Spin Polarization in high energy heavy-ion collisions

Qun Wang

Department of Modern Physics
Univ of Science & Technology of China (USTC)

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Outline

- **Introduction**
- **Theoretical description for particle polarization: kinetic theory in Wigner functions**
- **A microscopic model for global polarization through spin-orbit couplings in particle scatterings, a non-equilibrium model**
- **Summary**

Introduction

Global OAM and Magnetic field in HIC

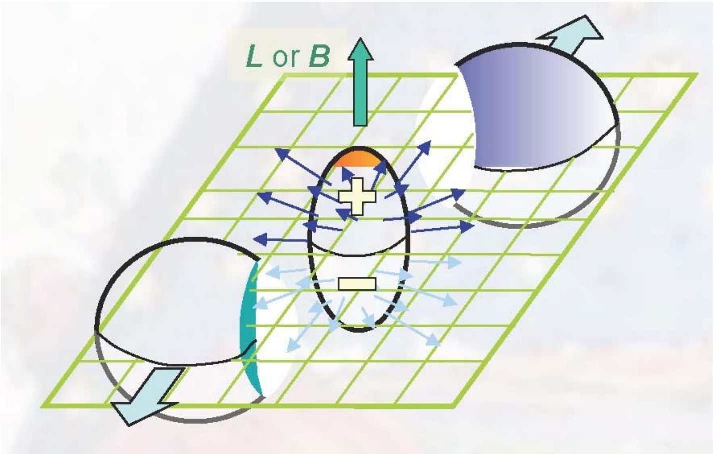
- **Huge global orbital angular momenta are produced**

$$L \sim 10^5 \hbar$$

- **Very strong magnetic fields are produced**

$$B \sim m_\pi^2 \sim 10^{18} \text{ Gauss}$$

- **How do orbital angular momenta be transferred to the matter created?**
- **Any way to measure angular momentum? How is spin coupled to local vorticity in a fluid?**



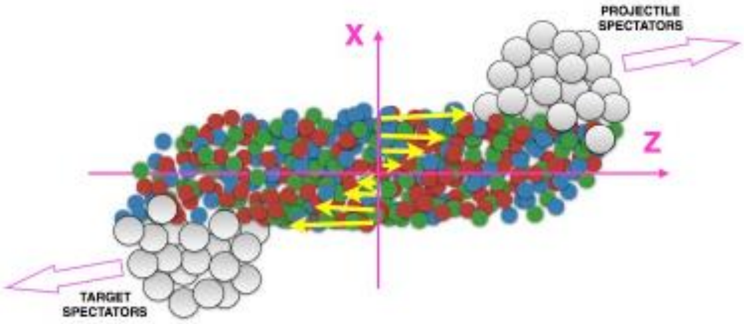


Figure taken from Becattini et al, 1610.02506

Rotation vs Polarization

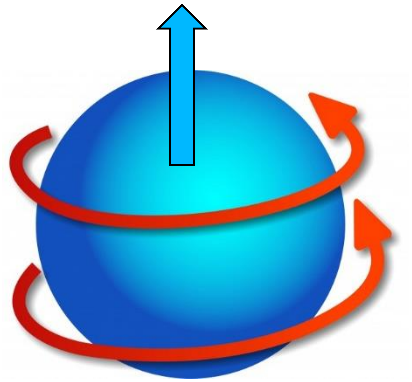
- **Barnett effect: rotation to polarization**

  uncharged object in rotation

  → spontaneous magnetization

  → polarization (spin-orbital coupling)

  [Barnett, Rev.Mod.Phys.7,129(1935)]



- **Einstein-de Haas Effect: polarization to rotation**

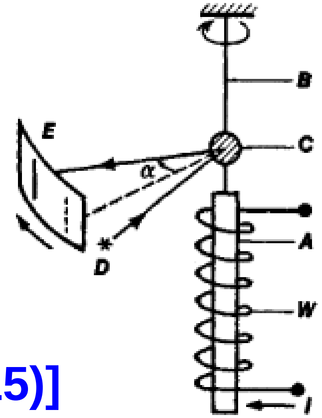
  magnetic field (impulse)

  → polarization of electrons

  → ΔL_electron

  → ΔL_mechanical = - ΔL_electron

[Einstein, de Haas, DPG Verhandlungen 17, 152(1915)]



Theoretical models and proposals: early works on global polarization in HIC

With such correlation between rotation and polarization in materials, we expect the same phenomena in heavy ion collisions. Some early works along this line:

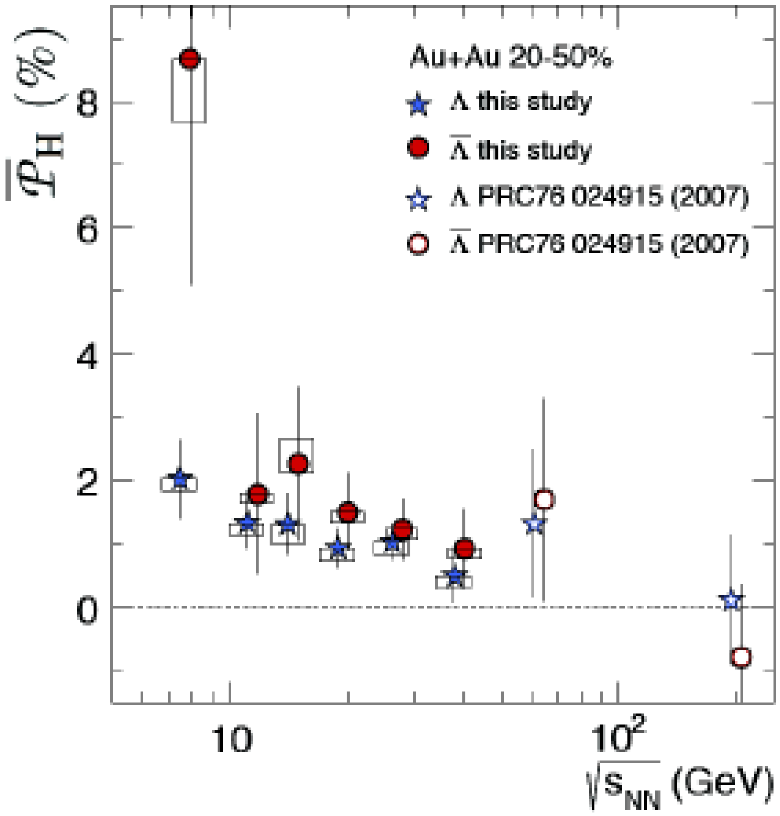
- **Polarizations of Λ hyperons and vector mesons through spin-orbital coupling in HIC from global OAM**
- **-- Liang and Wang, PRL 94,102301(2005), PRL 96, 039901(E) (2006) [nucl-th/0410079]**
- **-- Liang and Wang, PLB 629, 20(2005) [nucl-th/0411101]**

- **Polarized secondary particles in un-polarized high energy hadron-hadron collisions**
- **-- Voloshin, nucl-th/0410089**

- **Polarization as probe to vorticity in HIC**
- **-- Betz, Gyulassy, Torrieri, PRC 76, 044901(2007) [0708.0035]**

- **Statistical model for relativistic spinning particles**
- **-- Becattini, Piccinini, Annals Phys. 323, 2452 (2008) [0710.5694]**

Rotation vs Polarization



STAR Collab., Nature 548 (2017) 62

Largest vorticity ever observed

- **The fluid vorticity may be estimated from the data using the hydrodynamic relation with a systematic uncertainty of a factor of 2, mostly due to uncertainties in the temperature**

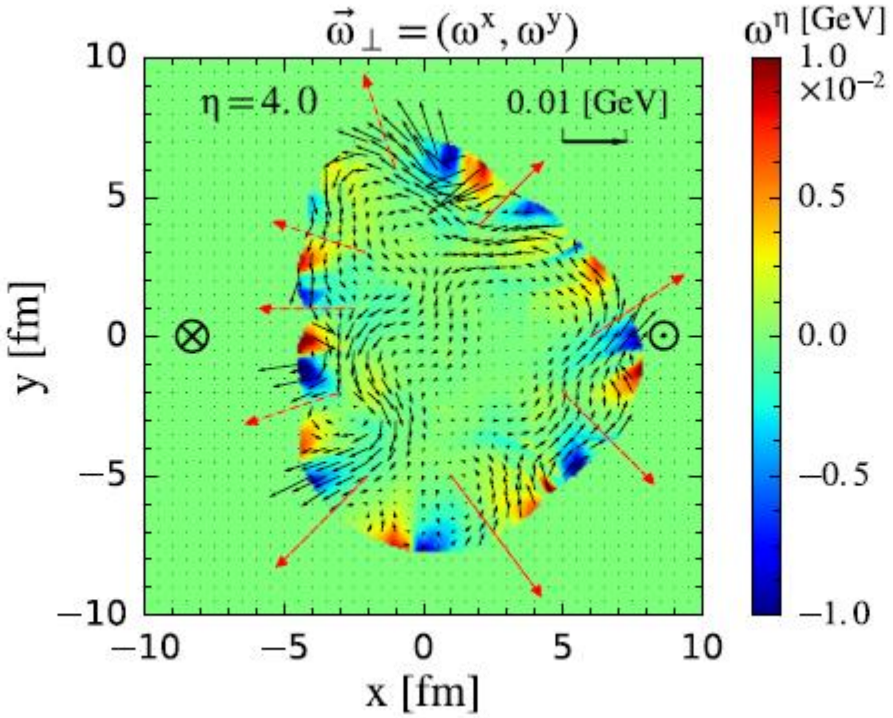
$$\omega \sim k_B T(\mathscr{P}_\Lambda + \mathscr{P}_{\bar{\Lambda}})/\hbar \approx (9 \pm 1) \times 10^{21}\ \mathrm{s}^{-1}$$

