中国科学技术大学
University of Science and Technology of China

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Escaping From Big Bang Singularity

Yi-Fu Cai

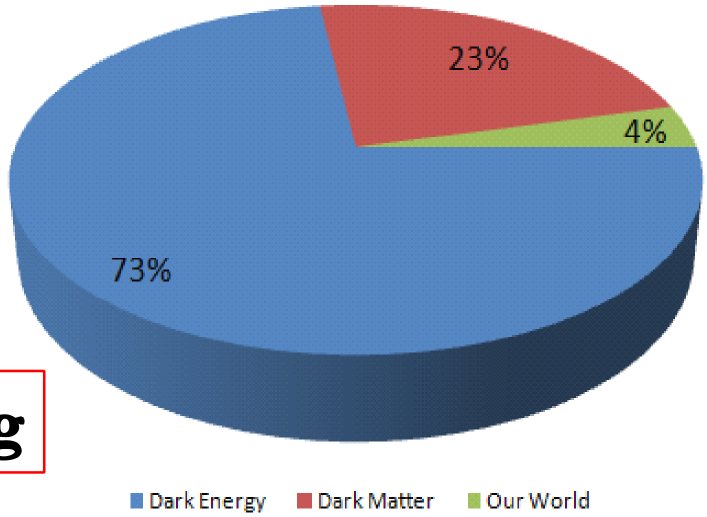
Cosmology Initiative @ ASU

April 26, 2012 in Hefei

Outline

- A brief review of standard model of modern cosmology
  - Big Bang + Inflation
- Model building of bounce cosmologies from Effective Field Theories
  - Lee-Wick Bounce
  - G(Galileon-like) Bounce
- Cosmological perturbations
  - Primordial Power Spectrum
  - Non-Gaussianities
- Curvaton/Preheating in bounce cosmology
- Summary

Cosmology: science based on observations



CMB: WMAP, Planck

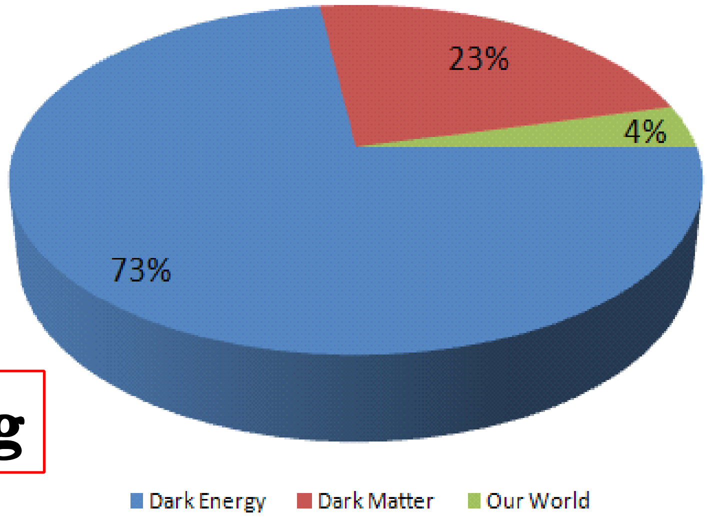
LSS: SDSS → **hot Big Bang**

SuperNovae

Theoretical Problems:

- Horizon
- Flatness
- Monopole
- Structure formation
- And singularity…

Cosmology: science based on observations



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SuperNovae

Theoretical Problems:

- Horizon
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- And singularity…

One of the most fundamental questions related to theories of quantum gravity

Solutions to cosmic singularity:	Benefits	Weak points
TOE	Solve everything	What is TOE?
String theory, AdS/CFT	Solid mathematics	Far away from reality
Matrix theory, Loop quantum gravity	An eye to quantum nature of spacetime	Out of control in computation
Effective field theory	Powerful to link theories with observations	Incomplete in UV limit

Why we use EFT to study early universe?
EFT may tell us potential signatures of QG near the singularity.

A first glance of this power from the simplest model

Consider the Lagrangian of a free massive scalar field:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$$

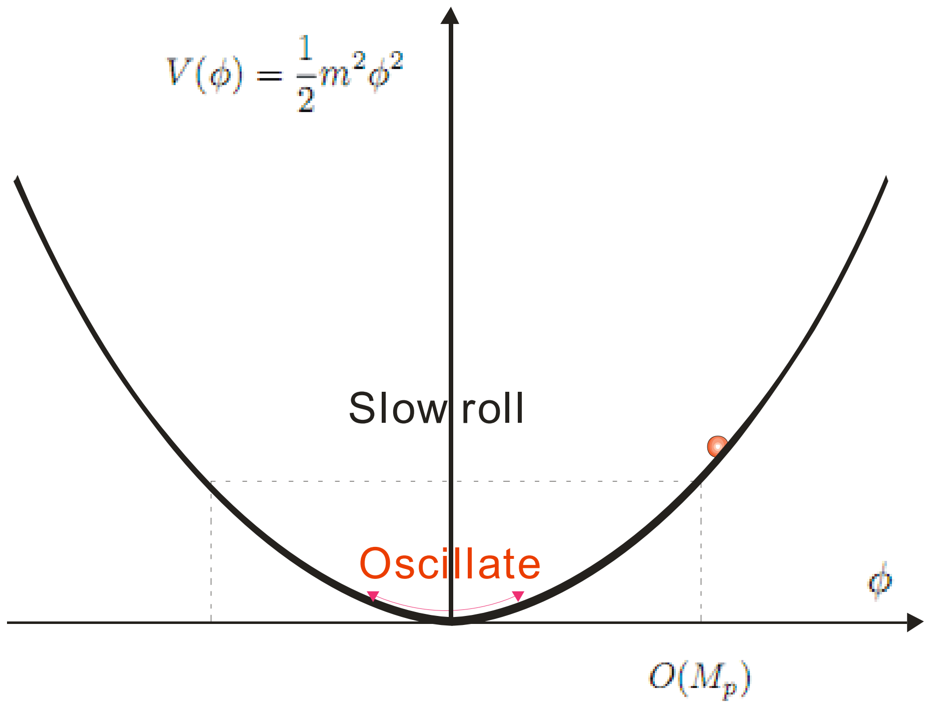
Its cosmological evolution follows the Friedmann equation and the Klein-Gordon equation:

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2\right)$$

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

This model yields an accelerating phase at high energy scale, when the amplitude of the scalar is larger than the Planck mass.

Inflation



Guth 1981; Sato 1981; Linde 1982;
Albrecht & Steinhardt, 1982;
Starobinsky 1980; Fang 1980; …

Benefits of inflation model:

- Acceleration at early times: no horizon problem;
- The universe should be flat: no flatness problem;
- Monopoles are diluted during inflation;
- Primordial fluctuations can lead to the formation of LSS

Main predictions:

- The universe should be homogeneous, isotropic and flat;
- The primordial power spectrum should be gaussian, adiabatic, and nearly scale-invariant

Problems:

- The initial singularity is still not addressed;

Borde and Vilenkin, PRL72:3305, 1994

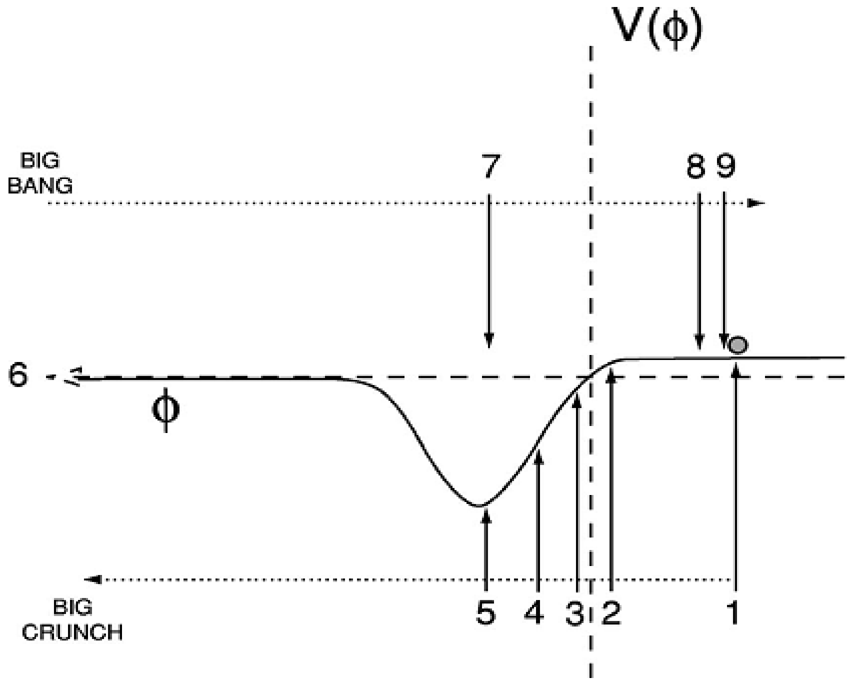
- Trans-Planckian problem exists for the inflationary fluctuations ;

Martin & Brandenberger, PRD63:123501, 2001

Ekpyrotic model

The collision of two M branes in 5D gives rise to a nonsingular cyclic universe, and the description of EFT in 4D is

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} \mathcal{R} + \frac{1}{2} (\partial\phi)^2 - V(\phi) + \beta^4(\phi)(\rho_M + \rho_R) \right)$$



1 DE domination
2 decelerated expansion
3 turnaround
4 ekpyrotic contracting phase
5 before big crunch
6 a singular bounce in 4D
7 after big bang
8 radiation domination
9 matter domination

