Gravitational Wave Echoes

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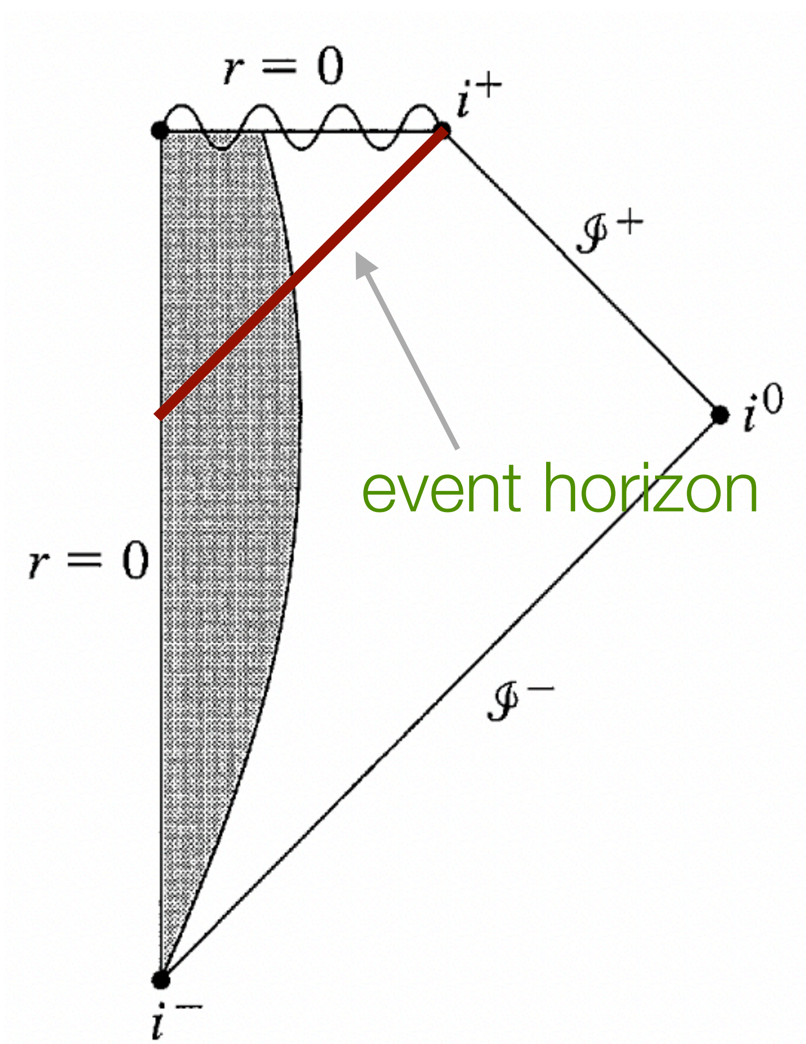
ICTS Workshop, Yichang, 24 November 2019

Zhang & **SYZ**, 1709.07503
Huang, Xu & **SYZ**, 1908.00189
Wang, Zhang, **SYZ** & Piao, 1802.02003, 1904.00212

The photo of a black hole

What defines a black hole?



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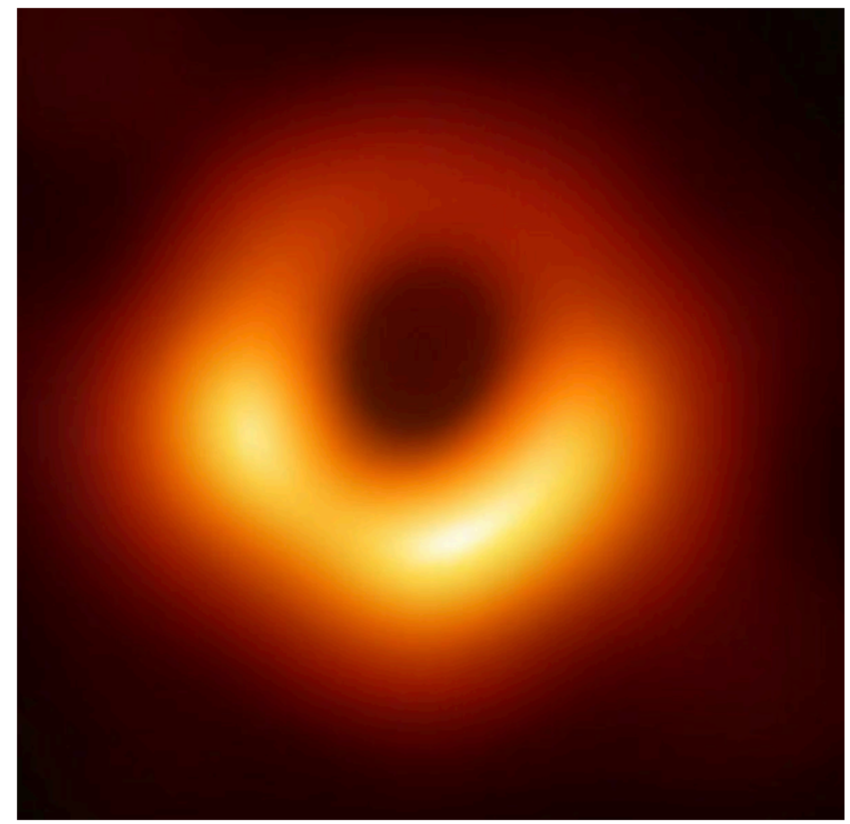
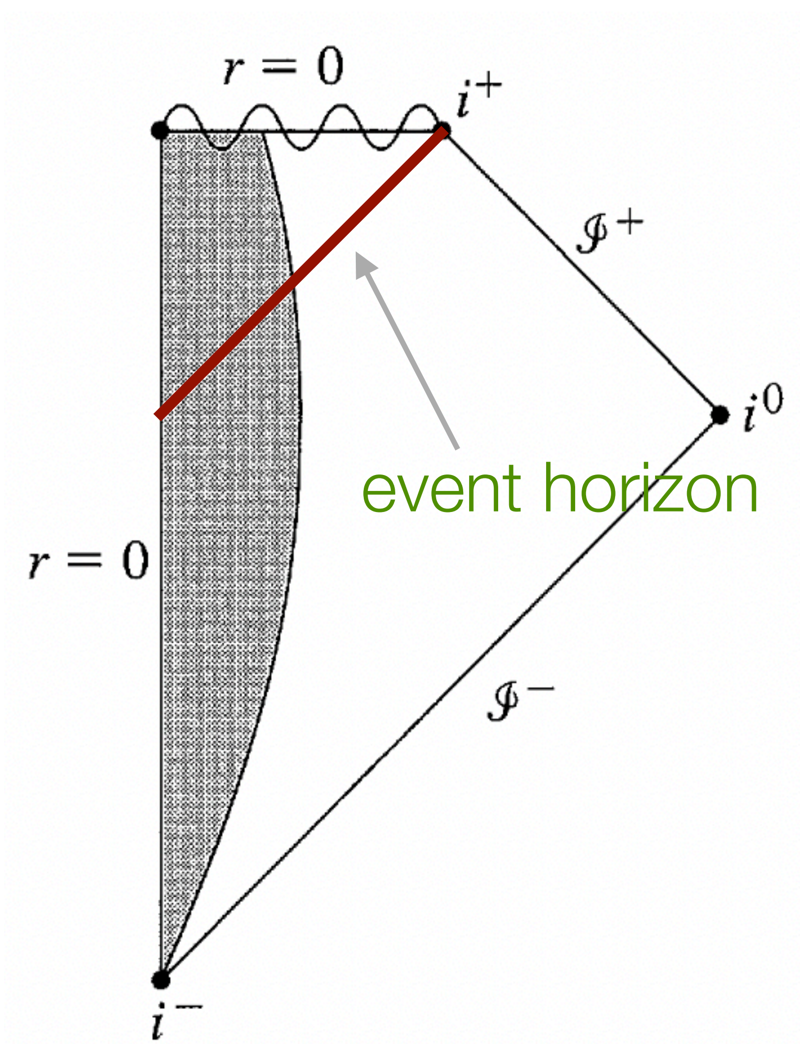


Event Horizon Telescope

good PR but atrocious, grotesque abuse!

The photo of a black hole

What defines a black hole?



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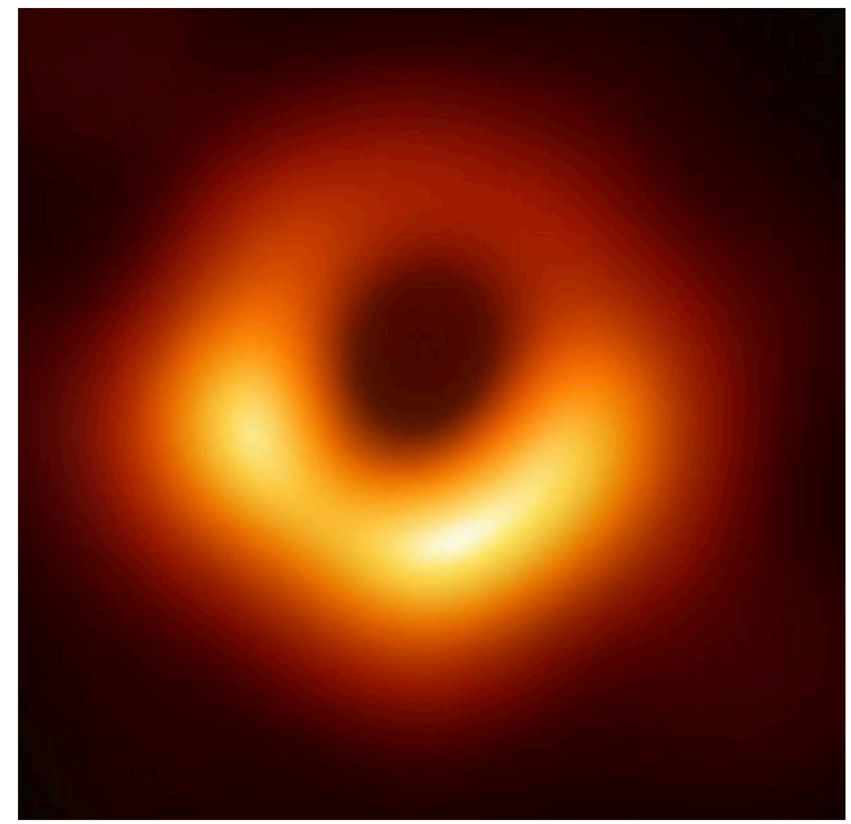


Photosphere Telescope

good PR but atrocious, grotesque abuse!

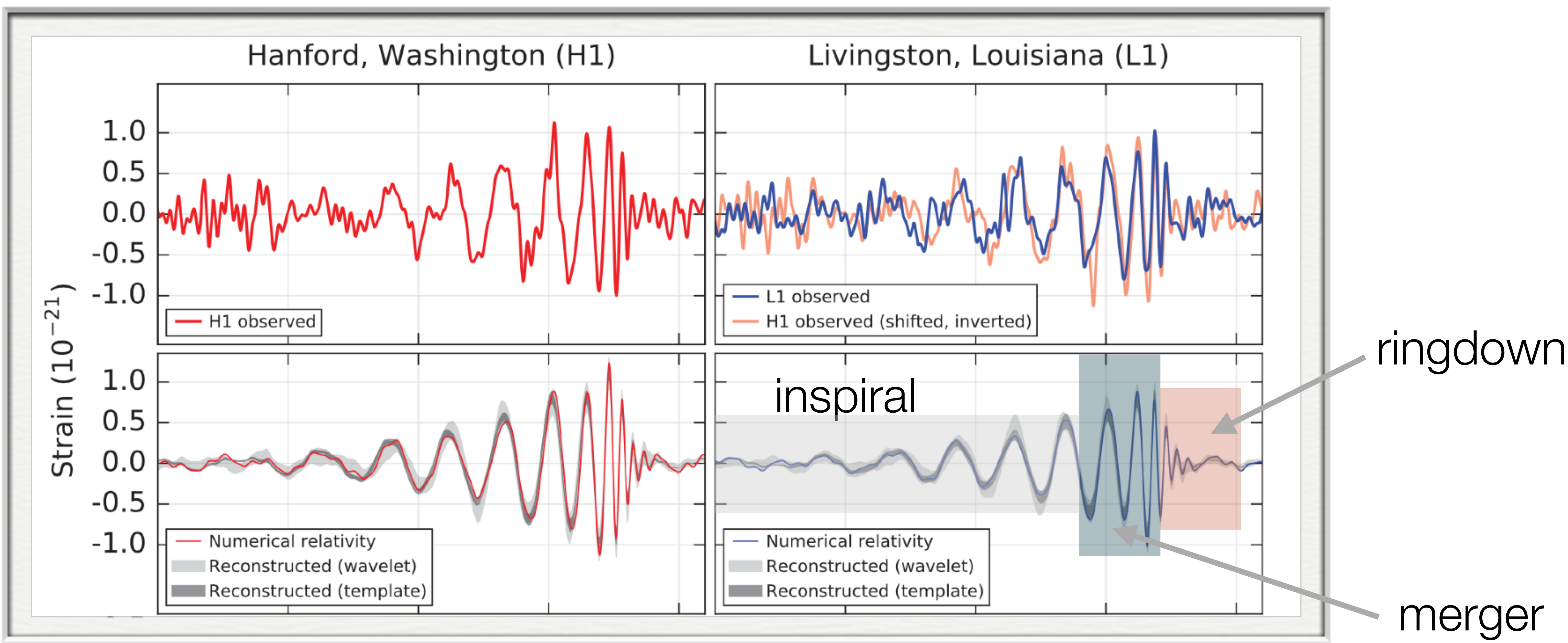
The sound of a black hole merger

Selected for a Viewpoint in *Physics*
PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

