Gedanken Experiments to Destroy a BTZ Black Hole

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Based on BN, Baoyi Chen & Feng-Li Lin, PRD 100 (2019) 044043, arXiv:1902.00949[gr-qc]

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Motivation

Weak Cosmic Censorship Conjecture (WCCC): Penrose 1969

"Nature abhors a naked singularity."

Gedanken experiments: Could we create a naked singularity by throwing matter into a (near-)extremal black hole?

Motivation

4D Kerr-Newman black holes:

$$M^2 \geq (J/M)^2 + Q^2$$

Extremal: Wald 1974

- Throw a test particle with large q and j/m ?

  Impossible, due to electromagnetic and centrifugal repulsion.

- Drop a spinning test particle with large s/m ?

  Impossible, due to gravitational spin-spin repulsive force.

  WCCC safe :)

Motivation

4D Kerr-Newman black holes:

$$M^2 \geq (J/M)^2 + Q^2$$

Near-extremal:

- Seems possible to make $M < Q$, by throwing matter with proper q into a slightly non-extremal Reissner-Nordstrom black hole. Hubeny 1999

  Loophole: violation of WCCC happens at the order q^2, test particle approximation not valid (back reaction, self energy, ...)

- By taking into account of contribution from 2nd order perturbation, no violation of WCCC can occur. Sorce-Wald 2017

  WCCC still safe :)

Motivation

How about 3D BTZ black holes?

Outline

- Motivation
- BTZ black holes and variational identites
- Gedanken experiments
  - Destroy an extremal BTZ
  - Destroy a near-extremal BTZ
- Conclusion and Discussion

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BTZ black holes

3D Mielke-Baekler gravity with torsion: Mielke-Baekler 1991

$$L = L_{\mathrm{EC}} + L_{\Lambda} + L_{\mathrm{CS}} + L_{\mathrm{T}} + L_{\mathrm{M}} \,,$$

$$\begin{aligned}
L_{\mathrm{EC}} &= \frac{1}{\pi}\, e^a \wedge R_a \,, \\
L_{\Lambda} &= -\frac{\Lambda}{6\pi}\, \epsilon_{abc}\, e^a \wedge e^b \wedge e^c \,, \\
L_{\mathrm{CS}} &= -\,\theta_{\mathrm{L}} \left(\omega^a \wedge \mathrm{d}\omega_a + \frac{1}{3}\, \epsilon_{abc}\, \omega^a \wedge \omega^b \wedge \omega^c \right) \,, \\
L_{\mathrm{T}} &= \frac{\theta_{\mathrm{T}}}{2\pi^2}\, e^a \wedge T_a \,,
\end{aligned}$$

Three well-defined limits: (on-shell $T_a \propto \mathcal{T} \equiv \frac{-\theta_{\mathrm{T}} + 2\pi^2 \Lambda \theta_{\mathrm{L}}}{2 + 4\theta_{\mathrm{T}}\theta_{\mathrm{L}}}$)

- Einstein gravity: $\theta_{\mathrm{L}} \to 0 \,,\ \theta_{\mathrm{T}} \to 0$
- Chiral gravity: torsionless, set $\mathcal{T} = 0$ then take $\theta_{\mathrm{L}} \to -1/(2\pi\sqrt{-\Lambda}\,)$
- Torsional chiral gravity: take $\theta_{\mathrm{L}} \to -1/(2\pi\sqrt{-\Lambda}\,)$ first, then obtain $\mathcal{T} \to \pi\sqrt{-\Lambda}\,/\,2$ hence torsion not vanishing

BTZ black holes

BTZ solutions in Mielke-Baekler gravity: Hehl *et al* 2003

dreibeins:

$$e^0 = N\mathrm{d}t, \qquad e^1 = \frac{\mathrm{d}r}{N}, \qquad e^2 = r\left(\mathrm{d}\phi + N^\phi \mathrm{d}t\right) \ ,$$

$$N^2(r) = -M - \Lambda_{\text{eff}}\, r^2 + \frac{J^2}{4r^2}, \qquad N^\phi(r) = -\frac{J}{2r^2}, \qquad \Lambda_{\text{eff}} \equiv -\frac{\mathcal{T}^2 + \mathcal{R}}{\pi^2} \ ,$$

