Studies on the primordial perturbations

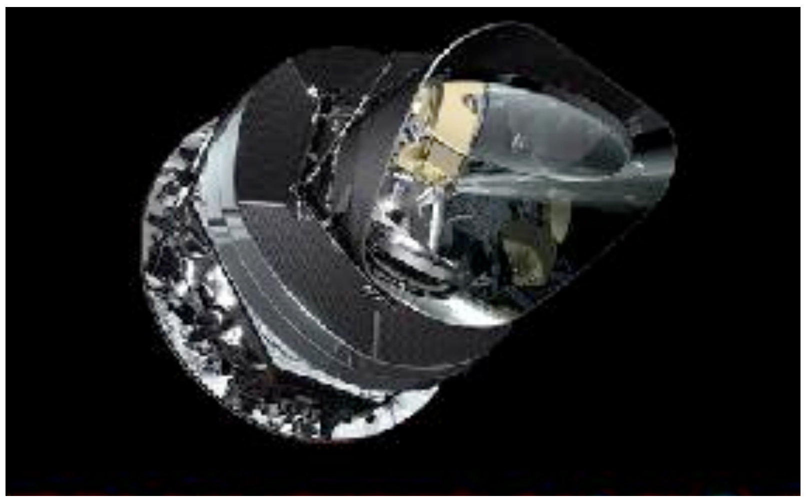
李明哲
中国科技大学 交叉学科理论研究中心
基金委“暗能量本质及其基本理论”高级研讨班，西安，2014-4-9

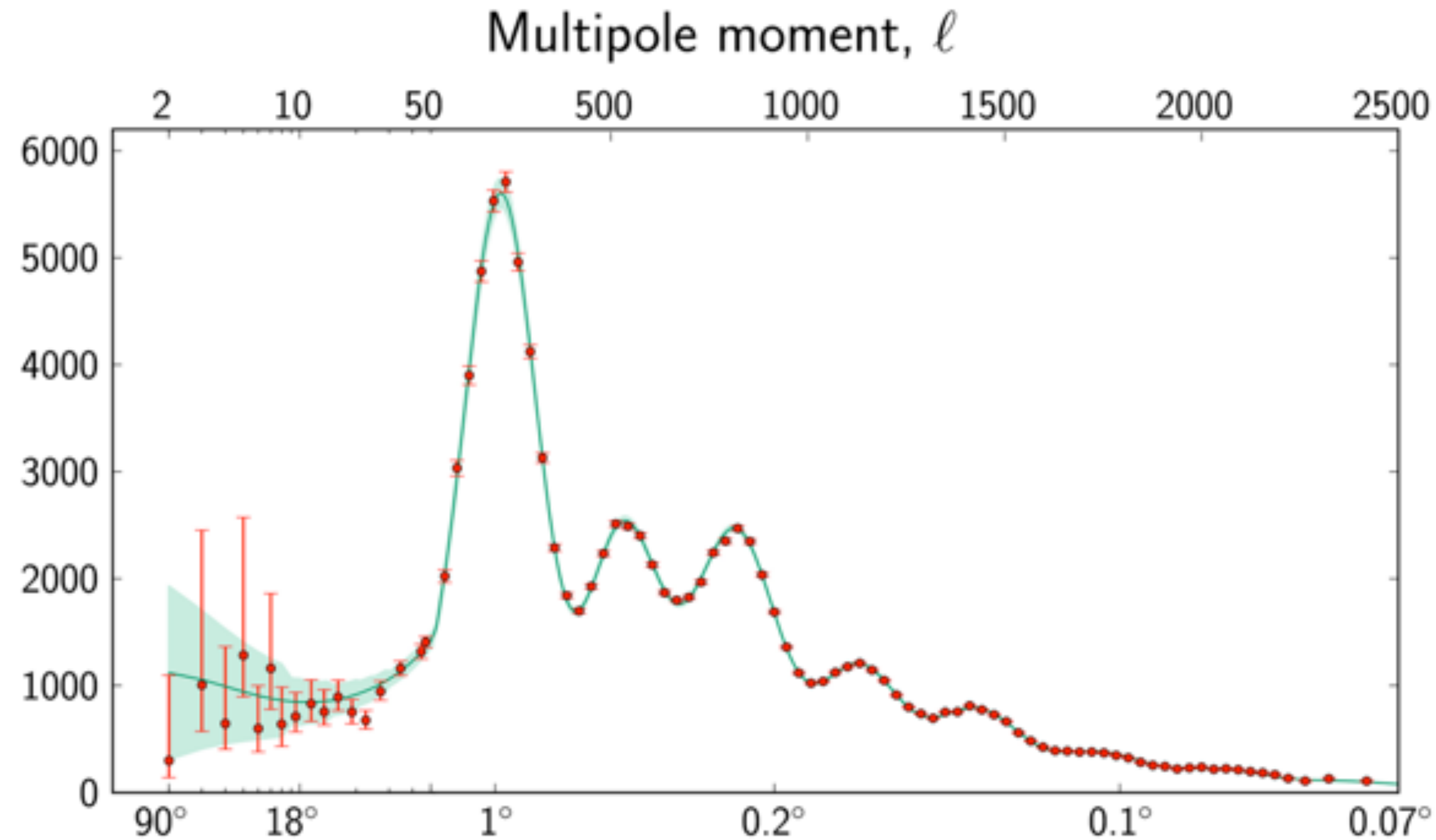
Outline

- Generating scale-invariant density perturbation in contracting universe
- Inflation models with flat potentials
- Conclusions

Generating Scale Invariant Density Perturbation in Contracting Universe

PLANCK





Primordial (scalar) perturbations: adiabatic, nearly scale-invariant, negligible non-Gaussianities

$$\mathcal{P}_\zeta = A_s (k/k_*)^{n_s - 1} \qquad n_s = 0.969 \pm 0.010$$

$$f_{NL} = 2.7 \pm 5.8 (Local), \ \ -42 \pm 75 (Equilateral), \ \ -25 \pm 39 (Orthogonal)$$

Consistent with single field slow-roll inflation!

Generation of scalar perturbation in single field model

$$S = \int d^4x\sqrt{g}(\frac{M_p^2}{2}R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi))$$

Background $ds^2 = a^2(\eta)\eta_{\mu\nu}dx^\mu dx^\nu$ $\quad \epsilon = -\frac{\dot{H}}{H^2} = \frac{3}{2}(1+w) = \frac{1}{2M_p^2}\frac{\phi'^2}{\mathcal{H}^2}$

Perturbation

$$S_2 = M_p^2 \int d^4x\sqrt{g}\epsilon g^{\mu\nu}\partial_\mu\zeta\partial_\nu\zeta$$

$\zeta = -\Psi - \frac{\mathcal{H}}{\phi'}\delta\phi$ curvature perturbation, gauge invariant

$$S_2 = \int d^4x(M_p^2\epsilon a^2)\eta^{\mu\nu}\partial_\mu\zeta\partial_\nu\zeta = \int d^4x[\eta^{\mu\nu}\partial_\mu u\partial_\nu u + \frac{z''}{z}u^2]$$

$$z = M_p a\sqrt{\epsilon}\ , \quad u = z\zeta$$

$$w = \text{constant} \qquad \frac{z''}{z} = \frac{a''}{a} = \frac{1-3w}{(1+3w)^2}\frac{2}{\eta^2}$$

Quantization

$$S_2 = \int d^4x[\eta^{\mu\nu}\partial_\mu \hat{u}\partial_\nu \hat{u} + \frac{z''}{z}\hat{u}^2] \qquad \hat{\zeta} = \frac{\hat{u}}{z}$$

$$\hat{u}_{\vec{k}} = \hat{a}_{\vec{k}} u_k + \hat{a}^\dagger_{\vec{k}} u_k^* \qquad [\hat{a}_{\vec{k}},\ \hat{a}^\dagger_{\vec{k}'}] = \delta^3(\vec{k}-\vec{k}') \qquad \hat{a}_{\vec{k}}|0\rangle = 0$$

$$u_k'' + (k^2 - \frac{z''}{z})u_k = 0$$

Deep inside horizon

$k|\eta| \gg 1$

Bunch-Davies vacuum

Outside horizon

$k|\eta| \sim 0$

$$u_k = \frac{e^{-ik\eta}}{\sqrt{2k}} \longrightarrow u_k = \sqrt{-\frac{\pi}{2}\eta}[C_1 H_\nu^{(1)}(-k\eta) + C_2 H_\nu^{(2)}(-k\eta)] \longrightarrow u_k \sim (-\eta)^{1/2}(-k\eta)^{-\nu}$$

$-\infty$ $\longrightarrow$ $\eta = 0$

$$C_1 = e^{i\frac{2\nu+1}{4}\pi}/\sqrt{2}\ ,\ C_2 = 0$$

$$\nu = \frac{3}{2}|\frac{w-1}{1+3w}|$$

Power spectrum

$$\mathcal{P}_u(k) = \frac{k^3}{2\pi^2}|u_k|^2 \to A(-\eta)^{1-2\nu}k^{3-2\nu}$$

$$\mathcal{P}_\zeta = \frac{1}{z^2}\mathcal{P}_u(k) \to B(-\eta)^{1-2\nu-\frac{4}{1+3w}}k^{3-2\nu}$$

$$n_s = 4 - 2\nu$$

Scale-invariance

$$\nu = \frac{3}{2}|\frac{w-1}{1+3w}|$$

$$n_s = 1,\ \nu = \frac{3}{2}$$

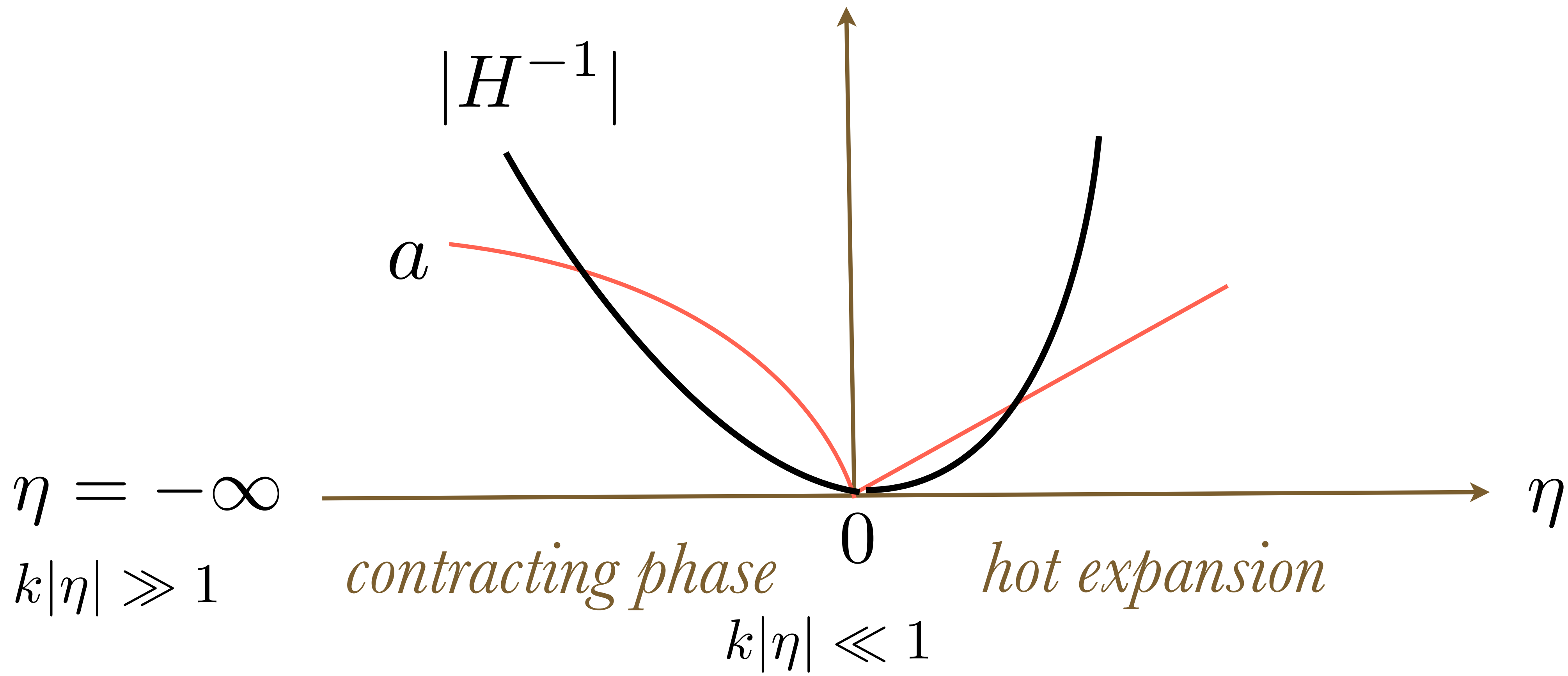
$$\frac{z''}{z} = \frac{a''}{a} = \frac{2}{\eta^2}$$

