Discrete Anomaly in 2D Conformal Field Theories

Yang Zhou(周洋)

Based on arXiv:1908.02918 Ken Kikuchi,YZ

&JHEP 1907 (2019) 091,YZ

2019 弦论、场论与宇宙学相关专题研讨会

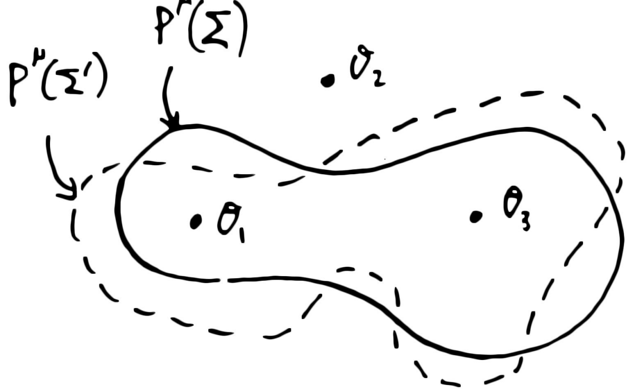
Global Symmetry

- Global symmetries are powerful in constraining correlation functions and other physical observables in quantum field theory (QFT).
  - Ward identity, representations, selection rules…

- `t Hooft anomaly is an obstruction for gauging a global symmetry. It constrains the dynamics and phases of QFT. ['t Hooft anomaly is not bad but physical->ABJ]
  - 't Hooft anomaly matching, RG flows, boundaries and interfaces…

- Symmetry transformations on operators can be implemented by topological operators $\rightarrow U_g(M)$ topological defects

Topological Defects



- Ward identity

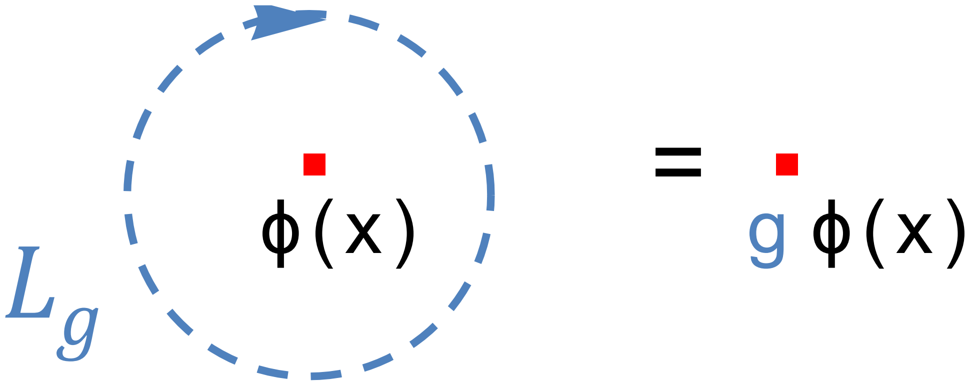
$$\partial_\mu \langle T^{\mu\nu}(x)\mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle = -\sum_i \delta(x-x_i)\partial_i^\nu \langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle$$

$$P^\nu(\Sigma) \equiv -\int_\Sigma dS_\mu T^{\mu\nu}(x)$$

- Continuous U(1) symmetry → $U_g(M) = e^{i\theta Q}$
  - topological property follows from the current conservation
- In general (continuous or discrete), a (0-form) global symmetry $g \in G$ is associated to a codimension-1 topological defect L_g.
- Symmetry transformation on local operators can be implemented by topological defect surrounding. In 2d, L_g is a line.

Topological Defects

- Continuous U(1) symmetry → $U_g(M) = e^{i\theta Q}$
  - topological property follows from the current conservation
- In general (continuous or discrete), a (0-form) global symmetry $g \in G$ is associated to a codimension-1 topological defect L_g.
- Symmetry transformation on local operators can be implemented by topological defect surrounding. In 2d, L_g is a line.



