Primordial Black Holes and Scalar Induced Gravitational Waves from Inflation with Nonminimal Derivative Coupling

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PRD 100, 063532 (2019)；in preparation

弦论、场论与宇宙学相关专题研讨会

宜昌 2019.11.23

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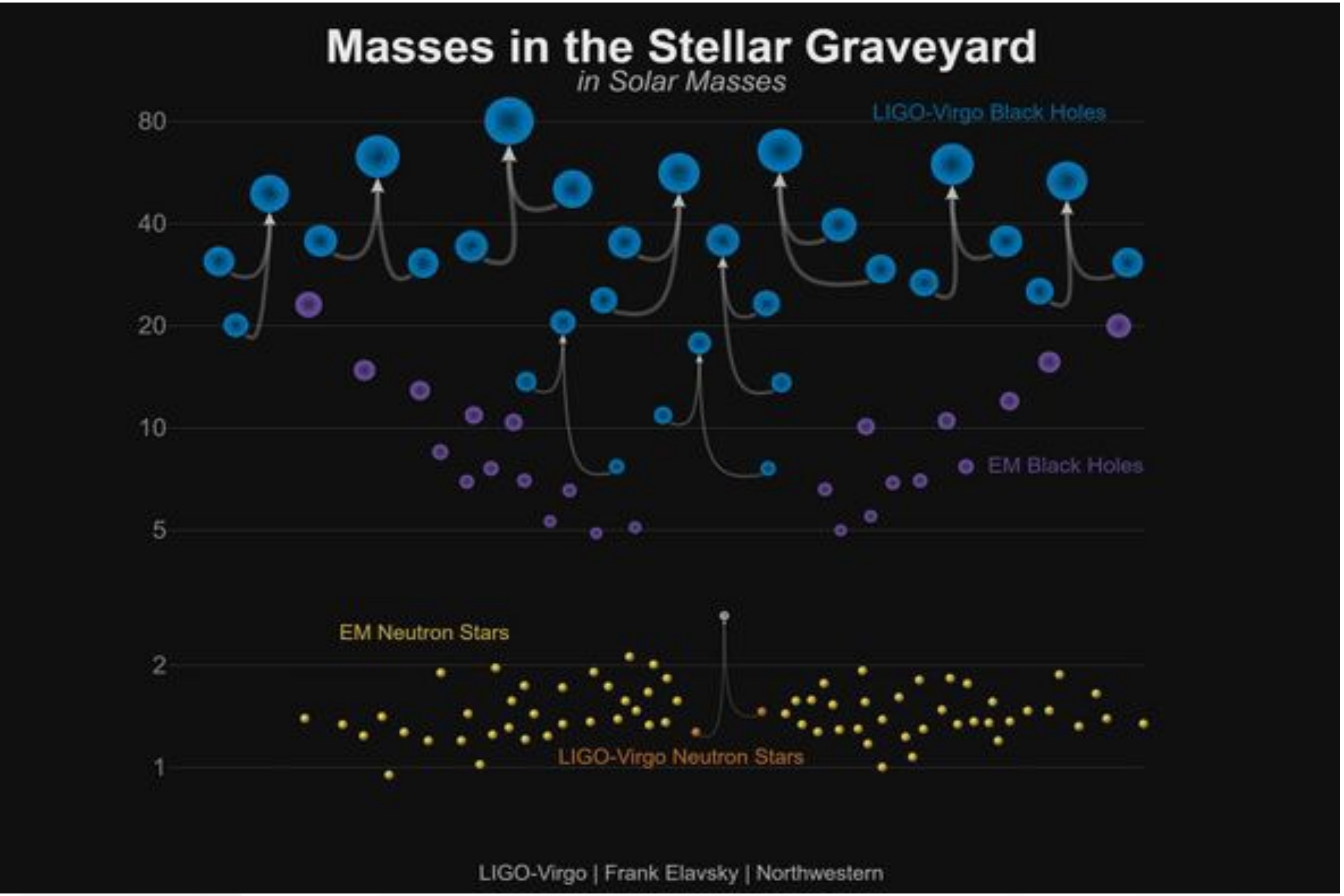
Motivation

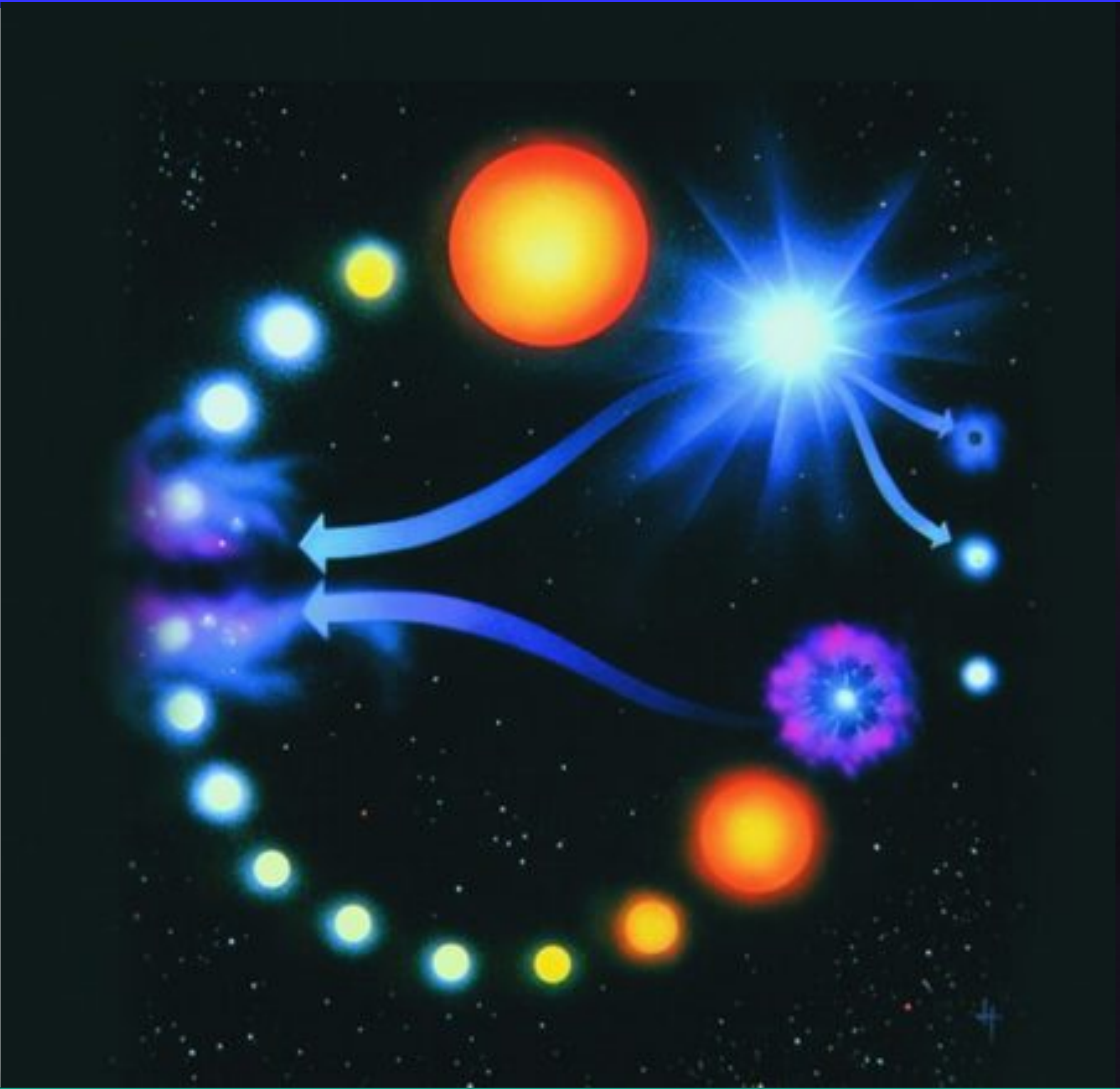
LIGO/Virgo gravitational wave events

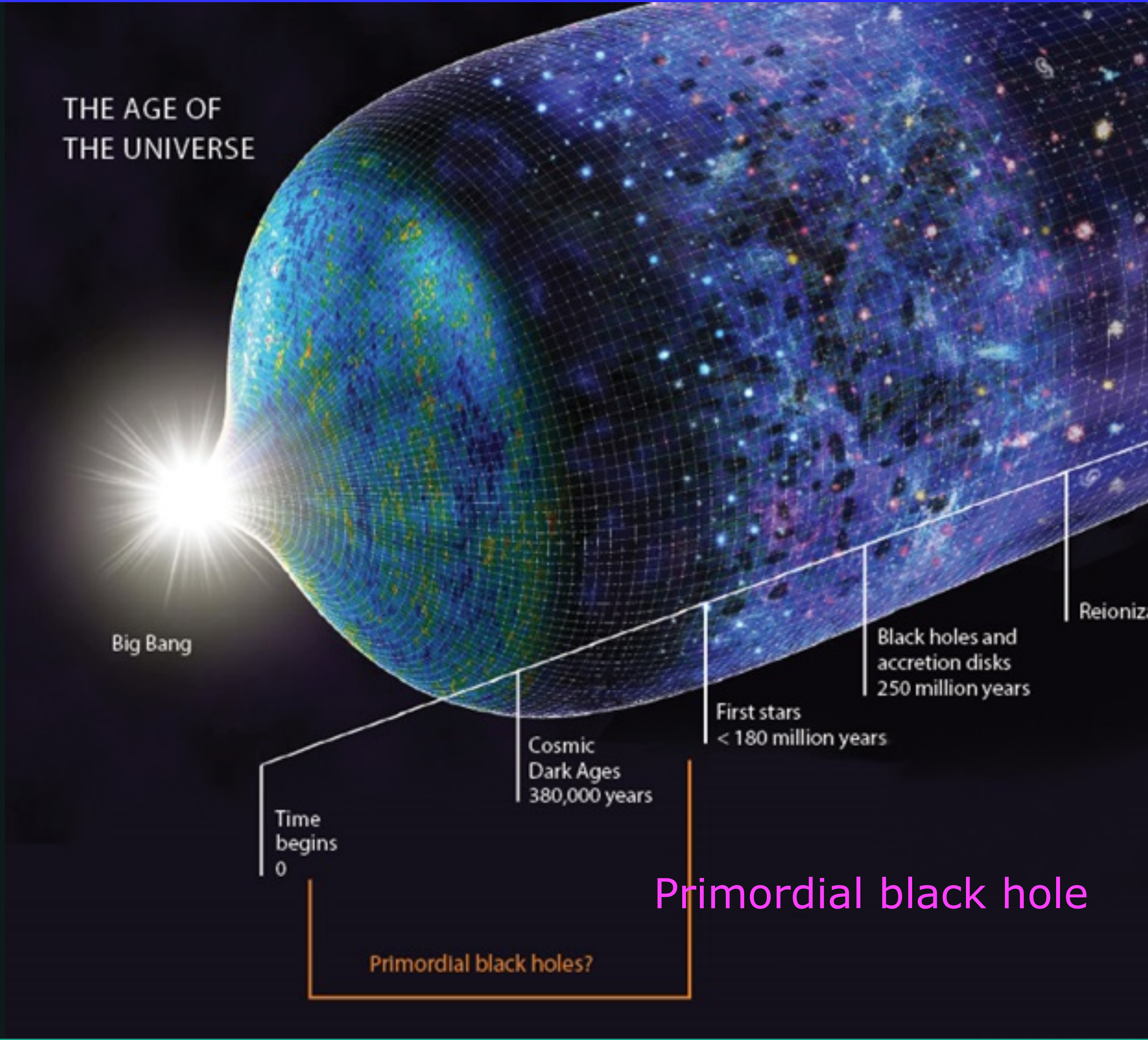


Event	$m_1/M_\odot$	$m_2/M_\odot$	$\mathcal{M}/M_\odot$
GW150914	$35.6^{+4.7}_{-3.1}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.7}_{-1.5}$
GW151012	$23.2^{+14.9}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.1}_{-1.2}$
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.5}$	$8.9^{+0.3}_{-0.3}$
GW170104	$30.8^{+7.3}_{-5.6}$	$20.0^{+4.9}_{-4.6}$	$21.4^{+2.2}_{-1.8}$
GW170608	$11.0^{+5.5}_{-1.7}$	$7.6^{+1.4}_{-2.2}$	$7.9^{+0.2}_{-0.2}$
GW170729	$50.2^{+16.2}_{-10.2}$	$34.0^{+9.1}_{-10.1}$	$35.4^{+6.5}_{-4.8}$
GW170809	$35.0^{+8.3}_{-5.9}$	$23.8^{+5.1}_{-5.2}$	$24.9^{+2.1}_{-1.7}$
GW170814	$30.6^{+5.6}_{-3.0}$	$25.2^{+2.8}_{-4.0}$	$24.1^{+1.4}_{-1.1}$
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$1.186^{+0.001}_{-0.001}$
GW170818	$35.4^{+7.5}_{-4.7}$	$26.7^{+4.3}_{-5.2}$	$26.5^{+2.1}_{-1.7}$
GW170823	$39.5^{+11.2}_{-6.7}$	$29.0^{+6.7}_{-7.8}$	$29.2^{+4.6}_{-3.6}$

