Flavor and modular symmetry

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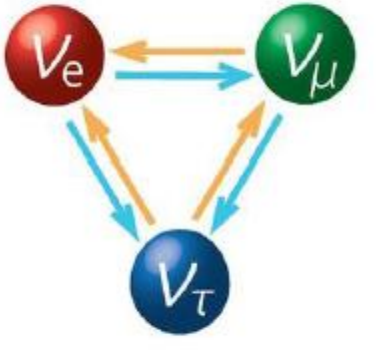
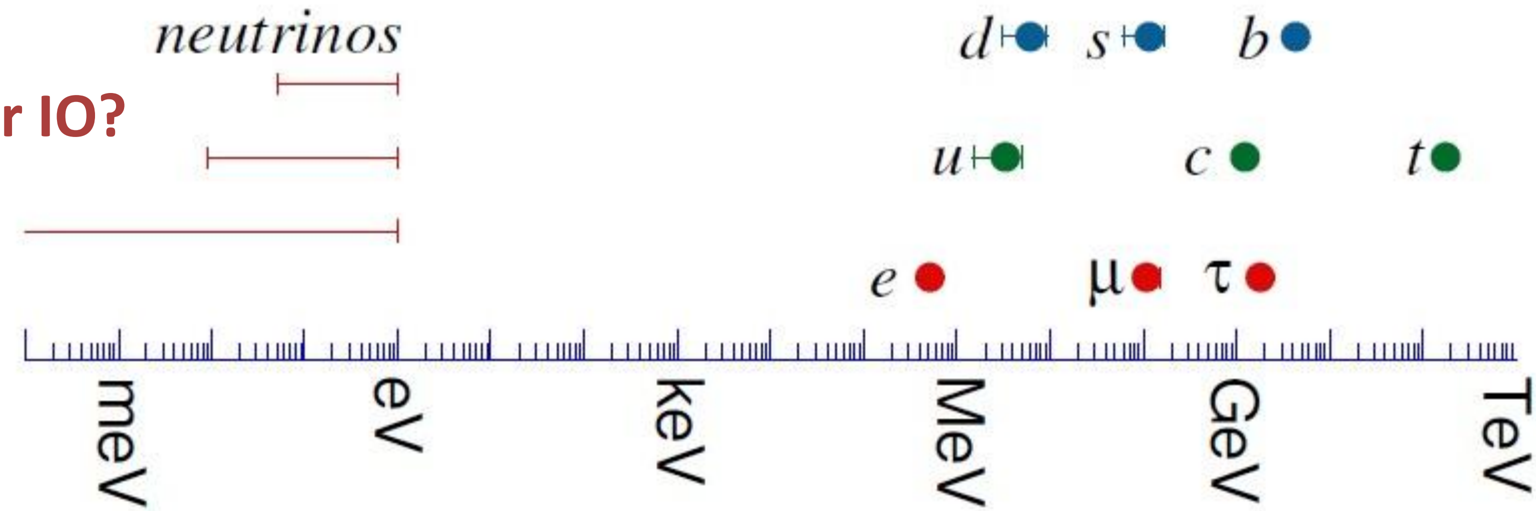
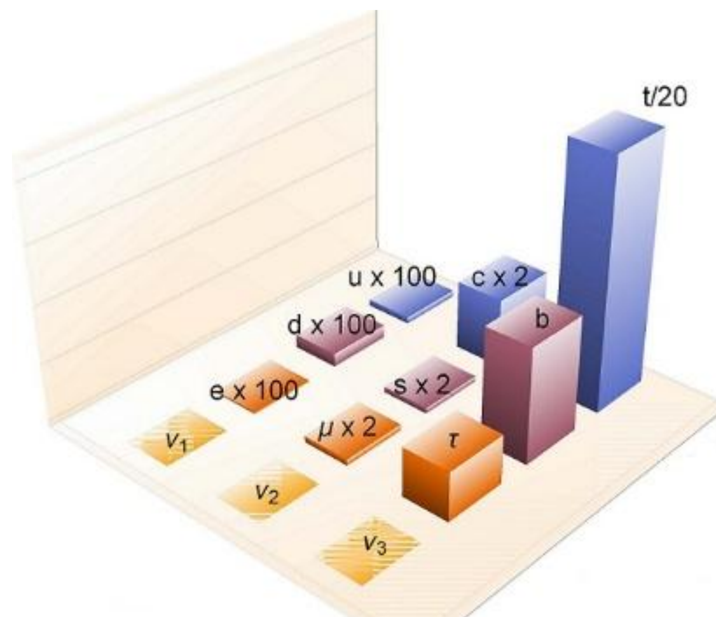


Fermion masses puzzle in SM

NO or IO?



- Why fermion mass hierarchies?
- Why are neutrino masses so small?

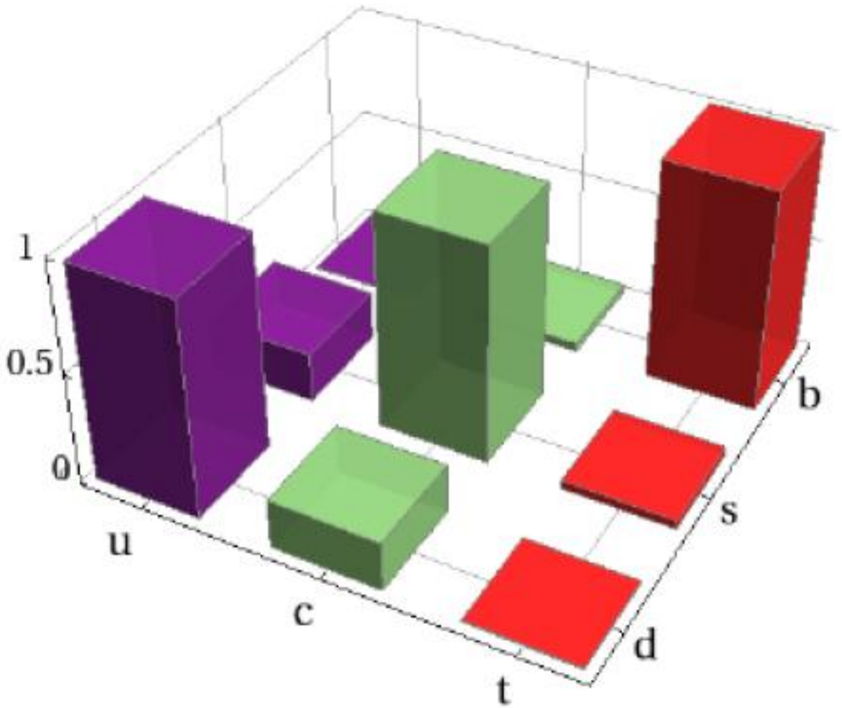


Flavor mixing puzzle in SM

Quark mixing

$$\|V_{CKM}\| \sim \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.009 & 0.04 & 1 \end{pmatrix}$$

