Wilson loops, M2-branes and AdS4/CFT3 duality

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Papers

- Based on
- Bin Chen, JW, NPB 25(2010)38, 0809.2863
- JW, Meng-Qi Zhu, 1312.3030
- Bin Chen, JW, Meng-Qi Zhu, to appear

Motivations

- Wilson loop plays an important role in the studies of confinement in quantum non-abelian gauge theories.
- In simplest AdS_5/CFT_4 correspondence, we have for Wilson loops
- Simplest loop <-----> Simplest embedding of F-Strings

- But for simplest AdS_4/CFT_3 duality
- simplest embedding <---NOT--> simplest loop
- We want to ask, what happens for complicated AdS4/CFT3 duality

Backgrounds

- Soon after the proposal of AdS/CFT correspondence, the gravity dual of Wilson loops in *N*=4 super Yang-Mills theory is proposed. *[Rey Yee, 98] [Maldacena 98]*
- This loop includes scalars in the definition. When the trace is taken in the fundamental representation.

$$W[C] = \frac{1}{N} \mathrm{Tr}\, \mathcal{P} \exp\left(\oint (i A_\mu \dot{x}^\mu + \Phi_i \dot{y}^i) ds \right),$$

- The gravity dual is a F-string. The boundary of the string worldvolume is the loop.

Backgrounds

- The simplest loop in the field theory side, is the ½-BPS Wilson lines (a straight line or a circular loop with a special y).
- In the classical SUGRA limit, the worldvolume gives a minimal surface in $AdS_5 * S^5$.
- The simplest embedding is AdS_2 inside AdS_5, which is half-BPS as well.
- Simplest loop <-----> Simplest embedding

- Same loops in 4d $\mathcal{N}=2$ SYM is ½-BPS, too.

Backgrounds

- Things are different when we turn to AdS4/CFT3 correspondence.
- Let us consider the simplest known example.
- Aharony-Bergman-Jafferis-Maldacena theory is a 3d Chern-Simons-matter theory with $\mathcal{N}$=6 supersymmetries. The gauge group is U(N)*U(N) with Chern-Simons levels k and –k.
- The matter fields are scalars, C_I and fermions Ψ^I in the bi-fundamental representation of the gauge group. Here I=1, …, 4 and the R symmetry group is SU(4).

Properties of ABJM theory

- 1/k is the coupling constant.
- The theory has a large N (planar or ‘t Hooft) limit:

$$N \to \infty, k \to \infty, \lambda \equiv \frac{N}{k} \text{ fixed}$$

- This theory is found to be the LEFT for N M2-branes putting on the tip of the orbifold $\mathbf{C}^4/\mathbf{Z}_k$.
- For k=1 or 2, supersymmetries are enhanced to $\mathcal{N}$=8 dynamically.

The gravity dual

- When $N >> k^5$, this theory is dual to M-theory on $AdS_4 * S^7/Z_k$.

- While when $k << N << k^5$, a better description is in terms of type IIA string theory on $AdS_4 * CP^3$.

- Let us consider F-string in $AdS_4 * CP^3$ dual to Wilson loops. The simplest embedding is still AdS_2 inside AdS_4 and still half-BPS.

BPS Wilson loops

- Before ABJM theory appeared, Gaiotto and Yin constructed BPS Wilson loops in general 3d *N*=2 and *N*=3 CSM theories. The loop is 1/2-BPS and 1/3-BPS, respectively.

$$P\exp\left[\int d\tau\left(A_\mu\dot{x}^\mu+\sigma|\dot{\vec{x}}|\right)\right]$$

$$P\exp\left[\int d\tau\left(A_\mu\dot{x}^\mu+s_i\dot{y}^i\right)\right]$$

1/6-BPS Wilson loops in ABJM theory

Several groups constructed BPS Wilson loop in ABJM theory via Gaiotto-Yin and found this loop is 1/6-BPS. *[Chen, Wu, 08]* *[Drukker etal, 08]* *[Rey etal, 08]* ($A_1=C_1$, $A_2=C_2$, $B_1{}^*=C_3$, $B_2{}^*=C_4$)

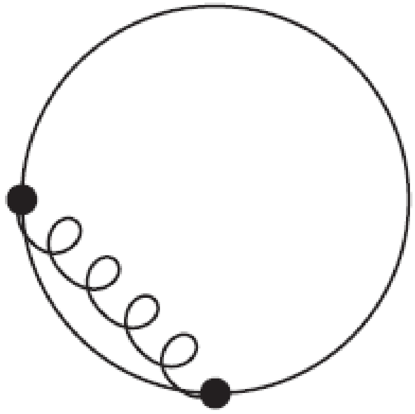
$$\frac{1}{N} Tr_{U_1(N)} P\exp\left(i\int d\tau (A_\mu \dot{x}^\mu + i\frac{2\pi}{k}(A_i A_i^\dagger - B_i^\dagger B_i)|\dot{\vec{x}}|)\right)$$

