Progress in Massive Gravity

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Beyond Einstein Theories of Gravity

Type I: UV Modifications:
eg. Quantum Gravity, string theory, extra dimensions, branes, supergravity

At energies well below the scale of new physics Λ , gravitational effects are well incorporated in the language of Effective Field Theories

$$S = M_{\rm Planck}^2 \int d^4x \sqrt{-g} \left[\frac{1}{2} R + \frac{a}{\Lambda^2} R^2 + \frac{b}{\Lambda^2} R_{\mu\nu}^2 + \dots + \frac{c}{\Lambda^4} R_{abcd} R^{cd}_{ef} R^{efab} + \dots + \mathcal{L}_{\rm matter} \right]$$

$$+ \frac{d}{\Lambda^6} (R_{abcd} R^{abcd})^2 + \dots$$

eg Cardoso et al 2018

Addition of Higher Dimension, (generally higher derivative operators), **no failure of well-posedness/ghosts** etc as all such operators should be treated perturbatively (rules of EFT)

Type 2: IR Modifications:

Why modify gravity (in the IR)?

Principle Motivation is Cosmological:

Dark Energy and Cosmological Constant

I: Old cosmological constant problem:

Why is the universe not accelerating at a gigantic rate determined by the vacuum energy?

II: New cosmological constant problem:

Assuming I is solved, what gives rise to the remaining vacuum energy or dark energy which leads to the acceleration we observe?

Why modify gravity (in the IR)?

III: Because it allows us to put better constraints on Einstein gravity!

Curvature

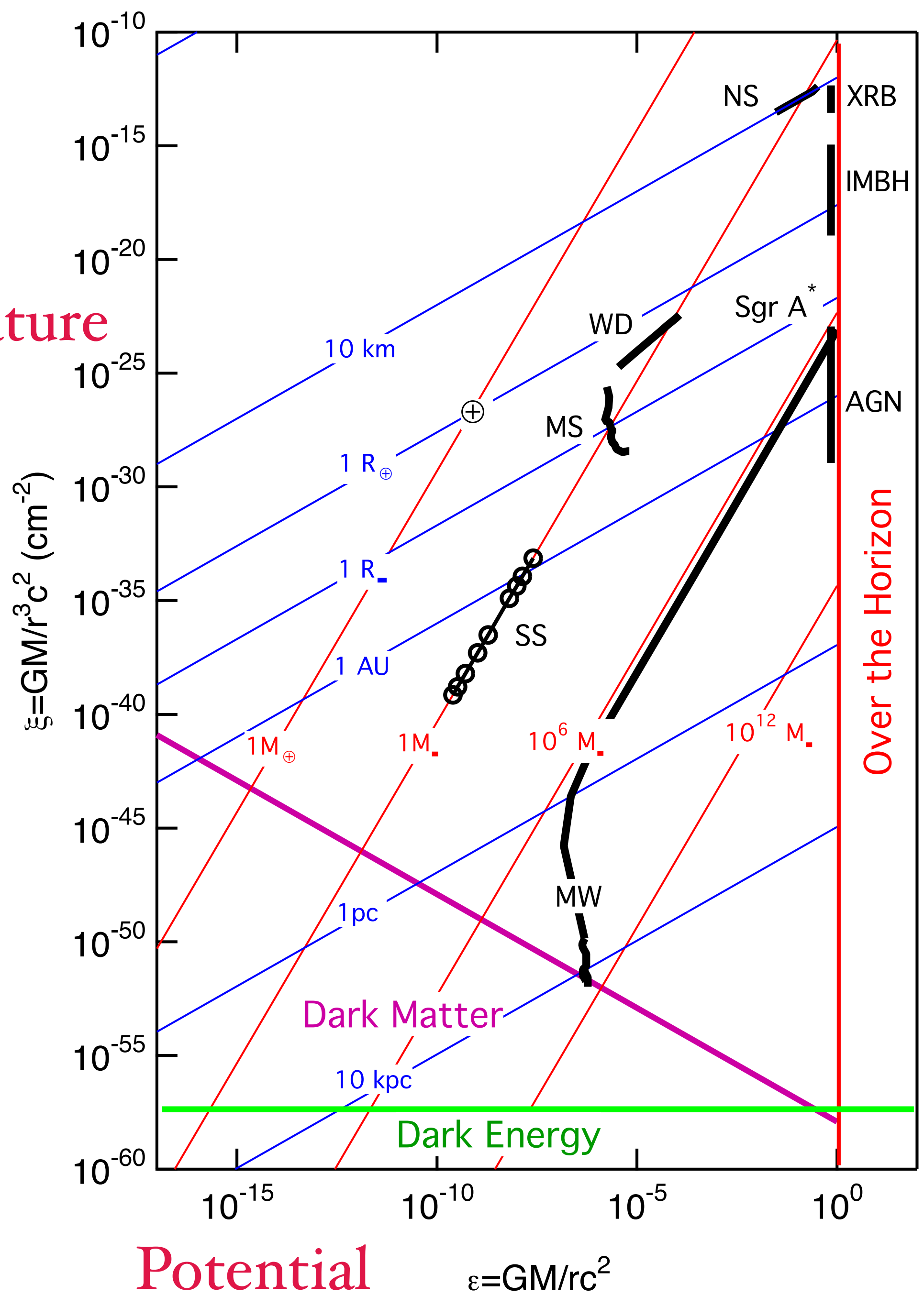


Figure 1: A parameter space for quantifying the strength of a gravitational field. The x-axis measures the potential $\epsilon \equiv GM/rc^2$ and the y-axis measures the spacetime curvature $\xi \equiv GM/r^3c^2$ of the gravitational field at a radius r away from a central object of mass M. These two parameters provide two different quantitative measures of the strength of the gravitational fields. The various curves, points, and legends are described in the text.

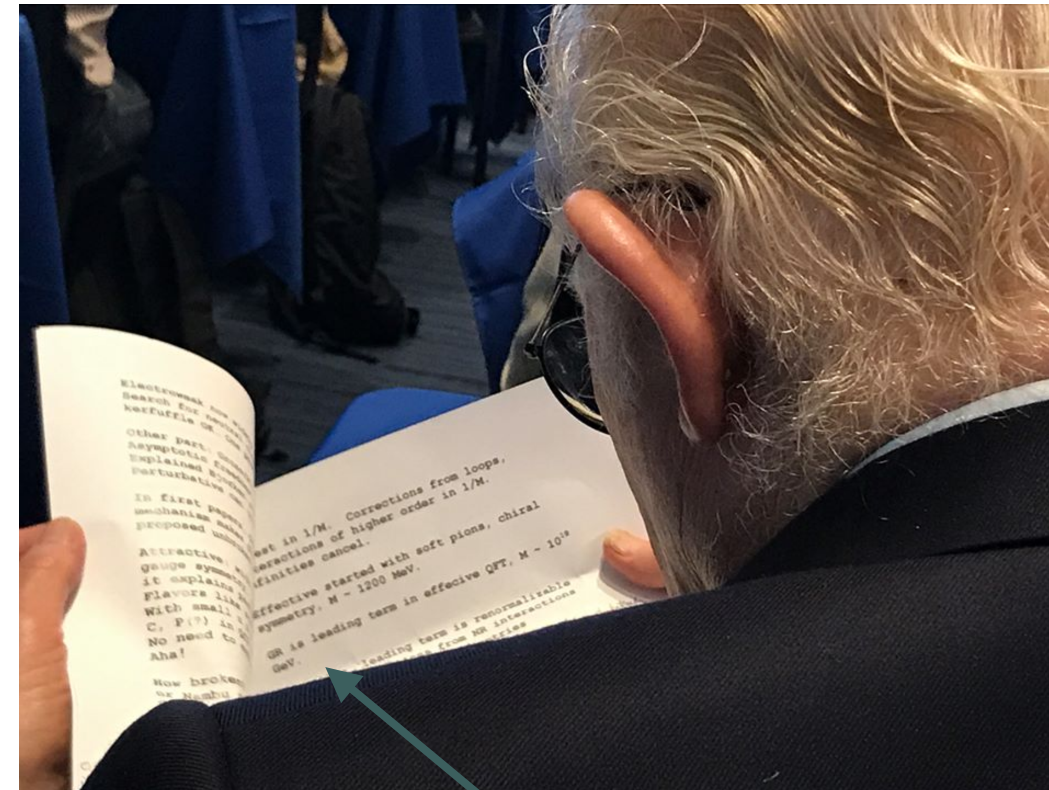
Gravity has only been tested over special ranges of scales and curvatures

e.g. Weinberg's nonlinear Quantum Mechanics-constructing to test linearity of QM

D. Psaltis, Living Reviews

Guiding Principle

Theorem: General Relativity is the Unique local and Lorentz invariant theory describing an interacting single massless spin two particle that couples to matter Weinberg, Deser, Wald, Feynman, ...



....GR is leading term in effective QFT ....

Locality

Massless ? Lorentz Invariant

Single Spin 2

Guiding Principle

Corollary: Any theory which preserves Lorentz invariance and Locality leads to new degrees of freedom!

Locality ✓

Massless ✗

?

Lorentz Invariant ✓

Single Spin 2 ✗

Theoretical Aspects of Massive Gravity

Massive Gravity: Hard or Soft?

Hard

A generic local, Lorentz invariant theory at the linearized level gives the following interaction between two stress energies

$$A \sim \frac{1}{M_{\rm Pl}^2}\int \frac{d^4k}{(2\pi)^4}\, T^{ab}(k)^* \left[\frac{P_{abcd}}{k^2} + \sum_{\rm pole} Z^{(2)}_{\rm pole} \frac{\mathcal{P}_{abcd}}{k^2+m^2_{\rm pole}} + \sum_{\rm pole} Z^{(0)}_{\rm pole} \frac{\eta_{ab}\eta_{cd}}{k^2+m^2_{\rm pole}} \right] T^{cd}(k)$$

$$+\frac{1}{M_{\rm Pl}^2}\int \frac{d^4k}{(2\pi)^4}\, T^{ab}(k)^* \left[\int d\mu\, \rho^{(2)}(\mu) \frac{\mathcal{P}_{abcd}}{k^2+\mu^2} + \rho^{(0)}(\mu) \frac{\eta_{ab}\eta_{cd}}{k^2+\mu^2} \right] T^{cd}(k)$$

$$P_{abcd} = \eta_{ac}\eta_{bd} + \eta_{bc}\eta_{ad} - \eta_{ab}\eta_{cd}$$

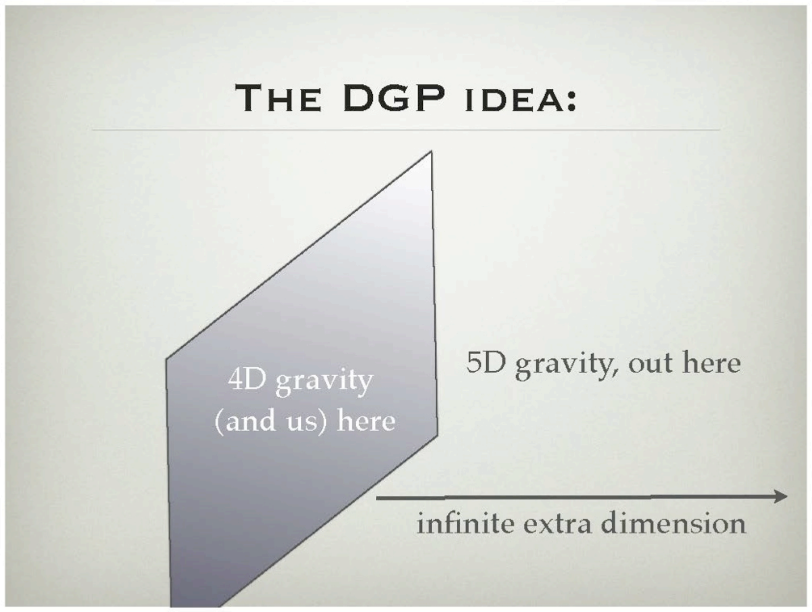
$$\mathcal{P}_{abcd} = \eta_{ac}\eta_{bd} + \eta_{bc}\eta_{ad} - \frac{2}{3}\eta_{ab}\eta_{cd}$$

Soft

Soft Massive Graviton is a **resonance**

Hard Massive Graviton is a **pole** (infinite lifetime)

Soft Massive Gravity: DGP Model



Soft Massive Gravity theories were constructed first! Naturally arise in Braneworld Models: **DGP**, **Cascading Gravity**: Soft Massive Graviton is a Resonance State localized on Brane

$$\Delta S \sim \frac{1}{M^2_{\text{Planck}}} \int \frac{d^4k}{(2\pi)^4} T^{\mu\nu}(k) \left[\int_0^\infty d\mu \frac{\rho(\mu) P_{\mu\nu\alpha\beta}(k)}{k^2+\mu} \right] T^{\alpha\beta}(k)$$

Soft

More irrelevant ⟷ More relevant

$$S = \int d^4x \sqrt{-g_4} \frac{M_4^2}{2} R_4 + \int d^4x \sqrt{-g_4} \mathcal{L}_M + \int d^5x \sqrt{-g_5} \frac{M_5^3}{2} R_5$$

Dominates in UV ⟷ Dominates in IR

What does HARD massive gravity mean?