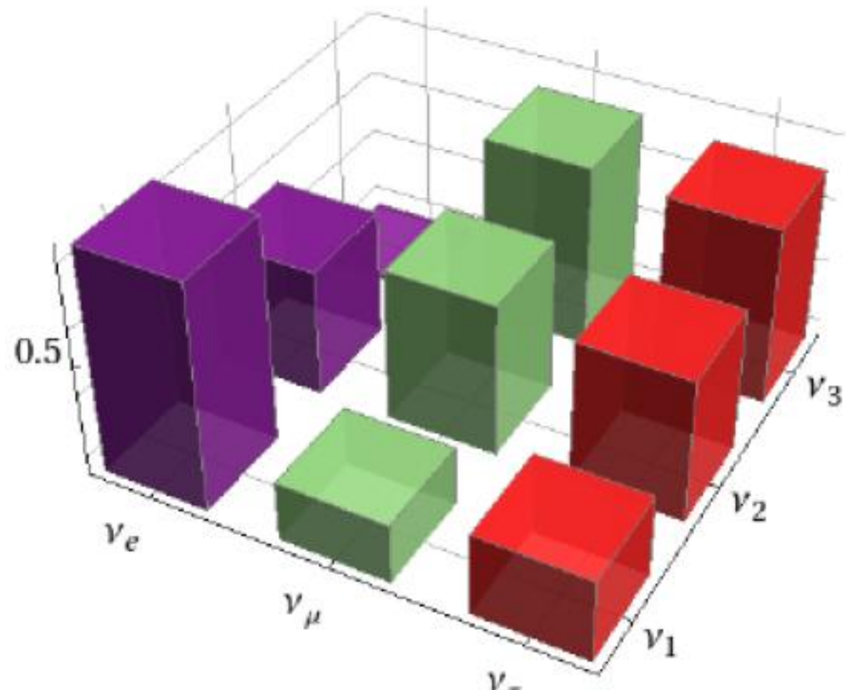
[CKMfitter, Summer 2018]



Lepton mixing

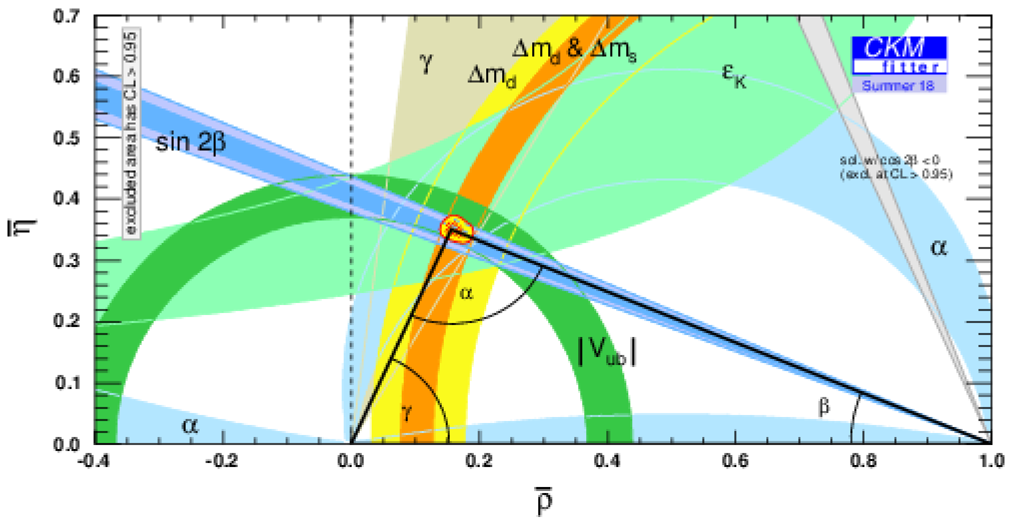
$$\|U_{PMNS}\| \sim \begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.29 & 0.59 & 0.75 \\ 0.49 & 0.59 & 0.64 \end{pmatrix}$$

[Gonzalez-Garcia et al., NuFIT4.1 (2019)]



CP violation puzzle in SM

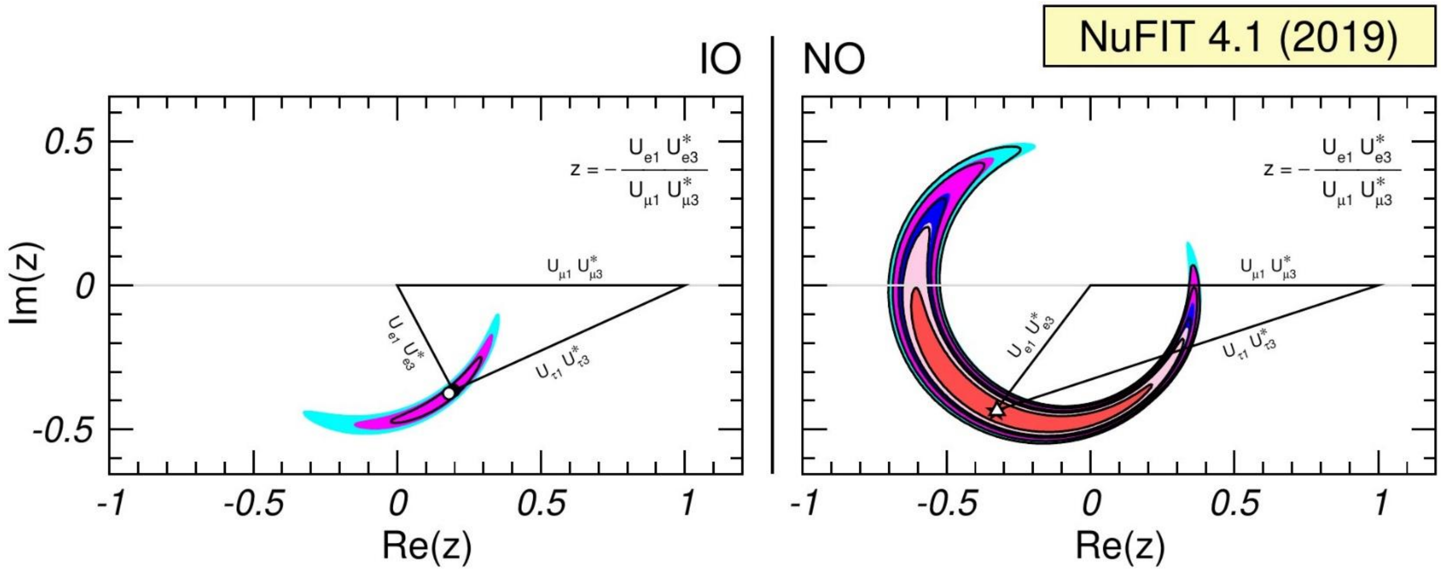
Quark CP violation [CKMfitter, 2018 Summer]



$$\alpha = (91.6^{+1.7}_{-1.1})^{\circ}$$

- What is the origin of CP violation?