B. P. Abbott *et al.* , Phys. Rev. X **9**, 031040 (2019)

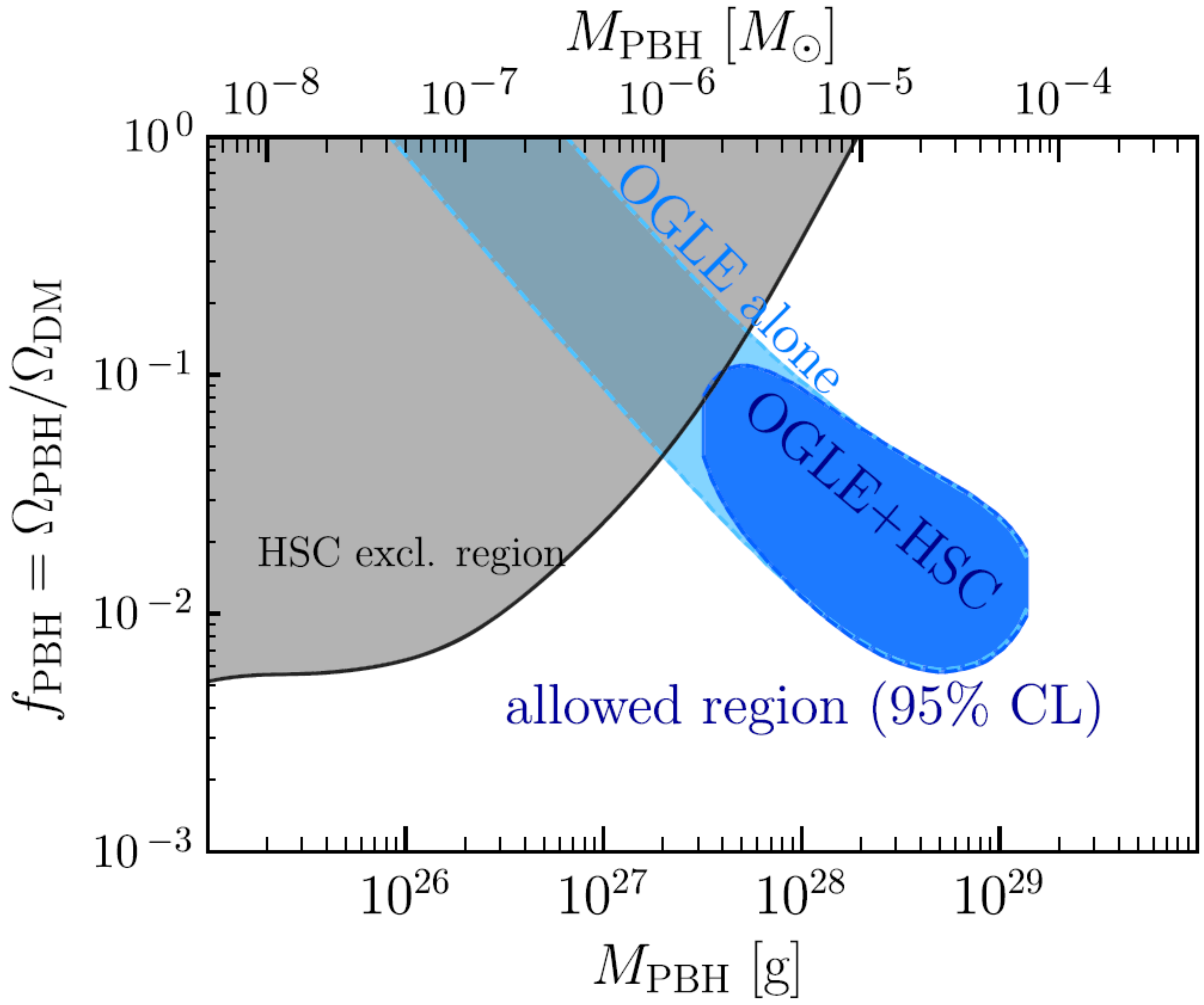
Formation of black hole





Primordial black hole

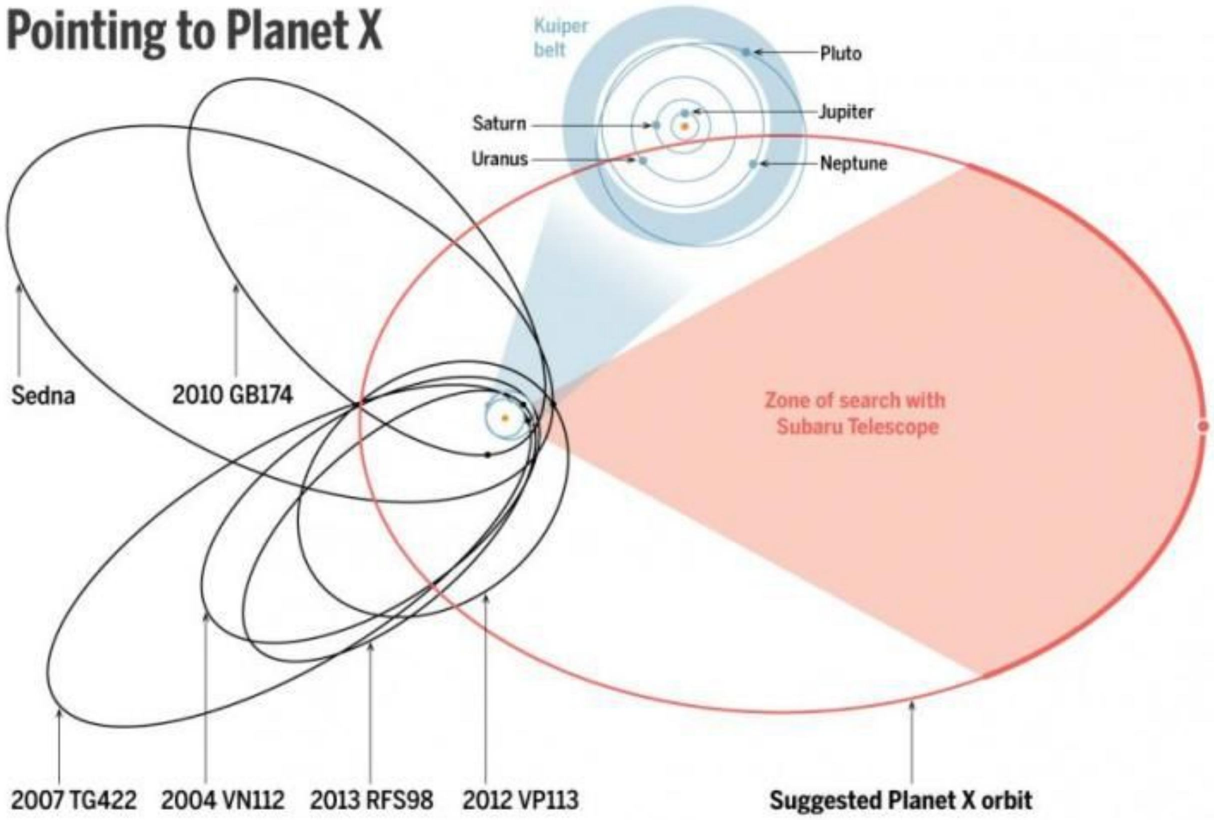
Motivation Ultrashort-timescale microlensing events in the OGLE data



[1] P. Mróz *et al.* , Nature (London) **548**, 183 (2017) [2] H. Niikura *et al.* , Phys. Rev. D **99**, 083503 (2019)

Motivation

What if Planet 9 is a Primordial Black Hole (PBH)?



$$M_9 \sim 5 - 15 M_\oplus$$

A. SIZE OF THE PBH

The Schwarzschild radius of a black hole is given by

$$r_{\rm BH} = \frac{2GM_{\rm BH}}{c^2} \simeq 4.5{\rm cm}\left(\frac{M_{\rm BH}}{5M_\oplus}\right) . \qquad (15)$$

In Figure 1 we provide an exact scale image of a $5M_\oplus$ PBH. The associated DM halo however extends to the stripping radius $r_{t,\odot} \sim 8$AU, this would imply a DM halo which extends roughly the distance from Earth to Saturn (both in real life and relative to the image).

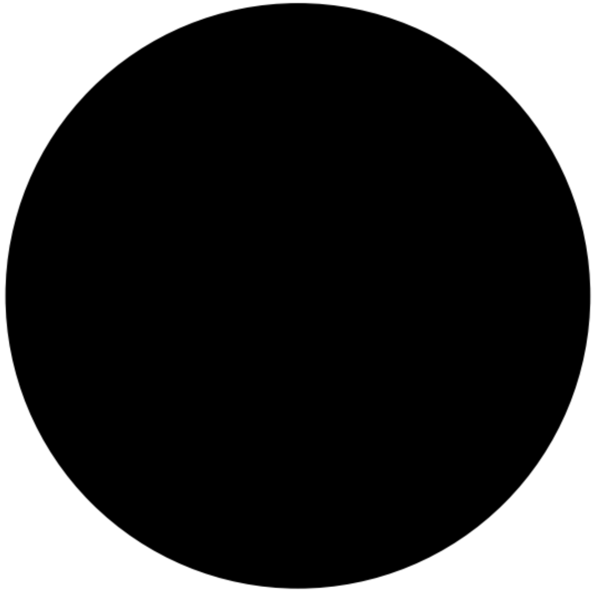
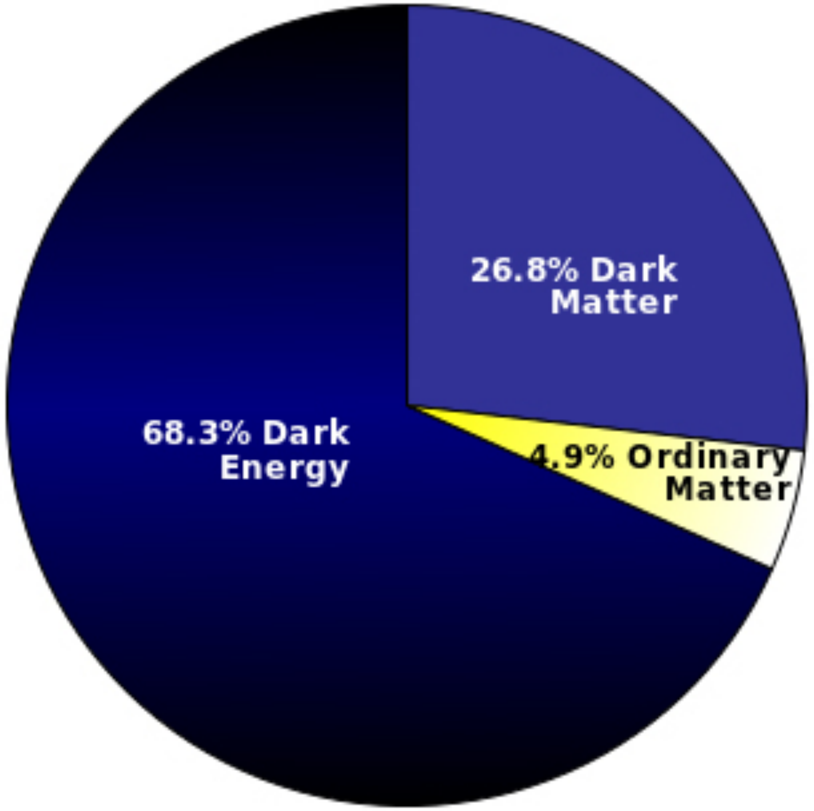


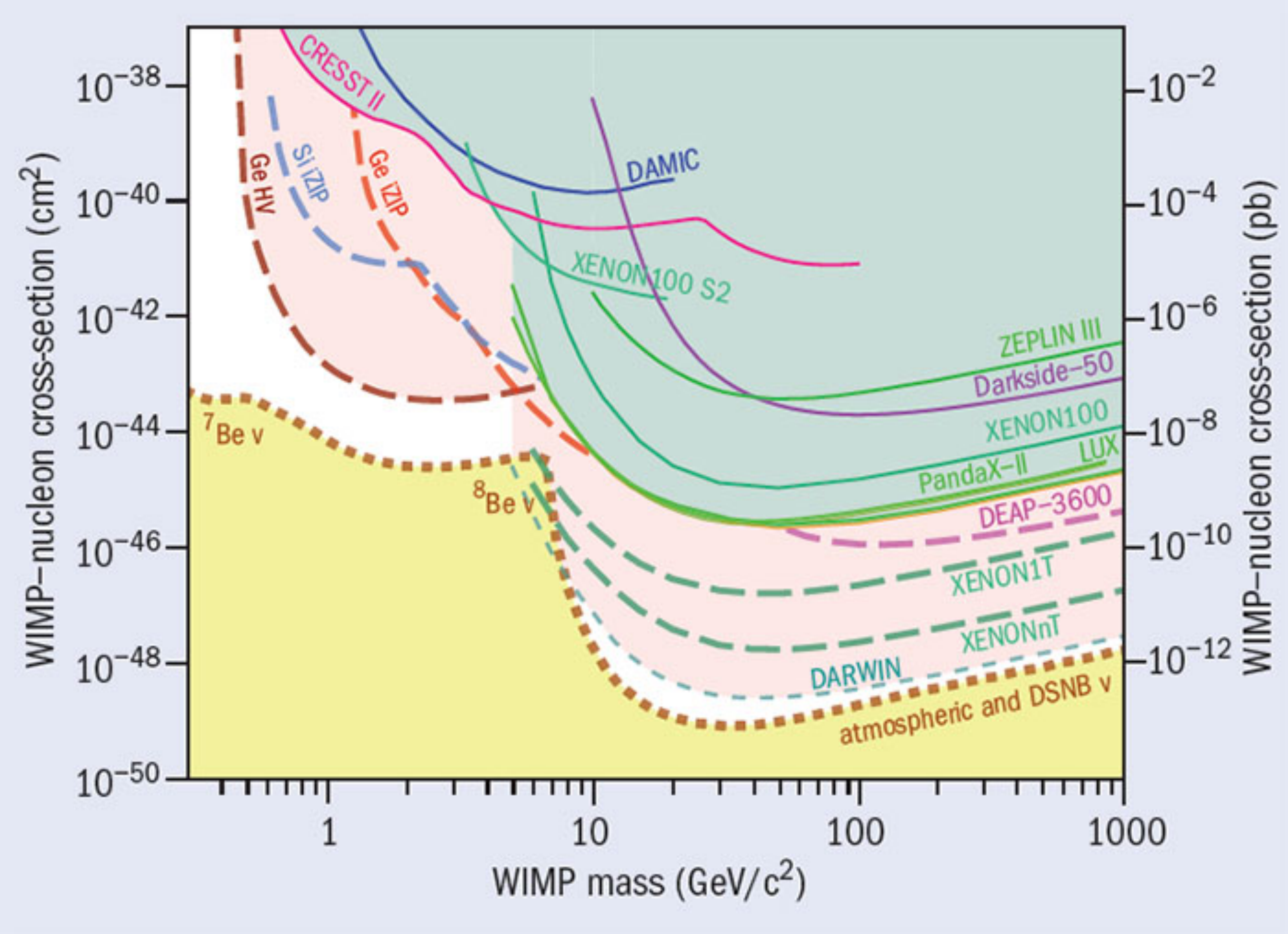
FIG. 1. Exact scale (1:1) illustration of a $5M_\oplus$ PBH. Note that a $10M_\oplus$ PBH is roughly the size of a ten pin bowling ball.