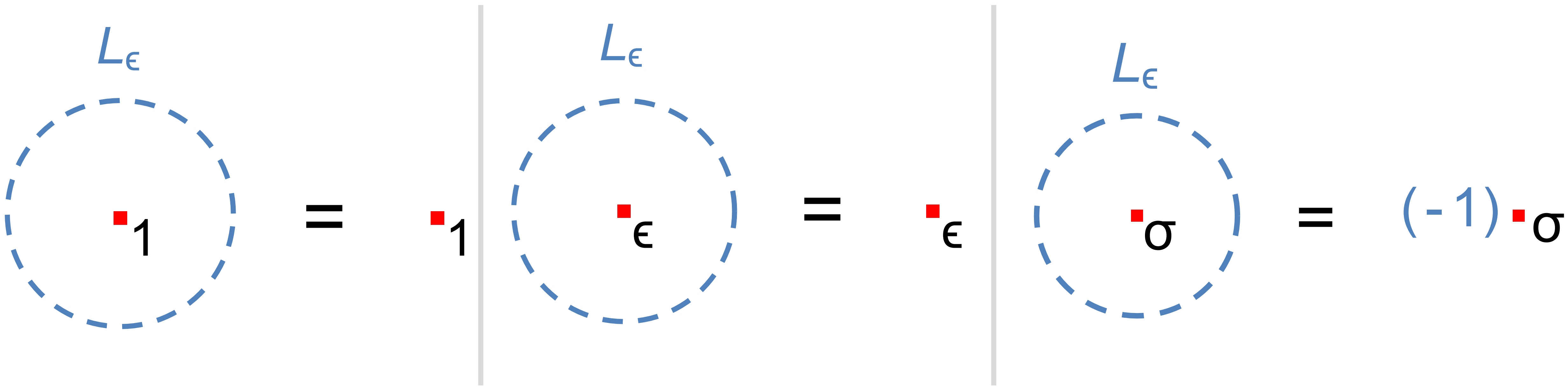
2D Ising Model (c=1/2)

- **3** primary operators in the Ising model and there is a Z_2 global symmetry.

Primary: $\mathbf{1}_{(0,0)}$ $\boldsymbol{\varepsilon}_{(\frac{1}{2},\frac{1}{2})}$ $\boldsymbol{\sigma}_{(\frac{1}{16},\frac{1}{16})}$