Khoury, Ovrut, Steinhardt & Turok, PRD64:123522, 2001

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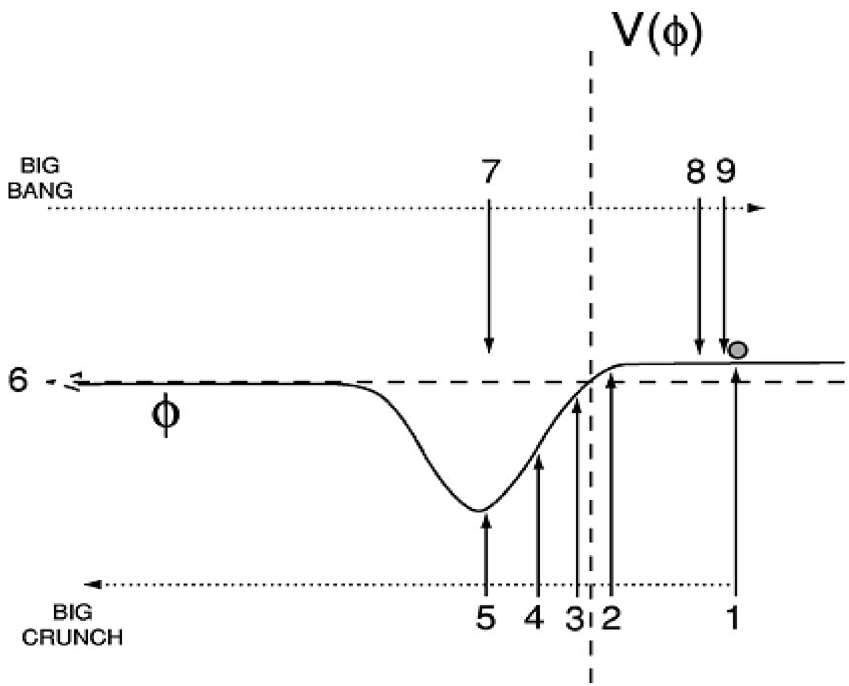

1 DE domination
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5 before big crunch
6 a **singular** bounce in 4D
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9 matter domination

Failure of effective field theory description, uncertainty involved in perturbations.

Nonsingular Bounce

- Pre-big-bang

  (non-perturbative effects)
- String gas cosmology

  (thermal non-local system)
- Mirage cosmology

  (braneworld)
- Modified gravity

  (high-order corrections)
- New Ekpyrotic model

  (ghost condenstate)
- …

Nonsingular bounces from EFT

- **Lee-Wick Bounce**

CYF, Qiu, Brandenberger, Zhang, PRD80:023511,2009.

History of Lee-Wick

- 1969: T. D. Lee and G. C. Wick proposed the Lee-Wick mechanism.
  - Higher derivative action → New Degree of Freedom.
  - Opposite kinetic term → Problem of Ghosts!
- 1970's: debates on the consistency of the theory.
- 1970's: super-symmetry discovered → interest in Lee-Wick theory wanes.
- 2007: Lee-Wick construction resurrected by Grinstein, O'Connell and Wise:

  Lee-Wick Standard Model.
  - No progress on conceptual issues related to ghosts.
  - Phenomenological studies (LHC).

Unsettled issue:

Quantization? Some attempts,

van Tonder, arXiv:0810.1928,

Shalaby, arXiv:0812.3419.

Considered as an effective field description of physics beyond SM.

Lee-Wick Bounce

The simplest Lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} - \frac{1}{2M^2}(\partial^2\hat{\phi})^2 - \frac{1}{2}m^2\hat{\phi}^2$$

A higher derivative term is involved.
Classically, a new degree of freedom is obtained.
(so-called LW partner)
A reflection of new physics in the description of EFT.

Equivalent Lagrangian :

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2(\phi - \tilde{\phi})^2 - \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{1}{2}M^2\tilde{\phi}^2$$

Regular Higgs $\phi \equiv \hat{\phi} + \tilde{\phi}$ LW partner $\tilde{\phi} \equiv \frac{\partial L}{\partial \Box \hat{\phi}}$

The mass terms can be diagonalized by rotating the field basis, and the rotation angle is very small when M>>m.

Equations of Motion

- Metric of FRW space-time:

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2$$

- Einstein action coupled to Lee-Wick Model leads to the following equations for cosmological dynamics:

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\tilde{\phi}}^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\tilde{\phi}^2\right]$$
$$\dot{H} = -4\pi G\left(\dot{\phi}^2 - \dot{\tilde{\phi}}^2\right)$$

- In addition, there are the Klein-Gordon equations.

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$
$$\ddot{\tilde{\phi}} + 3H\dot{\tilde{\phi}} + M^2\tilde{\phi} = 0$$

Sketch

A heavier field is much more stable than a lighter one at low energy densities and curvatures.

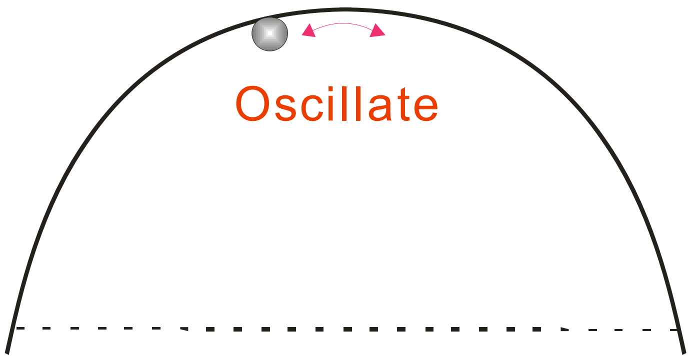
Contracting: ϕ dominates

Near the bounce:

$\phi_f \simeq \frac{M_{pl}}{\sqrt{12\pi}}$ ϕ freezes $\tilde{\phi}$ still oscillates

A bounce happens:

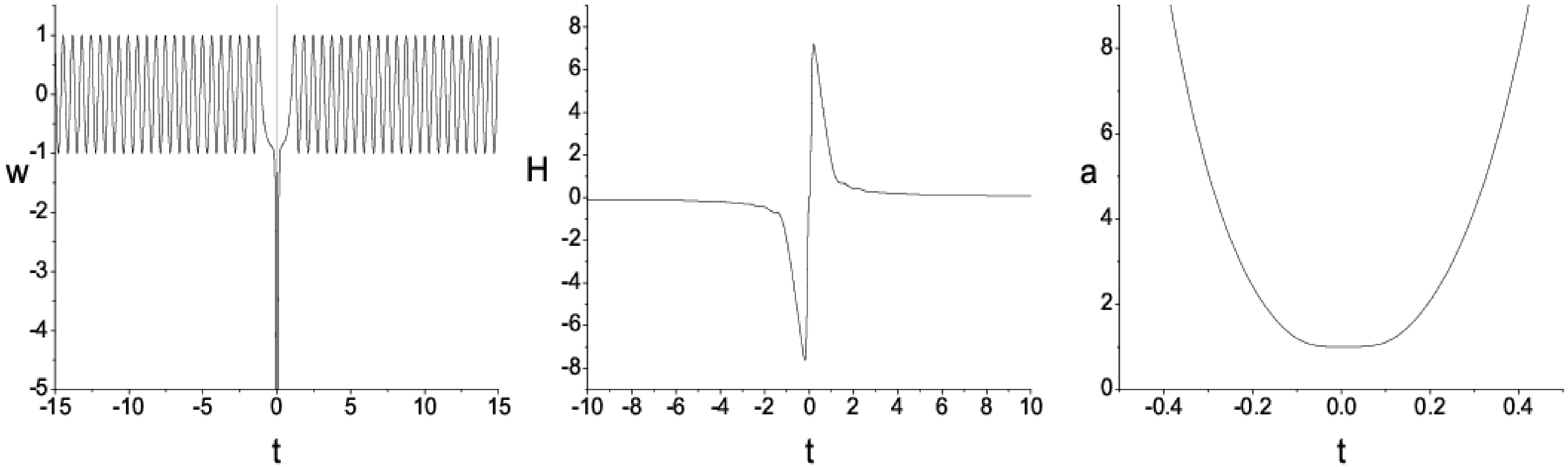
when the contribution of LW scalar to the energy density catches up to that of the normal scalar



Expanding: ϕ dominates again

Numerical Results

The plots of the equation-of-state, Hubble parameter, and scale factor in the model:



Matter Bounce

Starobinsky, JETP Lett.30:682,1979;
Wands, PRD60:023507,1999;
Finelli & Brandenberger, PRD65:103522,2002

Weak point:

•The ghost mode leads to quantum instability in this model, therefore we have to justify the reliability of this EFT.

Check for availability:

•The energy scale has to be much lower than the mass scale M.
•In Lee-Wick bounce, the maximum of hubble parameter and the bounce duration reach

$$H_m \sim m \quad \Delta t_b \sim m^{-1}$$

which is much smaller than M.

Nonsingular bounces from EFT

- G(alileon-like) Bounce

Qiu, Evslin, CYF, Li, Zhang, JCAP 1110 (2011) 036;
Easson, Sawicki, Vikman, JCAP 1111 (2011) 021;
CYF, Easson, Brandenberger, 2012

What is a Galileon?

➢**Definition:** the Lagrangian involves higher derivative operators, but the equation of motion remains second order, so the model can have NEC violation without ghosts.