J. Scholtz1 and J. Unwin, 1909.11090

Motivation

Primordial black hole is a possible candidate of dark matter

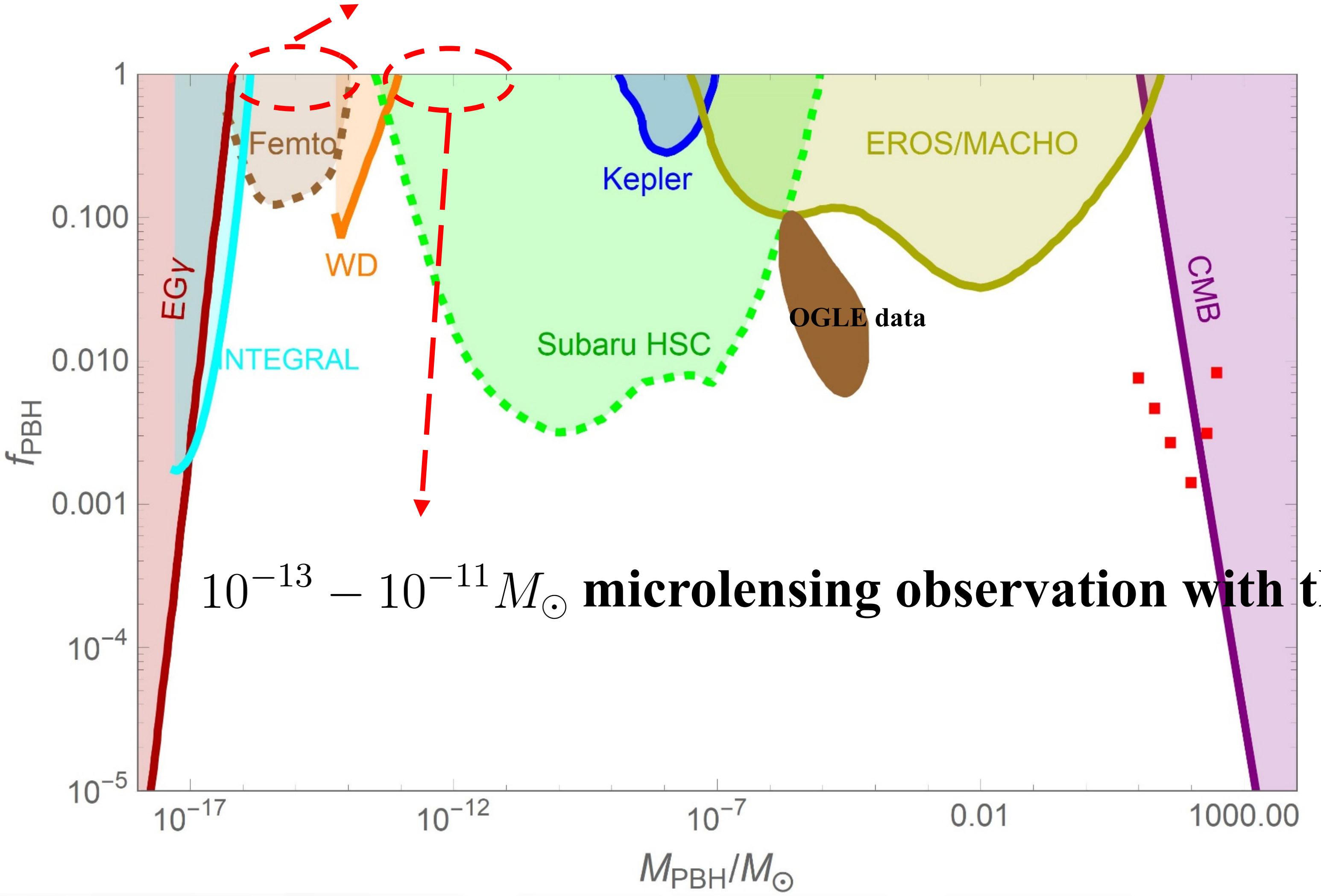




Motivation

Primordial Black Holes as all dark matter

$10^{-16} - 10^{-14} M_{\odot}$ **gravitational femtolensing of gamma-ray bursts** [1]



$10^{-13} - 10^{-11} M_{\odot}$ **microlensing observation with the Subaru HSC** [2]

[1] A. Barnacka *et al.* ,Phys. Rev. D **86**, 043001 (2012); A. Katz *et al.* , JCAP 12 (2018) 005

[2] H. Niikura *et al.* , Nat. Astron. **3**, 524 (2019)

Motivation

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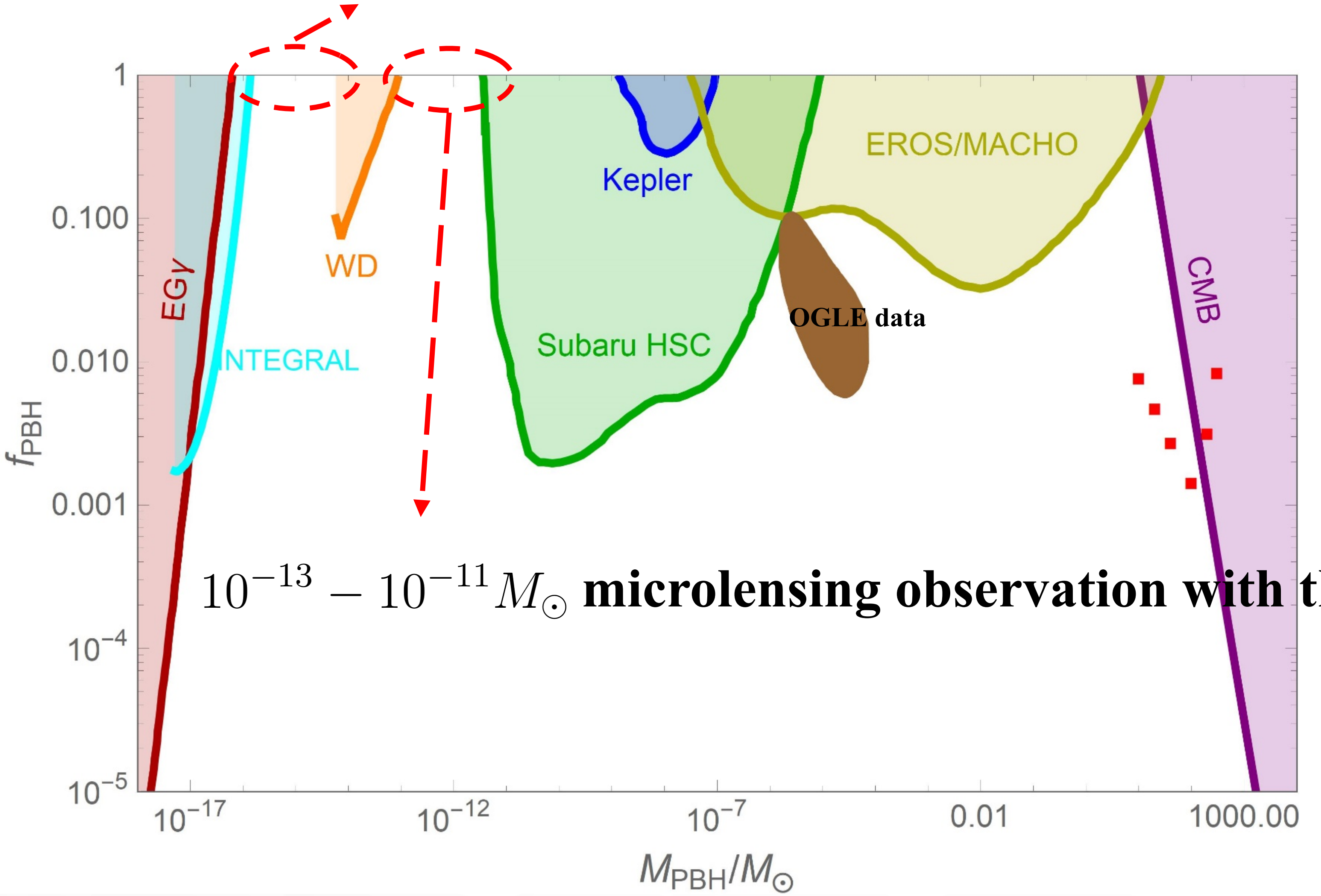

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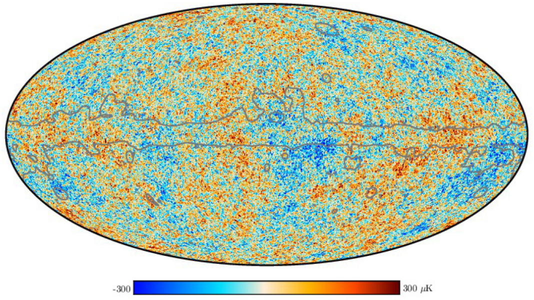
[2] H. Niikura *et al.* , Nat. Astron. **3**, 524 (2019)

Seed for PBHs: Primordial curvature perturbations

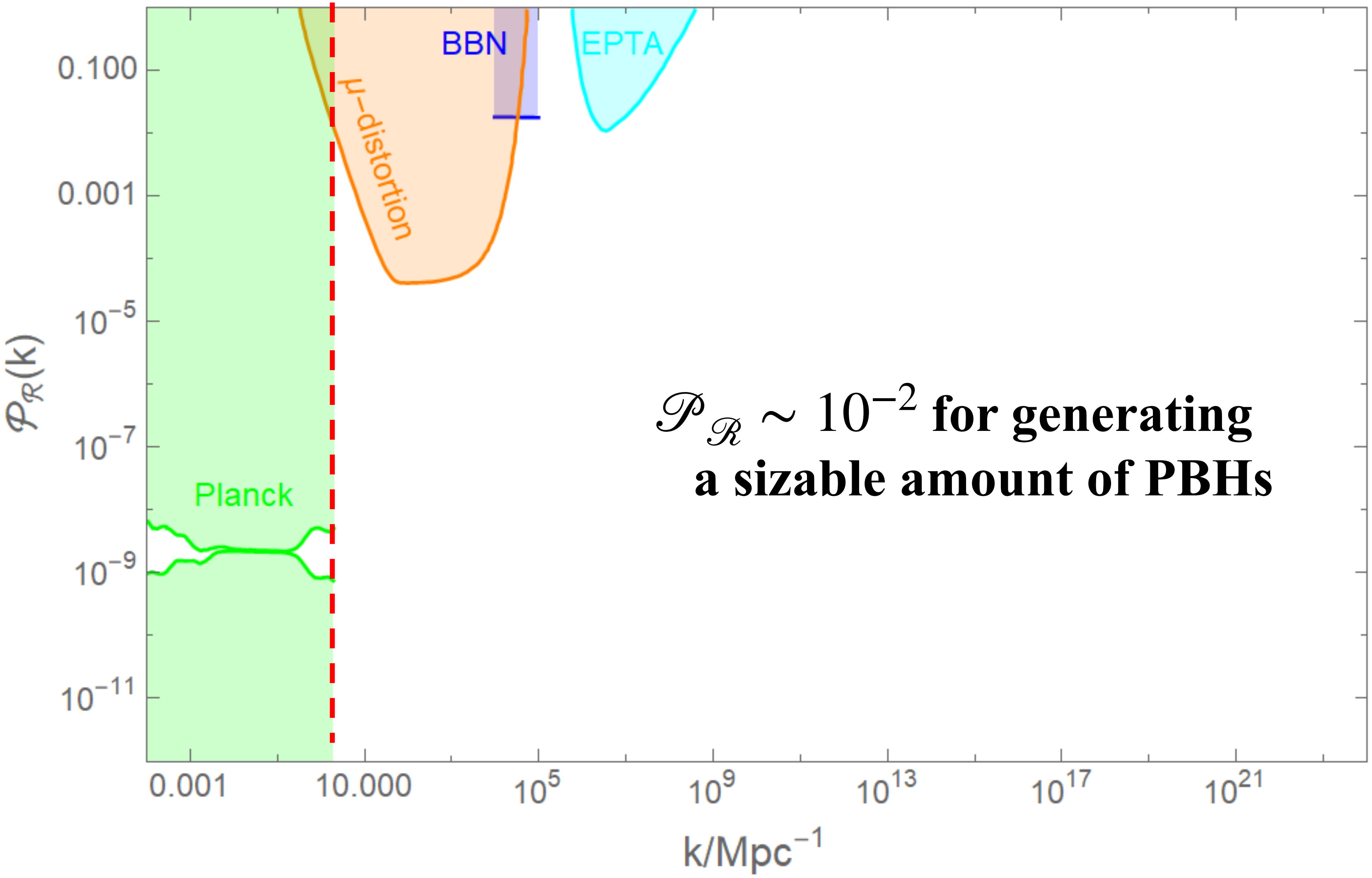
large scales

$k \lesssim 1\mathrm{Mpc}^{-1}$

$\mathcal{P}_{\mathcal{R}} \sim 10^{-9}$



small scales $k \gtrsim 1\mathrm{Mpc}^{-1}$



$\mathscr{P}_{\mathscr{R}} \sim 10^{-2}$ **for generating a sizable amount of PBHs**

