Secondary gravitational waves

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Outline

- Primordial perturbations
- Second order tensor perturbations
- PBH DM constraints on primordial curvature perturbation with piecewise power law parametrization
- Upper bound on secondary Gravitational Waves (GWs)
- Conclusions

Lu, Gong, Yi, Zhang, 1907.11896

Scalar Perturbation

- ADM decomposition

$$ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

- Uniform field gauge $\delta\phi = 0$ Inflationary era

$$N = 1 + N_1, \quad N^i = \psi_{,i} + N_T^i, \quad \gamma_{ij} = a^2(1+2\zeta)\delta_{ij},$$

$$\gamma^{ij} = a^{-2}(1-2\zeta)\delta_{ij}, \quad N_i = a^2(\psi_{,i} + N_T^i), \quad \partial_i N_T^i = 0$$

3 Independent DOFs $N_1,\ \psi,\ \zeta$

- Constraints and solutions

Hamiltonian and momentum constraints

$$N_1 = \dot{\zeta}/H$$

$$\psi = -\frac{\zeta}{a^2 H} + \chi, \quad \nabla^2\chi = \frac{\dot{\phi}^2}{2H^2}\dot{\zeta}$$

Primordial curvature perturbation

- Curvature perturbations

$$\delta_2 S = \frac{1}{2}\int dt d^3x \frac{\dot{\phi}^2}{H^2}\left[a^3\dot{\zeta}^2 - a(\zeta_{,i})^2\right]$$
$$= \frac{1}{2}\int d\tau d^3x \frac{a^2\phi'^2}{\mathscr{H}^2}\left[\zeta'^2 - (\zeta_{,i})^2\right]$$
$$= \frac{1}{2}\int d\tau d^3x \left[v'^2 - (v_{,i})^2 + \frac{z''}{z}v^2\right],$$

$$v = a\phi'\zeta/\mathscr{H},\ \phi' = d\phi/d\tau,\ \mathscr{H} = d\ln a/d\tau$$

- Mode equation $$v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0$$

Primordial Tensor Perturbation

- Primordial GWs

$$ds^2 = a^2[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j],$$

$$h_{ii} = 0,\ \partial_i h_{ij} = 0$$ Transverse traceless gauge

$$\delta_2 S = \frac{1}{64\pi G}\int d\tau d^3x[(h'_{ij})^2 - (\partial_l h_{ij})^2]a^2$$

- Momentum space $h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 0$

$$h_{ij}(\boldsymbol{x},\eta) = \frac{1}{(2\pi)^{3/2}}\int d^3k e^{i\boldsymbol{k}\cdot\boldsymbol{x}}[h_{\boldsymbol{k}}(\eta)e_{ij}(\boldsymbol{k}) + \tilde{h}_{\boldsymbol{k}}(\eta)\tilde{e}_{ij}(\boldsymbol{k})]$$

- Polarization tensor

$+$ polarization $e_{ij}(\boldsymbol{k}) = \frac{1}{\sqrt{2}}[e_i(\boldsymbol{k})e_j(\boldsymbol{k}) - \tilde{e}_i(\boldsymbol{k})\tilde{e}_j(\boldsymbol{k})],$

$\times$ polarization $\tilde{e}_{ij}(\boldsymbol{k}) = \frac{1}{\sqrt{2}}[e_i(\boldsymbol{k})\tilde{e}_j(\boldsymbol{k}) + \tilde{e}_i(\boldsymbol{k})e_j(\boldsymbol{k})],$

Orthonormal basis $\boldsymbol{e}\cdot\tilde{\boldsymbol{e}} = \boldsymbol{e}\cdot\boldsymbol{k} = \tilde{\boldsymbol{e}}\cdot\boldsymbol{k} = 0$

Tensor perturbations

- Mode equation

$$\frac{d^2u_k^s}{d\tau^2} + \left(k^2 - \frac{a''}{a}\right)u_k^s = 0$$

$$u_k^s(\tau) = \frac{a}{\sqrt{16\pi G}} h_k^s(\tau) \qquad s = +,\ \times$$

- Properties
  - Scalar and tensor perturbations are independent
  - Quantum fluctuations
  - Evaluated at horizon crossing (in general)

  Most cases, perturbations are frozen on super-horizon scales

Matter Perturbations

- Possible issues
  - Growth on super-horizon scales (inflation)

    Evaluated at the end of inflation?

  - Initial value at RD and MD eras

    1) Reheating physics is uncertain (unknown)
    2) The start time of RD is uncertain
    3) Evolutions of perturbations on super-horizon scales
    4) Fluctuations from quantum to classical transition
    5) Horizon reentry?

- Horizon reentry (=horizon crossing)

Late time perturbations originated from seeds during infaltion

The induced GWs and PBHs

- The generation of secondary GWs from large density perturbation at small scales

  Scalar tensor mix

- The formalism for the induced GWs
  - Numerical result
  - Semi-analytical result

Ananda, Clarkson, Wands, PRD 75 (07) 123518
Baumann, Steinhardt, Takahashi, PRD 76 (07) 084019
Nakama, Suyama, PRD 94 (16) 043507
Inomata, Kawasaki, Mukaida, Tada, Yanagida, PRD 95 (17) 123510
Kohri, Terada, PRD 97 (18) 123532
Espinosa, Racco, Riotto, JCAP 1809, 012
Lu, Gong, Yi, Zhang, 1907.11896

Secondary GWs

- Tensor-scalar mixing (to 2nd order)

$$ds^2 = a^2[-(1+2\Phi^{(1)})d\eta^2 + [(1-2\Psi^{(1)})\delta_{ij} + h_{ij}^{(2)}]dx^i dx^j]$$

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4S_{ij}$$

$$S_{ij} = 2\Phi\Phi_{,ij} + 2\Psi\Psi_{,ij} + \Phi_{,i}\Phi_{,j} + 3\Psi_{,i}\Psi_{,j} - \Psi_{,i}\Phi_{,j} - \Phi_{,i}\Psi_{,j}$$
$$- \frac{4}{3(1+w)\mathcal{H}^2}(\Psi' + \mathcal{H}\Phi)_{,i}(\Psi' + \mathcal{H}\Phi)_{,j}$$

$$\mathcal{H} = a'/a$$

$$h_{ij} = h_{ij}^{(2)}, \quad \Phi = \Phi^{(1)}, \quad \Psi = \Psi^{(1)}$$

Take $\Phi = \Psi$

Bardeen potential

- Bardeen equation

$$w = c_s^2$$

$$\Phi'' + \frac{6(1+w)}{(1+3w)\eta}\Phi' + wk^2\Phi = 0$$

- Radiation Dominated (inside horizon)