Basically 5 kinds of Galileon model:

$$
\begin{aligned}
\mathcal{L}_1 &= \Pi \quad \& \quad \mathcal{L}_2 = X \equiv -\nabla_\mu \Pi \nabla^\mu \Pi / 2 \\
\mathcal{L}_3 &= X(\Box\Pi) \\
\mathcal{L}_4 &= 2X[(\Box\Pi)^2 - (\nabla_\mu \nabla_\nu \Pi)(\nabla^\mu \nabla^\nu \Pi)] + X^2 R \\
\mathcal{L}_5 &= 2X[(\Box\Pi)^3 - 3(\Box\Pi)(\nabla_\mu \nabla_\nu \Pi)(\nabla^\mu \nabla^\nu \Pi) + 2(\nabla_\mu \nabla^\nu \Pi)(\nabla_\nu \nabla^\rho \Pi)(\nabla_\rho \nabla^\mu \Pi)] \\
&\quad -6X^2 G_{\mu\nu}(\nabla^\mu \nabla^\nu \Pi)
\end{aligned}
$$

Nicolis et al., Phys.Rev.D79:064036,2009

But can be generalized…

$$
\begin{aligned}
\mathcal{L} &= K(X, \Pi) \\
\mathcal{L}_3 &= G_1(X, \Pi)(\Box\Pi) \\
\mathcal{L}_4 &= G_{2,X}(X, \Pi)[(\Box\Pi)^2 - (\nabla_\mu \nabla_\nu \Pi)(\nabla^\mu \nabla^\nu \Pi)] + G_2(X, \Pi) R \\
\mathcal{L}_5 &= G_{3,X}(X, \Pi)[(\Box\Pi)^3 - 3(\Box\Pi)(\nabla_\mu \nabla_\nu \Pi)(\nabla^\mu \nabla^\nu \Pi) + 2(\nabla_\mu \nabla^\nu \Pi)(\nabla_\nu \nabla^\rho \Pi)(\nabla_\rho \nabla^\mu \Pi)] \\
&\quad -6G_3(X, \Pi) G_{\mu\nu}(\nabla^\mu \nabla^\nu \Pi)
\end{aligned}
$$

where R is the Ricci scalar and $G_{\mu\nu}$ is the Einstein tensor.

Deffayet et al., Phys.Rev. D84 (2011) 064039
Horndeski, Int. J. Theor. Phys. 10:363,1974

What is a Galileon?

➢ **Phenomenology:** (in less than one year)

Galileon as dark energy models:

- R. Gannouji,M. Sami, Phys.Rev.D82:024011,2010;
- A. De Felice, S. Tsujikawa, Phys.Rev.Lett.105:111301,2010;
- C. Deffayet,O. Pujolas,I. Sawicki, A. Vikman, JCAP 1010:026,2010;

… … … …

Galileon as inflation and slow expanstion models:

- P. Creminelli, A. Nicolis, E. Trincherini, JCAP 1011:021,2010;
- T. Kobayashi,M. Yamaguchi,J. Yokoyama, Phys.Rev.Lett.105:231302,2010;
- C. Burrage,C. de Rham,D. Seery,A. Tolley, JCAP 1101:014,2011;

… … … …

Observational constraints on Galileon models:

- S. Nesseris,A. De Felice, S. Tsujikawa, Phys.Rev.D82:124054,2010;
- A. Ali,R. Gannouji, M. Sami, Phys.Rev.D82:103015,2010;

… … … …

Spherically symmetric solutions in Galileon models:

- D. Mota, M. Sandstad,T. Zlosnik, JHEP 1012:051,2010.

… … … … …

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- ➢C. Deffayet,O. Pujolas,I. Sawicki, A. Vikman, JCAP 1010:0

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- ➢C. Burrage,C. de Rham,D. Seery,A. Tolley, JCAP 1101:01

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- ➢S. Nesseris,A. De Felice, S. Tsujikawa, Phys.Rev.D82:124
- ➢A. Ali,R. Gannouji, M. Sami, Phys.Rev.D82:103015,2010;

… … … …

Spherically symmetric solutions in Galileon models:

- ➢D. Mota, M. Sandstad,T. Zlosnik, JHEP 1012:051,2010.

… … … … …

Can we get a nonsingular bounce from this model?

Conformal Galileon Bounce

The action of conformal galileon model:

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + F^2 e^{2\Pi}(\partial\Pi)^2 + \frac{F^3}{M^3}(\partial\Pi)^2\Box\Pi + \frac{F^3}{2M^3}(\partial\Pi)^4\right]$$

See also 1007.0027 for "Galileon Genesis".

Stress energy tensor:

$$\begin{aligned} T_{\mu\nu} &= -F^2 e^{2\Pi}[2\partial_\mu\Pi\partial_\nu\Pi - g_{\mu\nu}(\partial\Pi)^2] - \frac{F^3}{2M^3}(\partial\Pi)^2[4\partial_\mu\Pi\partial_\nu\Pi - g_{\mu\nu}(\partial\Pi)^2] \\ &\quad -\frac{F^3}{M^3}[2\partial_\mu\Pi\partial_\nu\Pi\Box\Pi - \partial_\mu\Pi\partial_\nu(\partial\Pi)^2 - \partial_\nu\Pi\partial_\mu(\partial\Pi)^2 + g_{\mu\nu}\partial_\sigma\Pi\partial^\sigma(\partial\Pi)^2]\,. \end{aligned}$$

From which we get energy density and pressure:

$$\rho = F^2[-e^{2\Pi}\dot{\Pi}^2 + \frac{1}{\bar{H}^2}(\dot{\Pi}^4 + 4H\dot{\Pi}^3)] \quad \text{where } \bar{H} \equiv \sqrt{\frac{2M^3}{3F}}$$

$$P = F^2[-e^{2\Pi}\dot{\Pi}^2 + \frac{1}{3\bar{H}^2}(\dot{\Pi}^4 - 4\dot{\Pi}^2\ddot{\Pi})]$$

Asymptotic solution

Equation of motion:

$$4\dot{\Pi}^2\ddot{\Pi} - 3\bar{H}^2 e^{2\Pi}\dot{\Pi}^2 + 2\dot{\Pi}^4 + 12\alpha\dot{\Pi}^6 = 6\dot{\Pi}^3\sqrt{B} + \frac{\dot{B}}{2\alpha\sqrt{B}} \qquad B = -\alpha\bar{H}^2 e^{2\Pi}\dot{\Pi}^2 + \alpha\dot{\Pi}^4 + 4\alpha^2\dot{\Pi}^6$$

Hubble parameter: $H = 2\alpha\dot{\Pi}^3 - \sqrt{-\alpha\bar{H}^2 e^{2\Pi}\dot{\Pi}^2 + \alpha\dot{\Pi}^4 + 4\alpha^2\dot{\Pi}^6}$

In contracting phase: $H < 0 \quad \Longrightarrow \quad \dot{\Pi} > \bar{H}e^{\Pi}$

Analysis of the asymptotic behavior in contracting phase $t \to -\infty$

A. $\dot{\Pi} = c_0\bar{H}e^{\Pi}$ ⇨ $\dot{\Pi} \sim (t_0 - t)^{-1}$

⇨ Terms in EoM has different orders of t *inconsistent* **!**

B. $\dot{\Pi} \gg \bar{H}e^{\Pi}$

1) $\alpha\dot{\Pi}^2 \gg 1$ ⇨ $\dot{\Pi} \sim (t_0 - t)^{-1}$ ⇨ $\dot{\Pi} \to 0$ *inconsistent* **!**

2) $\alpha\dot{\Pi} = const.$ ⇨ $\dot{\Pi} = const.\quad \ddot{\Pi} \to 0$ ⇨ $3\alpha\dot{\Pi}^2 + 1 = 0$ *inconsistent* **!**

3) $\alpha\dot{\Pi}^2 \ll 1$ ⇨ $\dot{\Pi} = \frac{1}{\alpha^{1/4}\sqrt{2(t_0 - t)}}$

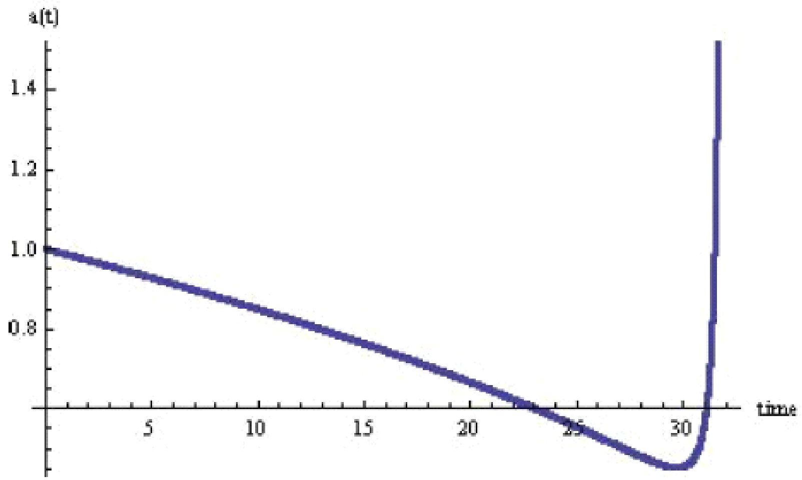
⇨ $\Pi \sim -\alpha^{-1/4}\sqrt{2(t_0 - t)} \quad H = \frac{1}{2(t - t_0)}$ **consistent** ***!***

Numerical Results

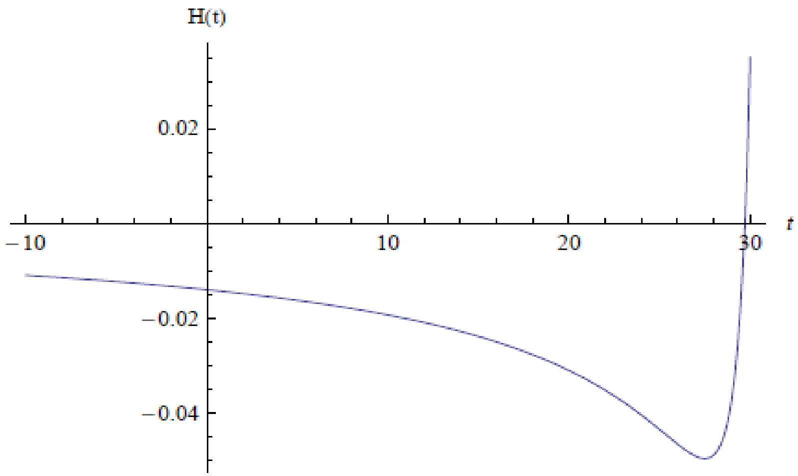
Plots of scale factor, Hubble parameter, Galileon scalar, and equation of state, respectively:

$$F = \bar{H} = 1 \ , \quad M_p^2 = \frac{1}{8\pi G} = 1 \ , \quad \alpha = \frac{1}{3}$$