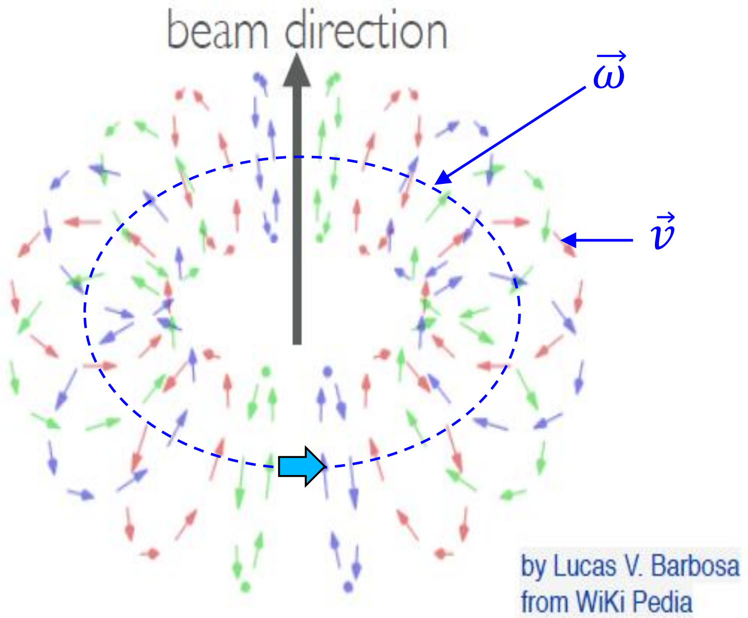
STAR Collab., 1701.06657; Becattini et al., 1610.02506; Pang et al., PRC 94, 024904(2016); Aristova, Frenklakh, Gorsky, Kharzeev, JHEP(2016);

- **This far surpasses the vorticity of all other known fluids**

solar subsurface flow	$10^{-7}\ \mathrm{s}^{-1}$
large scale terrestrial atmospheric patterns	$10^{-7} - 10^{-5}\ \mathrm{s}^{-1}$
Great Red Spot of Jupiter	$10^{-4}\ \mathrm{s}^{-1}$
supercell tornado cores	$10^{-1}\ \mathrm{s}^{-1}$
rotating, heated soap bubbles	$100\ \mathrm{s}^{-1}$
turbulent flow in bulk superfluid He-II	$150\ \mathrm{s}^{-1}$
superfluid nanodroplets	$10^{7}\ \mathrm{s}^{-1}$

Turbulence and vortices in high energy HIC





Spin-spin correlation of Λ can probe the vortical structure of sQGP

Pang, Petersen, QW, et al., PRL 117, 192301 (2016)

Theoretical models for global polarization

- **Spin-orbit coupling or microscopic models**
- **[Liang and Wang, PRL 94,102301(2005); Gao, Chen, Deng, Liang, QW, Wang, PRC 77, 044902(2008); Zhang, Fang, QW, Wang, arXiv:1904.09152.]**

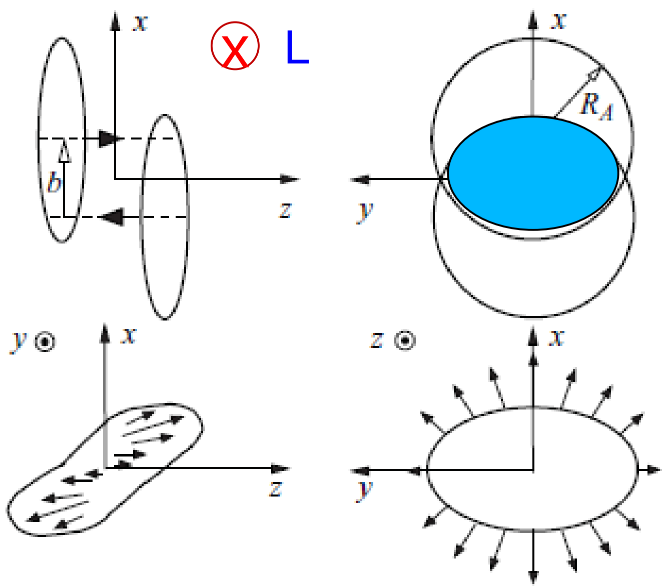
- **Statistical-hydro models**
- **[Zubarev (1979); Weert (1982); Becattini et al. (2012-2015); Hayat, et al. (2015); Floerchinger (2016).]**

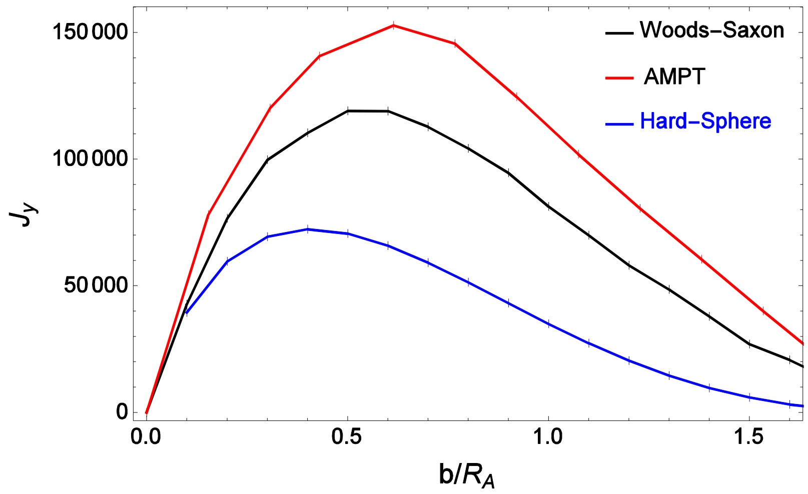
- **Spin hydrodynamic model**
- **[Florkowski,Friman,Jaiswal,Ryblewski,Speranza (2017-2018); Montenegro,Tinti,Torrieri (2017-2019).]**

- **Kinetic theory for massive fermions with Wigner functions**
- **[Early works: Heinz (1983); Vasak, Gyulassy and Elze (1987); Elze, Gyulassy, Vasak (1986); Zhuang, Heinz (1996).]**
- **[Recent developments: Fang, Pang, QW, Wang (2016); Weickgenannt, Sheng, Speranza, QW, Rischke (2019); Gao, Liang (2019); Wang, Guo, Shi, Zhuang (2019); Hattori, Hidaka, Yang (2019).]**

Global OAM in HIC

- **Non-central collisions produce global orbital angular momentum**

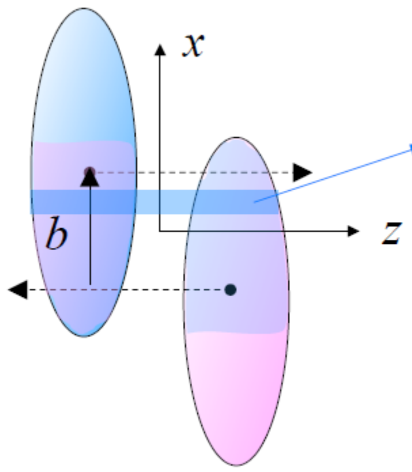




$$L_y = -p_{\rm in} \int x dx \left(\frac{dN^P_{\rm part}}{dx} - \frac{dN^T_{\rm part}}{dx} \right)$$

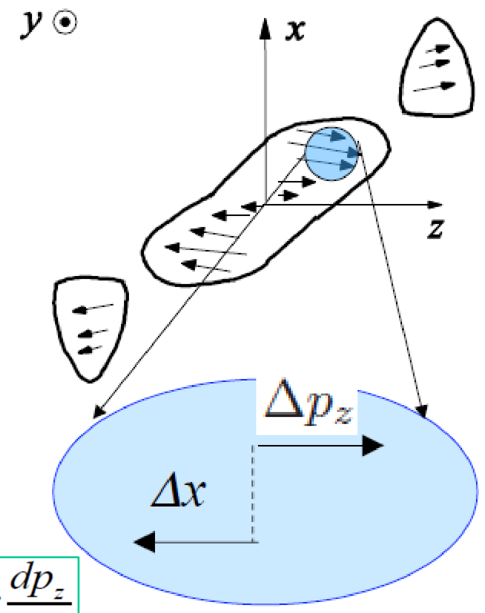
Liang & Wang, PRL 94, 102301(2005); PLB 629, 20(2005); Gao, Chen, Deng, Liang, QW, Wang, PRC 77, 044902(2008); Huang, Huovinen, Wang,PRC 84,054910(2011); Jiang, Lin, Liao, PRC 94,044910(2016); many others ……