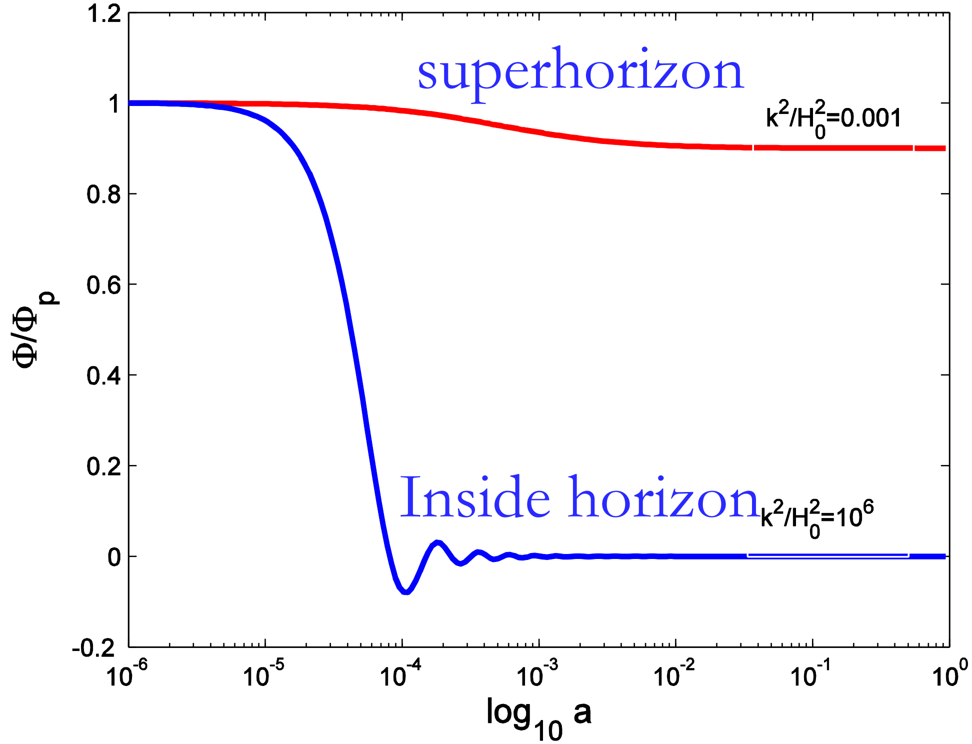
$$\Phi = 3\Phi_p \left[\frac{\sin(k\eta/\sqrt{3}) - (k\eta/\sqrt{3})\cos(k\eta/\sqrt{3})}{(k\eta/\sqrt{3})^3} \right]$$

- Matter Dominated (inside horizon)

$$\Phi = A + \frac{B}{(k\eta)^5}$$

The property of Bardeen Potential

- The evolution of Bardeen potential



Constant on super-horizon scales

It decays for sub-horizon scales during RD, but keeps to be a constant during MD

Radiation domination

- Evolutions

The Fourier component of the Bardeen potential $\Phi_{\boldsymbol{k}}$ is related with the primordial value $\phi_{\boldsymbol{k}}$ by the transfer function $\Psi(k\eta)$

$$\Phi_{\boldsymbol{k}}(\eta) = \phi_{\boldsymbol{k}}\Psi(k\eta)$$

$$\Psi(x) = \frac{9}{x^2}\left(\frac{\sin(x/\sqrt{3})}{x/\sqrt{3}} - \cos(x/\sqrt{3})\right)$$

$$\langle \phi_{\boldsymbol{k}}\phi_{\tilde{\boldsymbol{k}}}\rangle = \delta^{(3)}(\boldsymbol{k}+\tilde{\boldsymbol{k}})\frac{2\pi^2}{k^3}\left(\frac{3+3w}{5+3w}\right)^2\mathcal{P}_\zeta(k),$$

$$\Phi = \frac{3(1+w)}{5+3w}\zeta$$

Secondary GWs

- Induced GWs during RD

$$h_{ij}(\boldsymbol{x},\eta)=\frac{1}{(2\pi)^{3/2}}\int d^3k e^{i\boldsymbol{k}\cdot\boldsymbol{x}}[h_{\boldsymbol{k}}(\eta)e_{ij}(\boldsymbol{k})+\tilde{h}_{\boldsymbol{k}}(\eta)\tilde{e}_{ij}(\boldsymbol{k})]$$

$$\langle h_{\boldsymbol{k}}(\eta)h_{\tilde{\boldsymbol{k}}}(\eta)\rangle=\frac{2\pi^2}{k^3}\delta^{(3)}(\boldsymbol{k}+\tilde{\boldsymbol{k}})\mathcal{P}_h(k,\eta)$$

$$\Omega_{\rm GW}(k,\eta)=\frac{1}{24}\left(\frac{k}{aH}\right)^2\overline{\mathcal{P}_h(k,\eta)}$$

$$h''_{\boldsymbol{k}}+\frac{2}{\eta}h'_{\boldsymbol{k}}+k^2h_{\boldsymbol{k}}=4S_{\boldsymbol{k}}$$

$$S_{\boldsymbol{k}}=\int\frac{d^3\tilde{k}}{(2\pi)^{3/2}}e_{ij}(\boldsymbol{k})\tilde{k}^i\tilde{k}^j\left[2\Phi_{\tilde{\boldsymbol{k}}}\Phi_{\boldsymbol{k}-\tilde{\boldsymbol{k}}}\right.$$
$$\left.+\frac{4}{3(1+w)\mathcal{H}^2}\left(\Phi'_{\tilde{\boldsymbol{k}}}+\mathcal{H}\Phi_{\tilde{\boldsymbol{k}}}\right)\left(\Phi'_{\boldsymbol{k}-\tilde{\boldsymbol{k}}}+\mathcal{H}\Phi_{\boldsymbol{k}-\tilde{\boldsymbol{k}}}\right)\right]$$

Green's function

- Method of Green's function $g = ah$

$$h_{\boldsymbol{k}}(\eta) = \frac{4}{a(\eta)} \int_{\eta_k}^{\eta} d\tilde{\eta} G_{\boldsymbol{k}}(\eta, \tilde{\eta}) a(\tilde{\eta}) S_{\boldsymbol{k}}(\tilde{\eta})$$

$$G''_{\boldsymbol{k}}(\eta, \tilde{\eta}) + \left(k^2 - \frac{a''(\eta)}{a(\eta)} \right) G_{\boldsymbol{k}}(\eta, \tilde{\eta}) = \delta(\eta - \tilde{\eta})$$