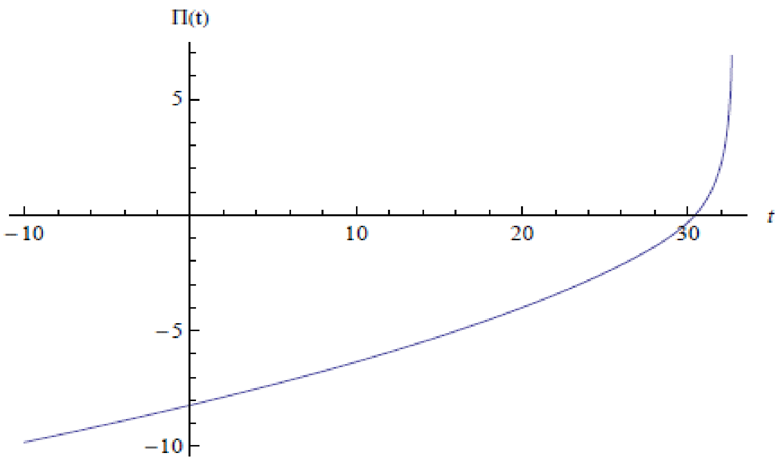
The scale factor a(t)



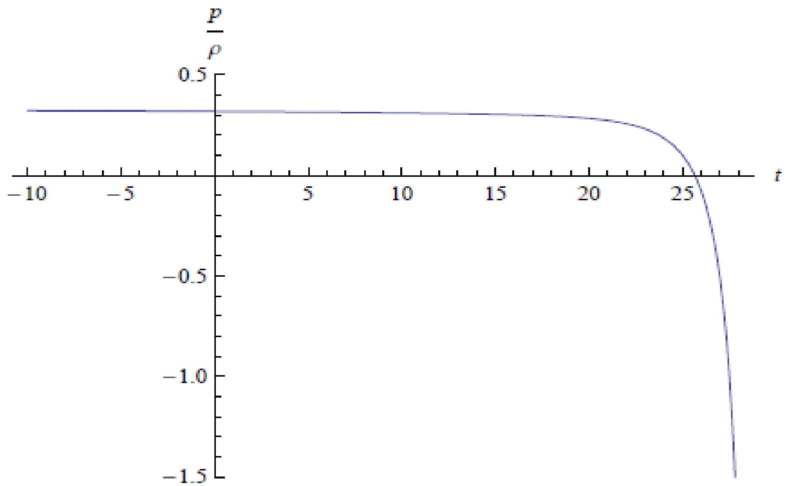
The Hubble Parameter in a Bouncing Solution



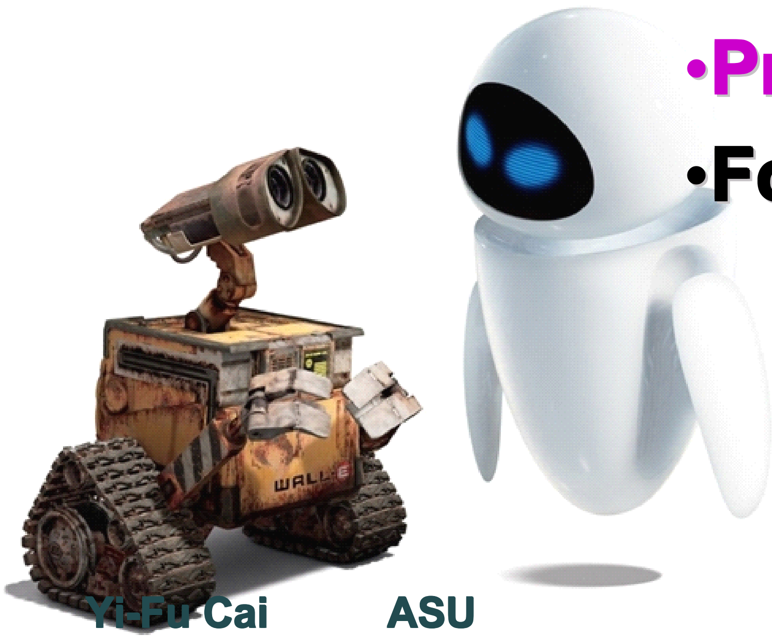
The Galileon field in a Bouncing Solution



The equation of state in a Bouncing Solution



Primordial perturbations in bounce cosmology



- Preliminaries
- Formalism

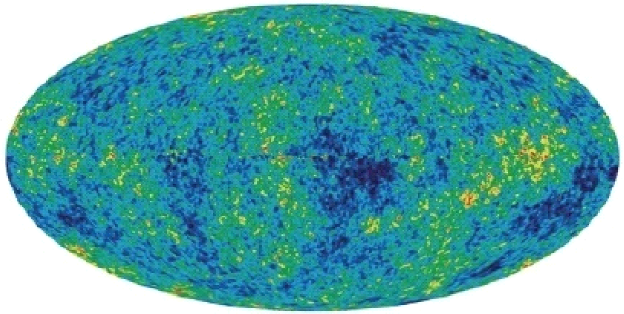
CYF, Qiu, Brandenberger, Piao, Zhang, JCAP 0803:013,2008;
CYF, Qiu, Brandenberger, Zhang, PRD80:023511,2009.

Yi-Fu Cai ASU April 26, 2012 Hefei

Cosmological Perturbation Theory

Why perturbations?

Primordial perturbations provide seeds for structure formation and explains why our current universe is not complete isotropic.



$$\frac{\Delta T}{T} \sim 10^{-5}$$

Two constraints for linear perturbations:

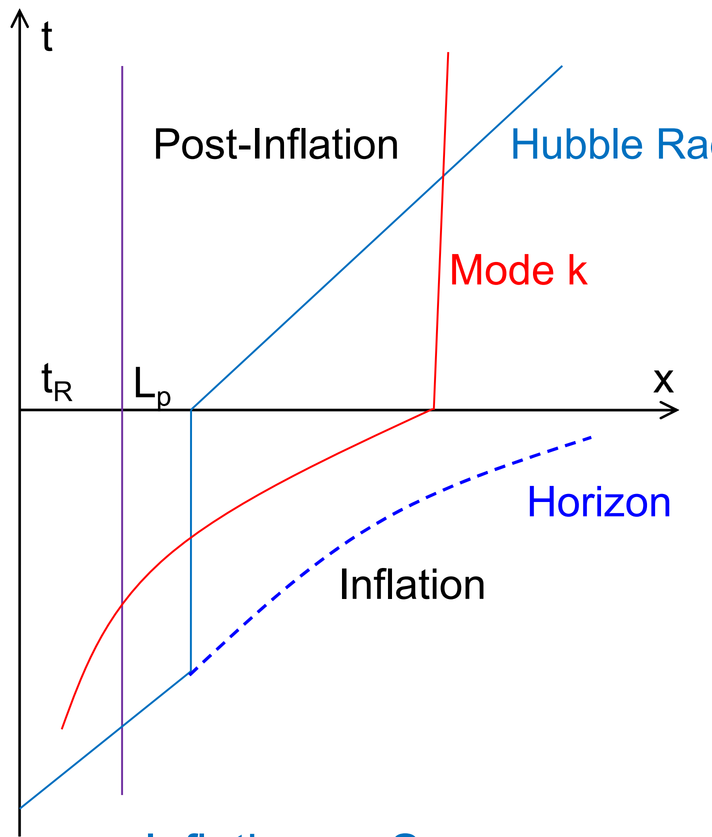
- <u>Theoretically:</u> stability must be guaranteed!
- <u>Observationally:</u> a (nearly) scale-invariant power spectrum and small tensor-to-scalar ratio

Sketch Plots

Crucial facts:

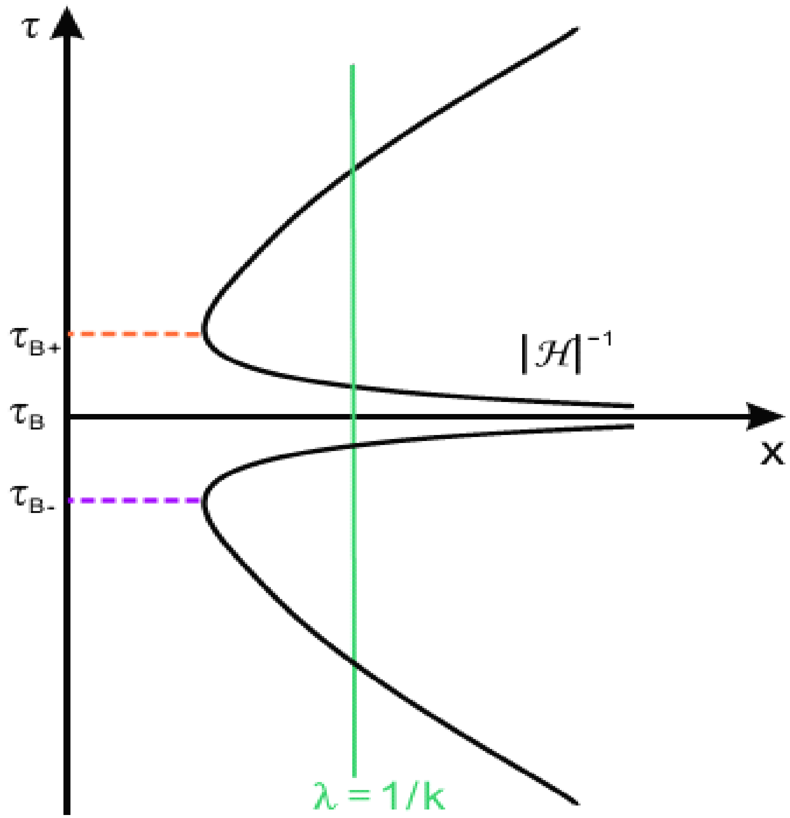
- •Fluctuations originate on sub-Hubble scales
- •Fluctuations propagate for a long time on super-Hubble scales
- •Trans-Planckian problem: Inflation; Bounce