dual spin connections:

$$\omega^a = \tilde{\omega}^a + \frac{\mathcal{T}}{\pi} e^a ,$$

$$\tilde{\omega}^0 = N\mathrm{d}\phi, \qquad \tilde{\omega}^1 = -\frac{N^\phi}{N}\mathrm{d}r, \qquad \tilde{\omega}^2 = -\Lambda_{\text{eff}}\, r\mathrm{d}t + rN^\phi \mathrm{d}\phi ,$$

$$\left(\Lambda_{\text{eff}} \equiv -\frac{\mathcal{T}^2 + \mathcal{R}}{\pi^2} \ , \qquad \mathcal{R} \equiv -\frac{\theta_{\mathsf{T}}^2 + \pi^2 \Lambda}{1 + 2\theta_{\mathsf{T}}\theta_{\mathsf{L}}}\right)$$

In torsion free limit $\mathcal{T} \to 0$, recover BTZ in Einstein and TMG with $\Lambda_{\text{eff}} = \Lambda$.

Variational identities

Main idea:

Consider general off-shell variations of fields, which incorporates all kinds of possible perturbations to a black hole (including throwing matter into it).

From variational identities, one could obtain general constrains for these perturbations so as to check WCCC. Sorce-Wald 2017

Variational identities: 1st order

First order variation of the MB Lagrangian (vacuum) gives

$$\begin{aligned}
\delta L &= \delta e^a \wedge E_a^{(e)} + \delta\omega^a \wedge E_a^{(\omega)} + \mathrm{d}\Theta(\phi, \delta\phi)\,, \\
\Theta(\phi, \delta\phi) &= \frac{1}{\pi}\,\delta\omega^a \wedge e_a + \frac{\theta_\mathrm{T}}{2\pi^2}\,\delta e^a \wedge e_a - \theta_\mathrm{L}\,\delta\omega^a \wedge \omega_a\,,
\end{aligned}$$

from the surface term, one defines the conserved symplectic current

$$\Omega(\phi, \delta_1\phi, \delta_2\phi) = \delta_1\Theta(\phi, \delta_2\phi) - \delta_2\Theta(\phi, \delta_1\phi)\,.$$

Noether current associated with a vector field ξ is

$$j_\xi = \Theta(\phi, \mathcal{L}_\xi\phi) - \mathrm{i}_\xi L\,, \tag{1}$$

which could be written in terms of Noether charge Q_ξ and constraints C_ξ :

$$j_\xi = \mathrm{d}Q_\xi + C_\xi\,, \tag{2}$$

$$\begin{aligned}
Q_\xi &= \frac{1}{\pi}\,(\mathrm{i}_\xi\omega^a) \wedge e_a + \frac{\theta_\mathrm{T}}{2\pi^2}\,(\mathrm{i}_\xi e^a) \wedge e_a - \theta_\mathrm{L}\,(\mathrm{i}_\xi\omega^a) \wedge \omega_a\,, \\
C_\xi &= -(\mathrm{i}_\xi e^a) \wedge E_a^{(e)} - (\mathrm{i}_\xi\omega^a) \wedge E_a^{(\omega)}\,.
\end{aligned}$$

Variational identities: 1st order

Variation of (1)(2) gives rise to off-shell linear variational identity:

$$\int_{\partial\Sigma} \delta Q_\xi \, - \, \mathrm{i}_\xi \Theta(\phi, \delta\phi) \;=\; \int_\Sigma \Omega(\phi, \delta\phi, \mathcal{L}_\xi \phi) - \int_\Sigma \delta C_\xi - \int_\Sigma \mathrm{i}_\xi (E\,\delta\phi)\,. \qquad (3)$$

For the case EOM satisfied and ξ is Killing field $\partial_t + \Omega_\mathrm{H}\partial_\varphi$, it reduce to

$$\delta\mathcal{M} - \Omega_\mathrm{H}\delta\mathcal{J} - T_\mathrm{H}\,\delta S \;=\; -\int_\Sigma \delta C_\xi\,. \qquad (4)$$

Consider $\delta\phi$ vanishes near internal boundary of Σ, then (4) with $T_\mathrm{H}\delta S = 0$ holds for both extremal and non-extremal black holes.