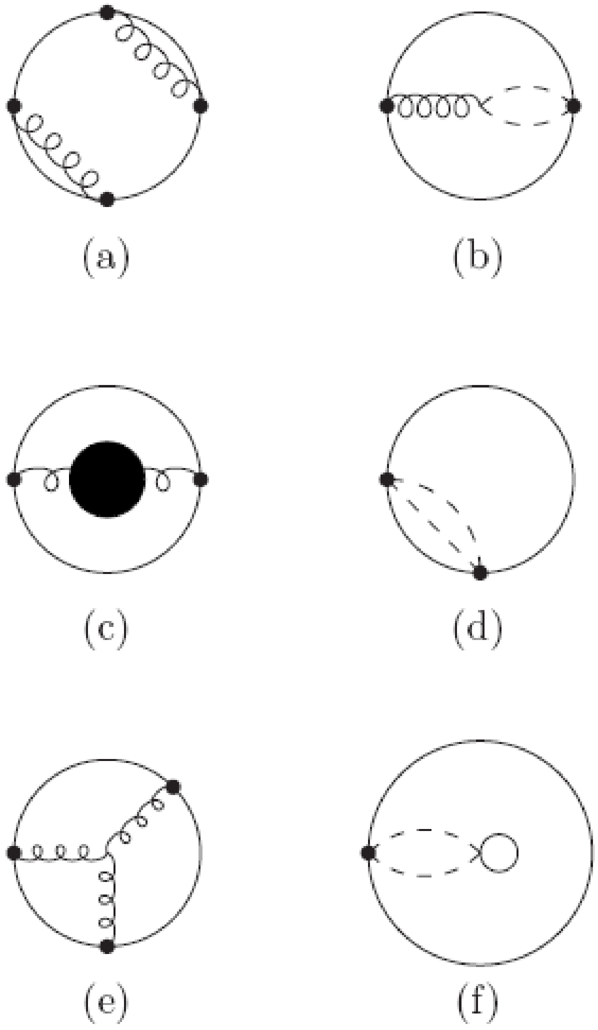
$$\times \quad \frac{1}{N} Tr_{U_2(N)} P\exp\left(i\int d\tau (\hat{A}_\mu \dot{x}^\mu + i\frac{2\pi}{k}(A_i^\dagger A_i - B_i B_i^\dagger)|\dot{\vec{x}}|)\right).$$

- GY loop operators seem to only preserve two supercharges for 3d *N*>1 theories.

Perturbative computations

- We consider a circular 1/6-BPS Wilson loop.
- The contribution at the order of λ vanishes.
- The contribution at the order of λ^2 is $\frac{5}{3}\pi^2\lambda^2$





Supersymmetric localization

- These results are recently confirmed by computations using **supersymmetric localization**.

  [A. Kapustin, B. Willett, I. Yaakov, 09]

½-BPS Wilson loops

- 1/2-BPS loops were constructed later in *[Drukker, Trancanelli, 09]*, the construction was further explained in *[K. Lee, S. Lee, 10]*

$$L \equiv \begin{pmatrix} A_\mu \dot{x}^\mu + \frac{2\pi}{k}|\dot{x}| M_J^I C_I \bar{C}^J & \sqrt{\frac{2\pi}{k}}\, |\dot{x}|\, \eta_I^\alpha \bar{\psi}_\alpha^I \\ \sqrt{\frac{2\pi}{k}}\, |\dot{x}|\, \psi_I^\alpha \bar{\eta}_\alpha^I & \hat{A}_\mu \dot{x}^\mu + \frac{2\pi}{k}|\dot{x}| \widehat{M}_J^I \bar{C}^J C_I \end{pmatrix},$$

$$W_{\mathcal{R}} \equiv \mathrm{Tr}_{\mathcal{R}}\, \mathcal{P} \exp\left(i \int L\, d\tau\right)$$

- So now, simplest embedding <---NOT--> simplest loop.
- How about the stories in 3d CSM theories with less supersymmetries?