Y. Akrami *et al.* , 1807.06211; D. J. Fixsen *et al.* , Astrophys. J. **473**, 576 (1996); K. Inomata *et al.* , PRD **94**, 043527 (2016); **99**, 043511 (2019)

Motivation — How to amplify the amplitude of power spectrum: Flatten potential

Simple single-field inflation model

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right]$$

Slow-roll approximation

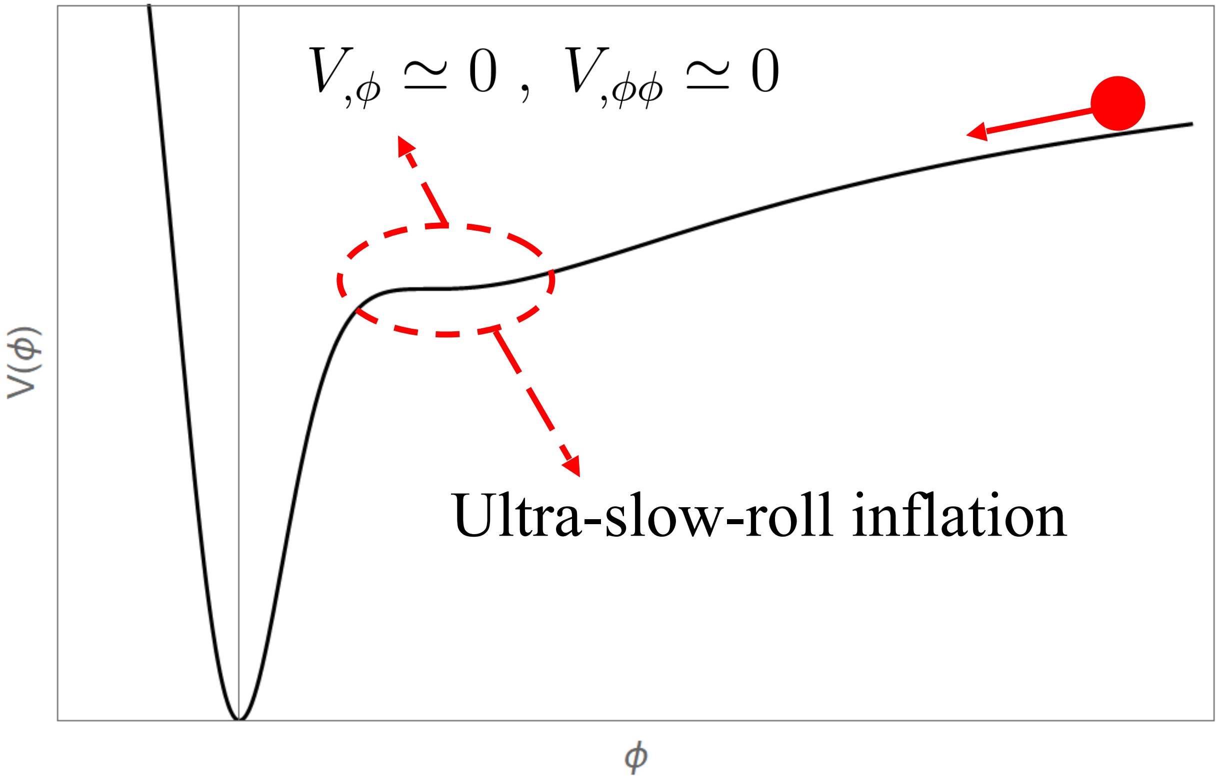
$$3H^2 \simeq \kappa^2 V(\phi) \qquad 3H\dot{\phi} \simeq -V_{,\phi}$$

Power spectrum

$$\mathcal{P}_{\mathcal{R}} = \frac{1}{8\pi^2}\left(\frac{H}{M_{\rm pl}}\right)^2\frac{1}{\epsilon} \qquad \left(\epsilon \equiv -\frac{\dot{H}}{H^2}\right)$$

$$\epsilon \simeq \epsilon_V \qquad \left(\epsilon_V \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2\right)$$

Near-inflection point



C. Germani and T. Prokopec, Phys. Dark Universe **18,** 6 (2017); Di and Gong, JCAP 07 (2018) 007.

Motivation

How to amplify the amplitude of power spectrum: Increase friction

$$\ddot{\phi} + 3H\dot{\phi} + V(\phi)_{,\phi} = 0$$

ultra-slow-roll inflation

flatten potential

increase friction

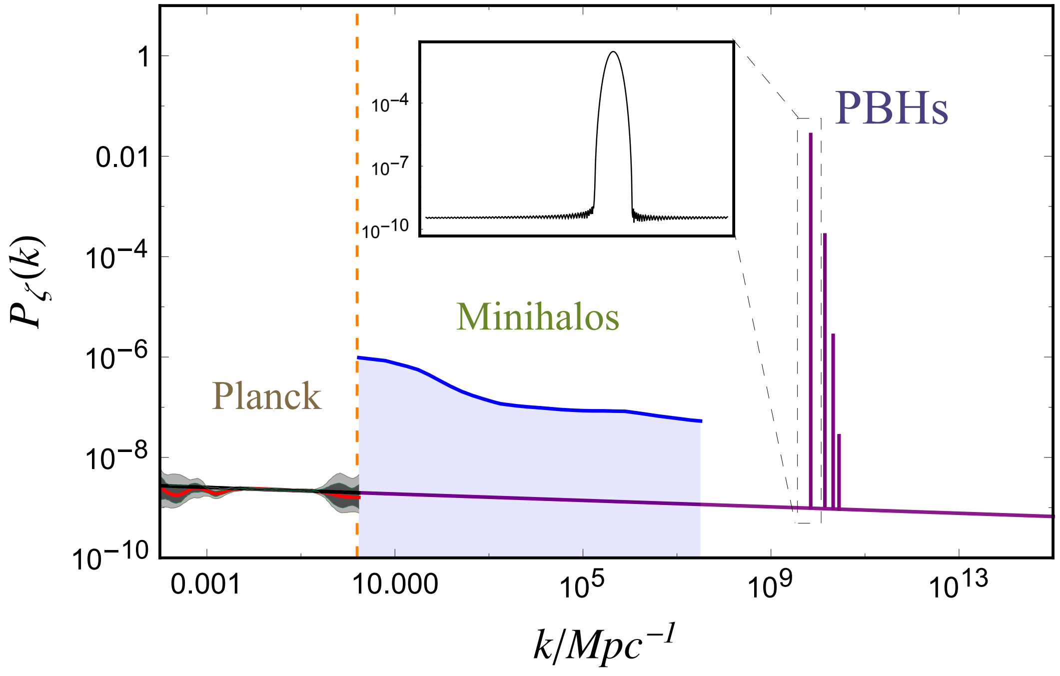
mechanism of gravitational enhanced friction

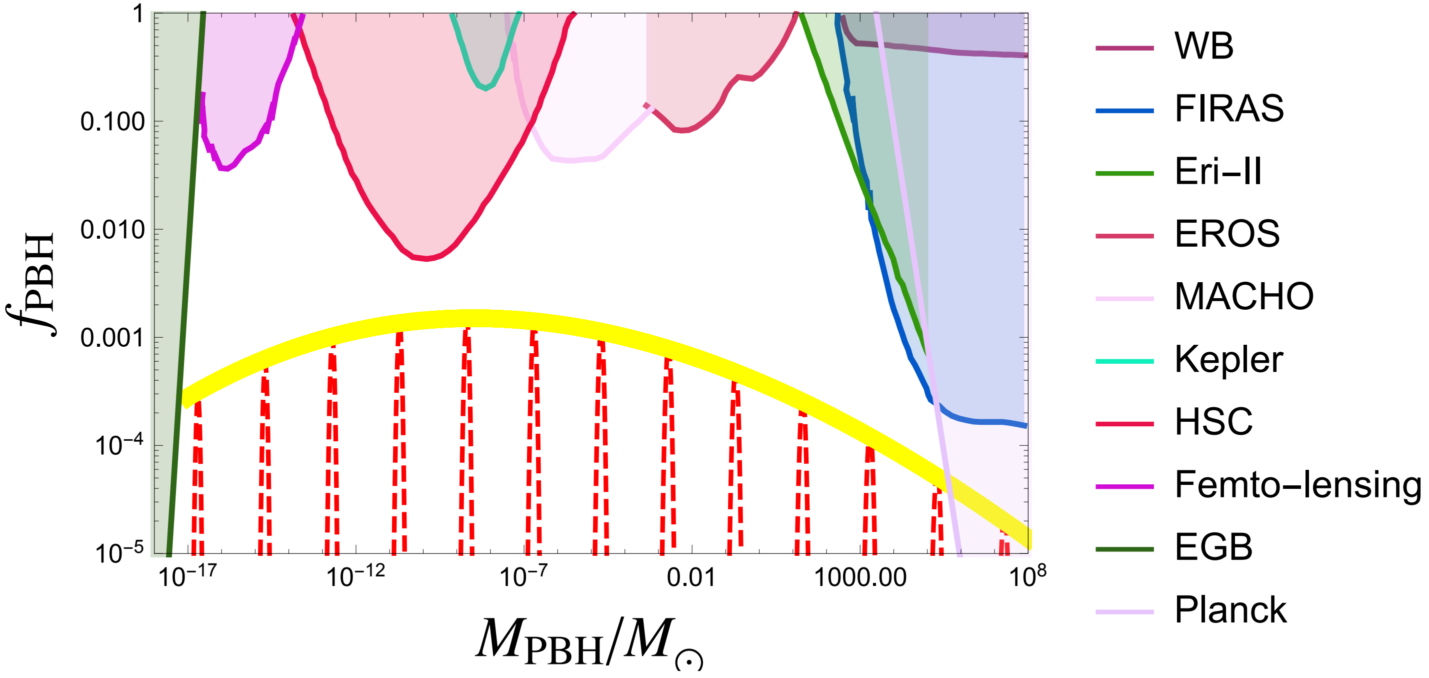
nonminimal derivative coupling (NDC)

Motivation

PBH from Sound Speed Resonance during Inflation

$$u_k'' + (c_s^2 k^2 - z''/z)u_k = 0 \qquad c_s^2 = 1 - 2\xi[1 - \cos(2k_*\tau)]$$





Cai, et al., PRL(2018)

Contents

Basic equations

The action

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}\left(g^{\mu\nu} - \kappa^2\xi G^{\mu\nu}\right)\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right]$$

$$\xi \equiv \theta(\phi) \qquad \Downarrow \qquad ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2$$

Hamiltonian constraint equation (HC)

$$3H^2 = \kappa^2\left[\frac{1}{2}\left(1 + 9\kappa^2\theta(\phi)H^2\right)\dot{\phi}^2 + V(\phi)\right]$$

Equation of motion for inflaton (EoM)

$$\left(1 + 3\kappa^2\theta(\phi)H^2\right)\ddot{\phi} + \left[1 + \kappa^2\theta(\phi)\left(2\dot{H} + 3H^2\right)\right]3H\dot{\phi} + \frac{3}{2}\kappa^2\theta_{,\phi}H^2\dot{\phi}^2 + V_{,\phi} = 0$$

Slow-roll inflation

Slow-roll parameters $\epsilon \equiv -\frac{\dot{H}}{H^2}$ $\delta_\phi \equiv \frac{\ddot{\phi}}{H\dot{\phi}}$ $\delta_X \equiv \frac{\kappa^2\dot{\phi}^2}{2H^2}$ $\delta_D \equiv \frac{\kappa^4\theta\dot{\phi}^2}{4}$

Slow-roll conditions $\{\epsilon, |\delta_\phi|, \delta_X, \delta_D\} \ll 1$

Operational condition $|\kappa^2\theta_{,\phi}H\dot{\phi}| \ll \mathcal{A} \equiv 1 + 3\kappa^2\theta(\phi)H^2$ **for simplicity**

Approximate HC and EoM $3H^2 \simeq \kappa^2 V(\phi)$ $3H\mathcal{A}\dot{\phi} + V_{,\phi} \simeq 0$

$$\epsilon \simeq \frac{\epsilon_V}{\mathcal{A}}$$

⇒

If $\mathcal{A} \simeq 1, \ \epsilon \simeq \epsilon_{\rm V}$

If $\mathcal{A} \gg 1, \ \epsilon \ll \epsilon_{\rm V}$

Approximate solutions

Under the slow-roll approximation, the power spectrum of the curvature perturbation has the form

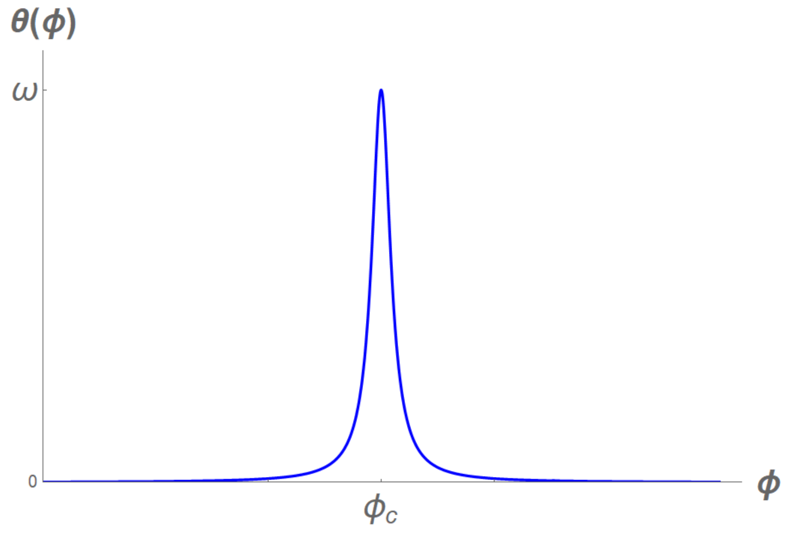
$$\mathcal{P}_{\mathcal{R}} = \frac{H^2}{8\pi^2 Q_s c_s^3} \simeq \frac{V^3}{12\pi^2 M_{\rm pl}^6 V_{,\phi}^2}\left(1 + \theta(\phi)\frac{V}{M_{\rm pl}^4}\right)$$

The scalar spectral index and the tensor-to-scalar ratio:

$$n_s \simeq 1 - \frac{1}{\mathcal{A}}\left[2\epsilon_V\left(4 - \frac{1}{\mathcal{A}}\right) - 2\eta_V\right] \qquad \left(\eta_V \equiv \frac{M_{\rm pl}^2}{V}\frac{d^2V}{d\phi^2}\right)$$

$$r \simeq \frac{16\epsilon_V}{\mathcal{A}}$$

How to achieve a large-amplitude curvature perturbations?