In SM, Electroweak symmetry
is spontaneously broken by the VEV of the Higgs field

$$SU(2) \times U(1)_Y \rightarrow U(1)_{\mathrm{EM}}$$

Result, W and Z bosons become massive

Would-be-Goldstone-mode in Higgs field becomes
Stuckelberg field which gives boson mass

Higgs Vev Higgs Boson Stuckelberg field

$$\Phi = (v + \rho) e^{i\pi}$$

e.g. for Abelian Higgs

$$A_\mu \rightarrow A_\mu + \partial_\mu \chi \qquad \pi \rightarrow \pi + \chi$$

Symmetry Breaking Pattern

In **Massive Gravity** - Local Diffeomorphism Group and an additional global Poincare group is broken down the diagonal subgroup

$$Diff(M) \times \text{Poincare} \to \text{Poincare}_{\text{diagonal}}$$

In **Bigravity** - Two copies of local Diffeomorphism Group are broken down to a single copy of Diff group

$$Diff(M) \times Diff(M) \to Diff(M)_{\text{diagonal}}$$

Higgs for Gravity

Despite much *blood, sweat and tears* an explicit Higgs mechanism for gravity is not known

However if such a mechanism exists, we DO know how to write down the low energy effective theory in the spontaneously broken phase

For Abelian Higgs this corresponds to integrating out the Higgs boson and working at energy scales lower that the mass of the Higgs boson

Higgs Boson → ρ ; Stuckelberg field → π

$$E \ll m_\rho \qquad \Phi = (v+\rho)e^{i\pi}$$

⇒ Stuckelberg formulation of massive vector bosons

Stuckelberg Formulation for Massive Gravity

Arkani-Hamed et al 2002
de Rham, Gabadadze 2009

Diffeomorphism invariance is spontaneously broken but maintained by introducing Stueckelberg fields

Vev of spin 2 Higgs field defines a 'reference metric'

$$f_{\mu\nu} = \langle \hat{O}_{\mu\nu} \rangle$$

reference metric

Stuckelberg fields

Dynamical Metric

$$g_{\mu\nu}(x)$$

$$F_{\mu\nu} = f_{AB}(\phi)\partial_\mu \phi^A \partial_\nu \phi^B$$

helicity-1 mode of graviton

$$\phi^a = x^a + \frac{1}{mM_P} A^a + \frac{1}{\Lambda^3} \partial^a \pi$$

$$\Lambda^3 = m^2 M_P$$

helicity-0 mode of graviton

Stuckelberg Formulation for Bigravity

Fasiello, AJT 1308.1647

$$Diff(M) \times Diff(M) \rightarrow Diff(M)_{\text{diagonal}}$$

Bigravity breaks the same amount of symmetry as massive gravity, need to introduce same number of Stuckelberg fields

Dynamical metric I

$$g_{\mu\nu}(x)$$

Dynamical metric II

$$F_{\mu\nu} = f_{AB}(\phi)\partial_\mu \phi^A \partial_\nu \phi^B$$

$$\phi^A = x^A + \frac{1}{\Lambda_3^3}\partial^A \pi$$

=

Fasiello, AJT 1308.1647

But there are two ways to introduce Stuckelberg fields!

Dynamical metric I

$$g_{\mu\nu}(x)$$

Dynamical metric II

$$F_{\mu\nu} = f_{AB}(\phi)\partial_\mu \phi^A \partial_\nu \phi^B$$

$$\tilde{x}^A = \phi^A(x) = x^A + \partial^A \pi(x)$$

OR

Dynamical metric I

$$\tilde{G}_{AB}(\tilde{x}) = g_{\mu\nu}(Z)\partial_A Z^\mu \partial_B Z^\nu$$

Dynamical metric II

$$f_{AB}(\tilde{x})$$

$$x^\mu = Z^\mu(\tilde{x}) = \tilde{x}^\mu + \partial^\mu \tilde{\pi}(\tilde{x})$$

Galileon Duality!!!!!

Discovering how to square root

$$F_{\mu\nu} = f_{AB}(\phi)\partial_\mu \phi^A \partial_\nu \phi^B$$

$$\phi^a = x^a + \frac{1}{mM_P}A^a + \frac{1}{\Lambda^3}\partial^a \pi$$

Helicity zero mode enters reference metric **squared**

$$F_{\mu\nu} \approx \eta_{\mu\nu} + \frac{2}{\Lambda^3}\partial_\mu\partial_\nu\pi + \frac{1}{\Lambda^6}\partial_\mu\partial_\alpha\pi\partial^\alpha\partial_\nu\pi$$

To extract dominant helicity zero interactions we need to take a square root

$$\left[\sqrt{g^{-1}F}\right]_{\mu\nu} \approx \eta_{\mu\nu} + \frac{1}{\Lambda^3}\partial_\mu\partial_\nu\pi$$

Branch uniquely chosen to give rise to 1 when Minkowski de Rham, Gabadadze, AJT 2010

Hard Λ_3 Massive Gravity

$Diff(M) \times \text{Poincare} \rightarrow \text{Poincare}_{\text{diagonal}}$

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left(M_P^2 R[g] - m^2 \sum_{n=0}^{4} \beta_n \mathcal{U}_n\right) + \mathcal{L}_M$$

$K = 1 - \sqrt{g^{-1} f}$

$$\text{Det}[1 + \lambda K] = \sum_{n=0}^{d} \lambda^n \mathcal{U}_n(K)$$

Characteristic Polynomials

Double epsilon structure!!!!!

Unique low energy EFT where the strong coupling scale is $\Lambda_3 = (m^2 M_P)^{1/3}$

5 propagating degrees of freedom
5 polarizations of gravitational waves!!!!

Hassan, Rosen 2011

Hard Massless plus Λ_3 Massive Gravity

$$\mathcal{L} = \frac{1}{2}\left(M_P^2 \sqrt{-g} R[g] + M_f^2 \sqrt{-f} R[f] - m^2 \sum_{n=0}^{d} \beta_n U_n(K) \right) + \mathcal{L}_M$$

$$\mathrm{Det}[1 + \lambda K] = \sum_{n=0}^{d} \lambda^n \mathcal{U}_n(K)$$

$$K = 1 - \sqrt{g^{-1} f}$$

decoupling limit

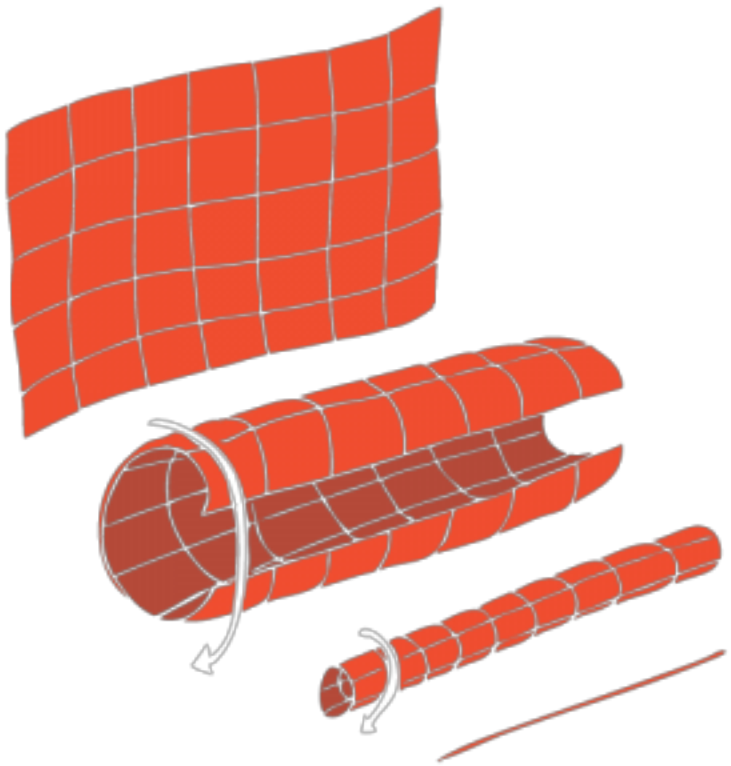
$M_f \to \infty$

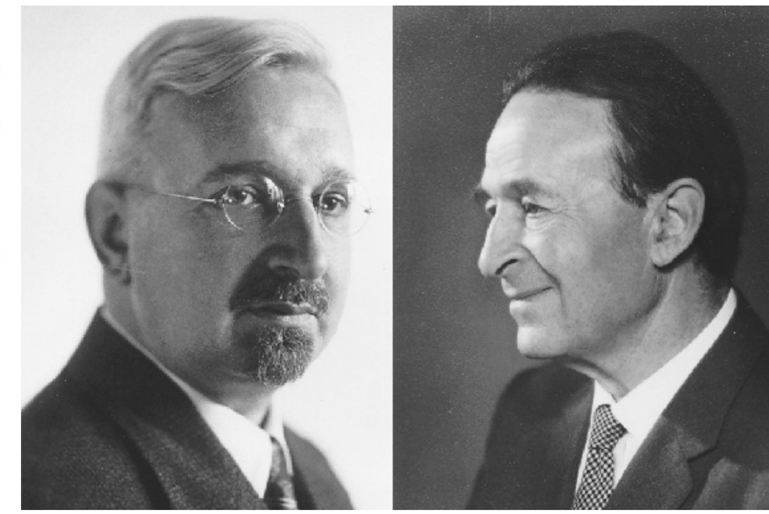
$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left(M_P^2 R[g] - m^2 \sum_{n=0}^{4} \beta_n \mathcal{U}_n \right) + \mathcal{L}_M$$

Bigravity=
massless graviton (2 d.o.f.)
+ massive graviton (5 d.o.f.)

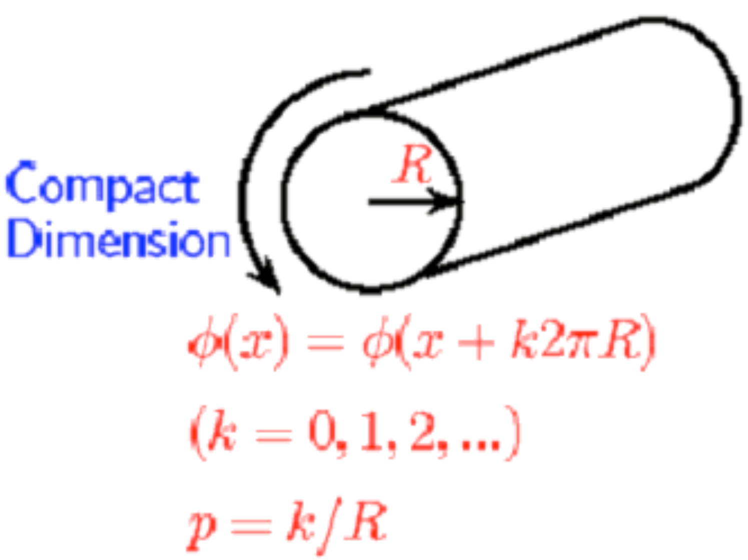
+decoupled massless graviton $f_{\mu\nu}$

The original (post Einstein) modified theory of gravity: *Kaluza-Klein theory*

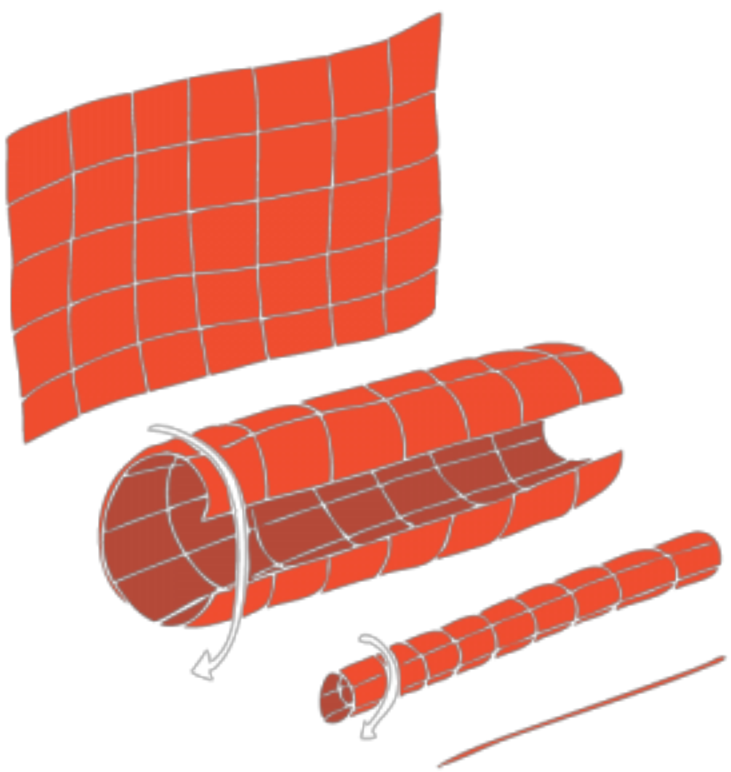




* 5 Dimensional Gravity compactified on a circle
* 5D massless graviton = 4D massless graviton + 4D massless photon + 4D massless scalar + N 4D massive gravitons
* Consistent UV modification at KK scale m= 1/R



Kaluza-Klein = theory of massive gravitons



$$ds^2 = dy^2 + [\eta_{\mu\nu} + h_{\mu\nu}(x,y)]dx^\mu dx^\nu$$
$$y \in [0, L]$$

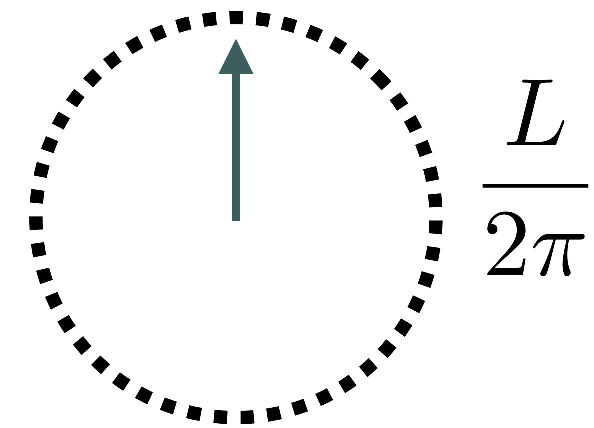
$$h_{\mu\nu}(x,y) = \sum_{n=-\infty}^{\infty} e^{2\pi i n y/L} h_{\mu\nu,n}(x)$$

Kaluza-Klein tower of massive graviton states with wavefunctions $h_{\mu\nu,n}(x)$

$$m_n = \frac{2\pi}{L} n$$

Finite number of weakly coupled gravitons $N \sim M_{\text{planck}} L$

Gravitational Deconstruction



Arkani-Hamed, Cohen, Georgi 2001

Arkani-Hamed, Schwartz 2004

de Rham, Matas, AJT 2013

$$ds^2 = dy^2 + [\eta_{\mu\nu} + h_{\mu\nu}(x,y)]dx^\mu dx^\nu$$

Now replace the continuous extra dimension by a **lattice**

$$y_k = k\frac{L}{N} \qquad \partial_y h_{\mu\nu}(x,y_k) = \frac{N}{L}\left(h_{\mu\nu}(x,y_{k+1}) - h_{\mu\nu}(x,y_k)\right)$$

$$k = 0 \dots N$$

Gives a theory of N massive and one massless graviton

This picture of the discrete Fourier transform of the KK picture

Gravitational Deconstruction

de Rham, Matas, AJT 2013

All of this may be performed at the non-linear level, easiest in Einstein-Cartan (vielbein formalism)

5D GR

$$S = M_5^2 \int dy \int E \wedge E \wedge E \wedge R_5$$

4D multigravity - **Ghost free** 1 massless + N massive gravitons

Hinterbichler and Rosen 2012

$$S = M_4^2 \sum_i \int e \wedge e \wedge R_4 + M_4^2 \sum_i m_i^2 e_i \wedge e_i \wedge (e_{i+1} - e_i) \wedge (e_{i+1} - e_i)$$