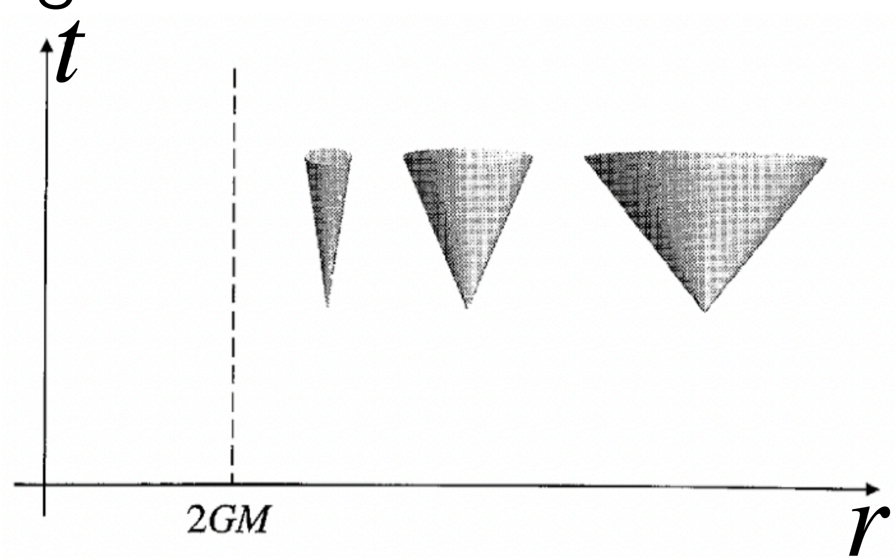
B. P. Abbott *et al.*[*]
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)

GW150914

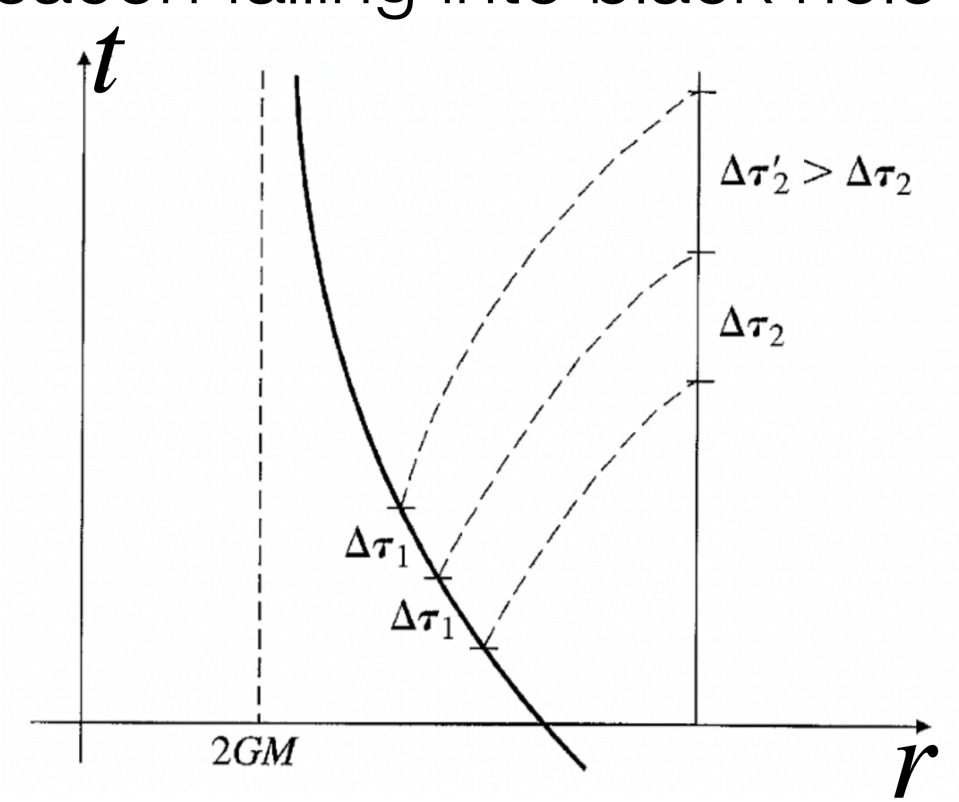


But have we observed an event horizon?

light cones near horizon



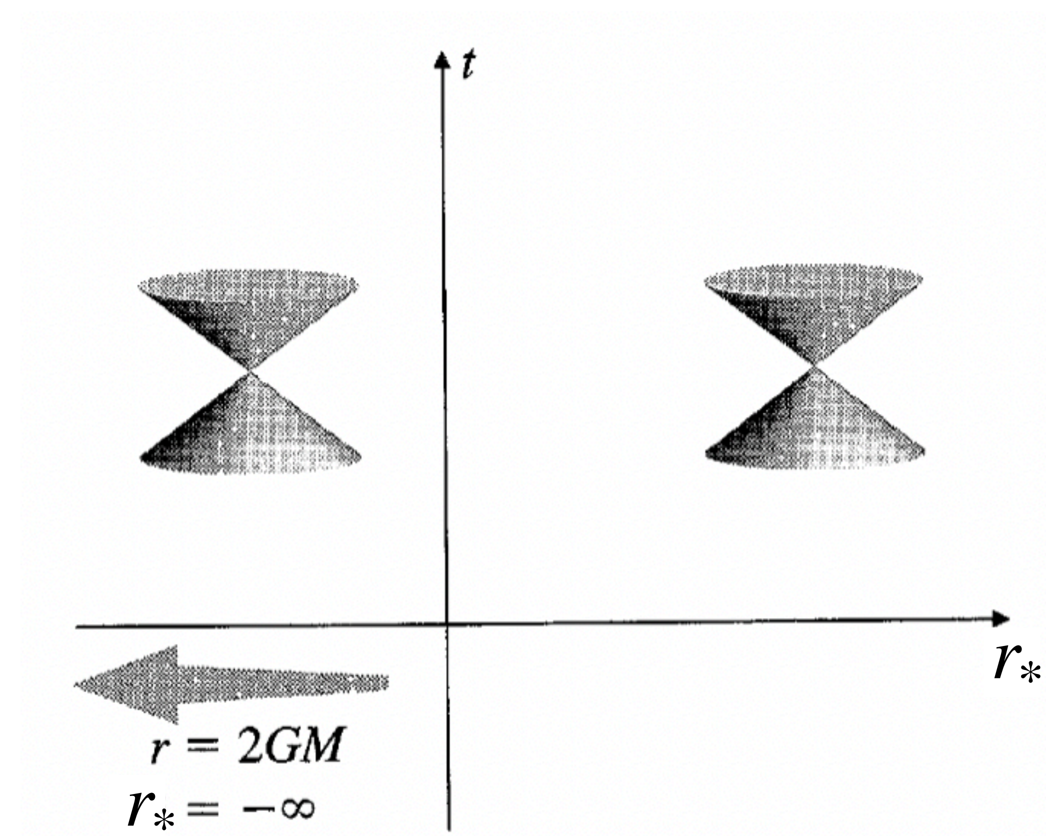
beacon falling into black hole



Tortoise coordinate

$$r_* = r + 2GM \ln\left(\frac{r}{2GM} - 1\right)$$

$$\mathrm{d}s^2 = \left(1 - \frac{2GM}{r}\right)\left(-\mathrm{d}t^2 + \mathrm{d}r_*^2\right) + r^2 d\Omega^2$$

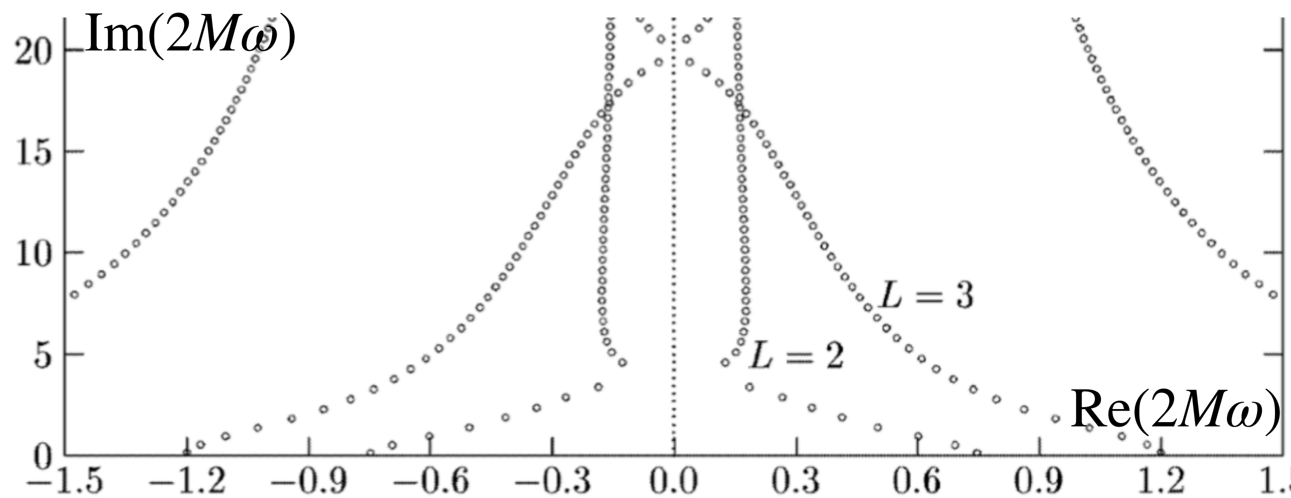


The magic of quasi-normal modes?

- QNMs of BH: characteristic oscillation modes of BH spacetime

Regge-Wheeler-Zerilli equation

$$\frac{d^2\psi_l(r_*)}{dr_*^2} + \left[\omega^2 - V_l(r_*)\right]\psi_l(r) = S_l$$



- damped: $\omega = \omega_R + i\omega_I$
- ringdown dominated by QNMs
- QNM spectrum determined by BH's M and a (Kerr)
- test of no-hair theorem

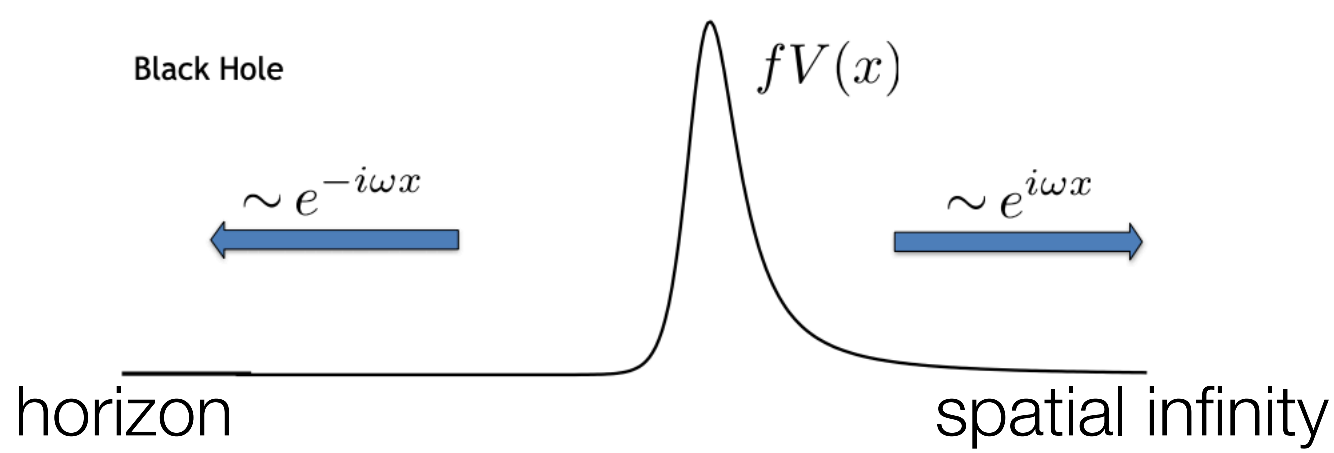
Ringdown: only conclusive test of BH(?)