R.Brandenberger, hep-ph/9910410



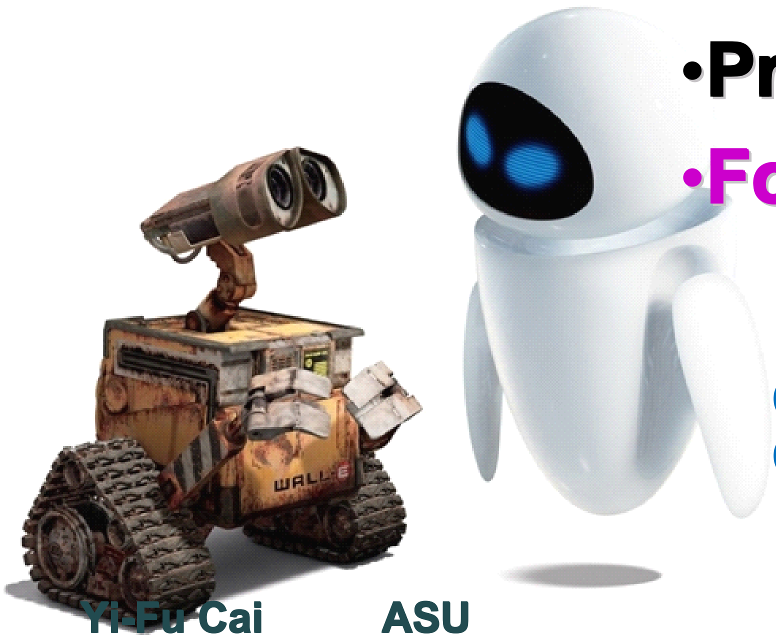
Inflationary Success:
a scale-invariant spectrum

CYF et al., 0810.4677



Can we obtain a scale-invariant spectrum as in inflation?

Primordial perturbations in bounce cosmology



- Preliminaries
- Formalism

CYF, Qiu, Brandenberger, Piao, Zhang, JCAP 0803:013,2008;
CYF, Qiu, Brandenberger, Zhang, PRD80:023511,2009.

Setup of Perturbations

Perturbed scalars: $\phi \to \phi + \delta\phi$

Perturbed metric: $ds^2 = a^2(\eta)[(1+2\Phi)d\eta^2 - (1-2\Psi)dx^i dx^i]$

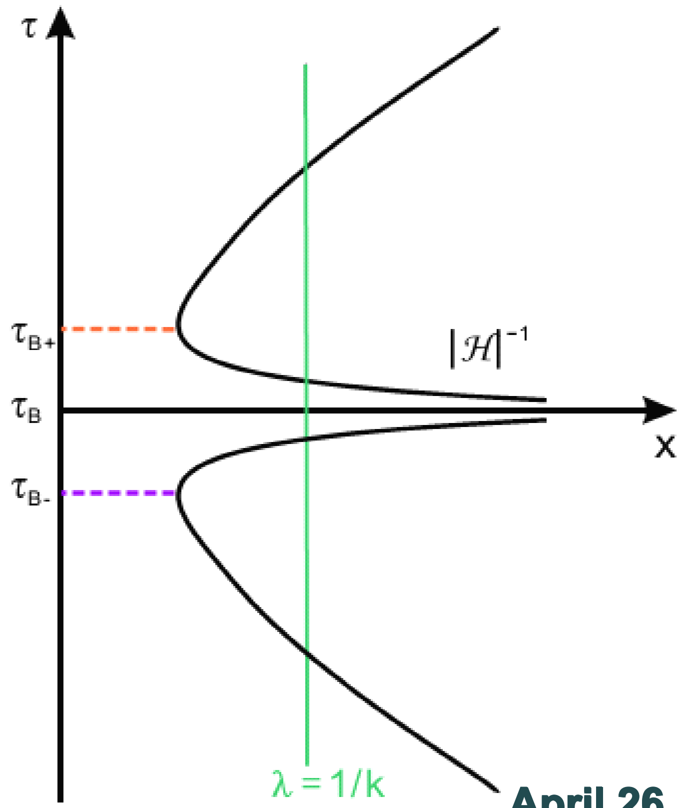
Pert. Equation: $\Phi'' + 2(\mathcal{H} - \frac{\phi''}{\phi'})\Phi' + 2(\mathcal{H}' - \mathcal{H}\frac{\phi''}{\phi'})\Phi - \nabla^2\Phi = 0$

Status:

Contracting phase

The Bounce

Expanding phase



Curvature perturbation on uniform density

Specifically, we consider a matter bounce

$$\zeta = \Phi + \frac{\mathcal{H}}{\mathcal{H}^2 - \mathcal{H}'}(\Phi' + \mathcal{H}\Phi)$$

Contracting: $\zeta_k = \frac{A}{\sqrt{2k^3}} X_k$

$$A = i\frac{4H_B}{\sqrt{3}\mathcal{H}_B^3}\ , \ \ X_k = \frac{e^{ik(\eta - \tilde{\eta}_B)}}{(\eta - \tilde{\eta}_B)^3}[1 - ik(\eta - \tilde{\eta}_B)]$$

Expanding: $\zeta_k = \zeta_k|_B \sim k^{-\frac{3}{2}}$

Scale-invariant and constant

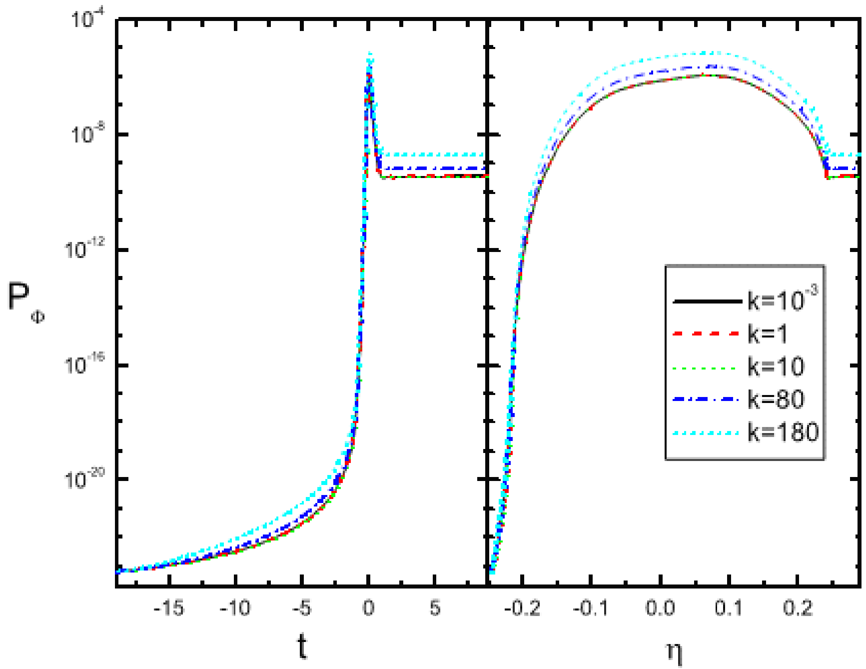
Comments:

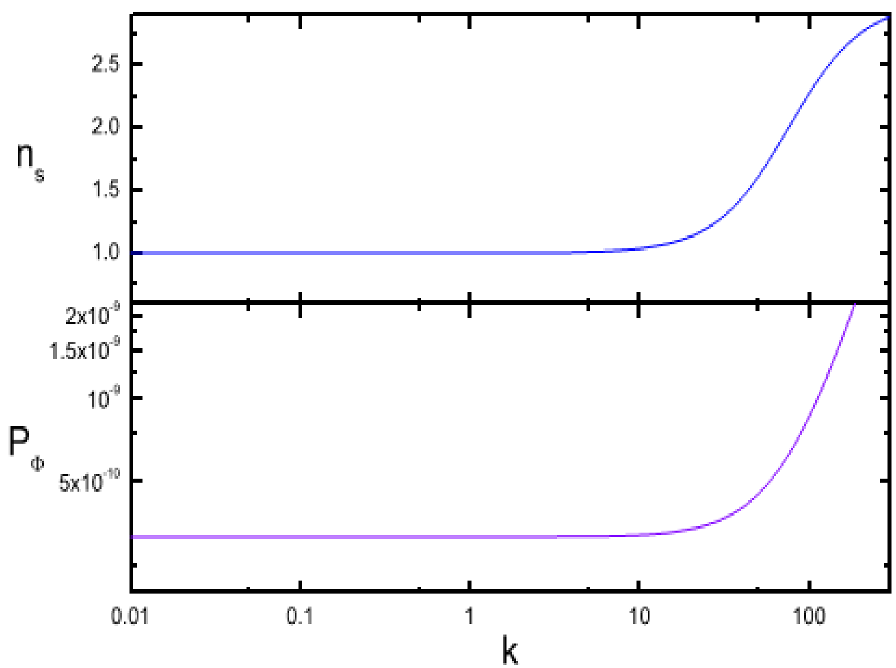
1, zeta is **no longer a conserved quantity** outside Hubble radius when the universe is contracting;

2, curv. pert. in contracting phase can be transferred into expanding phase smoothly.