By weighting the discretization we may generate all allowed ghost free mass terms

Universal Decoupling Limit: Galileon

At energies $m \ll E \ll M_{\text{PLanck}}$ $\Lambda_3 = (m^2 M_{\text{Planck}})^{1/3}$

All Lorentz invariant Hard and Soft and Multi-graviton theories look like **Galileon theories** (plus massless spin 2 plus Maxwell)

$$\pi \to \pi + v_\mu x^\mu + c \qquad K_{\mu\nu} = \frac{\partial_\mu \partial_\nu \pi}{\Lambda_3^3}$$

$$S = \int d^4x \left[-\frac{1}{4} h^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} - \frac{1}{4} v^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} v_{\alpha\beta} \right] + S_{\text{Galileon}} + S_{\text{mattercoupling}}$$

$$S_{\text{Galileon}} = \sum_{n=0}^{4} \pi \, c_n \, \mathcal{U}_n(K) \qquad \text{Det}[1 + \lambda K] = \sum_{n=0}^{d} \lambda^n \mathcal{U}_n(K)$$

Novel feature, matter has `disformal' couplings

$$S_{\text{matter coupling}} = \int d^4x \frac{1}{M_P} (\pi T + \partial_\mu \pi \partial_\nu \pi T^{\mu\nu} + \dots)$$

Explicitly Decoupling limit for Bigravity

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_P} h_{\mu\nu} \qquad f_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_P} v_{\mu\nu}$$

de Rham, Gabadadze 2009
Fasiello, AJT 2013

massless helicity 2

massless helicity 0

$$S_{\text{helicity}-2/0} = \int \mathrm{d}^4 x \left[-\frac{1}{4} h^{\mu\nu} \hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} - \frac{1}{4} v^{\mu\nu} \hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu} v_{\alpha\beta} + \frac{\Lambda_3^3}{2} h^{\mu\nu}(x) X^{\mu\nu} + \frac{M_{\rm p}\Lambda_3^3}{2M_{\rm f}} v_{\mu A}[x^a + \Lambda_3^{-3}\partial^a \pi](\eta^A_\nu + \Pi^A_\nu) Y^{\mu\nu} \right]$$

$$X^{\mu\nu} = -\frac{1}{2} \sum_{n=0}^{4} \frac{\hat{\beta}_n}{(3-n)!n!} \varepsilon^{\mu\ldots}\varepsilon^{\nu\ldots}(\eta + \Pi)^n \eta^{3-n}$$

$$\Pi_{ab} = \frac{\partial_a \partial_b \pi}{\Lambda_3^3}$$

$$Y^{\mu\nu} = -\frac{1}{2} \sum_{n=0}^{4} \frac{\hat{\beta}_n}{(4-n)!(n-1)!} \varepsilon^{\mu\ldots}\varepsilon^{\nu\ldots}(\eta + \Pi)^{(n-1)} \eta^{4-n}$$

Generalized Galileons and Generalized Massive Gravity

de Rham, Keltner, AJT, 2014

The massive gravity action can be generalized to a covariant theory whose decoupling limit corresponds to the generalized Galileons

$$S = M_P^2 \int d^4x \sqrt{-g} \left[\Phi(\phi^a \phi_a) R - \sum_{n=0}^{d} \beta_n(\phi^a \phi_a) \mathcal{U}_n(K) \right]$$

inclusion of potentials for Stuckelberg fields, in the decoupling limit corresponds to

$$\mathcal{L} = \sum_{n=0}^{d} A_n(X) \mathcal{U}_n(K)$$

$$K^\mu_\nu = \partial^\mu \partial_\nu \pi$$

$$X = -\frac{1}{2} (\partial \pi)^2$$

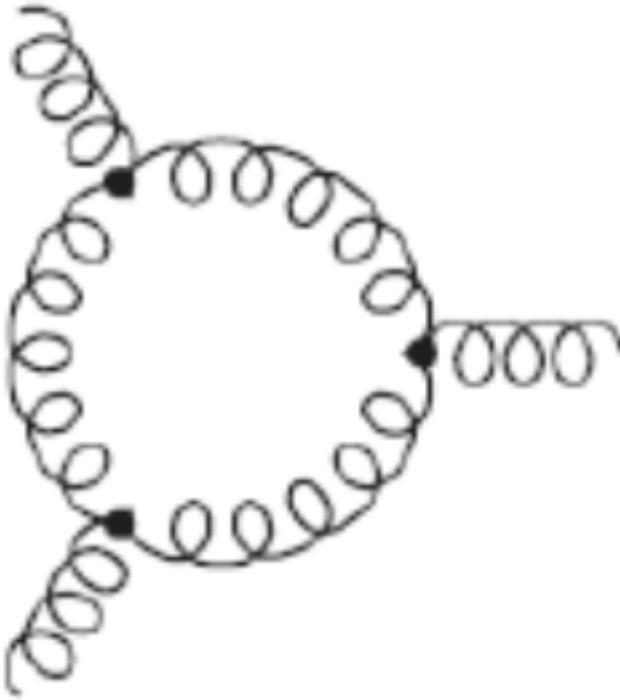
N.B. these models **allow** for flat FRW solutions!!!

Massive Gravity as an EFT

Ghost free massive gravity, bigravity and multigravity are Effective Field Theories (EFT), which breaks down at the scale $\Lambda_3 = (m^2 M_{\text{Planck}})^{1/3}$

Generic one-loop Graviton diagram needs counter-terms at the scale (principally due to helicity zero mode interactions)

$$\Lambda_3 = (m^2 M_{\text{Planck}})^{1/3}$$



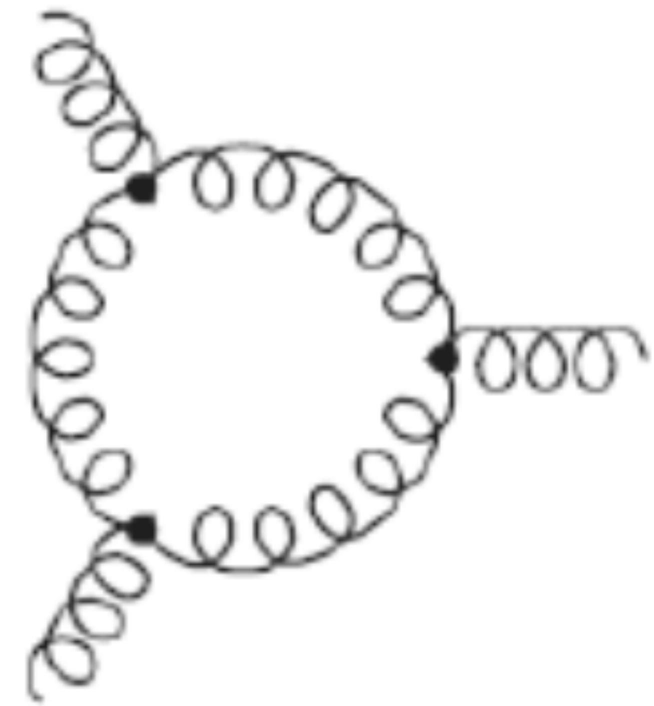
Counter-terms which are not needed in GR!

Vainshtein radius LARGER than Schwarzschild radius

Massive Gravity/Galileons etc as an EFT

One-loop Graviton diagram needs counter-terms at the scale

$$K = 1 - \sqrt{g^{-1}f} \qquad \Lambda_3 = (m^2 M_{\rm Planck})^{1/3}$$



In decoupling limit: $M_{\rm PLanck} \to \infty, m \to 0$

$$K_{\mu\nu} \to \frac{\partial_\mu \partial_\nu \pi}{\Lambda_3^3}$$

EFT corrections then take the form

(even away from the decoupling limit)

de Rham, Melville, AJT 2017

$$\Lambda^4 L_0 = \left[\frac{M^2}{2} R - \Lambda^3 M \sum_n \alpha_n \mathcal{E}\mathcal{E} g^{4-n} K^n \right] + \Lambda^4 \sum \beta_{p,q,r} \left(\frac{\nabla}{\Lambda} \right)^p K^q_{\mu\nu} \left(\frac{R_{\mu\nu\rho\sigma}}{\Lambda^2} \right)^r$$

Infinite number of derivative suppressed operators

HOW DO WE GO BEYOND THIS?

Gauge Invariance and Mass

Schwinger taught us that you can have a mass without violating gauge invariance!

Example: Proca theory

$$\mathcal{L} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu^2$$

Introduce Stueckelberg fields

$$\mathcal{L} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2(A_\mu - \partial_\mu\phi)^2$$

Integrate out Stueckelberg fields

$$\mathcal{L} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}m^2 F_{\mu\nu}\frac{1}{\Box}F^{\mu\nu}$$

Such a nonlocal mass term can be obtained from loop effects from integrating out a light field

Schwinger Model (2D)

Gauge Invariance and Mass

We can do the same for massive gravity - integrating out Stueckelberg fields generates non-local mass terms which are nevertheless manifestly gauge invariant

Ghost-free massive gravity can be written in the manifestly gauge invariant form

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}R - \frac{1}{2}m^2 R^{\mu\nu}\frac{1}{\Box^2}(R^{\mu\nu} - g^{\mu\nu}R) + \dots$$

+ Infinite number
higher curvature similar terms

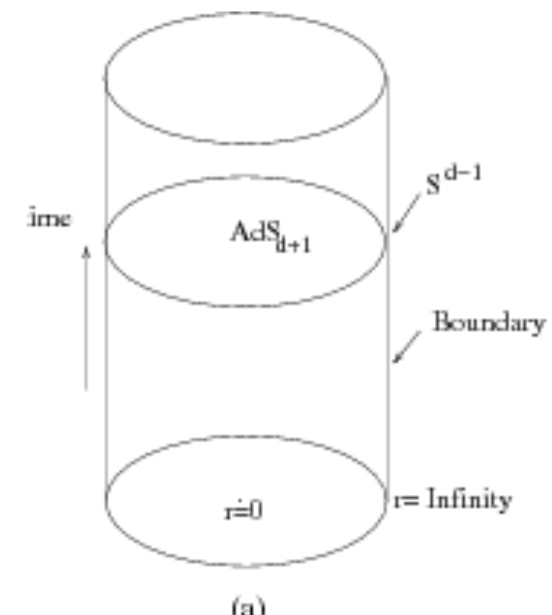
These non-local terms could be viewed as arising from integrating out quantum effects from light fields

UV complete Massive Gravity

Porrati, 2001

A local, causal, unitary theory UV completion of a theory of a single massive (with no massless!) graviton exists in AdS

UV completion: 4d gravity coupled to a CFT in AdS



$$\mathcal{L} = \sqrt{-g}(R + 2|\Lambda|) + \mathcal{L}_{\mathrm{CFT}}$$

CFT = e.g. conformally coupled scalar field

Integrating out the CFT with mixed boundary conditions at the boundary of AdS generates a **non-local** contribution to the action

Non-local term precisely generates a **mass for the graviton**!

Higgs Mechanism for Gravity or Schwinger mechanism for Mass!

Raising the Cutoff- <u>the Third Way</u>?

$$\Delta S \sim \frac{1}{M_{\text{Planck}}^2} \int \frac{d^4k}{(2\pi)^4} T^{\mu\nu}(k) \left[\sum_{\text{pole}} Z_{\text{pole}} \frac{P_{\mu\nu\alpha\beta}(k)}{k^2 + m_{\text{pole}}^2} + \int_0^\infty d\mu \frac{\rho(\mu) P_{\mu\nu\alpha\beta}(k)}{k^2 + \mu} \right] T^{\alpha\beta}(k)$$

Hard

Soft

Hard and Soft

No vDVZ discontinuity on AdS

Its an old result, that on AdS you can take the massless limit of massive gravity and recover GR plus a decoupled sector

= NO vDVZ discontinuity!

$$K^\mu_\alpha K^\alpha_\nu = \delta^\mu_\mu - g^{\mu\alpha} f_{ab}(\phi)\partial_\alpha \phi^a \partial_\nu \phi^b$$

$$\mathcal{L} = \frac{1}{2} M_{\text{Planck}}^{d-2} R - \frac{1}{2} m^2 M_{\text{Planck}}^{d-2} (K_{\mu\nu}^2 - K^2)$$

$\frac{d(d-3)}{2}$ d.o.f. $\qquad$ $(d-2)+1$ d.o.f.

On AdS $f_{ab}(\phi) = \frac{L^2}{\phi_d^2}\eta_{ab}$ we can take

$M_{\text{Planck}} \to \infty \qquad \Lambda = (m^2 M_{\text{Planck}}^{d-2})^{1/d}$ fixed

$\Lambda_2 \gg \Lambda_3$

Only Problem: We don't live in AdS!!!!!!