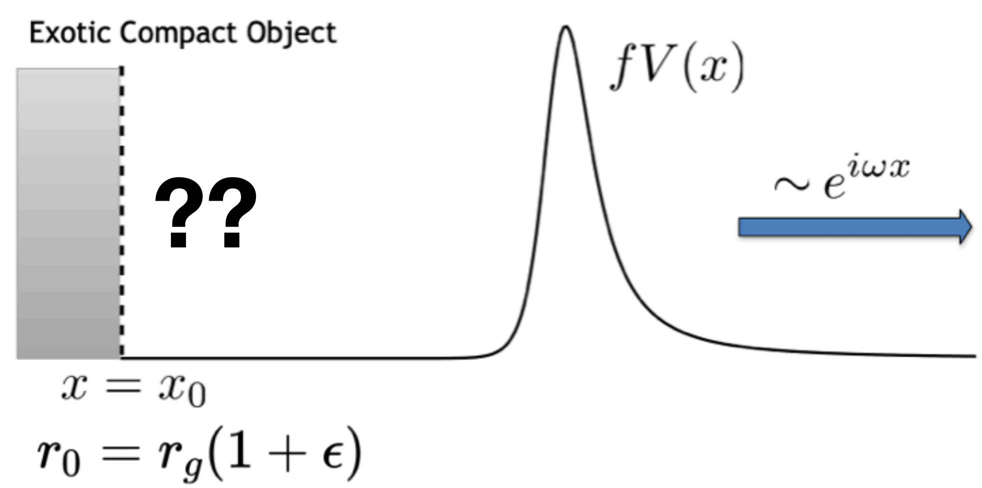
Not necessarily!

- This is based on assumption: **Ringdown $\simeq$ QNMs**
- Ringdown simply determined by photosphere

frequency & damping time ~ circular photo orbit

- QNMs are sensitive to boundary conditions





Ringdown $\simeq$ QNMs

Ringdown $\neq$ QNMs!

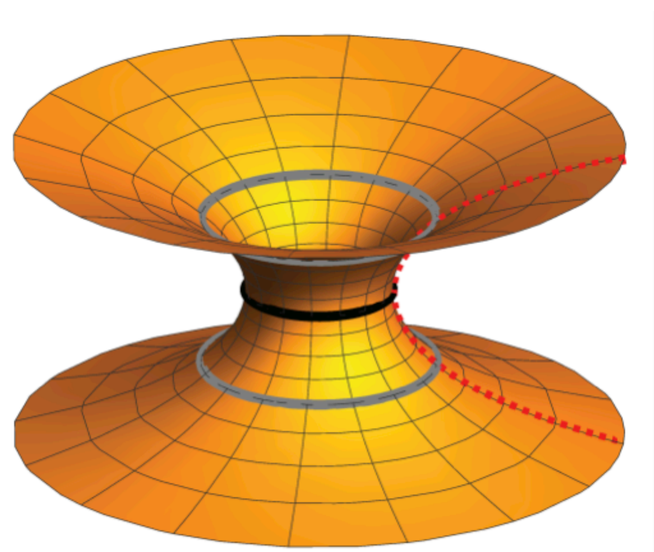
Exotic compact object (ECO): an example

Featured in Physics | Editors' Suggestion

Is the Gravitational-Wave Ringdown a Probe of the Event Horizon?

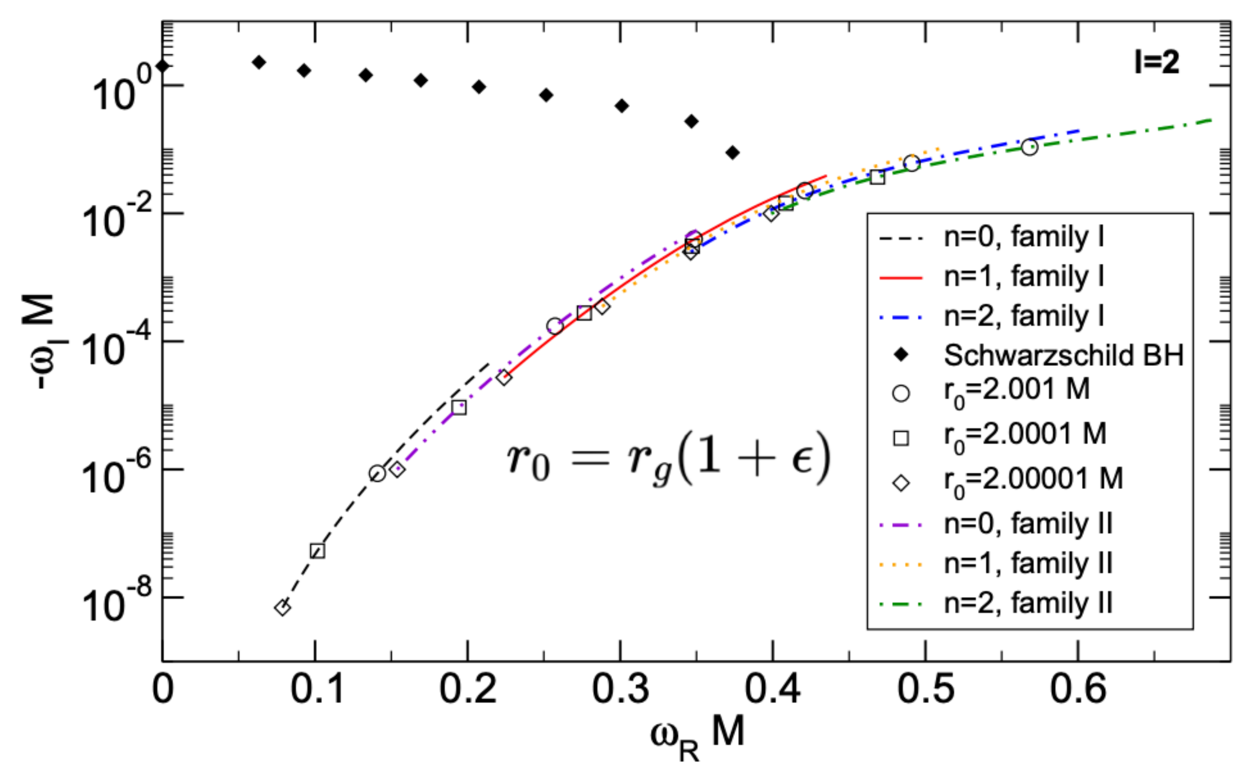
Vitor Cardoso, Edgardo Franzin, and Paolo Pani
Phys. Rev. Lett. **116**, 171101 – Published 27 April 2016; Erratum Phys. Rev. Lett. **117**, 089902 (2016)

