熵力在黑洞中的应用及实验检验

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Theories of gravity

1. The first apple: Newton's gravity
2. The second apple: Einstein's gravity
3. The third apple: String theory? Loop quantum gravity? Extra dimensions? Verlinde entropic force?

1. Introduction

E.P. Verlinde, "On the Origin of Gravity and the Laws of Newton", arXiv:1001.0785[hep-th].

1. Gravity can be explained as an entropic force—gravity is not fundamental
2. The holographic principle + the equipartition law of energy $\Rightarrow$ Newton's law of gravitation
3. A relativistic generalization leads to the Einstein equations

1. Introduction

In fact, the similar idea can be traced back to

1. **Sakharov:** (1968 "Vacuum quantum fluctuations in curved space and the theory of gravitation")
2. **Jacoboson:** (1995 "Thermodynamics of space-time: The Einstein equation of state")
3. **Padmanabhan:** (arXiv:gr-qc/0209088, arXiv:0706.1654, arXiv:0802.1798 , arXiv:0911.5004, arXiv:0912.3165, arXiv:1003.5665 etc.)

Applications, criticisms and developments (116 papers, China 32)

1. **FRW universe** (Y.G. Gong, R.G. Cai, Y. Ling, Y.F. Cai, J. Liu, H. Wei, Y.X. Chen, B. Liu, Y.X. Liu etc.)
2. **Black holes** (R.G. Cai, B.Q. Ma, X.G. He, X.N. Wu, Y.X. Liu etc.)
3. **Dark energy** (M. Li, Y. Wang, H. Wei etc.)
4. **Others** (C.J. Gao, T. Wang, L. Zhao, M. Li, B. Wang, Z. Chang, S.F. Wu etc.)

2. Review the idea of entropic force

In order to define the temperature, we first need to introduce the potential ϕ via the timelike Killing vector ξ^μ:

$$\phi = \frac{1}{2}\log(-\xi^\mu \xi_\mu), \tag{1}$$

where ξ_μ satisfies the Killing equation

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0. \tag{2}$$

The redshift factor is denoted by e^ϕ, and the reference point with $\phi = 0$ is taken to be at infinity.
The holographic screens are put at surfaces of constant redshift. So the entire screen has the same time coordinate.

2. Review the idea of entropic force

The four velocity u^μ and the acceleration a^μ of a particle located very close to the screen are

$$
\begin{aligned}
u^\mu &= e^{-\phi}\xi^\mu, &(3)\\
a^\mu &= u^\nu\nabla_\nu u^\mu = e^{-2\phi}\xi^\nu\nabla_\nu\xi^\mu = -\nabla^\mu\phi. &(4)
\end{aligned}
$$

Note that the acceleration is perpendicular to screen $\mathcal{S}$.

The local Unruh-Verlinde temperature T on the screen is defined by

$$
T = -\frac{\hbar}{2\pi}e^{\phi}n^\mu a_\mu = \frac{\hbar}{2\pi}e^{\phi}n^\mu\nabla_\mu\phi = \frac{\hbar}{2\pi}e^{\phi}\sqrt{\nabla^\mu\phi\nabla_\mu\phi}. \tag{5}
$$

2. Review the idea of entropic force

Assuming that the change of entropy at the screen is 2π for a displacement by one Compton wavelength normal to the screen, one has

$$\nabla_\mu S = -2\pi \frac{m}{\hbar} n_\mu. \tag{6}$$

The entropic force is turned out to be

$$F_\mu = T\nabla_\mu S = -me^\phi \nabla_\mu \phi, \tag{7}$$

where $-\nabla_\mu \phi$ is the relativistic analogue of Newton's acceleration, and the additional factor e^ϕ is due to the redshift.