Warped Massive Gravity

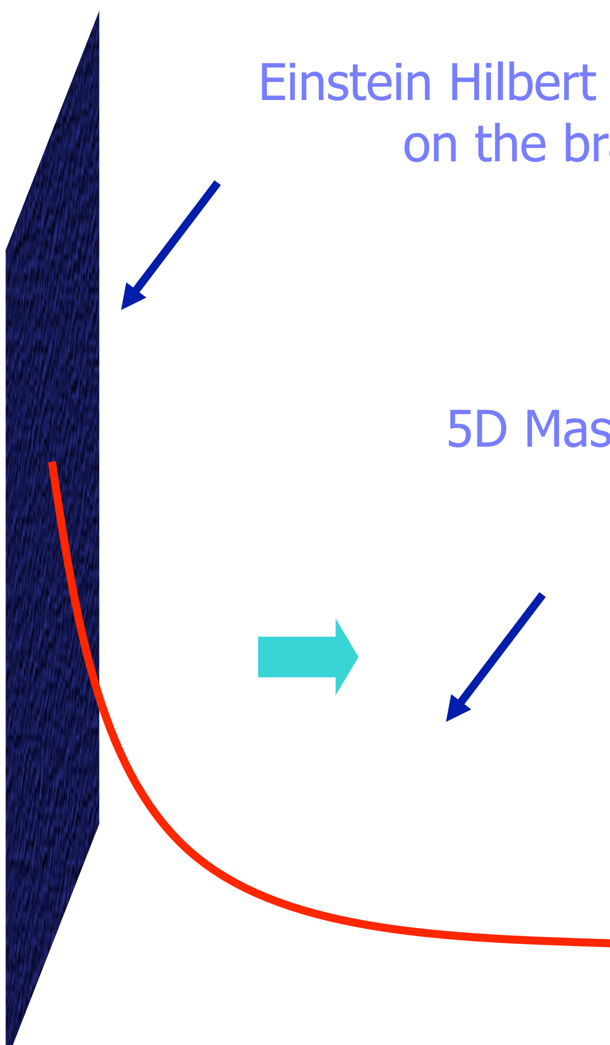
Gabadadze 2017

Solution: Do AdS Massive gravity in 5 dimensions, with our universe localized on a 3+1 brane

Einstein Hilbert + mass term on the brane

$$\mathcal{L}_{\rm brane} = \frac{1}{2} M_4^2 R_4 - \frac{\alpha}{2} m^2 M_4^2 (k_{\mu\nu}^2 - k^2)$$

5D Massive Gravity on AdS in the Bulk



$$f_{ab}(\phi) = \frac{L^2}{\phi_5^2} \eta_{ab}$$

$$\mathcal{L}_{\rm bulk} = \frac{1}{2} M_5^3 R_5 - \frac{1}{2} m^2 M_{\rm Planck}^3 (K_{\mu\nu}^2 - K^2)$$

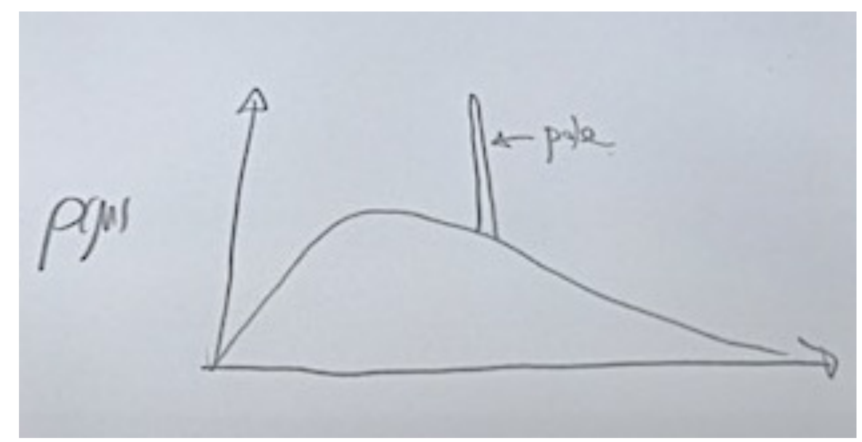
Soft and Hard (nonlocal) massive gravity

Cutoff is raise to

Gabadadze 2017

$$\Lambda_2 = (m^2 M_4^2)^{1/4} \sim \frac{1}{L} \sim (M_5^3 m^2)^{1/5} \gg \Lambda_3$$

This is achieved because of a continuum/resonance of soft gravitons whose masses are smaller than usual hard mass graviton



Result: Low energy effective theory is more non-local (although full theory is completely local)

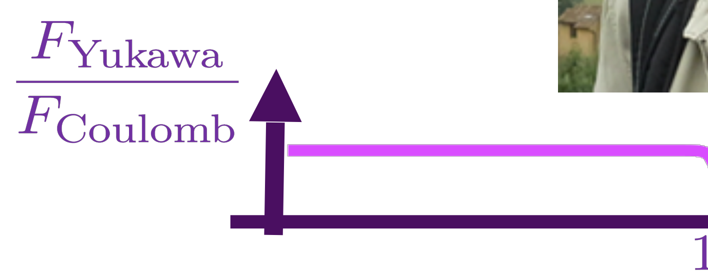
Observational Aspects of Massive Gravity

Constraints on the Graviton Mass

de Rham, Deskins, AJT, Zhou, Reviews of Modern Physics

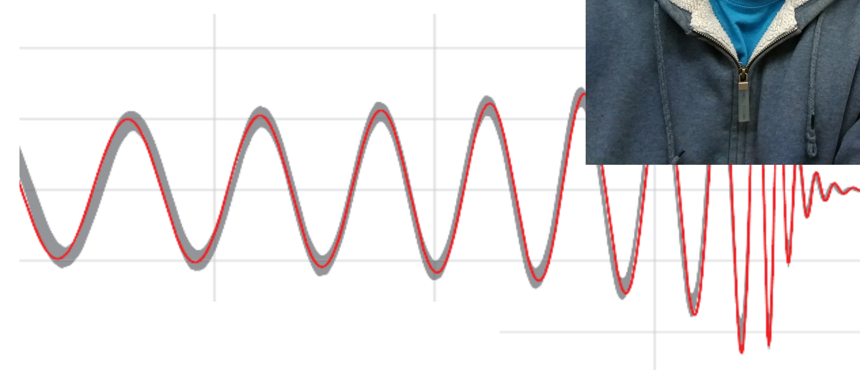
Yukawa

m_g (eV)	λ_g (km)	
10^{-23}	10^{12}	Solar System tests
10^{-32}	10^{21}	Weak lensing
10^{-29}	10^{19}	Bound clusters



Dispersion Relation

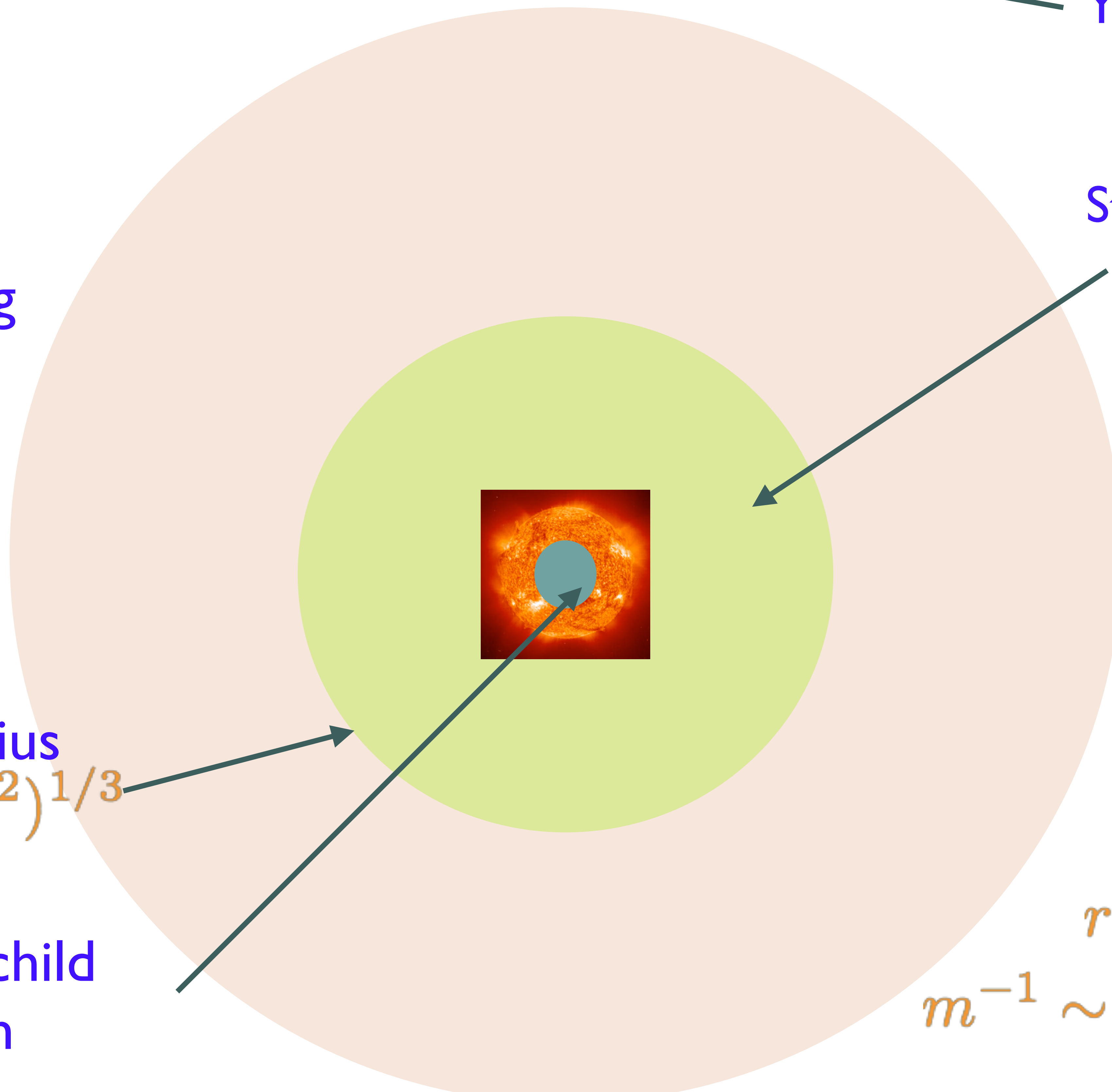
m_g (eV)	λ_g (km)	
10^{-22}	10^{11}	aLIGO bound
10^{-20}	10^{9}	Pulsar timing
10^{-30}	10^{20}	B–mode's in CMB



Fifth Force

m_g (eV)	λ_g (km)	
10^{-32}	10^{22}	Lunar Laser Ranging
10^{-27}	10^{17}	Binary pulsar
10^{-32}	10^{22}	Structure formation

Vainshtein effect is strongly scale and density dependent



Yukawa region

$r > m^{-1}$

Strong coupling region

$r \ll r_V$

$Z \gg 1$

Weak coupling region

$r \gg r_V$

$Z \sim 1$

Vainshtein radius

$r_V = (r_s m^{-2})^{1/3}$

Schwarzschild region

$r < r_s$

For Sun

$r_V \sim 250pc$

$m^{-1} \sim 4000Mpc$

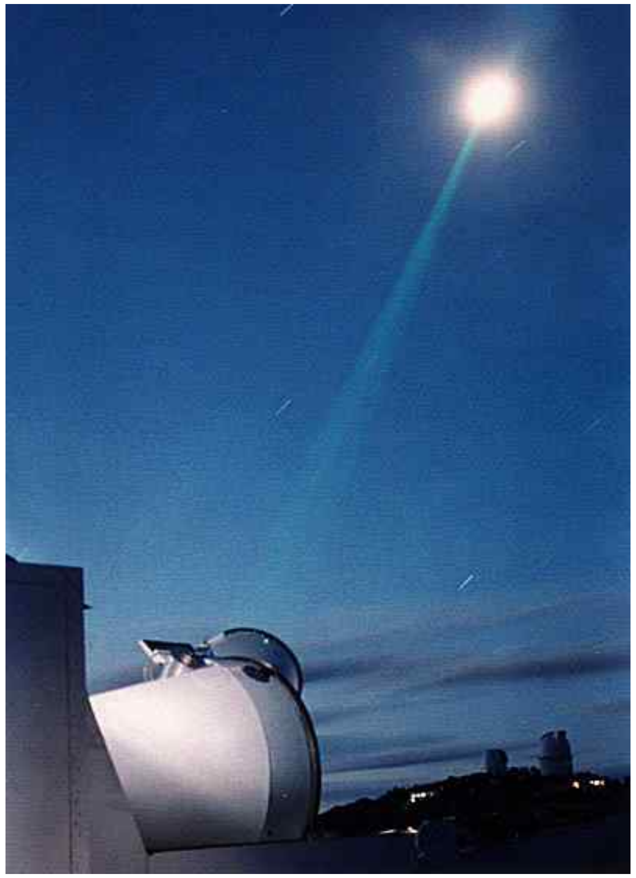
$r_s \sim 3km$

Fifth Force

m_g (eV)	λ_g (km)	
10^{-32}	10^{22}	Lunar Laser Ranging
10^{-27}	10^{17}	Binary pulsar
10^{-32}	10^{22}	Structure formation

Fifth Force Bounds: Lunar Laser ranging

Traditionally Strongest Constraint on Mass of Graviton

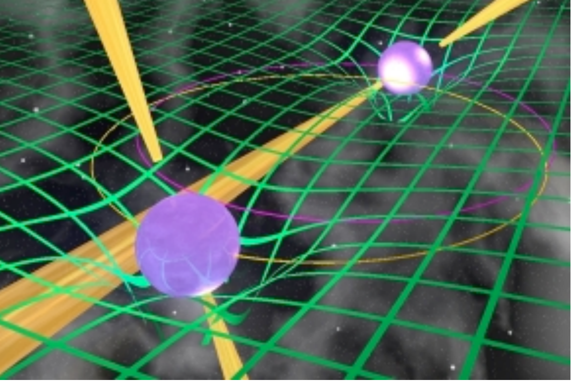


For DGP, (cubic Galileon)

$$m_g < \delta\phi \left(\frac{r_{S,\oplus}}{a^3}\right)^{1/2} \qquad m_g \lesssim 10^{-32}\text{eV}$$

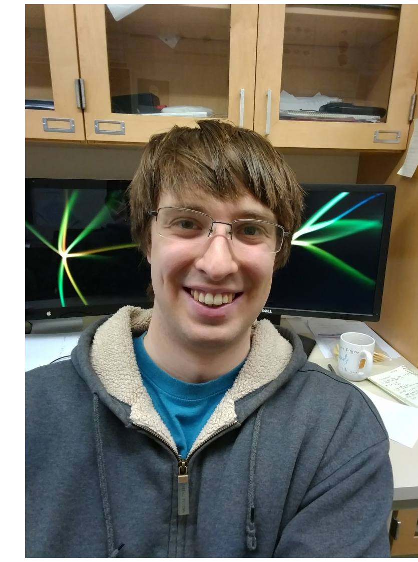
For hard mass graviton, (~ quartic Galileon)

$$m_g < \delta\phi^{3/4} \left(\frac{r_{S,\oplus}}{a^3}\right)^{1/2} \qquad m_g \lesssim 10^{-30}\text{eV}$$



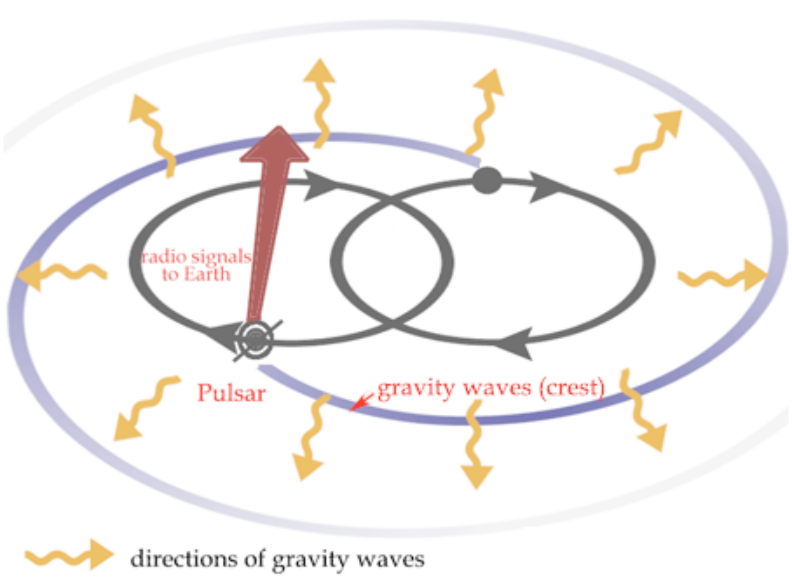
Binary Pulsars

de Rham, AJT, Wesley 2012
de Rham, Matas, AJT 2013
Dar, de Rham, Deskins, Giblin, AJT 2018