Generation of secondary GWs starts well outside the horizon $\eta_k = 0$

Generation of secondary GWs at horizon reentry $\eta_k = 1/k$

- Green's function

$$G_{\boldsymbol{k}}(\eta, \tilde{\eta}) = \frac{\sin[k(\eta - \tilde{\eta})]}{k}, \ \mathrm{RD}$$

$$G_{\boldsymbol{k}}(\eta, \tilde{\eta}) = -k\eta\tilde{\eta} \left[j_1(k\eta) y_1(k\tilde{\eta}) - j_1(k\tilde{\eta}) y_1(k\eta) \right], \ \mathrm{MD}$$

The power spectrum

- The second-order tensor power spectrum

$$\mathcal{P}_h(k,\eta) = 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left[\frac{4v^2-(1-u^2+v^2)^2}{4uv}\right]^2 I_{\rm RD}^2(u,v,x)\mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku)$$

RD

$$u = |\boldsymbol{k}-\tilde{\boldsymbol{k}}|/k, \quad v = \tilde{k}/k, \quad x = k\eta$$

$$I_{\rm RD}(u,v,x) = \frac{1}{9x}(I_s \sin x + I_c \cos x)$$

$$I_c(u,v,x) = -4\int^x y\sin(y) f(u,v,y)dy$$

$$I_s(u,v,x) = 4\int^x y\cos(y) f(u,v,y)dy$$

The lower limit of the integral

$$f(u,v,x) = 2\Psi(vx)\Psi(ux) + [\Psi(vx)+vx\Psi'(vx)]\,[\Psi(ux)+ux\Psi'(ux)]$$

Generation of Perturbations

- Outside vs inside horizon
  - Initial value at RD

1) Reheating physics is uncertain (unknown)
2) The start time of RD is uncertain
3) Evolutions of perturbations on super-horizon scales
4) Fluctuations from quantum to classical transition
5) Horizon reentry?

- Horizon reentry (=horizon crossing)

Late time perturbations originated from seeds during infaltion

Induced GWs

- GW density

$$\Omega_{\rm GW}(k,\eta)=\frac{1}{6}\left(\frac{k}{aH}\right)^2\int_0^\infty dv\int_{|1-v|}^{1+v}du\left[\frac{4v^2-(1-u^2+v^2)^2}{4uv}\right]^2\overline{I^2_{\rm RD}(u,v,x)}\mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku)$$

$$\Omega_{GW}=A_{GW}P_\zeta^2\begin{cases}\frac{a(\eta)}{a_{eq}}\frac{k}{k_{eq}}, & k<k_{eq}\\ \frac{a(\eta)}{a_{eq}}\left(\frac{k}{k_{eq}}\right)^{2-2\gamma}, & k_{eq}<k<k_c(\eta)\\ \frac{a_{eq}}{a(\eta)}, & k>k_c(\eta)\end{cases}$$

Production after horizon reentry

$$k_c(\eta)=\left(\frac{a(\eta)}{a_{eq}}\right)^{1/(\gamma-1)}k_{eq},\ A_{GW}\approx 10,\ \gamma\approx 3$$

Baumann, Steinhardt, Takahashi, PRD 76 (07) 084019

Induced GWs

$$\mathcal{P}_h(k,\eta) = \frac{2(216)^2}{\pi^4\eta^2}\int_0^\infty dv \int_{|v-1|}^{|v+1|} du\, \frac{1}{(uv)^8}\left[4v^2-(u^2-v^2-1)^2\right]^2 \mathcal{P}_\Phi(uk)\mathcal{P}_\Phi(vk)$$
$$\times\left[\sin(x)\int_{x_0}^{x} d\tilde{x}_1\, \mathcal{I}_1(\tilde{x}_1) - \cos(x)\int_{x_0}^{x} d\tilde{x}_1\, \mathcal{I}_2(\tilde{x}_1)\right]$$
$$\times\left[\sin(x)\int_{x_0}^{x} d\tilde{x}_2\, \mathcal{I}_3(\tilde{x}_2) - \cos(x)\int_{x_0}^{x} d\tilde{x}_2\, \mathcal{I}_4(\tilde{x}_2)\right],$$

$$\mathcal{I}_j(x) = \sum_{m=1}^{5}\sum_{n=1}^{8} \sin(\alpha_n x + \phi_n)\,\frac{M^j_{nm}}{x^m}$$

Production well outside horizon

$$x_0 = k\eta = 0$$

$$X_j(u,v,x,x_0) = \int_{x_0}^{x} d\tilde{x}\, \mathcal{I}_j(\tilde{x})$$
$$= \sum_{m=1}^{5}\sum_{n=1}^{8} M^j_{nm}\left\{\left[\sum_{k=1}^{m-2}\frac{(m-k-2)!}{(m-1)!}\alpha_n^k \right.\right.$$
$$\left.\times\sin\left(\alpha_n\tilde{x}+\phi_n+\frac{(k+2)}{2}\pi\right)\tilde{x}^{(1+k-m)}\right]_{x_0}^{x}$$
$$\left.-\frac{\alpha_n^{(m-1)}}{(m-1)!}\int_{x_0}^{x} d\tilde{x}\,\frac{1}{\tilde{x}}\sin\left(\alpha_n\tilde{x}+\phi_n+\frac{(m+1)}{2}\pi\right)\right\}$$

Ananda, Clarkson, Wands, PRD 75 (07) 123518

Induced GWs

- Semi-analytic formulae

Starts generation well outside the horizon

$$h_{\boldsymbol{k}}(\eta) = \frac{4}{a(\eta)} \int_0^{\eta} d\tilde{\eta} G_{\boldsymbol{k}}(\eta, \tilde{\eta}) a(\tilde{\eta}) S_{\boldsymbol{k}}(\tilde{\eta})$$

$$I_{\mathrm{RD}}(v, u, x \to \infty) = \frac{3(u^2+v^2-3)}{4u^3v^3x} \left[\sin x \left(-4uv + (u^2+v^2-3) \log \left| \frac{3-(u+v)^2}{3-(u-v)^2} \right| \right) - \pi (u^2+v^2-3) \Theta(v+u-\sqrt{3}) \cos x \right]$$

$$\overline{I^2_{\mathrm{RD}}(v, u, x \to \infty)} = \frac{1}{2} \left(\frac{3(u^2+v^2-3)}{4u^3v^3x} \right)^2 \left[\left(-4uv + (u^2+v^2-3) \log \left| \frac{3-(u+v)^2}{3-(u-v)^2} \right| \right)^2 + \pi^2 (u^2+v^2-3)^2 \Theta(v+u-\sqrt{3}) \right]$$

$$\mathcal{P}_h(k,\eta) = 4 \int_0^{\infty} dv \int_{|1-v|}^{1+v} du \left[\frac{4v^2 - (1-u^2+v^2)^2}{4uv} \right]^2 I^2_{\mathrm{RD}}(u,v,x) \mathcal{P}_\zeta(kv) \mathcal{P}_\zeta(ku)$$

$$\Omega_{\mathrm{GW}}(k,\eta) = \frac{1}{24} \left(\frac{k}{aH} \right)^2 \overline{\mathcal{P}_h(k,\eta)}$$