Consider the following special functional form

$$\theta(\phi) = \frac{\omega}{\sqrt{\kappa^2\left(\frac{\phi-\phi_c}{\sigma}\right)^2+1}}$$

Consider a simple potential

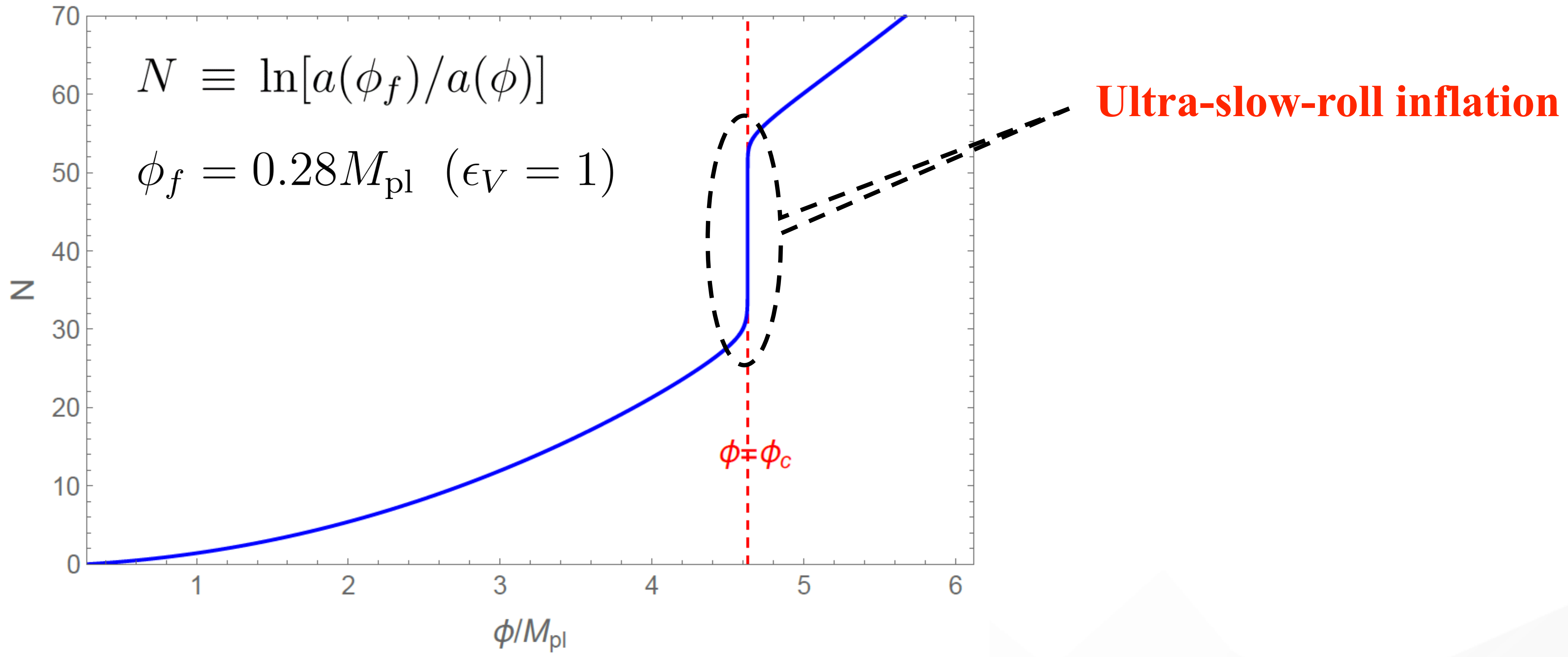
$$V(\phi) = \lambda M_{\rm pl}^{4-p}|\phi|^p \quad (p=2/5)$$

Enhanced Power spectrum

$$\mathcal{P}_{\mathcal{R}} \simeq \frac{\lambda}{12\pi^2 p^2}\left|\frac{\phi}{M_{\rm pl}}\right|^{2+p}\left(1+\frac{\omega\lambda}{\sqrt{\kappa^2\left(\frac{\phi-\phi_c}{\sigma}\right)^2+1}}\left|\frac{\phi}{M_{\rm pl}}\right|^p\right)$$

Inflationary dynamics

Concrete case $\phi_c = 4.63 M_{\rm pl}\ ,\ \sigma = 2.6\times 10^9\ ,\ \omega\lambda = 1.33\times 10^7$



Exact results: Mukhanov-Sasaki equation

$$u_k'' + \left(c_s^2 k^2 - \frac{z''}{z}\right) u_k = 0$$

where variable u is related to the curvature perturbation $\mathcal{R}$

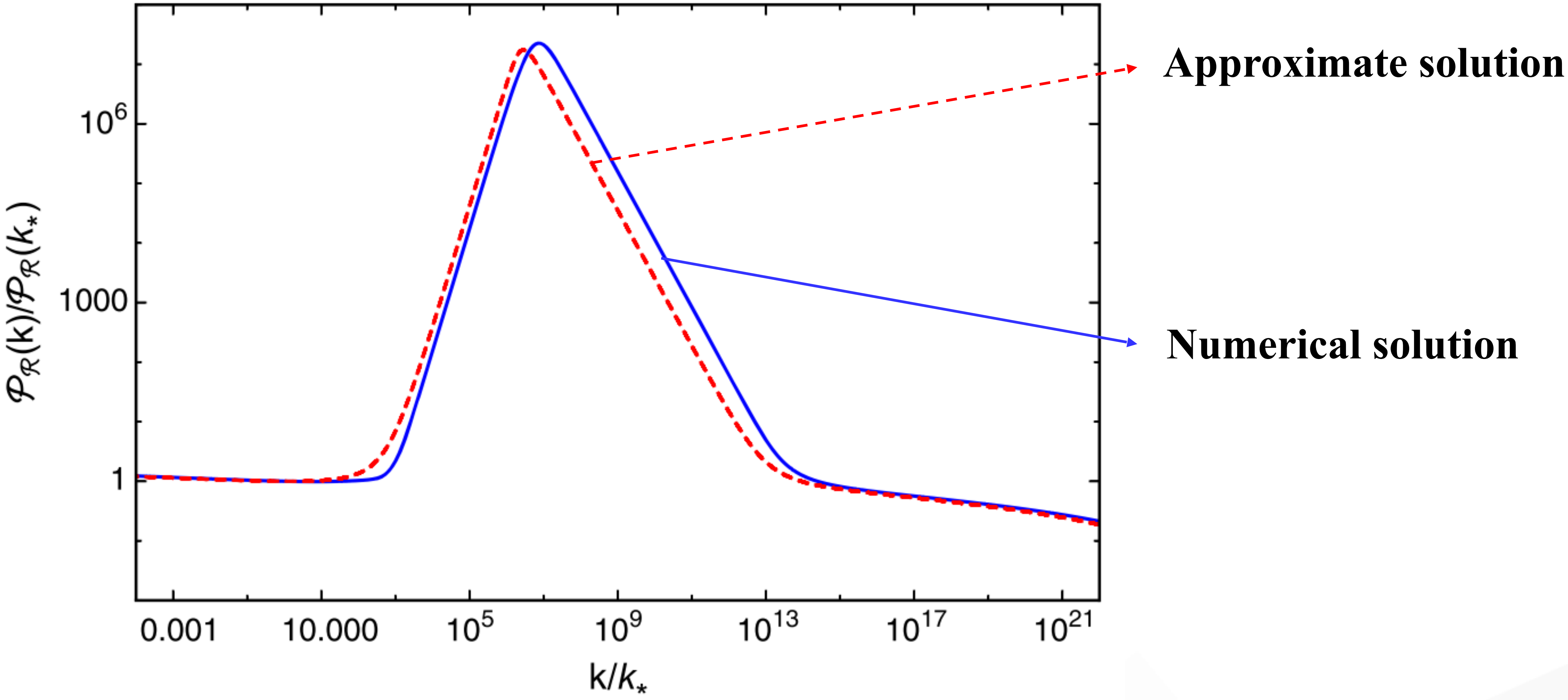
$$u \equiv z\mathcal{R} \quad (z \equiv \sqrt{2Q_s}a)$$

$$Q_s = \frac{w_1(4w_1w_3 + 9w_2^2)}{3w_2^2} \qquad c_s^2 = \frac{3(2w_1^2w_2H - w_2^2w_4 + 4w_1\dot{w}_1w_2 - 2w_1^2\dot{w}_2)}{w_1(4w_1w_3 + 9w_2^2)}$$

$$w_1 = M_{\rm pl}^2(1-2\delta_D) \quad w_2 = 2HM_{\rm pl}^2(1-6\delta_D) \quad w_3 = -3H^2M_{\rm pl}^2(3-\delta_X-36\delta_D) \quad w_4 = M_{\rm pl}^2(1+2\delta_D)$$

Power spectrum of the curvature perturbations $\mathcal{P}_\mathcal{R} \equiv \frac{k^3}{2\pi^2}\left|\frac{u_k}{z}\right|^2$