Global OAM in HIC



$$\frac{dN^{P,T}_{part}}{dx}$$

Number of participant nucleons per unit x in projectile or target



Collective longitudinal momentum per produced parton

$$p_z(x,b) = \frac{\sqrt{s}}{2c(s)} \frac{\frac{dN^P_{\rm part}}{dx} - \frac{dN^T_{\rm part}}{dx}}{\frac{dN^P_{\rm part}}{dx} + \frac{dN^T_{\rm part}}{dx}}$$

$$\Delta p_z = \Delta x \frac{dp_z}{dx}$$

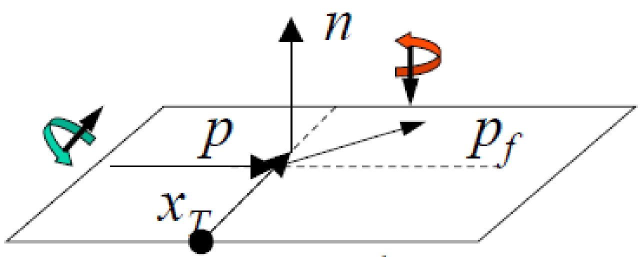
$$L_y = -\Delta x \Delta p_z = -(\Delta x)^2 \frac{dp_z}{dx} \sim \beta\omega_y$$

Liang & Wang (2005); Gao, et al. (2008); Betz, Gyulassy, Torrieri (2007); Jiang, Lin, Liao (2016); many others ……

Spin-orbital coupling model

Quark scatterings in potential

- **Quark scatterings at small angle in static potential with screening mass**
- **Unpolarized and polarized cross sections**



$$\frac{d\sigma}{d^2\vec{x}_T} = \frac{d\sigma_+}{d^2\vec{x}_T} + \frac{d\sigma_-}{d^2\vec{x}_T} = 4C_T\alpha_s^2 K_0(\mu x_T)$$

$$\frac{d\Delta\sigma}{d^2\vec{x}_T} = \frac{d\sigma_+}{d^2\vec{x}_T} - \frac{d\sigma_-}{d^2\vec{x}_T} \propto \vec{n}\cdot(\vec{x}_T \times \vec{p})$$

$$A^0(q_T) = \frac{1}{q_T^2 + \mu^2}$$

screening mass

$$\mu \sim T\sqrt{\alpha_S}$$

Polarization vector **OAM** **Spin-Orbit coupling**

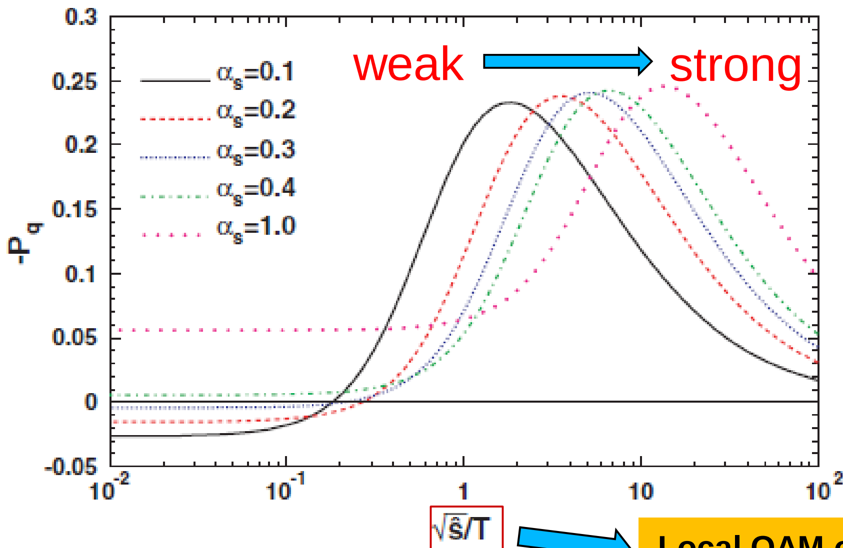
- **Polarization for small angle scattering and** $m_q \gg p, \mu$

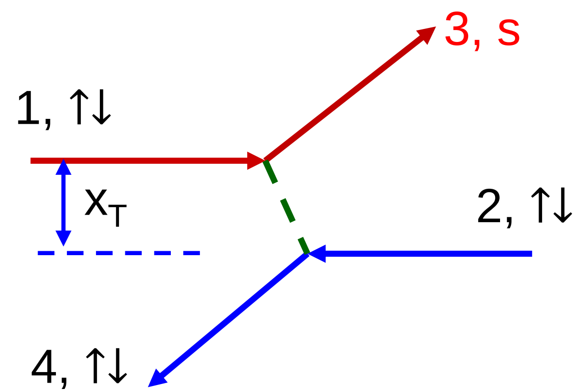
$$P_q \approx -\pi \frac{\mu p}{4m_q^2} \sim -\frac{\Delta E_{LS}}{E_0}$$

Liang, Wang, PRL 94, 102301(2005)

Quark-quark scattering

- **Beyond small angle approximation with HTL gluon progagator**





Local OAM or vorticity

$$L \sim \langle x \rangle p \sim \frac{p}{\mu} \sim \beta\omega$$

Quark polarization as functions of the square root of parton-parton scattering energy over T [≈ local OAM or vorticity] which increases with α_S

Liang, Wang, PRL 94, 102301(2005); PLB 629, 20(2005);
Gao, Chen, Deng, Liang, QW, Wang, PRC 77, 044902(2008)

Kinetic model with Wigner function

- **Kinetic approach**
- **Classical kinetic approach: f(t,x,p)**
- **Quantum kinetic approach: W(t,x,p)**

Wigner functions for fermions in background EM field

- **The Wigner function for spin 1/2 fermions in constant EM field satisfies EOM, which can be solved perturbatively in Planck constant $\hbar$.**
- **Wigner function can be decomposed in 16 generators of Clifford algebra**

$$W=\frac{1}{4}\left[\mathscr{F}+i\gamma^{5}\mathscr{P}+\gamma^{\mu}\mathscr{V}_{\mu}+\gamma^{5}\gamma^{\mu}\mathscr{A}_{\mu}+\frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}\right]$$

4x4 matrix | scalar | p-scalar | vector | axial-vector | tensor

$$j^{\mu}=\int d^{4}p\mathscr{V}^{\mu},\qquad j_{5}^{\mu}=\int d^{4}p\mathscr{A}^{\mu},\qquad T^{\mu\nu}=\int d^{4}pp^{\mu}\mathscr{V}^{\nu}$$

Heinz, Phys.Rev.Lett. 51, 351 (1983);
Vasak, Gyulassy and Elze, Annals Phys. 173, 462 (1987);
Elze, Gyulassy and Vasak, Nucl. Phys. B 276, 706(1986);
Zhuang, Heinz, Annals Phys. 245, 311(1996).
Many others ......

Wigner functions for chiral fermions in background EM field

- 魏格纳函数的解耦定理**(DCW Theorem)**

在电磁场中，手征费米子的魏格纳函数有多个分量，满足多个耦合方程。我们证明了在$\hbar$的任意阶，这些方程都可以约化为一个分量的一个约束方程和一个演化方程。此定理大大简化了对手征效应的计算和模拟。

Gao, Liang, QW, et. al., "Disentangling covariant Wigner functions for chiral fermions", PRD (2018).

- 手征涡旋效应的"**1/3**"难题的解决

在**3**维手征动力学理论中，由涡旋导致的守恒流的普通部分只占全部贡献的**1/3**，另外**2/3**贡献丢失(后来发现由磁化流给出)。由于我们理论中使用的是协变魏格纳函数，可以证明通过选取参考系，可以自然给出手征涡旋流的普通部分和磁化部分，每一部分都依赖于参考系(不是协变的)，而两者之和是协变的，不依赖于参考系。

Gao, Pang, QW, "The chiral vortical effect in Wigner function approach", PRD (2019).

Local Polarization Effect

Consider 3-flavor quark matter (u,d,s), the axial baryonic current

$$j_5^\sigma = n_5 u^\sigma + \xi_5 \omega^\sigma + \xi_{5B} B^\sigma$$

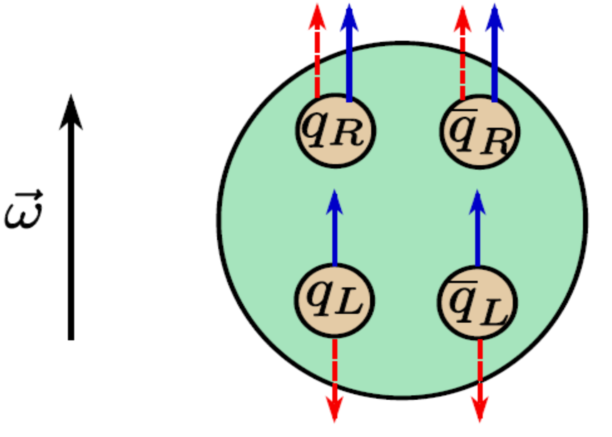
$$\xi_5 = N_c \left[\frac{1}{6} T^2 + \frac{1}{2\pi^2} (\mu^2 + \mu_5^2) \right],$$

$$\xi_{B5} = \frac{N_c}{6\pi^2} \mu \sum_f Q_f = 0.$$

→ Quadratic in temperature, chemical potential, chiral chemical potential → No cancellation!

Leading to Local Polarization Effects!
(either for high or low energy HIC)

The LPE can be measured in heavy ion collisions by the hadron (e.g. hyperon) polarization along the vorticity direction once it is fixed in the event.



