} = \frac{\eta}{2\epsilon} > 1$

$$\left|\frac{d(\Gamma - 1)}{H dt}\right| \ll |\Gamma - 1|$$

$$V(\phi) = V_0(\phi/m_{pl})^{-6}$$

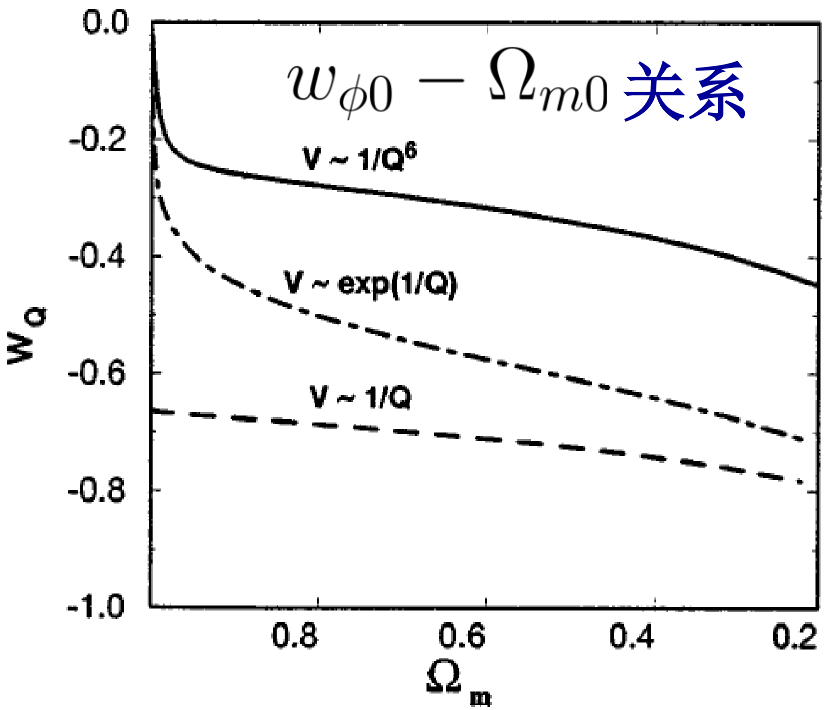




追踪解

- 追踪解：不太依赖于初始条件，在辐射或物质为主时期追踪场追踪背景辐射或物质（并不是最后的相同轨迹），这并不是我们感兴趣的
- 标量场在演化后期占主导后走相同轨迹，具体的轨迹只依赖于能标M
- 普遍$w_{\phi 0}-\Omega_{\phi 0}$关系

$$w_\phi \approx \frac{w_B - 2(\Gamma - 1)}{2\Gamma - 1} < w_B$$



追踪解

- 追踪解：为什么w_ϕ追踪背景
- 如何理解追踪场不追踪背景且占主导后走相同轨迹及 $w_{\phi 0}-\Omega_{\phi 0}$ 关系
- 指数势的吸引子解是特殊的追踪解，但其追踪解不唯一，而且依赖于参数 λ

$$V(\phi) = V_0 \exp(-\lambda\phi)$$

$$\Gamma = 1$$

$$\Gamma - 1 \approx \frac{w_B - w_\phi}{2(1+w_\phi)}(1-\Omega_\phi)$$

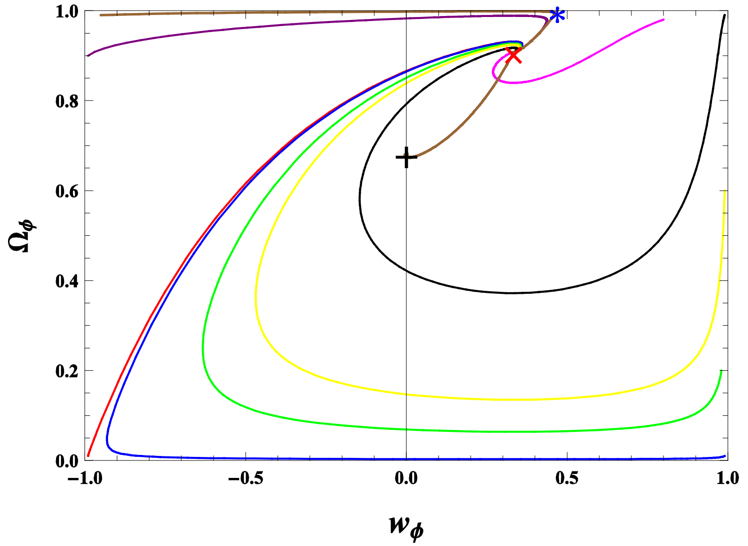
Rubano etal., PRD 69 (04) 103510

指数势

- 吸引子解 $V(\phi) = V_0 \exp(-\lambda\phi)$

CP	Existence	Stability	Ω_ϕ	w_ϕ
$(0,0)$	/	$-1 < w < 1$ Unstable	0	/
$(1,0)$	/	$\lambda < \sqrt{6}$ Unstable $\lambda > \sqrt{6}$ Saddle	1	1
$(-1,0)$	/	$\lambda > -\sqrt{6}$Unstable $\lambda < -\sqrt{6}$ Saddle	1	1
C	$\lambda^2 < 6$	$\lambda^2 < 3(1+w)$ Stable $3(1+w) < \lambda^2 < 6$ Saddle	1	$\lambda^2/3 - 1$
D	$\lambda^2 > 3(1+w)$	$3(1+w) < \lambda^2 < \Gamma$ Stable $\lambda^2 > \Gamma$ Spiral	$3(1+w)/\lambda^2$	w

Copeland etal, PRD 57 (98) 4686



负幂次势

- 幂次势 $V(\phi) = V_0(\phi/m_{pl})^{-\alpha}$ 吸引子解

$$\rho_\phi \ll \rho_b \quad \rho_\phi \sim a^{-3(1+w_\phi)}, \quad \rho_b \sim a^{-3(1+w_b)}$$

$$\gamma_\phi = \frac{\alpha}{\alpha+2}\gamma_b,$$

Liddle & Scherrer, PRD 59 (99) 023509

$$\phi(t) = At^{2/(\alpha+2)} = At_0^{2/(\alpha+2)}\left(\frac{a}{a_0}\right)^{3(1+w_b)/(\alpha+2)},$$

$$\rho_\phi = \left(\frac{2}{(\alpha+2)^2}A^2 + \Lambda^{4+\alpha}A^{-\alpha}\right)t_0^{-2\alpha/(\alpha+2)}\left(\frac{a}{a_0}\right)^{-3\alpha(1+w_b)/(\alpha+2)}$$

$$A = (\alpha\Lambda^{4+\alpha}/[4/(\alpha+2)(1+w_b) - 2\alpha/(\alpha+2)^2])^{1/(\alpha+2)}$$

幂次势

- 吸引子解 $V(\phi) = V_0(\phi/m_{pl})^\beta$

$$\rho_\phi \ll \rho_b \qquad \rho_\phi \sim a^{-3(1+w_\phi)}, \qquad \rho_b \sim a^{-3(1+w_b)}$$

$$1 + w_\phi = \frac{\beta}{\beta - 2}(1 + w_b), \qquad \beta > 2, \beta < 0$$

- 稳定性

$$\beta < 0, \qquad \beta < 2\left(\frac{6+m}{6-m}\right) \qquad m = 3(1+w_b)$$

$$m = 3,\ \ 4$$

$$\beta > 0, \qquad \beta > 2\left(\frac{6+m}{6-m}\right)$$

$$\beta > 2,\ \ w_\phi > w_b$$

负幂次势

- 负幂次势具有近似吸引子解

$$\rho_\phi \ll \rho_b \quad \gamma_\phi = \frac{\alpha}{\alpha+2}\gamma_b, \alpha > 0$$

- 破坏吸引子解 $\rho_\phi > \rho_b$
- 追踪解 $V(\phi) = \Lambda^{4+\alpha}\phi^{-\alpha}$

$$\rho_\phi = \left(\frac{2}{(\alpha+2)^2}A^2 + \Lambda^{4+\alpha}A^{-\alpha}\right) t_0^{-2\alpha/(\alpha+2)} \left(\frac{a}{a_0}\right)^{-3\alpha(1+w_b)/(\alpha+2)}$$

$$A = (\alpha\Lambda^{4+\alpha}/[4/(\alpha+2)(1+w_b) - 2\alpha/(\alpha+2)^2])^{1/(\alpha+2)}$$

- 追踪背景不依赖于初始条件，轨迹依赖于参数 Λ

标量场宇宙学

- 宇宙学方程

$$\Omega'_\phi = 3(\gamma_b - \gamma_\phi)\Omega_\phi(1-\Omega_\phi),$$

$$\gamma'_\phi = (2-\gamma_\phi)(-3\gamma_\phi + |\lambda|\sqrt{3\gamma_\phi\Omega_\phi}) \qquad \gamma = 1 + w$$

$$\lambda' = -\sqrt{3\gamma_\phi\Omega_\phi}\lambda|\lambda|(\Gamma - 1). \qquad \Gamma > 1, \lambda \text{ 初值很大}$$

$$\lambda = -\frac{V_{,\phi}}{V} = -\frac{1}{V}\frac{dV}{d\phi} \qquad \Gamma = \frac{VV_{,\phi\phi}}{V_{,\phi}^2} \qquad \tilde{x} = \ln[(1+w_\phi)/(1-w_\phi)]$$

$$\Gamma - 1 = \frac{3(w_b - w_\phi)(1-\Omega_\phi)}{(1+w_\phi)(6+\tilde{x}')} - \frac{(1-w_\phi)\tilde{x}'}{2(1+w_\phi)(6+\tilde{x}')} - \frac{2\tilde{x}''}{(1+w_\phi)(6+\tilde{x}')^2},$$

$$w_\phi \approx \frac{w_b(1-\Omega_\phi) - 2(\Gamma - 1)}{2\Gamma - 1 - \Omega_\phi} < w_b, \qquad (\Gamma > 1)$$

追踪解定义

- 追踪条件（ w_ϕ 近似为常数）

$$\gamma_\phi' = (2-\gamma_\phi)(-3\gamma_\phi + |\lambda|\sqrt{3\gamma_\phi\Omega_\phi})$$

$$\gamma_\phi = 1 + w_\phi = \frac{1}{3}\lambda^2\Omega_\phi,\ \ \Omega_\phi \approx 0, \lambda \text{ 比较大}$$

$$w_\phi = w_\phi^{trk} = \frac{w_b - 2(\Gamma-1)}{2\Gamma-1} \qquad \lambda' = -\sqrt{3\gamma_\phi\Omega_\phi}\lambda|\lambda|(\Gamma-1).$$

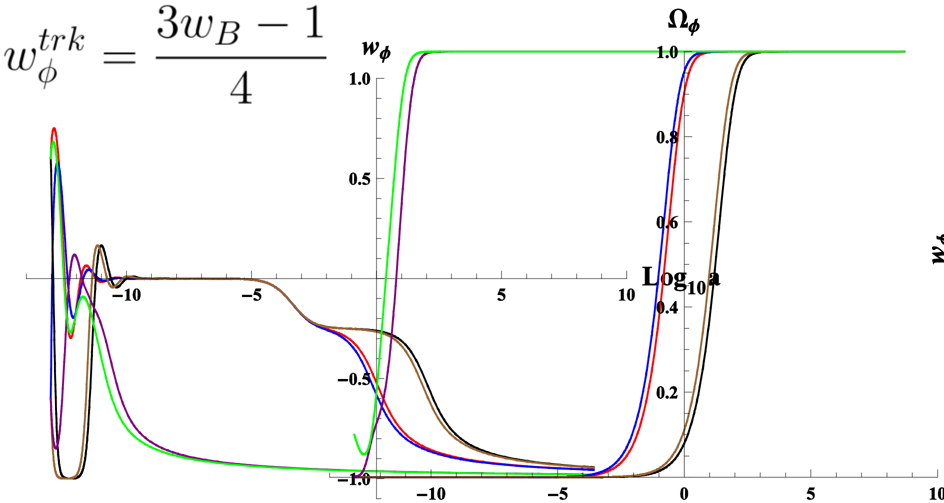
- 追踪解：不但追踪背景，而且开始占主导后遵从相同的 $w_\phi - \Omega_\phi$ 轨迹，追踪条件可作为追踪解的初始条件，且滚动参数 λ 绝对值应该随时间减小，且初始值应该很大