Schwarzschild Wormhole

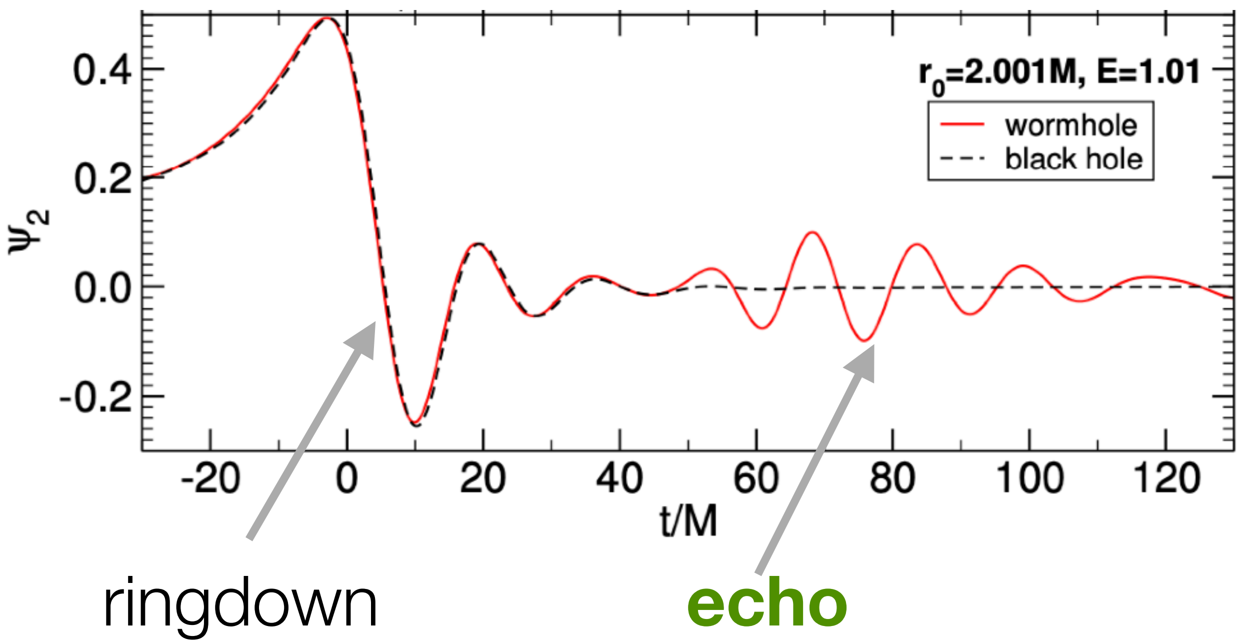


"almost" Schwarzschild BH

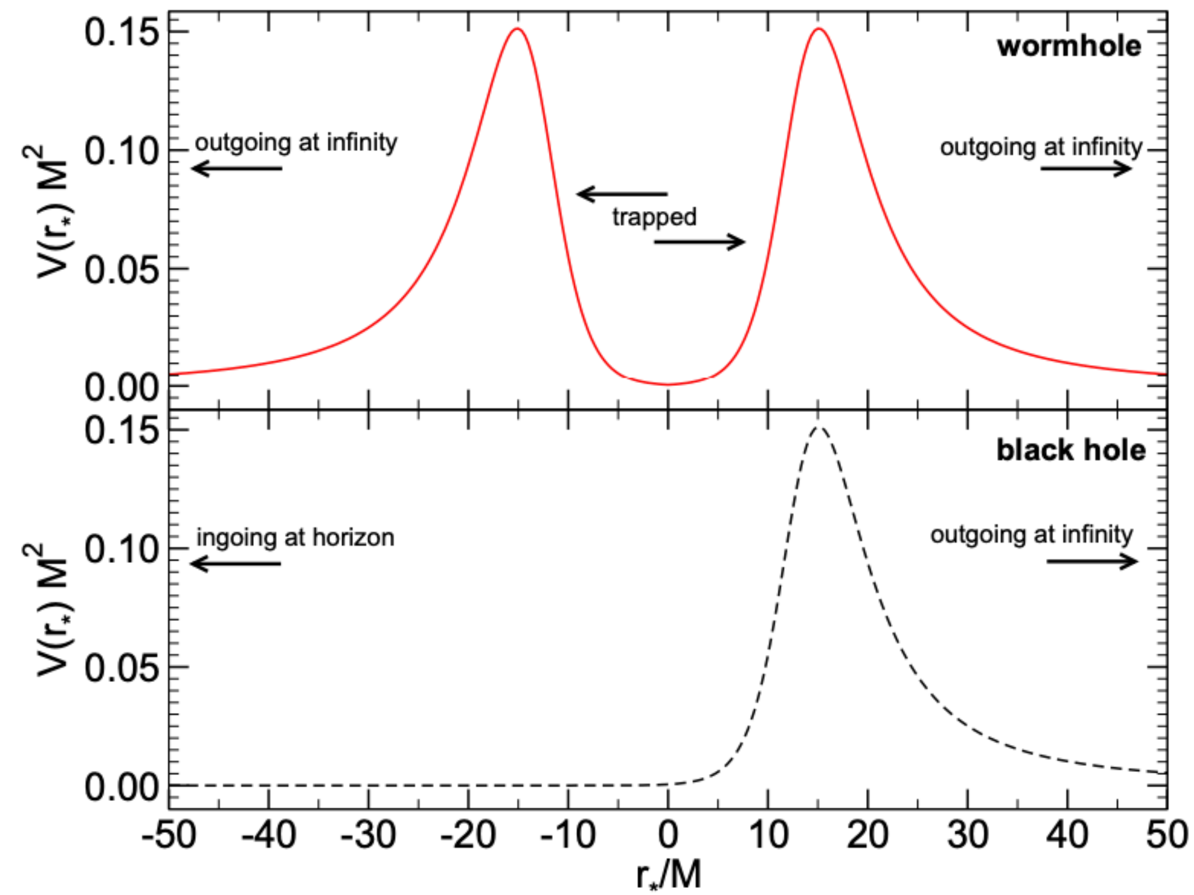
QNMs

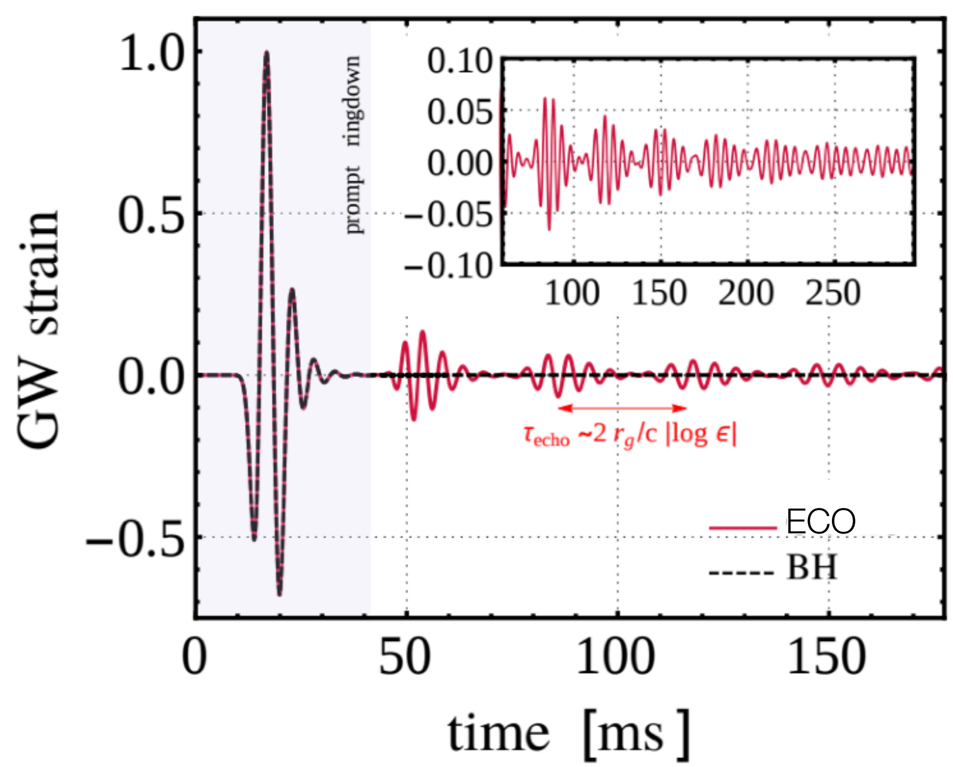


GW waveform



Effective potential

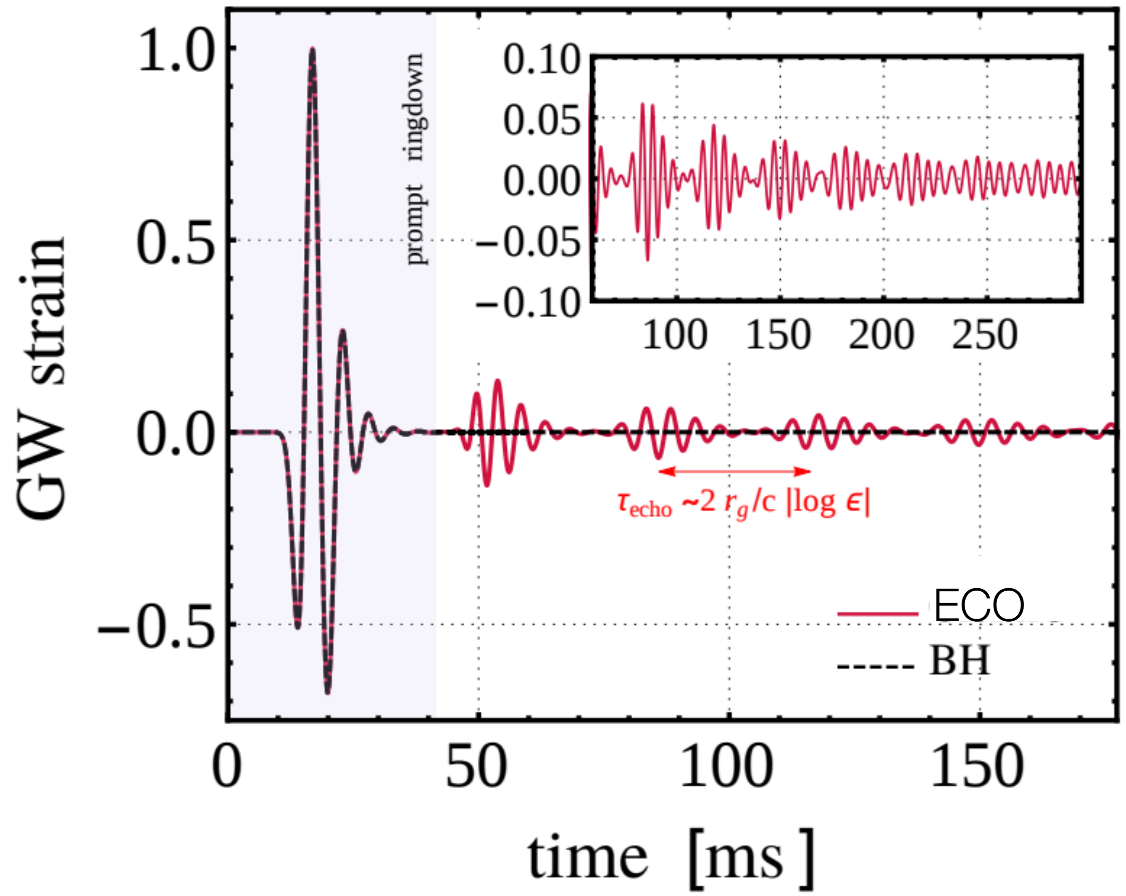




For an ECO

1. Ringdown is same as BH
2. There are extra echoes

Smoking gun for new physics: echoes



New physics radius

$$r_0 = r_g(1+\epsilon)$$

Echo interval

$$\tau_{\rm echo} \sim \frac{|r_*(1.5r_g) - r_*(r_0)|}{c} \sim \frac{2r_g}{c}|\log(\epsilon)|$$

What kind of new physics?

black hole mimickers: fuzzball, gravastar, boson stars, …

quantum gravity effects: firewall

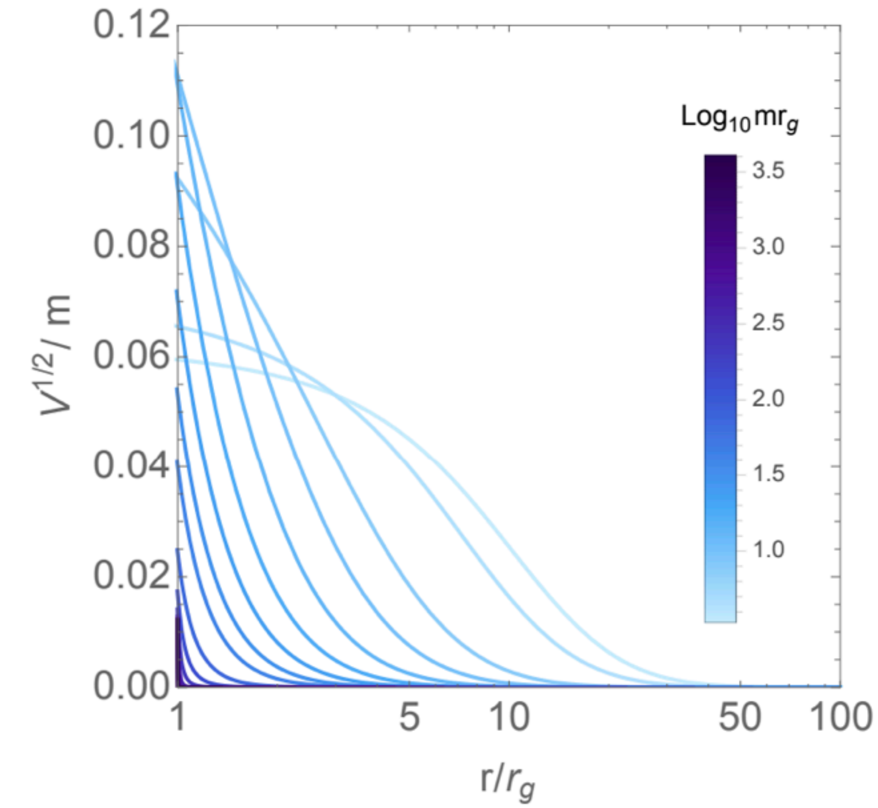
Planck length $\epsilon \sim 10^{-40}$ ➡ $\tau_{\rm echo}(60M_\odot) \sim 50\,{\rm ms}$

Echoes from MVMG black hole

Zhang & **SYZ**, 1709.07503

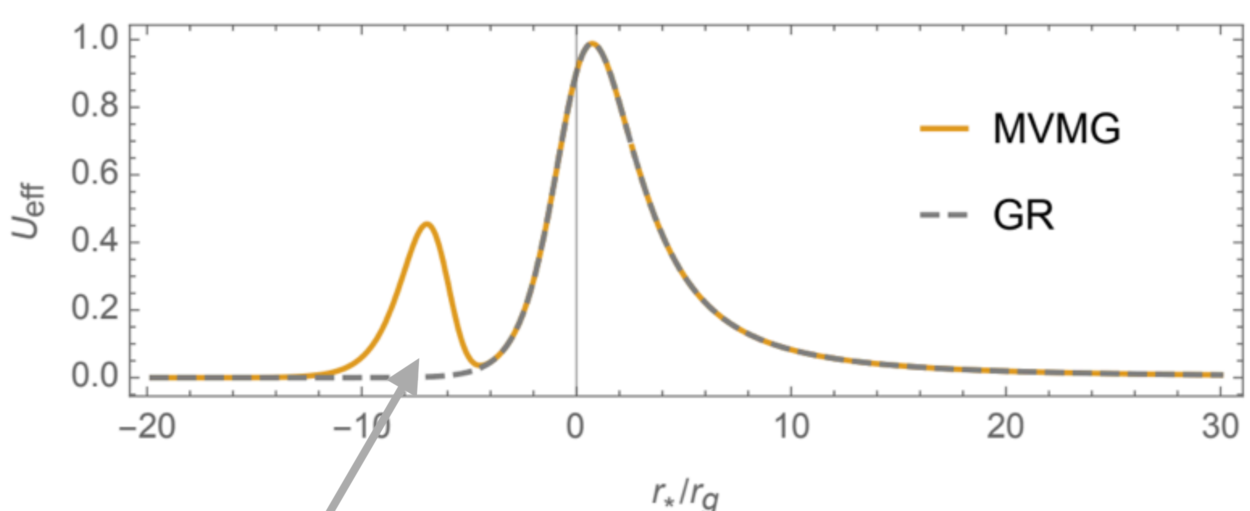
Mass-varying massive gravity

$$S = M_P^2 \int d^4x \sqrt{-g} \left[\frac{R}{2} + V(\sigma)\mathcal{U} - \frac{1}{2}(\partial\sigma)^2 - W(\sigma) \right]$$

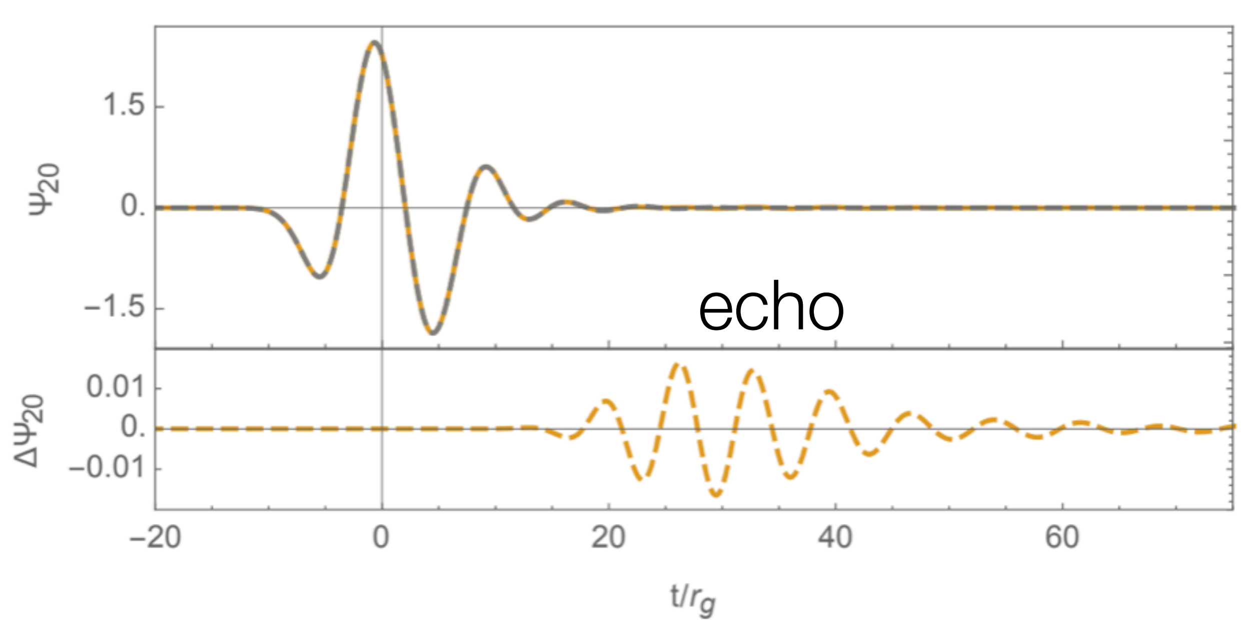


Hairy black hole

graviton mass becomes very big close to the horizon



graviton mass barrier



How to characterize GW echoes?