For BTZ solution in MB gravity,

$$\begin{aligned}
\mathcal{M} \;&\equiv\; \int_\infty \delta Q_t \, - \, \mathrm{i}_t\Theta(\phi, \delta\phi) \;=\; M - 2\theta_\mathrm{L}\left(\mathcal{T}M + \pi\Lambda_\mathrm{eff}J\right)\,, \\
\mathcal{J} \;&\equiv\; \int_\infty \delta Q_\varphi \, - \, \mathrm{i}_\varphi\Theta(\phi, \delta\phi) \;=\; J \, + \, 2\theta_\mathrm{L}\left(\pi M - \mathcal{T}J\right)\,, \\
T_\mathrm{H}\delta S \;&\equiv\; \int_B \delta Q_\xi \, - \, \mathrm{i}_\xi\Theta(\phi, \delta\phi)\,, \qquad S \;=\; 4\pi r_+ \, - \, 8\pi\theta_\mathrm{L}\left(\mathcal{T}r_+ - \pi\sqrt{-\Lambda_\mathrm{eff}}\,r_-\right)\,.
\end{aligned}$$

Variational identities: 1st order

Although (4) is derived from Lagrangian without matter, since we didn't enforce linearized EOM, it should also be valid for perturbations due to matter.

Due to EOM in the presence of matter:

$$E_a^{(e)} = -\Sigma_a \,, \qquad E_a^{(\omega)} = -\tau_a \,,$$

in which $\Sigma_a \equiv \delta L_{\mathrm{M}}/\delta e^a$, $\tau_a \equiv \delta L_{\mathrm{M}}/\delta \omega^a$ are the energy-momentum tensor and spin angular momentum tensor of matter, linear variational identity for BTZ in MB model turns out to be:

$$\begin{aligned}
& \delta\mathcal{M} - \Omega_{\mathrm{H}}\delta\mathcal{J} \\
= \;& (1 - 2\theta_{\mathrm{L}}\mathcal{T} - 2\pi\theta_{\mathrm{L}}\Omega_{\mathrm{H}})(\delta M - \Omega_{\mathrm{H}}\delta J) - 2\pi\theta_{\mathrm{L}}\Lambda_{\mathrm{eff}}\left(\frac{r_+^2 - r_-^2}{r_+^2}\right)\delta J \\
= \;& \int_\Sigma d^2x\sqrt{-\gamma}\left\{\xi_\mu k_\nu \delta\Sigma^{\mu\nu} - \left(\kappa_{\mathrm{H}} n_{\mu\nu} + \frac{\mathcal{T}}{\pi}\epsilon_{\mu\nu}{}^{\sigma}\xi_\sigma\right)k_\lambda \delta\tau^{\mu\nu\lambda}\right\} .
\end{aligned}$$

Variational identities: 2nd order

For near-extremal black holes it would not be sufficient to consider just 1st order. Variation of (3) gives

$$\mathcal{E}_\Sigma(\phi;\delta\phi) \;=\; \int_{\partial\Sigma} \left[\delta^2 Q_\xi \;-\; \mathrm{i}_\xi \delta\Theta(\phi,\delta\phi)\right] \;+\; \int_\Sigma \delta^2 C_\xi \;+\; \int_\Sigma \mathrm{i}_\xi \left(\delta E \wedge \delta\phi\right)\,,$$

in which Wald's canonical energy

$$\mathcal{E}_\Sigma(\phi;\delta\phi) \;\equiv\; \int_\Sigma \Omega(\phi,\delta\phi,\mathcal{L}_\xi\delta\phi)\,.$$

For ϕ a stationary black hole solution and ξ the horizon Killing field, assuming no perturbation near interior boundary, we obtain the identity for second order variation:

$$\begin{aligned} & \delta^2\mathcal{M} \;-\; \Omega_{\mathrm{H}}\delta^2\mathcal{J} \\ = \quad & \mathcal{E}_\Sigma(\phi;\delta\phi) \;+\; \delta^2 \int_\Sigma d^2x\sqrt{-\gamma}\left\{\xi_\mu k_\nu \Sigma^{\mu\nu} \;-\; \left(\kappa_{\mathrm{H}} n_{\mu\nu} + \frac{\mathcal{T}}{\pi}\,\epsilon_{\mu\nu}{}^{\sigma}\xi_\sigma\right) k_\lambda \tau^{\mu\nu\lambda}\right\}\,. \end{aligned}$$