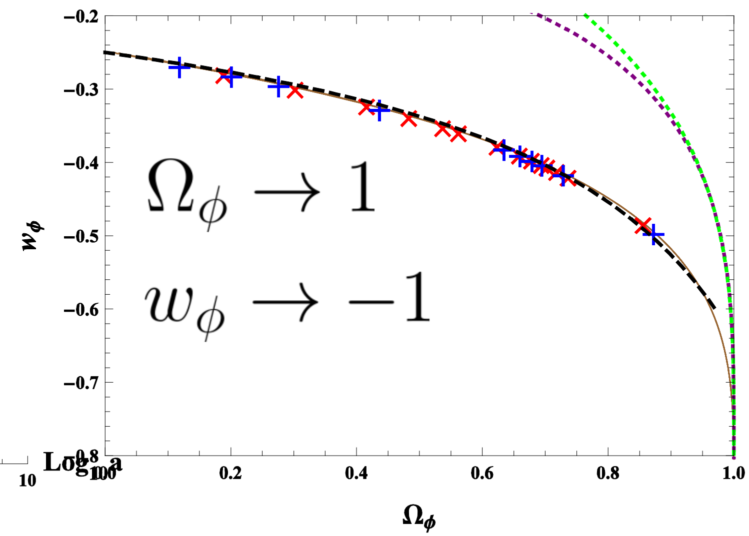
Gong, PLB 731 (14) 342

负幂次势追踪解

- 追踪解 $V(\phi) = V_0(\phi/m_{pl})^{-6}$ $\Gamma = 7/6$

$$w_\phi^{trk} = \frac{3w_B - 1}{4}$$





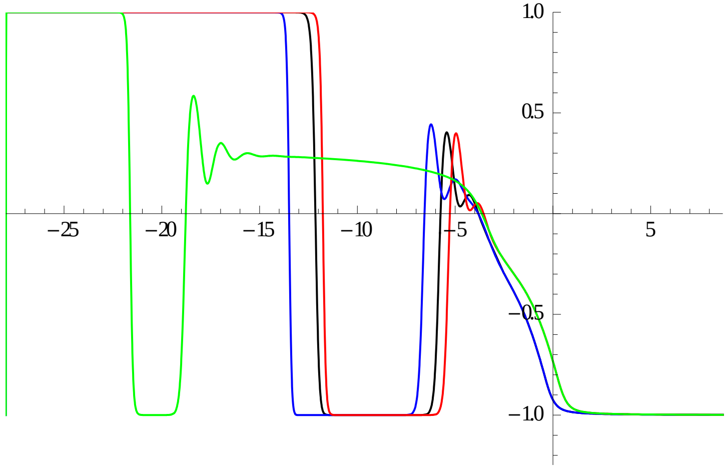
- $w - \Omega_\phi$ 关系 **Watson & Scherrer, PRD 68 (03) 123524**

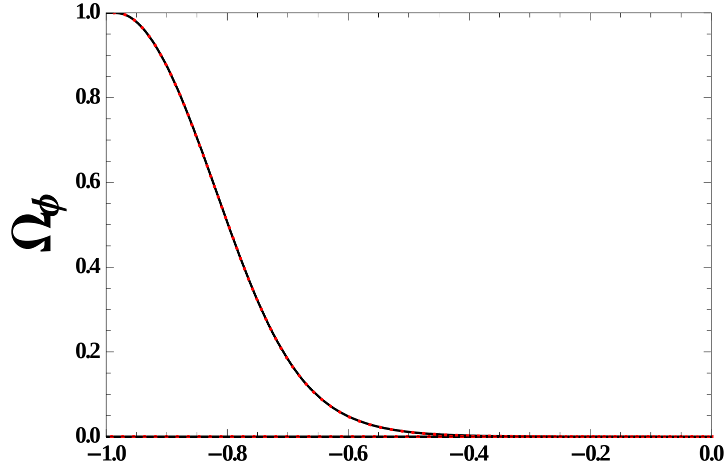
$$\Omega_\phi = \frac{1 - 2w_i + 4w_i^2}{(1 - w_i^2)w_i}(w_\phi - w_i) \qquad w_i = w_\phi^{trk}, \Omega_\phi \ll 1$$

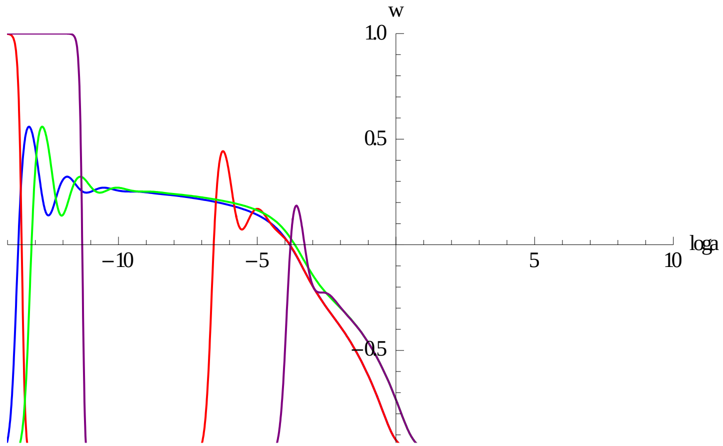
$$\Omega_\phi = \left[1 - \frac{(1 - w_i^2)w_i}{1 - 2w_i + 4w_i^2}\frac{1}{\gamma_i - \gamma_\phi}\left(\frac{\gamma_\phi}{\gamma_i}\right)^{1/2 + \gamma_i\gamma_b/4(\gamma_b - \gamma_i)} \exp\left[\frac{\gamma_i\gamma_b}{\gamma_i - \gamma_b}\left(\frac{1}{\gamma_\phi} - \frac{1}{\gamma_i}\right)\right]\right]^{-1}.$$

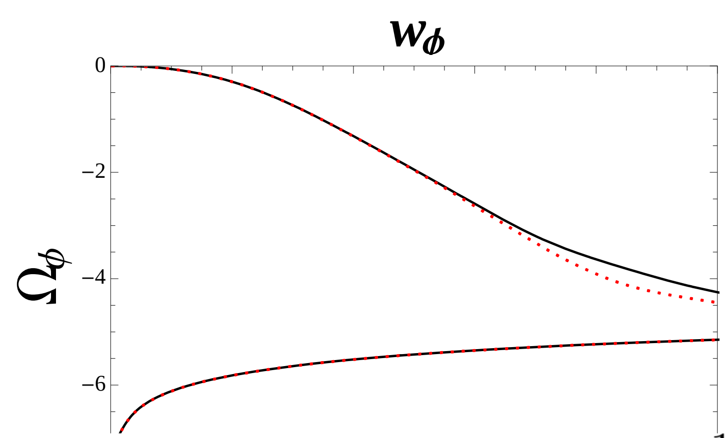
追踪解

- 追踪势 $V(\phi) = V_0 \exp(M/\phi)$ $\Gamma = 1 + 2/\sqrt{\lambda}$









$$\Omega_\phi = \left[1 + 3\gamma_\phi^{1/4}(1-\gamma_\phi)^{-15/2} \exp[-(3\gamma_\phi)^{1/4} - (3\gamma_\phi)^{-3/4}]\right]^{-1}$$

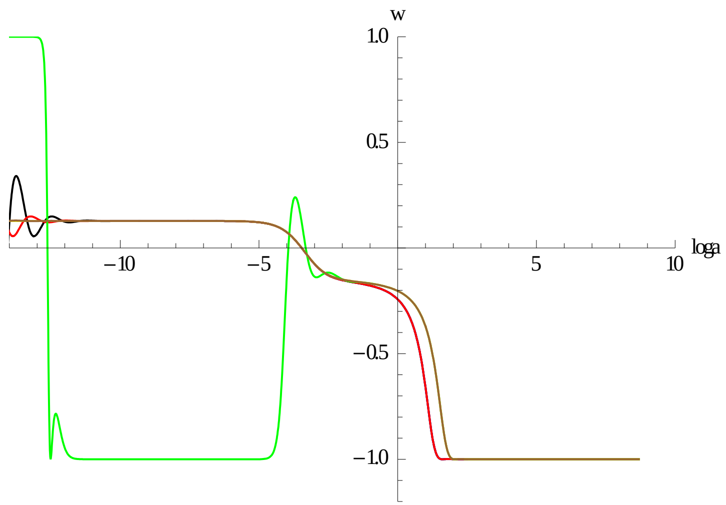
追踪解

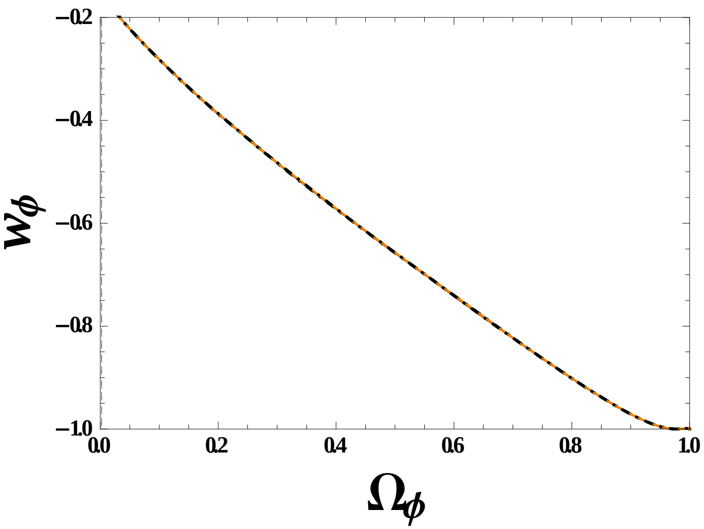
- 超引力势SUGRA

$$\lambda = \frac{\alpha}{\phi} - \phi$$

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^{\alpha}} \exp\left[\frac{1}{2}\left(\frac{\phi}{M_{pl}}\right)^2\right]$$

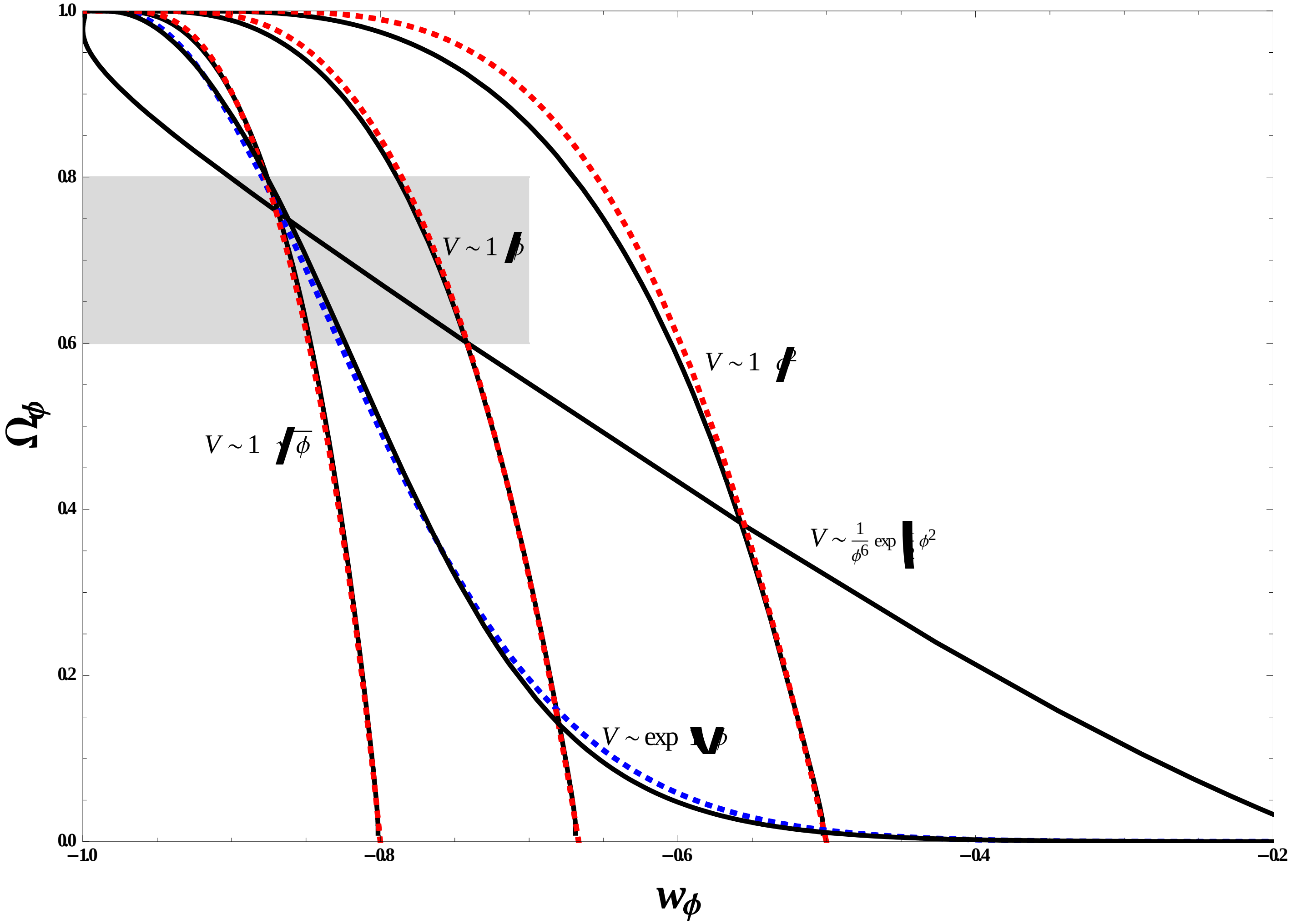
$$\Gamma = 1 + \frac{1+\alpha\phi^{-2}}{\lambda^2}$$