Mark, Zimmerman, Du & Chen, 1709.07503

Parametrize new physics with a reflective boundary

$$\psi \propto e^{-i\omega(x-x_0)} + \tilde{\mathscr{R}}(\omega) e^{i\omega(x-x_0)}, \quad x \to x_0$$

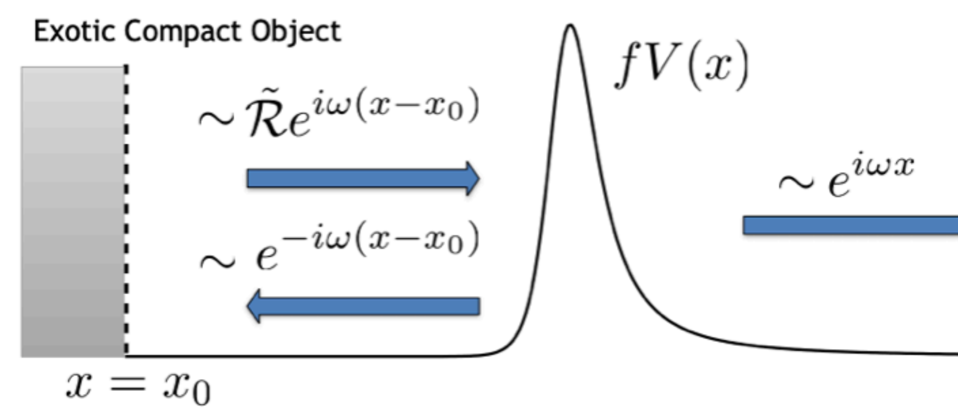
$$\psi(x) = \int_{-\infty}^{\infty} dx' \tilde{g}_{\rm ref}(x,x') \tilde{S}(x')$$

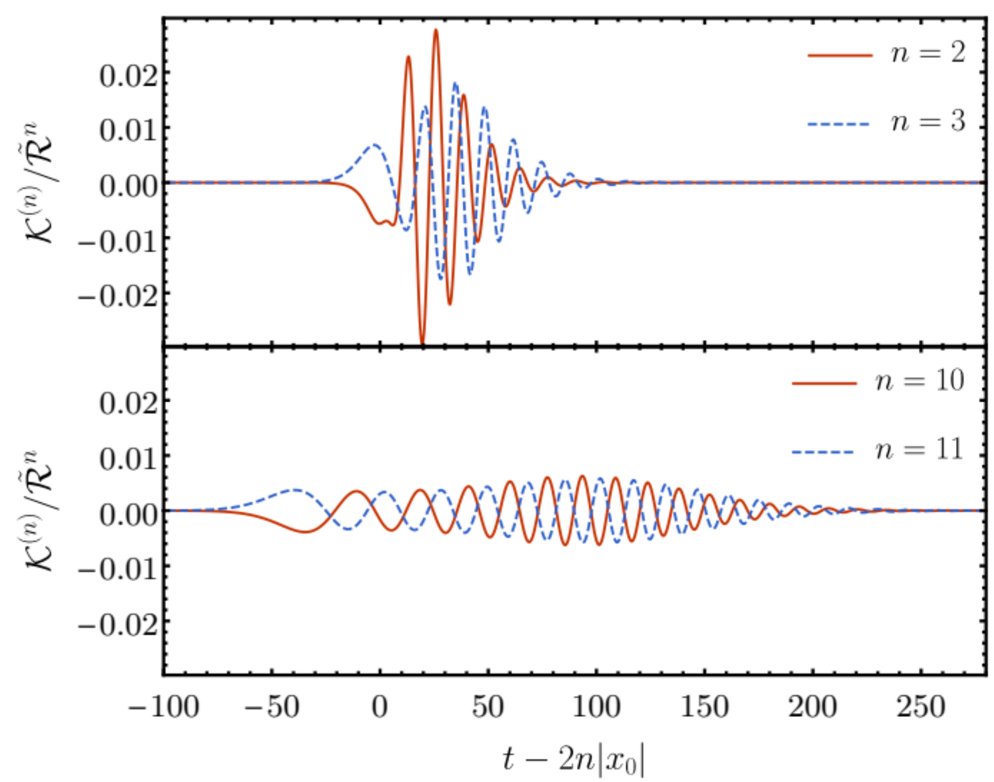
$$\tilde{g}_{\rm ref}(x,x') = \tilde{g}_{\rm BH}(x,x') + \tilde{\mathcal{K}} \frac{\tilde{\psi}_{\rm up}(x)\tilde{\psi}_{\rm up}(x')}{W_{\rm BH}}$$

GR part echo part

$$\tilde{\mathcal{K}} = \tilde{\mathcal{T}}_{\rm BH} \tilde{\mathcal{R}} e^{-2i\omega x_0} \sum_{n=1}^{\infty} (\tilde{\mathcal{R}}_{\rm BH}\tilde{\mathcal{R}})^{(n-1)} e^{-2i(n-1)\omega x_0}$$

n-th term corresponds to n-th echo





identity individual echoes

The Fredholm approach to echoes

Huang, Xu & **SYZ**, 1908.00189

Regge-Wheeler-Zerilli equation

$$\phi''(\omega, x) + (\omega^2 - V(x))\phi(\omega, x) = \mathcal{I}(\omega, x)$$

Fredholm integral equation

$$\boldsymbol{\phi(x) = f(x) + \lambda \int \mathrm{d}y K(x, y)\phi(y)}$$

$$K(x, y) = G(x, y)V(y) \qquad \left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \omega^2\right) G(x, y) = \delta(x - y) \qquad f(x) = \int \mathrm{d}y G(x, y)\mathcal{I}(y)$$

Separate kernel function

$$K(x, y) = \sum_{j=1}^{\infty} \alpha_j(x)\beta_j(y)$$

Solution

$$\phi(x) = f(x) - \lambda \int \mathrm{d}y f(y) \frac{\Delta_K(x,y,\lambda)}{\Delta(\lambda)}$$

$$\boldsymbol{\Delta(\lambda) = \det(c)}$$

$$\boldsymbol{c_{ij} = \delta_{ij} - \lambda \int \mathrm{d}x \beta_i(x) \alpha_j(x)}$$

$$\Delta_K(x,y,\lambda) = \begin{vmatrix} 0 & \alpha_1(x) & \alpha_2(x) & \cdots \\ \beta_1(y) & c_{11} & c_{12} & \cdots \\ \beta_2(y) & c_{21} & c_{22} & \cdots \\ & \cdots & & \vdots \end{vmatrix}$$

Example: BH boundary condition with potential $V(x)$

$$G(x,y) = \frac{e^{i\omega|x-y|}}{2i\omega}$$

Observer at large x

$$K(x,y) = \alpha(x)\beta(y) \qquad \alpha(x) = \frac{e^{i\omega x}}{2i\omega}, \quad \beta(y) = e^{-i\omega y} V(y).$$

$$\phi(x) = f(x) + \int \mathrm{d}x_1 \frac{e^{i\omega(x-x_1)}}{2i\omega} \frac{V(x_1) f(x_1)}{1 - \frac{1}{2i\omega} \int \mathrm{d}y V(y)}$$

Perturbation scheme

If separation is difficult, expand in $K(x, y)$

$$\phi(x) = f(x) + \lambda \int \mathrm{d}y f(y) \frac{\sum_{n=0}^{\infty} \frac{(-\lambda)^n}{n!} A_n(x, y)}{\sum_{n=0}^{\infty} \frac{(-\lambda)^n}{n!} D_n}$$

$$A_n(x, y) = \int \prod_{i=1}^{n} \mathrm{d}x_i \begin{vmatrix} K(x, y) & K(x, x_1) & K(x, x_2) & \cdots & K(x, x_n) \\ K(x_1, y) & K(x_1, x_1) & K(x_1, x_2) & \cdots & K(x_1, x_n) \\ & & \cdots & & \vdots \\ K(x_n, y) & K(x_n, x_1) & K(x_n, x_2) & \cdots & K(x_n, x_n) \end{vmatrix}$$

$$D_n = \int \prod_{i=1}^{n} \mathrm{d}x_i \begin{vmatrix} K(x_1, x_1) & K(x_1, x_2) & K(x_1, x_3) & \cdots & K(x_1, x_n) \\ K(x_2, x_1) & K(x_2, x_2) & K(x_2, x_3) & \cdots & K(x_2, x_n) \\ & & \cdots & & \vdots \\ K(x_n, x_1) & K(x_n, x_2) & K(x_n, x_3) & \cdots & K(x_n, x_n) \end{vmatrix}$$

Convergence and error estimate

Huang, Xu & **SYZ**, 1908.00189

- Approximate solution

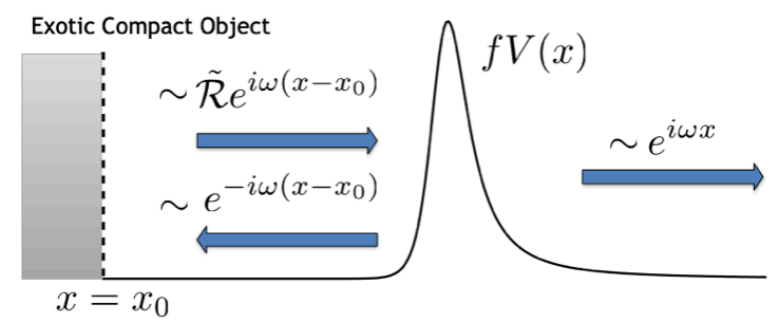
$$\phi_{\text{app}}(x) = f(x) + \int_a^b \frac{\sum_{n=0}^{N} \frac{(-\lambda)^n}{n!} A_n(x,y)}{\sum_{n=0}^{N} \frac{(-\lambda)^n}{n!} D_n} f(y) \mathrm{d}y$$

  - absolute convergent if $V(x)$ is normalizable
  - For Schwarzschild potential
    - estimated errors from truncating N
    - estimated errors from neglecting $(-L, a) \cup (b, +\infty)$

Echoes in Fredholm formalism

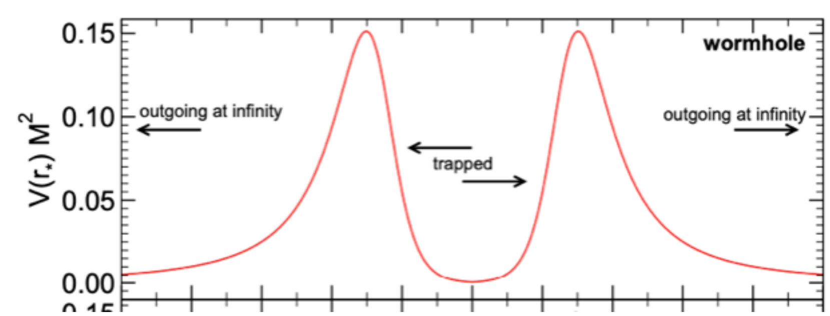
1. Reflective "mirror"

$$G(x,y) = \frac{e^{i\omega|x-y|}}{2i\omega} + R(\omega)\frac{e^{i\omega(x+y)}}{2i\omega}$$



2. Extra potential barrier

$$V(x) = \bar{V}(x) + R(\omega)V_{\rm Q}(x)$$



For either case

$$K(x,y) = \bar{K}(x,y) + R(\omega)Q(x,y)$$

GR kernel echo kernel

$$\phi(x) = f(x) + \lambda \int \mathrm{d}y f(y) \frac{\sum_{n=0}^{\infty} \frac{(-\lambda)^n}{n!} A_n(x,y)}{\sum_{n=0}^{\infty} \frac{(-\lambda)^n}{n!} D_n}$$

$$D_n = \sum_{j=0}^{n} R^j D_n^j, \quad A_n(x,y) = \sum_{j=0}^{n+1} R^j A_n^j(x,y)$$