$\downarrow$

Z_2 Line: $\boldsymbol{L_{\varepsilon}}$

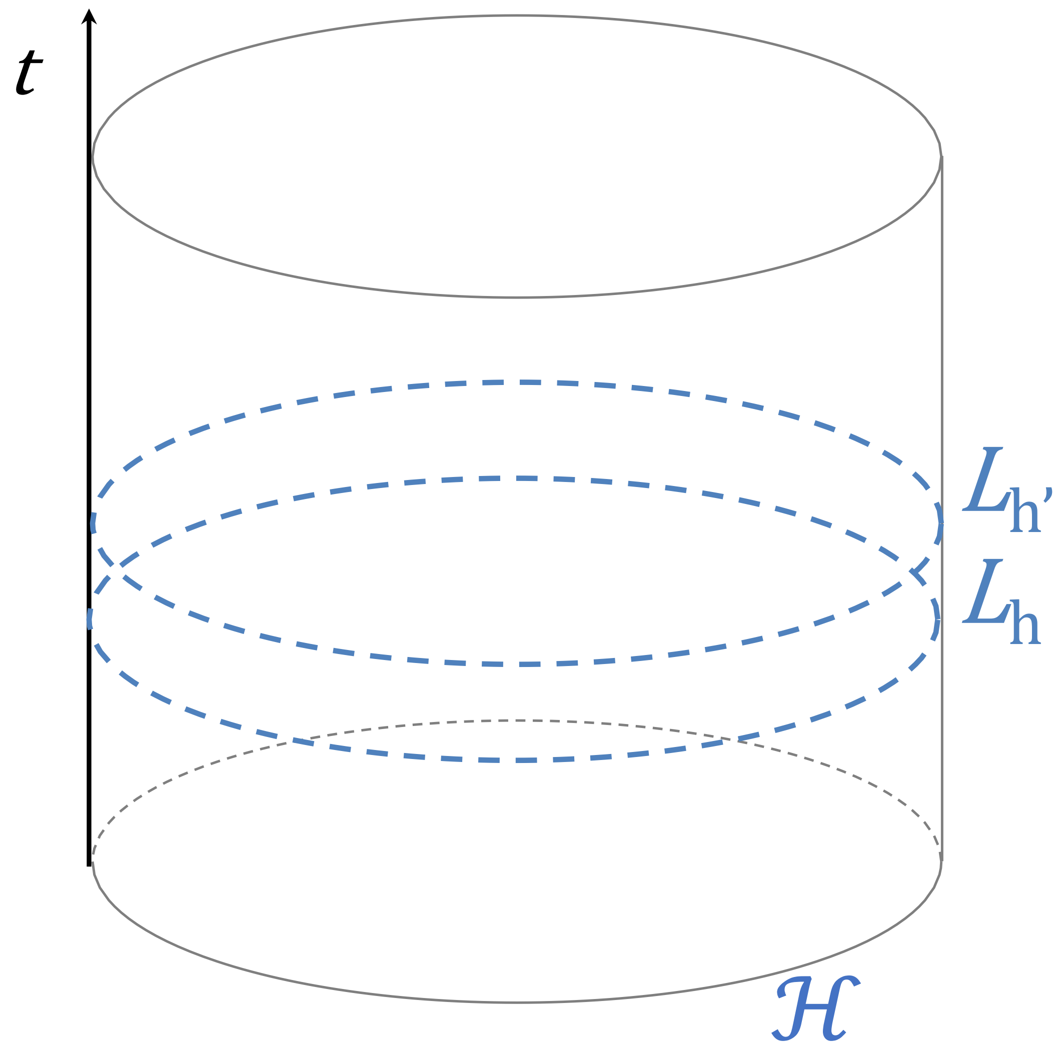


- Z_2 symmetry transformation is implemented by $\boldsymbol{L_\varepsilon}$.

Symmetry Lines and Partition Function

- Symmetry lines act as symmetry transformation of local operator.
- The fusion of symmetry lines follows the group multiplication rules.
- The (inserted) torus partition function is

$$Z_{(h,1)}(\tau) = \mathrm{Tr}_{\mathcal{H}}[\hat{h} q^{L_0 - c/24} \bar{q}^{\bar{L}_0 - c/24}]$$



Symmetry Lines and Partition Function

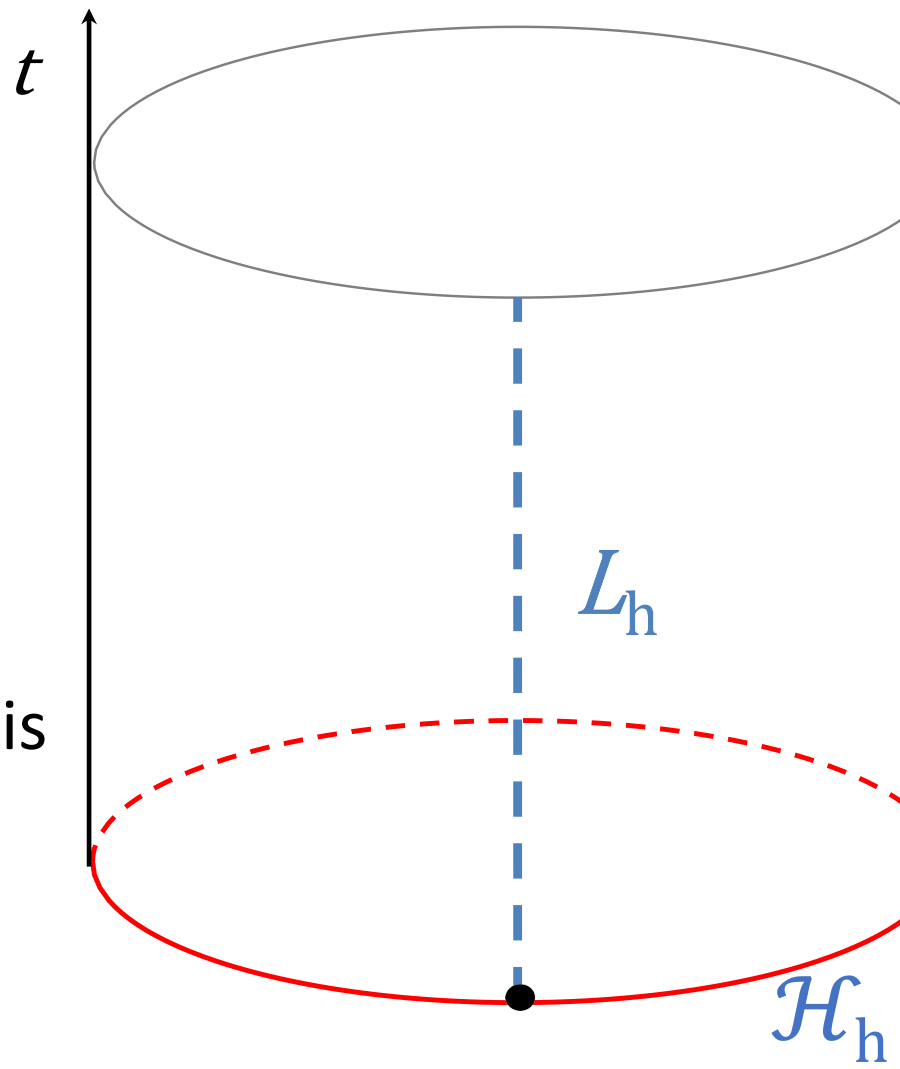
- Symmetry lines twist the boundary condition in quantization, therefore modified the Hilbert space.