Extra polarizations of graviton = extra modes of gravitational wave

Binary pulsars lose energy faster than in GR so the orbit slows down more rapidly



	A	B	C	D	E
Pulsar	1913+16 Taylor-Hulse	B2127+11	B1534+12	J0737–3039 double pulsar	J1738+0333
$M_1/M_\odot$	1.386	1.358	1.345	1.338	1.46
$M_2/M_\odot$	1.442	1.354	1.333	1.249	0.181
T_P/days	0.323	0.335	0.420	0.102	0.355
e	0.617	0.681	0.274	0.088	3.4×10^{-7}
$\frac{dT_P}{dt}\vert_\pi$ Monopole	9.8×10^{-22}	1.4×10^{-21}	1.1×10^{-22}	5.1×10^{-23}	8.1×10^{-24}
$\frac{dT_P}{dt}\vert_\pi$ Dipole	10^{-30}	10^{-32}	10^{-33}	10^{-32}	10^{-31}
$\frac{dT_P}{dt}\vert_\pi$ Quadrupole	9.1×10^{-21}	1.0×10^{-20}	6.1×10^{-21}	4.3×10^{-21}	1.1×10^{-21}
$\frac{dT_P}{dt}\vert_{GR}$	1.1×10^{-12}	1.7×10^{-12}	8.5×10^{-14}	5.6×10^{-13}	10^{-14}
σ	5.1×10^{-15}	1.3×10^{-13}	2.0×10^{-15}	1.7×10^{-14}	10^{-15}
Ref.	[29, 30]	[31]	[32, 33]	[34]	[35]

Table 1. The predicted contribution to the orbital period derivative $\dot{T}_P$ from π alone in the monopole, dipole and quadrupole channels (taking $m = 1.54 \times 10^{-33}$eV) for four known DNS pulsars (A to D) and one pulsar-white dwarf binary (E) with the GR result. The experimental uncertainty σ is given using [36].

Perturbation Theory

Cubic Galileon Action

$$S = \int \mathrm{d}^4 x \left(-\frac{3}{4}(\partial\pi)^2 \left(1 + \frac{1}{3\Lambda^3}(\Box\pi)\right) + \frac{1}{2M_{\mathrm{Pl}}}\pi T \right)$$

Orbiting Point Source

$$T^\mu_\nu = -\left[\sum_{i=1,2} M_i \delta^3(\vec{x} - \vec{x}_i(t))\right] \delta^\mu_0 \delta^0_\nu$$

Spherically Symmetric Background

$$\partial_r \bar{\pi} = \frac{\Lambda^3}{4r}\left[\sqrt{9r^4 + \frac{32 r_*^3 r}{\pi}} - 3r^2\right] \qquad r_* = \frac{1}{\Lambda}\left(\frac{M}{16 M_{\mathrm{Pl}}}\right)$$

Scalar Gravitational Waves: Power Radiated

$$P = \frac{\pi}{3M_{\rm Pl}^2} \sum_{n=0}^{\infty} \sum_{lm} \frac{n}{T_P} |\mathcal{M}_l mn|^2 \qquad \mathcal{M}_{lmn} = \frac{1}{T_P} \int_0^{T_p} dt \int d^3x u_{ln}(r) Y_{lm}(\theta,\phi) e^{-int/T_p} \delta T(x,t)$$

Dominated by Quadrupole Radiation:

$$P_{\rm quadrupole} = 2^{7/2} \frac{5\lambda_1^2}{32} \frac{(\Omega_P \bar{r})^3}{(\Omega_P r_*)^{3/2}} \frac{M_Q^2}{M_{\rm pl}^2} \Omega_P^2$$

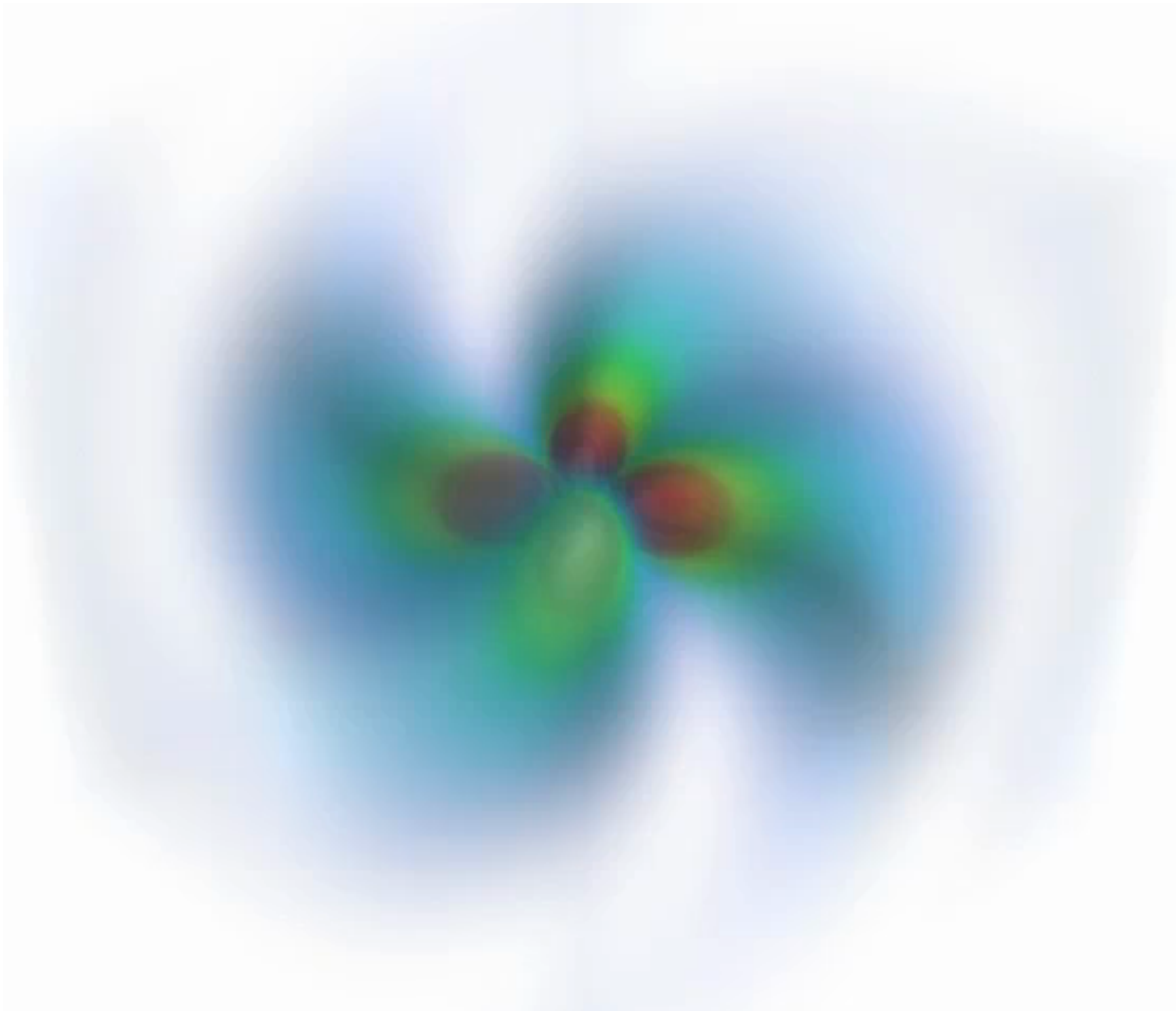
relative to GR result:

$$\frac{P_{\rm quadrupole}^{\rm Galileon}}{P_{\rm quadrupole}^{\rm GR}} = q(\Omega_P r_*)^{-3/2} (\Omega_P \bar{r})^{-1}$$

For realistic binary pulsars suppressed by 10^{-9}-10^{-7}

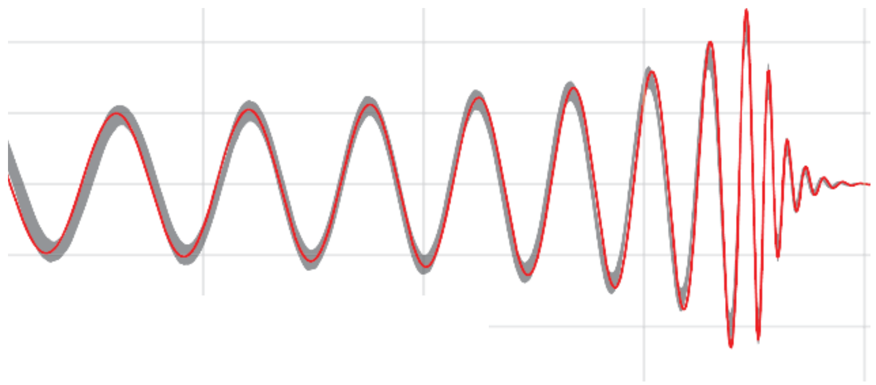
Static Suppression $\propto (\Omega_P r_*)^{-5/2}$

Movie



m_g (eV)	λ_g (km)	Dispersion Relation
10^{-22}	10^{11}	aLIGO bound
10^{-20}	10^{9}	Pulsar timing
10^{-30}	10^{20}	B–mode's in CMB

Direct Detection of GW

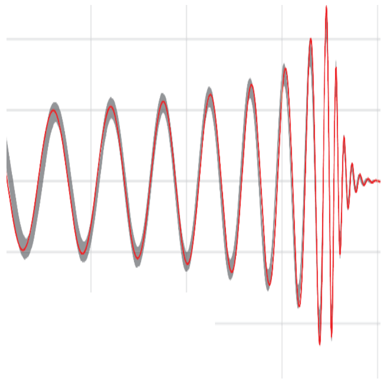


Constraints modifications of the dispersion relation

$$E^2 = \mathbf{k}^2 + m_g^2$$

Generic for the helicity-2 modes of any Lorentz invariant model of massive gravity

GW signal would be more squeezed than in GR



Speed increases with frequency

$$v_g/c \approx 1 - \frac{1}{2}(c/\Lambda_g f)^2$$

$$1 - \frac{v_g}{c} = 5 \times 10^{-17} \left(\frac{200\text{Mpc}}{D}\right) \left(\frac{\Delta t}{1\text{s}}\right)$$

$$m_g \lesssim 4 \times 10^{-22}\text{eV} \left(f\Delta t \frac{f}{100\text{Hz}} \frac{200\text{Mpc}}{D}\right)^{1/2}$$

For GW150914,

$D \sim 400\text{Mpc}, \ f \sim 100\text{Hz}, \ \rho \sim 23 \quad \Rightarrow \quad m_g \lesssim 10^{-22}\text{eV}$

Will 1998
Abbott et al., 2016

Do we know all the constraints on graviton mass from aLIGO??

No! Many other effects to consider

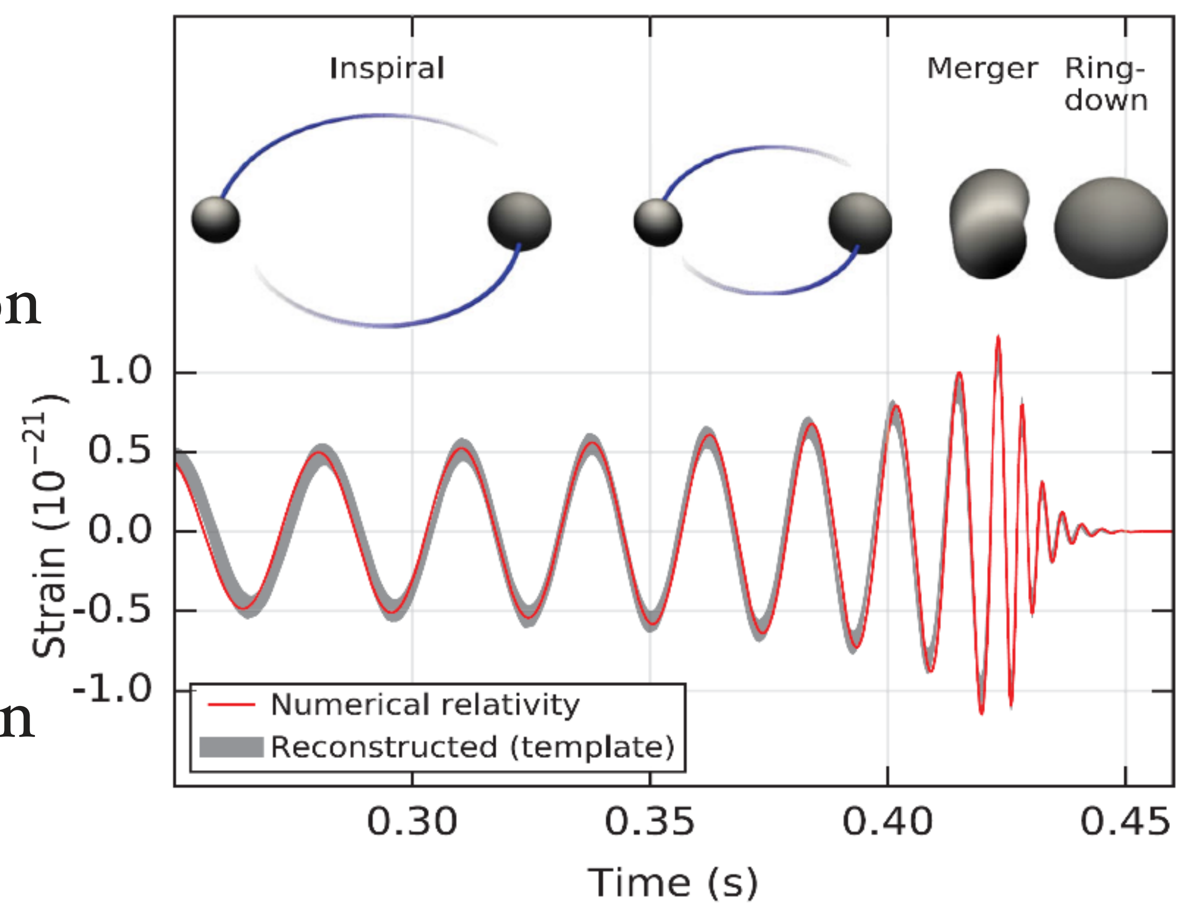
regime and, bound, for the first time several high-order post-Newtonian coefficients. We constrain the graviton Compton wavelength in a hypothetical theory of gravity in which the graviton is massive and place a 90%-confidence lower bound of 10^{13} km. Within our statistical uncertainties, we find no evidence for violations of

LIGO & VIRGO, PRL116, 221101 (2016)

$$m_{\rm graviton} < 10^{-22}{\rm eV}$$

GW150914

- Graviton Mass *depends on environment,* for instance it *depends on distance to black holes*
- Graviton Mass likely to vary non-adiabatically during merger creating additional non-adiabatic effects in the waveform
- Additional scalar (and vector) gravitational radiation. Scalar radiation may dominate effects on tensors.
- Black hole/NS solution modified, in particular quasi-normal modes may be different
- Vainshtein suppression may not be active in merger region - needs proper numerical simulation
- PN expansion almost certainly doesn't work in Vainshtein region



AJT Conjecture: Likely real constraints on LI MG are stronger!