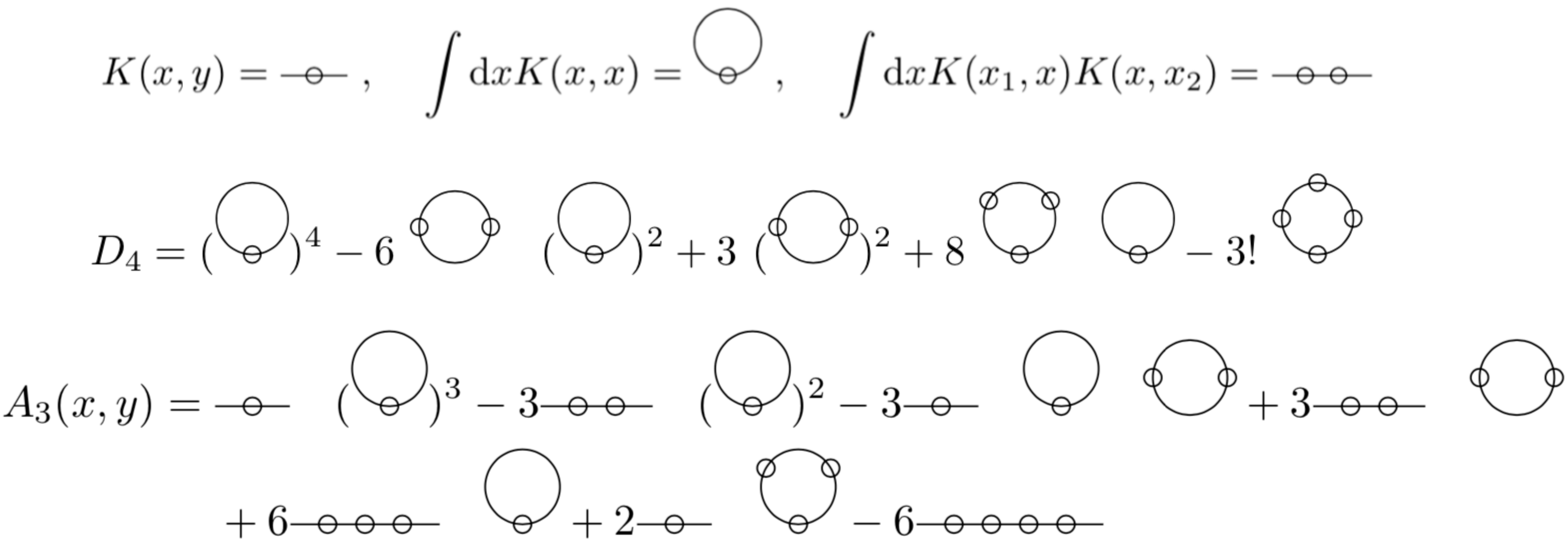
$$D_n^j = \sum_{1\le i_1<\cdots<i_j\le n} \int \prod_{i=1}^{n} \mathrm{d}x_i \begin{vmatrix} \bar{K}(x_1,x_1) & \cdots & Q(x_1,x_{i_1}) & \cdots & Q(x_1,x_{i_j}) & \cdots & \bar{K}(x_1,x_n) \\ & \cdots & & \cdots & & \vdots & \\ \bar{K}(x_n,x_1) & \cdots & Q(x_n,x_{i_1}) & \cdots & Q(x_n,x_{i_j}) & \cdots & \bar{K}(x_n,x_n) \end{vmatrix},$$

$$A_n^j(x,y) = \sum_{1\le i_1<\cdots<i_j\le n} \int \prod_{i=1}^{n} \mathrm{d}x_i \begin{vmatrix} \bar{K}(x,y) & \bar{K}(x,x_1) & \cdots & Q(x,x_{i_1}) & \cdots & Q(x,x_{i_j}) & \cdots & \bar{K}(x,x_n) \\ \bar{K}(x_1,y) & \bar{K}(x_1,x_1) & \cdots & Q(x_1,x_{i_1}) & \cdots & Q(x_1,x_{i_j}) & \cdots & \bar{K}(x_1,x_n) \\ & & \cdots & & \cdots & & \vdots & \\ \bar{K}(x_n,y) & \bar{K}(x_n,x_1) & \cdots & Q(x_n,x_{i_1}) & \cdots & Q(x_n,x_{i_j}) & \cdots & \bar{K}(x_n,x_n) \end{vmatrix}$$

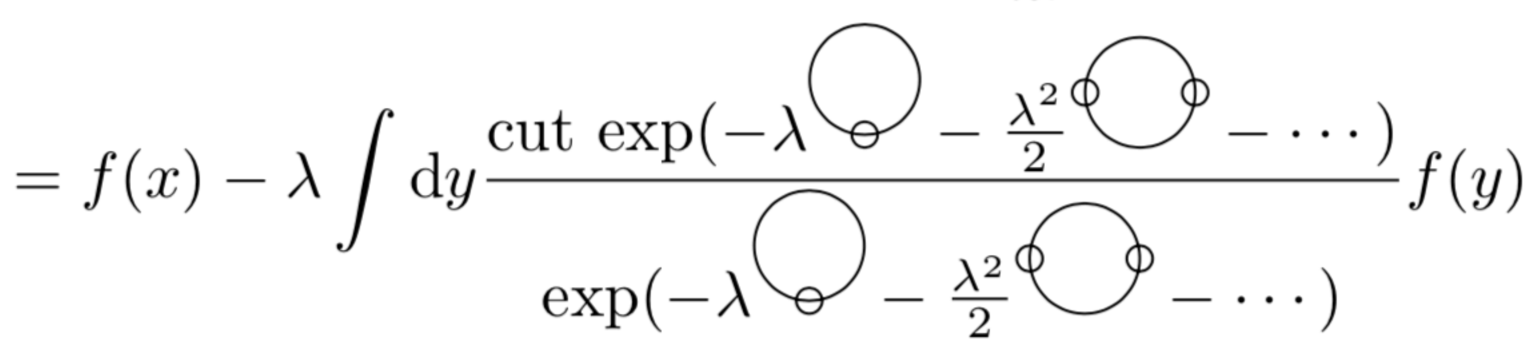
Huang, Xu & **SYZ**, 1908.00189

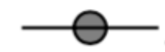
Diagrammatica for perturbations

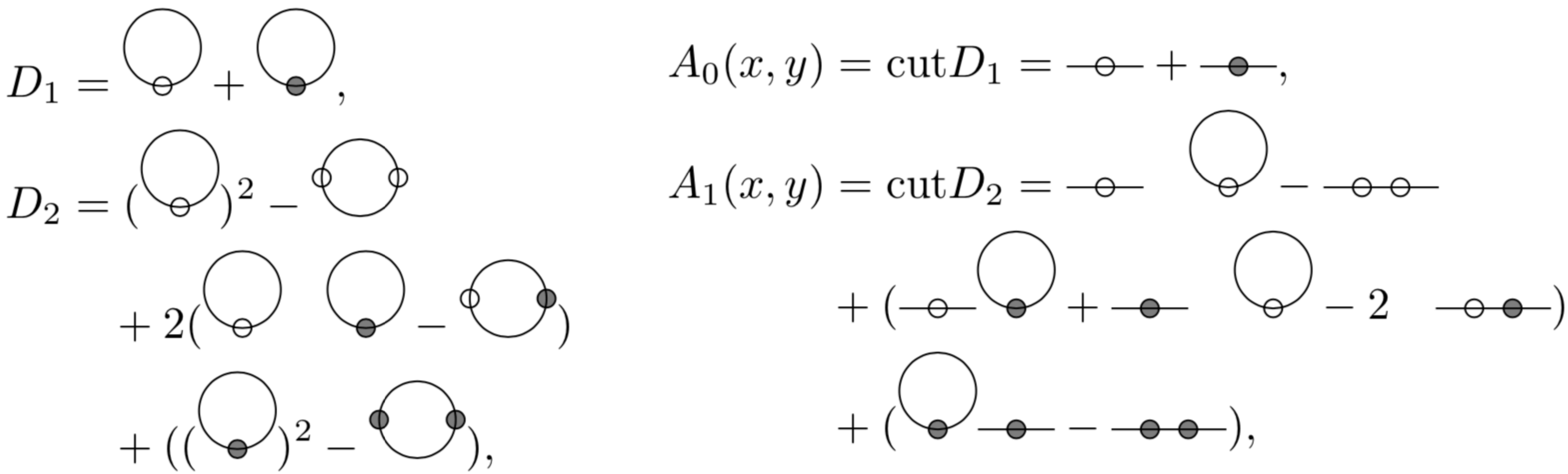
Huang, Xu & **SYZ**, 1908.00189



$$\phi(x) = f(x) + \lambda \int \mathrm{d}y f(y) \frac{\sum_{n=0}^{\infty} \frac{(-\lambda)^n}{n!} A_n(x, y)}{\sum_{n=0}^{\infty} \frac{(-\lambda)^n}{n!} D_n}$$



For echoes, add $R(\omega)Q(x,y) =$ .



For n-th echo, collect all vacuum and 2-point diagrams with n solid vertices

Diagrammatical rules

- $\bar{K}(x_i, x_j)$ is represented as a (2-point) circle vertex;
- $R(\omega)Q(x_i, x_j)$ is represented as a (2-point) solid vertex;
- Diagrams with n solid vertices contribute to the n-th echo waveform;
- $\int \mathrm{d}x$ is represented as a "propagator";
- For each diagram with an odd number of loops, assign a minus sign;
- D_n includes all loop diagrams with n vertices;
- A_n is obtained by cutting open one of the loops in D_{n+1}.

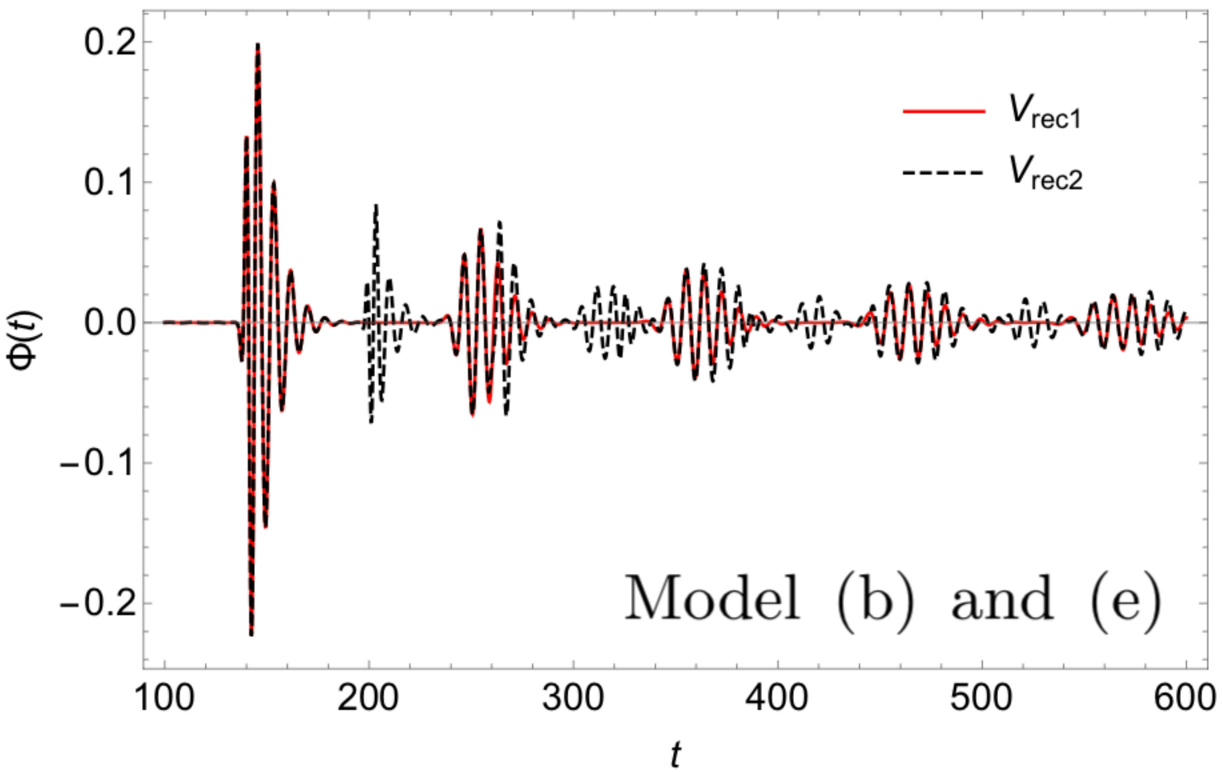
Huang, Xu & **SYZ**, 1908.00189

Numerical results

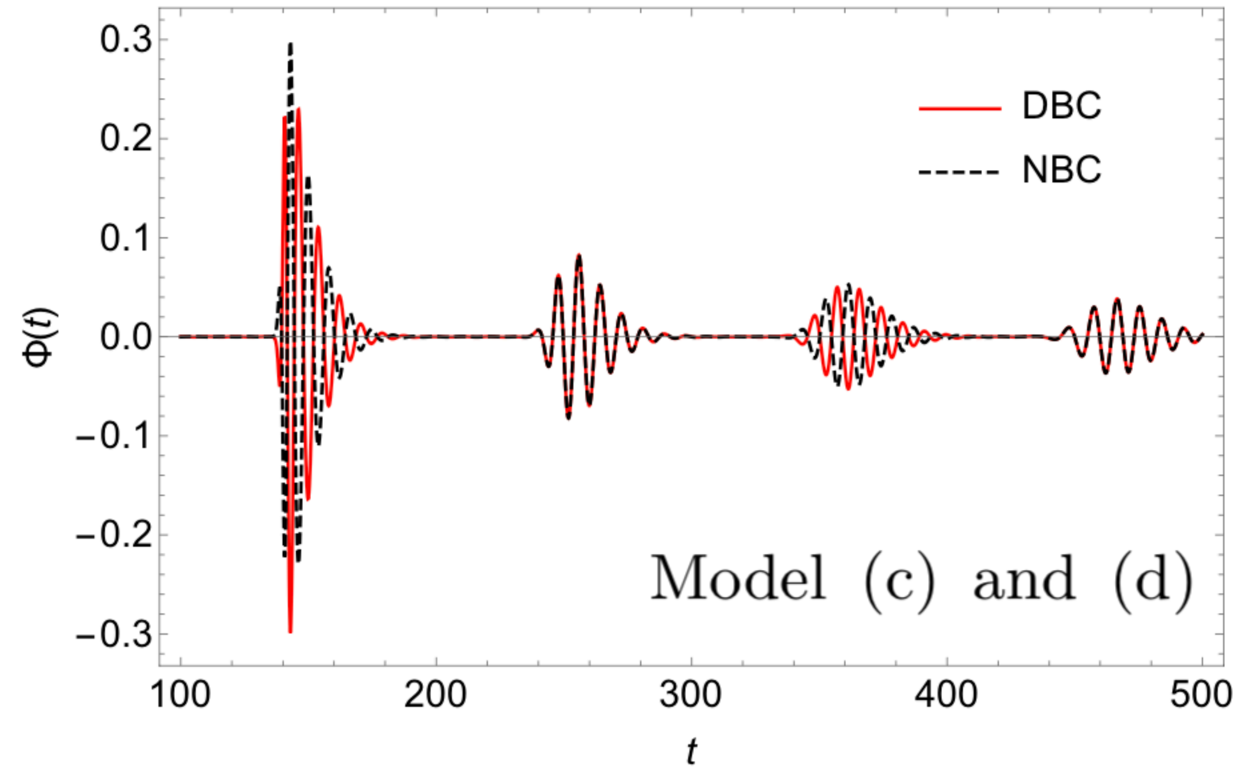
Huang, Xu & **SYZ**, 1908.00189

Model	Potential $V(x)$	Left Boundary
(a)	$V_{gr}(x)$	open
(b)	$V_{gr}(x) + V_{rec1}(x)$	open
(c)	$V_{gr}(x)$	Dirichlet $x = -20$
(d)	$V_{gr}(x)$	Neumann $x = -20$
(e)	$V_{gr}(x) + V_{rec2}(x)$	open