J.H.Gao, Z.T.Liang, S.Pu, et al., PRL109, 232301(2012)

Spin pseudo-vector component for massive spin-1/2 fermions

- **Spin pseudo-vector is given by axial vector component of WF (Pauli-Lubanski pseudo-vector)**

$$\mathscr{A}_{\mu}^{(0)} = -\frac{1}{2m}\epsilon_{\mu\beta\nu\sigma}p^{\beta}\underline{\mathscr{S}_{(0)}^{\nu\sigma}}$$

tensor component

- **where in the zeroth order $\mathbf{O}(\hbar^{\mathbf{0}})$**

$$A^{(0)}(x,p) \equiv \frac{2}{(2\pi\hbar)^3}\sum_{es} s\,\theta(e\,p^0)f_s^{(0)e}(x,e\,\mathbf{p})$$

$$\begin{aligned}\mathscr{A}_{\mu}^{(0)}(x,p) &= m\,n_{\mu}^{(0)}(x,\mathbf{p})\delta(p^2-m^2)\underline{A^{(0)}(x,p)}\\ \mathscr{S}_{\mu\nu}^{(0)}(x,p) &= m\,\Sigma_{\mu\nu}^{(0)}(x,\mathbf{p})\delta(p^2-m^2)\underline{A^{(0)}(x,p)}\end{aligned}$$

Fang, Pang, QW, Wang, PRC (2016);
Weickgenannt, Sheng, Speranza, QW, Rischke, PRD(2019);
Gao, Liang, PRD(2019);
Hattori, Hidaka, Yang, PRD(2019);
Wang, Guo, Shi, Zhuang, PRD(2019).

Polarization (spin) vector in WF at $O(\hbar)$

- **Polarization at zeroth order is vanishing if we assume that the chemical potential for spin-up and spin-down fermions are equal.**
- **Polarization vector at the first order**

Fang, Pang, QW, et al., PRC(2016);
Yang, Fang, QW, et al., PRC(2018);
QW, NPA(2017)

$$P_{\pm}^{\mu}(x,p) \approx \frac{1}{2m}\left(\tilde{\omega}_{\text{th}}^{\mu\nu} \pm \frac{1}{E_p T} Q\tilde{F}^{\mu\nu}\right) p_\nu$$

$$p^\mu = (E_p, \pm\mathbf{p})$$

- **Applying to s and $\bar{s}$ quark, we obtain the polarization along the y-direction**

$$\boldsymbol{\varepsilon} = -\frac{1}{2}[\partial_t(\beta\mathbf{u}) + \nabla(\beta u^0)]$$

$$\begin{aligned} P_{s/\bar{s}}^{y}(t,\mathbf{x},\mathbf{p}_{s/\bar{s}}) &= \frac{1}{2}\omega_{\text{th}}^{y} \pm \frac{1}{2m_s}\hat{\mathbf{y}}\cdot(\boldsymbol{\varepsilon}\times\mathbf{p}_{s/\bar{s}}) \\ &\pm \frac{Q_s}{2m_s T}B_y + \frac{Q_s}{2m_s^2 T}\hat{\mathbf{y}}\cdot\left(\mathbf{E}\times\mathbf{p}_{s/\bar{s}}\right) \end{aligned}$$

electric part of vorticity tensor

Wigner functions for fermions in background EM field

- **In the coalescence model we have**

$$
\begin{aligned}
P^y_{\Lambda/\bar{\Lambda}}(t,\mathbf{x}) &= \frac{1}{3}\int \frac{d^3\mathbf{r}}{(2\pi)^3}\frac{d^3\mathbf{q}}{(2\pi)^3}\left|\psi_{\Lambda/\bar{\Lambda}}(\mathbf{q},\mathbf{r})\right|^2 \\
&\quad \times \left[P^y_{s/\bar{s}}(\mathbf{p}_1,\mathbf{x}) + P^y_{s/\bar{s}}(\mathbf{p}_2,\mathbf{x}) + P^y_{s/\bar{s}}(\mathbf{p}_3,\mathbf{x})\right] \\
&= \frac{1}{2}\omega^y_{\mathrm{th}} \pm \frac{Q_s}{2m_s T}B_y
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{p}_1 &= \frac{1}{3}\mathbf{p} + \frac{1}{2}\mathbf{r} + \mathbf{q} \\
\mathbf{p}_2 &= \frac{1}{3}\mathbf{p} + \frac{1}{2}\mathbf{r} - \mathbf{q} \\
\mathbf{p}_3 &= \frac{1}{3}\mathbf{p} - \mathbf{r}
\end{aligned}
$$

- **We see that only magnetic field appears.**
- **The product of P^y_Λ and $P^y_{\bar{\Lambda}}$**

$$
P_\Lambda P_{\bar{\Lambda}} = \frac{1}{4}(\omega^y_{\mathrm{th}})^2 - \frac{Q_s^2}{4m_s^2 T^2}B_y^2
$$

Yang, Fang, QW, et al., PRC(2018); Sheng, Oliva, QW (2019)

Wigner functions for fermions in background EM field

- **Spin density matrix element for ϕ meson in coalescence model**

$$\rho_{00}^{\phi}(t,\mathbf{x}) \approx \frac{1}{3} - \frac{4}{9}\int \frac{d^3\mathbf{p}}{(2\pi)^3} P_s^y(\mathbf{p}) P_{\bar{s}}^y(-\mathbf{p}) \left|\psi_\phi(\mathbf{p})\right|^2$$

- **where we have applied $\mathbf{p}_s = \mathbf{p}$ and $\mathbf{p}_{\bar{s}} = -\mathbf{p}$**

Liang, Wang (2005)
Yang, Fang, QW, et al. (2018)

- **Inserting $P^y_{s/\bar{s}}(t,\mathbf{x},\mathbf{p}_{s/\bar{s}})$ and taking an average of $\rho_{00}^{\phi}(t,\mathbf{x})$ over the fireball volume V and the polarization time t with an effective temperature T_{eff}, we obtain**

Sheng, Oliva, QW (2019)

$$\rho_{00}^{\phi} \approx \frac{1}{3} - \frac{4}{9}\left\langle P_{\bar{\Lambda}}^y P_{\Lambda}^y \right\rangle - \frac{1}{27 m_s^2}\left\langle \mathbf{p}^2 \right\rangle_\phi \left\langle \varepsilon_z^2 + \varepsilon_x^2 \right\rangle + \frac{e^2}{243 m_s^4 T_{\mathrm{eff}}^2}\left\langle \mathbf{p}^2 \right\rangle_\phi \left\langle E_z^2 + E_x^2 \right\rangle$$

$c_\Lambda \sim 6\times 10^{-5}$

$c_\varepsilon \sim 10^{-5} - 10^{-6}$

$c_E \sim 10^{-5}$

$\sqrt{s} \leq 200$ GeV

Mean field of ϕ

- So with vorticity and EM field, we have

$$\rho_{00}^{\phi} \approx \frac{1}{3}$$

Coherent vector meson field was proposed to explain $P_{\Lambda} - P_{\bar{\Lambda}}$
Csernai, Kapusta, Welle (2019)

- How to accommodate a large positive deviation for ρ_{00}^{ϕ} ?
- Coherent ϕ field may be the key:
- There may exist non-zero strangeness current [e.g. $s(x) \neq \bar{s}(x)$ in nucleon]:

$$\partial_{\mu} J_s^{\mu} = 0$$

Sheng, Oliva, QW (2019)

$$J_s^{\mu} = (0, \mathbf{J}_s(t, \mathbf{x})) = (0, j_s^{(x)}(y,z,t), j_s^{(y)}(x,z,t), j_s^{(z)}(x,y,t))$$

- The electric and magnetic part of ϕ field that contribute to the spin alignment along +y direction are given by

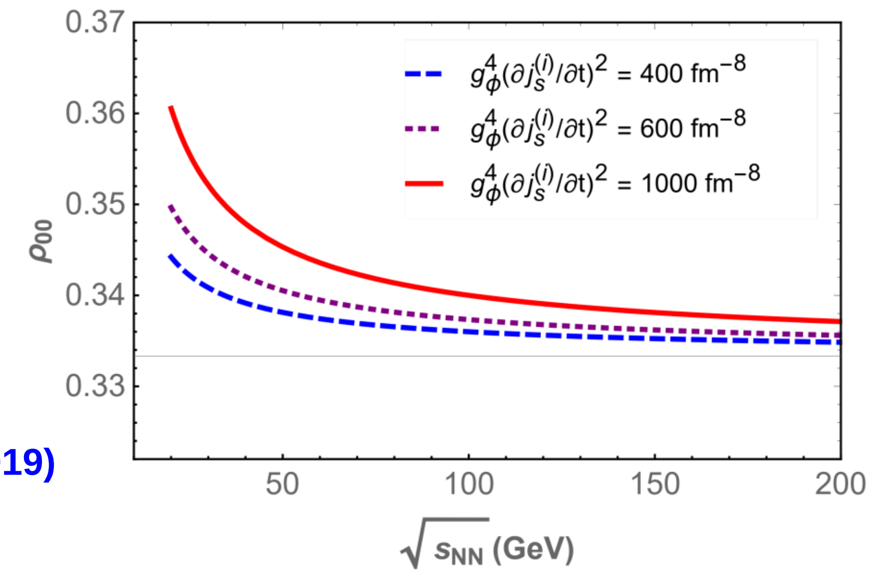
$$\mathbf{E}_{\phi} = \hat{\mathbf{z}} \frac{g_{\phi}}{m_{\phi}^2} \frac{\partial j_s^{(z)}}{\partial t} + \hat{\mathbf{x}} \frac{g_{\phi}}{m_{\phi}^2} \frac{\partial j_s^{(x)}}{\partial t} \qquad \mathbf{B}_{\phi} = -\frac{g_{\phi}}{m_{\phi}^2} \nabla \times \mathbf{J}_s = \hat{\mathbf{y}} \frac{g_{\phi}}{m_{\phi}^2} \left[\frac{\partial j_s^{(z)}}{\partial x} - \frac{\partial j_s^{(x)}}{\partial z} \right]$$