2. Review the idea of entropic force

We now consider a holographic screen on a closed surface of constant redshift ϕ. The number of bit N of the screen is assumed to proportional to the area of the screen and is given by

$$N = \frac{A}{\hbar}. \tag{8}$$

Then, by assuming that each bit on the holographic screen contributes an energy $T/2$ to the system, and by using the equipartition law of energy, we get

$$E = \frac{1}{2}\int_{\mathcal{S}} TdN \tag{9}$$

with dN the bit density on the screen. Inserting the expressions of T and N into (9) results in

$$E = \frac{1}{4\pi}\int_{\mathcal{S}} e^{\phi}\nabla\phi \; dA. \tag{10}$$

3. Unruh-Verlinde Temperature and energy for static spherically symmetric black holes

Consider a 4D static spherically symmetric black holes

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2, \tag{11}$$

where $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\varphi^2$. We assume

$$\lim_{r\to\infty} f(r) = 1. \tag{12}$$

The event horizon radius r_{EH} is usually determined by the largest solution of $f(r) = 0$.

3. Unruh-Verlinde Temperature and energy for static spherically symmetric black holes

By using the Killing equation

$$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu - 2\Gamma^\lambda_{\mu\nu}\xi_\lambda = 0, \tag{13}$$

and the static spherically symmetric properties of the metric (11)

$$\partial_t \xi_\mu = \partial_\varphi \xi_\mu = 0, \tag{14}$$

as well as the condition $\xi_\mu \xi^\mu = -1$ at infinity, the timelike Killing vector of the general black hole is solved as

$$\xi_\mu = \left(-f(r), 0, 0, 0\right), \tag{15}$$

which is zero at the event horizon.

3. Unruh-Verlinde Temperature and energy for static spherically symmetric black holes

The potential, acceleration and temperature on the holographic screen put at a spherical surface are

$$\phi = \frac{1}{2}\ln f(r), \tag{16}$$

$$a^{\mu} = \left(0, -\frac{1}{2}f'(r), 0, 0\right), \tag{17}$$

$$T = \frac{\hbar}{4\pi}|f'(r)|. \tag{18}$$

The energy on the screen is

$$E = \frac{1}{2}|f'(r)|r^2. \tag{19}$$

3.1 The Schwarzschild black hole

For the Schwarzschild black hole,

$$f(r) = 1 - \frac{2M}{r}. \tag{20}$$

The event horizon is located at $r_{EH} = 2M$, and the Hawking temperature T_H on the event horizon is given by

$$T_H = \frac{\hbar}{4\pi r_{EH}} = \frac{\hbar}{8\pi M}. \tag{21}$$

The Unruh-Verlinde temperature and the energy on the screen are

$$T = \frac{\hbar M}{2\pi r^2}. \tag{22}$$

$$E = M. \tag{23}$$

Note that, on the event horizon $r = r_{EH}$, we have

$$T|_{r=r_{EH}} = T_H. \tag{24}$$

3.2 The RN black hole

For the RN black hole,

$$f(r) = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right). \qquad (25)$$

There are two horizons $r_\pm = M \pm \sqrt{M^2 - Q^2}$.
The Hawking temperatures $T_{H\pm}$ on the horizons are

$$T_{H\pm} = \frac{\hbar}{4\pi}\frac{(r_+ - r_-)}{r_\pm^2} = \frac{\hbar}{2\pi}\frac{\sqrt{M^2 - Q^2}}{r_\pm^2}. \qquad (26)$$

The Unruh-Verlinde temperature and the energy on the screen are

$$T = \frac{\hbar}{2\pi}\frac{|Mr - Q^2|}{r^3}, \qquad E = \left|M - \frac{Q^2}{r}\right|. \qquad (27)$$

E is the Komar energy. On the two horizons, we also have

$$T\big|_{r=r_\pm} = T_{H\pm}. \qquad (28)$$

4. Unruh-Verlinde Temperature and energy for the Kerr black hole

The Kerr metric reads

$$ds^2 = -\left(1 - \frac{2Mr}{\varrho^2}\right) dt^2 + \frac{\varrho^2}{\Delta} dr^2 + \varrho^2 d\theta^2$$
$$+ \left((a^2 + r^2)\sin^2\theta + \frac{2Mra^2\sin^4\theta}{\varrho^2}\right) d\varphi^2 - \frac{4Mra\sin^2\theta}{\varrho^2} dt d\varphi, \quad (29)$$

where $\Delta = r^2 - 2Mr + a^2$**, and** $\varrho = r^2 + a^2\cos^2\theta$**.**
The two horizons are

$$r_\pm = M \pm \sqrt{M^2 - a^2}. \quad (30)$$

The Hawking temperatures on the horizons are

$$T_{H\pm} = \frac{\hbar}{4\pi} \frac{(r_+ - r_-)}{(r_\pm^2 + a^2)} = \frac{\hbar}{4\pi} \frac{\sqrt{M^2 - a^2}}{Mr_\pm}. \quad (31)$$