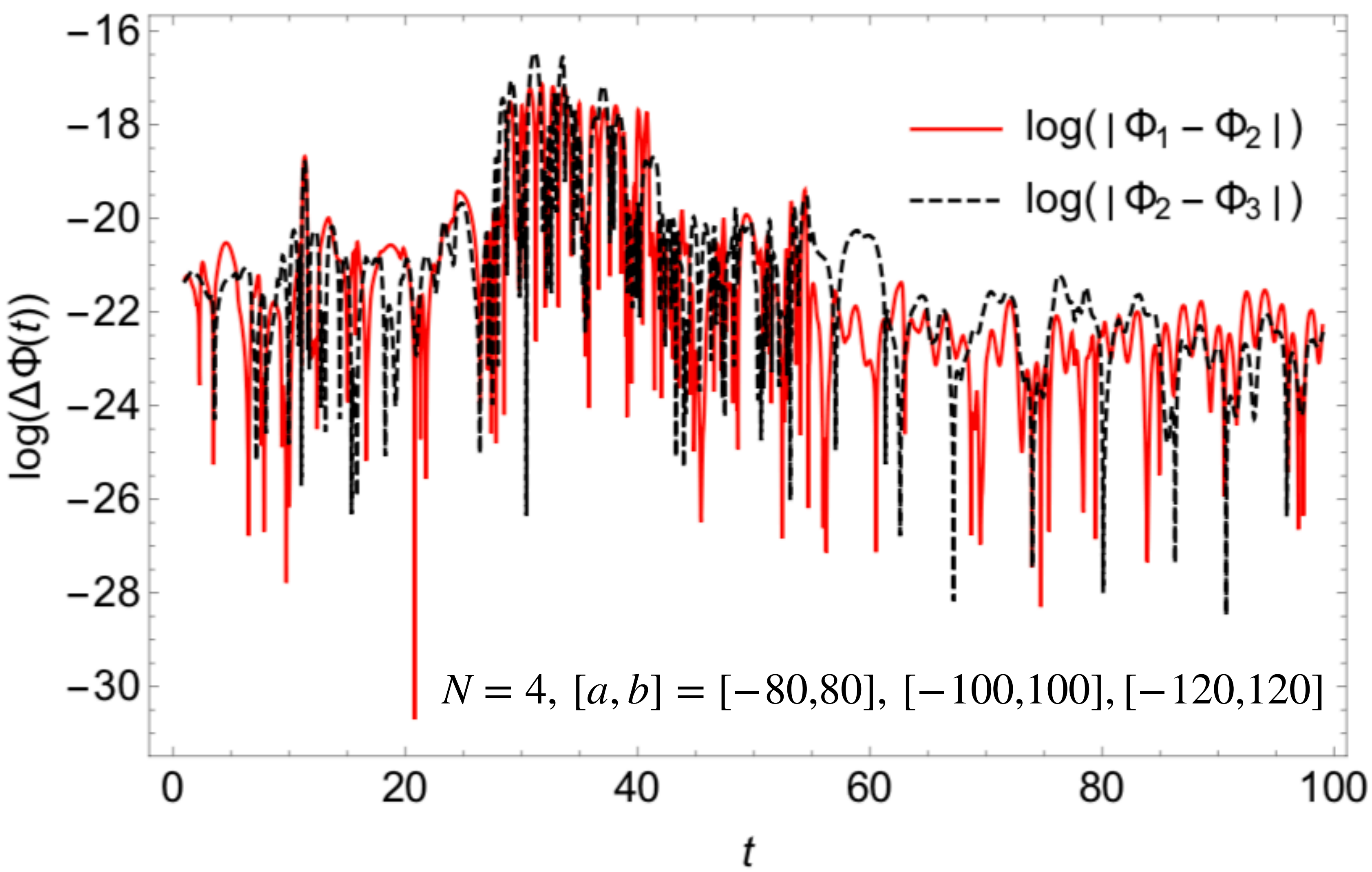
extra barrier



reflective boundary



Numerical accuracy



Echoes from rotating (Kerr-like) ECOs

Teukolsky master equation

Huang, Xu & **SYZ**, 1908.00189

$$\Delta^{-s}\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta^{s+1}\frac{\mathrm{d}R(r)}{\mathrm{d}r}\right)+U(\omega,r)R(r)=\mathcal{I}_1(\omega,r)$$

$$\phi=\Delta^{\frac{s}{2}}(r^2+a^2)^{\frac{1}{2}}R,\qquad \frac{\mathrm{d}}{\mathrm{d}x}=\frac{\Delta}{r^2+a^2}\frac{\mathrm{d}}{\mathrm{d}r}$$

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2}+(\omega^2-V(\omega,x))\phi=\mathcal{I}(\omega,x)$$

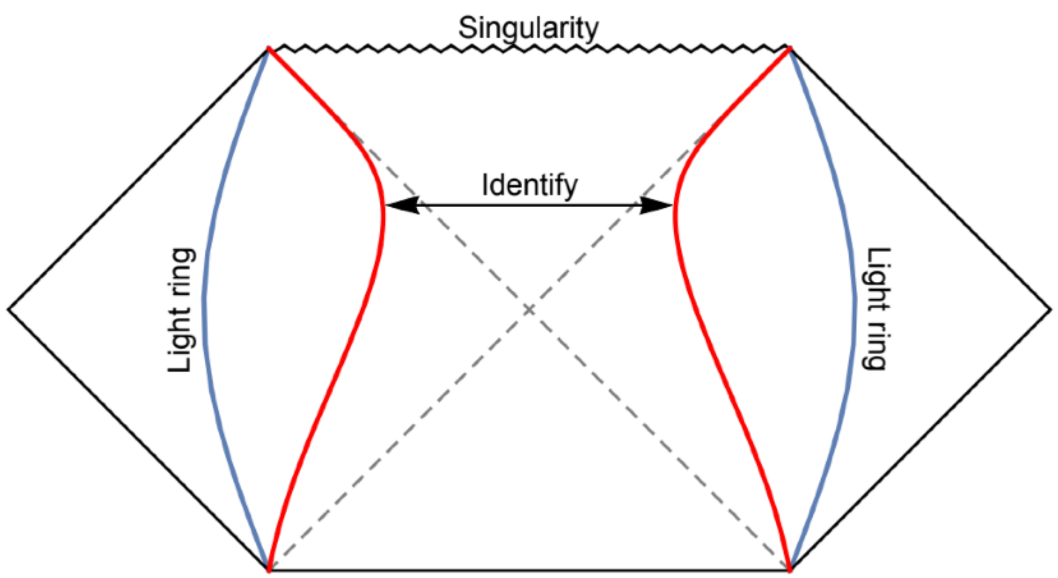
$$V(\omega,x)=\frac{3}{4}\frac{\Delta^2 r}{(r^2+a^2)^5}-\frac{2\Delta}{(r^2+a^2)^3}(\Delta+2r^2)+\frac{a^2m^2}{(r^2+a^2)^2}-\frac{2am\lambda}{(r^2+a^2)}+\frac{(4i\omega r-\lambda)\Delta}{(r^2+a^2)^2}-\frac{s\Delta+s^2(r-M)^2}{(r^2+a^2)^2}$$

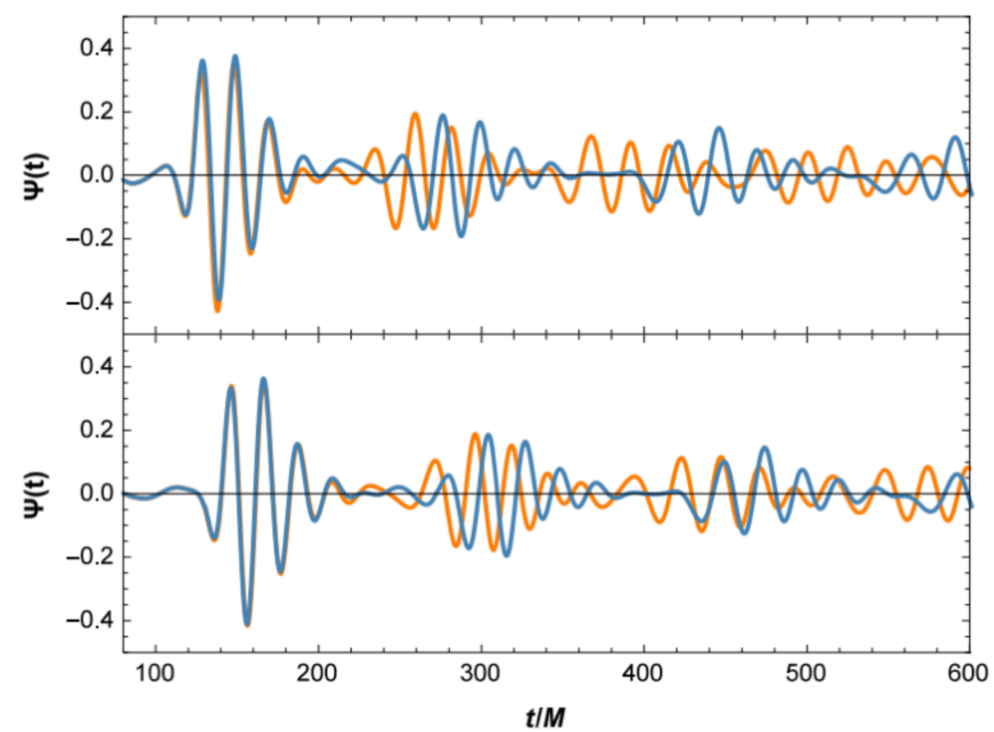
complex, frequency-dependent effective potential
but Fredholm formalism still applies

Unequal interval echoes?