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Destroy an extremal BTZ

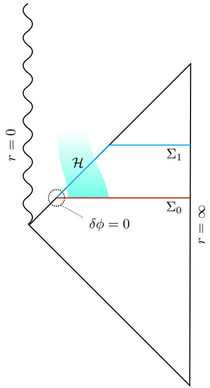
Considering a 1-parameter family of solutions $\phi(\lambda)$ with $\phi(0)$ an extremal BTZ.

Since horizons are located at

$$r_\pm^2 = \frac{1}{2\Lambda_{\mathsf{eff}}}\left(-M \mp \sqrt{M^2 + \Lambda_{\mathsf{eff}} J^2}\right),$$

defining $f(\lambda) = M(\lambda)^2 + \Lambda_{\mathsf{eff}} J(\lambda)^2$, criterion for preserving WCCC is

$$f(\lambda) \geq 0$$



Destroy an extremal BTZ

To 1st order in λ,

$$f(\lambda) = 2\lambda\sqrt{-\Lambda_{\text{eff}}}\,|J|\left(\delta M - \sqrt{-\Lambda_{\text{eff}}}\,\delta J\right) + \mathcal{O}(\lambda^2)\,.$$

linear variational identity for extremal BTZ:

$$\begin{aligned}&\left(1 - 2\theta_{\text{L}}\mathcal{T} - 2\pi\theta_{\text{L}}\sqrt{-\Lambda_{\text{eff}}}\right)\left(\delta M - \sqrt{-\Lambda_{\text{eff}}}\,\delta J\right)\\ = &\int_\Sigma d^2x\sqrt{-\gamma}\left\{\xi_\mu k_\nu \delta\Sigma^{\mu\nu} - \frac{\mathcal{T}}{\pi}\,\epsilon_{\mu\nu}{}^{\sigma}\xi_\sigma k_\lambda \delta\tau^{\mu\nu\lambda}\right\}.\end{aligned}$$

- Torsional chiral gravity: unclear, due to torsional coupling
- Chiral gravity: $f(\lambda) \geq 0$ provided Null Energy Condition ($k_\mu k_\nu \delta\Sigma^{\mu\nu} \geq 0$)
- Einstein gravity: $f(\lambda) \geq 0$ provided Null Energy Condition

WCCC preserved for the latter two :)

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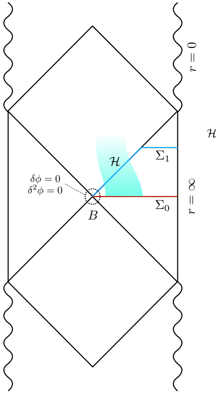
Destroy a near-extremal BTZ

Assumption: The non-extremal BTZ black hole is *linearly stable* to perturbations, i.e., any source-free linear perturbation $\delta\phi$ approaches a perturbation $\delta\phi^{\mathrm{BTZ}}$ towards another BTZ solution at sufficiently late times.

This does not indicate WCCC, since finite perturbation is needed to overspin a nonextremal black hole, while a linear perturbation can always be scaled down.



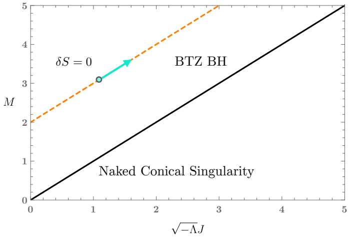
Destroy a near-extremal BTZ

Chiral gravity: linear variation identity and NEC gives

$$\delta\mathcal{M} \, - \, \Omega_{\mathrm{H}}\delta\mathcal{J} \geq 0 \,,$$

hence $T_{\mathrm{H}}\,\delta S \geq 0$ according to 1st law.