The CSM theory dual to AdS4*Y7 (From Jafferis' talk@Strings 2009)

Y7	SUSY	CSM theory
S^7 or $S^7/Z2$	$\mathcal{N}$=8	ABJM with k=1, 2
S^7/Z_k	$\mathcal{N}$=6	ABJM with k>2
S^7/D_k	$\mathcal{N}$=5	$O(2N)_{2k}*Usp(N)_{-k}$
S^7/Γ	$\mathcal{N}$=4	Orbifolds Necklace quiver theories
3-Sasakian manifolds	$\mathcal{N}$=3	1-loop N=3 quivers
Sasaki-Einstein	$\mathcal{N}$=2	CY3 quivers (sum of CS level is zero)

Wilson loops

SUSY	Field theory	M-theory (some results)
N=6	1/6 BPS (GY-type) 1/2 BPS(DT)	1/6 BPS (from smearing) *[Drukker Plefka Young 08]* 1/2 BPS
N=5	2/5 BPS(*Lee-Lee*) GY type	
N=4	Construction of DT/LL type?	? BPS
N=3	1/3 BPS (GY)	? BPS (Main topics)
N=2	1/2 BPS (GY)	½ BPS (Farquet, Sparks; *JW, Zhu* for $Y7=Q^{111}$)

AdS4*N(1, 1)

- The metric on *AdS₄* N(1, 1)* is

$$ds^2 = R^2(\frac{1}{4}ds^2_{AdS_4} + ds^2_{N(1,1)}),$$

$$ds^2_{AdS_4} = \cosh^2 u(-\cosh^2\rho dt^2 + d\rho^2) + du^2 + \sinh^2 u d\phi^2,$$

$$ds^2_{N(1,1)} = \frac{1}{2}(d\alpha^2 + \frac{1}{4}\sin^2\alpha(\sigma_1^2 + \sigma_2^2) + \frac{1}{4}\sin^2\alpha\cos^2\alpha\sigma_3^2$$

$$+ \quad \frac{1}{2}(\Sigma_1 - \cos\alpha\sigma_1)^2 + \frac{1}{2}(\Sigma_2 - \cos\alpha\sigma_2)^2 + \frac{1}{2}(\Sigma_3 - \frac{1}{2}(1 + \cos^2\alpha)\sigma_3)^2)$$

1-forms

$$
\begin{aligned}
\sigma_1 &= \sin\phi_1 d\theta_1 - \cos\phi_1 \sin\theta_1 d\psi_1, \\
\sigma_2 &= \cos\phi_1 d\theta_1 + \sin\phi_1 \sin\theta_1 d\psi_1, \\
\sigma_3 &= d\phi_1 + \cos\theta_1 d\psi_1, \\
\Sigma_1 &= \sin\phi_2 d\theta_2 - \cos\phi_2 \sin\theta_2 d\psi_2, \\
\Sigma_2 &= \cos\phi_2 d\theta_2 + \sin\phi_2 \sin\theta_2 d\psi_2, \\
\Sigma_3 &= d\phi_2 + \cos\theta_2 d\psi_2.
\end{aligned}
$$

4-form flux

$$H = -\frac{3}{8} R^3 \cosh^2 u \cosh \rho \sinh u dt \wedge d\rho \wedge du \wedge d\phi.$$

KSE

- The Killing spinor equation is

$$\nabla_{\underline{m}}\eta + \frac{1}{576}(3\Gamma_{\underline{n}\underline{p}\underline{q}\underline{r}}\Gamma_{\underline{m}} - \Gamma_{\underline{m}}\Gamma_{\underline{n}\underline{p}\underline{q}\underline{r}})H^{\underline{n}\underline{p}\underline{q}\underline{r}}\eta = 0.$$

$$\nabla_{\underline{m}}\eta = e^{\mu}_{\underline{m}}\partial_{\underline{\mu}}\eta + \frac{1}{4}\omega^{\underline{ab}}_{\underline{m}}\Gamma_{\underline{ab}}\eta$$

- The spin connection is obtained from the Cartan structure equation

$$de^{\underline{m}} + \omega^{\underline{m}}_{\underline{n}} \wedge e^{\underline{n}} = 0$$

- Integrability of KSE give

$$C^{\underline{abcd}}\Gamma_{\underline{ab}}\eta = 0,$$

$$\Gamma_{\underline{4567}}\eta = \eta.$$

Killing spinors

$$\eta = e^{\frac{\phi_2}{2}(\Gamma_{\underline{47}}+\Gamma_{\underline{89}})} e^{\frac{\theta_2}{2}(\Gamma_{\underline{46}}-\Gamma_{\underline{8\sharp}})} e^{\frac{\psi_2}{2}(\Gamma_{\underline{47}}+\Gamma_{\underline{89}})}$$
$$e^{-\frac{u}{2}\Gamma_{\underline{2}}\hat{\Gamma}} e^{-\frac{\rho}{2}\Gamma_{\underline{1}}\hat{\Gamma}} e^{-\frac{t}{2}\Gamma_{\underline{0}}\hat{\Gamma}} e^{\frac{\phi}{2}\Gamma_{\underline{23}}}\eta_0,$$

$$\Gamma_{\underline{4567}}\eta_0 = -\eta_0,\ (\Gamma_{\underline{58}} + \Gamma_{\underline{69}} + \Gamma_{\underline{7\sharp}} - \Gamma_{\underline{4}}\hat{\Gamma}) = 0.$$

- This give 12 supercharges [Page-Pope]. This is consistent with the duality with 3d N=3 superconformal field theory.