The power spectrum: approximate and exact solution



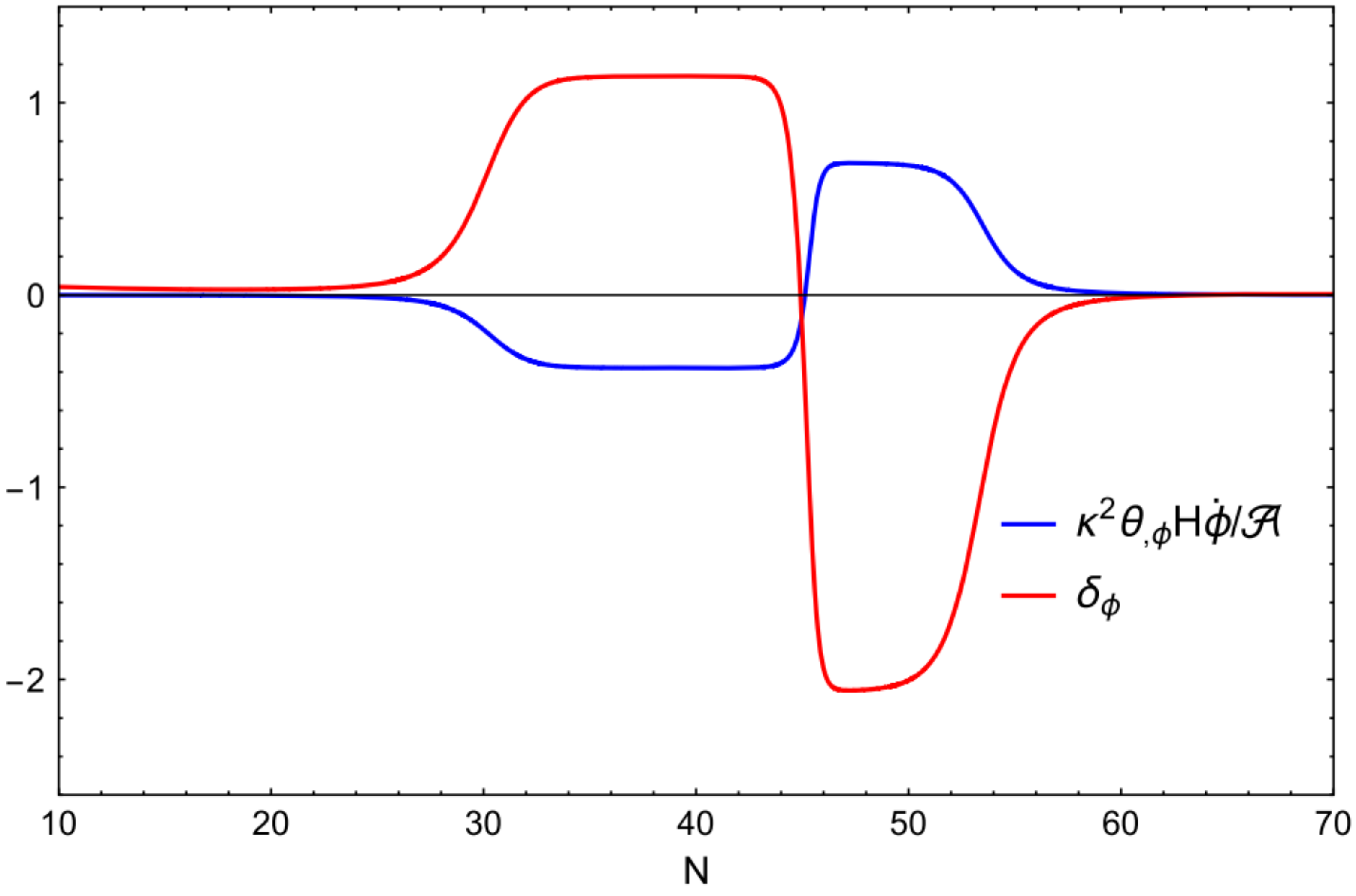
Slow-roll violation

One of slow-roll conditions

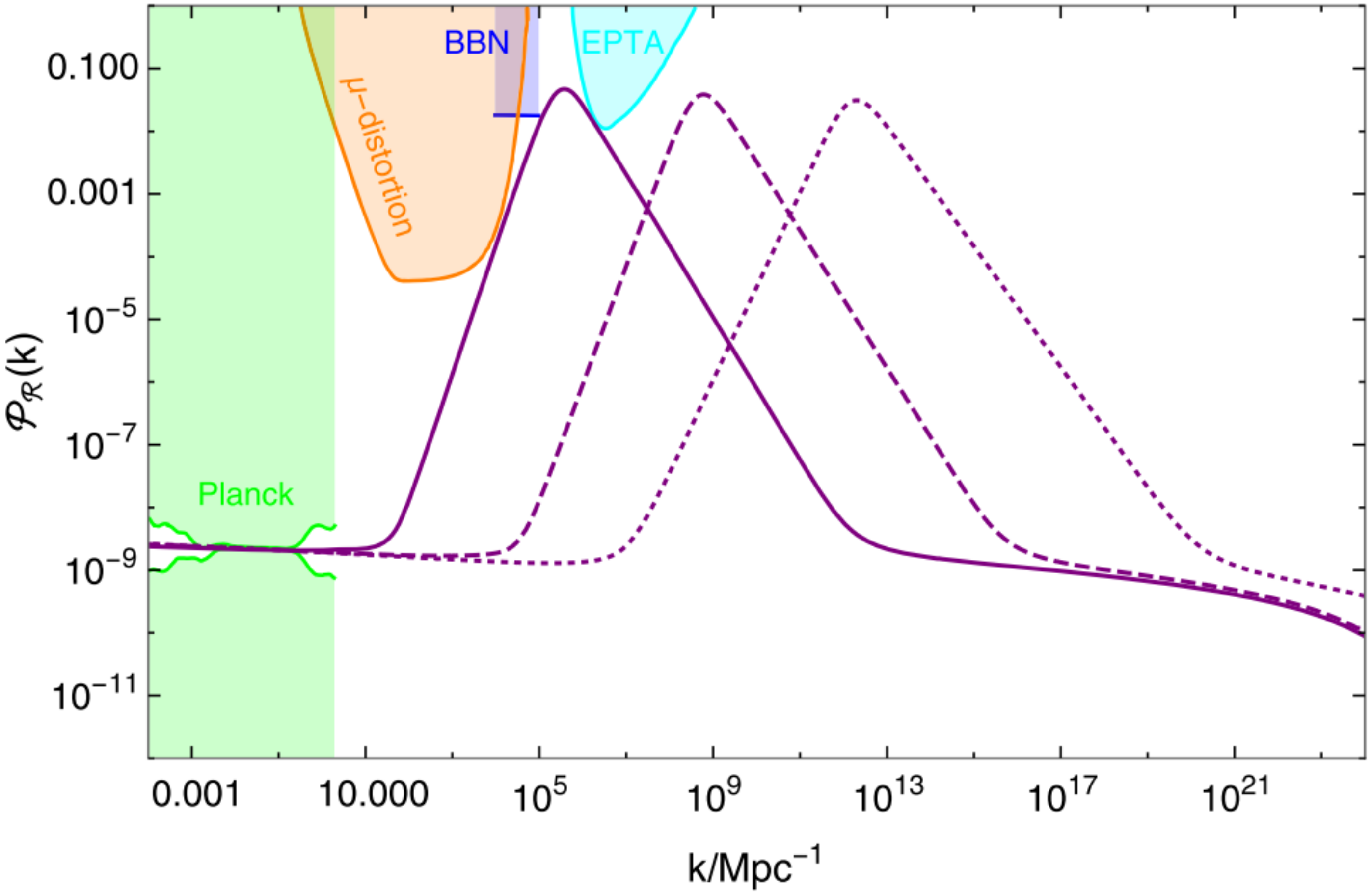
$$|\delta_\phi| = \left|\frac{\ddot{\phi}}{H\dot{\phi}}\right| << 1$$

Operational condition

$$|\kappa^2\theta_{,\phi}H\dot{\phi}| << \mathcal{A}$$



The power spectrum



λ	n_s	r
7.09×10^{-10}	0.9666	0.0431
8.23×10^{-10}	0.9618	0.0497
8.52×10^{-10}	0.9607	0.0512

Contents

Basic formulas for PBH formation during radiation-dominated era

Under the assumption that the probability distribution function of perturbations is Gaussian, the production rate of PBHs with mass M based on the Press-Schechter theory is[1]

$$\beta(M) = \int_{\delta_c} \frac{d\delta}{\sqrt{2\pi\sigma^2(M)}} e^{-\frac{\delta^2}{2\sigma^2(M)}} = \frac{1}{2}\mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2\sigma^2(M)}}\right)$$

where δ_c ($\simeq 0.4$ [2]) is the threshold of the density perturbations for the PBH formation

The mass M of formed PBHs is related to the horizon mass at the horizon entry of the perturbations with the comoving wave number k

$$M(k) = \gamma \frac{4\pi}{\kappa^2 H}\bigg|_{k=aH} \simeq M_\odot \left(\frac{\gamma}{0.2}\right)\left(\frac{g_*}{10.75}\right)^{-\frac{1}{6}}\left(\frac{k}{1.9\times 10^6\ \mathrm{Mpc}^{-1}}\right)^{-2} \qquad (g_* \simeq 106.75)$$

where γ ($\simeq 0.2$)[3] is the ratio of the PBH mass to the horizon mass and indicates the efficiency of collapse

[1] S. Young *et al.* , JCAP 07 (2014) 045 [2] T. Harada *et al.* , PRD **88**, 084051 (2013); [3] B. J. Carr, APJ **201**, 1 (1975)

Basic formulas for PBH formation during radiation-dominated era

$\sigma^2(M)$ represents the variance of density contrast for the PBH mass M [1]

$$\sigma^2(M(k)) = \int d\ln q\; W^2(qk^{-1})\frac{16}{81}(qk^{-1})^4\mathcal{P}_{\mathcal{R}}(q)$$

where W is the window function, which is taken to be the Gaussian function $W(x) = e^{-x^2/2}$

The abundance of PBHs with mass M over logarithmic mass interval is estimated as

$$f(M) \equiv \frac{1}{\Omega_{\rm DM}}\frac{d\Omega_{\rm PBH}}{d\ln M} \simeq \frac{\beta(M)}{1.84\times10^{-8}}\left(\frac{\gamma}{0.2}\right)^{\frac{3}{2}}\left(\frac{10.75}{g_*}\right)^{\frac{1}{4}}\left(\frac{0.12}{\Omega_{\rm DM}h^2}\right)\left(\frac{M}{M_\odot}\right)^{-\frac{1}{2}}$$

Total abundance of PBH $\quad \dfrac{\Omega_{\rm PBH}}{\Omega_{\rm DM}} = \displaystyle\int \frac{dM}{M} f(M) \quad (\Omega_{\rm DM}h^2 \simeq 0.12$[2]$)$

Abundance of PBHs is exponentially sensitive to small variations of the power spectrum

[1] S. Young *et al.* , JCAP 07 (2014) 045 [2] N. Aghanim *et al.* , 1807 .06209

Three parameters sets for three interesting motivations of PBHs

- ➢ **stellar-mass** $(\sim \mathcal{O}(10) M_\odot)$ **PBHs: LIGO/Virgo GW events**
- ➢ **earth-mass** $(\sim \mathcal{O}(10^{-5}) M_\odot)$ **PBHs: OGLE microlensing events**
- ➢ **asteroid-mass** $(\sim \mathcal{O}(10^{-12}) M_\odot)$ **PBHs: most of dark matter**

TABLE I: The successful parameter sets for producing the PBHs with mass around $\mathcal{O}(10) M_\odot$ (*Case 1*), $\mathcal{O}(10^{-5}) M_\odot$ (*Case 2*) and $\mathcal{O}(10^{-12}) M_\odot$ (*Case 3*).

#	$\phi_c/M_{\rm pl}$	$\omega\lambda$	σ
Case 1	4.63	1.33×10^7	2.6×10^{-9}
Case 2	3.9	1.53×10^7	3×10^{-9}
Case 3	3.3	1.978×10^7	3.4×10^{-9}

Observational constraints

Scalar spectral index and the tensor-to-scalar ratio

$$n_s \simeq 1 - \frac{1}{\mathcal{A}}\left[2\epsilon_V\left(4 - \frac{1}{\mathcal{A}}\right) - 2\eta_V\right] \qquad r \simeq \frac{16\epsilon_V}{\mathcal{A}} \qquad \left(\eta_V \equiv \frac{M_{\rm pl}^2}{V}\frac{d^2V}{d\phi^2}\right)$$

#	N_*	λ	n_s	r
Case 1	60	7.09×10^{-10}	0.9666	0.0431
Case 2	60	8.23×10^{-10}	0.9618	0.0497
Case 3	65	8.52×10^{-10}	0.9607	0.0512

Planck 2018 results [1] $(k_* = 0.05\mathrm{Mpc}^{-1})$

$$\ln(10^{10}\mathcal{P}_\mathcal{R}) = 3.044 \pm 0.014 \qquad (68\%\ \mathrm{C.L.})$$

$$n_s = 0.9649 \pm 0.0042 \qquad (68\%\ \mathrm{C.L.})$$

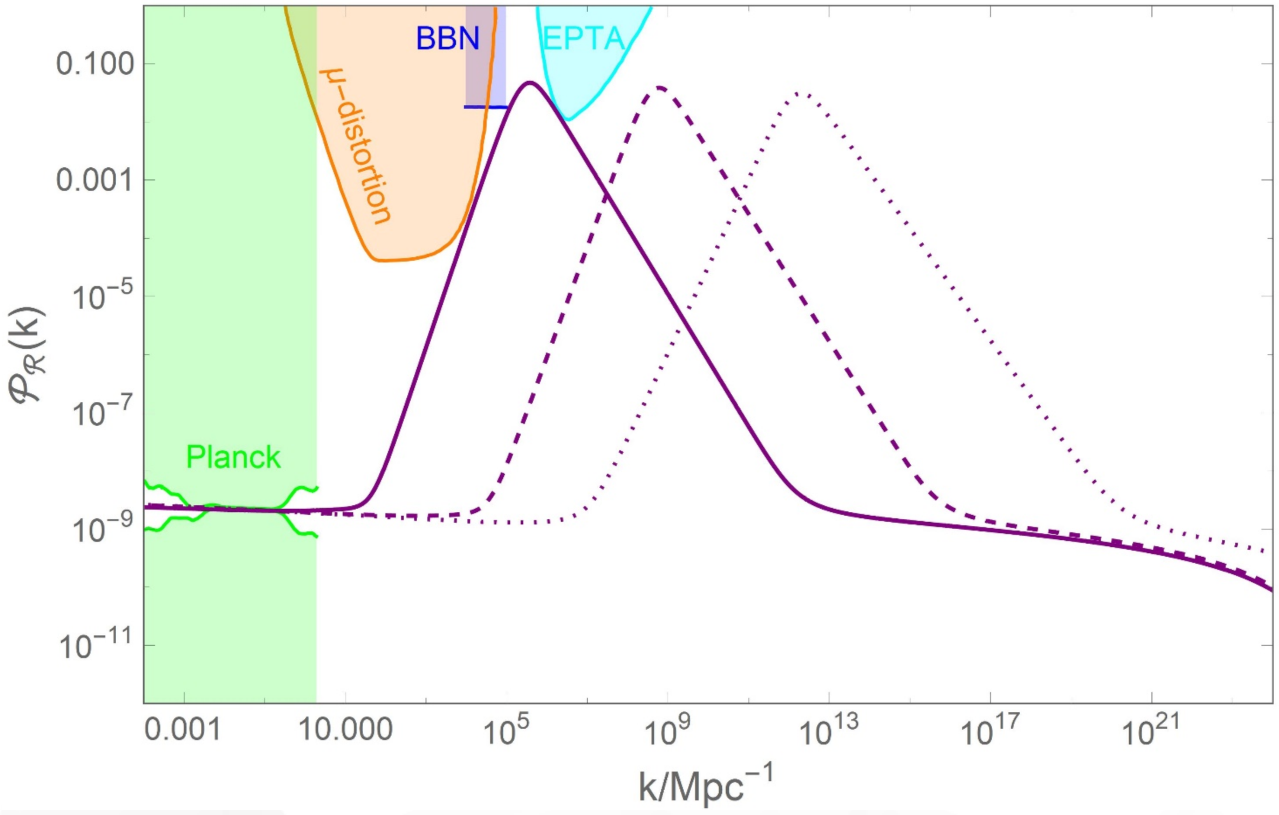
$$r < 0.07 \qquad (95\%\ \mathrm{C.L.})$$