4. Unruh-Verlinde Temperature and energy for the Kerr black hole

The solution for the timelike Killing vector ξ_μ is [Tian and Wu, PRD81(2010)104013]

$$\xi_\mu = \left(-1 + \frac{2Mr(\Omega - 2a^2 Mr\sin^2\theta)}{\Omega\varrho^2}, 0, 0, 0\right), \tag{32}$$

where $\Omega = (r^2 + a^2)^2 - a^2\Delta\sin^2\theta$. The corresponding potential is

$$\phi(r,\theta) = -\frac{1}{2}\log\left(1 + \frac{2Mr(r^2 + a^2)}{\Delta\varrho^2}\right). \tag{33}$$

The holographic screens are axisymmetric.
However, when $r \gg r_+$ or $r \to r_\pm$, the holographic screens are approximate spherically symmetric.

4. Unruh-Verlinde Temperature and energy for the Kerr black hole

The non-zero components of the acceleration are

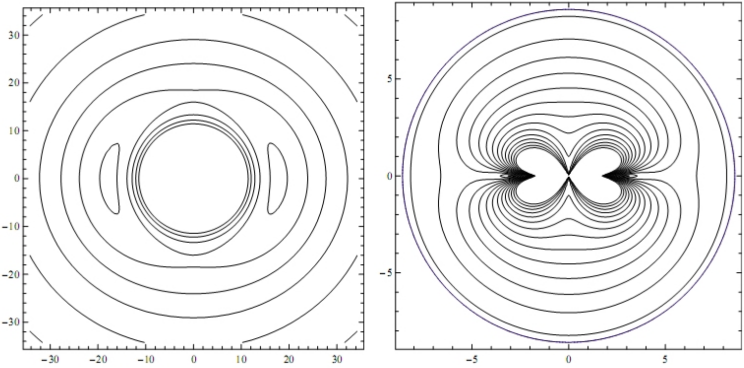
$$a^r = \frac{2Mr(a^2+r^2)\left(M - r - \frac{\Delta r}{\varrho^2}\right) + M\Delta(a^2+3r^2)}{\varrho^2[\Delta\varrho^2 + 2Mr(a^2+r^2)]},$$

$$a^\theta = \frac{Mra^2(a^2+r^2)\sin(2\theta)}{\varrho^4[2Mr(a^2+r^2)+\Delta\varrho^2]}. \quad (34)$$

The Unruh-Verlinde temperature is given by

$$T = \frac{h}{2\pi}\left\{\frac{M^2\left[\left(a^2+3r^2\right)\Delta\varrho^2 + 2r\left(a^2+r^2\right)\left(M\varrho^2 - r\left(\Delta+\varrho^2\right)\right)\right]^2}{\varrho^4\left[2Mr\left(a^2+r^2\right)+\Delta\varrho^2\right]^3} + \frac{\Delta\left[a^2 Mr(a^2+r^2)\sin(2\theta)\right]^2}{\varrho^4\left[2Mr\left(a^2+r^2\right)+\Delta\varrho^2\right]^3}\right\}^{\frac{1}{2}}. \quad (35)$$

4. Unruh-Verlinde Temperature and energy for the Kerr black hole



(a) $r \geq r_+ = 11.4$ (b) $r \leq r_- = 8.6$

Figure: A contour plot of the Unruh-Verlinde temperature ϕ in the y-z plane for the Kerr black hole with $M = 10$ and $a = 9.9$.

4. Unruh-Verlinde Temperature and energy for the Kerr black hole

On the horizons $r = r_{\pm}$ ($\Delta = 0$), the Unruh-Verlinde temperatures become

$$\begin{aligned} T\big|_{r=r_{\pm}} &= \frac{\hbar}{2\pi}\sqrt{\frac{(M-r)^2}{2Mr(a^2+r^2)}}\ \Bigg|_{r=r_{\pm}} \\ &= \frac{\hbar}{4\pi}\frac{\sqrt{M^2-a^2}}{Mr_{\pm}} = T_{H\pm}. \end{aligned} \tag{36}$$

So, the Unruh-Verlinde temperatures on both horizons are equal to the Hawking temperatures.