Contribution to ρ_{00} from mean field of ϕ

- **The main contribution is from the electric part of the coherent ϕ field and positive definite!**

$$\rho_{00}^{\phi} \approx \frac{1}{3} + \frac{g_\phi^4}{27 m_s^4 m_\phi^4 T_{\text{eff}}^2} \left\langle \mathbf{p}^2 \right\rangle_\phi \left\langle \left(\frac{\partial j_s^{(z)}}{\partial t} \right)^2 + \left(\frac{\partial j_s^{(x)}}{\partial t} \right)^2 \right\rangle$$

- **Our prediction:**



Sheng, Oliva, QW (2019)

Comparison with data for global polarization

Theoretical results for global polarization in y direction (OAM)

- **AMPT transport model**
- **-- Li, Pang, QW, Xia, PRC96,054908(2017)**

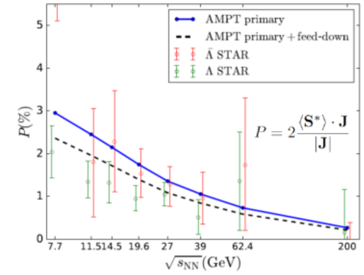
- **UrQMD + vHLLE hydro**
- **-- Karpenko, Becattini, EPJC 77, 213(2017)**

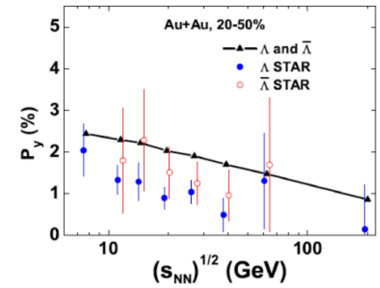
- **PICR hydro**
- **-- Xie, Wang, Csernai, PRC 95,031901(2017)**

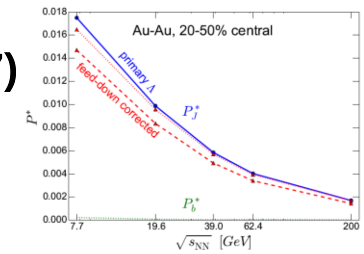
- **Chiral Kinetic Approach + Collisions**
- **-- Sun, Ko, PRC96, 024906(2017)**

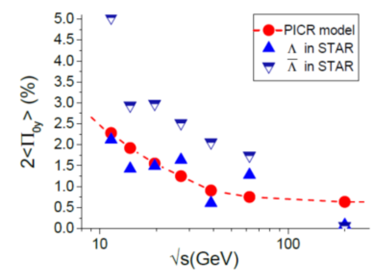
- **Many other works …………**

- **Take-home message I: All models can describe data: the global polarization is a robust and significant effect!**





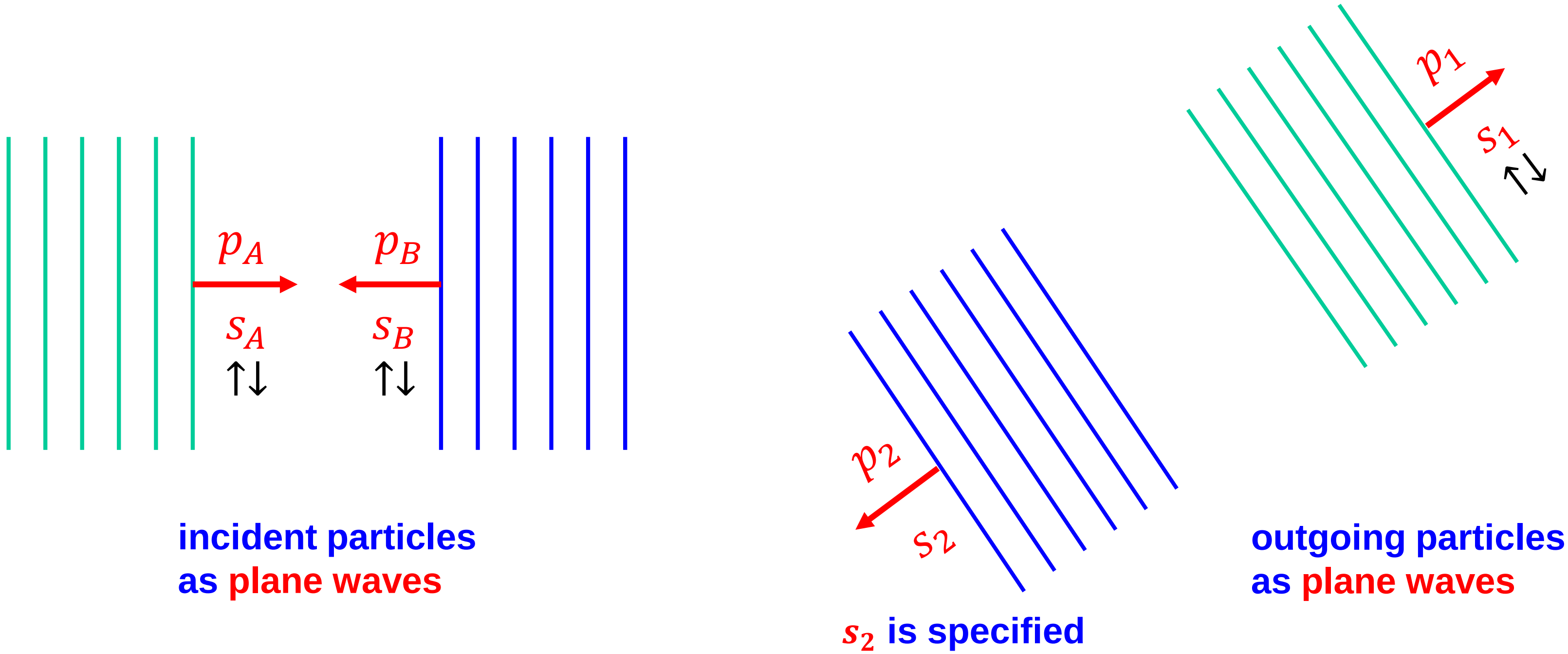




A microscopic model for global polarization through spin-orbit couplings in particle scatterings

[J.-J. Zhang, R.-H. Fang, QW, et al., to appear in PRC, 1904.09152]

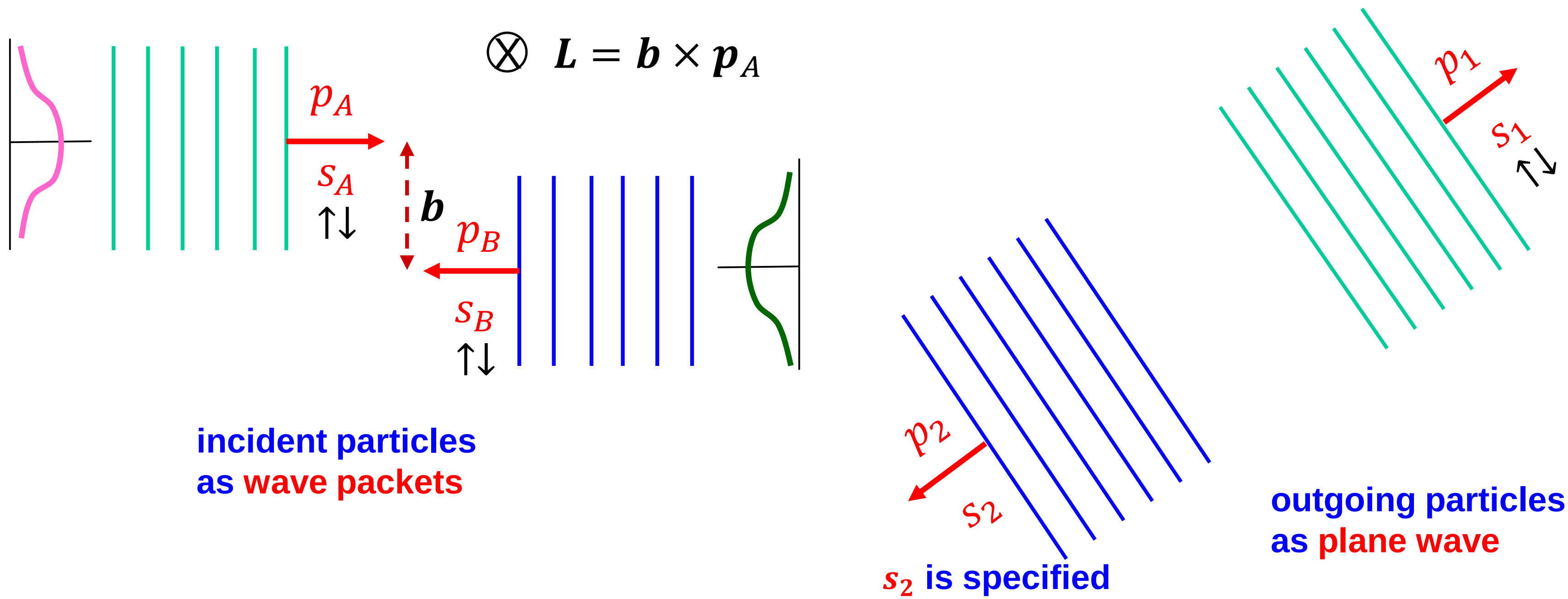
Collisions of particles as plane waves



Particle collisions as plane waves:
since there is no preferable position for particles, so there is no OAM and polarization

$$\langle \hat{\boldsymbol{x}} \times \hat{\boldsymbol{p}} \rangle = \mathbf{0} \quad \Longrightarrow \quad \left(\frac{d\sigma}{d\Omega}\right)_{s_2=\uparrow} = \left(\frac{d\sigma}{d\Omega}\right)_{s_2=\downarrow}$$