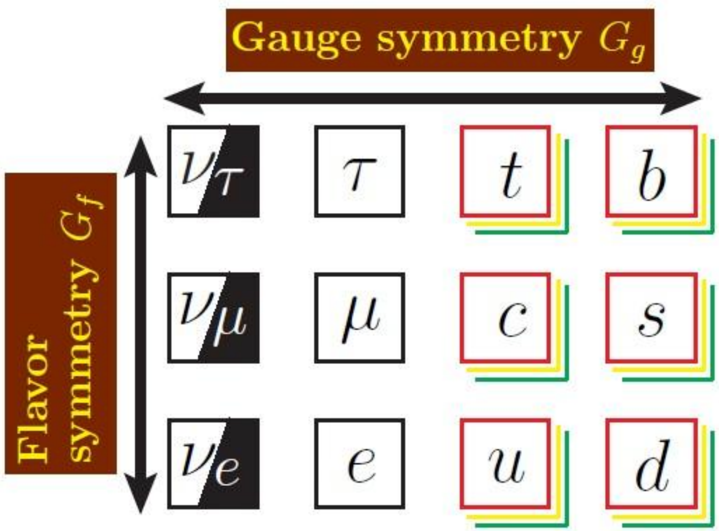
Leptonic CP violation [Gonzalez-Garcia et al., NuFIT4.1 (2018)]



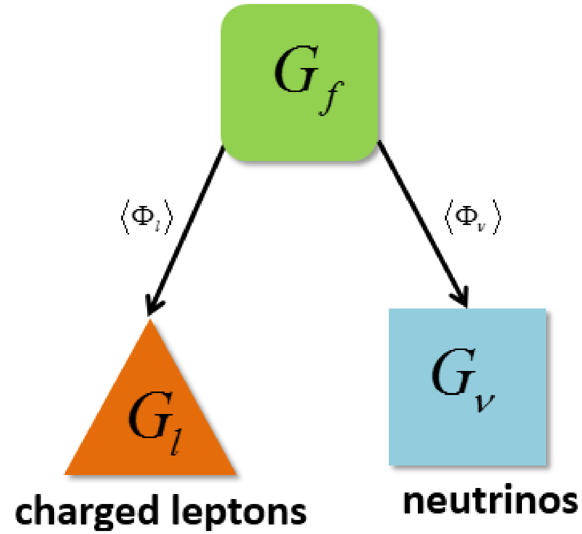
$\delta_{CP} \approx -90^{\circ}$?

Flavor symmetry approach to flavor puzzle

One of the few tools we have, but with several obstacles



$$\begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \\ \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \\ \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \end{pmatrix} \sim \mathbf{3}$$



For lepton sector, at leading order [Altarelli, Feruglio,1002.0211]

$$\mathcal{L}_m = -Y^e_{ij}(\langle \Phi_e \rangle)\overline{L_i} H e_{Rj} - \frac{1}{2} Y^\nu_{ij}(\langle \Phi_\nu \rangle)\overline{L^c_i} H H^T L_j$$

✓ alignment of flavon VEVs (**complicated dynamics**)

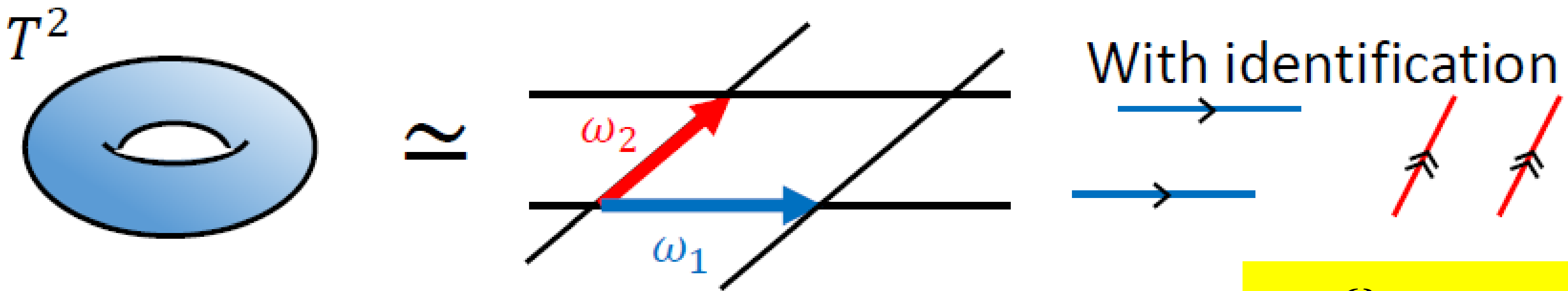
For flavor group A_4: $\langle \Phi_l \rangle \propto (1, 0, 0)^T, \ \langle \Phi_\nu \rangle \propto (1, 1, 1)^T$

✓ extra symmetry Z_n or $U(1)_R$

✓ higher dimensional operators

Modular invariance as flavor symmetry

Torus compactification in string theory leads to **Modular Symmetery**

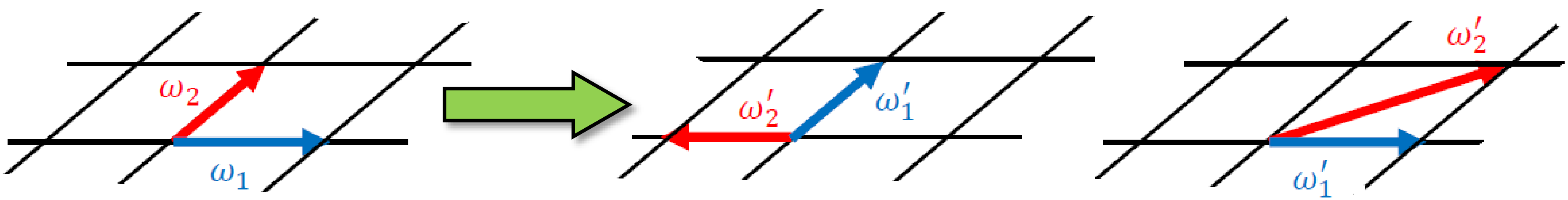


The shape of a torus T^2 is characterized by a modulus $\tau = \frac{\omega_2}{\omega_1}, \ \mathrm{Im}\ \tau > 0$

The lattice (torus) is left invariant by modular transformations

$$\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}, \qquad ad - bc = 1, \quad a, b, c, d \ \text{ integers} \qquad \bar{\Gamma} \simeq PSL(2, Z)$$

generated by two independent lattice transformations



$$S^2 = (ST)^3 = 1$$

$$S : \tau \to -1/\tau, \qquad T : \tau \to \tau + 1$$

Finite modular group

Infinite normal subgroups: Principal congruence subgroups

$$\Gamma(N)=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix}\in SL(2,Z),\ \begin{pmatrix} a & b \\ c & d \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\ (\mathrm{mod}\ \ N)\right\}$$

Projective congruence subgroups $T^N \in \Gamma(N)$

$$\overline{\Gamma}(2)\equiv\Gamma(2)/\{I,-I\},\qquad \overline{\Gamma}(N)\equiv\Gamma(N),\quad N>2$$

- Inhomogeneous finite modular groups: **quotient group**

$$\Gamma_N \equiv PSL(2,Z)/\overline{\Gamma}(N) \Longrightarrow \Gamma_N \cong \left\{S,T \mid S^2=(ST)^3=1,\ T^N=1\right\}$$

$$\Gamma_2 \cong S_3,\quad \Gamma_3 \cong A_4,\quad \Gamma_4 \cong S_4,\quad \Gamma_5 \cong A_5$$

[Feruglio, 1706.08749]

- Homogeneous finite modular groups: **quotient group**

$$\Gamma'_N \equiv SL(2,Z)/\Gamma(N)$$

[Liu,Ding,1907.01488]

$$\Gamma'_N \cong \left\{S,T \mid S^2=R,\ (ST)^3=R^2=T^N=1,\ TR=RT\right\}$$

Γ'_N is the double covering group of Γ_N ,i.e. $\Gamma'_3 \equiv T'$

Crucial element: Modular forms

Modular forms are **holomorphic** functions transforming under

$$f(\gamma\tau) = (c\tau + d)^k f(\tau), \quad \forall \gamma \in \overline{\Gamma}(N)$$

N: level, positive integer

k: modular weight, even integer

Modular forms of weight k and level N form a linear space, they can be decomposed into irreducible representations of finite modular group ,

$$f_i(\gamma\tau) = (c\tau + d)^k \rho_{ij}(\gamma) f_j(\tau), \quad \gamma \in \overline{\Gamma}$$

[Feruglio, 1706.08749]

ρ is unitary representation of Γ_N.