- This twisting is related to automorphism of operator algebra

- The (twisted') torus partition function is

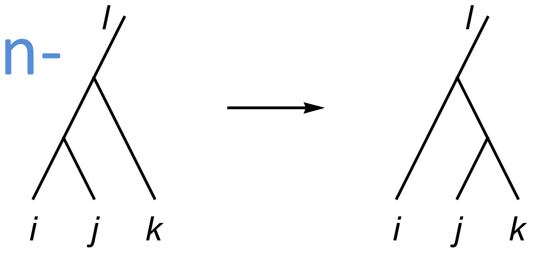
$$Z_{(1,h)}(\tau) = \mathrm{Tr}_{\mathcal{H}_h}[q^{L_0-c/24}\bar{q}^{\bar{L}_0-c/24}]$$

$$SZ_{(h,1)}(\tau) = Z_{(1,h)}(\tau)$$

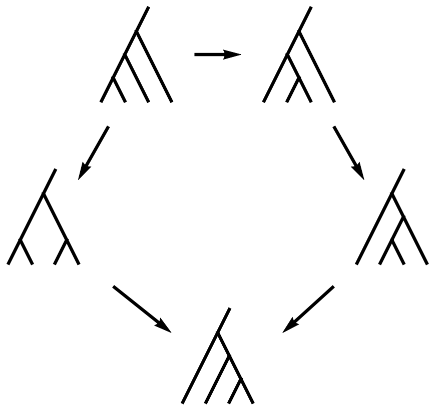


How to detect ‘t Hooft anomaly for a global symmetry G in 2d CFT?

- Solving crossing phases (F-symbol) of TDLs [Chang-Lin-Shao-Yin, Bhardwaj-Tachikawa]



- F-symbol anomaly of G are classified/constrained by the pentagon identity → group cohomology $H^3(G,U(1))$ [Dijkgraaf-Witten,Chen-Gu-Liu-Wen]



- Our main result is that: [K.Kikuchi-YZ]

A new mixed anomaly=noncommutativity of symmetry line insertions on torus

Consistency Conditions I

- Untwisted partition function is modular S and T invariant.