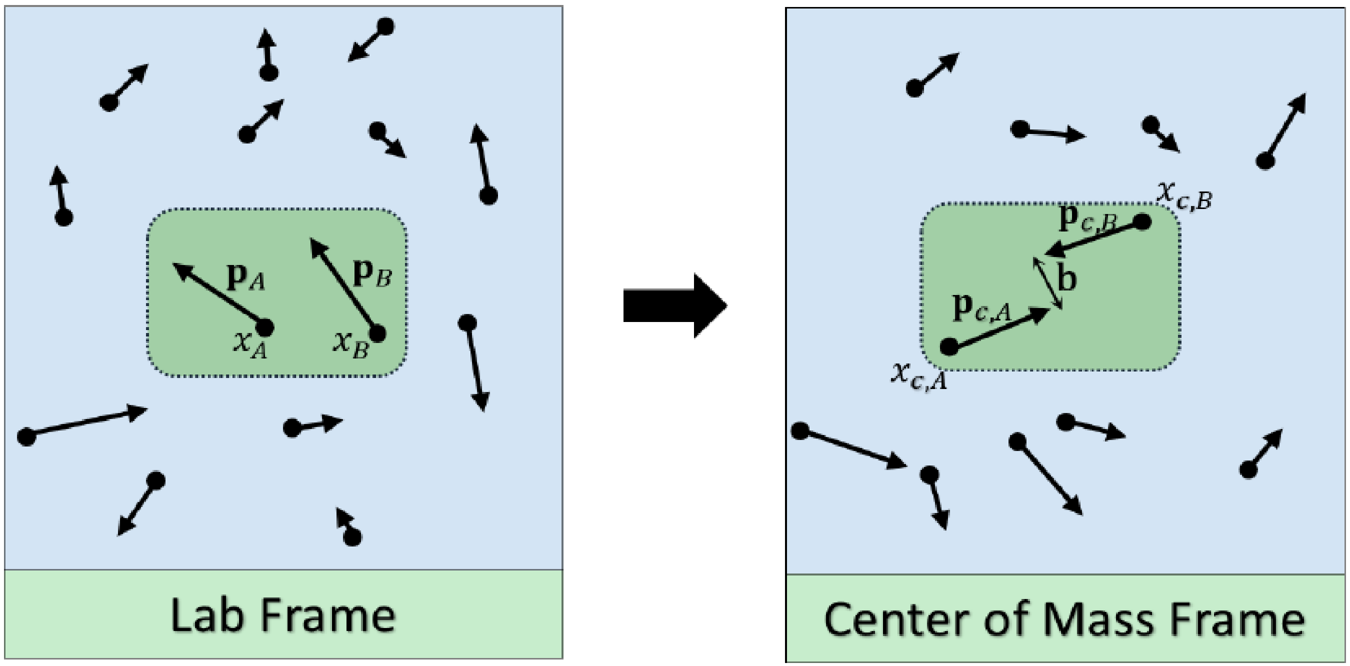
Collisions of particles as wave packets



Particle collisions as wave packets: there is a transverse distance between two wave packets (impact parameter) giving non-vanishing OAM and then the polarization of one final particle

$$\boldsymbol{L} = \boldsymbol{b} \times \boldsymbol{p}_A \quad \Longrightarrow \quad \left(\frac{d\sigma}{d\Omega}\right)_{s_2=\uparrow} \neq \left(\frac{d\sigma}{d\Omega}\right)_{s_2=\downarrow}$$

Collisions of particles at different space-time points



(1) Momentum distributions depend on $u^\alpha(x)$ in Lab frame

(2) Collisions of momentum states at one space-time point does not contain information about gradient of $u^\alpha(x)$

(3) The gradient of $u^\alpha(x)$ can only be probed by collisions of particles at different space-time points

Collisions of particles at different space-time points

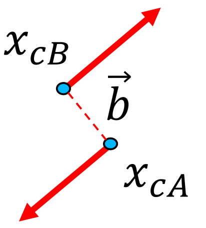
- **Two incident particles at** $x_A = (t_A, \mathbf{x}_A)$ **and** $x_B = (t_B, \mathbf{x}_B)$
- **We have**

$$\boxed{\mathbf{x}_A \neq \mathbf{x}_B}$$

$$t_A = t_B \Longrightarrow t_{c,A} \neq t_{c,B}$$

$$t_A \neq t_B \Longrightarrow t_{c,A} = t_{c,B}$$

$$\boxed{\mathbf{x}_{c,A} \neq \mathbf{x}_{c,B}}$$



$\left|\vec{b}\right| < b_0$

CM frame

- **We impose the causality condition in CM frame for scattering of particles at two different space-time points (the time interval and longitudinal distance of two space-time points should be small enough for scattering to take place)**

$$\Delta t_c = t_{c,A} - t_{c,B} = 0$$

$$\Delta x_{c,L} = \hat{\mathbf{p}}_{c,A} \cdot (\mathbf{x}_{c,A} - \mathbf{x}_{c,B}) = 0$$

$\left|\vec{b}\right| < b_0$

Collisions of particles at different space-time points

- **Collision rate of two particles at two space-time points in CMS**

$$R_{AB\to 12} = \int \frac{d^3p_A}{(2\pi)^3 2E_A}\frac{d^3p_B}{(2\pi)^3 2E_B}\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}$$
$$\times \frac{1}{C_{AB}}\int d^4x_A d^4x_B \delta(\Delta t)\delta(\Delta x_L)$$
$$\times f_A(x_A,p_A) f_B(x_B,p_B) G_1 G_2 |v_A - v_B|$$
$$\times (2E_A)(2E_B)|_{\text{out}}\langle p_1 p_2|\phi_A(x_A,p_A)\phi_B(x_B,p_B)\rangle_{\text{in}}|^2$$

$C_{AB} \equiv \int d^4X = t_X \Omega_{\text{int}}$

equal time and L-distance

distributions for incident particles At two points

scattering amplitude

- **We carry out integral over x_A and x_B**

$$I = \int d^4x_A d^4x_B \delta(\Delta t)\delta(\Delta x_L) f_A(x_A,p_A) f_B(x_B,p_B)$$
$$\times \exp\left(-i\mathbf{k}_A\cdot\mathbf{x}_A - i\mathbf{k}_B\cdot\mathbf{x}_B + i\mathbf{k}'_A\cdot\mathbf{x}_A + i\mathbf{k}'_B\cdot\mathbf{x}_B\right)$$
$$\approx \int d^4X d^2\mathbf{b} f_A\left(X+\frac{y_T}{2},p_A\right) f_B\left(X-\frac{y_T}{2},p_B\right)$$
$$\times \exp\left[i(\mathbf{k}'_A - \mathbf{k}_A)\cdot\mathbf{b}\right]$$

$$X = \frac{1}{2}(x_A + x_B)$$
$$y = x_A - x_B.$$

$\vec{b} = \vec{x}_A - \vec{x}_B$

phase from impact parameter

all variables are defined in CMS but we suppress index 'c' for simplicity

Polarization of spin-1/2 particles from scatterings (general formula)

- **Polarization from particle scatterings $A+B\rightarrow 1+2$ at different space-time points**

all variables are defined in CMS index 'c'

$$
\begin{array}{lcl}
(s_A,p_A)+(s_B,p_B) & \rightarrow & (s_1,p_1)+(s_2,p_2) \\
\textit{wave packets} & & \textit{plane waves} \\
\textit{sum over } (s_A,s_B,s_1) & & s_2 \textit{ is open}
\end{array}
$$