- k > 0 odd/even integer, modular forms fall in representations of homogeneous finite modular group Γ'_N

N	$\dim \mathcal{M}_k(\Gamma(N))$	Γ_N	$\lvert\Gamma_N\rvert$	Γ'_N	$\lvert\Gamma'_N\rvert$
2	$k/2+1$ (k even)	S_3	6	S_3	6
3	$k+1$	A_4	12	T'	24
4	$2k+1$	S_4	24	S'_4	48
5	$5k+1$	A_5	60	A'_5	120

[Liu,Ding,1907.01488]

Formalism: modular invariant theory

For N=1 global SUSY, the modular invariant action

[Ferrara et al, 1989; Feruglio, 1706.08749]

$$S = \int d^4x d^2\theta d^2\bar{\theta}\; K(\Phi_I, \bar{\Phi}_I, \tau, \bar{\tau}) + \int d^4x d^2\theta\, W(\Phi_I, \tau) + \text{h.c.}$$

- Minimal Kahler potential

$$K = -h\ln(-i\tau + i\bar{\tau}) + \sum_I (-i\tau + i\bar{\tau})^{-k_I} |\Phi_I|^2 \longrightarrow \text{kinetic terms}$$

- Modular invariant superpotential

$$W = \sum_n Y_{I_1 I_2 \ldots I_n}(\tau)\Phi_{I_1}\Phi_{I_2}\ldots\Phi_{I_n}$$

$Y_{I_1 I_2 \ldots I_n}(\tau)$ are modular forms

$$\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d},$$

$$\Phi_I \to (c\tau + d)^{-k_I}\rho_I(\gamma)\Phi_I$$

$$Y_{I_1 I_2 \ldots I_n}(\tau) \to Y_{I_1 I_2 \ldots I_n}(\gamma\tau) = (c\tau + d)^{k_Y}\rho_Y(\gamma) Y_{I_1 I_2 \ldots I_n}(\tau)$$

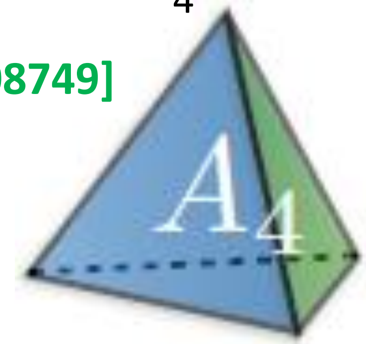
Modular invariance requires

$$\begin{cases} k_Y = k_{I_1} + k_{I_2} + \ldots + k_{I_n} \\ \rho_Y \otimes \rho_{I_1} \otimes \ldots \otimes \rho_{I_n} \supset 1 \end{cases}$$

Example: a minimal model based on $\Gamma_3=A_4$

Γ_3 is isomorphic to A_4, smallest non-abelian finite with 3-dimensional irreducible representation.

- Three weight 2 modular forms transforming as a triplet 3 of A_4

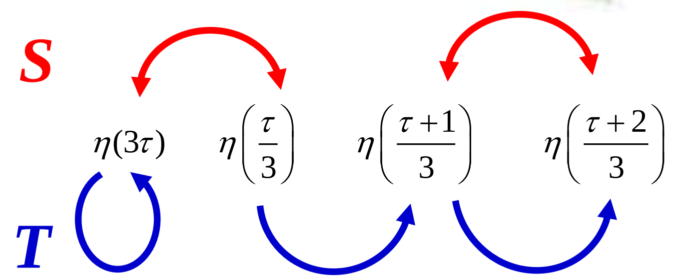


$$Y(\tau)=\left(Y_1(\tau),\quad Y_2(\tau),\quad Y_3(\tau)\right)^T \sim 3$$ **A_4 triplet** [Feruglio, 1706.08749]

$$Y_1(\tau) = \frac{i}{2\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)} + \frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)} + \frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)} - \frac{27\eta'(3\tau)}{\eta(3\tau)}\right]$$

$$Y_2(\tau) = \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)} + \omega^2\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)} + \omega\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right]$$

$$Y_3(\tau) = \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)} + \omega\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)} + \omega^2\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right]$$



Dedekind eta function: $\eta(\tau)=q^{1/24}\prod_{n=1}^{\infty}(1-q^n)\,,\quad q\equiv e^{2\pi i\tau}$

$$Y_1(\tau)=1+12q+\ldots,\ \ Y_2(\tau)=-6q^{1/3}(1+7q+\ldots),\ \ Y_3(\tau)=-18q^{2/3}(1+2q+\ldots)$$

Tensor products of $Y_{1,2,3}$ generate higher weight modular forms

- Field content

	N^c	(e^c, μ^c, τ^c)	L	H_d	H_u
$SU(2)_L \times U(1)_Y$	$(1,0)$	$(1,1)$	$(2,-1/2)$	$(2,-1/2)$	$(2,+1/2)$
$\Gamma_3 \cong A_4$	$\mathbf{3}$	$(\mathbf{1}', \mathbf{1}', \mathbf{1}'')$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$
k_I	1	$(1,3,1)$	1	0	0

Charged lepton mass terms

[Ding, King, Liu, 1907.11714]

$$W_e = \alpha e^c (LY)_{1''} H_d + \beta \mu^c (LY^2)_{1''} H_d + \gamma \tau^c (LY)_{1'} H_d$$

$$M_e = \begin{pmatrix} \alpha Y_3 & \alpha Y_2 & \alpha Y_1 \\ \beta(Y_2^2 - Y_1 Y_3) & \beta(Y_3^2 - Y_1 Y_2) & \beta(Y_1^2 - Y_2 Y_3) \\ \gamma Y_2 & \gamma Y_1 & \gamma Y_3 \end{pmatrix} v_d$$

NO flavons

The couplings α, β and γ are fixed by charged lepton masses.

Neutrino mass terms

$$W_\nu = g_1((N^c L)_{3_S} Y)_1 H_u + g_2((N^c L)_{3_A} Y)_1 H_u + \Lambda (N^c N^c Y)_1$$

$$M_D = \begin{pmatrix} 2g_1 Y_1 & (-g_1+g_2)Y_3 & (-g_1-g_2)Y_2 \\ (-g_1-g_2)Y_3 & 2g_1 Y_2 & (-g_1+g_2)Y_1 \\ (-g_1+g_2)Y_2 & (-g_1-g_2)Y_1 & 2g_1 Y_3 \end{pmatrix} v_u, \quad M_N = \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ -Y_3 & 2Y_2 & -Y_1 \\ -Y_2 & -Y_1 & 2Y_3 \end{pmatrix} \Lambda$$