$(1)\ w = 0,\ \mathcal{P}_\zeta \sim (-\eta)^{-6}$ Finelli & Brandenberger, PRD (2002); Wands, PRD (1999)

matter bounce, unstable background

$(2)\ w = -1,\ \mathcal{P}_\zeta \to const.$ $\quad a = -\frac{1}{H\eta}$ de Sitter space, inflation

Bouncing universe: alternative to inflation



$$|H^{-1}| \text{ shrinks faster than } \frac{a}{k} \text{ as long as } w > -1/3$$

$$\mathcal{H} = \frac{a'}{a} = aH \propto \frac{1}{\eta} \qquad |\frac{k}{aH}| \sim k|\eta|$$

Flattening and isotropizing the universe by the contracting phase

Without w>1 component

$$H^2 = \frac{1}{3M_p^2}(\frac{\rho_m}{a^3} + \frac{\rho_r}{a^4} + \frac{\sigma^2}{a^6}) - \frac{k}{a^2}$$

dominates the universe, highly anisotropic

Belinsky, Khalatnikov & Lifshitz, Adv. Phys. (1970)

With w>1 component

$$H^2 = \frac{1}{3M_p^2}(\frac{\rho_m}{a^3} + \frac{\rho_r}{a^4} + \frac{\sigma^2}{a^6} + \frac{\rho_\phi}{a^{3(1+w_\phi)}}) - \frac{k}{a^2}$$

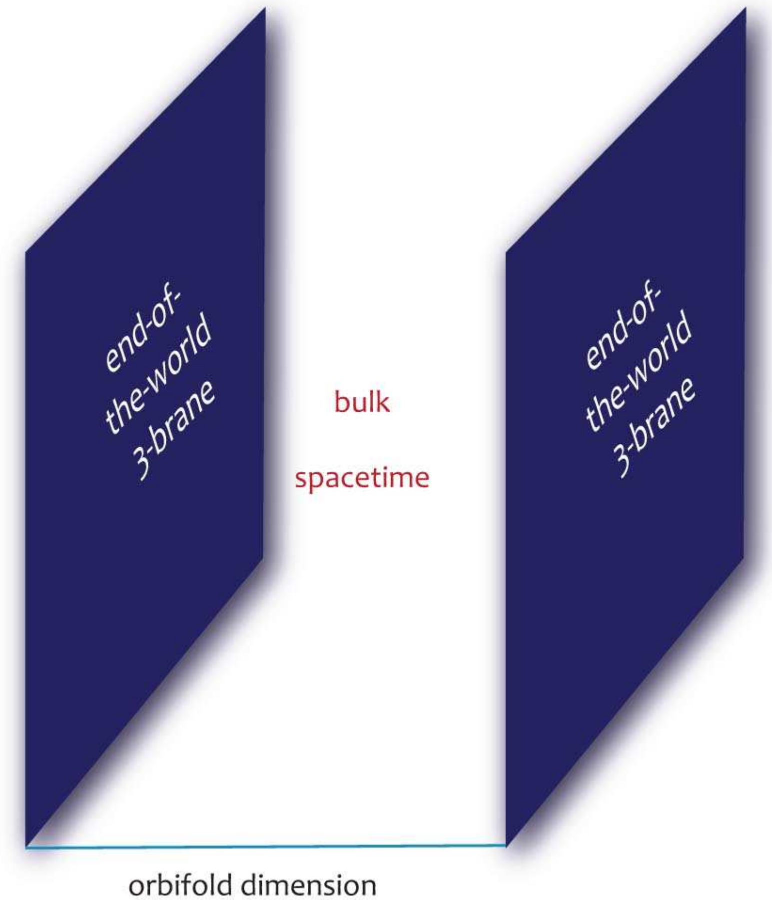
dominant, others are suppressed

Erickson, Wesley, Steinhardt, Turok, PRD(2004)

Ekpyrotic phase: slow contraction with w>1

Khoury, Ovrut, Steinhardt, Turok, PRD (2001); Steinhardt & Turok, PRD (2002)

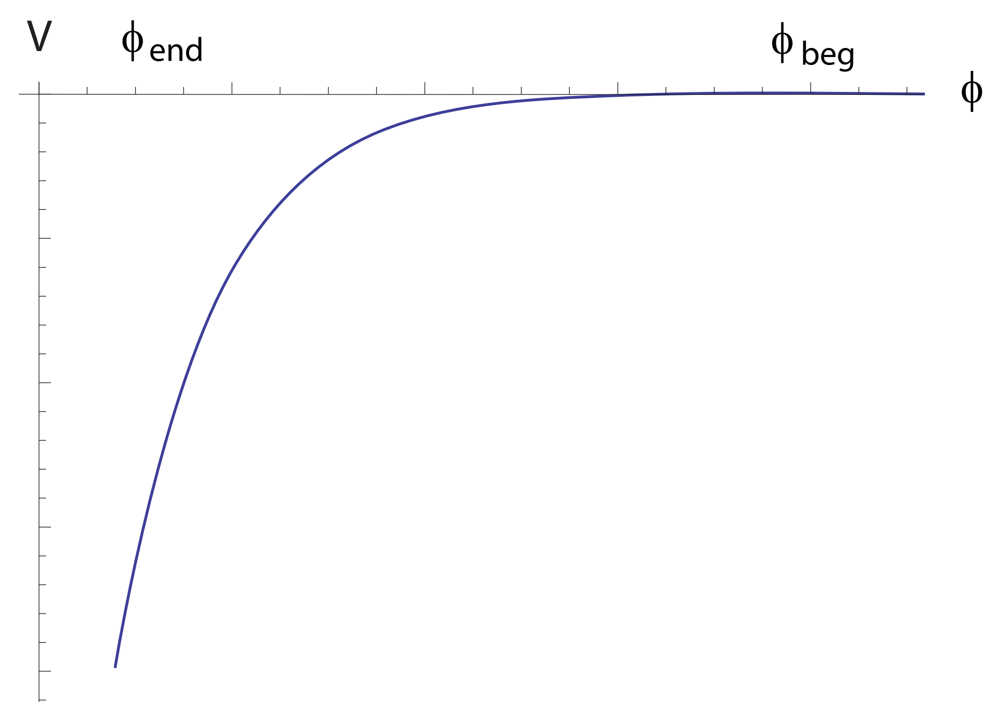
$$S = \int d^4x\sqrt{g}\left(\frac{M_p^2}{2}R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right)$$



Negative steep potential

$$V = -V_0 e^{-\frac{c}{M_p}\phi}$$

$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V} > 1$$



Scaling solution (stable)

$$w = \frac{c^2}{3} - 1 > 1$$

The curvature perturbation produced in single field Ekpyrotic model has a strongly blue spectrum, excluded by observations

$$n_s = 3 - \frac{2}{1+3w} > 2.5$$

Rapid varying w model

$$V = V_0(1 - e^{-\frac{c}{M}\phi})$$ Khoury & Steinhardt, PRL (2010)

Scale-invariant perturbations are generated during the transition

$$w \simeq -1 \to w \gg 1$$

Large non-Gaussianities, not compatible with results of PLANCK

Standard entropic mechanism

Lehners, McFadden, Turok & Steinhardt, PRD (2007), Lehners & Steinhardt, arXiv:1304.3122

Multi fields

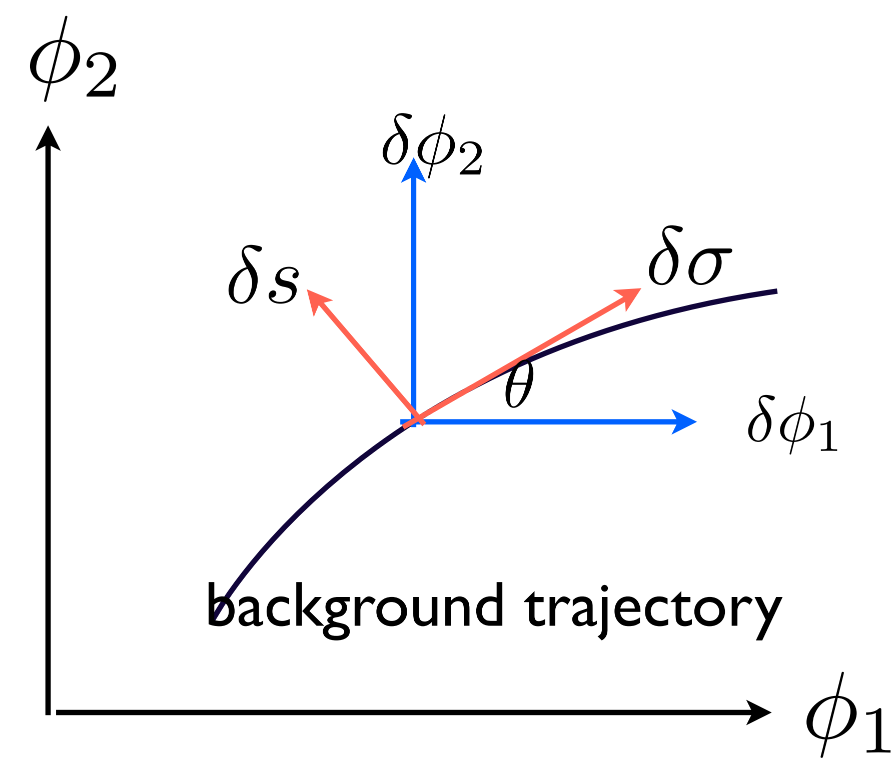
$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 + V_1 e^{-\frac{c_1}{M_p}\phi_1} + V_2 e^{-\frac{c_2}{M_p}\phi_2}$$

Projection into directions along and orthogonal to the background trajectory

$$\sigma'^2 = \phi_1'^2 + \phi_2'^2 \qquad \cos\theta \equiv \phi_1'/\sigma' \qquad \sin\theta \equiv \phi_2'/\sigma'$$

$$\sigma' = \phi_1'\cos\theta + \phi_2'\sin\theta$$

$$s' = -\phi_1'\sin\theta + \phi_2'\cos\theta = \frac{-\phi_1'\phi_2' + \phi_2'\phi_1'}{\sigma'} = 0$$



adiabatic perturbation $\quad \delta\sigma = \delta\phi_1\cos\theta + \delta\phi_2\sin\theta$

entropy perturbation $\quad \delta s = -\delta\phi_1\sin\theta + \delta\phi_2\cos\theta$

Gordon, Wands, Bassett, Maartens, PRD (2001)

$$\delta\sigma'' + 2\mathcal{H}\delta\sigma' + (k^2 + a^2 V_{\sigma\sigma} - \theta'^2)\delta\sigma = -2a^2 V_\sigma \Psi + 4\sigma'\Psi' + 2a(\frac{\theta'\delta s}{a})' - 2a^2 V_\sigma \frac{\theta'}{\sigma'}\delta s$$

$$\delta s'' + 2\mathcal{H}\delta s' + (k^2 + a^2 V_{ss} + 3\theta'^2)\delta s = 4M_p^2 k^2 \frac{\theta'}{\sigma'}\Psi$$

$$V = -\sum_i V_i e^{-\frac{c_i}{M_p}\phi_i}$$

$$V_\sigma \equiv \cos\theta V_{\phi_1} + \sin\theta V_{\phi_2}$$

$$V_{\sigma\sigma} \equiv \sin^2\theta V_{\phi_2\phi_2} + \sin 2\theta V_{\phi_1\phi_2} + \cos^2\theta V_{\phi_1\phi_1}$$

$$V_{ss} \equiv \sin^2\theta V_{\phi_1\phi_1} - \sin 2\theta V_{\phi_1\phi_2} + \cos^2\theta V_{\phi_2\phi_2}$$

Adiabatic and entropy perturbations are decoupled if $\theta' = 0$
Otherwise the entropy perturbation can source the adiabatic perturbation.
The reprocessed adiabatic perturbation inherits the same shape of the entropy perturbation

Scaling solution $\theta' = 0$

background trajectory is a straight line

$$\frac{\phi_1'}{\phi_2'} = \frac{c_2}{c_1} \ , \ w = \frac{c_1^2 c_2^2}{3(c_1^2 + c_2^2)} - 1 \qquad \sin\theta = \frac{c_1}{\sqrt{c_1^2 + c_2^2}} \ , \ \cos\theta = \frac{c_2}{\sqrt{c_1^2 + c_2^2}}$$

$$\delta s'' + 2\mathcal{H}\delta s' + (k^2 + a^2 V_{ss})\delta s = 0 \qquad v \equiv a\delta s$$

$$v'' + (k^2 - \frac{a''}{a} + a^2 V_{ss})v = 0 \qquad -\frac{a''}{a} + a^2 V_{ss} \to -\frac{2}{\eta^2} \ when \ w \gg 1$$

scale-invariant entropy perturbation!

