Thermodynamic geometry for the Schwarzschild AdS and Reissner-Nordström black holes

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arXiv:1910.12182 [gr-qc]

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Outline I

1. Introduction

2. Our works
   - For the Reissner-Nordström black hole
   - For the Schwarzschild AdS black hole
   - For the Schwarzschild black hole

3. Summary

Introduction

Thermodynamic geometry

- Thermodynamics is considered to be one of the finest descriptions of real phenomena.
- Gibbs and Carathéodory noticed by the end of the 19th century and the beginning of the 20th, respectively, it lacks the mathematical precision of other areas, such as classical mechanics or electrodynamics.
- This problem has inspired a line of research whose main goal is to describe thermodynamic systems geometrically.
- Frank Weinhold introduced the first thermodynamic geometry, where the internal energy is chosen to be the thermodynamic potential.

F. Weinhold: J. Chem. Phys. 63 (1975) 2479

Thermodynamic geometry

This approach was further explored by George Ruppeiner, starting from the Boltzmann entropy formula, introduced thermodynamic entropy geometry.

G. Ruppeiner: Phys. Rev. A 20 (1979) 1608; Rev. Mod. Phys. 67 (1995) 605

- The theory of fluctuations of equilibrium thermodynamics
  - **Equilibrium states can be represented by points on two-dimensional surface**
  - **Distance between these equilibrium states is related to the fluctuation between them.**
- Ruppeiner metric $\xlongequal{\text{conformally equivalent to}}$ Weinhold metric

Thermodynamic geometry

- Boltzmann's entropy formula

$$S = k_{\mathrm{B}} \ln \Omega$$

where the Boltzmann constant $k_{\mathrm{B}} \approx 1.38 \times 10^{-23} J/K$, and Ω denotes the number of the microscopic states of the corresponding thermodynamic system

- Its inversion

$$\Omega = e^{\frac{S}{k_{\mathrm{B}}}}$$

which is the starting point for the thermodynamic fluctuation theory.

Thermodynamic geometry

- Thermodynamic fluctuation theory gives the probability
  - Einstein (1904):
$$\text{probability} \propto e^{S}$$
  - Fluctuations take place about the state of maximum entropy. Expand entropy S to second-order about that maximum entropy state:
$$\text{probability} \propto \exp\left(-\frac{1}{2} g^{R}_{\mu\nu} \Delta x^{\mu} \Delta x^{\nu}\right)$$

x^{α} : the independent fluctuating thermodynamic variables
Δx^{α} : the difference between x^{α} and its equilibrium value x^{α}_{0}

Thermodynamic geometry

- $g^{R}_{\mu\nu}$: thermodynamic entropy metric

$$g^{R}_{\mu\nu} = -\frac{\partial^2 S}{\partial x^\mu \partial x^\nu}$$

- $g^{R}_{\mu\nu}$ (Ruppeiner metric) and $g^{W}_{\alpha\beta}$ (Weinhold metric)

$$-\frac{\partial^2 S}{\partial x^\mu \partial x^\nu} = g^{R}_{\mu\nu} = \frac{1}{T} g^{W}_{\alpha\beta} = \frac{1}{T} \frac{\partial^2 U}{\partial Y^\alpha \partial Y^\beta}$$

- $\Delta l^2 = g^{R}_{\mu\nu} \Delta x^\mu \Delta x^\nu$: probability “distance”—the less probable a fluctuation between states, the further apart they are
- **Distance yields thermodynamic curvature** R

Statistical models

Model	n	d	R sign	$\|R\|$ divergence
Ideal Bose gas	2	3	$-$	$T \to 0$
Mean-field theory	2	$\cdots$	$-$	critical point
van der Waals (critical regime)	2	3	$-$	critical point
Ideal gas	2	3	0	$\|R\|$ small
Ideal gas paramagnet	3	3	$+$	$\|R\|$ small
Ideal Fermi gas	2	2, 3	$+$	$T \to 0$
Ideal gas Fermi paramagnet	3	3	$+$	$T \to 0$
Takahashi gas	2	1	$\pm$	$T \to 0$
Gentile's statistics	2	3	$\pm$	$T \to 0$
M-statistics	2	2, 3	$\pm$	$T \to 0$
Anyons	2	2	$\pm$	$T \to 0$
Ising ferromagnet	2	1	$-$	$T \to 0$
Ising on Bethe lattice	2	$\cdots$	$-$	critical point
Ideal paramagnet	2	$\cdots$	0	$\|R\|$ small