追踪解的普遍特性

- $w-\Omega_\phi$ 关系



追踪解的普遍特性

- w'_ϕ 限制 **Chiba, PRD 73 (06) 063501**

$$w' > 3w(1-w^2)/(1-2w)$$

$$\tilde{x} = \ln[(1+w_\phi)/(1-w_\phi)]$$

$$\Gamma - 1 = \frac{3(w_b - w_\phi)(1-\Omega_\phi)}{(1+w_\phi)(6+\tilde{x}')} - \frac{(1-w_\phi)\tilde{x}'}{2(1+w_\phi)(6+\tilde{x}')} - \frac{2\tilde{x}''}{(1+w_\phi)(6+\tilde{x}')^2},$$

$$\tilde{x}'' = 0$$

$$x'_m = -6\frac{w(1-\Omega_\phi) + 2(1+w)(\Gamma-1)}{(1-w) + 2(1+w)(\Gamma-1)} > -6\frac{2(1+w)(\Gamma-1)}{(1-w)+2(1+w)(\Gamma-1)}.$$

$$x'_m > \frac{6w}{1-2w}.$$

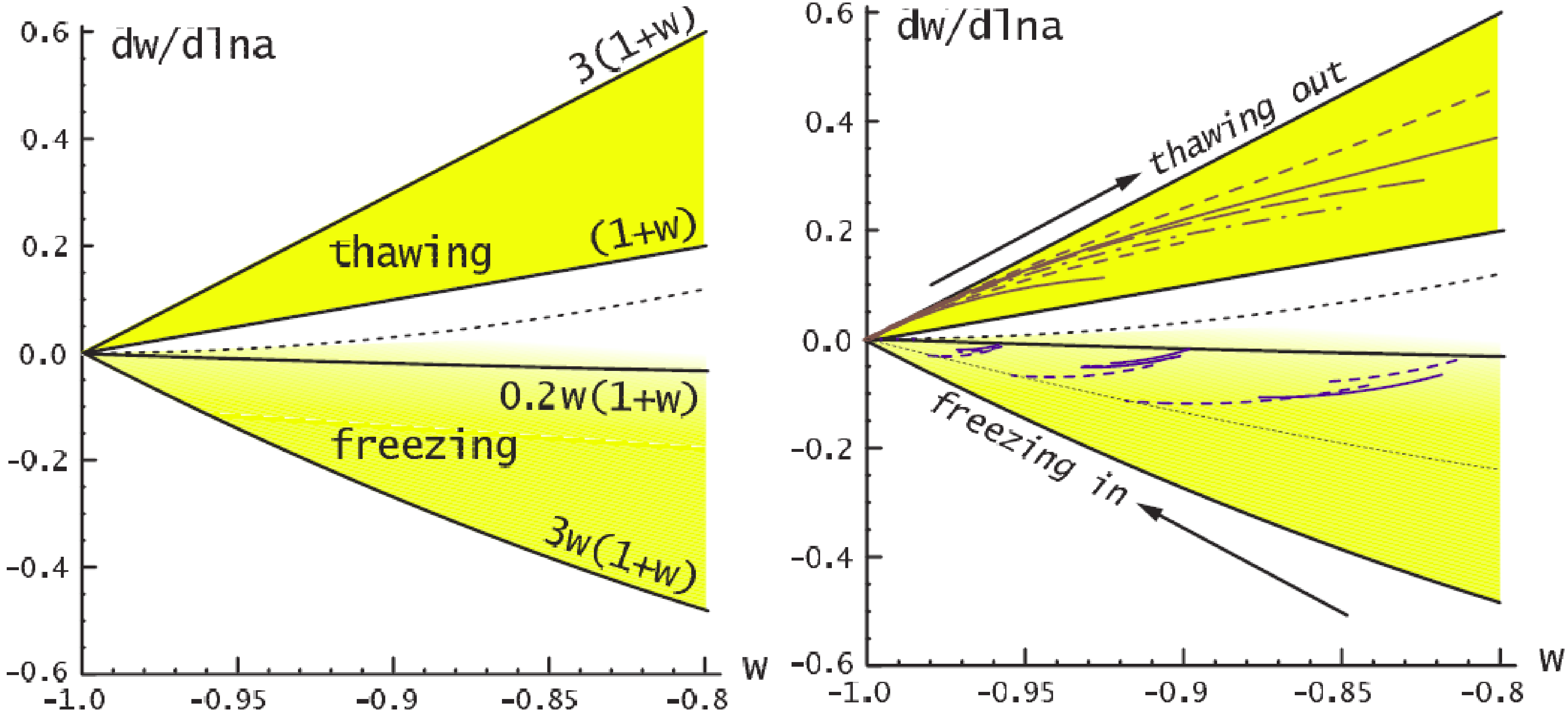
$$w' > \frac{3w}{1-2w}(1-w)(1+w) \geq -(1-w)(1+w).$$

追踪解的特性

- 追踪解的特性：基本与初始条件无关
- $w-\Omega_\phi$关系：$\phi,\ \dot{\phi}$由追踪条件确定
- 在最近开始占主导作用，现在的行为接近于宇宙学常数（观测限制 $w\sim -1$）
- w'_ϕ 有一定的限制（ w'_ϕ 下限）

$w_\phi - w'_\phi$ 关系

- thawing及freezing解



Caldwell & Linder, PRL95 (05) 141301

模型分类

- thawing解：w_ϕ 从-1开始缓慢增加并偏离-1 $1+w_\phi \sim 0, w'_\phi > 0$
- freezing解：w_ϕ从大于-1开始缓慢减小到-1（包括tracking解）$1+w_\phi \sim 0, w'_\phi < 0$

Caldwell & Linder, PRL95 (05) 141301

- cooling解：w_ϕ从大于-1开始缓慢减小（包括freezing及tracking解）

$$-3(1-w_\phi^2) < w'_\phi < 0.2w_\phi(1+w_\phi)$$

Barger etal., PLB 635 (06) 61

$w - w'$ 关系(quintessence)

- Quintessence $w'_\phi \geq -3(1-w_\phi^2)$ $\dot{V} < 0$
- Thawing model

$$0 < w'_\phi < 3(1+w_\phi)(2+w_\phi) \qquad 1 + w < w' < 3(1+w)$$

- Freezing model

Caldwell and Linder, PRL 95, 141301

$$3w_\phi(1+w_\phi) < w'_\phi < 0 \qquad 3w(1+w) < w' \lesssim 0.2w(1+w)$$

- Tracking model

$$-(1-w)(1+w) < w' \qquad w' > 3w(1-w^2)/(1-2w)$$

Scherrer, PRD 73 (06) 043502

Chiba, PRD 73 (06) 063501

$w-w'$ 关系

- 所有标量场满足的限制

$$\gamma_\phi' = (2-\gamma_\phi)(-3\gamma_\phi + |\lambda|\sqrt{3\gamma_\phi\Omega_\phi})$$

$$w_\phi' \geq -3(1-w_\phi^2)$$

- thawing及freezing模型

$$w_\phi' = 3(1+w_\phi)\left(1+w_\phi+\frac{2\ddot{\phi}}{3H\dot{\phi}}\right)$$

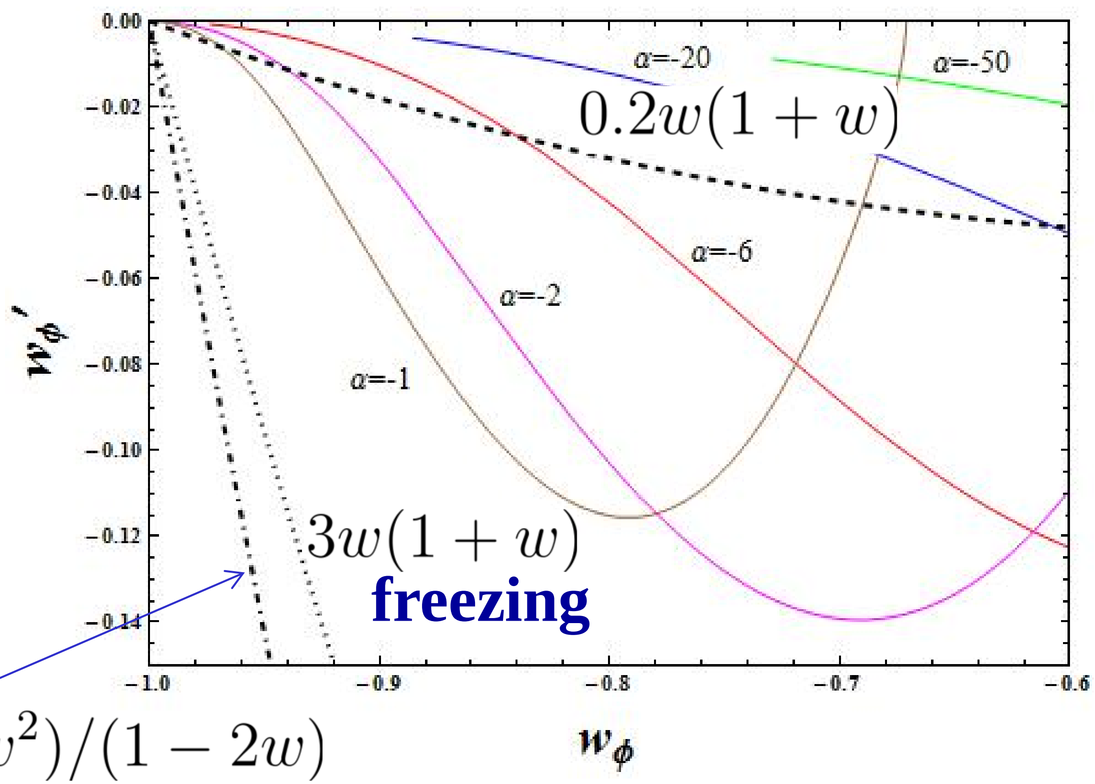
$$|\ddot{\phi}| \lesssim \frac{|\dot{\phi}|}{t} = \frac{3}{2}H|\dot{\phi}|$$

$$0 < w_\phi' < 3(1+w_\phi)(2+w_\phi) \qquad 1+w < w' < 3(1+w)$$

$$3w_\phi(1+w_\phi) < w_\phi' < 0 \qquad 3w(1+w) < w' \lesssim 0.2w(1+w)$$

$w - w'$ 关系

- 追踪解



$3w(1-w^2)/(1-2w)$

tracking

thawing模型

- thawing解 $w \sim -1$ $w_\phi - \Omega_\phi$ 关系

$$\lambda' = -\sqrt{3\gamma_\phi \Omega_\phi}\lambda|\lambda|(\Gamma - 1).$$

$$\gamma_\phi \approx 0, \ \lambda \sim \text{常数}$$

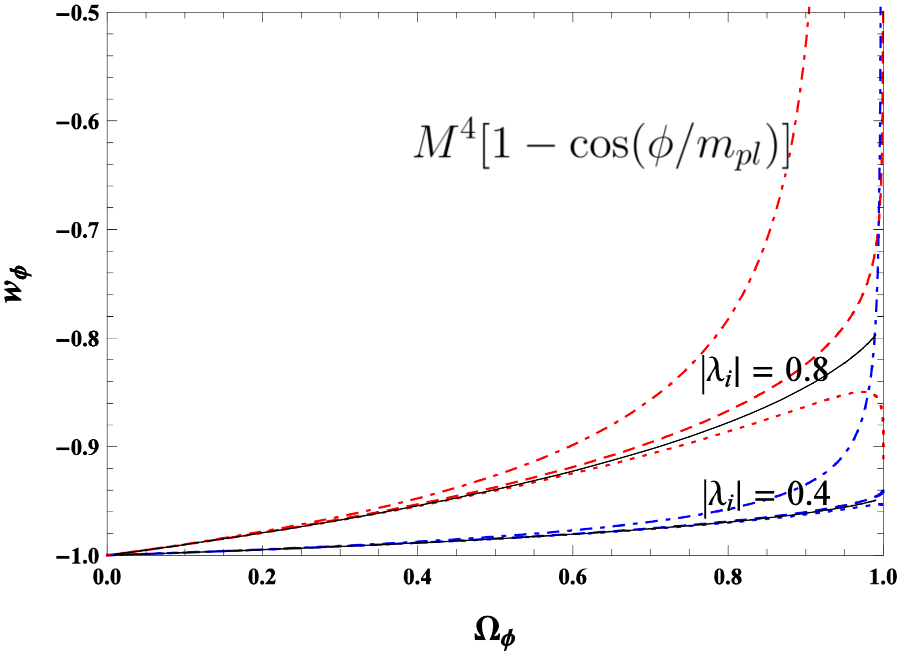
$$\frac{d\gamma_\phi}{d\Omega_\phi} = \frac{-6\gamma_\phi + 2\lambda\sqrt{3\gamma_\phi\Omega_\phi}}{3\gamma_b\Omega_\phi(1-\Omega_\phi)}$$