During the Ekpyrotic phase, adiabatic perturbation is negligibly small, entropy perturbation is scale-invariant.
After the Ekpyrotic phase, entropy perturbation converts into the adiabatic perturbation. The conversion happens when $\theta' \neq 0$

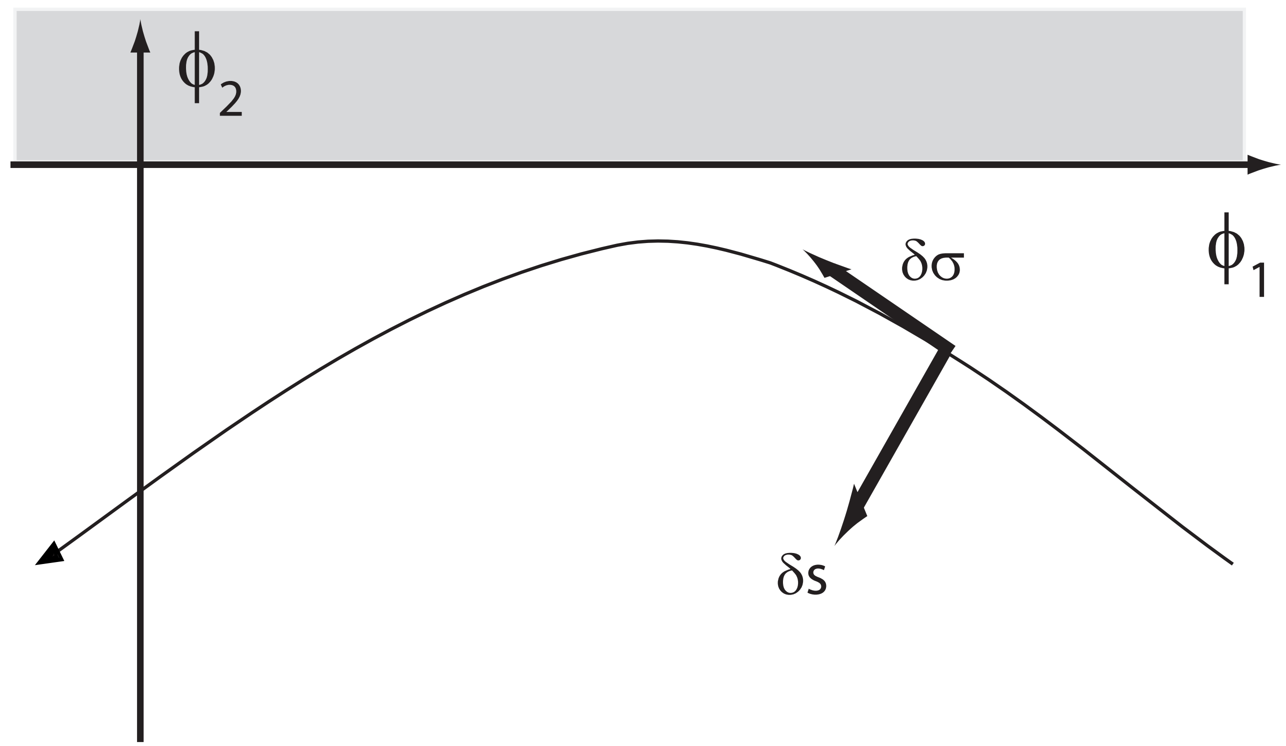


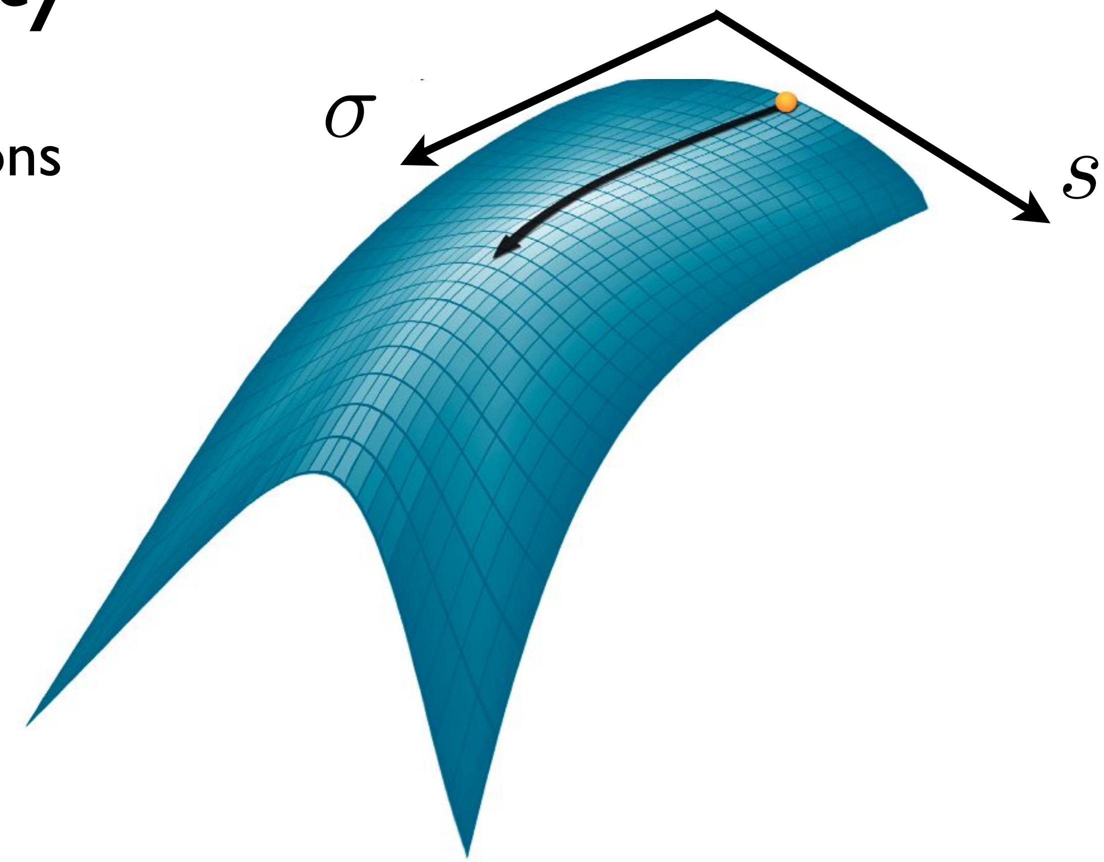
Fig. 6. The trajectory in field space reflects off a boundary at $\phi_2 = 0$. The entropy perturbation, denoted δs, is orthogonal to the trajectory. The bending causes the conversion of entropy modes into adiabatic modes $\delta\sigma$, which are perturbations tangential to the trajectory.

Lehners, McFadden, Turok & Steinhardt, PRD (2007)

Difficulty: tachyonical instability

Fields (including background and perturbations) rotations

$$\sigma = \phi_1 \cos\theta + \phi_2 \sin\theta$$
$$s = -\phi_1 \sin\theta + \phi_2 \cos\theta$$



only valid when $\theta' = 0$

$$S_s = \int d^4x \sqrt{g} [\frac{1}{2} g^{\mu\nu} \partial_\mu s \partial_\nu s - V(\sigma,\ s)]$$

$$V(\sigma,\ s) = -\exp(-\frac{c_1 c_2}{\sqrt{c_1^2 + c_2^2} M_p}\sigma)[V_1 \exp(\frac{c_1^2}{\sqrt{c_1^2 + c_2^2} M_p} s) + V_2 \exp(-\frac{c_2^2}{\sqrt{c_1^2 + c_2^2} M_p} s)]$$

Koyama & Wands, JCAP (2007); Koyama, Mizuno, Wands, Class. Quant. Grav. (2007)
Buchbinder, Khoury & Ovrut, JHEP (2007)

General argument

$$S_s = \frac{1}{2}\int d^4x a^2[\eta^{\mu\nu}\partial_\mu s \partial_\nu s - a^2 m_s^2 s^2]$$

$$m_s^2 = \frac{\partial^2 V(\sigma,\ s)}{\partial s^2} \qquad\qquad \frac{a''}{a} = \frac{1-3w}{(1+3w)^2}\frac{2}{\eta^2}$$

$$v'' + (k^2 - \frac{a''}{a} + a^2 m_s^2)v = 0$$

Scale-invariance requires $\frac{a''}{a} - a^2 m_s^2 = \frac{2}{\eta^2}$ $\qquad a^2 m_s^2 = [\frac{1-3w}{(1+3w)^2} - 1]\frac{2}{\eta^2}$

$$w > 1,\ m_s^2 < 0$$

Scaling solution $\frac{\phi_1'}{\phi_2'} = \frac{c_2}{c_1}\ ,\ w = \frac{c_1^2 c_2^2}{3(c_1^2 + c_2^2)} - 1$ is unstable

Non-minimal couplings

Mingzhe Li, arXiv:1306.0191, Physics Letters B 724 (2013) 192-197

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V_0 e^{-\frac{\lambda}{M_p}\phi} + \frac{1}{2}e^{-\frac{\alpha}{M_p}\phi}\partial_\mu\chi\partial^\mu\chi$$

In our case $\lambda = \alpha$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + e^{-\frac{\lambda}{M_p}\phi}(\frac{1}{2}\partial_\mu\chi\partial^\mu\chi + V_0)$$

EOM

$$\Box\phi + \frac{\lambda}{M_p}e^{-\frac{\lambda}{M_p}\phi}(\frac{1}{2}\partial_\mu\chi\partial^\mu\chi + V_0) = 0$$

$$\Box\chi - \frac{\lambda}{M_p}\partial_\mu\phi\partial^\mu\chi = 0$$

$$\mathcal{H}^2 = \frac{1}{6M_p^2}(\phi'^2 + \chi'^2 e^{-\frac{\lambda}{M_p}\phi} - 2a^2 V_0 e^{-\frac{\lambda}{M_p}\phi})$$

$$x \equiv \frac{\phi'}{\sqrt{6} M_p \mathcal{H}} \ , \ y \equiv \frac{e^{-\frac{\lambda}{2M_p}\phi} \chi'}{\sqrt{6} M_p \mathcal{H}} \qquad z \equiv \frac{a\sqrt{V_0} e^{-\frac{\lambda}{2M_p}\phi}}{\sqrt{3} M_p \mathcal{H}}$$

$$x^2 + y^2 - z^2 = 1 \qquad x^2 + y^2 \geq 1$$

Di Marco, Finelli, Brandenberger, PRD (2003)

$$\dot{x} = 3x(x^2 + y^2 - 1) - \frac{\sqrt{6}}{2}\lambda(x^2 + 2y^2 - 1)$$
$$\dot{y} = 3y(x^2 + y^2 - 1) + \frac{\sqrt{6}}{2}\lambda xy$$

$$\dot{x} \equiv dx/d\ln a, \ \ \dot{y} \equiv dy/d\ln a$$

Critical points (scaling solutions)

$$x = x_0, \ \ y = y_0 \text{ with } \dot{x}_0 = 0, \ \ \dot{y}_0 = 0$$

$(i)\ (x_0 = -1,\ y_0 = 0)$ $\qquad (ii)\ (x_0 = 1,\ y_0 = 0)$ $\qquad (iii)\ (x_0 = \frac{\lambda}{\sqrt{6}},\ y_0 = 0)$

(In)stabilities

$$X = x - x_0 \qquad Y = y - y_0$$

$$\dot{X} = (9x_0^2 - \sqrt{6}\lambda x_0 - 3)X \ , \ \dot{Y} = (3x_0^2 + \frac{\sqrt{6}}{2}\lambda x_0 - 3)Y$$

$$|X| \sim \exp[(9x_0^2 - \sqrt{6}\lambda x_0 - 3)\ln a] \ , \ |Y| \sim \exp[(3x_0^2 + \frac{\sqrt{6}}{2}\lambda x_0 - 3)\ln a]$$

$$Contracting\ universe, \ \ln a\ decreases$$

$$Stable\ solutions, \ 9x_0^2 - \sqrt{6}\lambda x_0 - 3 > 0, \ 3x_0^2 + \frac{\sqrt{6}}{2}\lambda x_0 - 3 > 0$$

Assuming positive λ

$(i)\ (x_0 = -1,\ y_0 = 0)$ unstable

$(ii)\ (x_0 = 1,\ y_0 = 0)$ stable if $\lambda < \sqrt{6}$

$z_0 = 0 \qquad w = 1 \qquad \phi \to +\infty$

dynamically equivalent to single field model, blue power spectrum

$(iii)\ (x_0 = \frac{\lambda}{\sqrt{6}},\ y_0 = 0)$ **stable if** $\lambda > \sqrt{6}$

$$w = \frac{\lambda^2}{3} - 1 > 1 \qquad x_0 = \frac{\lambda}{\sqrt{6}} > 1,\ y_0 = 0 \qquad z_0 = -\sqrt{\frac{\lambda^2}{6} - 1}$$

This is what we need!