What about Black hole solution, is horizon modified?

Many attempts to construct Black Hole solutions of massive (bi) gravity have focused on special symmetric solutions many in non-standard branches.

Babichev, Brito, Volkov, Comelli, Pilo… many more

There should be a solution with

Yukawa asymptotics!

= Schwarzschild as $m \to 0$

Nonsingular Black Holes in Massive Gravity: Time-Dependent Solutions

Rachel A. Rosen

coordinate-invariant singularities at the horizon. In this work we investigate the possibility of black hole solutions which can accommodate both a nonsingular horizon and Yukawa asymptotics. In particular, by adopting a time-dependent ansatz, we derive perturbative analytic solutions which possess nonsingular horizons. These black hole solutions are indistinguishable from Schwarzschild black holes in the massless limit. At finite mass, they depend explicitly on time. However, we demonstrate that the location of the apparent horizon is not necessarily time-dependent, indicating that these black holes are not necessarily accreting or evaporating (classically). In deriving these

Black Hole Mechanics for Massive Gravitons

Rachel A. Rosen[1]

[1] *Department of Physics, Columbia University, New York, NY 10027, USA*

It has been argued that black hole solutions become unavoidably time-dependent when the graviton has a mass. In this work we show that, if the apparent horizon of the black hole is a null surface with respect to a fiducial Minkowski reference metric, then the location of the horizon is necessarily time-*independent*, despite the dynamical metric possessing no time-like Killing vector. This result is non-perturbative and model-independent. We derive a second law of black hole mechanics for these black holes and determine their surface gravity. An additional assumption establishes a zeroth and a first law of black hole mechanics. We apply these results to the specific model of dRGT ghost-free massive gravity and show that consistent solutions exist which obey the required assumptions. We determine the time-dependent scalar curvature at the horizon of these black holes.

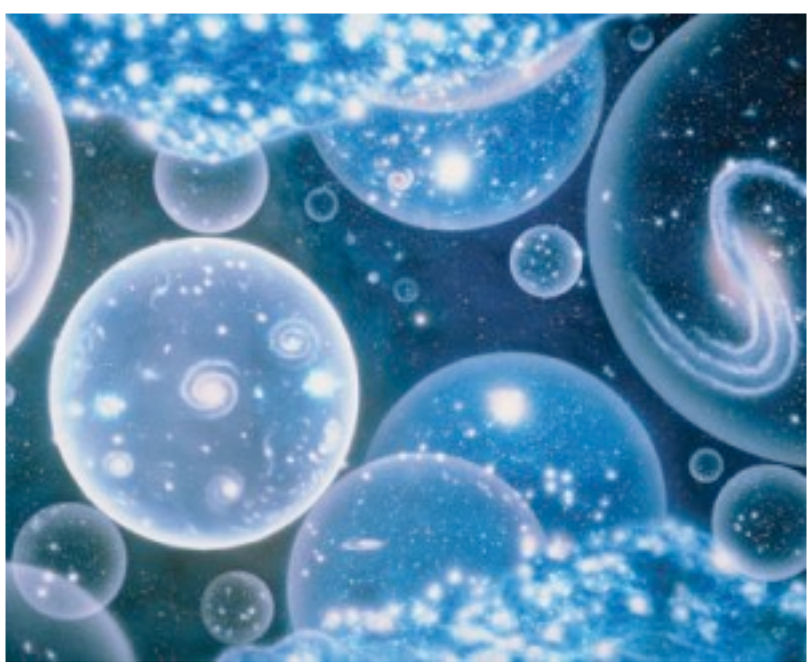
Cosmological Solutions

D'Amico et al. 2011

Perfect Homogeneous and Isotropic solutions (FRW) are forbidden in the simplest form of Massive Gravity

Possible to find inhomogeneous models that are locally indistinguishable from FRW over scales set by the graviton mass

COMPTON WAVELENGTH of GRAVITON = COHERENCE LENGTH



$$d \leq m^{-1}$$

In each bubble the Vainshtein mechanism ensures the cosmology is close to Einstein GR

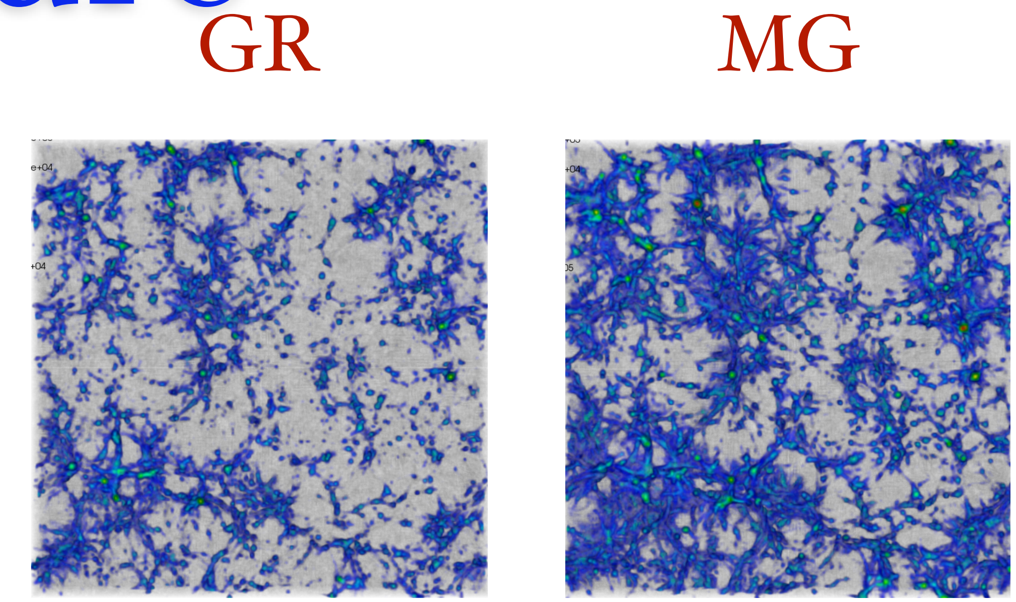
Cosmological Solutions

- Previously described *Generalized Massive gravity* **does** admit FRW solutions
- *Bi-gravity* and multi-gravity do admit FRW solutions
- *Quasi-dilaton* and other extensions where mass term depends on a field admit FRW solutions
- In general allowing the **mass to depend on other fields** (be they scalars or additional metrics) is the solution to this problem!

Growth of Structure

These theories have a modified growth of structure which is highly nonlinear

$$\rho = \bar{\rho} + \delta\rho$$



Khoury & Wyman, 2009

In early universe when $\bar{\rho} \gg \Lambda^3 M_P$

Vainshtein mechanism at work, fifth force is screened - GR recovered e.g. inflation is essentially unchanged

In late universe when $\bar{\rho} \ll \Lambda^3 M_P$

Vainshtein mechanism switches off and linearized fluctuations know about fifth force

As structure grows

Vainshtein mechanism turns on in high density (potential) regions and not in low density

Existential Crisis of MG: Does a UV completion exist?

Can I describe theories of massive gravity/multi-gravity at energy scales higher than Λ_3 ?

Is there a UV completion?

Is there a Lorentz Invariant Higgs mechanism for gravity?

If not, what do we give up? Lorentz invariance? Locality?

Are all EFTs allowed?

aka Swampland!

With typical assumption that:
UV completion is Local, Causal, Poincare Invariant and Unitary

$$[\hat{O}(x), \hat{O}(y)] = 0 \quad \text{if } (x-y)^2 > 0$$

Answer: NO! Certain low energy effective theories do not admit well defined UV completions

Recent Recognition: **Positivity Bounds!**

Asymptotic (Sub)Luminality):

Positive Wigner-Eisenbud time delay $T \sim \dfrac{d\delta(E)}{dE} > 0$

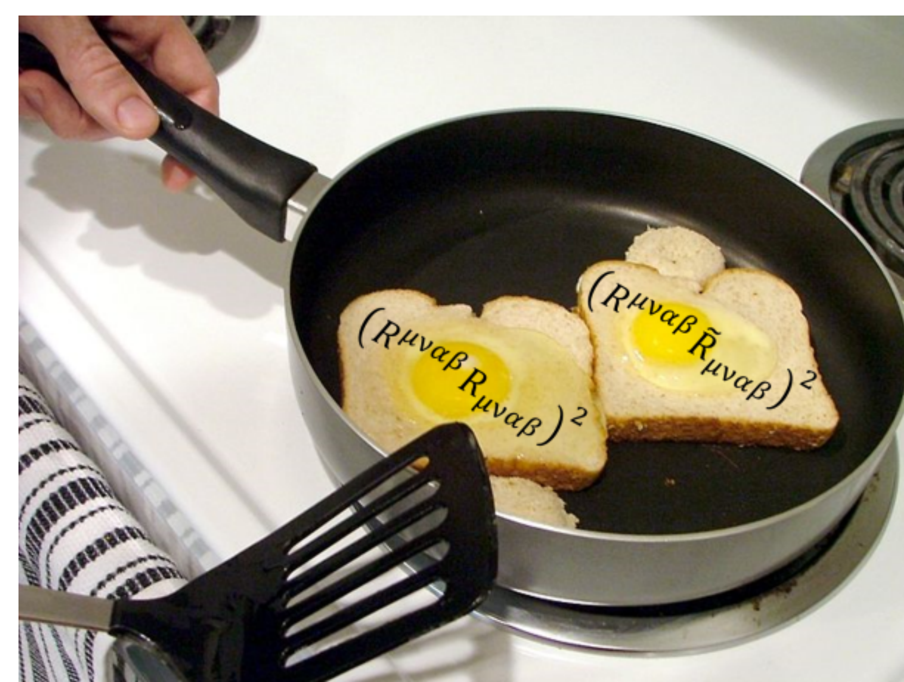
Don’t Panic - Think Positive!

Scott Melville

Shuang-Yong Zhou

Claudia de Rham

Positivity Bounds!



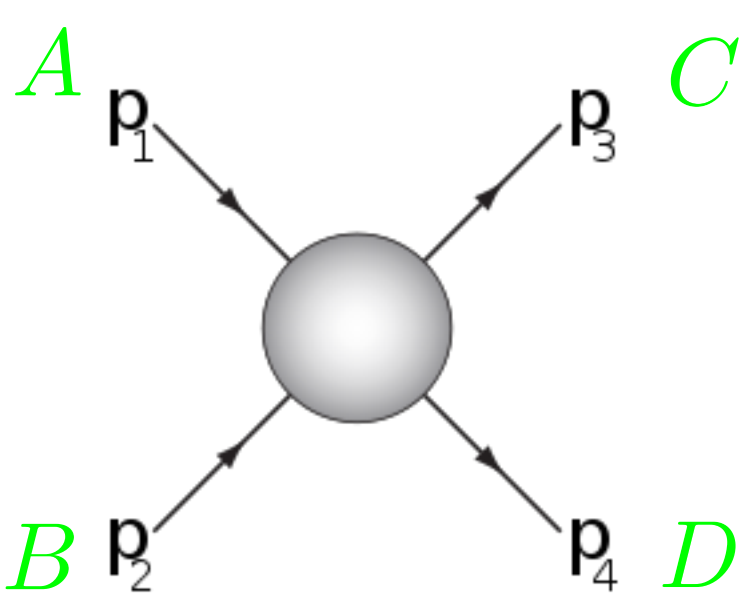
Recently featured in CQG+ ...
https://cqgplus.com/2018/05/30/low-energy-think-positive/

1960's S-matrix assumptions

1. Unitarity $S^\dagger S = 1$ $|A(k)| < \alpha e^{\beta|k|}$
2. Locality: Scattering Amplitude Polynomially (Exponentially) Bounded
3. Causality: Analytic Function of Mandelstam variables (modulo poles+cuts)
4. Poincare Invariance
5. Crossing Symmetry: Follows from above assumptions
6. Mass Gap: Existence of Mandelstam Triangle and Validity of Froissart Bound