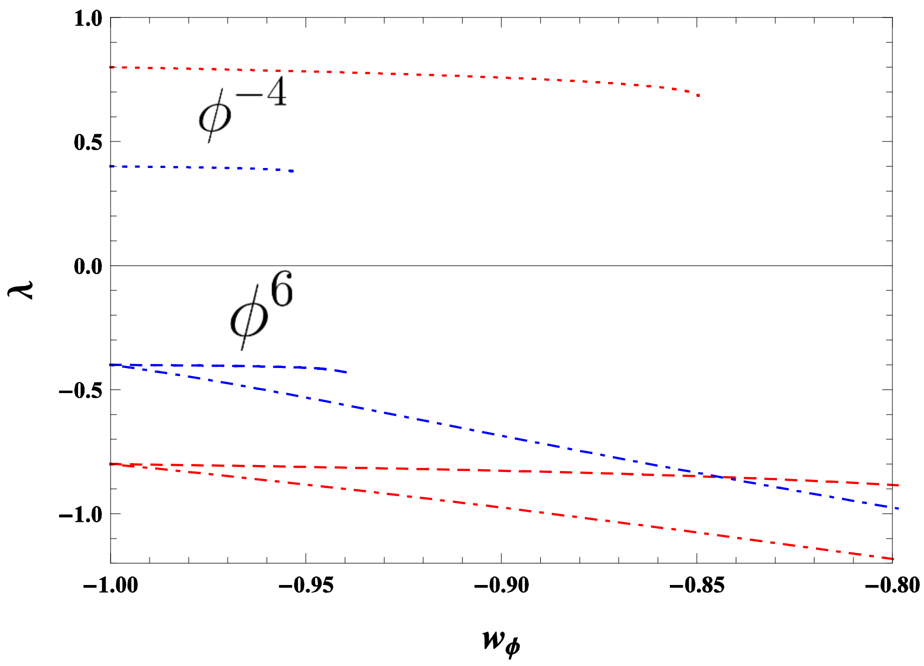
$$\gamma_\phi = \frac{\lambda_i^2}{3}\left(1+\frac{1}{2}\gamma_b\right)^{-2}\Omega_\phi(1-\Omega_\phi)^{2/\gamma_b}\,{}_2F_1^2\left(\frac{1}{\gamma_b}+\frac{1}{2}, \frac{1}{\gamma_b}+1, \frac{1}{\gamma_b}+\frac{3}{2}; \Omega_\phi\right)$$

$$\gamma_\phi = \frac{\lambda_i^2}{3}\left[\frac{1}{\sqrt{\Omega_\phi}} - \left(\frac{1}{\Omega_\phi} - 1\right)\tanh^{-1}(\sqrt{\Omega_\phi})\right]^2, \ \gamma_b = 1$$

thawing解特性

- $w-\Omega_\phi$ 近似





Gong, PLB 731 (14) 342

thawing模型的 $w - w'$ 关系

- thawing模型

$$\gamma_\phi = \frac{\lambda_i^2}{3}\left[\frac{1}{\sqrt{\Omega_\phi}} - \left(\frac{1}{\Omega_\phi} - 1\right)\tanh^{-1}(\sqrt{\Omega_\phi})\right]^2$$

$$\Omega_\phi \to 0, w_\phi \to -1, F \to 4/27$$

$$\Omega_\phi \to 1, F \to 1/3$$

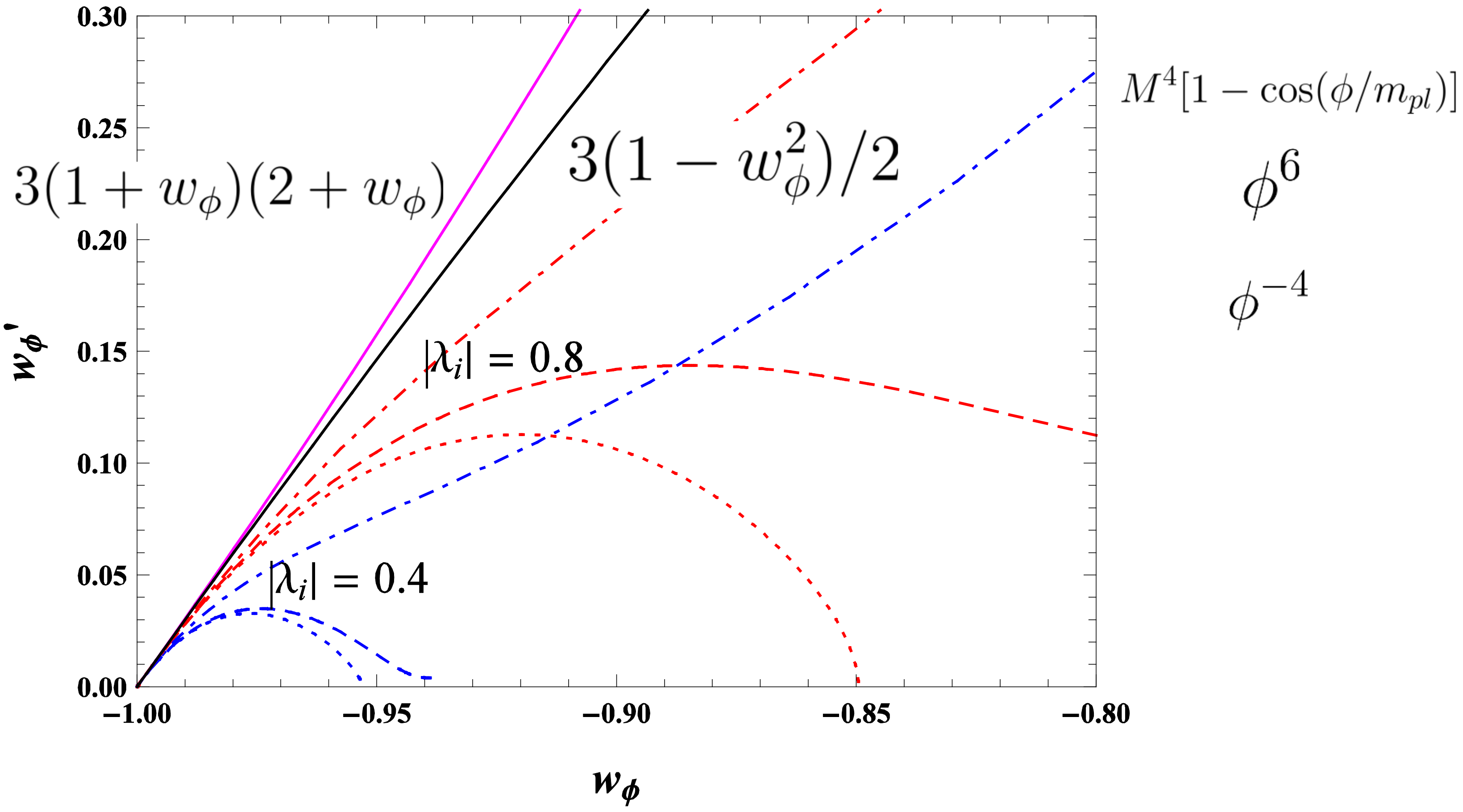
$$4/27 \le F = \gamma_\phi/(\Omega_\phi \lambda^2) \le 1/3$$

$$\gamma'_\phi = (2-\gamma_\phi)(-3\gamma_\phi + |\lambda|\sqrt{3\gamma_\phi\Omega_\phi})$$

$$0 \le w'_\phi \le 3(1-w_\phi^2)/2$$

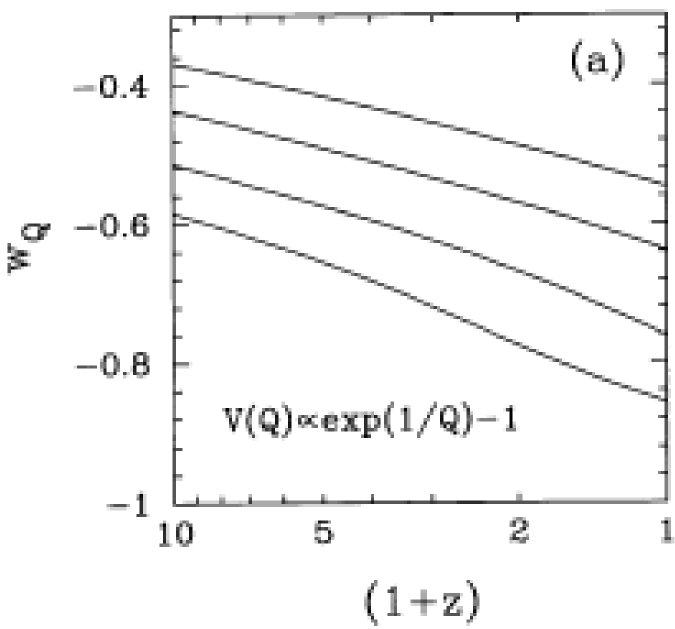
thawing模型

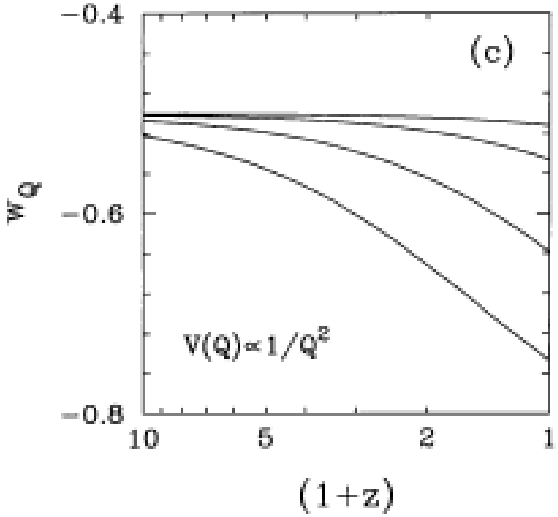
- $w - w'$ 关系



参数化（动力学性质近似）

- **Dark energy parametrization: capture the main dynamics of scalar field**

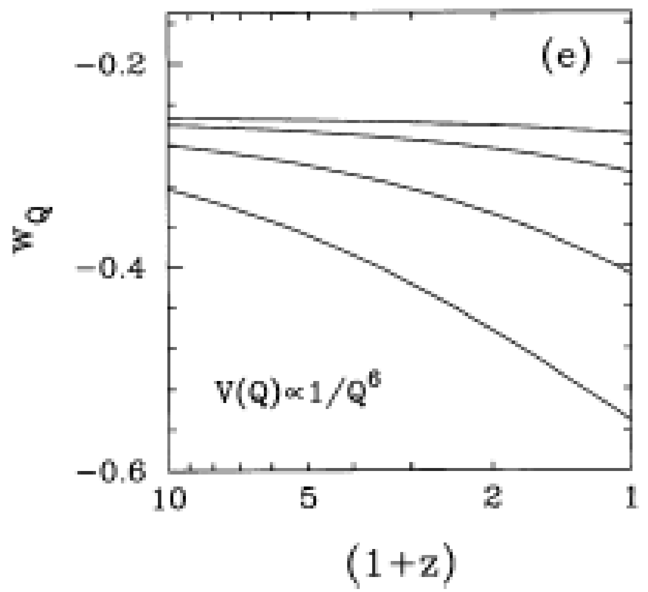




$$w(z) = w(0) - \alpha \ln(1+z)$$

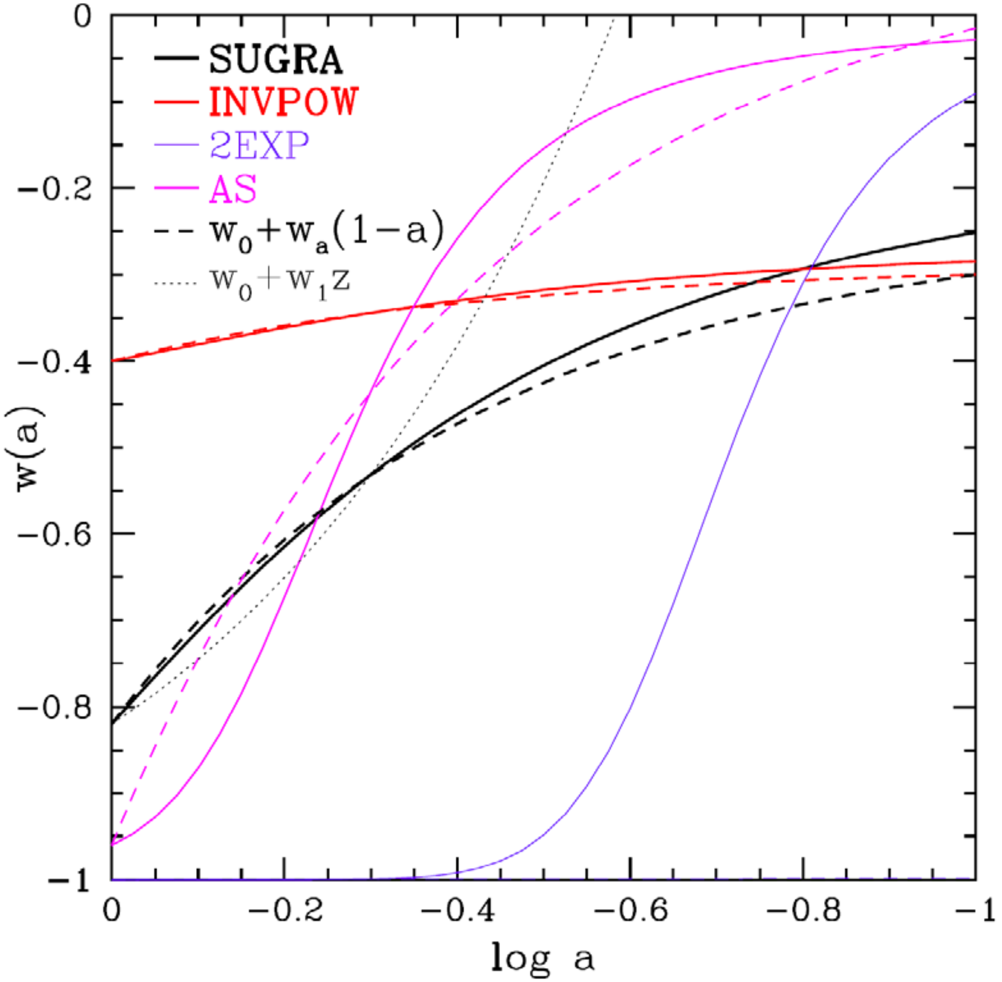
$$z \lesssim 4$$

MNRAS 383, 879 (1999)