Damping by the non-minimal coupling

$$\chi'' + 2\mathcal{H}\chi' - \frac{\lambda}{M_p}\phi'\chi' = \chi'' - (\sqrt{6}\lambda x_0 - 2)\mathcal{H}\chi' = \chi'' - (\lambda^2 - 2)\mathcal{H}\chi' = 0$$

$$\sigma'^2 = \phi'^2 + \chi'^2 e^{-\frac{\lambda}{M_p}\phi}$$

$$\sigma' = \phi' \cos\theta + \chi' e^{-\frac{\lambda}{2M_p}\phi} \sin\theta$$

$$\cos\theta = \frac{\phi'}{\sigma'}\ ,\ \ \sin\theta = \frac{\chi' e^{-\frac{\lambda}{2M_p}\phi}}{\sigma'}$$

$$\chi' = 0\ ,\ \theta = 0 \qquad \sigma = \phi$$

$$s \propto \chi$$

χ is a spectator, always corresponds to entropy direction

Entropy perturbation

$$S_\chi = \frac{1}{2}\int d^4x(\sqrt{g}+\delta\sqrt{g})e^{-\frac{\lambda}{M_p}(\phi+\delta\phi)}(g^{\mu\nu}+\delta g^{\mu\nu})\partial_\mu(\chi+\delta\chi)\partial_\nu(\chi+\delta\chi)$$

Quadratic action, considering $\partial_\mu\chi = 0$

$$S_{\delta\chi} = \frac{1}{2}\int d^4x\sqrt{g}e^{-\frac{\lambda}{M_p}\phi}g^{\mu\nu}\partial_\mu\delta\chi\partial_\nu\delta\chi$$

$$z_0^2 = \frac{a^2V_0e^{-\frac{\lambda}{M_p}\phi}}{3M_p^2\mathcal{H}^2} = \frac{\lambda^2}{6}-1\ ,\ e^{-\frac{\lambda}{M_p}\phi} = \frac{(\lambda^2-6)M_p^2\mathcal{H}^2}{2a^2V_0}$$

$$\begin{aligned} S_{\delta\chi} &= \frac{1}{2}\int d^4x a^2 e^{-\frac{\lambda}{M_p}\phi}\eta^{\mu\nu}\partial_\mu\delta\chi\partial_\nu\delta\chi \\ &= \frac{1}{2}\int d^4x\frac{(\lambda^2-6)M_p^2\mathcal{H}^2}{2V_0}\eta^{\mu\nu}\partial_\mu\delta\chi\partial_\nu\delta\chi \end{aligned}$$

$$\mathcal{H} = \frac{2}{1+3w}\frac{1}{\eta} = \frac{2}{\lambda^2-2}\frac{1}{\eta}$$

$$S_{\delta\chi} = \frac{1}{2}\int d^4x \frac{1}{h^2\eta^2}\eta^{\mu\nu}\partial_\mu\delta\chi\partial_\nu\delta\chi \qquad h = \frac{\lambda^2-2}{M_p}\sqrt{\frac{V_0}{2(\lambda^2-6)}}$$

Massless field living in an effective de Sitter space $\bar{g}_{\mu\nu} = \frac{1}{h^2\eta^2}\eta_{\mu\nu}$

$$S_{\delta\chi} = \frac{1}{2}\int d^4x \sqrt{\bar{g}}\bar{g}^{\mu\nu}\partial_\mu\delta\chi\partial_\nu\delta\chi$$

$\mathcal{P}_{\delta\chi}^{1/2} = \frac{h}{2\pi}$ scale-invariant, do not need w>>1

The conversion to adiabatic perturbation can only take place after the bounce via

(1) curvaton mechanism   Lyth & Wands, PLB (2002)

(2) modulated preheating

$$V_{eff} \sim g(\chi)\xi\bar{\psi}\psi \qquad \Gamma \propto g^2$$

$$\zeta \sim \frac{\delta\Gamma}{\Gamma} \sim 2\frac{d\ln g}{d\chi}\delta\chi$$

Dvali, Gruzinov, Zaldarriaga, PRD (2004); Kofman, astro-ph/0303614; Battefeld, PRD (2008)

$$More\ general\ case,\ \lambda \neq \alpha$$

$$Attractor\ solution\ is\ unchanged$$

$$S_{\delta\chi} = \frac{1}{2}\int d^4x a^2 e^{-\frac{\alpha}{M_p}\phi}\eta^{\mu\nu}\partial_\mu\delta\chi\partial_\nu\delta\chi = \frac{1}{2}\int d^4x q^2\eta^{\mu\nu}\partial_\mu\delta\chi\partial_\nu\delta\chi$$

$$a \propto \eta^\beta\ with\ \beta = -\frac{\lambda\alpha - 2}{\lambda^2 - 2} \qquad u \equiv q\delta\chi$$

$$u_k'' + (k^2 - \frac{q''}{q})u_k = u_k'' + (k^2 - \frac{\beta(\beta-1)}{\eta^2})u_k = 0$$

$$u_k = \sqrt{-\frac{\pi}{2}\eta}H_\nu^{(1)}(-k\eta) \qquad \nu = \frac{1}{2} - \beta$$

$$\mathcal{P}_{\delta\chi} \sim k^{2+2\beta}$$

$$p = \alpha - \lambda \qquad n_s = 1 - \frac{2\lambda p}{\lambda^2 - 2}$$

small red tilt spectrum $0 < p \ll 1$

Another non-minimal coupling model

$$S_\chi = -\frac{1}{2}\int d^4x\sqrt{g}\,\frac{R}{M^2}\,g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$$

$$R = 6a''/a^3 = 6(\mathcal{H}' + \mathcal{H}^2)/a^2 = \frac{12(1-3w)}{(1+3w)^2a^2\eta^2}$$

$$S_{\delta\chi} = \frac{1}{2}\int d^4x\,\frac{1}{\tilde{h}^2\eta^2}\,\eta^{\mu\nu}\partial_\mu\delta\chi\partial_\nu\delta\chi$$

$$\mathcal{P}_{\delta\chi}^{1/2} = \frac{\tilde{h}}{2\pi} \qquad \tilde{h} = \frac{(1+3w)M}{2\sqrt{3(3w-1)}}$$

Non-Gaussianities were small, Fertig, Lehners and Mallwitz, arXiv:1310.8133

A general mechanism for producing scale-invariant perturbations and small non-Gaussianity in ekpyrotic models

arXiv:1404.1265

Anna Ijjas*
Max-Planck-Institute for Gravitational Physics (Albert-Einstein-Institute), 14476 Potsdam, Germany and Rutgers University, New Brunswick, NJ 08901, USA

Jean-Luc Lehners
Max-Planck-Institute for Gravitational Physics (Albert-Einstein-Institute), 14476 Potsdam, Germany

Paul J. Steinhardt
Department of Physics and Princeton Center for Theoretical Science, Princeton University, Princeton, NJ 08544, USA

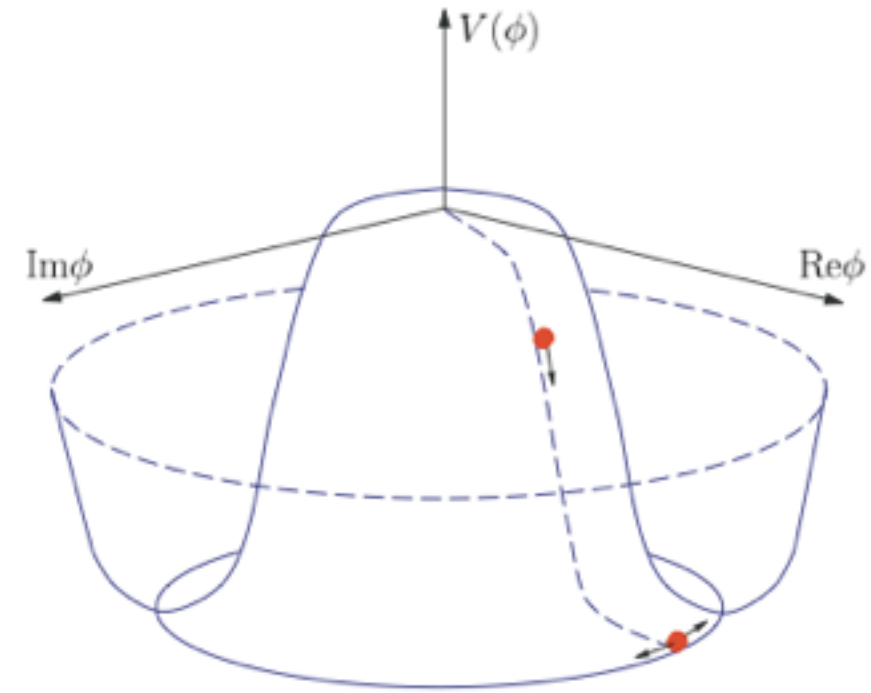
(Dated: April 7, 2014)

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{2}\Omega^2(\phi)\partial_\mu\chi\partial^\mu\chi\right).$$

Analogies I: Conformal rolling model

Rubakov, JCAP (2009)

$$S = \int d^4x\sqrt{g}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi^* - \frac{R}{6}|\phi|^2 - (-h^2|\phi|^4)]$$



Conformal symmetry + $U(1)$ symmetry

$$\mathcal{L} = \frac{1}{2}\partial_\mu f\partial^\mu f - \frac{R}{12}f^2 + \frac{h^2}{4}f^4 + \frac{1}{2}f^2\partial_\mu\theta\partial^\mu\theta \qquad \phi = \frac{1}{\sqrt{2}}fe^{i\theta}$$

f : conformal weight 1 $\qquad$ θ : conformal weight 0

$$F = af \quad F'' = h^2F^3$$

Attractor solution $\quad F = \frac{\sqrt{2}}{-h\eta}$

It breaks conformal symmetry spontaneously!

$$S_\theta = \int d^4x\sqrt{g}\mathcal{L}_\theta = \int d^4x\frac{F^2}{2}\eta^{\mu\nu}\partial_\mu\theta\partial_\nu\theta = \int d^4x\frac{1}{h^2\eta^2}\eta^{\mu\nu}\partial_\mu\theta\partial_\nu\theta \qquad \mathcal{P}_\theta^{1/2} = \frac{h}{2\pi}$$

Analogies II: Galilean Genesis

Creminelli, Nicolis, Trincherini, JCAP (2010)

$$\mathcal{L} = f^2 e^{2\pi}(\partial\pi)^2 + \frac{f^3}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{f^3}{2\Lambda^3}(\partial\pi)^4$$

Solution, emergent universe

$$e^{\pi} = -\frac{1}{H_0 t}, \; H_0^2 = \frac{2\Lambda^3}{3f} \; , -\infty < t < 0, \; g_{\mu\nu} = \eta_{\mu\nu}$$