$$
\begin{aligned}
\frac{d^4\mathrm{P}_{AB\rightarrow 12}(X)}{dX^4} = & \frac{1}{(2\pi)^4}\int\frac{d^3p_{c,A}}{(2\pi)^3 2E_{c,A}}\frac{d^3p_{c,B}}{(2\pi)^3 2E_{c,B}}\frac{d^3p_{c,1}}{(2\pi)^3 2E_{c,1}}\frac{d^3p_{c,2}}{(2\pi)^3 2E_{c,2}} \\
& \times|v_{c,A}-v_{c,B}|G_1G_2\int d^3k_{c,A}d^3k_{c,B}d^3k'_{c,A}d^3k'_{c,B} \\
& \times\phi_A(\mathbf{k}_{c,A}-\mathbf{p}_{c,A})\phi_B(\mathbf{k}_{c,B}-\mathbf{p}_{c,B})\phi^*_A(\mathbf{k}'_{c,A}-\mathbf{p}_{c,A})\phi^*_B(\mathbf{k}'_{c,B}-\mathbf{p}_{c,B}) \\
& \times\delta^{(4)}(k'_{c,A}+k'_{c,B}-p_{c,1}-p_{c,2})\delta^{(4)}(k_{c,A}+k_{c,B}-p_{c,1}-p_{c,2}) \\
& \times\int d^2\mathbf{b}_c f_A\left(X_c+\frac{y_{c,T}}{2},p_A\right)f_B\left(X_c-\frac{y_{c,T}}{2},p_B\right)\exp\left[i(\mathbf{k}'_{c,A}-\mathbf{k}_{c,A})\cdot\mathbf{b}_c\right] \\
& \times\sum_{s_A,s_B,s_1,s_2}2s_2\mathbf{n}_c\mathcal{M}\left(\{s_A,k_{c,A};s_B,k_{c,B}\}\rightarrow\{s_1,p_{c,1};s_2,p_{c,2}\}\right) \\
& \times\mathcal{M}^*\left(\{s_A,k'_{c,A};s_B,k'_{c,B}\}\rightarrow\{s_1,p_{c,1};s_2,p_{c,2}\}\right)
\end{aligned}
$$

wave packet momenta

wave packet

distributions for incident particles displaced by b

phase factor

scattering amplitude

polarization direction $\vec{n}_c=\vec{b}\times\vec{p}_{cA}$

scattering amplitude

Application: quark polarization in 22 parton scatterings in QGP (locally thermalized in p)

- **Asumptions:**

(1) local equilibrium in momentum but not in spin

(2) $f(x,p)$ depends on x^{μ} through $f(x,p)=f[\beta(x)p\cdot u(x)]$

(3) All 22 scatterings with at least one quark the in final state

- **Expansion of $f_A(x_{cA},p_{cA})f_B(x_{cB},p_{cB})$ in small $y_{c,T}=(0,\vec{b})$**

$$
\begin{aligned}
&f_A\left(X_c+\frac{y_{c,T}}{2},p_{c,A}\right)f_B\left(X_c-\frac{y_{c,T}}{2},p_{c,B}\right)\\
=\;& f_A(X_c,p_{c,A})f_B(X_c,p_{c,B})+\frac{1}{2}y_{c,T}^{\mu}\frac{\partial(\beta u_{c,\rho})}{\partial X_c^{\nu}}\\
&\times\left[p_{c,A}^{\rho}f_B(X_c,p_{c,B})\frac{df_A(X_c,p_{c,A})}{d(\beta u_c\cdot p_{c,A})}-p_{c,B}^{\rho}f_A(X_c,p_{c,A})\frac{df_B(X_c,p_{c,B})}{d(\beta u_c\cdot p_{c,B})}\right]
\end{aligned}
$$

$$
\begin{aligned}
y_{c,T}^{\mu}p_{c,A}^{\rho}\frac{\partial(\beta u_\rho)}{\partial X_c^{\mu}}
&=-\frac{1}{2}y_{c,T}^{[\mu}p_{c,A}^{\rho]}\omega_{\mu\rho}^{(c)}\\
&\quad+\frac{1}{4}y_{c,T}^{\{\mu}p_{c,A}^{\rho\}}\left[\frac{\partial(\beta u_{c,\rho})}{\partial X_c^{\mu}}+\frac{\partial(\beta u_{c,\mu})}{\partial X_c^{\rho}}\right]
\end{aligned}
$$

local OAM
L-ω coupling

non-zero

Quark polarization rate

- **Quark polarization per unit volume: 10D + 6D integration**

$$\frac{d^4\mathbf{P}_{AB\to 12}(X)}{dX^4} = \frac{\pi}{(2\pi)^4}\frac{\partial(\beta u_\rho)}{\partial X^\nu}\int \frac{d^3p_A}{(2\pi)^3 2E_A}\frac{d^3p_B}{(2\pi)^3 2E_B} \times |v_{c,A}-v_{c,B}|[\Lambda^{-1}]^\nu_j \mathbf{e}_{c,i}\epsilon_{ikh}\hat{\mathbf{p}}^h_{c,A} \times f_A(X,p_A)f_B(X,p_B)(p^\rho_A - p^\rho_B)\Theta_{jk}(\mathbf{p}_{c,A}) \equiv \frac{\partial(\beta u_\rho)}{\partial X^\nu}\mathbf{W}^{\rho\nu}$$

6D integral

Lorentz boost

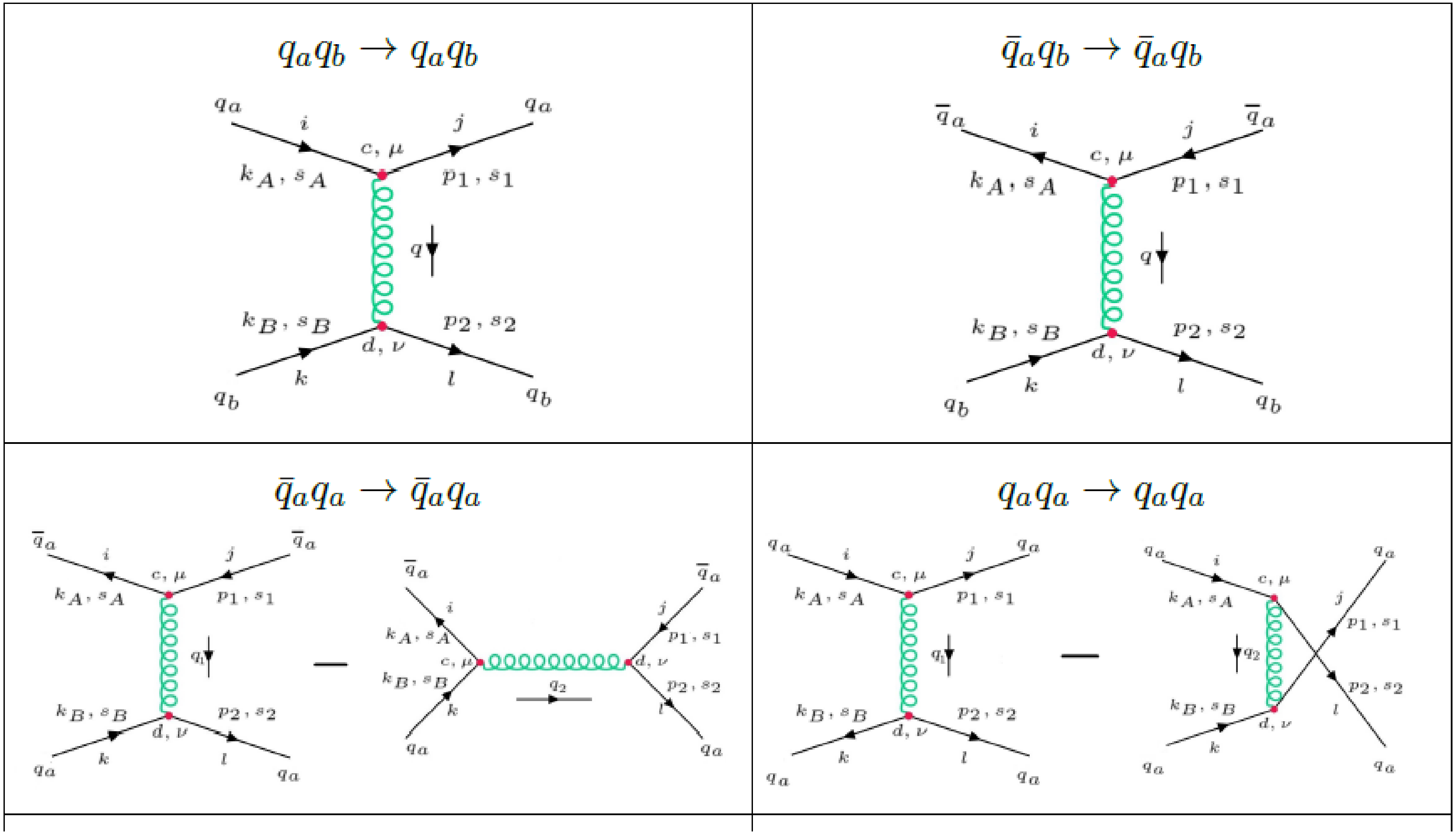
10D integral

16D integral !!