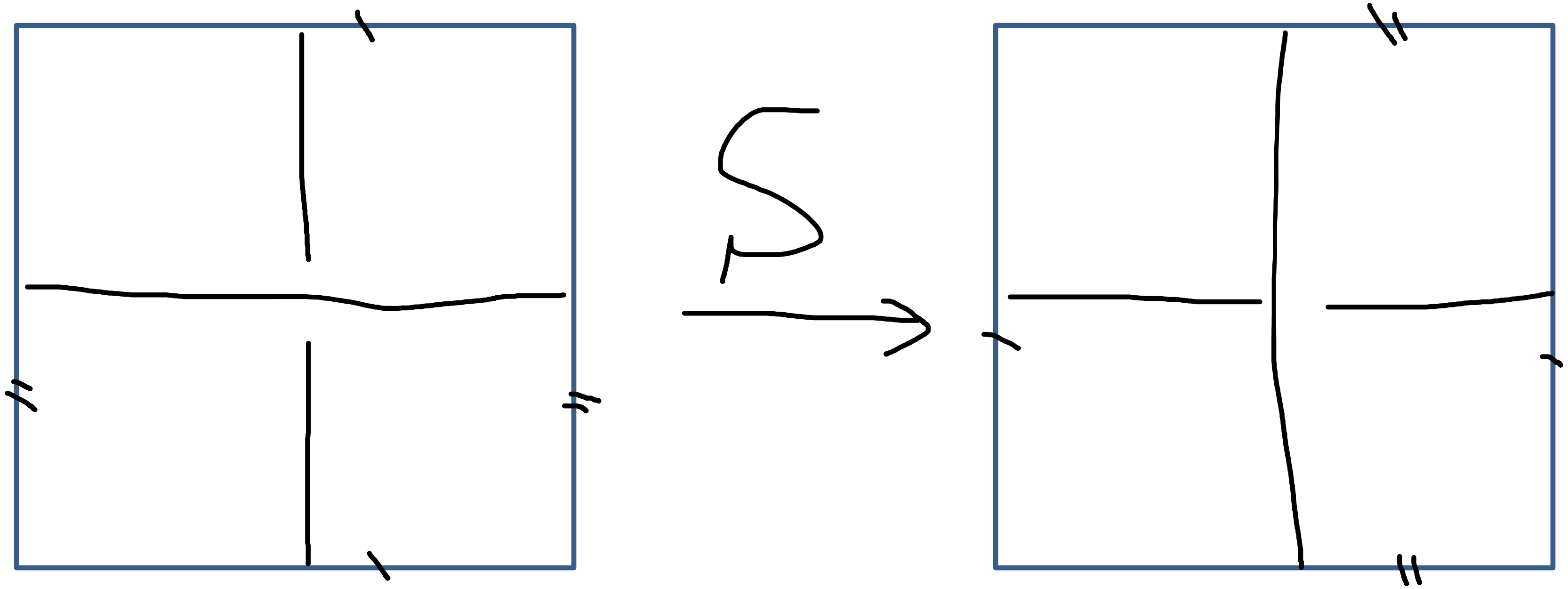
- If **2** circles on torus inserted by the same line, it is expected to be modular S-invariant

$$SZ_{(h,h)}(\tau) = Z_{(h,h)}(\tau)$$

- In general, when **2** circles inserted by two different lines, we should have

$$SZ_{(h,h')} = Z_{(h',h)}$$

$\mathbb{Z}_2$



Ising Model (c=1/2)

$$Z_{(h,1)}(\tau) = |\chi_{id}(\tau)|^2 + |\chi_{\varepsilon}(\tau)|^2 - |\chi_{\sigma}(\tau)|^2$$

$$Z_{(1,h)}(\tau) = \bar{\chi}_{id}(\bar{\tau})\chi_{\varepsilon}(\tau) + \bar{\chi}_{\varepsilon}(\bar{\tau})\chi_{id}(\tau) + |\chi_{\sigma}(\tau)|^2$$

$$Z_{(h,h)}(\tau) = -\bar{\chi}_{id}(\bar{\tau})\chi_{\varepsilon}(\tau) - \bar{\chi}_{\varepsilon}(\bar{\tau})\chi_{id}(\tau) + \bar{\chi}_{\sigma}(\bar{\tau})\chi_{\sigma}(\tau)$$