$$SO(4,2) \to SO(4,1)$$

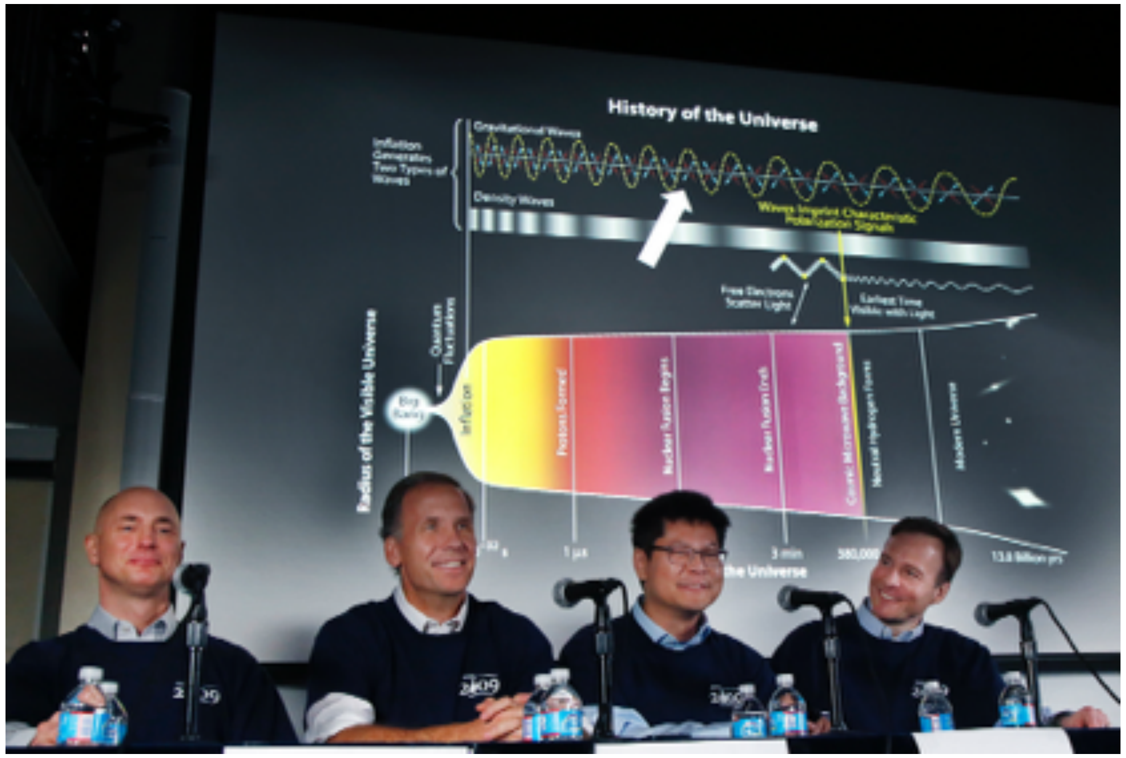
Coupling to a massless scalar

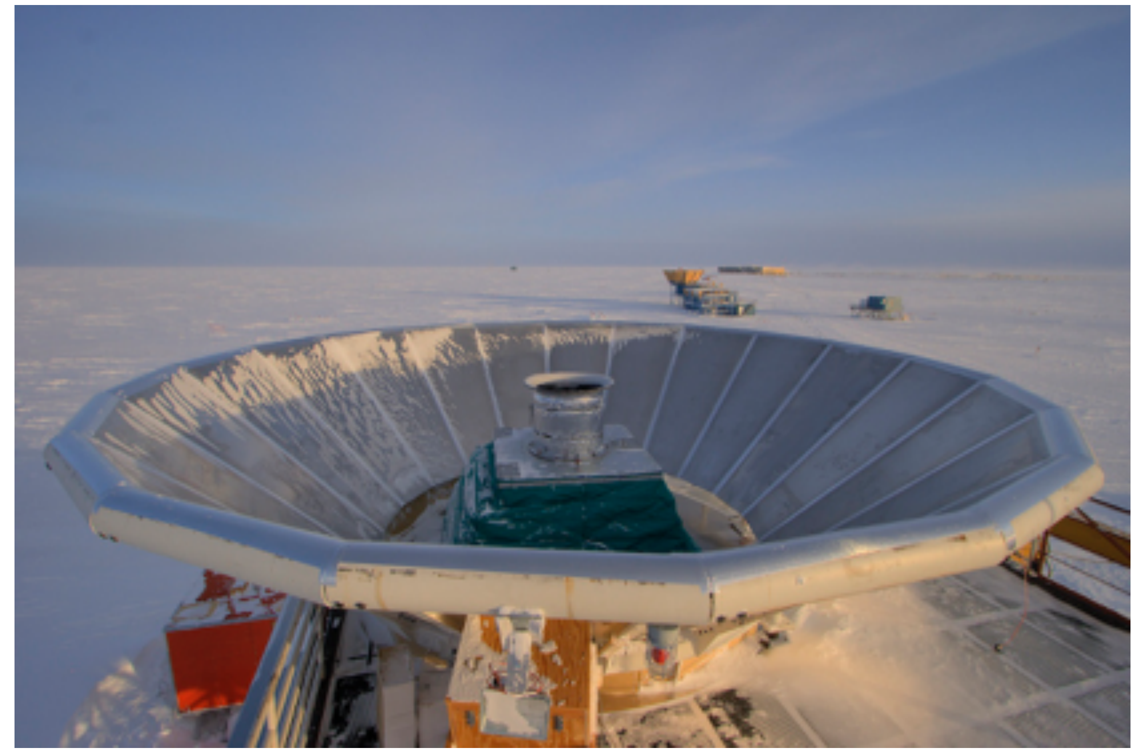
$$S_\sigma = \int d^4x e^{2\pi}(\partial\sigma)^2 = \int d^4x \frac{1}{H_0^2 t^2}\eta^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma \qquad \mathcal{P}_\sigma^{1/2} = \frac{H_0}{2\pi}$$

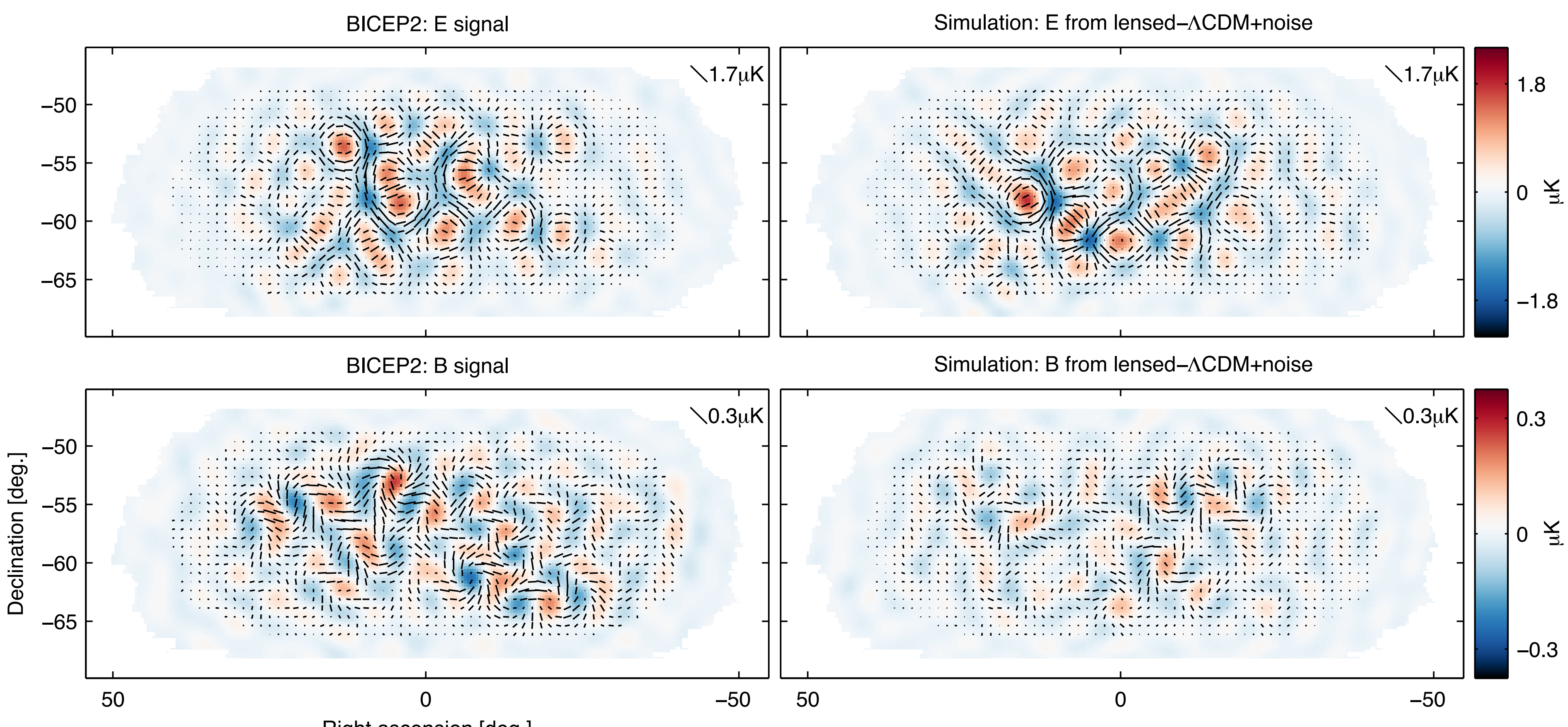
Phenomenological Lagrangian non-linearly realized conformal symmetry was constructed in Hinterbichler & Khoury, JCAP (2012); Hinterbichler, Joyce, Khoury, JCAP (2012)

Inflation Models with Flat Potentials

BICEP：Background Imaging of Cosmic Extragalactic Polarization
（宇宙河外偏振背景成像）







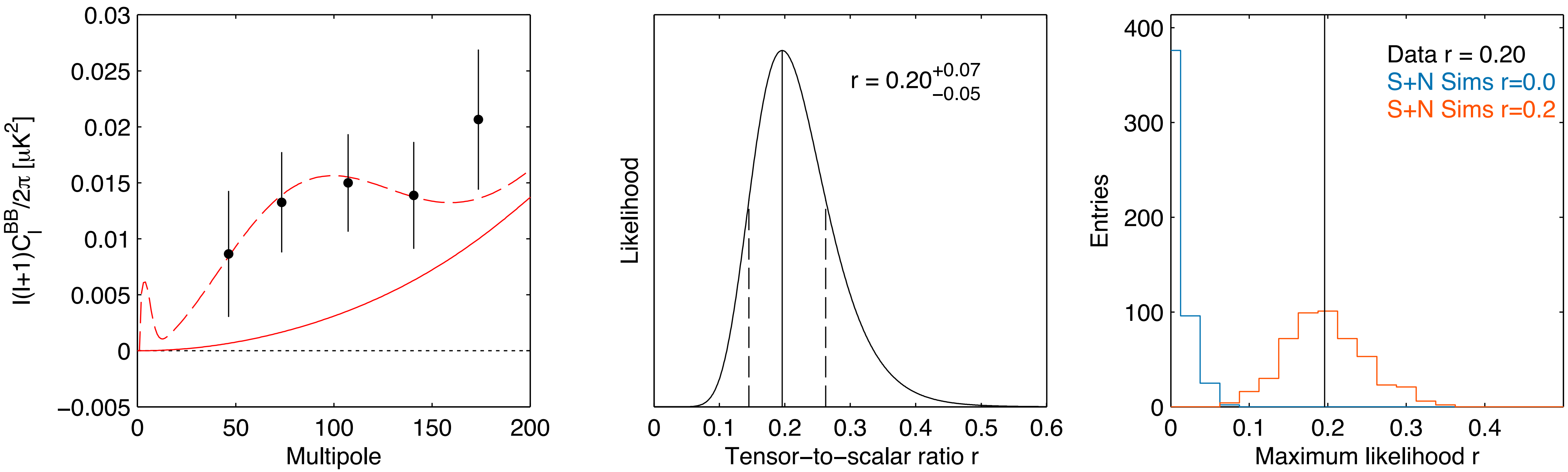


FIG. 10.— *Left:* The BICEP2 bandpowers plotted with the maximum likelihood lensed-ΛCDM+$r = 0.20$ model. The uncertainties are taken from that model and hence include sample variance on the r contribution. *Middle:* The constraint on the tensor-to-scalar ratio r. The maximum likelihood and $\pm 1\,\sigma$ interval is $r = 0.20^{+0.07}_{-0.05}$, as indicated by the vertical lines. *Right:* Histograms of the maximum likelihood values of r derived from lensed-ΛCDM+noise simulations with $r = 0$ (blue) and adding $r = 0.2$ (red). The maximum likelihood value of r for the real data is shown by the vertical line.

Well fit to single field slow roll inflation model

Primordial perturbations:
initial conditions seeded anisotropies and large scale structures

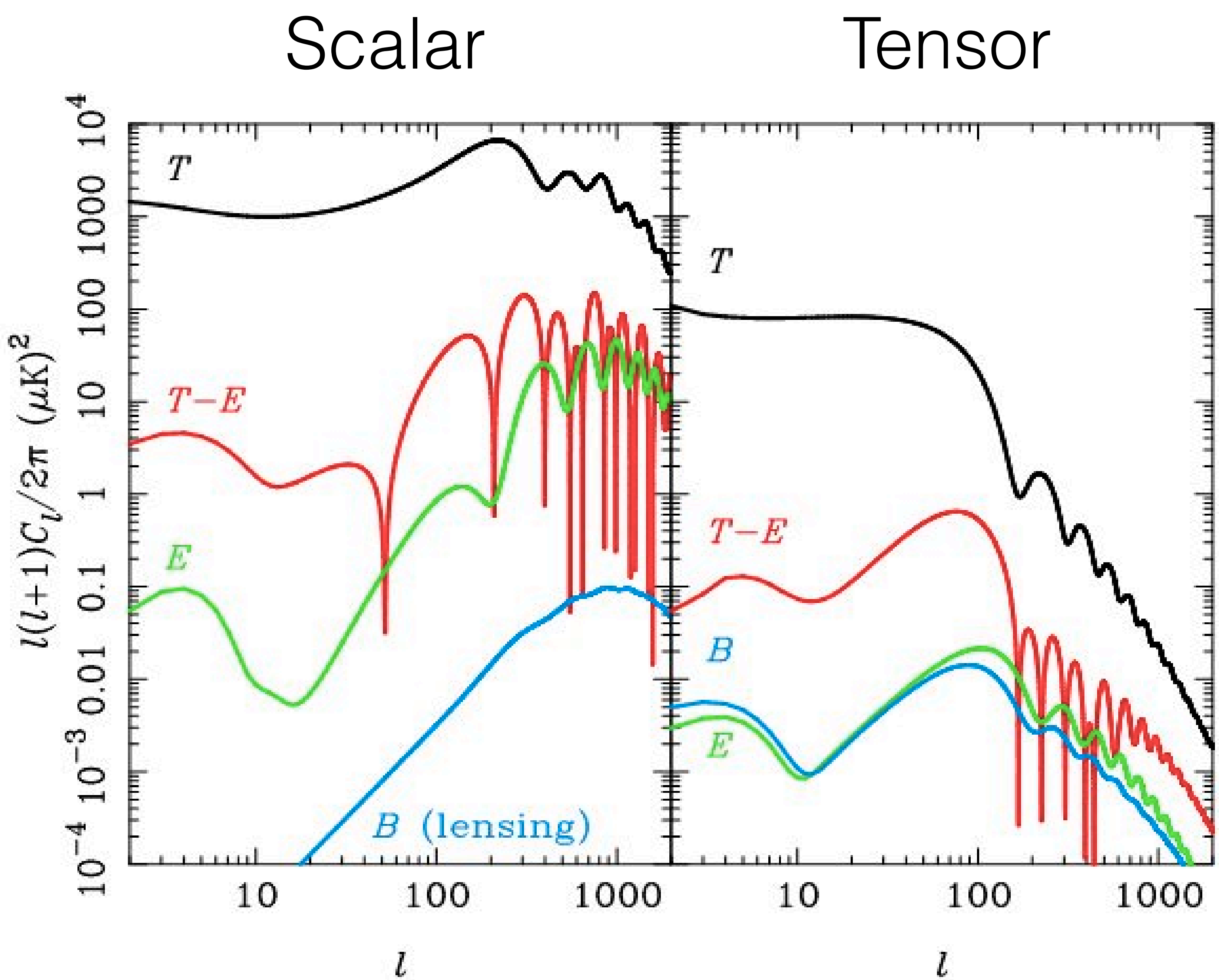
Scalar $$P_s(k) = A_s(k_*)\left(\frac{k}{k_*}\right)^{n_s(k_*)-1+\frac{1}{2}\alpha_s(k_*)\ln(k/k_*)}$$