Kohri, Terada, PRD 97 (18) 123532

Induced GWs

- Generation starts at horizon reentry

$$I_c(u,v,x) = -4\int_1^x y\sin(y) f(u,v,y)dy = T_c(u,v,x) - T_c(u,v,1)$$

$$I_s(u,v,x) = 4\int_1^x y\cos(y) f(u,v,y)dy = T_s(u,v,x) - T_s(u,v,1)$$

$$T_c(u,v,x) = -4\int_0^x y\sin(y) f(u,v,y)dy$$

$$T_s(u,v,x) = 4\int_0^x y\cos(y) f(u,v,y)dy$$

$$f(u,v,x) = 2\Psi(vx)\Psi(ux) + \left[\Psi(vx) + vx\Psi'(vx)\right]\left[\Psi(ux) + ux\Psi'(ux)\right]$$

Espinosa, Racco, Riotto, JCAP 1809, 012
Lu, Gong, Yi, Zhang, 1907.11896

Induced GWs

$$\overline{I^2_{\rm RD}(u,v,x\to\infty)} = \frac{1}{2x^2}\left[\left(\frac{3\pi(u^2+v^2-3)^2\Theta(u+v-3)}{4u^3v^3}+\frac{T_c(u,v,1)}{9}\right)^2 + \left(\frac{\tilde{T}_s(u,v,1)}{9}\right)^2\right]$$

$$\tilde{T}_s(u,v,1) = T_s(u,v,1) + \frac{27(u^2+v^2-3)}{u^2v^2} - \frac{27(u^2+v^2-3)^2}{4u^3v^3}\ln\left|\frac{3-(u+v)^2}{3-(u-v)^2}\right|$$

- Scale invariant power spectrum

$$\mathcal{P}_\zeta(k) = A_\zeta$$

$$\Omega(k,\eta) \approx 0.7859 A_\zeta^2 \qquad \Omega(k,\eta) \approx 0.8222 A_\zeta^2$$

$$x_0 = k\eta = 1 \qquad x_0 = k\eta = 0$$

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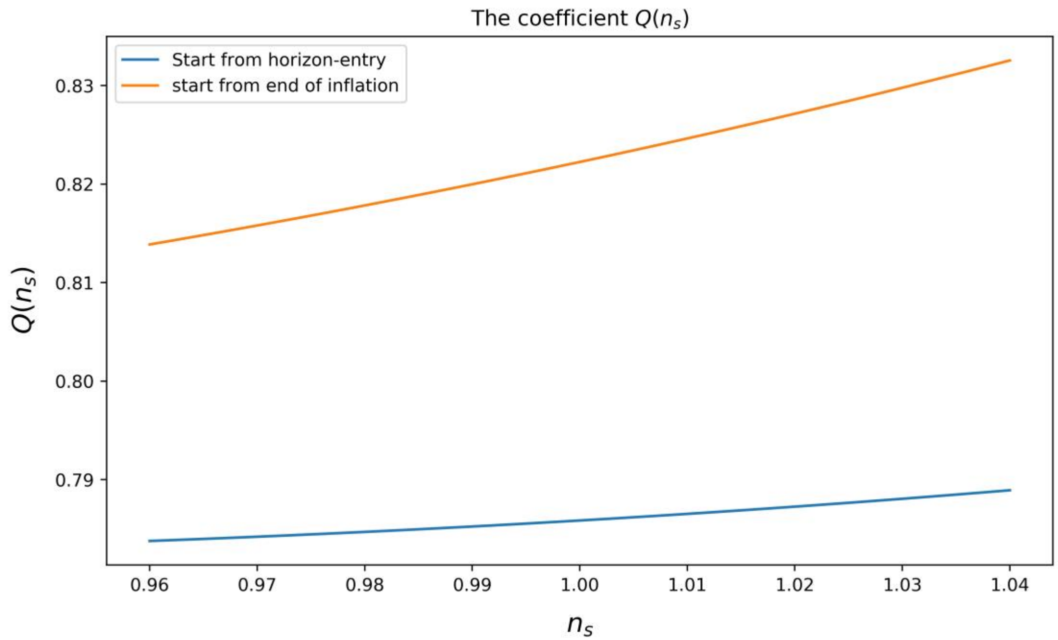
Difference

- The power law power spectrum

$$\mathcal{P}_\zeta(k) = A_\zeta \left(\frac{k}{k_p}\right)^{n_s-1}$$

$$\Omega_{\rm GW}(k,\eta) = Q(n_s) A_\zeta^2 \left(\frac{k}{k_p}\right)^{2(n_s-1)}$$



The monochromatic power spectrum

- Delta function

$$\mathcal{P}_\zeta(k) = A_\zeta \delta\left(\ln \frac{k}{k_p}\right)$$

$$\Omega_{\rm GW} = A_\zeta^2 \times \frac{\bar{k}^2}{192}\left(\frac{4}{\bar{k}^2}-1\right)^2 \Theta(2-\bar{k}) \left[\left(\frac{\tilde{T}_s(\bar{k}^{-1},\bar{k}^{-1},1)}{9}\right)^2 \right.$$
$$\left. + \left(\frac{3\bar{k}^6\pi}{4}\frac{2}{\bar{k}^2}-3^2\,\Theta(2-3\bar{k}) + \frac{T_c(\bar{k}^{-1},\bar{k}^{-1},1)}{9}\right)^2\right], \qquad \bar{k} \equiv k/k_p$$

Lu, Gong, Yi, Zhang, 1907.11896

$$\Omega_{\rm GW} = \frac{3A_\zeta^2}{64}\left(\frac{4-\tilde{k}^2}{4}\right)^2 \tilde{k}^2 \left(3\tilde{k}^2-2\right)^2 \times \left[\pi^2(3\tilde{k}^2-2)^2\Theta(2\sqrt{3}-3\tilde{k})\right.$$
$$\left. + \quad \left(4+(3\tilde{k}^2-2)\log\left|1-\frac{4}{3\tilde{k}^2}\right|\right)^2\right]\Theta(2-\tilde{k})$$