CPL Parametrization

- Approximation $w(z) = w_0 + w_a(1-a) = w_0 + w_a \frac{z}{1+z}$



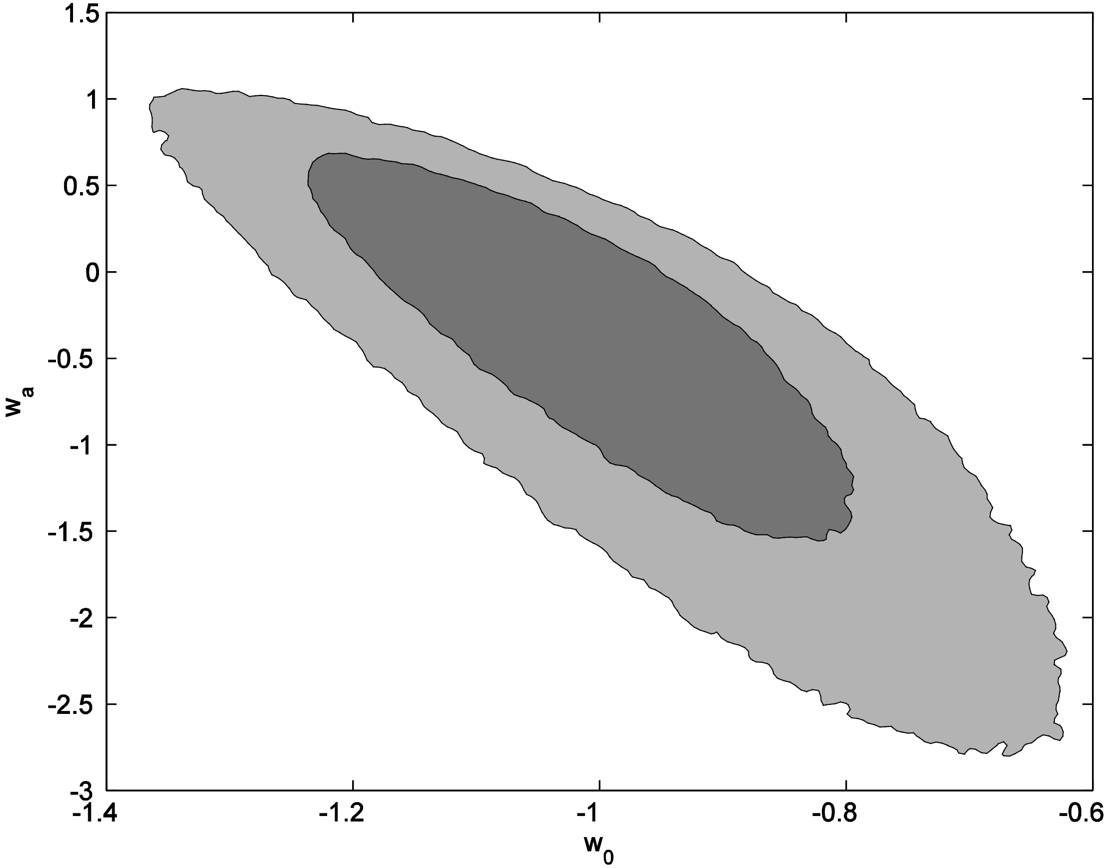
E. Linder

astro-ph/0210217

PRL90 (03) 091301

CPL模型中的简并性

- w_0 and w_a is degenerated



- What is the degeneracy? related with $w-\Omega_\phi$?

$w-\Omega_\phi$ 关系的应用

- Thawing解的近似关系

$$1+w=(1+w_0)\left[\frac{1}{\sqrt{\Omega_\phi}}-\left(\frac{1}{\Omega_\phi}-1\right)\tanh^{-1}(\sqrt{\Omega_\phi})\right]^2\times$$

$$\left[\frac{1}{\sqrt{\Omega_{\phi0}}}-(\Omega_{\phi0}^{-1}-1)\tanh^{-1}\sqrt{\Omega_{\phi0}}\right]^{-2}$$

Scherrer & Sen 2008, PRD 77, 083515

- 0^{th} order approximation $|1+w|\ll 1$

$$\Omega_\phi'=-3w\Omega_\phi(1-\Omega_\phi) \qquad \Omega_\phi=\Omega_\Lambda=\left[1+(\Omega_{\phi0}^{-1}-1)a^{-3}\right]^{-1}$$

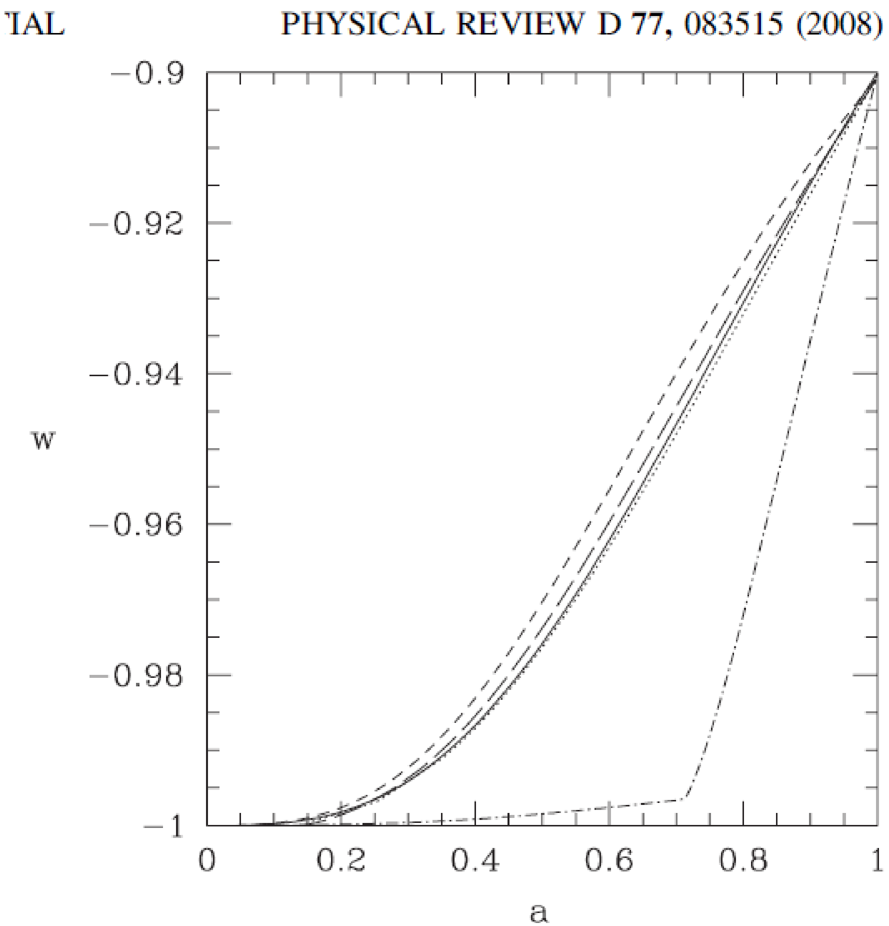
$$w(a)=-1+(1+w_0)\left[\frac{1}{\sqrt{\Omega_{\phi0}}}-(\Omega_{\phi0}^{-1}-1)\tanh^{-1}\sqrt{\Omega_{\phi0}}\right]^{-2}\times$$

$$\left[\sqrt{1+(\Omega_{\phi0}^{-1}-1)a^{-3}}-(\Omega_{\phi0}^{-1}-1)a^{-3}\tanh^{-1}[1+(\Omega_{\phi0}^{-1}-1)a^{-3}]^{-1/2}\right]^2$$

Thawing scalar fields

- Approximate w(a)

$$w(a) = -1 + (1+w_0)\left[\frac{1}{\sqrt{\Omega_{\phi 0}}} - (\Omega_{\phi 0}^{-1} - 1)\tanh^{-1}\sqrt{\Omega_{\phi 0}}\right]^{-2} \times \left[\sqrt{1 + (\Omega_{\phi 0}^{-1} - 1)a^{-3}} - (\Omega_{\phi 0}^{-1} - 1)a^{-3}\tanh^{-1}[1 + (\Omega_{\phi 0}^{-1} - 1)a^{-3}]^{-1/2}\right]^2$$



$V(\phi) = \phi^2$ **dotted curve**

$V(\phi) = \phi^{-2}$ **short dash curve**

$V(\phi) = \exp(-\lambda\phi)$ **long dash curve**

SSLCPL模型

- 近似行为

$$w(a) = -1+(1+w_0)\left[\frac{1}{\sqrt{\Omega_{\phi0}}} - (\Omega_{\phi0}^{-1}-1)\tanh^{-1}\sqrt{\Omega_{\phi0}}\right]^{-2} \times \left[\sqrt{1+(\Omega_{\phi0}^{-1}-1)a^{-3}} - (\Omega_{\phi0}^{-1}-1)a^{-3}\tanh^{-1}[1+(\Omega_{\phi0}^{-1}-1)a^{-3}]^{-1/2}\right]^2$$

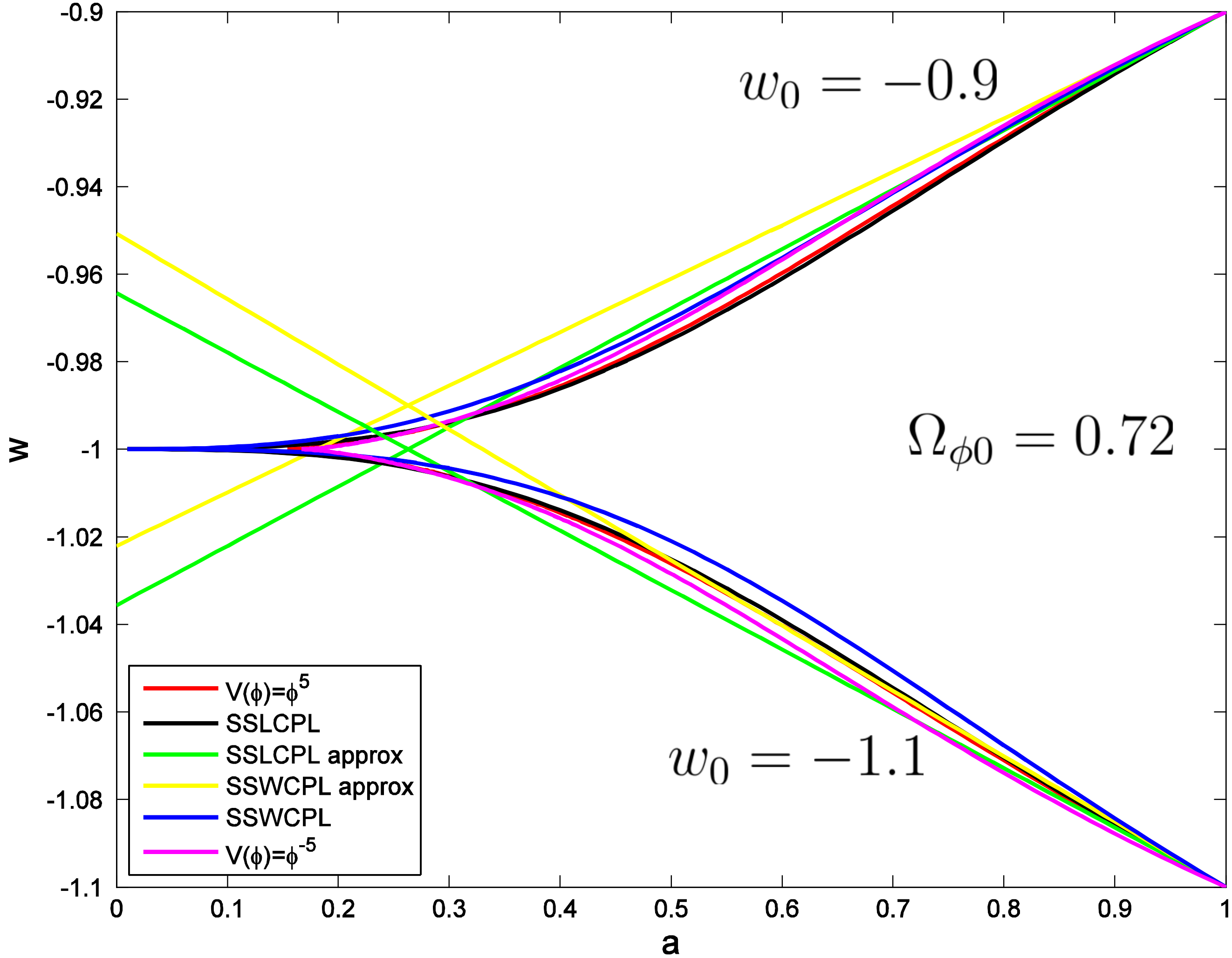
$$w = w_0 + 6(1+w_0)\frac{\Omega_{\phi0}^{-1/2} - \sqrt{\Omega_{\phi0}} - (\Omega_{\phi0}^{-1}-1)\tanh^{-1}(\sqrt{\Omega_{\phi0}})}{\Omega_{\phi0}^{-1/2} - (\Omega_{\phi0}^{-1}-1)\tanh^{-1}(\sqrt{\Omega_{\phi0}})}(1-a)$$

$$w_a = 6(1+w_0)\frac{(\Omega_{\phi0}^{-1}-1)[\sqrt{\Omega_{\phi0}} - \tanh^{-1}(\sqrt{\Omega_{\phi0}})]}{\Omega_{\phi0}^{-1/2} - (\Omega_{\phi0}^{-1}-1)\tanh^{-1}(\sqrt{\Omega_{\phi0}})}$$