Tensor $$P_t(k) = A_t(k_*)\left(\frac{k}{k_*}\right)^{n_t(k_*)}$$

$$r = P_t(k_*)/P_s(k_*)$$

spectral indices: n_s , n_t

scale invariance: $n_s = 1,\ n_t = 0$



Challinor & Peiris, arXiv:0903.5158 r=0.22

T, E: dominated by scalar perturbation

B: large scales (l<100) dominated by primordial gravitational waves; small scales dominated by lensing effect

Primordial perturbations from quantum fluctuations

Gravitational waves, spacetime ripples, quantized!

Inflation (single field)

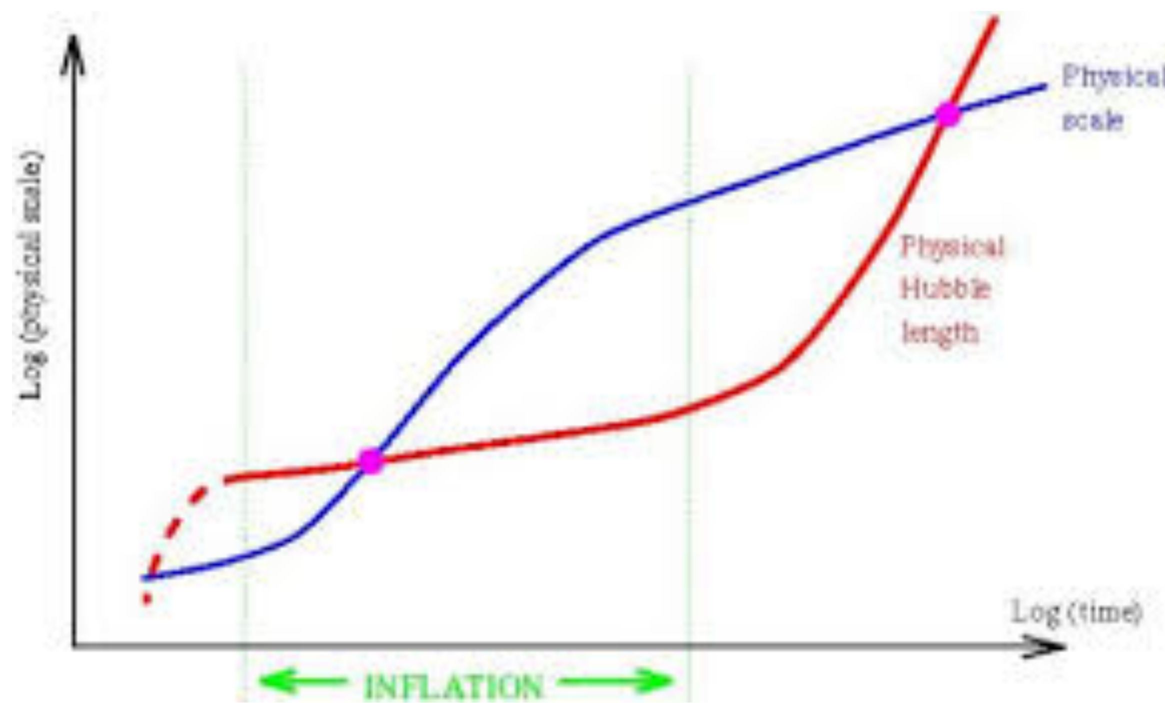
$$P_s(k) = \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2\Bigg|_{k=aH}$$

$$P_t(k) = \frac{8}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2\Bigg|_{k=aH} \qquad r = -8n_t$$

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$r = 0.2$ $\qquad V^{1/4} \sim 2\times 10^{16}\ \mathrm{GeV}$

$$H_{in} \sim 10^{14}\mathrm{GeV}$$



Slow-roll parameters $\epsilon,\ \eta,\ \xi$

$$M_p^2 = \frac{1}{8\pi G}$$

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{dH^{-1}}{dt} = \frac{3}{2}(1+w) \ll 1,\ \ |\delta| \equiv \left|\frac{\ddot{\phi}}{H\dot{\phi}}\right| \ll 1 \qquad \eta \equiv \epsilon - \delta,\ \ |\eta| \ll 1$$

$$H^2 = \frac{1}{3M_p^2}(\frac{\dot{\phi}^2}{2} + V) \simeq \frac{V}{3M_p^2} \qquad \ddot{\phi} + 3H\dot{\phi} + V_\phi = 0,\ 3H\dot{\phi} + V_\phi \simeq 0$$

$$\epsilon \simeq \frac{M_p^2}{2}\left(\frac{V_\phi}{V}\right)^2,\ \eta \simeq M_p^2\frac{V_{\phi\phi}}{V} \qquad \xi \equiv M_p^4\frac{V_\phi V_{\phi\phi\phi}}{V^2} \sim \mathcal{O}(\epsilon^2, \eta^2)$$

e-folding number

$$N \equiv \ln(\frac{a_f}{a_*}) = \int_{t_*}^{t_f} H(t')dt' \simeq \frac{1}{M_p^2}\int_{\phi_f}^{\phi_*} \frac{V}{V_\phi} d\phi$$

$$\epsilon, \eta \sim N^{-1}, \ \xi \sim N^{-2}$$

Pivot scale $k_* = a_* H_*$ $\frac{k_*}{a_0} = 0.002\mathrm{Mpc}^{-1} \text{ or } 0.05\mathrm{Mpc}^{-1}$

$$\ln(\frac{k_*}{a_0 H_0}) = \ln\frac{H_*}{H_0} + \ln\frac{a_*}{a_0} = \ln\frac{H_*}{H_0} + \ln\frac{a_*}{a_f} + \ln\frac{a_f}{a_{re}} + \ln\frac{a_{re}}{a_{eq}} + \ln\frac{a_{eq}}{a_0}$$

$$N = -\ln(\frac{k_*}{a_0 H_0}) + \ln(\sqrt{\frac{V_*}{3M_p^2}} H_{eq}^{-1}) + \ln(219\Omega_0 h) + \frac{1}{4}\ln\frac{\rho_{eq}}{\rho_{re}} + \frac{1}{3(1+w_{re})}\ln\frac{\rho_{re}}{\rho_f}$$

$N \sim 50 - 60$ Uncertainty is from reheating

$$(a\delta\phi_k)'' + (k^2 - \frac{z''}{z})(a\delta\phi_k) = 0,\ \mathrm{spatially-flat\ gauge}\ \psi = 0$$

$$(ah^{+,\times})'' + (k^2 - \frac{a''}{a})(ah^{+,\times}) = 0 \qquad z = a\frac{\dot{\phi}}{H}$$

Vacuum fluctuations, Bunch-Davies vacuum

$$a\delta\phi_k,\ ah^{+,\times} \to \frac{1}{\sqrt{2k}}e^{-ik\eta},\ \frac{k}{aH} \to \infty$$

Power spectra

$$P_s(k) = \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2\Bigg|_{k=aH} \qquad r = \frac{P_t}{P_s} = 16\epsilon$$

$$P_t(k) = \frac{8}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2\Bigg|_{k=aH}$$

Slow-roll

$$P_s = \frac{V}{24\pi^2 M_p^4 \epsilon}\Bigg|_{k=aH} \qquad P_t = \frac{2V}{3\pi^2 M_p^4}\Bigg|_{k=aH}$$

$$n_s - 1 = \frac{dP_s}{d\ln k} = -6\epsilon + 2\eta \qquad n_t = \frac{dP_t}{d\ln k} = -2\epsilon \qquad \alpha_s = \frac{dn_s}{d\ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\xi$$

Consistence relation $\quad r = -8n_t$

Slow-rolling gives $\alpha_s \sim 10^{-3}$

Planckian excursion

$$|\frac{d\phi}{dN}| = M_p^2 |\frac{V_\phi}{V}| = \sqrt{\frac{r}{8}} M_p$$

Lyth bound (Lyth, PRL (1997)) $|\Delta\phi| = |\phi_f - \phi_*| = N\sqrt{\frac{r}{8}} M_p$

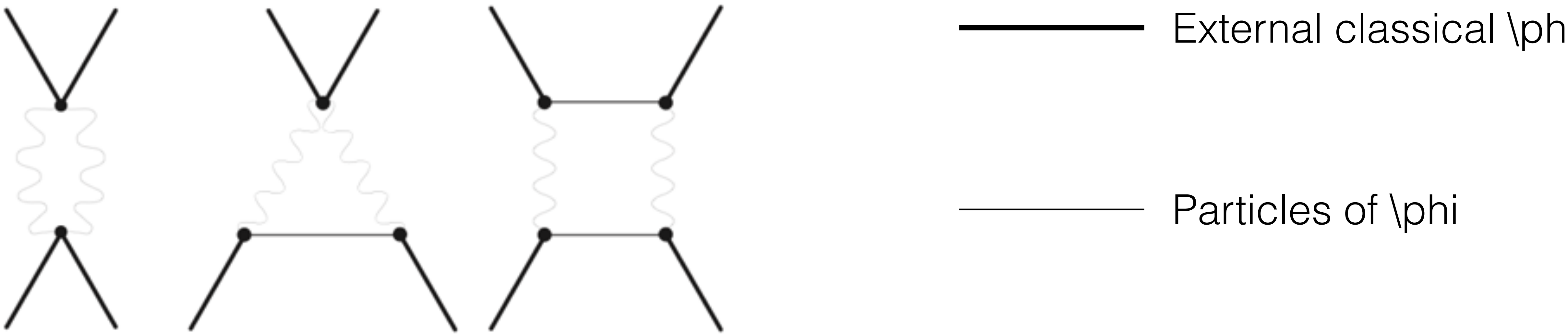
$$r = 0.2,\ |\Delta\phi| \simeq (8.0 - -9.5) M_p$$

$$\phi > M_p,\ V(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4}\phi^4 + \sum_n \lambda_n \frac{\phi^{4+n}}{M_p^n}$$

Out of control, spoils the flatness of the potential

Wilson coefficients: $\lambda_n \sim \mathcal{O}(1)$

Linde, hep-th/0503203; arXiv:0705.0164;
Kehagias and Riotto, arXiv:1403.4811



One loop corrections from quantum gravity   Smolin, PLB (1980)

$$\Delta V = \lambda_1 V_{\phi\phi} \frac{V}{M_p^2} \ln \frac{\Lambda^2}{M_p^2} + \lambda_2 \frac{V^2}{M_p^4} \ln \frac{\Lambda^2}{M_p^2} = (\tilde{\lambda}_1 \frac{V_{\phi\phi}}{M_p^2} + \tilde{\lambda}_2 \frac{V}{M_p^4})V$$

ϕ itself does not have physical meaning,
enters in the theory only through V and $V_{\phi\phi}$

$V_{\phi\phi} \ll M_p^2,\ V \ll M_p^4$ $\tilde{\lambda}_1,\ \tilde{\lambda}_2 \sim \mathcal{O}(1)$

Symmetries to protect the flatness of the inflaton potential

1, supersymmetry

SUSY provides flat directions, but is broken during inflation due to positive vacuum energy.