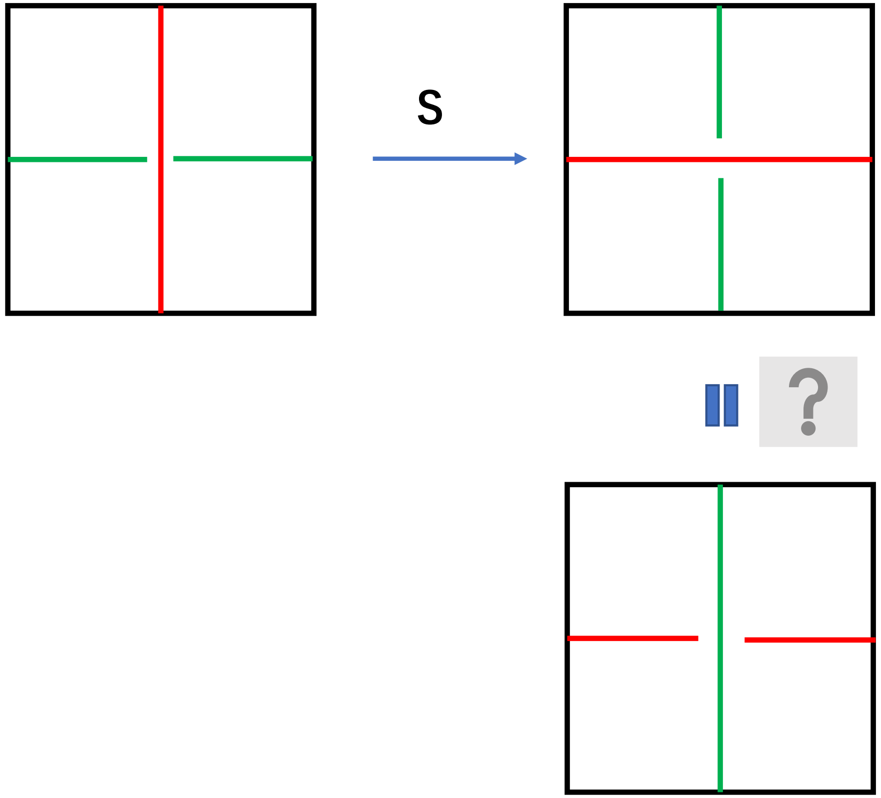
$$SZ_{(h,h)}(\tau) = Z_{(h,h)}(\tau)$$ [Lin-Shao, Kikuchi-YZ]

SU(2)_k Wess-Zumino-Witten (WZW)

$$Z_{(\mathbf{1},h)} = \mathcal{S} Z_{(h,\mathbf{1})} = \sum_{\hat{\mu}\in P_+^k} \bar{\chi}_{A\hat{\mu}} \chi_{\hat{\mu}}$$

$$\begin{aligned}
SZ_{(h,h)}(\tau) &= S \sum_{j=0,1/2,\dots,k/2} (-i)^k(-)^{2j} \chi_j(\tau)\bar{\chi}_{\frac{k}{2}-j}(\bar{\tau}) \\
&= \sum_{j,j_1,j_2} (-i)^k(-)^{2j} S_{jj_1} S_{\frac{k}{2}-j,j_2} \chi_{j_1}(\tau)\bar{\chi}_{j_2}(\bar{\tau}) \\
&= \sum_{j,j_1,j_2} (-i)^k(-)^{2j} S_{jj_1} (-)^{2j_2} S_{jj_2} \chi_{j_1}(\tau)\bar{\chi}_{j_2}(\tau) \\
&= \sum_{j,j_1,j_2} (-i)^k(-)^{2j_2} S_{jj_1} S_{j,\frac{k}{2}-j_2} \chi_{j_1}(\tau)\bar{\chi}_{j_2}(\bar{\tau}) \\
&= \sum_{j_1,j_2} (-i)^k(-)^{2j_2} \delta_{j_1,\frac{k}{2}-j_2} \chi_{j_1}(\tau)\bar{\chi}_{j_2}(\tau) \\
&= \sum_{j} (-i)^k(-)^{2(\frac{k}{2}-j)} \chi_j(\tau)\bar{\chi}_{\frac{k}{2}-j}(\tau) \\
&= (-)^k Z_{(h,h)}(\tau),
\end{aligned}$$