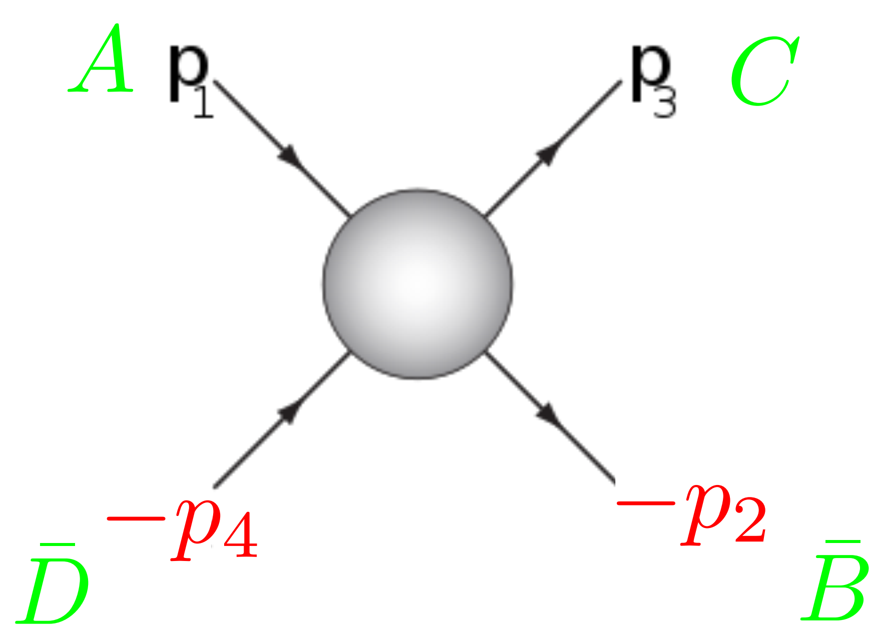
s-channel

$$A + B \to C + D$$



u-channel

$$A + \bar{D} \to C + \bar{B}$$



$$s + t + u = 4m^2$$

$$s = (p_1 + p_2)^2$$

$$t = (p_1 - p_3)^2$$

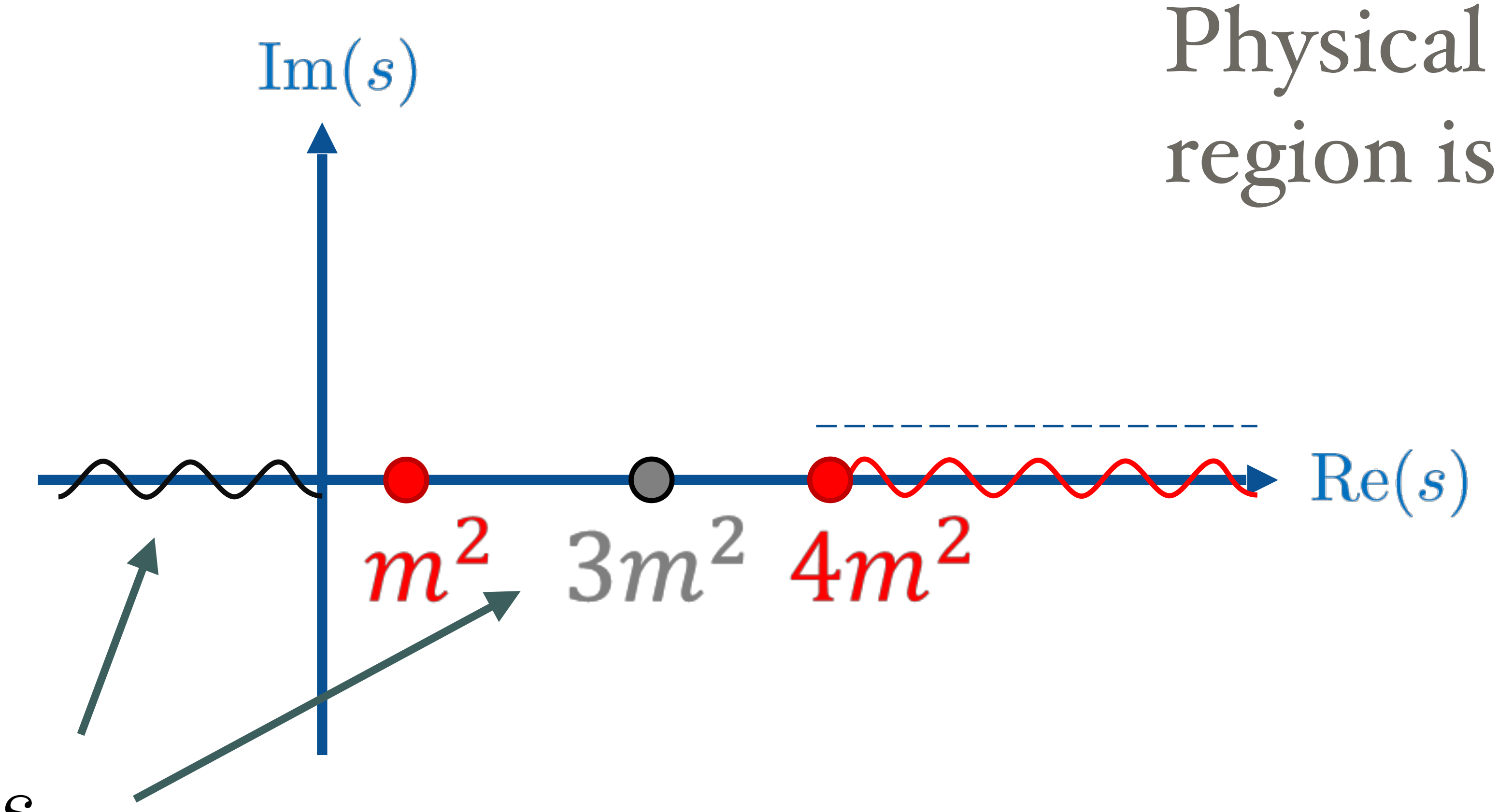
$$u = (p_1 - p_4)^2$$

$$s \leftrightarrow u$$

Forward Scattering Limit Dispersion relation

Complex s plane

$t = 0$



Physical scattering region is $s \geq 4m^2$

crossing:

$$u = 4m^2 - s$$

$$\mathcal{A}_s(s,0) = \frac{\lambda_s}{m^2 - s} + \frac{\lambda_u}{m^2 - u} + (a + bs) + s^2 \int_{4m^2}^{\infty} \frac{\rho_s(\mu)}{\mu^2(\mu - s)} + u^2 \int_{4m^2}^{\infty} \frac{\rho_u(\mu)}{\mu^2(\mu - u)}$$

Positivity/Unitarity

$$\rho(s) = \frac{1}{\pi} Im[A(s,0)] = \frac{\sqrt{s(s-4m^2)}\sigma(s)}{\pi} > 0$$

No. of subtractions =2

$$\sigma(s) < \frac{c}{m^2}(\log(s/s_0))^2$$

Forward Limit Positivity Bounds

Recipe: Subtract pole, differentiate to remove subtraction constants

$$\mathcal{A}'_s(s,t) = A_s(s,t) - \frac{\lambda_s}{m^2 - s} - \frac{\lambda_u}{m^2 - u}$$

$$\frac{1}{M!}\frac{d^M}{ds^M}\mathcal{A}'_s(2m^2,0) = \int_{4m^2}^{\infty} \frac{\rho_s(\mu)}{(\mu - 2m^2)^{M+1}} + \int_{4m^2}^{\infty} \frac{\rho_u(\mu)}{(\mu - 2m^2)^{M+1}} > 0$$

RH Cut   LH Cut $M \geq 2$

Adams et. al. 2006

Assume Weak Coupling

$$\frac{1}{M!}\frac{d^M}{ds^M}\mathcal{A}'^{\text{tree}}_s(2m^2,0) = \int_{\Lambda^2}^{\infty} \frac{\rho_s^{\text{tree}}(\mu)}{(\mu - 2m^2)^{M+1}} + \int_{\Lambda^2}^{\infty} \frac{\rho_u^{\text{tree}}(\mu)}{(\mu - 2m^2)^{M+1}} > 0$$

Directly translates into constraints on Wilsonian action

Extension away from forward scattering limit

de Rham, Melville, AJT, Zhou 1702.06134

$$\mathcal{A}(s,t) = 16\pi\sqrt{\frac{s}{s-4m^2}}\sum_{\ell=0}^{\infty}(2\ell+1)P_\ell(\cos\theta)a_\ell(s)$$

Unitarity

$$\mathrm{Im}\, a_\ell(s) = |a_\ell(s)|^2 + \cdots$$

$$\mathrm{Im}\, a_l(s) > 0\,, \quad s \geq 4m^2$$

$$\frac{d^n}{dt^n}\mathrm{Im}A(s,t)\Big|_{t=0} > 0$$

using $\frac{d^n}{dx^n}P_l(x)\Big|_{x=1} > 0$

$$\mathrm{Im}\, A(s,t) > 0\,, \quad 0 \leq t < 4m^2\,,\ s \geq 4m^2$$

$$M \geq 2$$

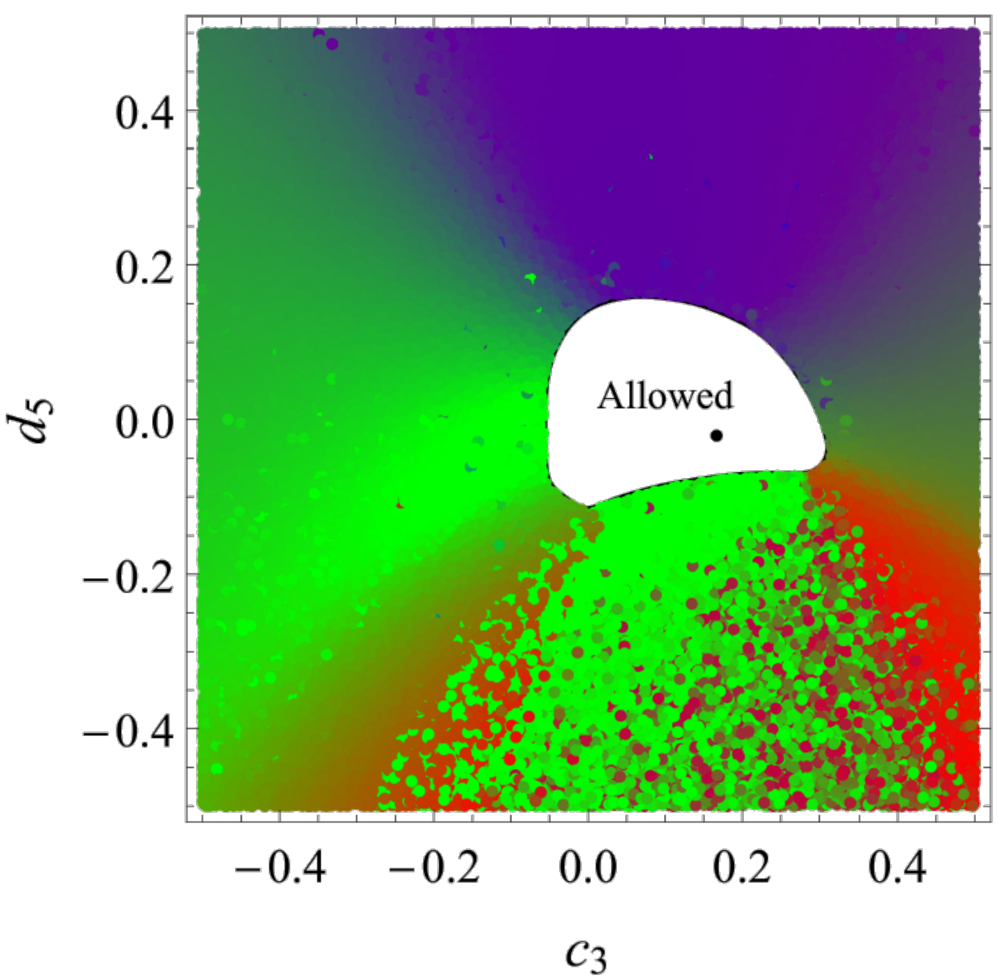
$$\frac{1}{M!}\frac{d^M}{ds^M}\mathcal{A}'_s(2m^2,t) = \frac{1}{\pi}\int_{4m^2}^{\infty}\frac{ImA_s(\mu,t)}{(\mu-2m^2+t/2)^{M+1}} + \frac{1}{\pi}\int_{4m^2}^{\infty}\frac{ImA_u(\mu,t)}{(\mu-2m^2+t/2)^{M+1}} > 0$$

What about general spins, e.g. spin 2 = massive gravity?

In forward limit, dispersion relation holds for helicity amplitudes

$A_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,0)$ has dispersion relation with 2 subtractions

Helicity: $\frac{\mathbf{J}\cdot\mathbf{p}}{|\mathbf{p}|}|\mathbf{p},S,\lambda\rangle = \lambda|\mathbf{p},S,\lambda\rangle$



Also applies to INDEFINITE helicity

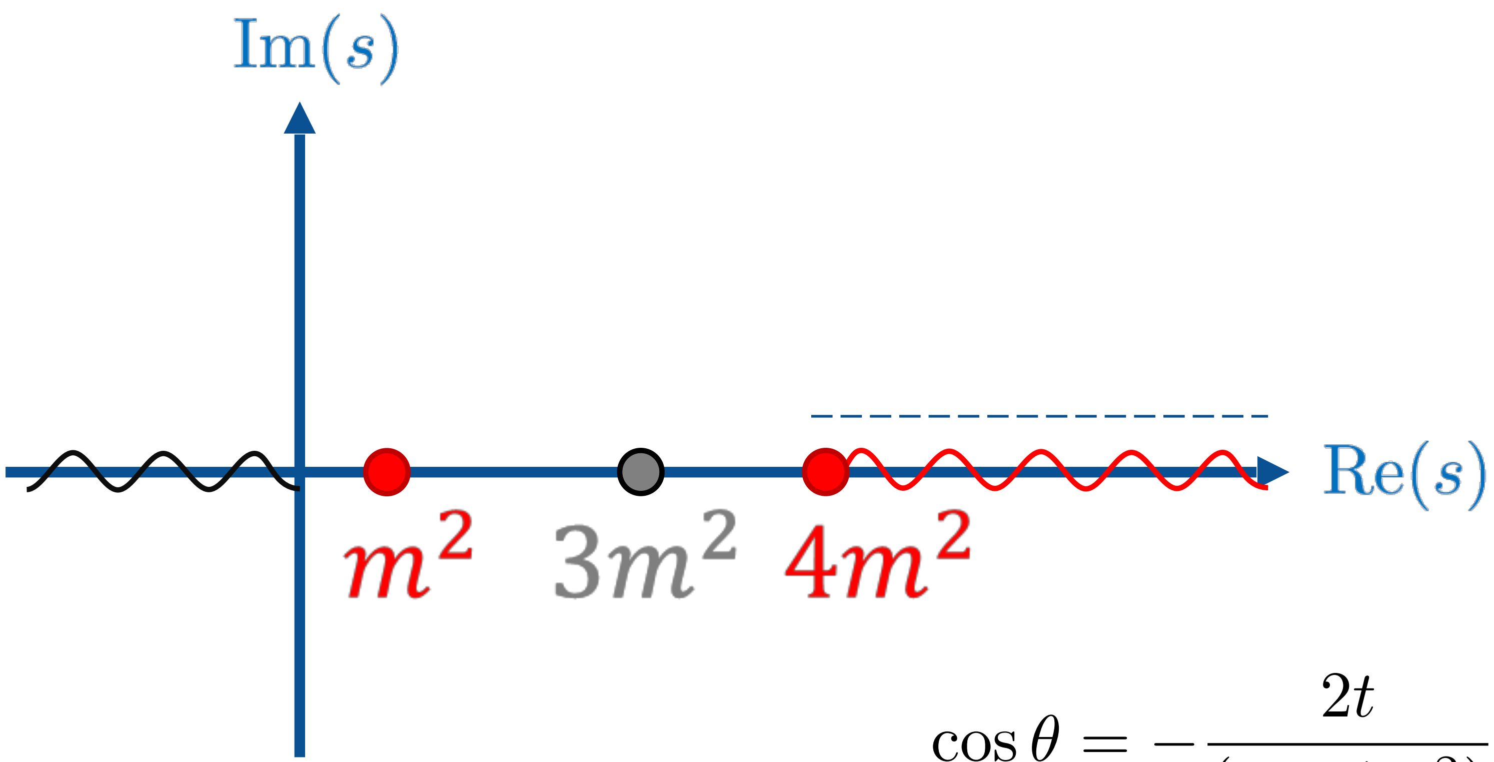
This has been used to place constraints on the mass parameters in massive gravity