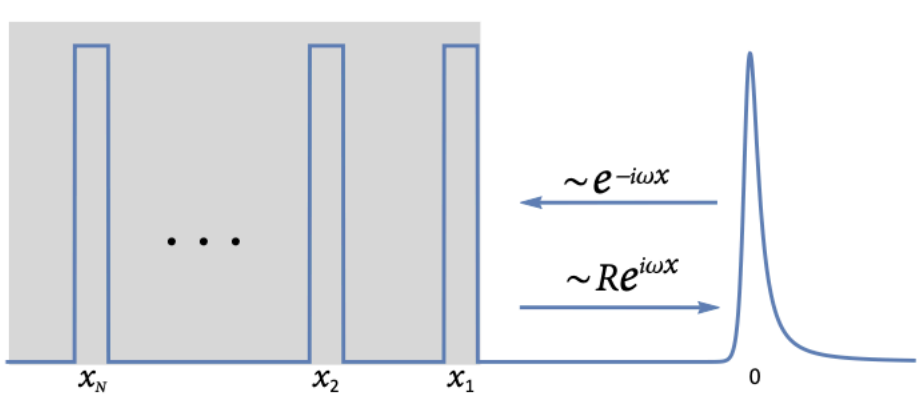
Slowly pinch-off worm hole

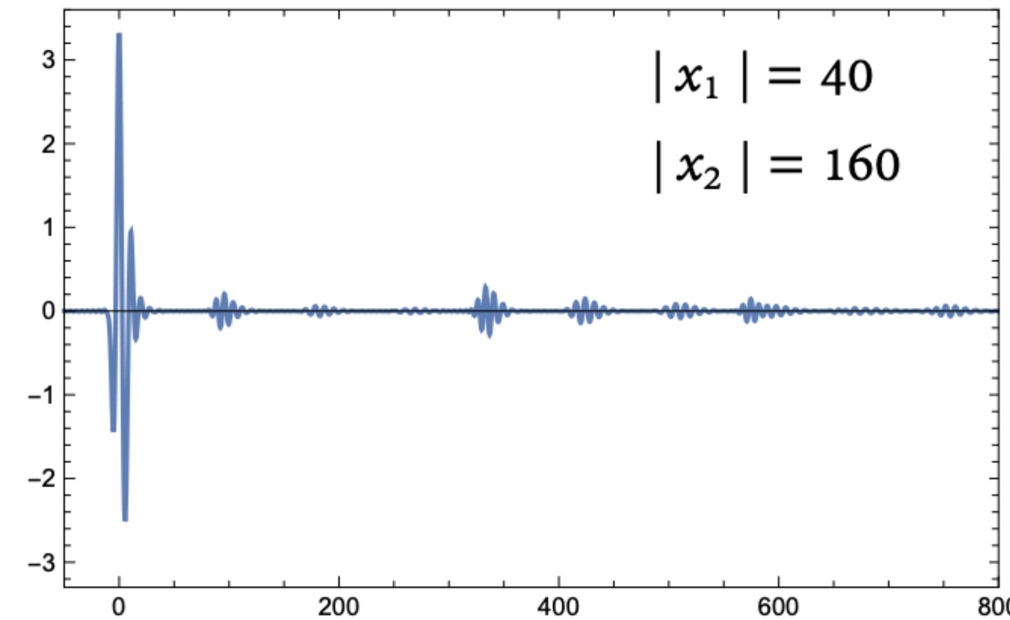
Wang, Zhang, **SYZ** & Piao, 1802.02003





Multiple extra barriers





Li & Piao, 1904.05652

Model selection: equal *vs* unequal

Wang, Zhang, **SYZ** & Piao, 1904.00212

Equal interval echoes

$$x_n = t - t_{echo} - n\Delta t_{echo}$$

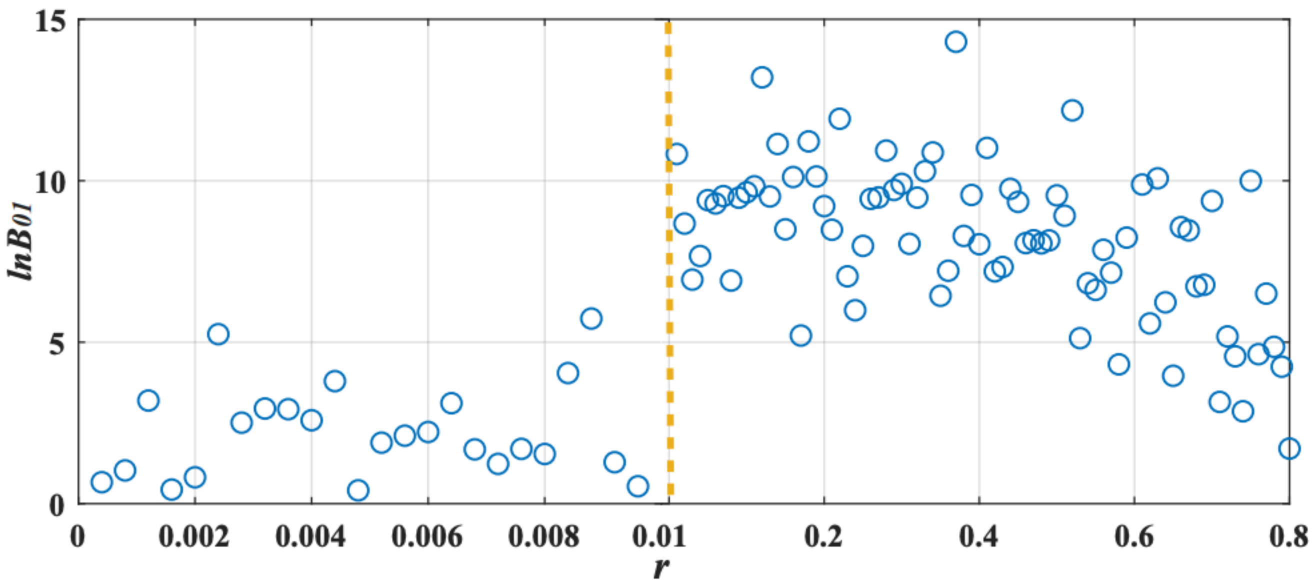
Unequal interval echoes

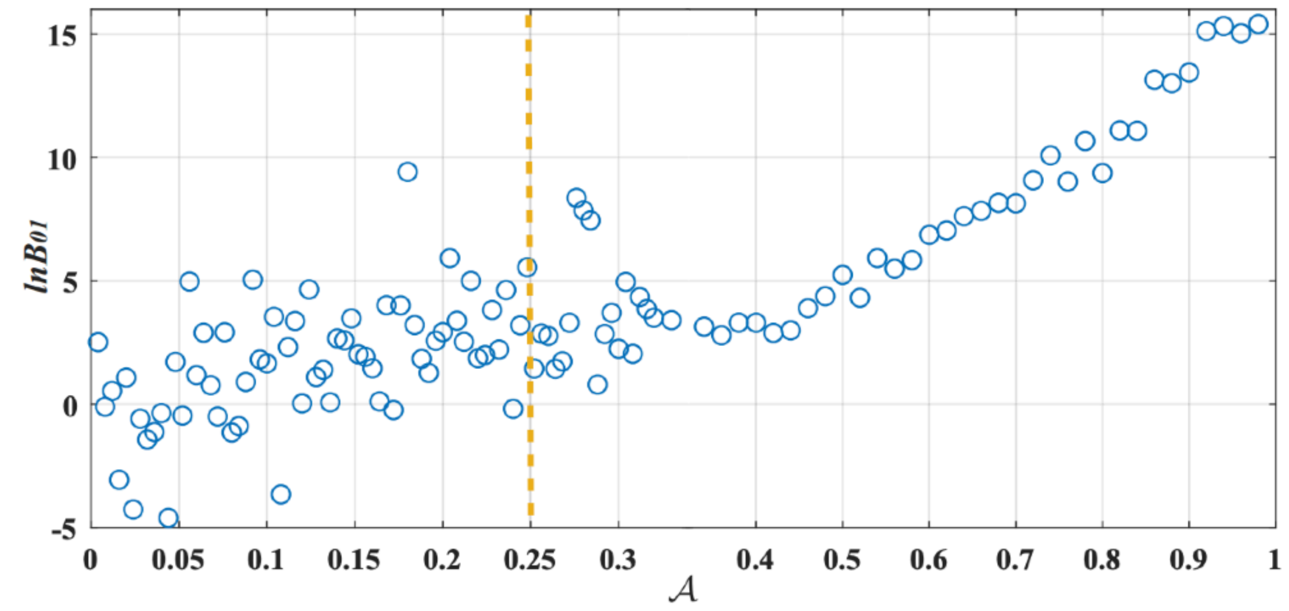
$$x_n = t - t_{echo} - n\Delta t_{echo} - \frac{n(n+1)}{2} r\Delta t_{echo}$$

use unequal echo templates if variation of echo intervals
is greater than statistical errors of intervals

Bayes factor

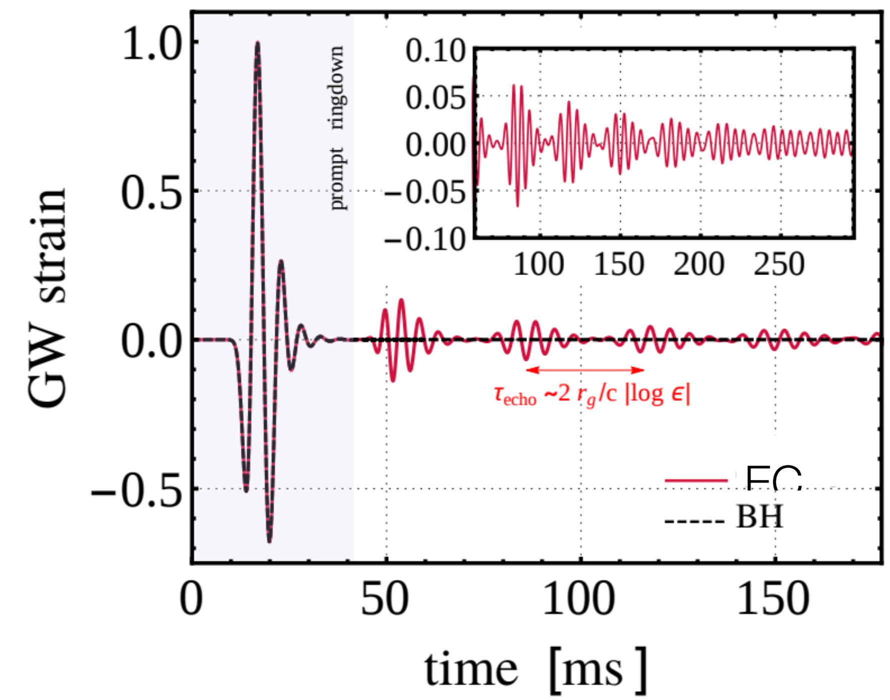
$$\ln B_{01} = \ln p(d|\mathcal{H}_1, I) - \ln p(d|\mathcal{H}_0, I)$$





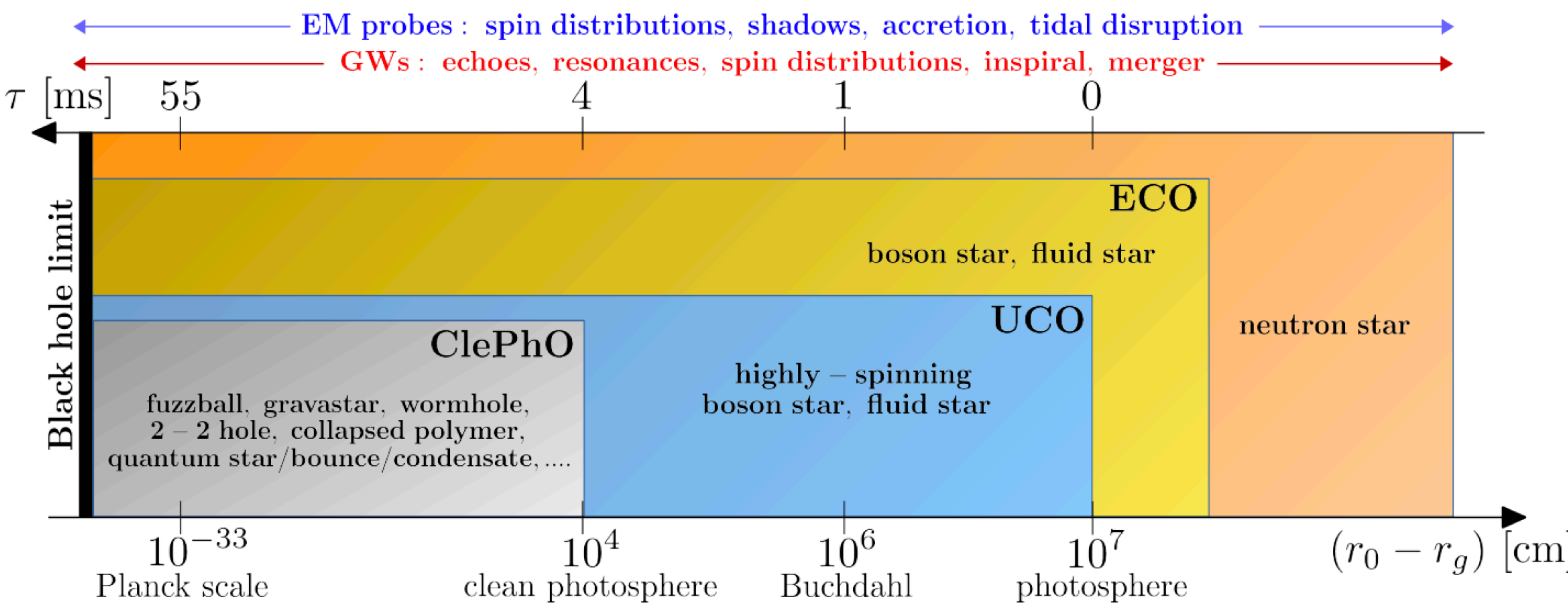
Summary

- We have not observed BH horizons.
- GW echoes encode new physics.
- Massive graviton → Echoes
- Possibility of unequal echo intervals



- We use the Fredholm approach to re-process GW echoes.
    - This approach can be presented diagrammatically.
    - Numerically, the Fredholm formalism is very accurate.

Thank you


EM probes : spin distributions, shadows, accretion, tidal disruption
GWs : echoes, resonances, spin distributions, inspiral, merger
τ [ms]
55
4
1
0
Black hole limit
ECO
boson star, fluid star
UCO
highly – spinning
boson star, fluid star
ClePhO
fuzzball, gravastar, wormhole,
2 – 2 hole, collapsed polymer,
quantum star/bounce/condensate, ...
neutron star
10−33
Planck scale
104
clean photosphere
106
Buchdahl
107
photosphere
(r0 − rg) [cm]