Two different lines inserted



SU(N)_k WZW

$$Z_{(h,h)}(\tau) = \sum_{\hat{\mu}\in P_+^k} e^{-\pi i k|A\hat{\omega}_0|^2 - 2\pi i(A\hat{\omega}_0,\hat{\mu})}\bar{\chi}_{A\hat{\mu}}(\bar{\tau})\chi_{\hat{\mu}}(\tau)$$

$$\begin{aligned}
SZ_{(h,h)}(\tau) &= S\sum_{\hat{\mu}\in P_+^k} e^{-\pi i k|A\hat{\omega}_0|^2 - 2\pi i(A\hat{\omega}_0,\hat{\mu})}\bar{\chi}_{A\hat{\mu}}(\bar{\tau})\chi_{\hat{\mu}}(\tau) \\
&= \sum_{\hat{\mu},\hat{\nu}_1,\hat{\nu}_2\in P_+^k} e^{-\pi i k|A\hat{\omega}_0|^2 - 2\pi i(A\hat{\omega}_0,\hat{\mu})} S^*_{\hat{\nu}_1,A\hat{\mu}}\bar{\chi}_{\hat{\nu}_1}(\bar{\tau}) S_{\hat{\nu}_2,\hat{\mu}}\chi_{\hat{\nu}_2}(\tau) \\
&= \sum_{\hat{\mu},\hat{\nu}_1,\hat{\nu}_2\in P_+^k} e^{-\pi i k|A\hat{\omega}_0|^2 + 2\pi i(A\hat{\omega}_0,\hat{\nu}_1)} S^*_{\hat{\nu}_1,\hat{\mu}}\bar{\chi}_{\hat{\nu}_1}(\bar{\tau}) S_{A\hat{\nu}_2,\hat{\mu}}\chi_{\hat{\nu}_2}(\tau) \\
&= \sum_{\hat{\mu}\in P_+^k} e^{-\pi i k|A\hat{\omega}_0|^2 + 2\pi i(A\hat{\omega}_0,A\hat{\mu})}\bar{\chi}_{A\hat{\mu}}(\bar{\tau})\chi_{\hat{\mu}}(\tau).
\end{aligned}$$

Consistency Conditions I: continued

- When the order of discrete Abelian G is higher than Z_2, we have to truncate the S-transformed twisted partition function to that including primaries operators corresponding to symmetry lines (Verlinde lines).

- We interpret the violation of consistency condition I as a signal of a mixed anomaly for RCFT

- This anomaly agrees precisely with 3d 1-form anomaly [Hung-Wu-Zhou]

Remarks:

- In diagonal RCFTs, primaries are 1-1 corresponding to Verlinde lines, among which there are symmetry lines we concern. [Verlinde, Petkova-Zuber, Moore-Seiberg]

- Our ‘t Hooft anomaly is a mixed anomaly between global symmetry G and the outer automorphism symmetry. Which is different from F-symbol ‘t Hooft anomaly.

Tests:

- Ising CFT: (Kramers-Wannier duality=anomaly free)

- SU(3)_1:

$$Z_{diag} = |\chi_{\mathbf{1}}|^2 + |\chi_{\mathbf{3}}|^2 + |\chi_{\bar{\mathbf{3}}}|^2. \qquad Z_{orb} = |\chi_{\mathbf{1}}|^2 + \chi_{\mathbf{3}}\bar{\chi}_{\bar{\mathbf{3}}} + \chi_{\bar{\mathbf{3}}}\bar{\chi}_{\mathbf{3}}.$$

- SU(N)_k:

Consistency Conditions II

- We have used modular S-transformation, but not yet T.

- If **2** circles on torus inserted by a generating line of Z_n, it is expected, after n times of T-twist [CFT yellow book]

$$T^n Z_{(1,h)}(\tau) = Z_{(h^n,h)}(\tau) = Z_{(1,h)}(\tau)$$

- In general, when **2** circles inserted by different lines,

$$Z_{(h^n,h')}(\tau) = Z_{(1,h')}(\tau)$$

Consistency Conditions II

- The violation of this condition is interpreted as a mixed anomaly between G and diffeomorphism [Numasawa-Yamaguchi]

- This consistency was considered as (generalized) orbifolding conditions when $G=Z_N$ in early days [CFT yellow book]

- We generalize this consistency condition to arbitrary abelian discrete symmetry $G= Z_M * Z_N \ldots$, and find consistency condition I is sufficient but not necessary condition for II.