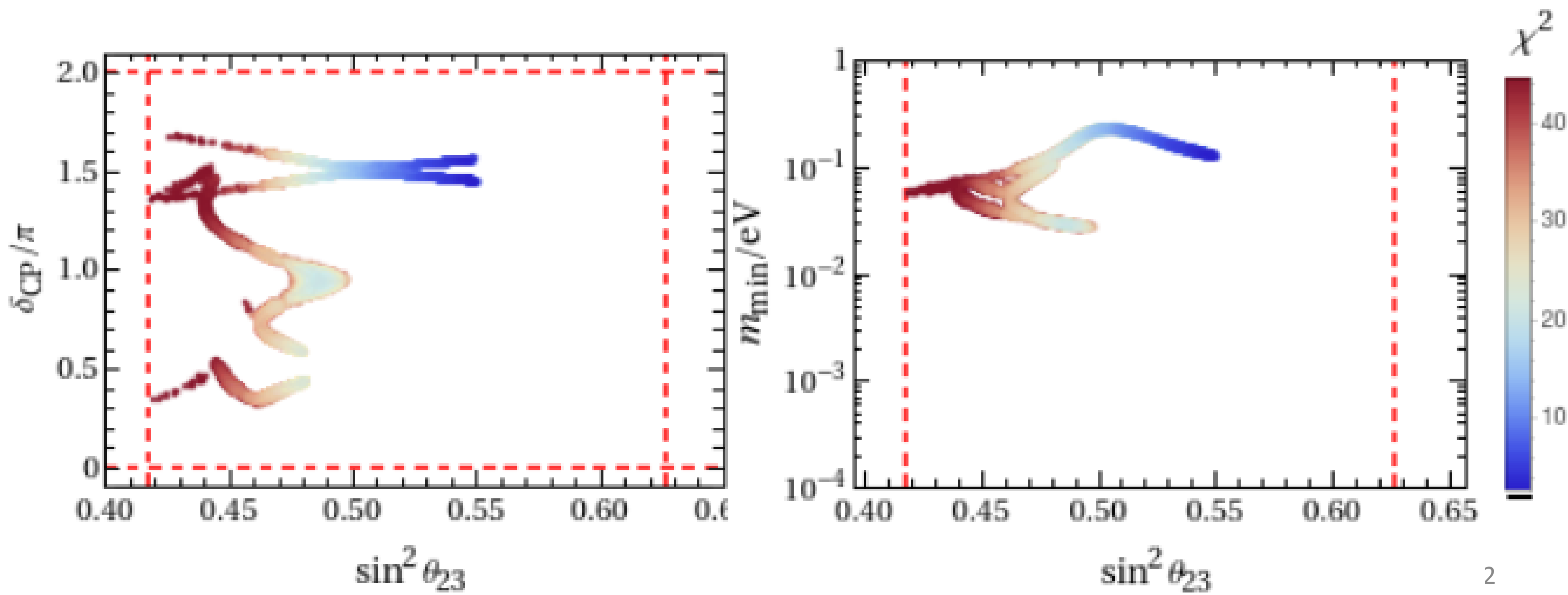
The complex modulus τ is the only source of modular symmetry breaking, best agreement with experimental data can be achieved for

Input parameters: $\langle \tau \rangle = 0.0428 + 2.105\,i, \quad g_2 / g_1 = 1.154 e^{0.625\pi i}$

Predictions:

$$\sin^2\theta_{12} = 0.310, \quad \sin^2\theta_{13} = 0.0224, \quad \sin^2\theta_{23} = 0.580,$$
$$\delta_{CP} = 1.602\pi, \quad \alpha_{21} = 1.992\pi, \quad \alpha_{31} = 0.986\pi, \quad \chi^2 = 0.0003$$
$$m_1 = 0.0805\text{eV}, \quad m_2 = 0.0810\text{eV}, \quad m_3 = 0.0949\text{eV}$$

[Ding, King, Liu, 1907.11714]



Classification of simplest A_4 modular models

charged lepton

$$W_e = E^c L H_d f_E(\tau)$$

neutrino

$$W_\nu = \begin{cases} LLH_uH_u f_W(\tau)/\Lambda, & \text{Weinberg operator} \\ N^c L H_u f_D(\tau) + \Lambda N^c N^c f_N(\tau), & \text{seesaw} \end{cases}$$

Models	mass matrices	A_4	modular weights		
			$k_{E^c_{1,2,3}}$	k_L	k_{N^c}
$\mathcal{A}_1$	W_1, C_1	$1,\ 1,\ 1$	$1, 3, 5$	1	—
$\mathcal{A}_2$	W_1, C_2	$1',\ 1',\ 1'$	$1, 3, 5$	1	—
$\mathcal{A}_3$	W_1, C_3	$1'',\ 1'',\ 1''$	$1, 3, 5$	1	—
$\mathcal{A}_4$	W_1, C_4	$1,\ 1,\ 1'$	$1, 3, 1$	1	—
$\mathcal{A}_5$	W_1, C_5	$1,\ 1,\ 1''$	$1, 3, 1$	1	—
$\mathcal{A}_6$	W_1, C_6	$1',\ 1',\ 1$	$1, 3, 1$	1	—
$\mathcal{A}_7$	W_1, C_7	$1'',\ 1'',\ 1$	$1, 3, 1$	1	—
$\mathcal{A}_8$	W_1, C_8	$1'',\ 1'',\ 1'$	$1, 3, 1$	1	—
$\mathcal{A}_9$	W_1, C_9	$1',\ 1',\ 1''$	$1, 3, 1$	1	—
$\mathcal{A}_{10}$	W_1, C_{10}	$1,\ 1'',\ 1'$	$1, 1, 1$	1	—
$\mathcal{B}_1(\mathcal{C}_1)[\mathcal{D}_1]$	$S_1(S_2)[S_3], C_1$	$1,\ 1,\ 1$	$0(3)[1], 2(5)[3], 4(7)[5]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_2(\mathcal{C}_2)[\mathcal{D}_2]$	$S_1(S_2)[S_3], C_2$	$1',\ 1',\ 1'$	$0(3)[1], 2(5)[3], 4(7)[5]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_3(\mathcal{C}_3)[\mathcal{D}_3]$	$S_1(S_2)[S_3], C_3$	$1'',\ 1'',\ 1''$	$0(3)[1], 2(5)[3], 4(7)[5]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_4(\mathcal{C}_4)[\mathcal{D}_4]$	$S_1(S_2)[S_3], C_4$	$1,\ 1,\ 1'$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_5(\mathcal{C}_5)[\mathcal{D}_5]$	$S_1(S_2)[S_3], C_5$	$1,\ 1,\ 1''$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_6(\mathcal{C}_6)[\mathcal{D}_6]$	$S_1(S_2)[S_3], C_6$	$1',\ 1',\ 1$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_7(\mathcal{C}_7)[\mathcal{D}_7]$	$S_1(S_2)[S_3], C_7$	$1',\ 1',\ 1''$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_8(\mathcal{C}_8)[\mathcal{D}_8]$	$S_1(S_2)[S_3], C_8$	$1'',\ 1'',\ 1$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_9(\mathcal{C}_9)[\mathcal{D}_9]$	$S_1(S_2)[S_3], C_9$	$1'',\ 1'',\ 1'$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_{10}(\mathcal{C}_{10})[\mathcal{D}_{10}]$	$S_1(S_2)[S_3], C_{10}$	$1,\ 1'',\ 1'$	$0(3)[1], 0(3)[1], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$