Numerical Results

The plots of the power spectrum and spectral index





Non-Gaussianities in Bounce Cosmology

CYF, Xue, Brandenberger, Zhang, JCAP 0905 (2009) 011
CYF, Xue, Brandenberger, Zhang, JCAP 0906 (2009) 037

Non-Gaussianities in early universe

- Non-gaussianity parameter:

  in spacetime (local limit) $\zeta(x) = \zeta_g(x) + \frac{3}{5} f_{NL} [\zeta_g^2 - \langle \zeta_g^2 \rangle]$

  in k-space (including shape)

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = \frac{6}{5} f_{NL} (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \left\{ \frac{2\pi^2}{k_2^3} \frac{2\pi^2}{k_3^3} P_\zeta(k_2)^2 + 2 \text{ perms.} \right\}$$

- WMAP5 data:

$$-9 < f_{NL}^{\text{local}} < 111 \ (95\% \ \text{CL}) \qquad -151 < f_{NL}^{\text{equil}} < 253 \ (95\% \ \text{CL})$$

- The contribution from redefinition:

$$\zeta \to \zeta - \epsilon \zeta^2 + \ldots \qquad \epsilon = \frac{3}{2}(1+w)$$

So it gives

$$f_{NL} \supseteq \frac{5}{3} \epsilon$$

- Also other contributions…

Non-Gaussianities in bounce cosmology

Three-point correlation function:

$$
\begin{aligned}
<\zeta(t,\vec{k}_1)\zeta(t,\vec{k}_2)\zeta(t,\vec{k}_3)> &= i\int_{t_i}^{t} dt' <[\zeta(t,\vec{k}_1)\zeta(t,\vec{k}_2)\zeta(t,\vec{k}_3), L_{int}(t')]> \\
&= (2\pi)^7\delta(\sum \vec{k}_i)\frac{P_\zeta^2}{\prod k_i^3} \times \mathcal{A}(\vec{k}_1,\vec{k}_2,\vec{k}_3)
\end{aligned}
$$

Lagrangian in cubic order:

$$
\mathcal{L}_3 = (\epsilon^2 - \frac{\epsilon^3}{2})a^3\zeta\dot{\zeta}^2 + \epsilon^2 a\zeta(\partial\zeta)^2 - 2\epsilon^2 a^3\dot{\zeta}(\partial\zeta)(\partial\chi) + \frac{\epsilon^3}{2}a^3\zeta(\partial_i\partial_j\chi)^2 + f(\zeta)\frac{\delta\mathcal{L}_2}{\delta\zeta}|_1
$$

with $\chi \equiv \partial^{-2}\dot{\zeta}$ and

$$
\begin{aligned}
f(\zeta) = &\frac{1}{4\mathcal{H}^2}(\partial\zeta)^2 - \frac{1}{4\mathcal{H}^2}\partial^{-2}\partial_i\partial_j(\partial_i\zeta\partial_j\zeta) \\
&-\frac{a}{\mathcal{H}}\zeta\dot{\zeta} - \frac{\epsilon a}{2\mathcal{H}}\partial_i\zeta\partial_i\partial^{-2}\dot{\zeta} \\
&+\frac{\epsilon a}{2\mathcal{H}}\partial^{-2}\partial_i\partial_j(\partial_i\partial^{-2}\dot{\zeta}\partial_j\zeta) ,
\end{aligned}
$$

Non-Gaussianities in bounce cosmology

Shape Function: $\frac{\mathcal{A}_T}{k_1 k_2 k_3}$

- of order ε

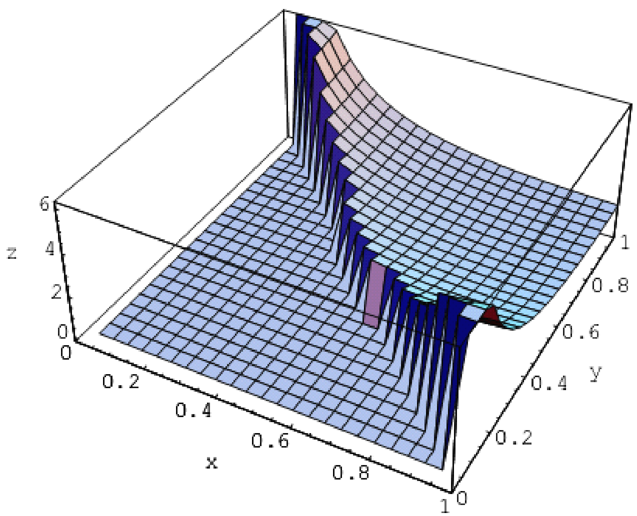
$$\mathcal{A}^{\epsilon} = -\frac{\epsilon}{2}\sum k_i^3$$

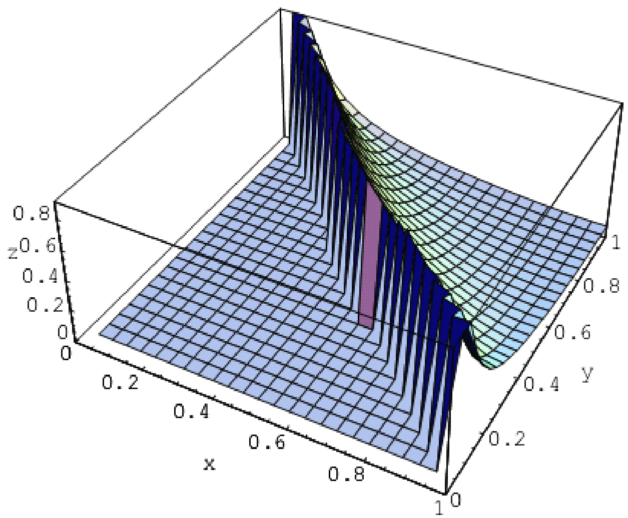
- of order ε 2

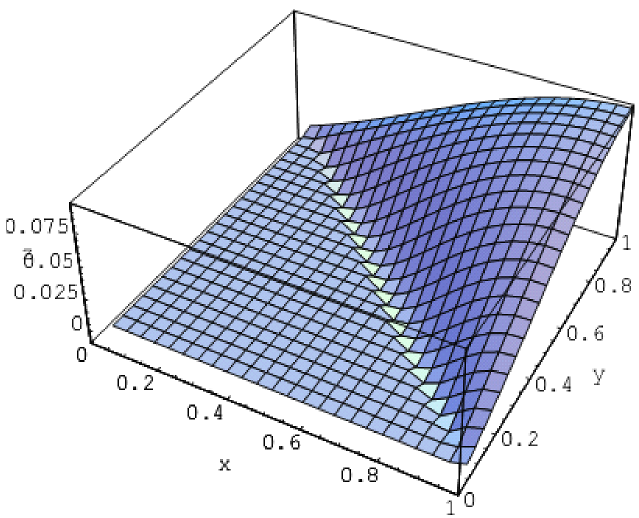
$$\mathcal{A}^{\epsilon^2} = -\frac{\epsilon^2}{24}\sum k_i^3 + \frac{\epsilon^2}{32}\sum_{i\neq j} k_i k_j^2 + \frac{\epsilon^2}{96\prod k_i^2}\left\{5\sum_{i\neq j} k_i^7 k_j^2 - 3\sum_{i\neq j} k_i^6 k_j^3 - 2\sum_{i\neq j} k_i^5 k_j^4\right\}$$

- of order ε 3

$$\mathcal{A}^{\epsilon^3} = \frac{\epsilon^3}{48}\sum k_i^3 + \frac{\epsilon^3}{96}\sum_{i\neq j} k_i k_j^2 + \frac{\epsilon^3}{96\prod k_i^2}\left\{\sum_i k_i^9 - 3\sum_{i\neq j} k_i^7 k_j^2 - \sum_{i\neq j} k_i^6 k_j^3 + 3\sum_{i\neq j} k_i^5 k_j^4\right\}$$







Non-Gaussianities in bounce cosmology

Main results:

- No slow roll —— a sizable amplitude;
- No slow roll —— new shapes;
- No conservation (zeta) —— new origins;
- Specifically, we consider a matter bounce

$$f_{NL}^{local} = -\frac{35}{8} \approx -4.4$$

Consequence:

- Detectable in Planck?

Preheating & Curvaton in bounce cosmology

Questions:

Large tensor-to-scalar ratio

$$r \equiv \frac{P_T}{P_\zeta} \sim O(1)$$

No particle production

the Universe is empty !

CYF, Brandenberger, Zhang, JCAP 1103 (2011) 003;
CYF, Brandenberger, Zhang, Phys.Lett. B703 (2011) 25;
CYF, PhD Thesis, June 2010

- Basic picture of preheating:

In the theory of preheating, the energy is quickly transferred from the primordial scalar (inflaton) to other field of which the mass is lighter than the primordial scalar but heavier than those particles at reheating. This process occurs after the primordial period, but earlier than thermal equilibrium and is accompanied with the parametric resonance effect.

Traschen, Brandenberger, PRD 42, 2491 (1990);
Kofman, Linde, Starobinsky, PRL 73, 3195 (1994);
…

- As a simple example, we consider a two-field model with a potential

$$V(\phi, \chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2$$

where in our model ϕ is the background scalar in nonsingular bounce cosmology, and χ is a second scalar field, which is so-called entropy field.