Killing spinors of the orbifolds

- We search for Killing vector K such that

$$\mathcal{L}_K \eta \equiv K^{\underline{m}} \nabla_{\underline{m}} \eta + \frac{1}{4} (\nabla_{\underline{m}} K_{\underline{n}}) \Gamma^{\underline{mn}} \eta.$$

vanishes.

$$\begin{aligned}
K_1 &= -\sin\psi_1 \partial_{\theta_1} + \frac{1}{\sin\theta_1} \cos\psi_1 \partial_{\phi_1} - \cot\theta_1 \cos\psi_1 \partial_{\psi_1}, \\
K_2 &= \cos\psi_1 \partial_{\theta_1} + \frac{1}{\sin\theta_1} \sin\psi_1 \partial_{\phi_1} - \cot\theta_1 \sin\psi_1 \partial_{\psi_1}, \\
K_3 &= \partial_{\psi_1}, \\
K_4 &= \partial_{\phi_1} + \partial_{\phi_2},
\end{aligned}$$

Probe M2-branes

- Bosonic part of M2 action

$$S_{M2} = T_2 \left(\int d^3\xi \sqrt{-\det g_{\mu\nu}} - \int P[C_3] \right),$$

- Equations of motion

$$\begin{aligned} & \frac{1}{\sqrt{-g}} \partial_m \left(\sqrt{-g} g^{mn} \partial_n X^{\underline{N}} \right) G_{\underline{MN}} + g^{mn} \partial_m X^{\underline{N}} \partial_n X^{\underline{P}} \Gamma^{\underline{Q}}_{\underline{NP}} G_{\underline{QM}} \\ = \quad & \frac{1}{3!} \epsilon^{mnp} (P[H_4])_{\underline{M}mnp}. \end{aligned}$$

M2 branes

- M2 branes dual to Wilson loops with worldvolume AdS_2*S^1 (AdS_2 in AdS_4, S^1 in N(1,1))
- Ansatz 1:

$$t = \tau, \rho = \xi, \psi_1 = \sigma.$$

- Equations of motion give $u = \alpha = 0,$ or $u = 0, \alpha = \frac{\pi}{2}, \theta_1 = 0, \frac{\pi}{2}.$
- Ansatz 2:

$$t = \tau, \rho = \xi, \phi_1 = \sigma, \phi_2 = \phi_0 + \sigma,$$

- Equations of motion give $u = 0, \alpha = 0, \pi/2.$

BPS condition

$$\Gamma_{M2}\eta = \eta,$$

with

$$\Gamma_{M2} = \frac{1}{\sqrt{-\det g_{\mu\nu}}}\partial_\tau X^{\mu_1}\partial_\xi X^{\mu_2}\partial_\sigma X^{\mu_3} e^{\underline{m}_1}_{\mu_1} e^{\underline{m}_2}_{\mu_2} e^{\underline{m}_3}_{\mu_3}\Gamma_{\underline{m}_1\underline{m}_2\underline{m}_3},$$

We found for ansatz 1, only the brane solutions put at

$$u = 0, \theta_1 = \theta_2 = 0, \alpha = 0, \pi/2.$$

are BPS and they are 1/3-BPS. The brane solutions in ansatz 2 put at θ_2=0are 1/3-BPS.

Wilson loops

SUSY	Field theory	M-theory (some results)
$\mathcal{N}$=6	1/6 BPS (GY-type) 1/2 BPS(DT)	1/6 BPS (from smearing) 1/2 BPS
$\mathcal{N}$=5	2/5 BPS(Lee-Lee) GY type	
$\mathcal{N}$=4	Construction of DT/LL type? (work in progress)	1/2 BPS for $Y7=S7/\Gamma$ (preliminary results)
$\mathcal{N}$=3	1/3 BPS (GY)	1/3 BPS for $Y7=N(1,1)$(Chen, JW, Zhu)
$\mathcal{N}$=2	½ BPS (GY)	½ BPS (Farquet, Sparks, JW, Zhu for Y7= $Y7=Q^{111}$)

Patterns (messages to take home)

- The Wilson loops seem to be at most half-BPS.

- For 3d SCFT with $\mathcal{N}$=3, 5 supersymmetries, the Wilson loops seem never to be half-BPS.

- Supersymmetries enhanced for the Wilson loops beyond Gaiotto-Yin type seems to start at $\mathcal{N}$=4.

Thanks for attentions!