40 simplest A_4 modular models without flavons

Models	NO	IO	Models	NO	IO	Models	NO	IO	Models	NO	IO
$\mathcal{A}_1$	✘	✘	$\mathcal{B}_1$	✔	✔	$\mathcal{C}_1$	✘	✘	$\mathcal{D}_1$	✔	✔
$\mathcal{A}_2$	✘	✘	$\mathcal{B}_2$	✔	✔	$\mathcal{C}_2$	✘	✘	$\mathcal{D}_2$	✔	✔
$\mathcal{A}_3$	✘	✘	$\mathcal{B}_3$	✔	✔	$\mathcal{C}_3$	✘	✘	$\mathcal{D}_3$	✔	✔
$\mathcal{A}_4$	✘	✘	$\mathcal{B}_4$	✘	✘	$\mathcal{C}_4$	✘	✘	$\mathcal{D}_4$	✘	✔
$\mathcal{A}_5$	✘	✘	$\mathcal{B}_5$	✘	✘	$\mathcal{C}_5$	✘	✘	$\mathcal{D}_5$	✔	✘
$\mathcal{A}_6$	✘	✘	$\mathcal{B}_6$	✘	✔	$\mathcal{C}_6$	✘	✘	$\mathcal{D}_6$	✔	✘
$\mathcal{A}_7$	✘	✘	$\mathcal{B}_7$	✘	✘	$\mathcal{C}_7$	✘	✘	$\mathcal{D}_7$	✔	✔
$\mathcal{A}_8$	✘	✘	$\mathcal{B}_8$	✘	✘	$\mathcal{C}_8$	✘	✘	$\mathcal{D}_8$	✔	✔
$\mathcal{A}_9$	✘	✘	$\mathcal{B}_9$	✔	✔	$\mathcal{C}_9$	✘	✘	$\mathcal{D}_9$	✔	✔
$\mathcal{A}_{10}$	✘	✘	$\mathcal{B}_{10}$	✔	✔	$\mathcal{C}_{10}$	✘	✘	$\mathcal{D}_{10}$	✔	✔

- ✓ **8** phenomenologically viable minimal models for both NO and IO as compared to **only one D_{10}** in previous analysis
- ✓ **3 free parameters** beside modulus τ for the neutrino sector (3 masses + 3 angles + 3 phases)
- ✓ Dirac CP phase **$\delta_{CP} \approx -\pi/2$** and large neutrino masses still allowed by data

Texture zeros

Imposing texture zeros: reduce the number of free parameters, thus lead to predictions for flavor mixing angles.

[S. Weinberg; H. Fritzsch; F. Wilczek & A. Zee, 1977]

- Fritzsch texture

$$M_u = \begin{pmatrix} 0 & A_U & 0 \\ A_U^* & 0 & B_U \\ 0 & B_U^* & C_U \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & A_D & 0 \\ A_D^* & 0 & B_D \\ 0 & B_D^* & C_D \end{pmatrix}, \quad \tan\theta_C \simeq \sqrt{\frac{m_d}{m_s}}$$

How to generate texture zero structure exactly?

Modular symmetry origin of texture zeros from $\Gamma'_3 = T'$

Γ'_3 is the double covering of A_4

$\Gamma'_3 \subset SU(2)$ $\quad\quad$ $A_4 \subset SO(3)$

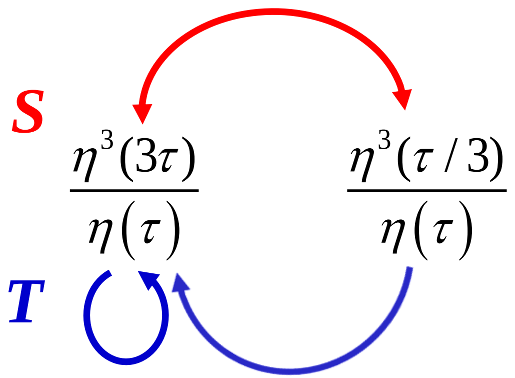
- Two weight 1 modular forms transforming as a doublet 2 of T'

$$Y_2^{(1)}(\tau) = \left(Y_1(\tau), \quad Y_2(\tau)\right)^T \sim 2$$

T' doublet [Liu,Ding,1907.01488]

$$Y_1(\tau) = \sqrt{2} e^{i7\pi/12} \frac{\eta^3(3\tau)}{\eta(\tau)}$$

$$Y_2(\tau) = \frac{\eta^3(3\tau)}{\eta(\tau)} - \frac{1}{3}\frac{\eta^3(\tau/3)}{\eta(\tau)}$$



$$Y_1(\tau) = \sqrt{2} e^{i7\pi/12} q^{1/3}(1 + q + 2q^2 + \ldots), \qquad Y_2(\tau) = 1/3 + 2q + 2q^3 + \ldots$$

Tensor products of $Y_{1,2}$ generate higher weight modular forms

Weight 2:

$$Y_{\mathbf{3}}^{(2)} = \begin{pmatrix} e^{i\pi/6}Y_2^2 \\ \sqrt{2}e^{i7\pi/12}Y_1Y_2 \\ Y_1^2 \end{pmatrix}$$

[Liu,Ding,1907.01488]

Weight 3: $$Y_{\mathbf{2}}^{(3)} = \begin{pmatrix} 3e^{i\pi/6}Y_1Y_2^2 \\ \sqrt{2}e^{i5\pi/12}Y_1^3 - e^{i\pi/6}Y_2^3 \end{pmatrix}, \quad Y_{\mathbf{2}''}^{(3)} = \begin{pmatrix} Y_1^3 + (1-i)Y_2^3 \\ -3Y_2Y_1^2 \end{pmatrix}$$

k	$\dim\mathcal{M}_k(\Gamma(3))$	Modular Forms
1	2	$Y_{\mathbf{2}}^{(1)}$
2	3	$Y_{\mathbf{3}}^{(2)}$
3	4	$Y_{\mathbf{2}}^{(3)}, Y_{\mathbf{2}''}^{(3)}$
4	5	$Y_{\mathbf{1}}^{(4)}, Y_{\mathbf{1}'}^{(4)}, Y_{\mathbf{3}}^{(4)}$
5	6	$Y_{\mathbf{2}}^{(5)}, Y_{\mathbf{2}'}^{(5)}, Y_{\mathbf{2}''}^{(5)}$
6	7	$Y_{\mathbf{1}}^{(6)}, Y_{\mathbf{3},I}^{(6)}, Y_{\mathbf{3},II}^{(6)}$

- ✓ Odd weight modular forms transform as T′ doublets 2,2′,2″
- ✓ Even weight modular forms transform as singlets 1,1′,1″ and triplet 3

Quark sector from modular symmetry $\Gamma'_3 = T'$

The 3rd generation quark is much heavier than the 1st and 2nd generation

Assignment:

$$Q_D \equiv \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \sim 2\,(\text{or } 2', 2''), \qquad Q_3 \sim 1\,(\text{or } 1', 1'')$$

Left-handed doublets

$$q^c_D \equiv \begin{pmatrix} q^c_1 \\ q^c_2 \end{pmatrix} \sim 2\,(\text{or } 2', 2''), \qquad q^c_3 \sim 1\,(\text{or } 1', 1'')$$

Right-handed singlets

Mass matrix:

vanishing for $k_{Q_D} + k_{q^c_D}$ odd

vanishing for $k_{Q_D} + k_{q^c_3}$ even

$$M_q = \begin{pmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{pmatrix}$$

[Lu, Liu, Ding, to appear]

vanishing for $k_{Q_3} + k_{q^c_D}$ even

vanishing for $k_{Q_3} + k_{q^c_3}$ odd

Texture zeros of quark mass matrices

Five texture zeros of quark mass matrices can be achieved from the T′ modular symmetry

$$\text{Case } \mathcal{A}: \quad M_q = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \qquad \text{Case } \mathcal{B}: \quad M_q = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & \times \end{pmatrix}$$

$$\text{Case } \mathcal{C}: \quad M_q = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix}, \qquad \text{Case } \mathcal{D}: \quad M_q = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & 0 \end{pmatrix}$$

$$\text{Case } \mathcal{E}: \quad M_q = \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & 0 & \times \end{pmatrix}.$$

[Lu, Liu, Ding, to appear]

New features:

✓ texture zeros arise from modular symmetry **not assumption**

✓ Non-vanishing entries are correlated with each other.