Entropy perturbation: setup

Perturbed scalars: $Q_i \equiv \delta\phi_i + \frac{\dot{\phi}_i}{H}\Phi$

Pert. Equation: $\ddot{Q}_i + 3H\dot{Q}_i - \frac{\nabla^2}{a^2}Q_i + \sum_j [V_{,ij} - \frac{8\pi G}{a^3}(\frac{a^3}{H}\dot{\phi}_i\dot{\phi}_j)^{\cdot}]Q_j = 0$

Curv. Pert.: $\zeta = H\frac{\sum_i \dot{\phi}_i Q_i}{\sum_j \dot{\phi}_j^2}$

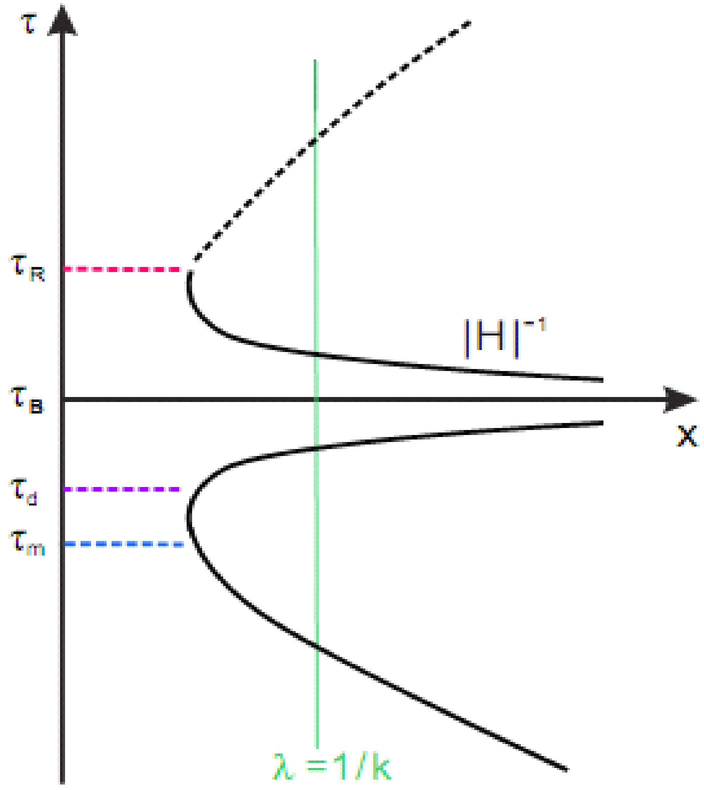
Status:

Contraction

Deflation

Bounce

Radiation Era



Entropy perturbation: kinetic amplification

- Before the bounce: $\delta\chi(t) = P_\chi^{\frac{1}{2}}(t) = \frac{|H(t)|}{4\pi}$
- During the bounce, we can parameterize H= α t under the approximation of linear bounce. In this case, the pert. Equation takes,

$$\ddot{Q}_\chi + 3H\dot{Q}_\chi + \frac{k^2}{a^2}Q_\chi + m^2_{\chi,\mathrm{eff}}Q_\chi \simeq 0$$

where $m^2_{\chi,\mathrm{eff}} = \frac{4g^2m^2_{pl}}{3\pi^3} - \frac{96\alpha}{\pi^2}(1+3\alpha t^2)$ becomes tachyonic in bouncing phase.

- Finally it yields an amplification factor $\delta\chi_b = \mathcal{F}\delta\chi_a$ with form of

$$\mathcal{F} \simeq e^{\sqrt{y(2+y)}+\frac{3}{\sqrt{2}}\sinh^{-1}(\frac{2\sqrt{y}}{3})} \qquad y = \frac{m^2}{\alpha}$$

Curvaton in bounce cosmology

- The entropy perturbation can be converted into curvature perturbation as a source term, through

$$\dot{\zeta} = -4\pi m_{\rm pl}^{-2} \sum_i \frac{H\delta\phi_i}{\dot{\phi}_i}\left(\frac{\dot{\phi}_i^2}{\dot{H}}\right)^{\cdot}$$

- As a consequence, the tensor-to-scalar ratio takes

$$r \equiv \frac{P_T}{P_\zeta} \simeq 35\mathcal{F}^{-2}$$

In the model of Lee-Wick bounce, r ~ 0.08. With the WMAP7 and BAO and SN, the latest limit is r < 0.2. Therefore large r problem is solved!

- Next to leading order, we study the non-Gaussian perturbation

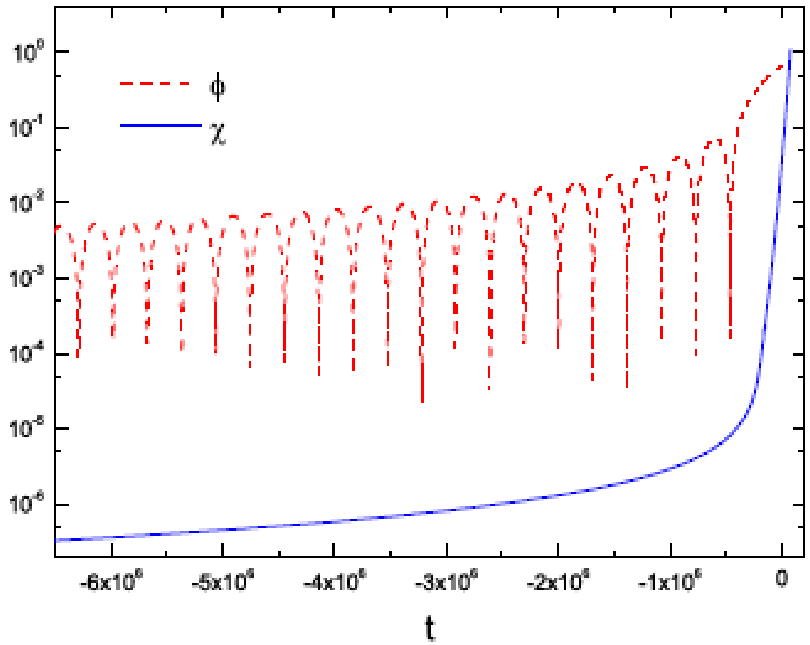
$$\dot{\zeta}_{\rm NL} = \frac{H}{\dot{\sigma}^2}(V_{,ss} + 4\dot{\theta}^2)\delta s^2 - \frac{H}{\dot{\sigma}^3}V_{,\sigma}\delta s\dot{\delta s}$$

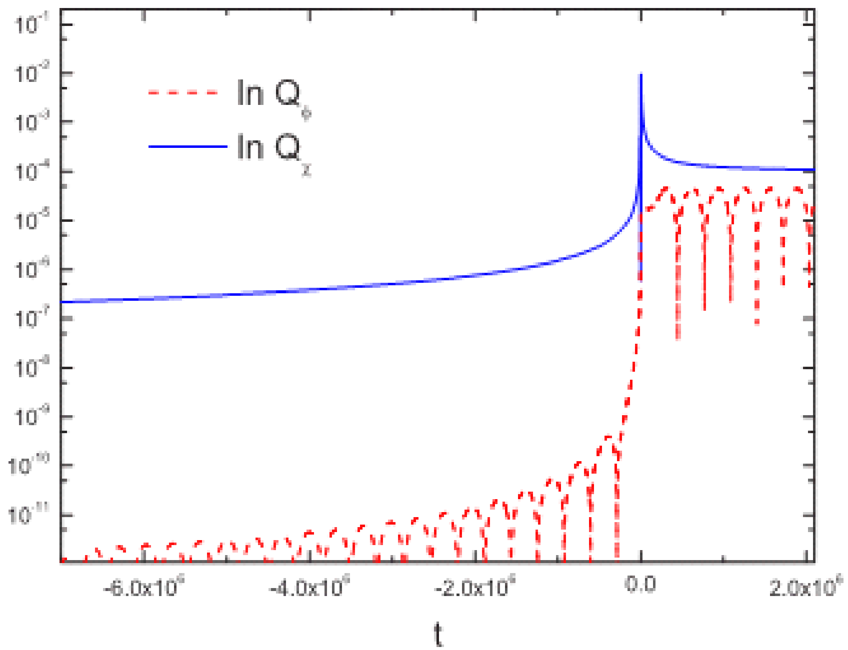
and the nonlinear parameter is $f_{\rm NL} \simeq -5.3\frac{m^4}{d^2 M^4}$

A typical value is in order of O(-5). Very sensitive to Planck experiment!

Numerical Results of Bounce Curvaton

- Evolution of the field fluctuations and the background fields:





- Results:
  - Entropy modes can be amplified kinetically during the bounce, and then converted into adiabatic modes in expanding phase;
  - Large non-gaussianities are generated.

Preheating a bouncing universe

- The modes of the □ field will undergo oscillations according to the equation:

$$\ddot{X}_k + w_k^2 X_k = 0$$

$$w_k^2 = \frac{k^2}{a^2} + g^2\phi^2 \qquad X = a^{3/2}\chi$$

- The background field □ oscillates essentially sinusoidally,

$$\phi(t) \simeq \tilde{\phi}(t)\sin mt \qquad \tilde{\phi}(t) = \frac{m_{pl}}{\sqrt{3\pi}m|t|}$$

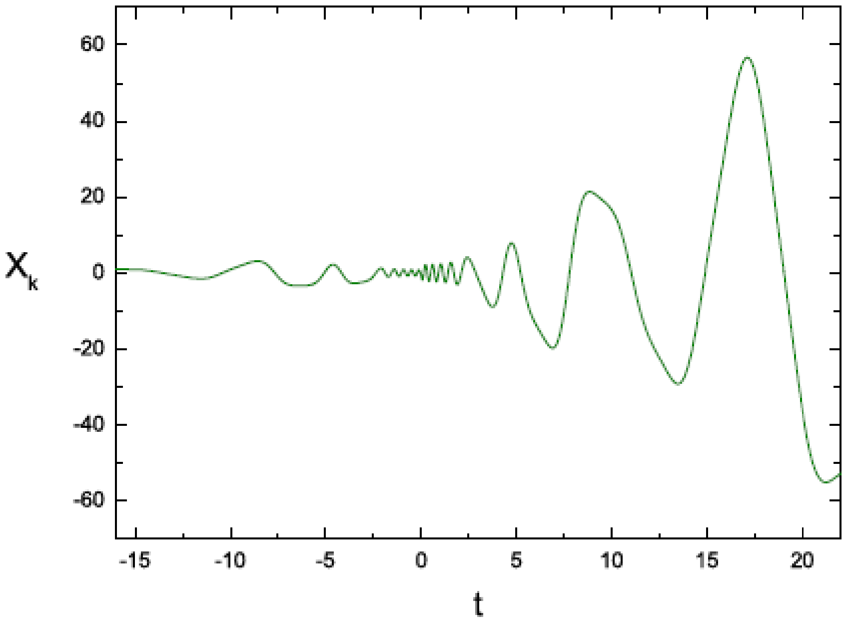
And thus leads to the Mathieu equation.