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The comparison

- Monochromatic and Gaussian power spectrum

$$\mathcal{P}_\zeta(k) = \frac{A_\zeta}{\sigma\sqrt{2\pi}} \exp\left(-\frac{\ln^2(k/k_p)}{2\sigma^2}\right)$$

$$\mathcal{P}_\zeta(k) = A_\zeta \delta\left(\ln \frac{k}{k_p}\right)$$

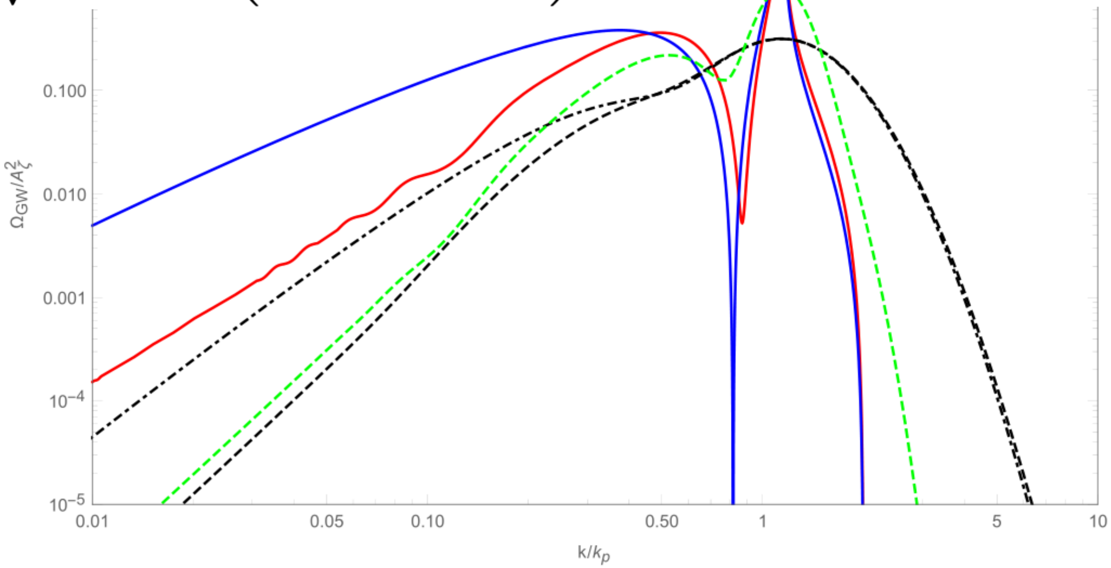


Figure 2. The induced GWs ($\Omega_G W/A_\zeta^2$) from monochromatic and Gaussian power spectra of curvature perturbations. The red solid line denotes the induced GWs from the monochromatic power spectrum, the green and black dashed lines denote the induced GWs from the Gaussian power spectrum with $\sigma = 0.2$ and $\sigma = 0.5$, respectively. For comparison, we also show the induced GWs from the monochromatic power spectrum and Gaussian power spectrum with $\sigma = 0.5$ by using the formulae derived in [32] with the blue solid line and the black dot dashed line, respectively.

The PBHs

- PBHs: Primordial Black hole (PBH) forms in the radiation era as a result of gravitational collapse of density perturbations generated during inflation
- The mass

Suppose that the mass of PBHs is of the same order of the horizon mass $M = \gamma M_H$

$$1M_\odot \approx 2 \times 10^{33}\mathrm{g}$$

$$M_H = \frac{4\pi\rho}{3H^3} = \frac{1}{2GH} \approx 2.02 \times 10^5 \left(\frac{t}{1\mathrm{s}}\right) M_\odot$$

$$M \approx 18.4\gamma M_\odot \left(\frac{g}{10.75}\right)^{-1/6} \left(\frac{k}{10^6\mathrm{Mpc}^{-1}}\right)^{-2} \qquad k = aH$$

$$M \approx 2 \times 10^5 \gamma M_\odot \left(\frac{g}{10.75}\right)^{-1/2} \left(\frac{T}{10^{10}\mathrm{K}}\right)^{-2}$$

PBH dark matter

- The energy fraction

After their formation, PBHs behave like matter, so the energy fraction of PBHs increases until the matter radiation equality, in terms of the current energy fraction of PBHs to dark matter

$$f_{\rm PBH} = \Omega_{\rm PBH}/\Omega_{\rm DM}$$

$$\beta(M_{\rm PBH}) = \frac{T_{\rm eq}}{T}\frac{\Omega_{\rm PBH}}{\Omega_{m0}}$$

$$= 4\times 10^{-9}\left(\frac{\gamma}{0.2}\right)^{-1/2}\left(\frac{g_*^i}{10.75}\right)^{1/4}\left(\frac{M_{\rm PBH}}{M_\odot}\right)^{1/2} f_{\rm PBH}$$

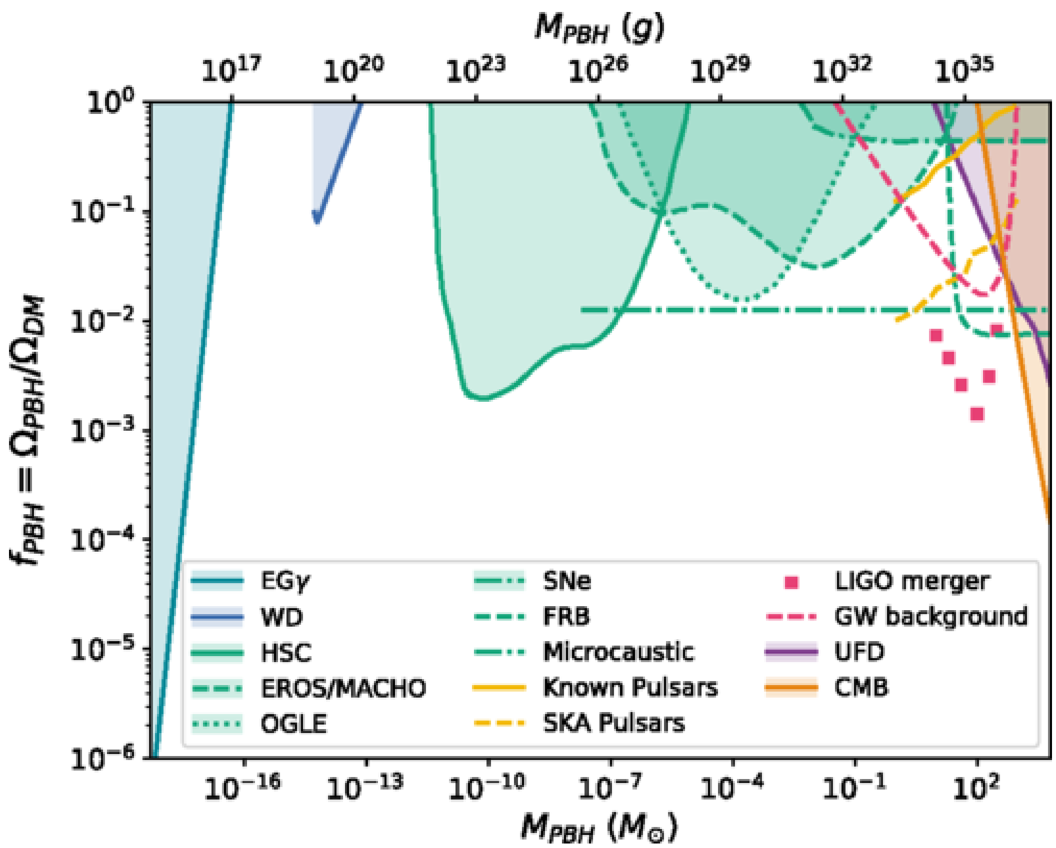
$$M_{\rm PBH} \approx 2\times 10^5 \gamma M_\odot \left(\frac{g}{10.75}\right)^{-1/2}\left(\frac{T}{10^{10}{\rm K}}\right)^{-2}$$