A viable model for quarks

- Field content

[Lu, Liu, Ding, to appear]

	Q_D	Q_3	(u^c, c^c, t^c)	d^c_D	b^c	$H_{u,d}$
$\Gamma'_3 \cong T'$	$\mathbf{2}$	$\mathbf{1'}$	$(\mathbf{1''}, \mathbf{1}, \mathbf{1''})$	$\mathbf{2''}$	$\mathbf{1''}$	$\mathbf{1}$
k_I	-1	2	$(2, 4, -2)$	5	-2	0

NO flavons

- **quark mass terms**

$$\mathcal{W}_u = y_1^u u^c Q_D Y_{\mathbf{2}}^{(1)} H_u + y_2^u u^c Q_3 Y_{\mathbf{1}}^{(4)} H_u + y_3^u c^c Q_D Y_{\mathbf{2''}}^{(3)} H_u + y_4^u t^c Q_3 H_u$$
$$\mathcal{W}_d = y_1^d d_D^c Q_D Y_{\mathbf{3}}^{(4)} H_d + y_2^d d_D^c Q_D Y_{\mathbf{1}}^{(4)} H_d + y_3^d b^c Q_3 H_d$$

$$M_u = \begin{pmatrix} y_1^u Y^{(1)}_{\mathbf{2},2} & -y_1^u Y^{(1)}_{\mathbf{2},1} & y_2^u Y^{(4)}_{\mathbf{1}} \\ y_3^u Y^{(3)}_{\mathbf{2''},2} & -y_3^u Y^{(3)}_{\mathbf{2''},1} & 0 \\ 0 & 0 & y_4^u \end{pmatrix}, \quad M_d = \begin{pmatrix} -\sqrt{2}e^{\frac{5i\pi}{12}} y_1^d Y^{(4)}_{\mathbf{3},3} & y_1^d Y^{(4)}_{\mathbf{3},1} - y_2^d Y^{(4)}_{\mathbf{1}} & 0 \\ y_1^d Y^{(4)}_{\mathbf{3},1} + y_2^d Y^{(4)}_{\mathbf{1}} & -\sqrt{2}e^{\frac{7i\pi}{12}} y_1^d Y^{(4)}_{\mathbf{3},2} & 0 \\ 0 & 0 & y_3^d \end{pmatrix}$$

7 zero elements

After removing unphysical phases, we have **8 input parameters** besides the modulus τ,

$$|y_1^u|,\ |y_2^u|,\ |y_3^u|,\ |y_4^u|,\ |y_1^d|, |y_2^d|, \arg(y_2^d), |y_3^d|$$

explain 10 obervables: 3 up quark masses+3 down quark masses+ 3 quark mixing angles+1 CP phase

Extension to lepton sector

[Ding, King, Liu, 1907.11714]

	L	(e^c, μ^c, τ^c)	N^c	$H_{u,d}$
$\Gamma'_3 \cong T'$	$\mathbf{3}$	$(\mathbf{1'}, \mathbf{1'}, \mathbf{1})$	$\mathbf{3}$	$\mathbf{1}$
k_I	1	$(1, 3, 1)$	1	0

Charged lepton mass terms

$$\mathcal{W}_e = \alpha e^c (L Y_{\mathbf{3}}^{(2)})_{\mathbf{1}''} H_d + \beta \mu^c (L Y_{\mathbf{3}}^{(4)})_{\mathbf{1}''} H_d + \gamma \tau^c (L Y_{\mathbf{3}}^{(2)})_{\mathbf{1}} H_d$$

$$\Rightarrow \quad M_e = \begin{pmatrix} \alpha Y^{(2)}_{\mathbf{3},3} & \alpha Y^{(2)}_{\mathbf{3},2} & \alpha Y^{(2)}_{\mathbf{3},1} \\ \beta Y^{(4)}_{\mathbf{3},3} & \beta Y^{(4)}_{\mathbf{3},2} & \beta Y^{(4)}_{\mathbf{3},1} \\ \gamma Y^{(2)}_{\mathbf{3},1} & \gamma Y^{(2)}_{\mathbf{3},3} & \gamma Y^{(2)}_{\mathbf{3},2} \end{pmatrix} v_d$$

The couplings α, β and γ are fixed by charged lepton masses.

Neutrino mass terms: seesaw mechanism

$$\mathcal{W}_\nu = g_1((N^c L)_{\mathbf{3}_S} Y_{\mathbf{3}}^{(2)})_{\mathbf{1}} H_u + g_2((N^c L)_{\mathbf{3}_A} Y_{\mathbf{3}}^{(2)})_{\mathbf{1}} H_u + \Lambda((N^c N^c)_{\mathbf{3}_S} Y_{\mathbf{3}}^{(2)})_{\mathbf{1}}$$

$$M_D = \begin{pmatrix} 2g_1 Y^{(2)}_{\mathbf{3},1} & (-g_1 + g_2) Y^{(2)}_{\mathbf{3},3} & (-g_1 - g_2) Y^{(2)}_{\mathbf{3},2} \\ (-g_1 - g_2) Y^{(2)}_{\mathbf{3},3} & 2g_1 Y^{(2)}_{\mathbf{3},2} & (-g_1 + g_2) Y^{(2)}_{\mathbf{3},1} \\ (-g_1 + g_2) Y^{(2)}_{\mathbf{3},2} & (-g_1 - g_2) Y^{(2)}_{\mathbf{3},1} & 2g_1 Y^{(2)}_{\mathbf{3},3} \end{pmatrix} v_u \,, \quad M_N = \Lambda \begin{pmatrix} 2Y^{(2)}_{\mathbf{3},1} & -Y^{(2)}_{\mathbf{3},3} & -Y^{(2)}_{\mathbf{3},2} \\ -Y^{(2)}_{\mathbf{3},3} & 2Y^{(2)}_{\mathbf{3},2} & -Y^{(2)}_{\mathbf{3},1} \\ -Y^{(2)}_{\mathbf{3},2} & -Y^{(2)}_{\mathbf{3},1} & 2Y^{(2)}_{\mathbf{3},3} \end{pmatrix}$$

The complex modulus τ is common in both quark and lepton sectors, and it is the unique source of modular symmetry breaking.