Cheung & Remmen (2016)

in the forward scattering limit

Analyticity for Spins

In addition to usual scalar poles and branch cuts we have ....



$$\cos\theta = -\frac{2t}{(s-4m^2)}$$

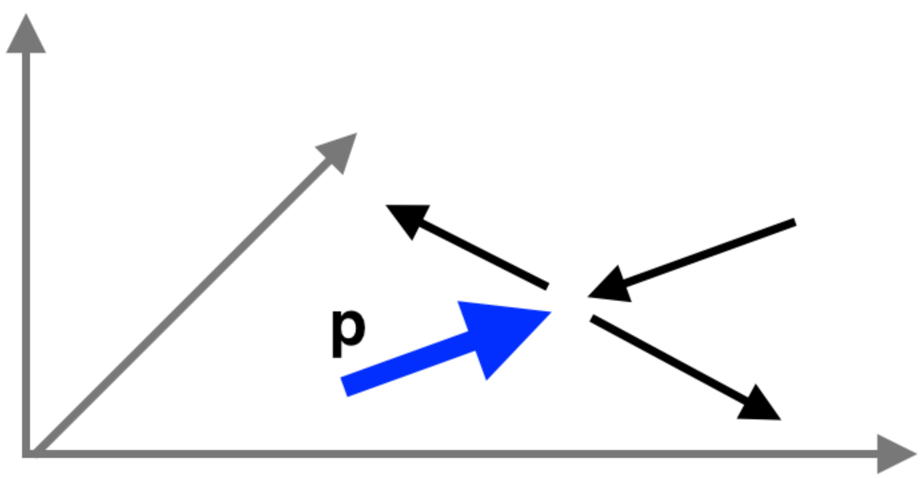
1. Kinematic (unphysical) poles at $s = 4m^2$
2. $\sqrt{stu}$ branch cuts
3. For Boson-Fermion scattering $\sqrt{-su}$ branch cuts

Origin: non-analyticities of polarization vectors/spinors

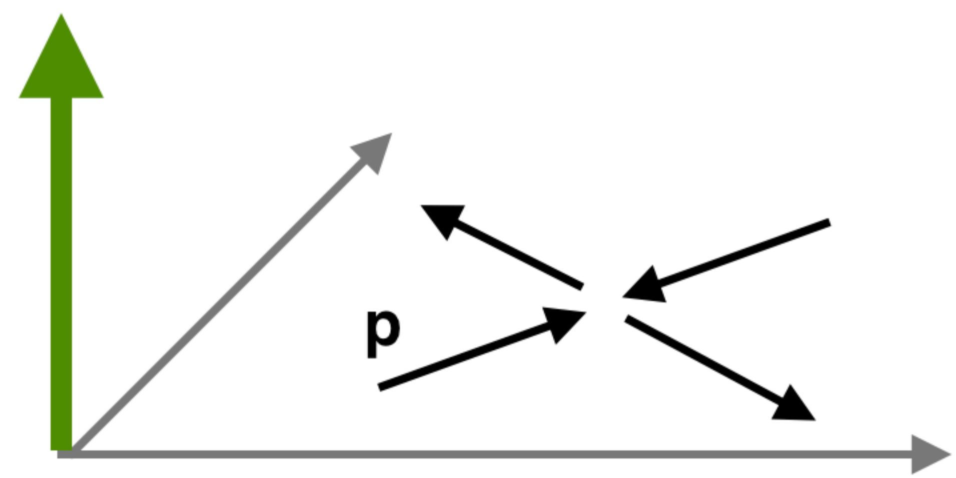
Transversitas, Transversitatum, et omnia Transversitas

Kotanski, 1965

Helicity



Transversity



$$\mathcal{T}_{\tau_1\tau_2\tau_3\tau_4} = \sum_{\lambda_1\lambda_2\lambda_3\lambda_4} u^{S_1}_{\lambda_1\tau_1} u^{S_2}_{\lambda_2\tau_2} u^{S_1 *}_{\tau_3\lambda_3} u^{S_2 *}_{\tau_4\lambda_4} \mathcal{H}_{\lambda_1\lambda_2\lambda_3\lambda_4}$$

Change of Basis $u^S_{\lambda\tau} = \langle S, \lambda | e^{-i\frac{\pi}{2}\hat{J}_z} e^{-i\frac{\pi}{2}\hat{J}_y} e^{i\frac{\pi}{2}\hat{J}_z} | S, \tau \rangle$

$$T^s_{\tau_1\tau_2\tau_3\tau_4}(s,t,u) = e^{-i\sum_i \tau_i \chi} T^u_{-\tau_1-\tau_4-\tau_3-\tau_2}(u,t,s)$$

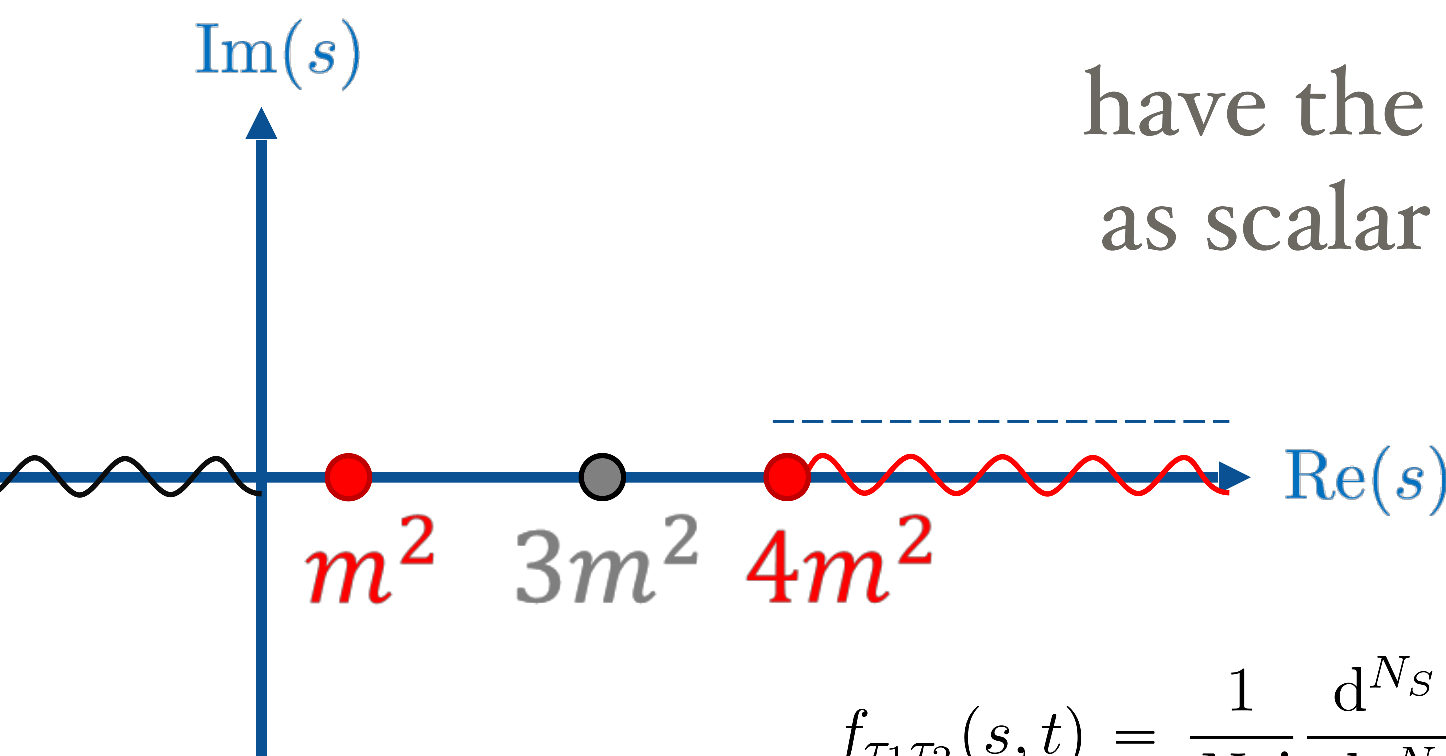
Crossing is Simple!!

Dispersion Relation with Positivity along <u>BOTH</u> cuts

de Rham, Melville, AJT, Zhou 1706.02712

Punch line: The specific combinations:

$$\mathcal{T}^{+}_{\tau_1\tau_2\tau_3\tau_4}(s,\theta) = \left(\sqrt{-su}\right)^{\xi}\mathcal{S}^{S_1+S_2}\left(\mathcal{T}_{\tau_1\tau_2\tau_3\tau_4}(s,\theta) + \mathcal{T}_{\tau_1\tau_2\tau_3\tau_4}(s,-\theta)\right)$$



have the same analyticity structure as scalar scattering amplitudes!!!!!!!

Implies Dispersion Relation

$$f_{\tau_1\tau_2}(s,t) = \frac{1}{N_S!}\frac{\mathrm{d}^{N_S}}{\mathrm{d}s^{N_S}}\tilde{\mathcal{T}}^{+}_{\tau_1\tau_2\tau_1\tau_2}(s,t)$$

$$f_{\tau_1\tau_2}(v,t) = \frac{1}{\pi}\int_{4m^2}^{\infty}\mathrm{d}\mu\frac{\mathrm{Abs}_s\mathcal{T}^{+}_{\tau_1\tau_2\tau_1\tau_2}(\mu,t)}{(\mu-2m^2+t/2-v)^{N_S+1}} + \frac{1}{\pi}\int_{4m^2}^{\infty}\mathrm{d}\mu\frac{\mathrm{Abs}_u\mathcal{T}^{+}_{\tau_1\tau_2\tau_1\tau_2}(4m^2-t-\mu,t)}{(\mu-2m^2+t/2+v)^{N_S+1}}$$

Application to Massive Gravity

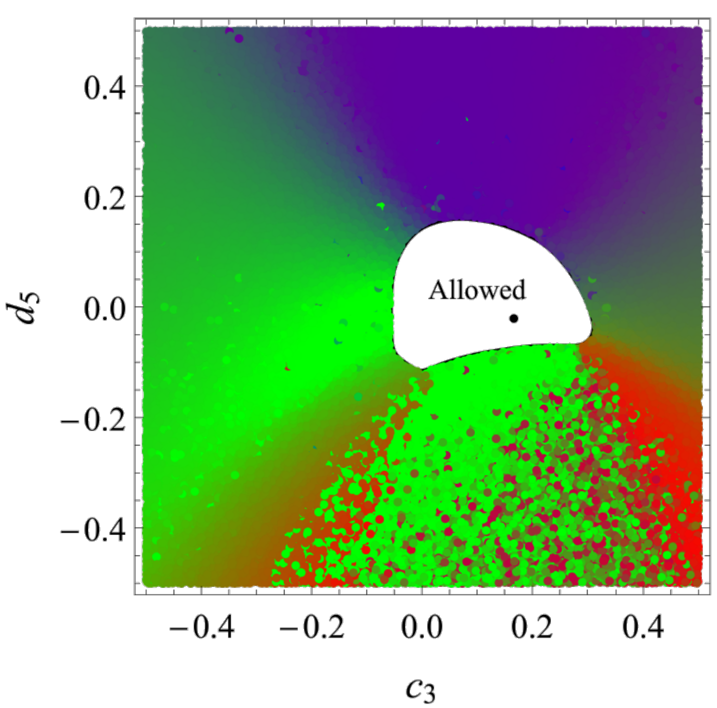
Unitary Gauge Massive Gravity

Einstein-Hilbert   Mass Term

$$\mathcal{L} \supset \frac{M_{\rm Pl}^2}{2}\left(R\,[g] - \frac{m^2}{4}V(g,h)\right)$$

Parameterize generic mass term (without dRGT tuning) as

$$V(g,h) \supset [h^2] - [h]^2 + (c_1-2)[h^3] + (c_2+\frac{5}{2})[h^2][h] + (d_1+3-3c_1)[h^4] + (d_3-\frac{5}{4}-c_2)[h^2]^2 + \ldots\,.$$



where $[h] = \eta^{\mu\nu}h_{\mu\nu},\ [h^2] = \eta^{\mu\nu}h_{\mu\alpha}\eta^{\alpha\beta}h_{\beta\nu};$

$$d_3 = -d_1/2 + 3/32 + \Delta d, \quad c_2 = -3c_1/2 + 1/4 + \Delta c$$

Application to Massive Gravity

Forward Limit

$$2M_{\rm Pl}^2 m^6 \frac{\partial^2}{\partial v^2} f_{\alpha\beta}|_{t=0} = \frac{352}{9}|\alpha_S \beta_S|^2 \left(\Delta c\left(-6+9c_1-4\Delta c\right)-6\Delta d\right) + \frac{176}{3}\alpha_S^* \beta_S^* (\alpha_{V_1}\beta_{V_1}-\alpha_{V_2}\beta_{V_2})\,\Delta c\left(3-3c_1+4\Delta c\right)$$

Positivity for general helicity implies: $\Delta c = 0$

Beyond forward

$$\frac{\partial}{\partial t} f_{\tau_1 \tau_2}(v,t) \;\propto\; \frac{v}{\Lambda_5^{10}}\Delta d + \mathcal{O}\left(\frac{m^2}{\Lambda_5^{10}}\right) \;>\; 0$$

$$\Delta d = 0$$

These are precisely the tunings that raise the cutoff from

$$\Lambda_5 = (m^4 M_{\rm Planck})^{1/5} \longrightarrow \Lambda_3 = (m^2 M_{\rm Planck})^{1/3}$$

Summary

We now know how to write down theories of multiple interacting massive spin 2 states and have examples of both **hard** and **soft** massive gravity theories

Galileons arise UNIVERSALLY in the decoupling limit

Phenomenology Dominated by **Vainshtein Mechanism**

Simulations of Binary Pulsars confirm that additional scalar radiation is suppressed

Biggest outstanding question is **UV completion**. Known examples appear to work on Anti-de Sitter spacetime

Positivity Bounds provide the most strongest constraints so far on the UV completion of the EFT - ongoing work with Prof. Zhou -

WATCH THIS SPACE!