$$\gamma = 3^{-3/2} \approx 0.2$$

B.J. Carr, APJ 201 (75) 1

PBH constraints

- Observational constraints



G. Sato-Polito, E.D. Kovetz, M. Kamionkowski, PRD 100 (19) 063521

The production of PBHs

The probability distribution

Suppose the density perturbations are Gaussian, the probability distribution of the smoothed density contrast $\delta(R)$ over a sphere with comoving radius R is

$$P(\delta(R)) = \frac{1}{\sqrt{2\pi\sigma^2(R)}} \exp\left(-\frac{\delta^2(R)}{2\sigma^2(R)}\right)$$

The smoothing scale R is the horizon size, $R = \mathcal{H}^{-1}$

The mass variance $\sigma^2(R) = \int_0^\infty W^2(kR)\frac{\mathcal{P}_\delta(k)}{k}dk = \frac{1}{2}\left(\frac{4}{9}\right)^2 \mathcal{P}_\zeta(1/R)$

The window function $W(kR) = \exp(-k^2R^2/2)$

Radiation domination $\mathcal{P}_\delta(k) = \frac{16}{81}\left(\frac{k}{aH}\right)^4 \mathcal{P}_\zeta(k)$

The production of PBHs

- The energy density (Press-Schechter)

$$\beta = \frac{\rho_{\rm PBH}}{\rho_{\rm tot}} = 2\int_{\delta_c}^{\infty} P(\delta)d\delta$$

$$\approx \mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2\mathcal{P}_\delta}}\right) = \mathrm{erfc}\left(\frac{9\delta_c}{4\sqrt{\mathcal{P}_\zeta}}\right)$$

$$= 4\times 10^{-9}\left(\frac{\gamma}{0.2}\right)^{-1/2}\left(\frac{g_*^i}{10.75}\right)^{1/4}\left(\frac{M_{\rm PBH}}{M_\odot}\right)^{1/2} f_{\rm PBH}$$

$\delta_c = 0.42$ T. Harada, C.-M. Yoo and K. Kohri, PRD 88 (13) 084051

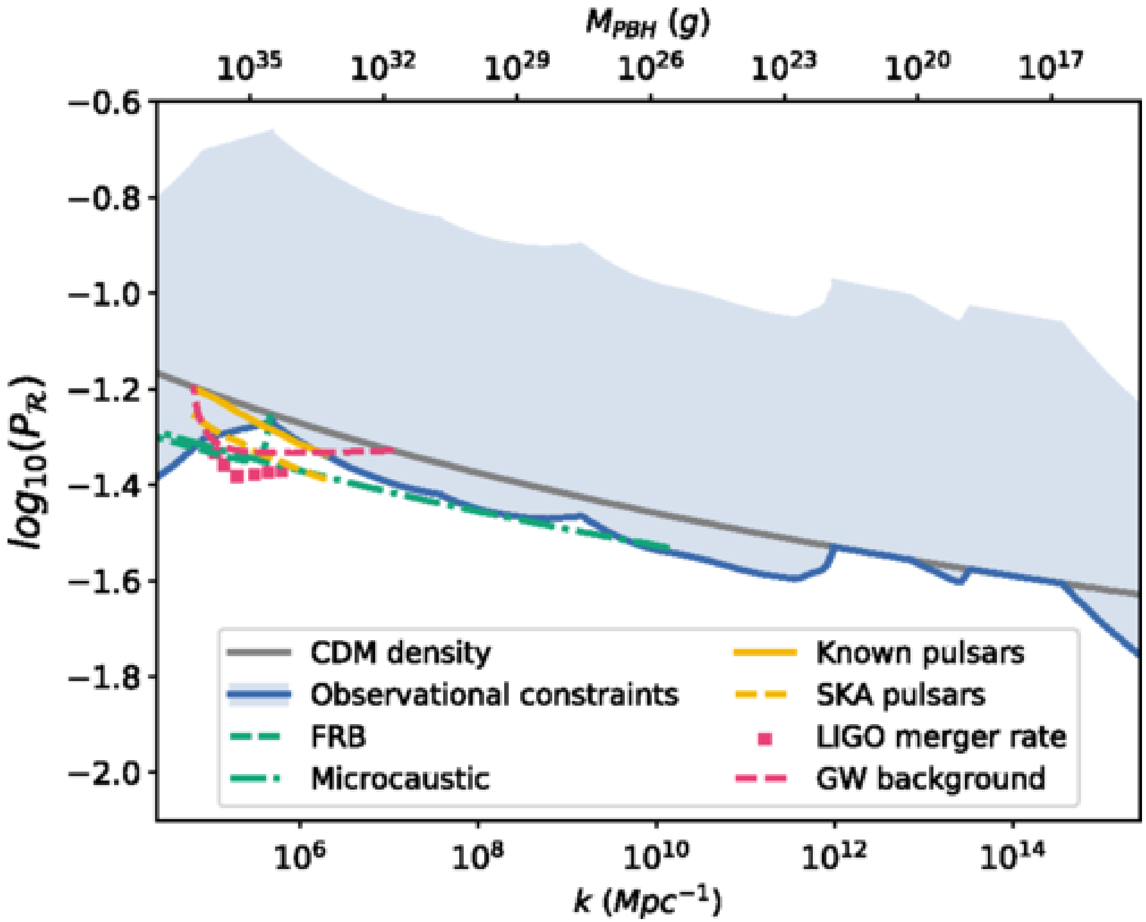
- Need an enhancement on the primordial power spectrum