Supergravity corrections give a mass~H to any flat directions, Coperland, Liddle, Lyth, Stewart and Wands, PRD (1994)

Spoils slow-roll condition $|\eta| < 1, \ m = \sqrt{V_{\phi\phi}} < H$

2, shift symmetry $\phi \to \phi + C$ PNGB

Natural inflation model, Freese, Frieman and Olinto, PRL (1990)

$$V = \Lambda^4 (1 - \cos\frac{\phi}{f})\ ,\ m \sim \frac{\Lambda^2}{f},\ H \simeq \frac{\Lambda^2}{\sqrt{3} M_p}$$

f : spontaneous breaking scale of global symmetry

$$\epsilon = \frac{M_p^2}{2f^2}\frac{1+\cos\frac{\phi}{f}}{1-\cos\frac{\phi}{f}} \qquad \eta = \frac{M_p^2}{f^2}\frac{\cos\frac{\phi}{f}}{1-\cos\frac{\phi}{f}} \qquad \delta = \epsilon - \eta = \frac{M_p^2}{2f^2}$$

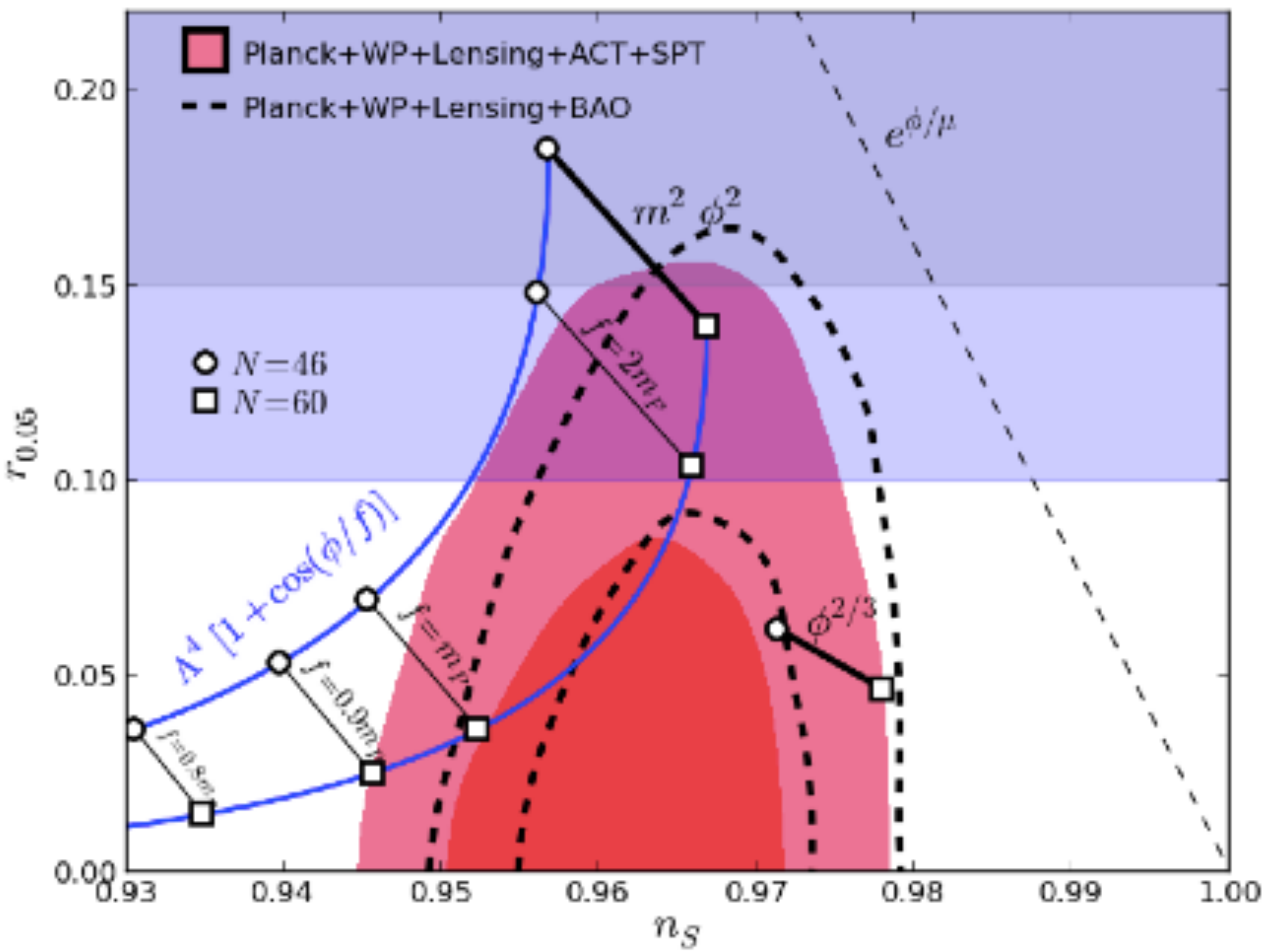
$|\delta| \ll 1,\ f \gg M_p/\sqrt{2}$ Outside the range of validity of EFT

Quantum gravity effects, e.g., virtual black holes, break global symmetries.

They are proportional to $(\frac{f}{M_p})^n$, unsuppressed.

R.Kallosh, A.Linde, D.Linde and L.Susskind, PRD(1995);
M.Kamionkowski and J.March-Russell, PLB(1992);
S.Barr and D.Seckel, PRD(1992)

Freese and Kinney, arXiv: 1403.5277



Extranatural inflation, extra dimensional version of natural inflation
Arkani-Hamed, Cheng, Creminelli and Randall, PRL(2003)

5d model, extra dimension compactified on a circle R
Abelian field A_a
Extra component A_5 propagates in the bulk
no local potential due to higher dimensional gauge invariance
shift symmetry, similar to 4d PNGB

Non-local potential for Wilson loop $e^{i\theta} = e^{i \oint A_5 dx^5}$ in the presence of charged fields in the bulk

$$\mathcal{L} = \frac{1}{2\, g_4^2 (2\pi R)^2} (\partial\theta)^2 - V(\theta) + \cdots \qquad g_4^2 = g_5^2/(2\pi R)$$

Massless charged fields, one-loop $\quad V(\theta) = -\frac{1}{R^4} \sum_I (-1)^{F_I} \frac{3}{64\pi^6} \sum_{n=1}^{\infty} \frac{\cos(nq\theta)}{n^5}$

Boson $F_I = 0$ Fermion $F_I = 1$

Hosotani, PLB(1983); Hatanaka, Inami and Lim, MPLA(1998);
Antoniadis, Benakli and Quiros, New J. Phys. (2001);
von Gersdorff, Irgens and Quiros, NPB(2002);
Cheng, Matchev and Schmaltz, PRD(2002)

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{3}{64\pi^6 R^4}(1 - \cos\frac{\phi}{f_{\text{eff}}}) \qquad \phi = f_{\text{eff}}\theta$$

$$f_{\text{eff}} = \frac{1}{2\pi g_{4\text{d}} R}$$

For sufficiently small g_4, $f_{\text{eff}} \gg M_p$

Quantum gravity corrections are negligible as long as $R^{-1} < M_p$

Virtual black holes cannot spoil gauge symmetry,
non-local effects suppressed by $e^{-2\pi M_5 R}$

Similar consideration for quintessence dark energy

"Quintessence and the rest of the world", S. Carroll, PRL (1998)

Nearly massless $m_\phi \sim \sqrt{V_{\phi\phi}} < H_0 \sim 10^{-33}\mathrm{eV}$

Hypothetical couplings (besides the gravity) to SM particles:
1, direct coupling

$$c\frac{\phi}{M}\mathcal{L}(\bar{\psi}\psi, F_{\rho\sigma}F^{\rho\sigma}, G_{\rho\sigma}G^{\rho\sigma}, ...)$$

A. Long range force, violates equivalence principle, constrained to $c \leq 10^{-4}(M/M_{pl})$;

B. Instability under quantum corrections, $\delta m_\phi \simeq \frac{\Lambda^2}{4\pi M} \sim 10^{-7}\mathrm{eV} >> m_\phi$, $M \sim M_{pl},\ \Lambda \sim \Lambda_{ew}$.

2, derivative coupling, pseudo-Goldstone originated from $U(1)$ symmetry breaking

$$\frac{c}{M}\partial_\mu\phi\mathcal{O}^\mu(\psi, F_{\rho\sigma}, G_{\rho\sigma}, ...)$$

A. shift symmetry $\phi \to \phi + const.$, guarantees the flatness of the potential;

B. propagates spin-dependent force, short range, much weaker constraint from astrophysics $M \geq 10^{10}\text{Gev}, PDG.$

Quintessence as a PNGB proposed by Frieman, Hill, Stbbins and Waga, PRL (1995)

$$V = \Lambda^4(1 - \cos\frac{\phi}{f})\ ,\ m \sim \frac{\Lambda^2}{f},\ H \simeq \frac{\Lambda^2}{\sqrt{3}M_p}$$

``Gauge Quintessence”, Pilo, Rayner and Riotto, PRD(2003

Massive charged fields in the bulk, one-loop

$$V(\theta) = \frac{1}{128\pi^6 R^4} \mathrm{Tr}\left[V(r_a^F, \theta) - V(r_a^B, \theta) \right]$$

Delgado, Pomarol and Quiros, PRD (1999)

$$\begin{aligned} V(r_a, \theta) &= x_a^2 \, \mathrm{Li}_3(r_a e^{-x_a}) + 3x_a \, \mathrm{Li}_4(r_a e^{-x_a}) \\ &+ 3 \, \mathrm{Li}_5(r_a e^{-x_a}) + h.c. \ , \end{aligned}$$

with

$$r_a = e^{i q_a \theta} \, , \quad x_a = 2\pi R M_a \, ,$$

and the poly-logarithm function $\mathrm{Li}_k(z)$ are

$$\mathrm{Li}_k(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^k} \ .$$

astro-ph/0302479, PRD(2003)

An inflation model with large variations in spectral index

Bo Feng[a], Mingzhe Li[a], Ren-Jie Zhang[b,c], and Xinmin Zhang[a]
[a] *Institute of High Energy Physics, Chinese Academy of Science, P.O. Box 918-4, Beijing 100039, P. R. China*
[b] *School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540, USA and*
[c] *Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, P. R. China.*

Consider two fields coupling to A_5 $\quad M_1 = 0,\ M_2 > R^{-1}$

$$V(\theta) = -\frac{3}{64\pi^6 R^4}\sum_{n=1}^{\infty}\frac{1}{n^3}\left[(-)^{F_1}\frac{\cos(nq_1\theta)}{n^2} + (-)^{F_2}e^{-nx_2}\left(\frac{x_2^2}{3}+\frac{x_2}{n}+\frac{1}{n^2}\right)\cos(nq_2\theta)\right]$$

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V_0\left[1-\cos\left(\frac{q_1\phi}{f_{\rm eff}}\right) - \sigma\cos\left(\frac{q_2\phi}{f_{\rm eff}}\right)\right]$$