4. Unruh-Verlinde Temperature and energy for the Kerr black hole

The energies on the horizons

$$E|_{r=r_\pm} = \frac{\sqrt{M^2 - a^2}}{2Mr_\pm}\left(r_\pm^2 + a^2\right) = \sqrt{M^2 - a^2}, \qquad (37)$$

which is the reduced mass M_0 of the Kerr black hole.
For $r \gg r_+$,

$$E \approx M\left(1 - 4\frac{a^2 M}{r^3}\right). \qquad (38)$$

For $r \to \infty$, the result is $E = M$, which is independent of the angular of the Kerr black hole.

The entropic force (7) for a static spherically symmetric black hole can be calculated as

$$F_\mu = (0, -\frac{mf'(r)}{2\sqrt{f(r)}}, 0, 0). \tag{39}$$

The magnitude of the force is

$$F = \sqrt{g^{\mu\nu} F_\mu F_\nu} = \frac{1}{2} mf'(r). \tag{40}$$

For the Schwarzschild black hole, the entropic force F is just the Newton force

$$F = \frac{GmM}{r^2}. \tag{41}$$

For the RN black hole, the entropic force is

$$F = \frac{GmM}{r^2}\left(1 - \frac{Q^2}{Mr}\right). \tag{42}$$

There is a corrected term caused by the charge of the black hole.
This force is related to the pure gravitational effect. The gravity or the geometry near the event horizon of the black hole is affected by the energy of the electric field.

The entropic force of the Kerr black hole is

$$F = \frac{Gm\sqrt{M^2 - a^2}}{r^2} \qquad (r = r_+) \tag{43}$$

$$F \approx \frac{GmM}{r^2}\left(1 - 4\frac{a^2 M}{r^3}\right) \qquad (r \gg r_+) \tag{44}$$

It also reduces at large r to the Newton force because the effect of the "field" caused by the angular momentum of the black hole becomes weaker with the increase of r.

5. The entropic force and its test

Discuss

1. The energy got from the holographic principle is indeed the Komar energy on the screen.
2. The energy E on the screen will trend to the ADM mass M when $r \to \infty$
3. According the entropic force idea, it is the reduced mass M_0 that takes the place of the black hole mass M.

Schwarzschild: $M_0 = M$

RN: $M_0 = M - \frac{Q^2}{r}$

Kerr: $M_0 = \sqrt{M^2 - a^2}$ at $r = r_+$

$$M_0 \approx M\left(1 - 4\frac{a^2 M}{r^3}\right) \text{ at } r \gg r_+$$

5. The entropic force and its test

Therefore, we can rewrite the entropic force of a black hole as

$$F = \frac{GmM_0}{r^2}. \tag{45}$$

This formula shows that, according entropic force idea, the force between a neutral particle and a charged particle would departure from the Newton's law of gravity, i.e.,

$$F = \frac{Gm}{r^2}\left(M - \frac{Q^2}{r}\right). \tag{46}$$

Hence, the idea of entropic force could be tested by gravity experiments.

6. Discussions and conclusions

The potential $\phi = \frac{1}{2}\log(-\xi^2)$ is divergent when the screen approaches the black hole horizon.

So, a new potential $\phi = -\frac{1}{2}(1+\xi^2)$ was introduced by Tan and Wu (arxiv:1002.1275).

For this new non-divergent potential, we have also the same entropic force:

$$F = \frac{Gm}{2}\sqrt{\frac{(\nabla^\mu\xi^2)(\nabla_\mu\xi^2)}{-\xi^2}}. \qquad (47)$$

6. Discussions and conclusions

1. The energy E on the screen is the reduce mass M_0 for a black hole
2. The Unruh-Verlinde temperatures on both horizons are equal to the Hawking temperatures
3. The idea of entropic force could be tested by gravity experiments

Thank you!