$$\mathcal{P}_\zeta = \frac{H^2}{8\pi^2\epsilon} \sim 0.01$$ Small scales

$$\mathcal{P}_\zeta \sim 10^{-9}$$ Larges scales

Observational constraints

- PBH fraction $f_{\mathrm{PBH}} \Rightarrow \mathcal{P}_\zeta$



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The constraints

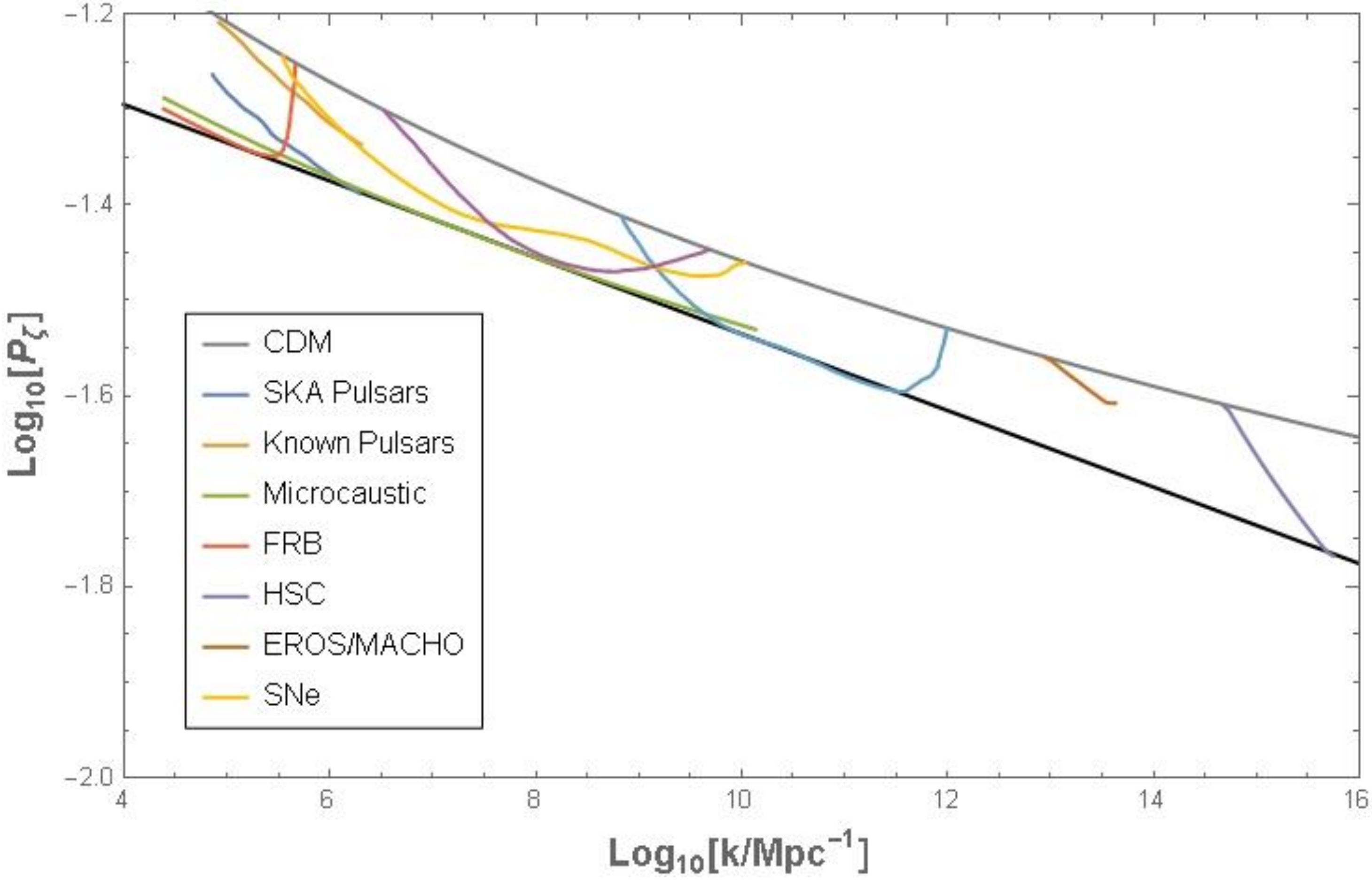
- Power law parametrization

$$
\mathcal{P}_\zeta(k) = \begin{cases} 2.1 \times 10^{-9} \left(\dfrac{k}{0.05\ \mathrm{Mpc}^{-1}} \right)^{0.9649-1}, & k \lesssim 1\ \mathrm{Mpc}^{-1} \\ 2.1 \times 10^{-9} \left(\dfrac{k}{0.05\ \mathrm{Mpc}^{-1}} \right)^{1.857}, & 1\ \mathrm{Mpc}^{-1} \lesssim k \lesssim 10^4\ \mathrm{Mpc}^{-1} \\ 5.1 \times 10^{-2} \left(\dfrac{k}{10^4\ \mathrm{Mpc}^{-1}} \right)^{0.960-1}, & k \gtrsim 10^4\ \mathrm{Mpc}^{-1} \end{cases}
$$