G. Ruppeiner: Springer proceedings in physics, 153 (2014) 179.

Thermodynamic curvature

- R is related to the correlation length ξ of the thermodynamic system

$$R \sim \kappa \xi^d$$

where κ is a dimensionless constant, and d denotes the physical dimensionality of the system

- The relationship between R and the interactions of the underlying microscopic model

  - $R < 0 \Rightarrow$ attractive interaction (like ideal Bose gas)
  - $R > 0 \Rightarrow$ repulsive interaction (like ideal Fermi gas)
  - $R = 0 \Rightarrow$ no interaction (like ideal gas)

Thermodynamic curvature for black holes

- Black hole can possess temperature and entropy $\xlongequal{\text{Mapping}}$ Thermodynamic system
- RN-AdS black hole $\xlongequal{\text{thermodynamic similarity}}$ van der Waals fluid.

D. Kubiznak and R.B. Mann: JHEP 07 (2012) 033

Black hole should have microscopic structure

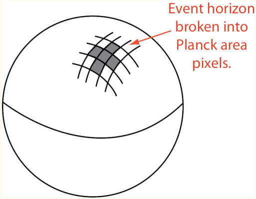
It is still unclear about the constituents of black holes

Black hole molecule!!!

S.-W. Wei and Y.-X. Liu: Phys. Rev. Lett. 115 (2015) 111302.

Thermodynamic curvature for black holes

- Physical explanation for thermodynamic curvature R



Microscopic degrees of freedom of BHs are carried by the Planck area pixels l_{P}^2. The dark pixels are portrayed as correlated. $|R|$ measures the average number of correlated pixels.

G. Ruppeiner: Phys. Rev. D 78 (2008) 024016

$\Rightarrow$ The big absolute value of the thermodynamic curvature implies strong interaction and the small one corresponds to weak interaction.

Y.-G. Miao and Z.-M. Xu: Phys. Rev. D 98 (2018) 044001

Thermodynamic curvature for black holes

Black hole	Dimension	R sign	$R=0$	$\lvert R\rvert$ divergence
Reissner-Nordström	$3+1$	0	-	none
Kerr	$3+1$	$+$	no	extremal
Kerr-Newman	$3+1$	$+$	no	extremal
Small black ring	$4+1$	$\pm$	yes	ext+crit line
BTZ	$2+1$	0	0	none
RN-AdS	$3+1$	$\pm$	yes	ext+crit line
K-AdS	$3+1$	$-$	no	critical line
Restricted KN	$3+1$	$+$	no	extremal
Large black ring	$4+1$	$-$	no	ext+crit line

G. Ruppeiner: Springer proceedings in physics, 153 (2014) 179.

Thermodynamic geometry for the Reissner-Nordström (RN) black hole

Background and Motivation

Puzzle: RN black hole is Ruppeiner flat $\Longrightarrow$ a non-interacting statistical system

J.E. Aman, et al.: Gen. Rel. Grav. 35 (2003) 1733.

- the phase space of extensive variables of the RN black hole maybe is incomplete
  $\Rightarrow$ the results of the RN black hole should be reduced from those of the Kerr-Newmann-AdS black hole
  $\Rightarrow$ RN black hole is an interacting system

  B. Mirza, M. Zamani-Nasab: JHEP 0706 (2007) 059.

Our viewpoint:

- The phase space of extensive variables is complete
- The electromagnetic interaction.

Thermodynamic geometry for the RN black hole

Our scheme:

- The phase space is (S, Q^2), not (S, Q)

$$f(r) = 1 - \frac{2M}{r} + \frac{r^2}{l^2} + \frac{Q^2}{r^2} \qquad \text{(RN-AdS)}$$

- Thermodynamic scalar curvature

$$R = \frac{1}{S - \pi Q^2}$$

- $R > 0 \Longrightarrow$ a repulsive interaction, i.e. electromagnetic repulsion interaction.

Thermodynamic geometry for the Schwarzschild AdS (SAdS) black hole

Background and Motivation

- For black holes in the AdS spacetime

$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi l^2}, \qquad \Rightarrow \qquad V = \frac{\partial M}{\partial P}.$$

- For static spherically symmetric AdS black holes:
  $C_V := T\frac{\partial S}{\partial T}\Big|_V = 0$
  - The entropy S and thermodynamic volume V are not independent: $S = \pi r_h^2, \quad V = \frac{4}{3}\pi r_h^3.$
- The line element of thermodynamic geometry singular
  $\Longrightarrow$ a divergent thermodynamic curvature.

Background and Motivation

One feasible solution: normalized thermodynamic curvature by treating the heat capacity at constant volume as a constant very close to zero.