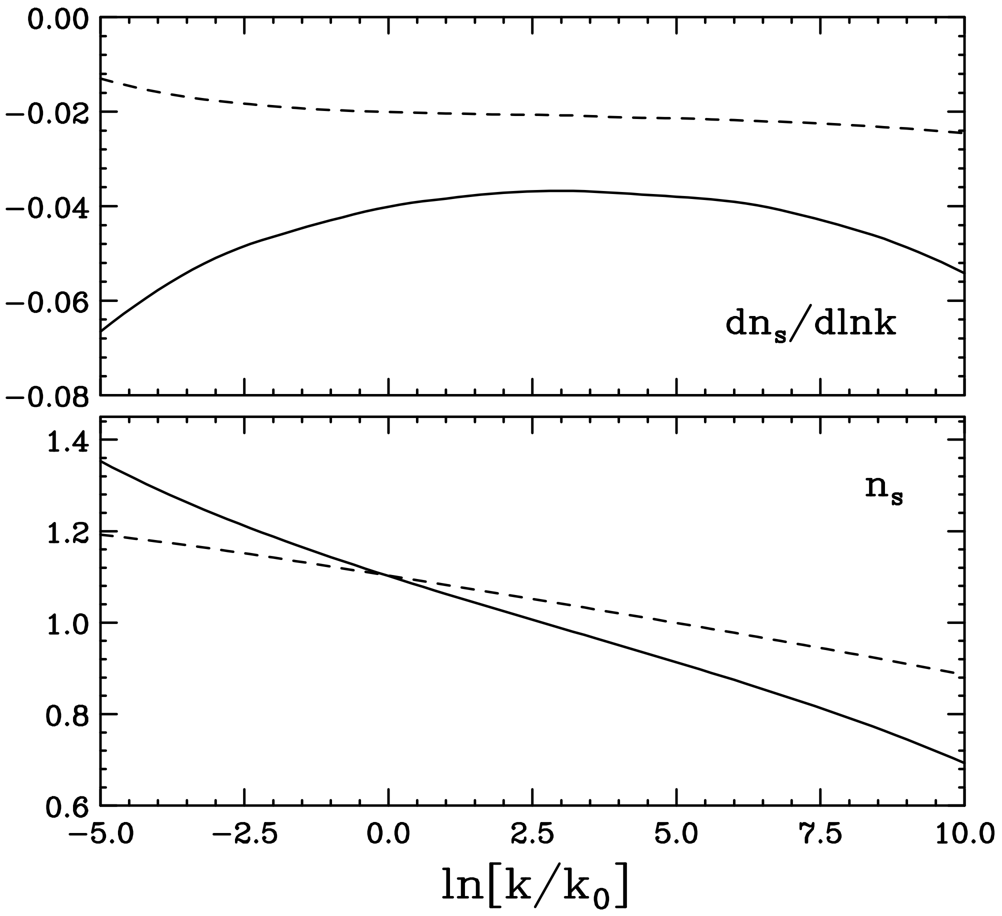
where

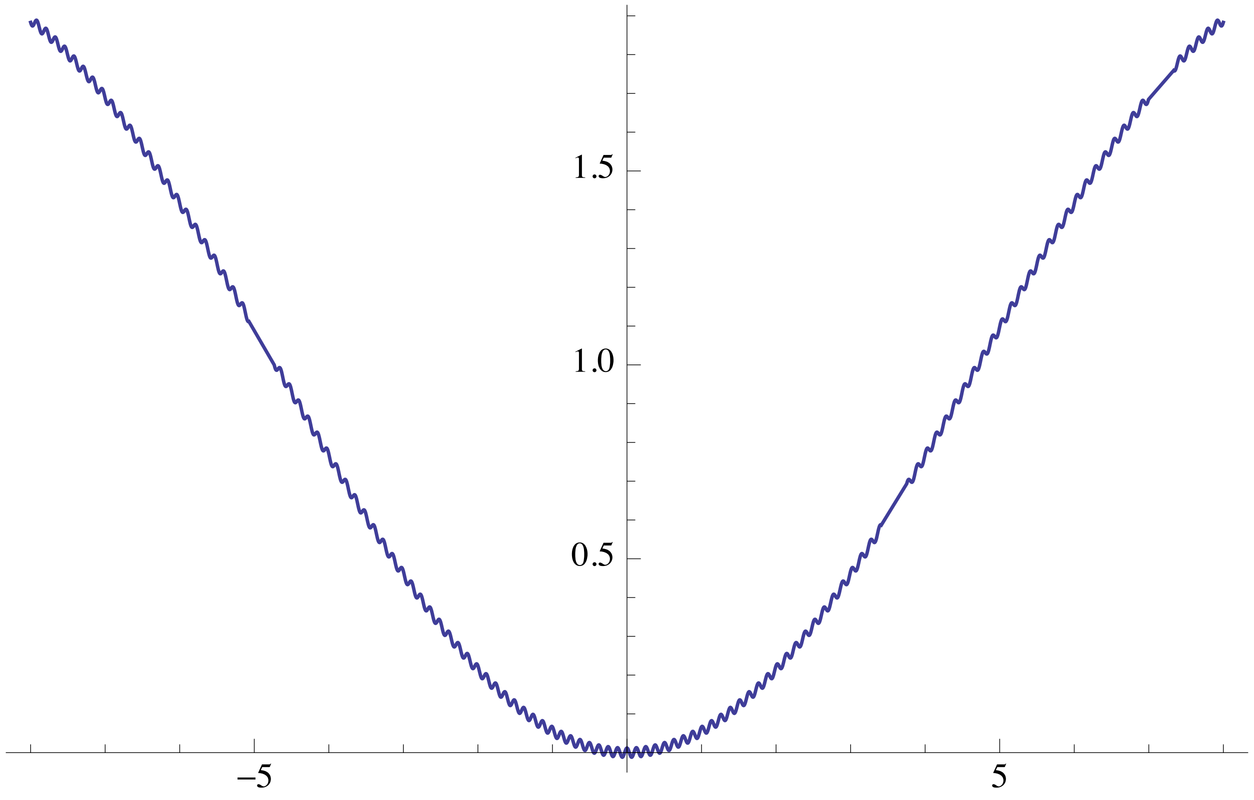
$$\sigma = (-)^{F_2+1}e^{-x_2}\left(\frac{x_2^2}{3}+x_2+1\right),\quad V_0 = \frac{3}{64\pi^6 R^4}$$

Parameters

$$\mu = q_1 M_p / f_{\text{eff}} \sim \mathcal{O}(0.1 - 1),\ \kappa = q_2/q_1 \gg 1,\ \sigma \ll 1$$

Extranatural inflation modulated by rapid oscillations
Slow-rolling is slightly broken





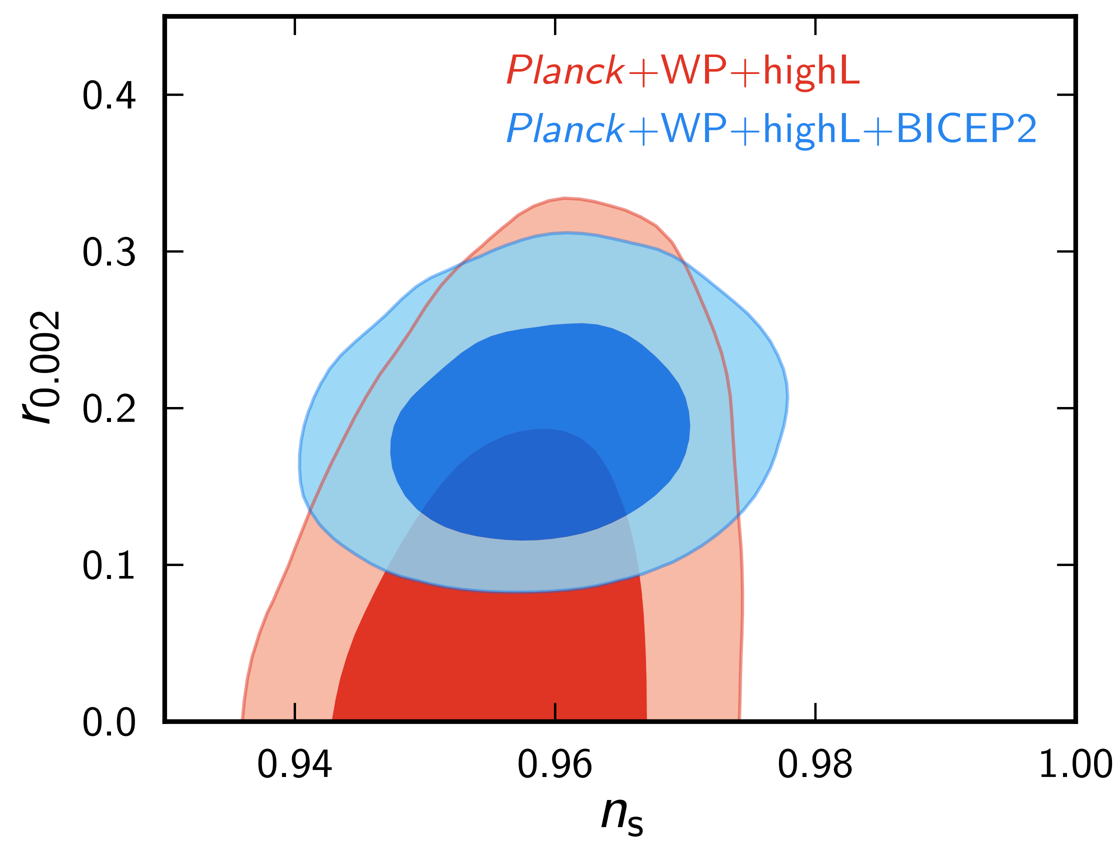
Tension between BICEP2 and Planck

$$r_{0.002} < 0.11 \quad (95\%; \text{no running})$$

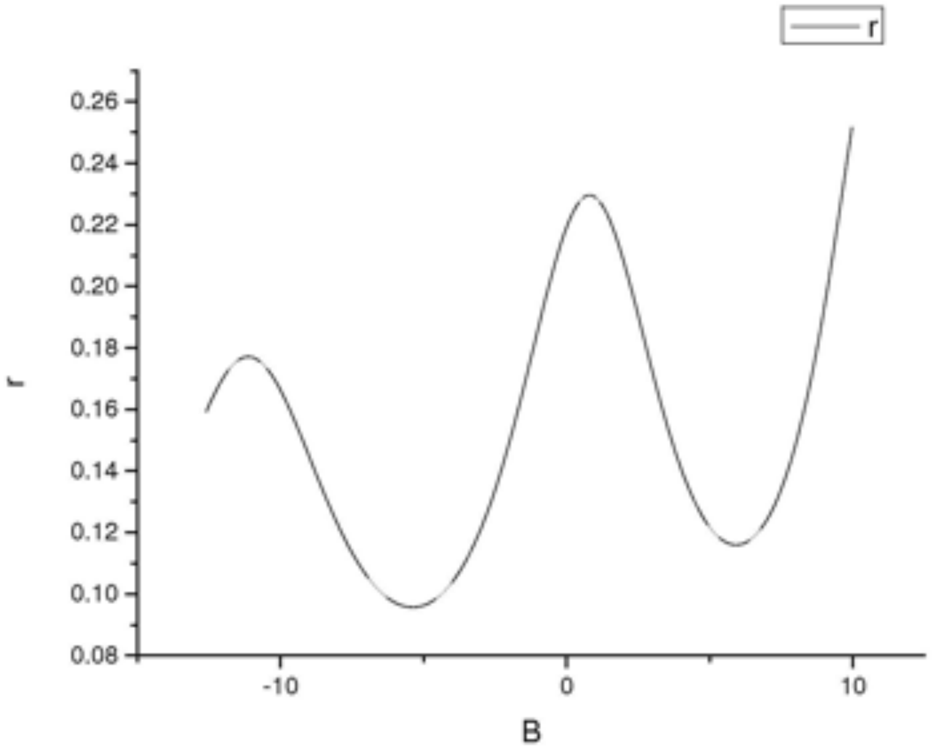
$$r_{0.002} < 0.26 \quad (95\%; \text{including running})$$

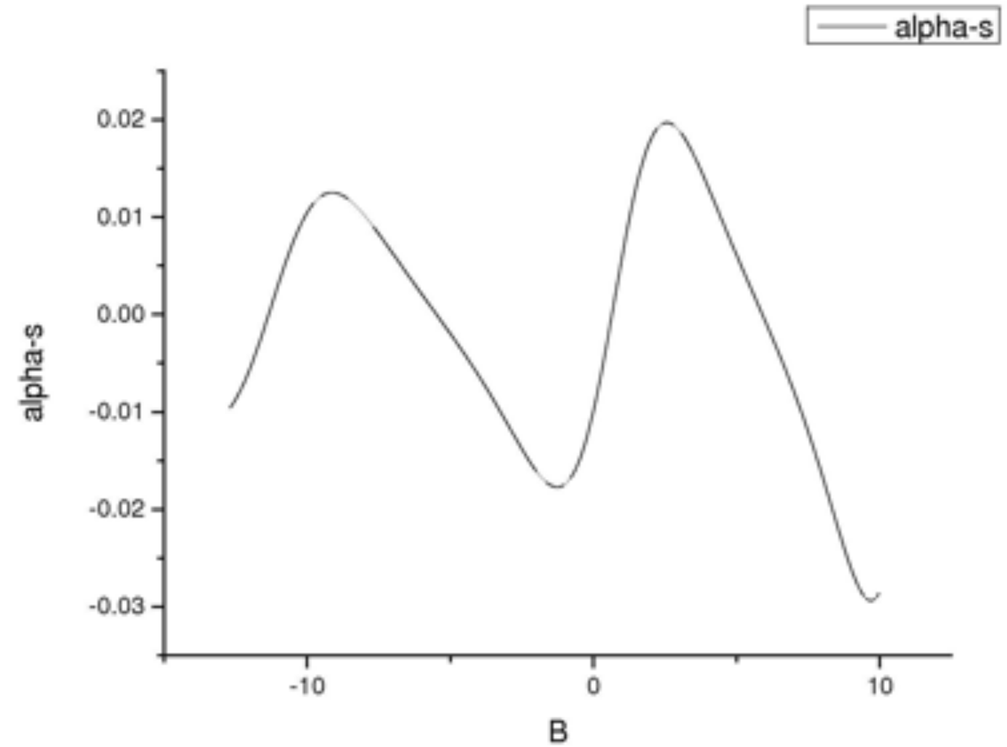
$$r_{0.002} = (\frac{A_t}{A_s})_{k=0.002Mpc^{-1}}$$

$$\text{Slow} - \text{roll}: \frac{dn_s}{d\ln k} \sim 10^{-3}$$



$$dn_s/d\ln k = -0.028 \pm 0.009(68\%)$$





Fitting to BICEP2 and Planck is ongoing

Conclusions

- If the results of BICEP2 are not true, alternatives to inflation are still alive and there are various mechanisms to generate nearly scale-invariant density perturbations

- If BICEP2 is confirmed, single field inflationary universe is the most natural scenario for the very early universe. The question left is its validity within the effective field theory.