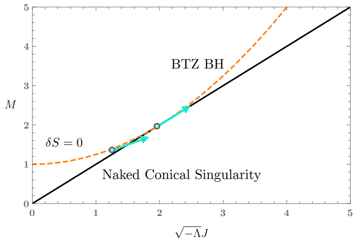
Tangent to constant entropy curve is a lower bound to all physically-realizable perturbations, a non-extremal BTZ black hole will at most be perturbed to another BTZ with the same entropy (no Hubeny-type violation of WCCC).



Destroy a near-extremal BTZ

Einstein gravity: linear variation identity and NEC also gives $\delta S \geq 0$.

Starting from a slightly non-extremal BTZ black hole, tangent to the curve of constant entropy is possible to move the original point to another point located in the region representing naked conical singularities.



(Hubeny-type violation of WCCC? requires 2nd order calculation)

Destroy a near-extremal BTZ: Einstein gravity

To 2nd order in λ,

$$\begin{aligned} f(\lambda) \quad &= \quad \left(M^2 - \alpha^2 J^2\right) + 2\lambda\left(M\delta M - \alpha^2 J\delta J\right) + \\ &\quad + \lambda^2\left[(\delta M)^2 - \alpha^2(\delta J)^2 + M\delta^2 M - \alpha^2 J\delta^2 J\right] + \mathcal{O}(\lambda^3)\,. \end{aligned} \tag{5}$$

2nd order variational identity and NEC gives

$$\delta^2 M - \Omega_{\mathrm{H}}\delta^2 J \;\geq\; \mathcal{E}_\Sigma(\phi;\delta\phi)\,, \tag{6}$$

where

$$\mathcal{E}_\Sigma(\phi;\delta\phi) \;=\; \mathcal{E}_{\mathcal{H}}(\phi;\delta\phi) + \mathcal{E}_{\Sigma_1}(\phi;\delta\phi)\,.$$

- $\mathcal{E}_{\mathcal{H}}(\phi;\delta\phi) = 0$, since it represents total flux of gravitational wave energy into the black hole, which vanishes in 3D Einstein gravity.
- $\mathcal{E}_{\Sigma_1}(\phi;\delta\phi) = \mathcal{E}_\Sigma(\phi;\delta\phi^{\mathrm{BTZ}})$, according to our assumption.

Destroy a near-extremal BTZ: Einstein gravity

Consider a one-parameter family of BTZ black holes $\phi^{\mathrm{BTZ}}(\beta)$,

$$M(\beta) = M + \beta\delta M^{\mathrm{BTZ}}\,, \quad J(\beta) = J + \beta\delta J^{\mathrm{BTZ}}\,,$$

δM^{BTZ} and δJ^{BTZ} are fixed by 1st order perturbation for $\phi(\lambda)$.
For this family of solutions, $\delta^2 M = \delta^2 J = \delta E = \delta^2 C = 0$, hence

$$\mathcal{E}_{\Sigma_1}(\phi;\delta\phi) = -T_{\mathrm{H}}\delta^2 S^{\mathrm{BTZ}}\,.$$

Assuming 1st order perturbation is optimally done, i.e., $\delta M = \Omega_{\mathrm{H}}\delta J$, use (6) to constrain $f(\lambda)$ in (5), we obtain

$$f(\lambda) \;\geq\; \left(M\epsilon - \lambda\frac{\alpha^2 J\delta J}{M}\right)^2 + \mathcal{O}(\lambda^3, \epsilon\lambda^2, \epsilon^2\lambda, \epsilon^3)\,.$$

$f(\lambda) \geq 0$ hence WCCC is preserved :)

Conclusion

- For Einstein gravity and chiral gravity, WCCC holds for both extremal and non-extremal BTZ black holes if the falling matter obeys the null energy condition.

- For torsional chiral gravity, WCCC hold or not depends on the additional null energy-like condition for the spin angular momentum tensor, which is not clear for us at present.

Discussion & future work

- Third law of black hole dynamics: one cannot turn a non-extremal black hole into an extremal one by throwing the matter in a finite time interval. Israel 1986

  In the context of AdS/CFT, our results can serve as an operational proof of thermodynamic third law by mapping our gedanken experiment around the BTZ black hole holographically to the cooling process of the boundary CFT toward zero temperature.

- WCCC for higher order derivative gravity? Working in progress...

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THANK YOU !