- the line element (based on $dU = TdS - PdV$)

$$\Delta l^2 = \frac{C_V}{T^2}\Delta T^2 + \frac{1}{T}\left(\frac{\partial P}{\partial V}\right)_T \Delta V^2$$

- van der Waals fluid $C_V = \frac{3}{2}k_B$ is of order 10^{-23} $\Rightarrow$ the black hole vanishing heat capacity as a $k_B \to 0^+$ limit.

- normalized thermodynamic curvature: $R_N = RC_V$

S.-W. Wei, Y.-X. Liu, and R.B. Mann: Phys. Rev. Lett. 123 (2019) 071103.

Thermodynamic geometry for the SAdS black hole

Our scheme: maybe more direct and natural

- internal energy: $dU = TdS - PdV$ is no longer hold
- the most basic thermodynamic differential relation is about that of enthalpy M,

$$dM = TdS + VdP$$

Universal form of line element

$$\Delta l^2 = \frac{1}{T}\Delta T \Delta S + \frac{1}{T}\Delta V \Delta P$$

Thermodynamic geometry for the SAdS black hole

- Phase space $\{S, P\}$

$$\Delta l^2 = \frac{1}{C_P}\Delta S^2 + \frac{2}{T}\left(\frac{\partial T}{\partial P}\right)_S \Delta S \Delta P + \frac{1}{T}\left(\frac{\partial V}{\partial P}\right)_S \Delta P^2$$

- Phase space $\{T, V\}$

$$\Delta l^2 = \frac{C_V}{T^2}\Delta T^2 + \frac{2}{T}\left(\frac{\partial P}{\partial T}\right)_V \Delta T \Delta V + \frac{1}{T}\left(\frac{\partial P}{\partial V}\right)_T \Delta V^2$$

Thermodynamic geometry for the SAdS black hole

For the SAdS black hole:

1. In the phase space $\{S, P\}$

$$R_{SP} = -\frac{1}{S(1+8PS)}.$$

2. In the phase space $\{T, V\}$

$$R_{TV} = -\frac{1}{3\pi TV}.$$

$R_{SP} = R_{TV} < 0 \quad \Rightarrow$ **attractive interaction**

Thermodynamic geometry for the Schwarzschild (Sch.) black hole

Thermodynamic geometry for the Sch. black hole

- The first law of thermodynamics

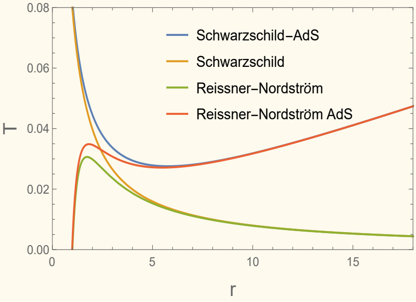
$$dU_{\text{Schwarzschild}} = T_{\text{Schwarzschild}} dS_{\text{Schwarzschild}}$$

- This renders the metric of thermodynamic geometry singular always.
- We have to analyze some of its micro-behavior with the help of the results of other black holes, like the RN and SAdS black holes.

Thermodynamic geometry for the Sch. black hole

The diagram of the temperature with respect to horizon radius for Schwarzschild, SAdS, RN and RN-AdS black holes.

- the behavior of temperature of SAdS black hole is close to that of Schwarzschild black hole in small scale
- the behavior of temperature of RN black hole is similar to that of Schwarzschild black hole in large scale
- the temperature curves of the Schwarzschild black hole and the RN-AdS black hole have intersection



Thermodynamic geometry for the Sch. black hole

The Schwarzschild black hole:

- **repulsion on low temperature region**
- **attraction on high temperature region**

$$R_{\text{Schwarzschild}} = 16\pi T^2_{\text{Schwarzschild}} \,\text{sgn}\left(2Pr - \frac{Q^2}{4\pi r^3}\right)$$

$$= \begin{cases} 16\pi T^2_{\text{Schwarzschild}}, & \text{large } r \text{ (low } T) \\ 0, & 2Pr = \dfrac{Q^2}{4\pi r^3} \\ -16\pi T^2_{\text{Schwarzschild}}, & \text{small } r \text{ (high } T) \end{cases}$$

Summary

Completely from the thermodynamic point of view

- RN black hole is an interaction system dominated by repulsion
- A basic and natural scheme for thermodynamic geometry of spherically symmetric AdS black holes
  - The general form of the line element of thermodynamic geometry of SAdS black hole and the specific forms in phase space
  - $R < 0 \Longrightarrow$ the attractive interaction for SAdS black hole
- Schwarzschild black hole is dominated by repulsion on low temperature region and by attraction on high temperature region

Thank you!