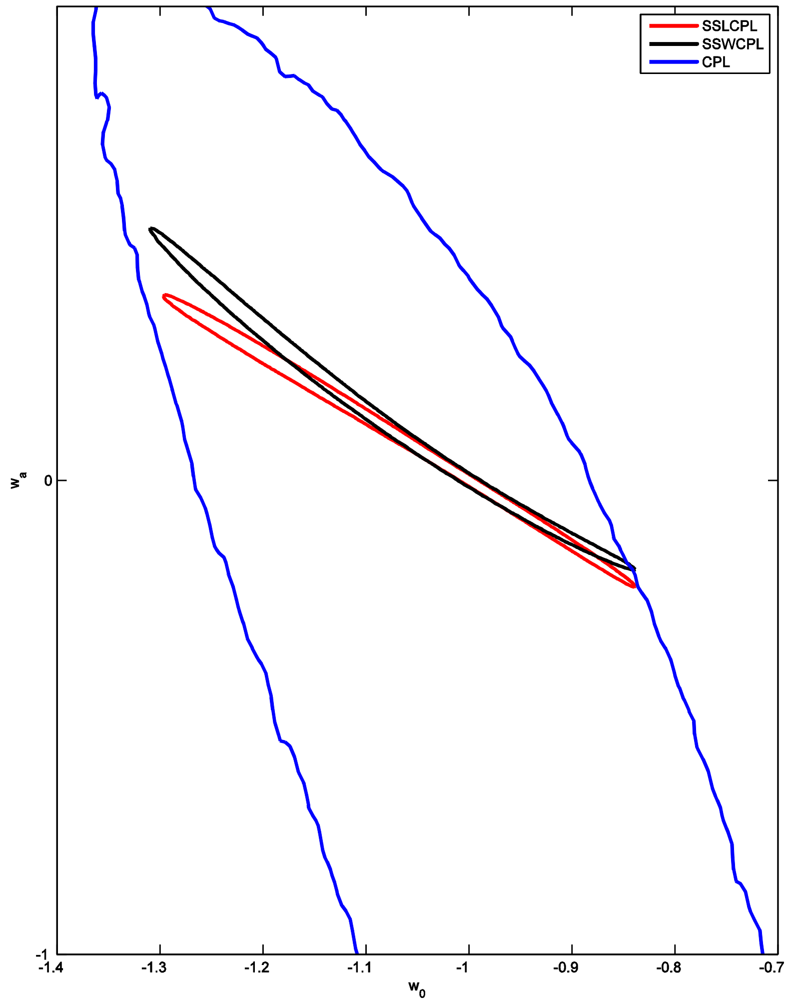
- 不是一个唯象的CPL模型，刻画了一类标量场的动力学行为（正则及快子场，适用范围更广）

Gong etal., IJMPD 22 (13) 1350035, EPJC 74 (14) 2729

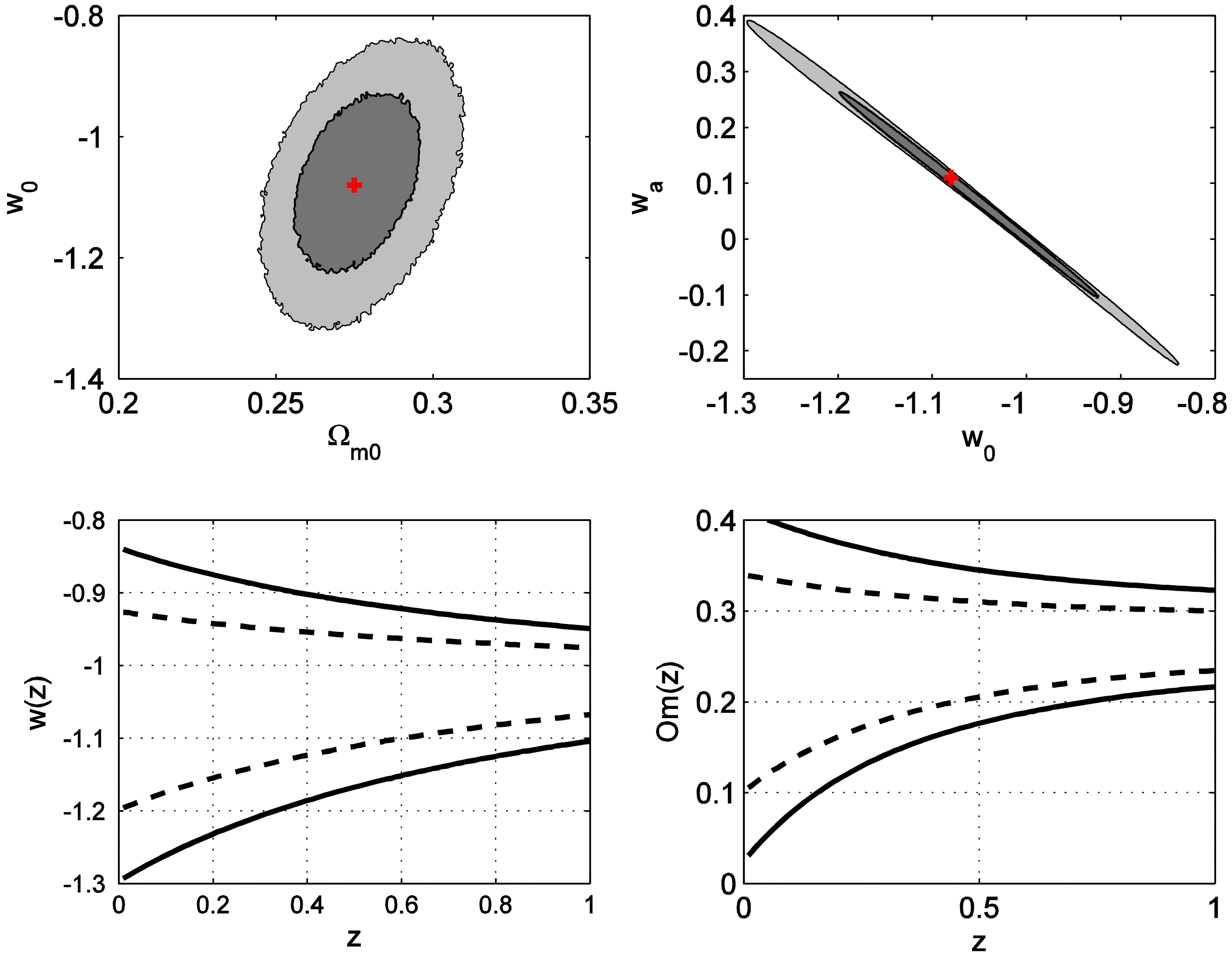
The accuracy of the approximation



简并性



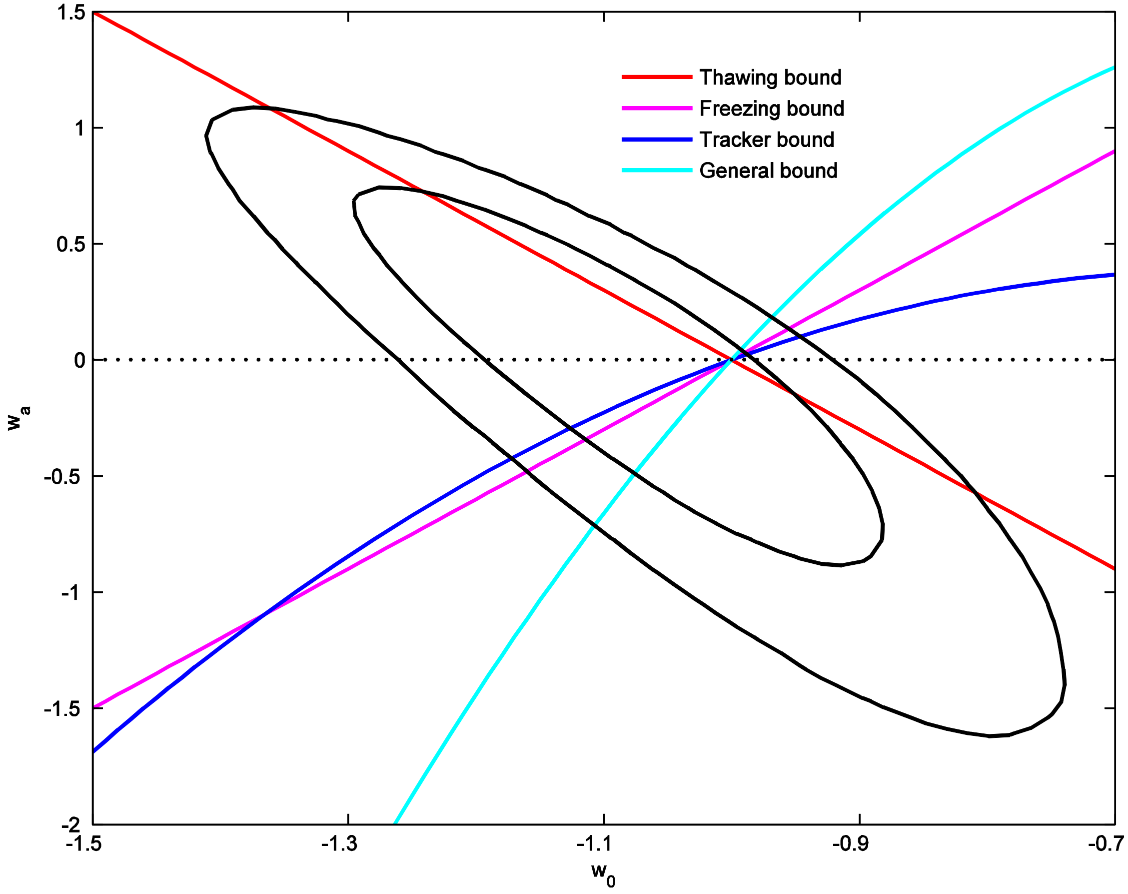
SSLCPL模型限制



-0.8
-1
-1.2
-1.4
0.2
0.25
0.3
0.35
Ω_{m0}
w_0
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-1.3
-1.2
-1.1
-1
-0.9
-0.8
w_0
w_a
-0.8
-0.9
-1
-1.1
-1.2
-1.3
0
0.2
0.4
0.6
0.8
1
z
w(z)
0.4
0.3
0.2
0.1
0
0
0.5
1
z
Om(z)

SSLCPL模型限制

- $w - w'$ 关系



SSLCPL模型重构

- 利用观测数据限制进行重构

$$\gamma_\phi = \frac{\lambda_i^2}{3}\left[\frac{1}{\sqrt{\Omega_\phi}} - \left(\frac{1}{\Omega_\phi} - 1\right)\tanh^{-1}(\sqrt{\Omega_\phi})\right]^2$$

$$\Omega'_\phi = 3(\gamma_b - \gamma_\phi)\Omega_\phi(1 - \Omega_\phi)$$

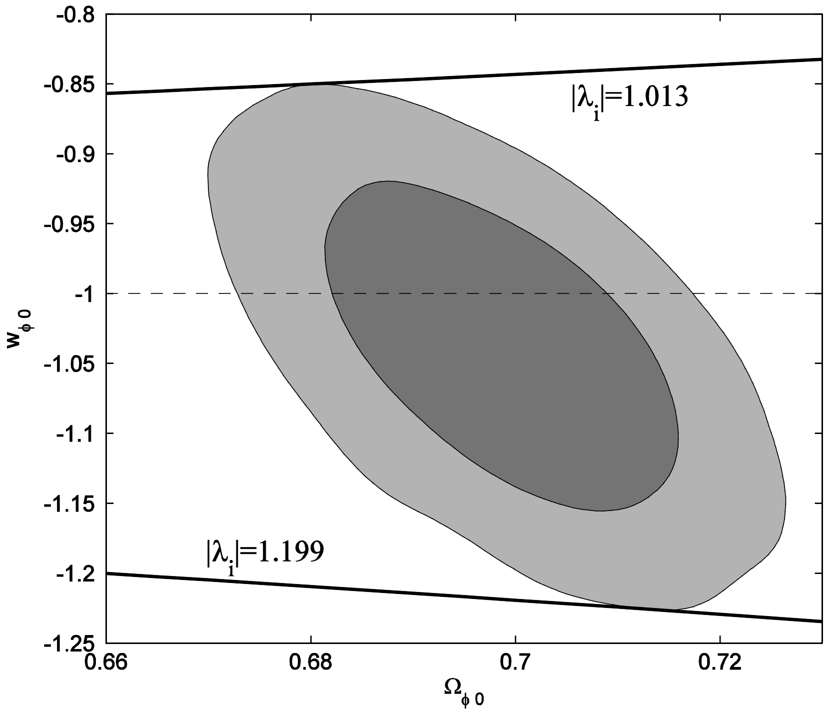
$$\Omega_m = 1 - \Omega_\phi = \frac{\Omega_{m0} a^{-3}}{E^2} \qquad E = H/H_0$$

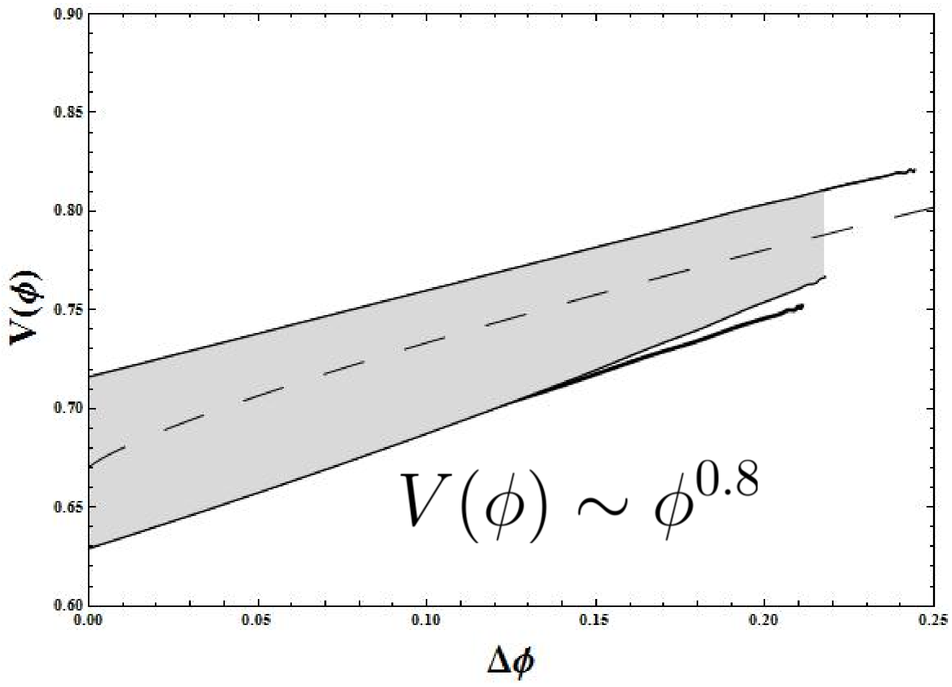
$$\Omega_\phi = x^2 + y^2, \quad w_\phi = \frac{x^2 - y^2}{x^2 + y^2}$$

$$x = \frac{\phi'}{\sqrt{6}} = \frac{1}{\sqrt{6}}\frac{d\phi}{d\ln a}, \quad y = \sqrt{\frac{V}{3H^2}}$$