Input parameters:

$$\langle\tau\rangle = -0.481 + 0.878i,\ \ y_2^u / y_1^u = 0.513,\ \ y_3^u / y_1^u = 1.080,$$

$$y_4^u / y_1^u = 4961.286,\ \ y_2^d / y_1^d = 3.535 e^{0.437i},\ \ y_3^d / y_1^d = 1.981,$$

$$\beta / \alpha = 0.375,\ \ \gamma / \alpha = 0.00423,\ \ g_2 / g_1 = 1.116 e^{1.974i}$$

Predictions: almost all observables are within the 1σ regions

$$m_u / m_c = 0.00185,\ \ m_c / m_t = 0.00282,\ \ m_d / m_s = 0.0506,\ \ m_s / m_b = 0.0182,$$

$$m_e / m_\mu = 0.00474,\ \ m_\mu / m_\tau = 0.0586,\ \ m_1 = 0.1287\ \text{eV},\ \ m_2 = 0.1290\ \text{eV},$$

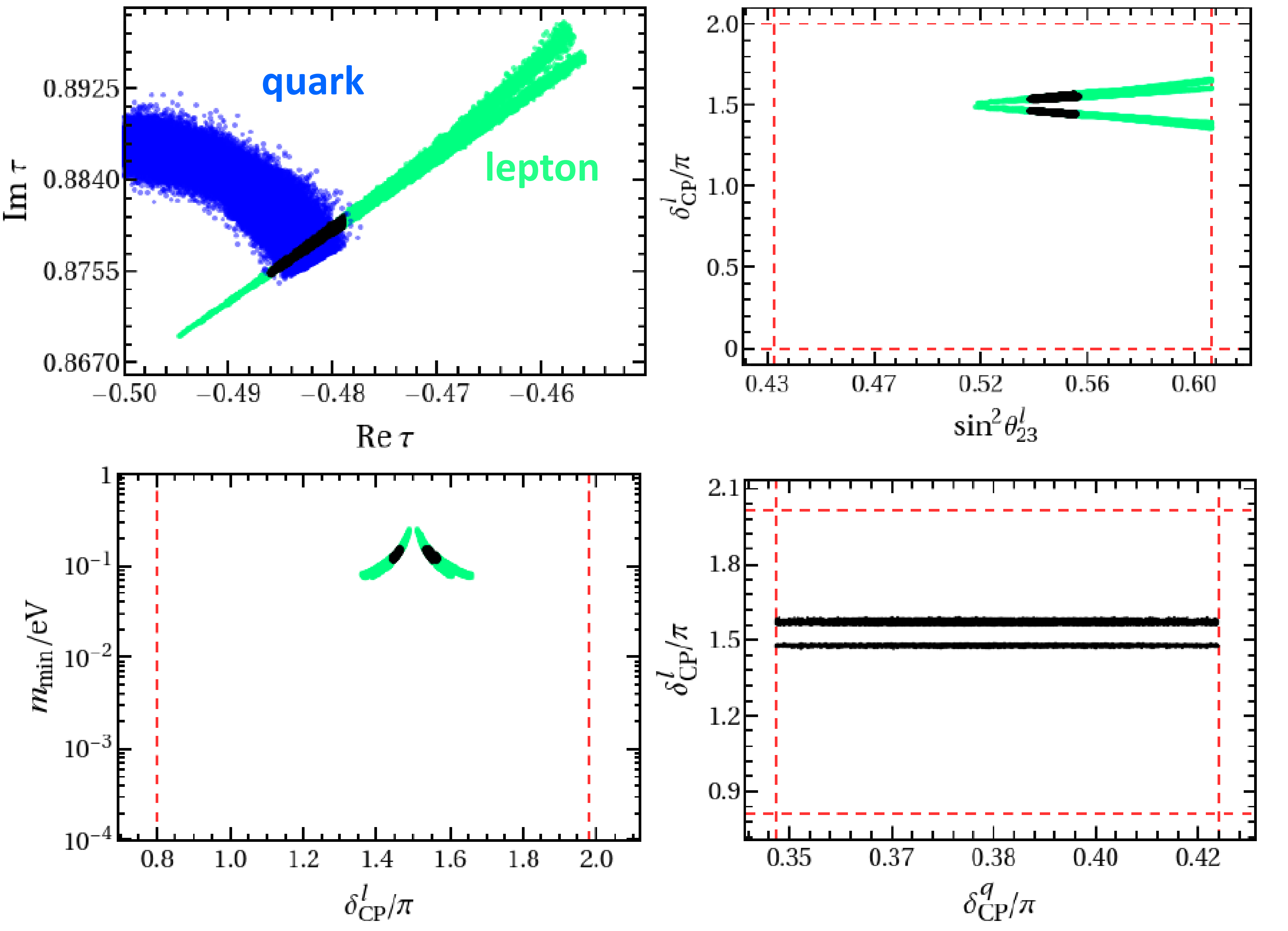
$$m_3 = 0.1382\ \text{eV},\ \ \Sigma_i m_i = 0.396\ \text{eV},\ \ m_{ee} = 0.129\ \text{eV},\ \ \sin\theta_{12}^q = 0.227,$$

$$\sin\theta_{13}^q = 0.00353,\ \ \sin\theta_{23}^q = 0.0379,\ \ \delta_{CP}^q = 69.212^\circ,\ \ \sin^2\theta_{12}^l = 0.310,$$

$$\sin^2\theta_{13}^l = 0.0223,\ \ \sin^2\theta_{23}^l = 0.549,\ \ \delta_{CP}^l = 278.260^\circ,\ \ \alpha_{21} = 359.461^\circ,$$

$$\alpha_{31} = 179.049^\circ$$

Precise measurements of θ_{23} , δ_{CP} and the effective mass m_{ee} in 0ν2β decay can exclude this model.


quark
lepton
Im τ
Re τ
0.8925
0.8840
0.8755
0.8670
−0.50
−0.49
−0.48
−0.47
−0.46
δ^l_{CP}/π
$\sin^2\theta^l_{23}$
2.0
1.5
1.0
0.5
0
0.43
0.47
0.52
0.56
0.60
m_{min}/eV
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0.8
1.0
1.2
1.4
1.6
1.8
2.0
δ^l_{CP}/π
δ^q_{CP}/π
2.1
1.8
1.5
1.2
0.9
0.35
0.37
0.38
0.40
0.42

Summary

Neutrino oscillation calls for convincing model of neutrino masses and mixings, with testable and confirmed predictions.

- Modular symmetry is a new promising approach to understand the fermion masses and flavor mixing puzzles with less free parameters. A systematic method of constructing the simplest modular symmetry models is proposed, A_4 as an example for illustration.

- Modular symmetry can generate quark mass matrices with texture zeros, and the measured values of quark and lepton masses, CKM matrix and lepton mixing angles can be accommodated by the T′ modular symmetry simultaneously.

Thank you for your attention!

Backup

Quark and lepton masses and mixing parameters

[Antusch,Maurer,1306.6879]

MSSM Quantity z_i	Values
$y_u/10^{-6}$	2.73325 ± 0.84731
$y_c/10^{-3}$	1.41719 ± 0.04960
y_t	0.50232 ± 0.01200
$y_d/10^{-6}$	5.12495 ± 0.56374
$y_s/10^{-4}$	1.01438 ± 0.05478
$y_b/10^{-3}$	5.56096 ± 0.06103
$y_e/10^{-6}$	2.07526 ± 0.01245
$y_\mu/10^{-4}$	4.38107 ± 0.02629
$y_\tau/10^{-3}$	7.48026 ± 0.03898
θ^q_{12}	0.22736 ± 0.00073
$\theta^q_{13}/10^{-2}$	0.34938 ± 0.01258
θ^q_{23}	0.04015 ± 0.00064
$\delta^q_{CP}/^\circ$	69.21330 ± 3.11460
$\sin^2\theta^l_{12}$	$0.310^{+0.013}_{-0.012}$
$\sin^2\theta^l_{23}$	$0.563^{+0.018}_{-0.024}$
$\sin^2\theta^l_{13}$	$0.02237^{+0.00066}_{-0.00065}$
$\delta^l_{CP}/^\circ$	221^{+39}_{-28}
$\frac{\Delta m^2_{21}}{10^{-5}\mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$
$\frac{\Delta m^2_{31}}{10^{-3}\mathrm{eV}^2}$	$2.528^{+0.029}_{-0.031}$