Lu, Gong, Yi, Zhang, 1907.11896

Constraints on power spectrum

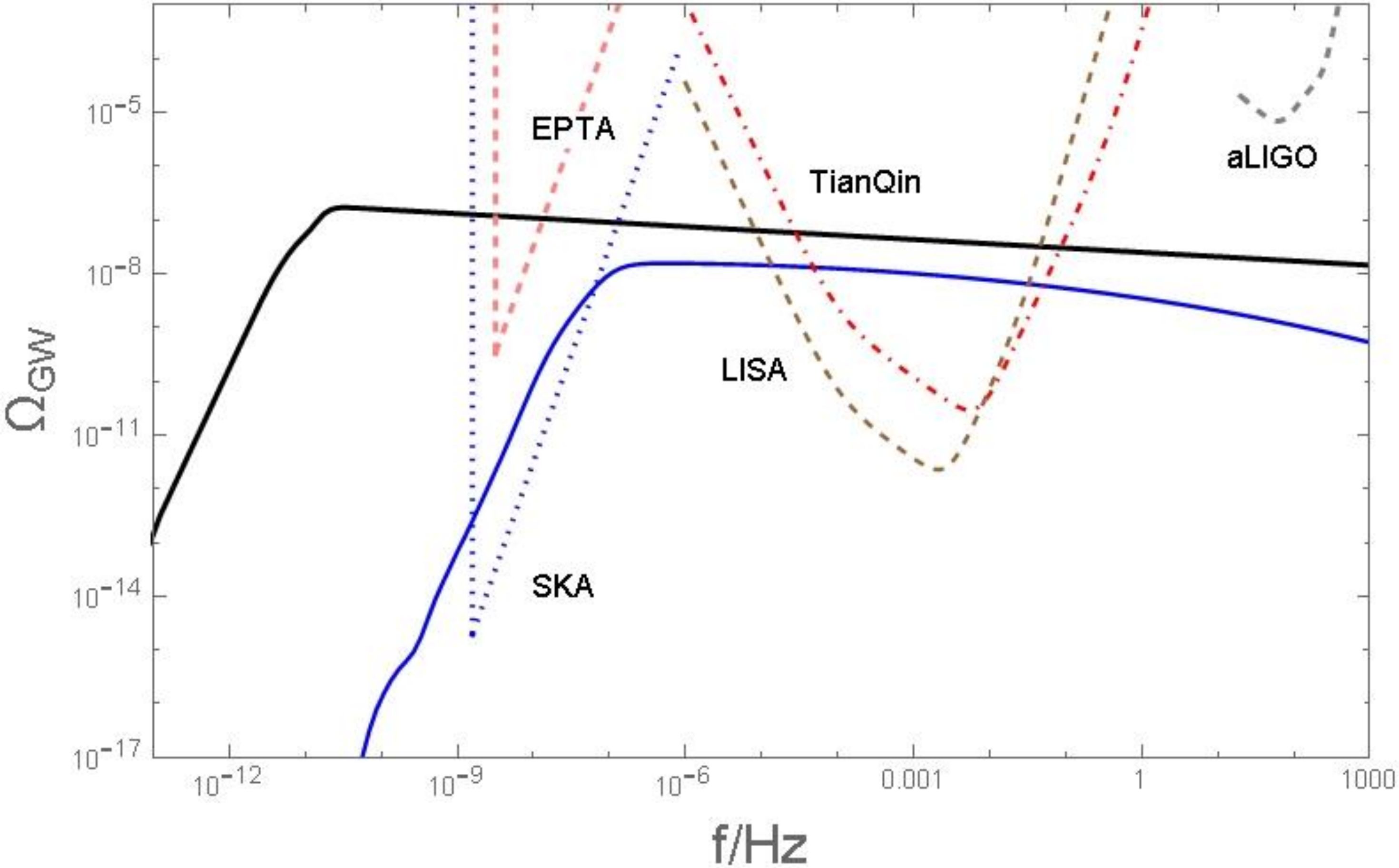
- Result



Lu, Gong, Yi, Zhang, 1907.11896

Possible Detection

- The secondary GWs



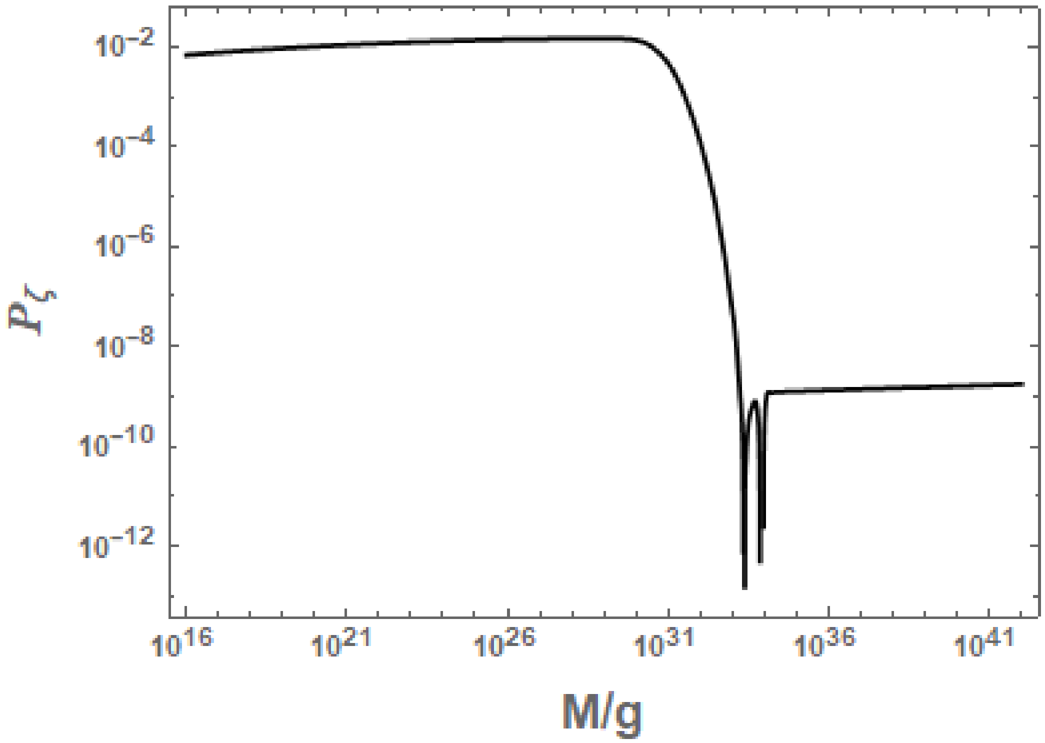
Non detection of induced GWs by LISA $\mathcal{P}_\zeta \lesssim 4 \times 10^{-4}$

The power spectrum

- A model with polynomial potential

$$V(\phi) = \begin{cases} V_0 \left| 1 + \sum_{m=1}^{m=5} \lambda_m \left(\frac{\phi}{M_{\rm Pl}} \right)^m \right|, & \phi \geq 0, \\ V_0 \left[1 + \sum_{m=1}^{m=3} \lambda_m \left(\frac{\phi}{M_{\rm Pl}} \right)^m \right], & \phi < 0, \end{cases}$$

Di & Gong, JCAP 07 (18) 007



Conclusions

- The contribution to the induced GWs by the production before horizon reentry is small for nearly scale invariant power spectrum, but it can be very large for monochromatic (Gaussian) power spectrum
- The upper bound on the primordial power spectrum by PBH DM constraints is $\mathcal{P}_\zeta \lesssim 0.05$
- The secondary GWs (generated by the upper limit of the power spectrum) may be detected by PTA or space-based GW detector

Thank You