SSLCPL模型限制

- thawing模型势重构（利用 $w_\phi - \Omega_\phi$ 关系）



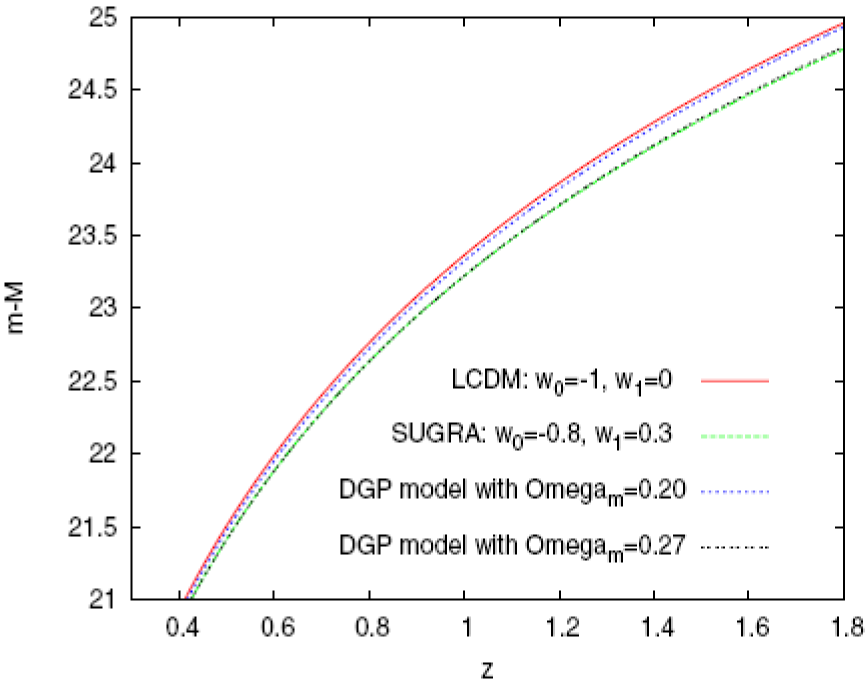


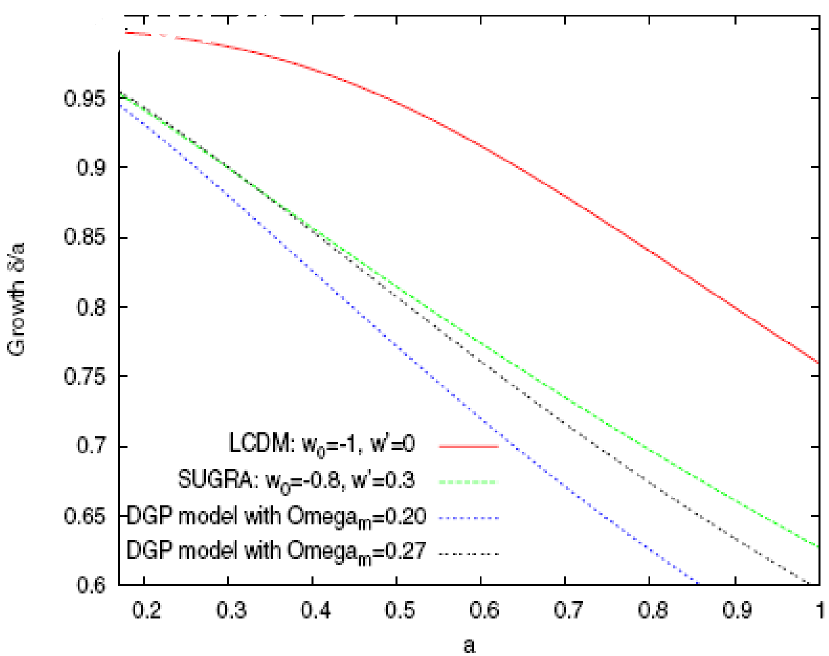
JLA+Planck+BAO+H(z)

物质密度扰动

- 尽管超新星数据测量无法区分暗能量效应和修改的引力效应，物质的密度扰动可以用来区分这些

Ishak, Upadhye & Spergel, PRD 74 (06) 043513





物质密度扰动

- 物质密度扰动 $\delta = \dfrac{\delta\rho}{\rho}$

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{eff}\rho\delta = 0$$ 长波极限

$$\frac{G_{eff}}{G} = \frac{4 + 2\omega}{3 + 2\omega}$$ Brans - Dicke Theory

$$\frac{G_{eff}}{G} = \frac{2(1 + 2\Omega^2)}{3(1 + \Omega^2)}$$ DGP model

$$G_{eff} = G$$ 爱因斯坦引力

增长因子

- 引入增长因子

$$f = \frac{d\ln\delta_+}{d\ln a}$$

$$\frac{df}{d\ln a} + f^2 + \left(\frac{\dot{H}}{H^2} + 2\right) f = \frac{3}{2}\frac{G_{eff}}{G}\Omega_m$$

- To the first order of approximation

$$f \approx \Omega_m^\gamma$$

- 增长指数 γ

Peeples, The Large-Scale Structure of the Universe, 1980

$$f(z) = \Omega_m^{0.6}$$

增长指数

- 长波近似

$$\delta_+(a) = \frac{5}{2}\Omega_{m0}H_0^2 H(a)\int_0^a \frac{da}{\dot{a}^3}$$

- 宇宙学常数为0模型

$$f(z=0) = -\frac{1}{2}\Omega_{m0} - 1 + \frac{5}{2}\Omega_{m0}^{3/2}/{}_2F_1\left(\frac{3}{2},\frac{5}{2},\frac{7}{2},1-\Omega_{m0}^{-1}\right) \approx \Omega_{m0}^{0.6}.$$

- 含宇宙学常数模型

Lahav etal., MNRAS 1991

$$\delta_+(a) \approx \frac{5}{2}a\Omega_m\left[\Omega_m^{4/7} - \Omega_\Lambda + \left(1+\frac{\Omega_m}{2}\right)\left(1+\frac{\Omega_\Lambda}{70}\right)\right]^{-1}$$

$$f(z=0) \approx \Omega_{m0}^{0.6} + \frac{\Omega_{\Lambda 0}}{70}(1+\frac{1}{2}\Omega_{m0})$$

$$f(z) = \Omega_m^{0.6}$$

增长因子

- 物质及物态方程参数为常数w的宇宙学模型

$$\delta_+(a) = a\ {}_2F_1\left[-\frac{1}{3w}, \frac{w-1}{2w}, 1-\frac{5}{6w}, -a^{-3w}\frac{1-\Omega_{m0}}{\Omega_{m0}}\right]$$

$$\gamma = \frac{3(1-w)}{5-6w} + \frac{3}{125}\frac{(1-w)(1-3w/2)}{(1-6w/5)^2(1-12w/5)}(1-\Omega_m) + O[(1-\Omega_m)^2]$$

Wang and Steinhardt 1998, ApJ 508, 483

- 宇宙学常数

$$\gamma = \frac{6}{11} + \frac{15}{1331}(1-\Omega)$$

增长指数

- 状态方程参数$w(z)$缓慢变化

$$\gamma = 0.55 + 0.05[1 + w(z=1)]$$

- DGP模型 $\gamma = \frac{11}{16} = 0.6875$
- DGP模型作为暗能量模型

$$w(z) = -\frac{\Omega_{m0}(1+z)^3 + 2\Omega_{r0}[\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{r0}^2} + \Omega_{r0}]}{2[\Omega_{m0}(1+z)^3 + \Omega_{r0}^2 + \Omega_{r0}\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{r0}^2}]}$$

$$\gamma = 0.55 + 0.05[1 + w(z=1)] \le 0.575$$

Linder, PRD 72 (05) 043529

Linder & Cahn, Astropart. Phys. 28 (07) 481

增长指数

- 物质为主（闭宇宙）$K=1$

$$a(\theta)=a_0q_0(2q_0-1)^{-1}(1-\cos\theta)$$

$$H_0t=q_0(2q_0-1)^{-3/2}(\theta-\sin\theta)$$

$$\rho=\frac{3H_0^2(2q_0-1)^3}{4\pi Gq_0^2(1-\cos\theta)^3}$$

Lightman & Schechter, ApJ 4 (1990) 831

$$(1-\cos\theta)\frac{d^2\delta}{d\theta^2}+\sin\theta\frac{d\delta}{d\theta}-3\delta=0$$

$$f=\frac{d\ln\delta_+}{d\ln a}$$

$$\delta_+\propto-\frac{3\theta\sin\theta}{(1-\cos\theta)^2}+\frac{5+\cos\theta}{1-\cos\theta}$$

$$f\approx1+\frac{1}{7}\theta^2\approx\Omega_m^{4/7}$$

$$\delta_-\propto\frac{\sin\theta}{(1-\cos\theta)^2}$$

$$\Omega_m=2(1-\cos\theta)/\sin^2\theta\approx1+\theta^2/4$$

增长指数

- 物质为主（开宇宙）$K=-1$

$$a(t)=a_0q_0(1-2q_0)^{-1}(\cosh\psi-1)$$

$$H_0t=q_0(1-2q_0)^{-3/2}(\sinh\psi-\psi)$$

$$\rho=\frac{3H_0^2(1-2q_0)^3}{4\pi Gq_0^2(\cosh\psi-1)^3}$$

$$(\cosh\psi-1)\frac{d^2\delta}{d\psi^2}+\sinh\psi\frac{d\delta}{d\psi}-3\delta=0$$

$$\delta_+\propto-\frac{3\psi\sinh\psi}{(1-\cosh\psi)^2}+\frac{5+\cosh\psi}{\cosh\psi-1}$$

$$f\approx\Omega_m^{4/7}$$

$$\delta_-\propto\frac{\sinh\psi}{(1-\cosh\psi)^2}$$

非平坦空间近似

- 曲率不为零：物质为主时期，$\Omega_m \approx 1, \quad f = 1$

$$f = \Omega_m^\gamma + (\gamma - 4/7)\Omega_k \qquad f = \Omega_m^\gamma, \ \gamma = \frac{4}{7}$$