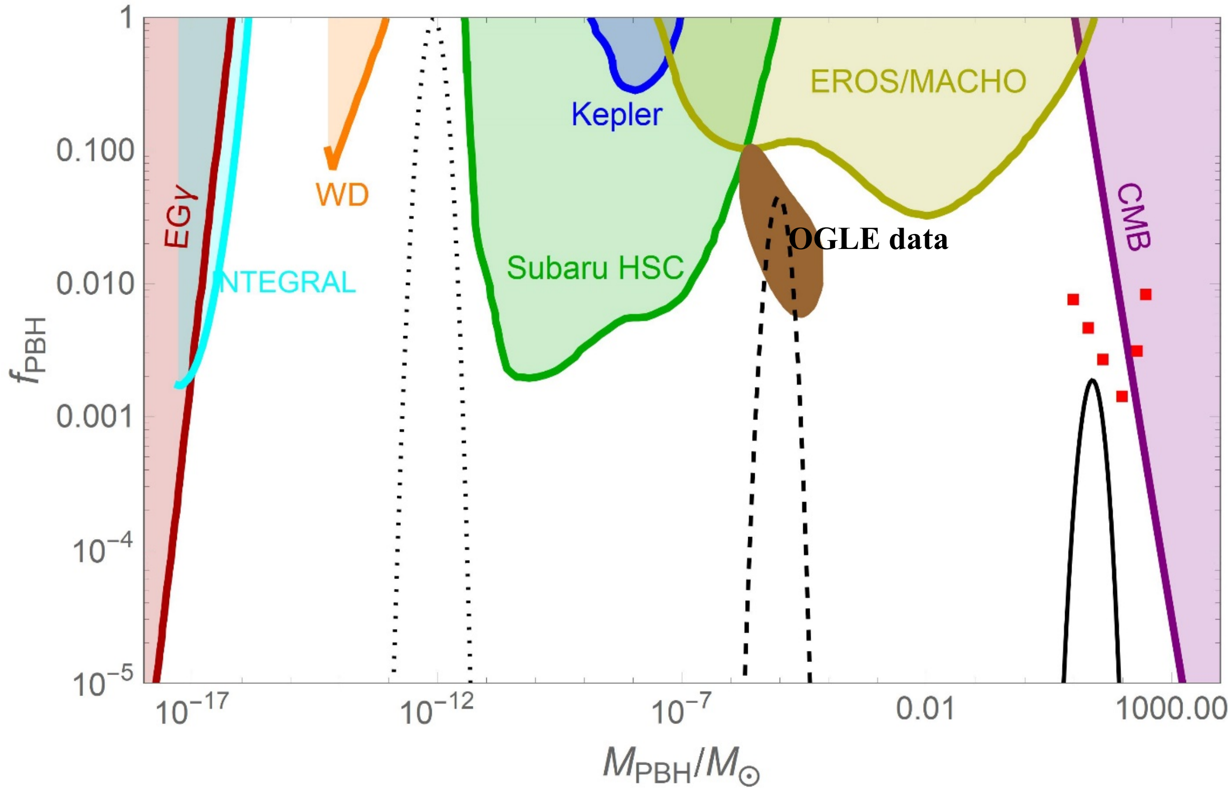
[1] Y. Akrami *et al.* , 1807.06211

Power spectra of curvature perturbations and mass spectra of PBHs

case 1 — solid line **case 2 — dashed line**

case 3 — dotted line





$M_{\mathrm{PBH}}^{\mathrm{peak}}/M_{\odot}$	$f_{\mathrm{PBH}}^{\mathrm{peak}}$	$\Omega_{\mathrm{PBH}}/\Omega_{\mathrm{DM}}$
23.5	1.88×10^{-3}	1.95×10^{-3}
9.02×10^{-6}	0.0452	0.044
8.1×10^{-13}	0.977	0.972

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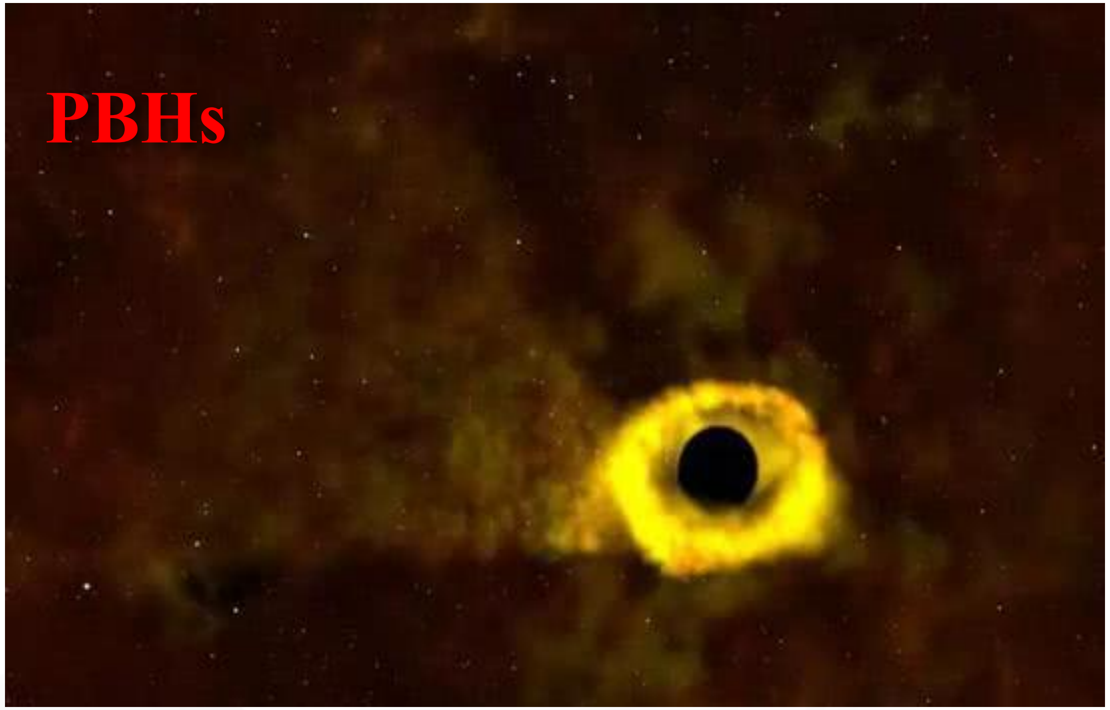
Scalar induced gravitational waves (SIGWs)

Relatively large curvature perturbations on small scales



K. Inomata and T. Nakama, Phys. Rev. D **99**, 043511 (2019)

Formalism of SIGWs

In the conformal Newtonian gauge, the perturbed FRW metric can be written as

$$ds^2 = a(\eta)^2 \left\{ -(1+2\Psi)d\eta^2 + \left[(1-2\Psi)\delta_{ij} + \frac{h_{ij}}{2}\right] dx^i dx^j \right\}$$

where $\eta \equiv \int a^{-1} dt$ is the conformal time, Ψ is the first-order scalar perturbation, and h_{ij} is the second-order transverse-traceless tensor perturbation

The equation of motion for second-order h_{ij} is given by

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\mathcal{T}^{lm}_{ij} S_{lm}$$

where $\mathcal{T}^{lm}_{ij}$ is the transverse-traceless projection operator and the source term has the form [1]

$$S^{(2)}_{ij} = 4\Psi\partial_i\partial_j\Psi + 2\partial_i\Psi\partial_j\Psi - \frac{1}{\mathcal{H}^2}\partial_i(\mathcal{H}\Psi + \Psi')\partial_j(\mathcal{H}\Psi + \Psi')$$

[1] K. N. Ananda *et al.* , Phys. Rev. D **75**, 123518 (2007)

Formalism of SIGWs

During the radiation dominated epoch $(aH = \eta^{-1})$

$$\Psi_k'' + \frac{4}{\eta}\Psi_k' + \frac{k^2}{3}\Psi_k = 0$$

This equation of motion has an attenuation solution given by [1]

$$\Psi_k(\eta) = \psi_k \frac{9}{(k\eta)^2}\left(\frac{\sin(k\eta/\sqrt{3})}{k\eta/\sqrt{3}} - \cos(k\eta/\sqrt{3})\right)$$

where ψ_k $(\Psi_k = \psi_k$ when $k\eta \ll 1)$ is the primordial perturbation characterized by the power spectrum,

$$\langle \psi_{\mathbf{k}} \psi_{\tilde{\mathbf{k}}} \rangle = \frac{2\pi^2}{k^3}\left(\frac{4}{9}\mathcal{P}_{\mathcal{R}}(k)\right)\delta(\mathbf{k} + \tilde{\mathbf{k}})$$

[1] D. Baumann *et al.* , Phys. Rev. D **76,** 084019 (2007)

Formalism of SIGWs

In the radiation-dominated era, the density parameter spectrum of GWs Ω_{GW} at η_c, which represents the time when Ω_{GW} stops growing, can be evaluated as [1]

$$\Omega_{\mathrm{GW}}(\eta_c,k)=\frac{1}{12}\int_0^\infty dv\int_{|1-v|}^{|1+v|}du\left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2\mathcal{P}_{\mathcal{R}}(ku)\mathcal{P}_{\mathcal{R}}(kv)$$
$$\left(\frac{3}{4u^3v^3}\right)^2(u^2+v^2-3)^2$$
$$\left\{\left[-4uv+(u^2+v^2-3)\ln\left|\frac{3-(u+v)^2}{3-(u-v)^2}\right|\right]^2+\pi^2(u^2+v^2-3)^2\Theta(v+u-\sqrt{3})\right\}$$

The current energy parameter and frequency of GWs are given, respectively, by

$$\Omega_{\mathrm{GW},0}=0.83\left(\frac{g_c}{10.75}\right)^{-1/3}\Omega_{\mathrm{r},0}\Omega_{\mathrm{GW}}(\eta_c,k)\qquad f=1.546\times10^{-15}\frac{k}{1\mathrm{Mpc}^{-1}}\mathrm{Hz}$$

$$(\Omega_{r,0}h^2\simeq4.2\times10^{-5}\ ,\ g_c\simeq106.75)$$

[1] K. Kohri and T. Terada, Phys. Rev. D **97**, 123532 (2018)

Approximate spectra index of power spectrum

Through analytical calculations, we find that

$$\mathcal{P}_{\mathcal{R}} \propto \begin{cases} k^{n_s^{(1)}} & , \ (k < k_p) \\ k^{n_s^{(2)}} & , \ (k > k_p) \end{cases}$$

where

$$n_s^{(1)} \simeq 3\left(1 - \sqrt{1 - \frac{4}{15}(\kappa\phi_c)^{-7/5}(\sigma\omega\lambda)^{-1}}\right)$$

$$n_s^{(2)} \simeq 3\left(1 - \sqrt{1 + \frac{4}{15}(\kappa\phi_c)^{-7/5}(\sigma\omega\lambda)^{-1}}\right)$$

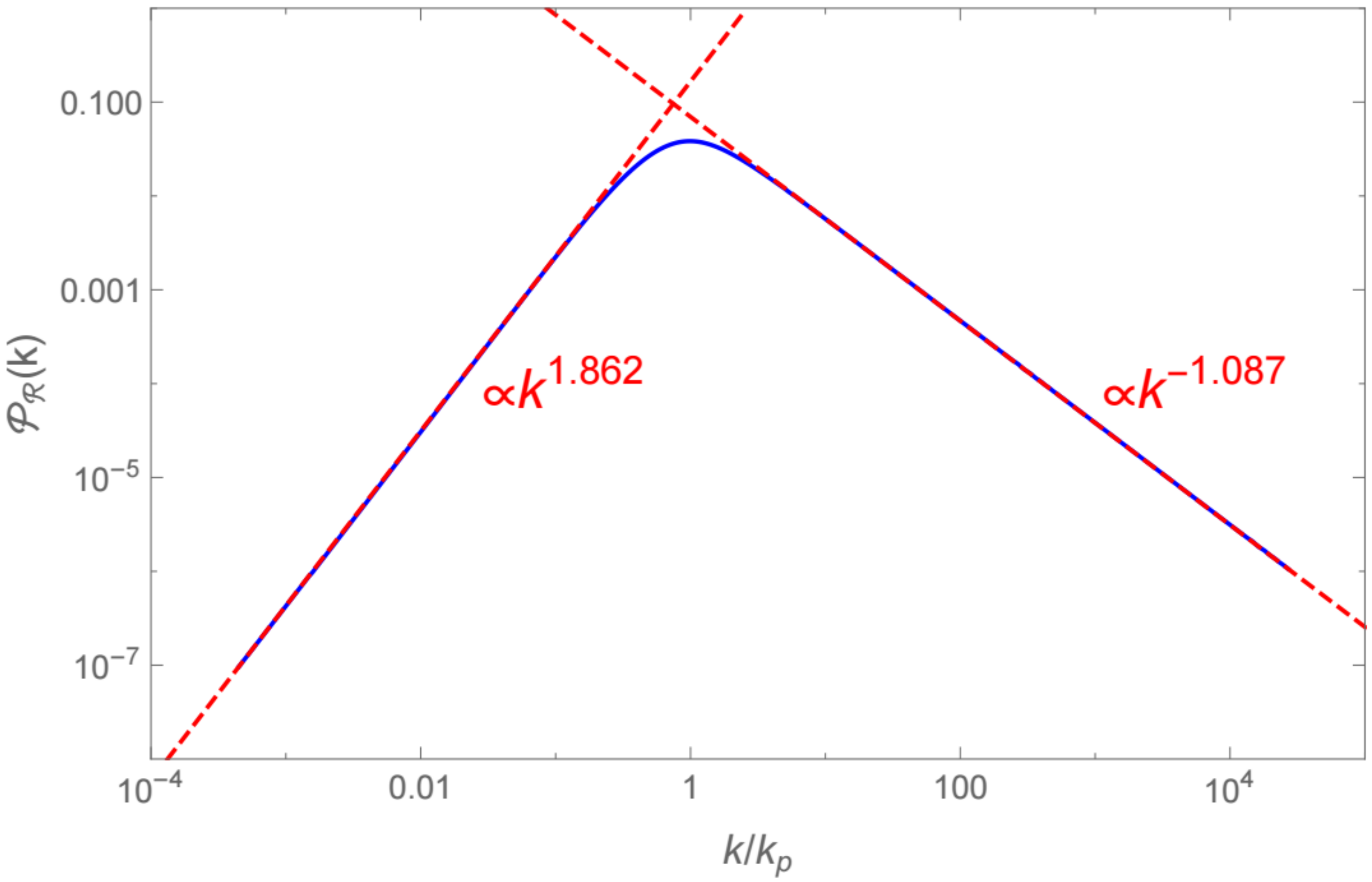
Approximate power-law power spectrum

Earth-mass PBHs: $\phi_c = 3.82 M_{\rm pl}\ ,\ \sigma = 3\times 10^9\ ,\ \omega\lambda = 1.59\times 10^7$