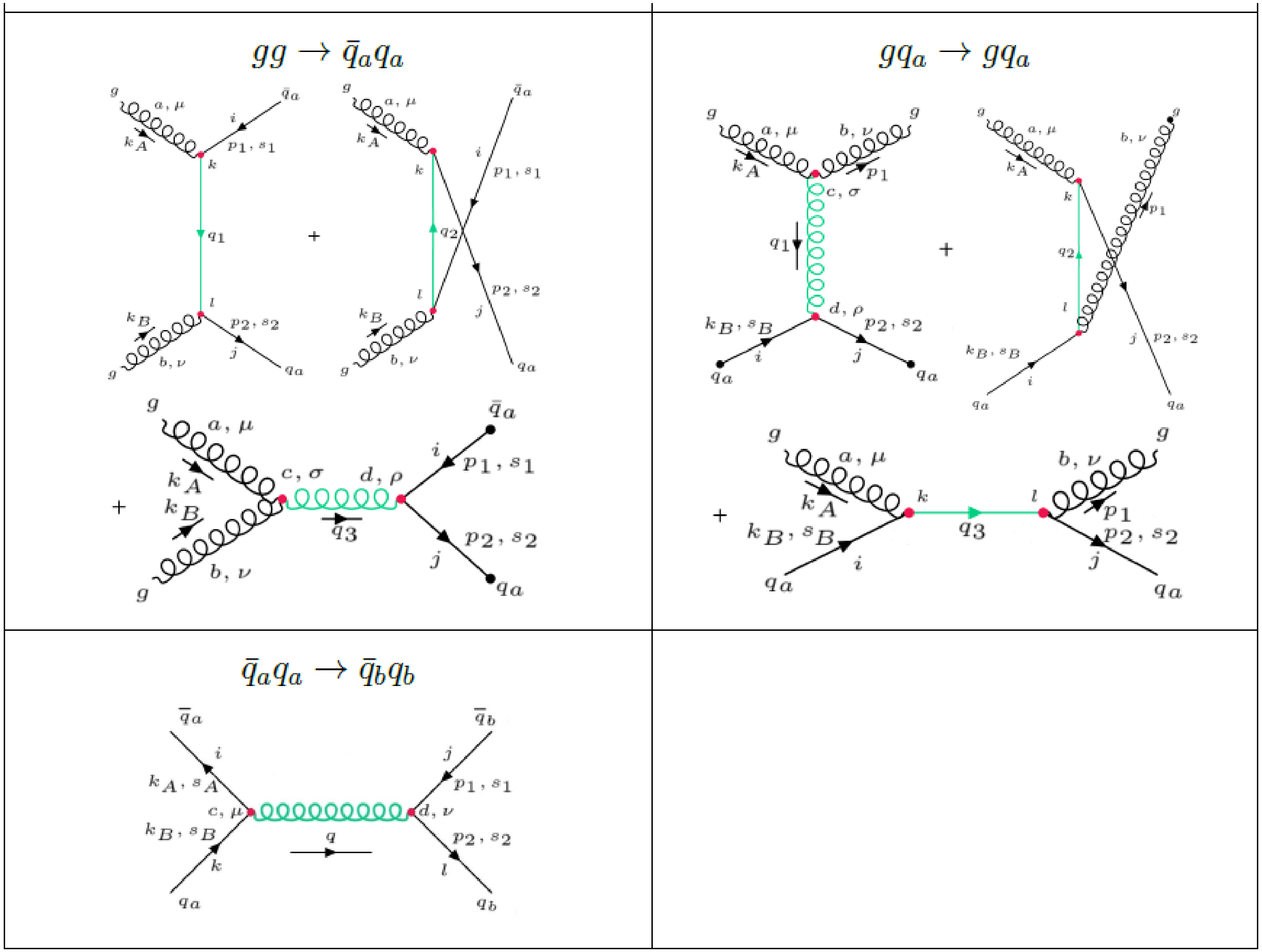
- **Numerical challenge !!! We use newly developed ZMCintegral-3.0, a Monte Carlo integration package that runs on multi-GPUs [Hong-zhong Wu, Junjie Zhang, Long-gang Pang, QW, Comp.Phys.Comm. (2019), 1902.07916.]**
- **Another challenge: there are more than 5000 terms in polarized amplitude squared**

$$I_M^{q_a q_b\to q_a q_b}(s_2) = \sum_{s_A,s_B,s_1}\sum_{i,j,k,l}\mathcal{M}(\{s_A,k_A;s_B,k_B\}\to\{s_1,p_1;s_2,p_2\})\mathcal{M}^*(\{s_A,k'_A;s_B,k'_B\}\to\{s_1,p_1;s_2,p_2\})$$

All 22 parton scaterings for quark polarization



All 22 parton scaterings for quark polarization



Numerical results for quark polarization

- **Numerical results show $W^{\rho\nu}$ has anti-symmetric structure**

$$\mathbf{W}^{\rho\nu} = W\epsilon^{0\rho\nu j}\mathbf{e}_j \quad \Rightarrow \quad \mathbf{W}^{\rho\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & W\mathbf{e}_z & -W\mathbf{e}_y \\ 0 & -W\mathbf{e}_z & 0 & W\mathbf{e}_x \\ 0 & W\mathbf{e}_y & -W\mathbf{e}_x & 0 \end{pmatrix}$$

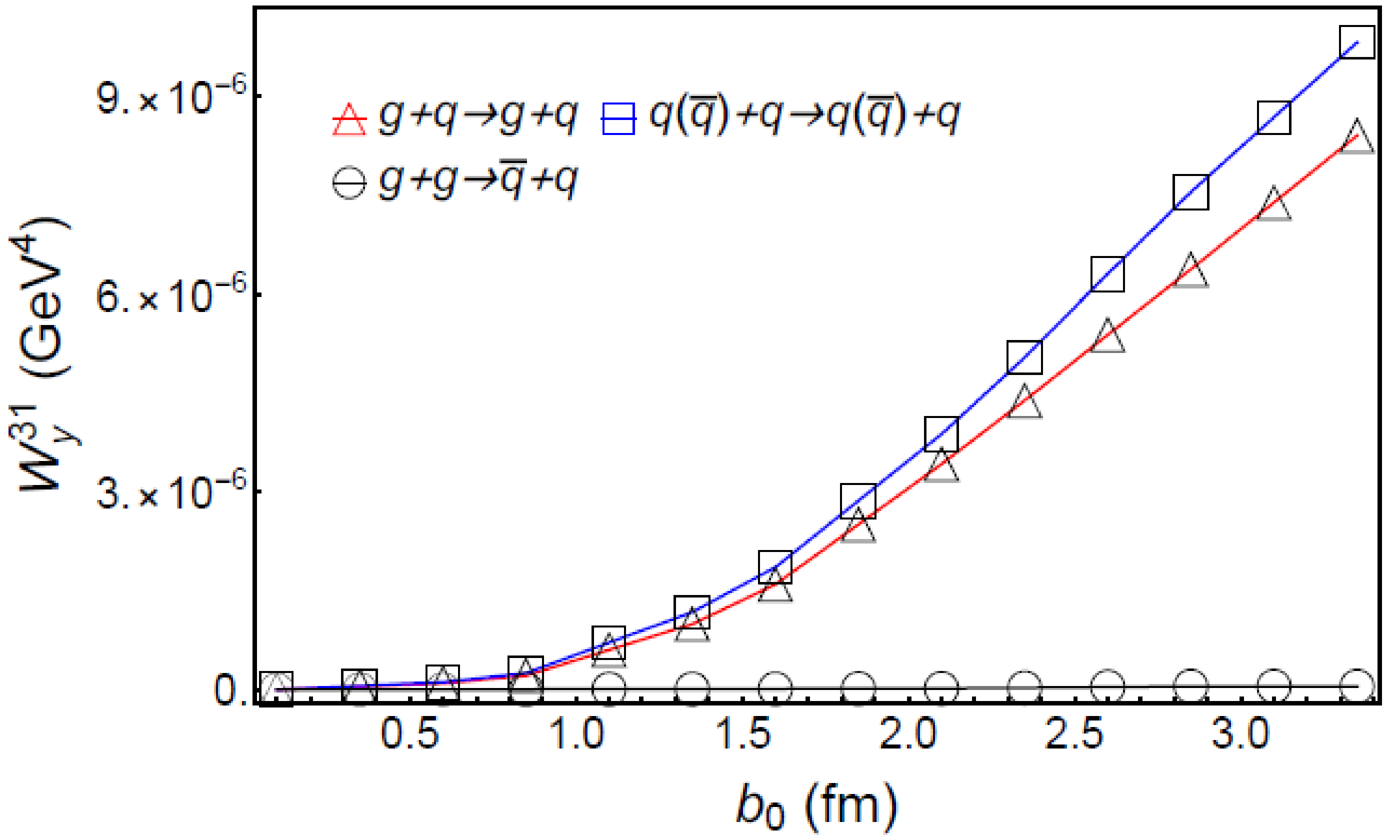
$$\begin{aligned} \frac{d^4\mathbf{P}_{AB\to 12}(X)}{dX^4} &= \epsilon^{0j\rho\nu}\frac{\partial(\beta u_\rho)}{\partial X^\nu}W\mathbf{e}_j = 2\epsilon_{jkl}\omega_{kl}W\mathbf{e}_j \\ &= 2W\nabla\times(\beta\mathbf{u}) \end{aligned}$$

$$\omega_{\rho\nu} = -(1/2)[\partial_\rho^X(\beta u_\nu) - \partial_\nu^X(\beta u_\rho)]$$

$$\omega_{kl} = (1/2)[\nabla_k(\beta\mathbf{u}_l) - \nabla_l(\beta\mathbf{u}_k)]$$

Polarization is given by the vorticity up to a coefficient W
W can be calculated numerically

Numerical results for quark polarization



The cutoff b_0 is of the order of hydro length scale $1/\partial u(x)$ and larger than interaction scale $1/m_D$: $b_0 \sim \frac{1}{\partial u(x)} > \frac{1}{m_D}$

$$\frac{d^4 P_{AB\to 12}(X)}{dX^4} = 2W\nabla_X \times (\beta \mathbf{u})$$

Summary

Take-home message:

- **"Discovery of global Λ polarization opens new directions in the study of the hottest, least viscous – and now, most vortical – fluid ever produced in the laboratory." --- from STAR Collab., Nature 548 (2017) 62.**
- **An emerging and rapidly expanding field!**