Y.G. Gong 等PRD 80 (2009) 023002

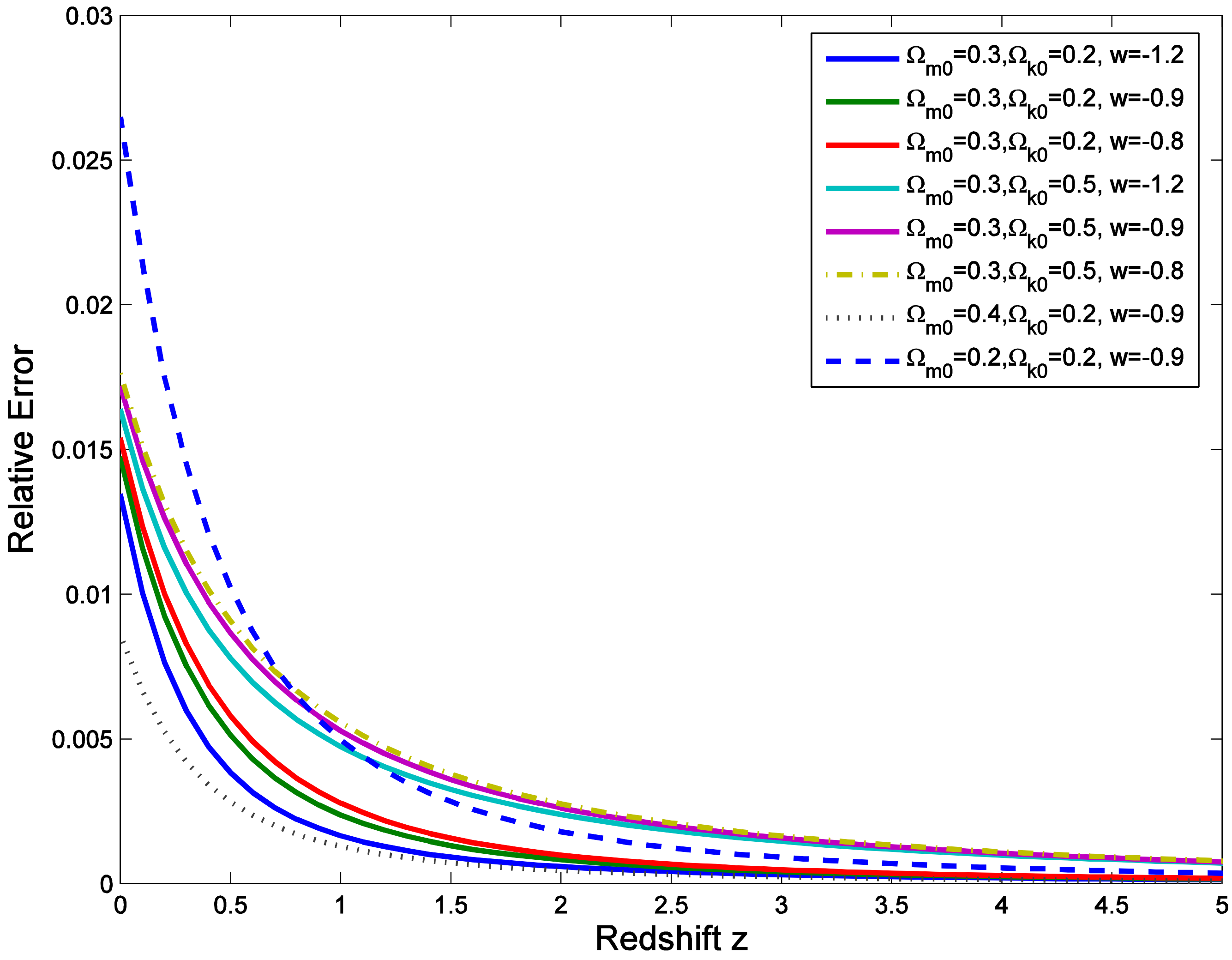
- DGP模型

$$\gamma = \frac{11}{16}$$

- 状态方程参数w为常数

$$\gamma = \frac{3(1-w)}{5-6w}$$

近似解



Relative Error
Redshift z
0.03
0.025
0.02
0.015
0.01
0.005
0
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
Ω_m0=0.3,Ω_k0=0.2, w=-1.2
Ω_m0=0.3,Ω_k0=0.2, w=-0.9
Ω_m0=0.3,Ω_k0=0.2, w=-0.8
Ω_m0=0.3,Ω_k0=0.5, w=-1.2
Ω_m0=0.3,Ω_k0=0.5, w=-0.9
Ω_m0=0.3,Ω_k0=0.5, w=-0.8
Ω_m0=0.4,Ω_k0=0.2, w=-0.9
Ω_m0=0.2,Ω_k0=0.2, w=-0.9

WMAP9结果

TABLE 3
WMAP Seven-year to Nine-year Comparison of the Six-Parameter ΛCDM Model[a]

Parameter	*WMAP*-only[b] Nine-year	Nine-year (MASTER)[c]	Seven-year	*WMAP*+BAO+H_0[b] Nine-year	Seven-year
Fit parameters					
$\Omega_b h^2$	0.02264 ± 0.00050	0.02243 ± 0.00055	$0.02249^{+0.00056}_{-0.00057}$	0.02266 ± 0.00043	0.02255 ± 0.00054
$\Omega_c h^2$	0.1138 ± 0.0045	0.1147 ± 0.0051	0.1120 ± 0.0056	0.1157 ± 0.0023	0.1126 ± 0.0036
Ω_Λ	0.721 ± 0.025	0.716 ± 0.028	$0.727^{+0.030}_{-0.029}$	0.712 ± 0.010	0.725 ± 0.016
$10^9 \Delta^2_\mathcal{R}$	2.41 ± 0.10	2.47 ± 0.11	2.43 ± 0.11	$2.427^{+0.078}_{-0.079}$	2.430 ± 0.091
n_s	0.972 ± 0.013	0.962 ± 0.014	0.967 ± 0.014	0.971 ± 0.010	0.968 ± 0.012
τ	0.089 ± 0.014	0.087 ± 0.014	0.088 ± 0.015	0.088 ± 0.013	0.088 ± 0.014
Derived parameters					
t_0 (Gyr)	13.74 ± 0.11	13.75 ± 0.12	13.77 ± 0.13	13.750 ± 0.085	13.76 ± 0.11
H_0 (km/s/Mpc)	70.0 ± 2.2	69.7 ± 2.4	70.4 ± 2.5	69.33 ± 0.88	70.2 ± 1.4
σ_8	0.821 ± 0.023	0.818 ± 0.026	$0.811^{+0.030}_{-0.031}$	0.830 ± 0.018	0.816 ± 0.024
Ω_b	0.0463 ± 0.0024	0.0462 ± 0.0026	0.0455 ± 0.0028	0.0472 ± 0.0010	0.0458 ± 0.0016
Ω_c	0.233 ± 0.023	0.237 ± 0.026	0.228 ± 0.027	$0.2408^{+0.0093}_{-0.0092}$	0.229 ± 0.015
$z_{\rm reion}$	10.6 ± 1.1	10.5 ± 1.1	10.6 ± 1.2	10.5 ± 1.1	10.6 ± 1.2

TABLE 9
Non-flat ΛCDM Constraints[a]

Parameter	*WMAP*	+eCMB	+eCMB+BAO	+eCMB+H_0	+eCMB+BAO+H_0
New parameter					
Ω_k	$-0.037^{+0.044}_{-0.042}$	-0.001 ± 0.012	$-0.0049^{+0.0041}_{-0.0040}$	0.0049 ± 0.0047	$-0.0027^{+0.0039}_{-0.0038}$
Related parameters					
$\Omega_{\rm tot}$	$1.037^{+0.042}_{-0.044}$	1.001 ± 0.012	$1.0049^{+0.0040}_{-0.0041}$	0.9951 ± 0.0047	$1.0027^{+0.0038}_{-0.0039}$
Ω_m	$0.19 < \Omega_m < 0.95$ (95% CL)	0.273 ± 0.049	0.292 ± 0.010	0.252 ± 0.017	$0.2855^{+0.0096}_{-0.0097}$
Ω_Λ	$0.22 < \Omega_\Lambda < 0.79$ (95% CL)	0.727 ± 0.038	0.713 ± 0.011	0.743 ± 0.015	0.717 ± 0.011
H_0 (km/s/Mpc)	$38 < H_0 < 84$ (95% CL)	71.2 ± 6.5	68.0 ± 1.0	$73.4^{+2.2}_{-2.3}$	$68.92^{+0.94}_{-0.95}$
t_0 (Gyr)	14.8 ± 1.5	13.71 ± 0.65	13.99 ± 0.17	13.46 ± 0.24	13.88 ± 0.16

观测现状

- Planck观测结果（1303.5076）

$$\Omega_{m0} = 0.315 \pm 0.017, \quad H_0 = 67.3 \pm 1.2\text{km/s/Mpc}$$

- 超新星观测结果

SNLS3 $\Omega_{m0} = 0.227^{+0.042}_{-0.035}$

Union2.1 $\Omega_{m0} = 0.295^{+0.043}_{-0.040}$

SNLS3-SDSS JLA $\Omega_{m0} = 0.295 \pm 0.034$

- 哈勃常数

ApJ 730 (2011) 119 $H_0 = 73.8 \pm 2.4\ \text{km/s/Mpc}$

$$H_0 = 62.3 \pm 1.3\ (\text{random}) \pm 4\ (\text{systematic})$$

A A Rev 15 (2008) 289

不同结果自洽性

- 哈勃常数问题 $H_0 = 67.3 \pm 1.2$

$$H_0 = 73.8 \pm 2.4$$

超新星定标修正 $H_0 = 70.6 \pm 3.3$

1311.3461

距离模型修正 $H_0 = 72.5 \pm 2.5$

$$H_0 = 69 \pm 6\ (\text{statistic}) \pm 4\ (\text{systematic})\ \text{km/s/Mpc}$$

引力透镜测量图像的时间延迟，1301.2251

Planck数据217GHzX217GHz修正，WMAP与Planck差别，1312.3313

$$\Omega_{m0} = 0.302 \pm 0.015,\ \ H_0 = 68.1 \pm 1.1$$

SNLS3-SDSS JLA $\Omega_{m0} = 0.295 \pm 0.034$

模型自洽性

- SSLCPL模型 $w = p/\rho$

$$w(a) = w_0 + 6(1+w_0)\frac{(\Omega_{\phi 0}^{-1}-1)[\sqrt{\Omega_{\phi 0}} - \tanh^{-1}(\sqrt{\Omega_{\phi 0}})]}{\Omega_{\phi 0}^{-1/2} - (\Omega_{\phi 0}^{-1}-1)\tanh^{-1}(\sqrt{\Omega_{\phi 0}})}(1-a)$$

Data	Ω_{m0}	w_0	H_0
Union2.1+BAO+$H(z)$	$0.291^{+0.018}_{-0.019}$	$-1.03^{+0.11}_{-0.12}$	$70.5^{+2.0}_{-1.9}$
SNLS3+BAO+$H(z)$	$0.277^{+0.018}_{-0.017}$	-1.12 ± 0.11	$72.1^{+1.9}_{-2.0}$
JLA+BAO+$H(z)$	$0.292^{+0.017}_{-0.019}$	-1.0 ± 0.09	70.1 ± 1.7
Planck+BAO+$H(z)$	$0.287^{+0.019}_{-0.023}$	$-1.17^{+0.17}_{-0.16}$	$70.6^{+2.7}_{-2.6}$
Union2.1+Planck+BAO+$H(z)$	$0.292^{+0.013}_{-0.015}$	$-1.12^{+0.10}_{-0.11}$	$69.9^{+1.7}_{-1.8}$
SNLS3+Planck+BAO+$H(z)$	$0.287^{+0.012}_{-0.013}$	-1.15 ± 0.09	$70.4^{+1.5}_{-1.4}$
JLA+Planck+BAO+$H(z)$	$0.302^{+0.011}_{-0.012}$	-1.04 ± 0.07	$68.6^{+1.1}_{-1.2}$

Gao and Gong, CQG in press

模型自洽性

CPL模型

Data	Ω_{m0}	w_0	w_a	H_0
Union2.1+BAO+$H(z)$	$0.298^{+0.023}_{-0.022}$	$-0.95^{+0.17}_{-0.21}$	$-0.58^{+1.27}_{-0.75}$	70.3 ± 2.0
SNLS3+BAO+$H(z)$	0.289 ± 0.020	-0.96 ± 0.18	-1.13 ± 1.20	$71.9^{+2.0}_{-1.9}$
JLA+BAO+$H(z)$	0.30 ± 0.02	$-0.89^{+0.13}_{-0.17}$	$-0.82^{+1.16}_{-0.75}$	70.0 ± 1.7
Planck+BAO+$H(z)$	$0.309^{+0.029}_{-0.036}$	$-0.86^{+0.34}_{-0.39}$	$-0.84^{+1.23}_{-0.90}$	$68.6^{+3.5}_{-3.6}$
Union2.1+Planck+BAO+$H(z)$	$0.298^{+0.015}_{-0.017}$	$-0.98^{+0.17}_{-0.2}$	$-0.49^{+0.79}_{-0.54}$	$69.5^{+1.8}_{-1.7}$
SNLS3+Planck+BAO+$H(z)$	$0.29^{+0.012}_{-0.013}$	$-1.02^{+0.13}_{-0.14}$	$-0.51^{+0.69}_{-0.48}$	70.5 ± 1.4
JLA+Planck+BAO+$H(z)$	0.304 ± 0.011	$-0.90^{+0.11}_{-0.12}$	$-0.68^{+0.60}_{-0.44}$	$68.8^{+1.1}_{-1.2}$

Data	Ω_{m0}	w_0	H_0
Union2.1+BAO+$H(z)$	$0.291^{+0.018}_{-0.019}$	$-1.03^{+0.11}_{-0.12}$	$70.5^{+2.0}_{-1.9}$
SNLS3+BAO+$H(z)$	$0.277^{+0.018}_{-0.017}$	-1.12 ± 0.11	$72.1^{+1.9}_{-2.0}$
JLA+BAO+$H(z)$	$0.292^{+0.017}_{-0.019}$	-1.0 ± 0.09	70.1 ± 1.7
Planck+BAO+$H(z)$	$0.287^{+0.019}_{-0.023}$	$-1.17^{+0.17}_{-0.16}$	$70.6^{+2.7}_{-2.6}$
Union2.1+Planck+BAO+$H(z)$	$0.292^{+0.013}_{-0.015}$	$-1.12^{+0.10}_{-0.11}$	$69.9^{+1.7}_{-1.8}$
SNLS3+Planck+BAO+$H(z)$	$0.287^{+0.012}_{-0.013}$	-1.15 ± 0.09	$70.4^{+1.5}_{-1.4}$
JLA+Planck+BAO+$H(z)$	$0.302^{+0.011}_{-0.012}$	-1.04 ± 0.07	$68.6^{+1.1}_{-1.2}$

不自洽性与模型无关，系统误差的原因，支持**LCDM**模型

DGP模型结果

- DGP模型观测限制（f(z)数据）

$$\Omega_m = 0.290^{+0.014}_{-0.012},$$

$$\Omega_k = 0.019 \pm 0.005,$$

$$\gamma = 0.46^{+0.12}_{-0.08} < 0.687$$

- Λ CDM模型

$$\Omega_m = 0.272^{+0.013}_{-0.010}, \quad \Omega_k = 0.002 \pm 0.004 \quad \gamma = 0.56^{+0.14}_{-0.09}$$

- 观测结果支持 ΛCDM模型

Gong etal., MNRAS 415 (11) 1943

Conclusions

- The dynamics of scalar fields has some common features
- The exists an approximate relation between w and Ω_ϕ which helps distinguish models
- The dynamics can be approximated by CPL parametrization with degenerated relation between w_0 and w_a
- The reduced degeneracy helps improve the constraint on w(z)
- w’ is limited for different class of models

THANK YOU！