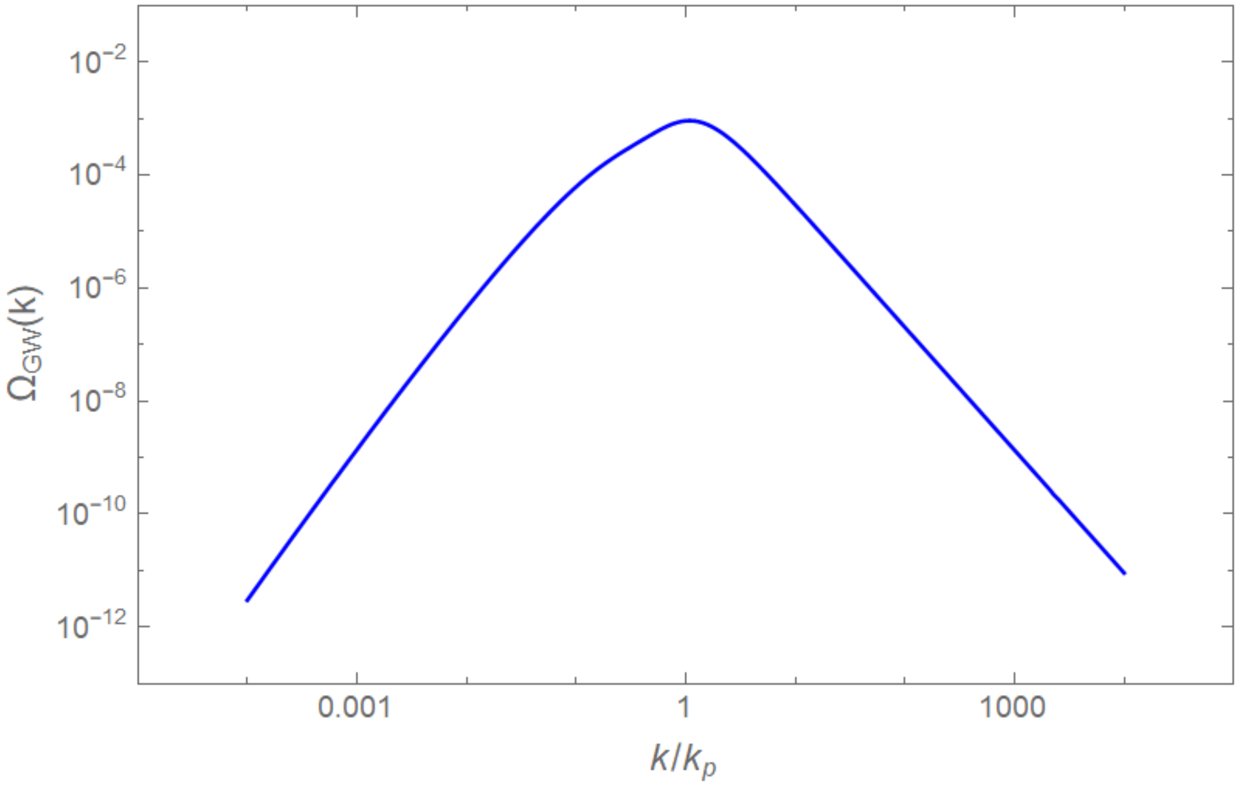
Analytic results

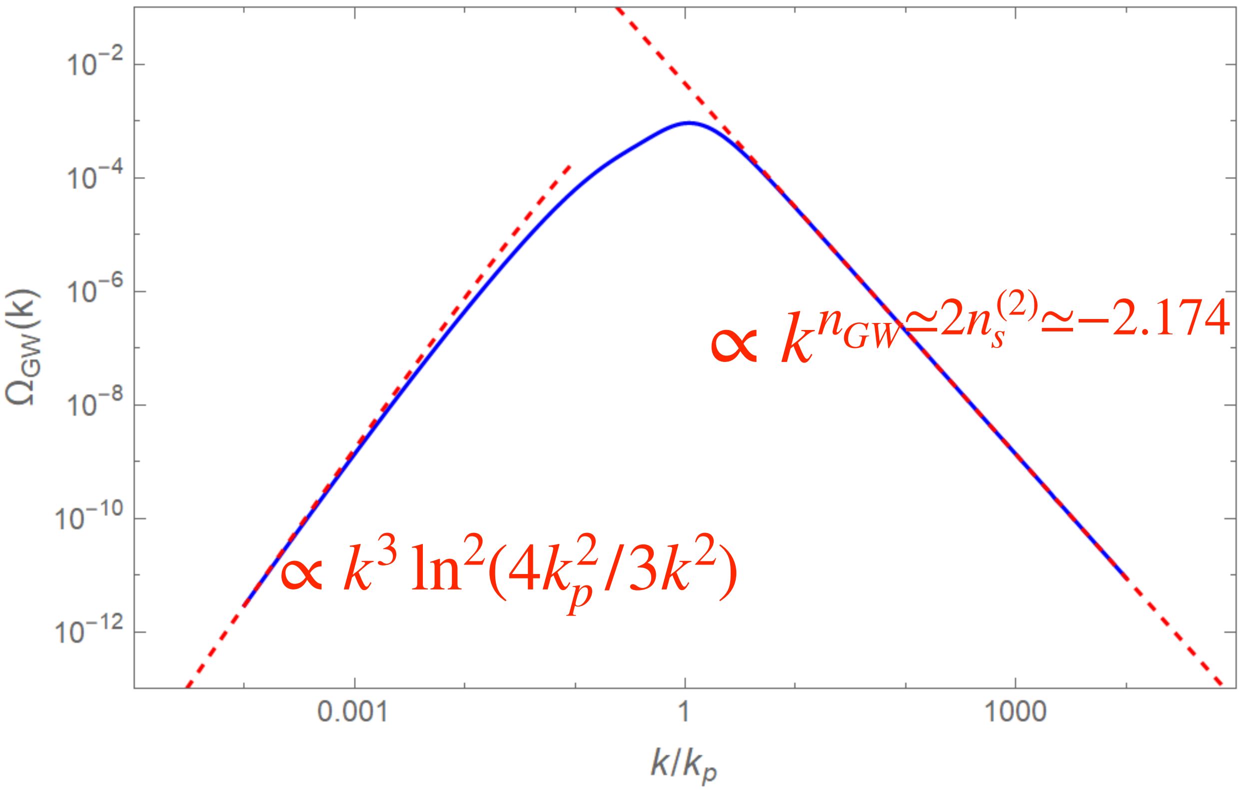
$$n_s^{(1)} \simeq 1.862$$

$$n_s^{(2)} \simeq -1.087$$



Density spectrum and scaling of SIGWs





Scaling of SIGWs

In the ultraviolet regions $(k > k_p)$, if $\mathcal{P}_{\mathcal{R}} \propto k^{n_s}$ with $n_s > -4$, the density spectrum of SIGWs is approximated by a power-law function of k [1]

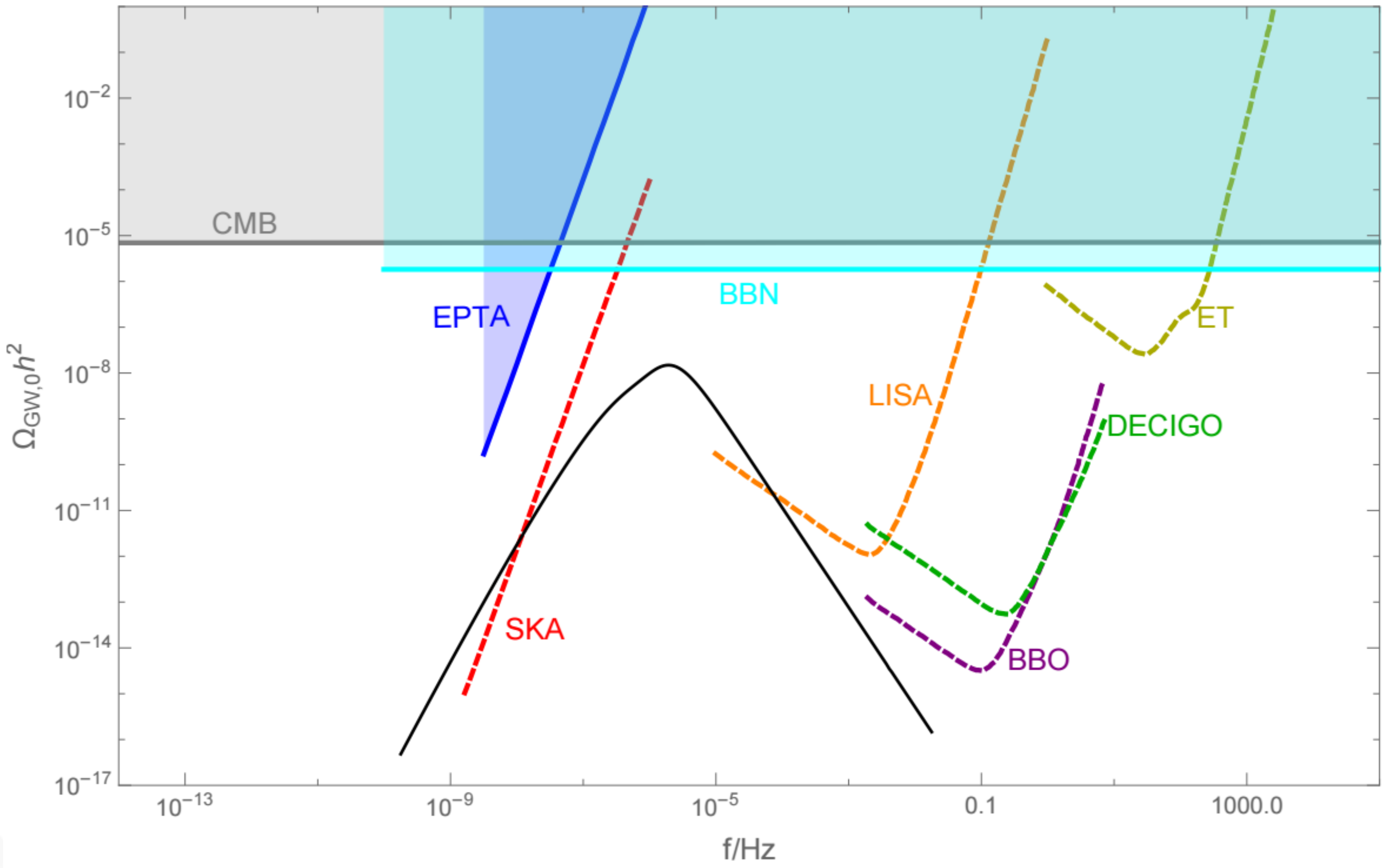
$$\Omega_{\mathrm{GW}}(k) \propto k^{n_{\mathrm{GW}}} \qquad n_{\mathrm{GW}} \simeq 2n_s$$

In the infrared regions $(k < k_p)$, the density spectrum of SIGWs has a log-dependent slope[2]

$$\Omega_{\mathrm{GW}}(k) \propto \left(\frac{k}{k_p}\right)^3 \ln^2\left(\frac{4k_p^2}{3k^2}\right) \qquad n_{\mathrm{GW}} \simeq 3 - \frac{4}{\ln \frac{4k_p^2}{3k^2}}$$

[1] Guo, *et al.* , 1907.05213 [2] Huang, *et al.* , 1910.09099

Density parameter spectrum of SIGWs



Contents

Conclusions

- The enhancement of the curvature perturbations can be realized in the nonminimal derivative coupling model with a coupling parameter related to the inflaton field.
- The obtained power spectrum of curvature perturbations has an enough large peak on the small scales and on the large scales satisfies the current observational constraints.
- The power spectrum in the vicinity of the peak can be well approximated by a power-law function of comoving wave number.
- Through fine-tuning two parameters, we can easily obtain a sharp mass spectrum of primordial black holes around specific mass such as $\mathcal{O}(10)M$, $\mathcal{O}(10^{-5})M_\odot$, and $\mathcal{O}(10^{-12})M_\odot$, which can explain the LIGO events, the ultrashort-timescale microlensing events in OGLE data, and the most of DM, respectively.
- The GW signal produced by scalar metric perturbations will be detected by SKA and LISA. Log-dependent slope of SIGWs in the infrared regions is confirmed, while in the ultraviolet regions a power-law scaling is obtained.

END

THANK YOU FOR LISTENING