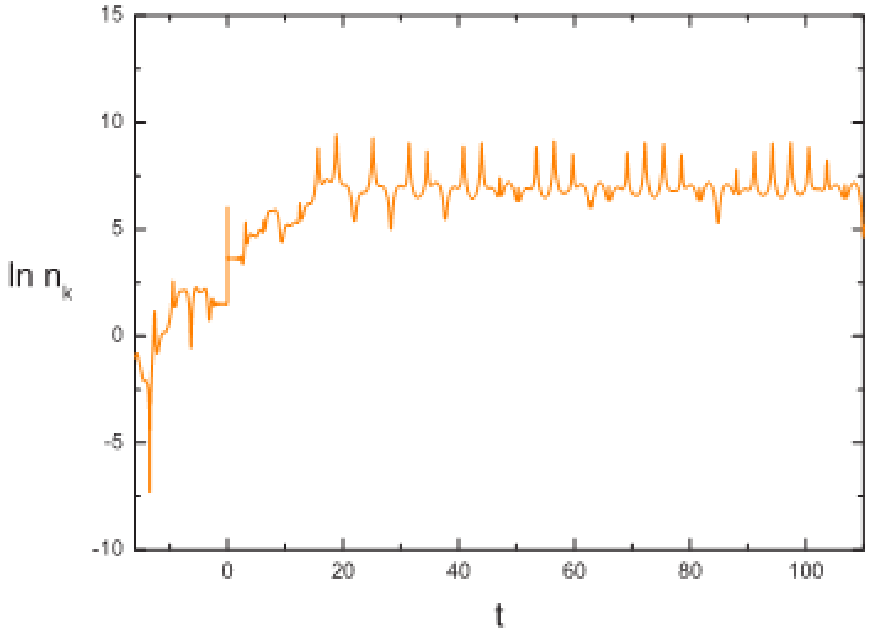
- The modes of □ undergo parametric resonance: Each time □ crosses zero the number density of □ particles will increase for all modes within the resonance band.

$$n_\chi(t) = \int (\frac{dk}{2\pi a})^3 n_k(t)$$
$$\simeq \frac{k_i^3 e^{2m\eta(t-t_i)}}{64\pi^2 a^3\sqrt{\pi m\eta(t-t_i)+2\pi^2}}$$

Numerical Results of Preheating

- Plots of the entropy field and comoving number density of particle production during broad resonance:





Numerical Results of Preheating

- Constraints on the coupling constant g:

  - Condition of broad resonance

$$g > \frac{3\pi m}{2 m_{pl}}$$

  - Constraint from □ backreaction

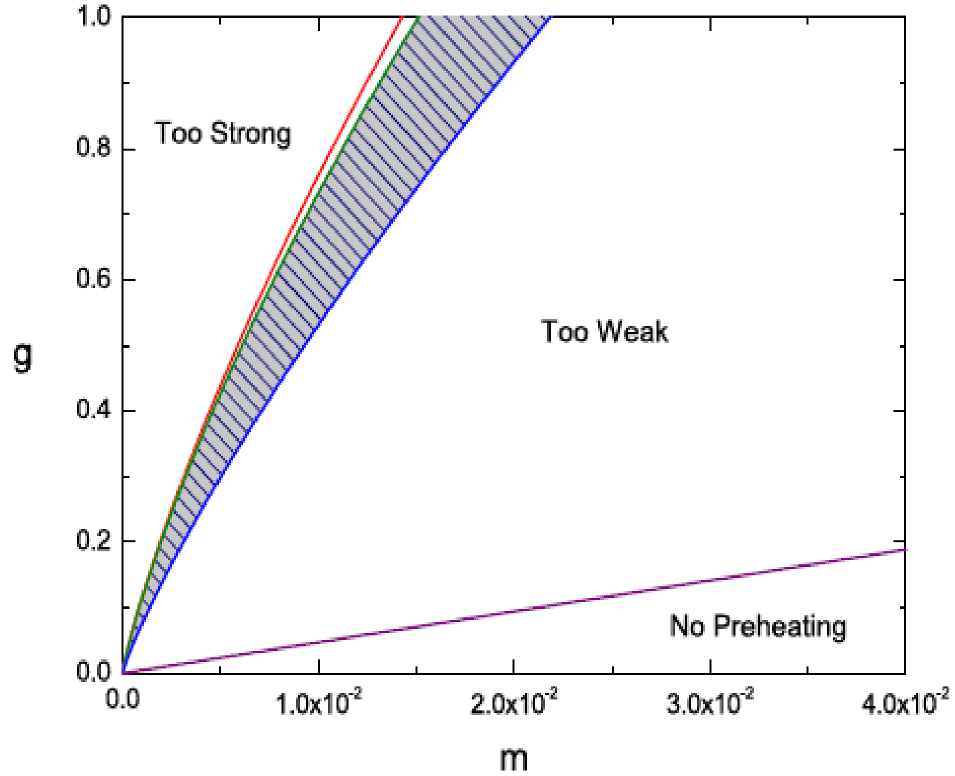
$$g < \frac{15m}{4\eta m_{pl}} \mathcal{W}\left(\frac{4\eta^{\frac{4}{5}} m_{pl}^{\frac{4}{5}}}{m^{\frac{4}{5}}} e^{\frac{2}{5}\pi\eta}\right)$$

  - Requirement that □ is subdominant before the bounce

$$g < \frac{13m}{4\eta m_{pl}} \mathcal{W}\left(\frac{3\pi\eta^{16/13} m_{pl}^{12/13}}{m^{12/13}}\right)$$

  - Requirement that □ is dominant after preheating

$$g >| \frac{3m}{2\eta m_{pl}} \mathcal{W}\left(\frac{19\pi^{3/2}\eta m_{pl}}{3m}\right)$$



Summary

- Using EFT to study physics of early universe
- Inflation and alternatives
- Avoiding initial singularity in the frame of EFT
    - Lee-Wick Bounce
    - G Bounce
- Scale-Invariant spectrum in bounce cosmology
- Large Non-Gaussianities
- Entropy perturbations and Particle productions (preheating)


"Everyone Can Bounce"
Thank You !

Model building of (nonsingular) bounce cosmology from EFT

- **Lee-Wick Bounce** CYF, Qiu, Brandenberger, Zhang,PRD80:023511,2009.
- **Non-relativistic gravitational bounce**
  Brandenberger, PRD80:043516,2009;
  CYF & Saridakis,0906.1789 [hep-th]

Non-relativistic gravitational Bounce

Motivations from EFT:

•Pioneer works by Lee and Wick suggest the UV behavior can be improved by adding higher order derivatives, but involves unbounded quantum states;
•Non-local field theory with single pole can be ghost-free, but out of control in computation;
•A model of power-counting renormalizable spin-2 field can be achieved by adding higher order spatial derivatives, as shown by Horava.

Particularities:

•The theory is perturbatively quantum stable;
•Lorentz symmetry is abandoned but as an emergent one at IR limit;
•Renormalizability requires k^6 term in the propagator for spin-2 field.

The model

The Einstein-Hilbert action:

$$S_g = \frac{1}{16\pi G}\int dt d^3x \sqrt{g} N \left(K_{ij}K^{ij} - K^2 + R\right)$$

where K_{ij} is the extrinsic curvature and R is 3d Ricci scalar.

The logic of EFT suggests that a complete action of gravity could include all possible terms consistent with the imposed symmetries, and the dimensions of these terms ought to be bounded due to renormalization. As a consequence, we add,

$$\begin{aligned}\Delta S_g \ = \ & \frac{1}{16\pi G}\int dt d^3x \sqrt{g} N \Big(\alpha_1 R_{ij}R^{ij} + \alpha_2 R^2 \\ & + \alpha_3 \nabla_i R_{jk} \nabla^i R^{jk} + \alpha_4 \nabla_i R_{jk} \nabla^j R^{ki} \\ & + \alpha_5 \nabla_i R \nabla^i R\Big) \ .\end{aligned}$$

which preserves parity and Galilean symmetries.

Equations of Motion

•By varying N and g_{ij}, we obtain the Friedmann equations:

$$H^2 = \frac{8\pi G}{3}\Big[\rho_m + \rho_k + \rho_{dr}\Big] ,$$
$$\dot{H} + \frac{3}{2}H^2 = -4\pi G\left[p_m - \frac{1}{3}\rho_k + \frac{1}{3}\rho_{dr}\right] ,$$

where we have derived a negative “dark radiation” which evolves proportional to a^{-4} for a curved spatial manifold.

•Therefore, there is a generic bounce for the matter component with EoS less than 1/3 and $k \neq 0$.

•If the matter component is realized by a free massive scalar, again we can obtain a matter bounce.