WZW Models (Condition I, ‘t Hooft anomaly)

type	center Γ	Smatrix	$SZ_{(h,h')}\big\| = Z_{(h',h)}\big\|$	$\|A\hat{\mu}\rangle_c = \|\hat{\mu}\rangle_c$
A_r	$\mathbb{Z}_{r+1}$	$k \in (r+1)\mathbb{Z}$	$k \in (r+1)\mathbb{Z}$	$k \in (r+1)\mathbb{Z}$
B_r	$\mathbb{Z}_2$	$k \in \mathbb{Z}$	$k \in \mathbb{Z}$	$k \in \mathbb{Z}$
C_r	$\mathbb{Z}_2$	$rk \in 2\mathbb{Z}$	$rk \in 2\mathbb{Z}$	$rk \in 2\mathbb{Z}$
D_{2l}	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$k \in 2\mathbb{Z}$	$k \in 2\mathbb{Z}$	$k \in 2\mathbb{Z}$
D_{2l+1}	$\mathbb{Z}_4$	$k \in 4\mathbb{Z}$	$k \in 4\mathbb{Z}$	$k \in 4\mathbb{Z}$
E_6	$\mathbb{Z}_3$	$k \in 3\mathbb{Z}$	$k \in 3\mathbb{Z}$	$k \in 3\mathbb{Z}$
E_7	$\mathbb{Z}_2$	$k \in 2\mathbb{Z}$	$k \in 2\mathbb{Z}$	$k \in 2\mathbb{Z}$

[Kikuchi-YZ]

WZW(Condition II, orbifolding)

Cartan matrix	Group G	center Γ	$\lvert A\hat{\omega}_0\rvert^2$	$e^{-\pi i N \lvert A\hat{\omega}_0\rvert^2}$	Anomaly Free
A_{n-1}	$\mathrm{SU}(n)$	$\mathbb{Z}_n$	$\lvert\omega_1\rvert^2 = \frac{n-1}{n}$	$(-1)^{n-1}$	$n \in 2\mathbb{Z}+1$ or $k \in 2\mathbb{Z}$
B_n	$\mathrm{Spin}(2n+1)$	$\mathbb{Z}_2$	$\lvert\omega_1\rvert^2 = 1$	1	$k \in \mathbb{Z}$
C_n	$\mathrm{USp}(n)$	$\mathbb{Z}_2$	$\lvert\omega_n\rvert^2 = \frac{n}{2}$	$(-1)^n$	$n \in 2\mathbb{Z}$ or $k \in 2\mathbb{Z}$
D_{2l+1}	$\mathrm{Spin}(4l+2)$	$\mathbb{Z}_4$	$\lvert\omega_1\rvert^2 = \frac{2l+1}{2}$	-1	$k \in 2\mathbb{Z}$
E_6	E_6	$\mathbb{Z}_3$	$\lvert\omega_5\rvert^2 = \frac{4}{3}$	1	$k \in \mathbb{Z}$
E_7	E_7	$\mathbb{Z}_2$	$\lvert\omega_6\rvert^2 = \frac{3}{2}$	-1	$k \in 2\mathbb{Z}$

$$T^n Z_{(1,h)}(\tau) = \boxed{Z_{(h^n,h)}(\tau) = Z_{(1,h)}(\tau)}$$ [Numasawa-Yamaguchi]

$$\boxed{Z_{(h^n,h')}(\tau) = Z_{(1,h')}(\tau)}$$ [Kikuchi-YZ]

Boundary state

- Conformal invariance finds Ishibashi states, $|\hat{\mu}\rangle\rangle$

- Physical conditions find Cardy states,

$$|\hat{\mu}\rangle_c := \sum_{\hat{\lambda}\in P_+^k} \frac{S_{\hat{\mu}\hat{\lambda}}}{\sqrt{S_{\hat{0}\hat{\lambda}}}}|\hat{\lambda}\rangle\rangle$$

- For the center symmetry of WZW, it is isomorphic to ``permutation'' of primaries

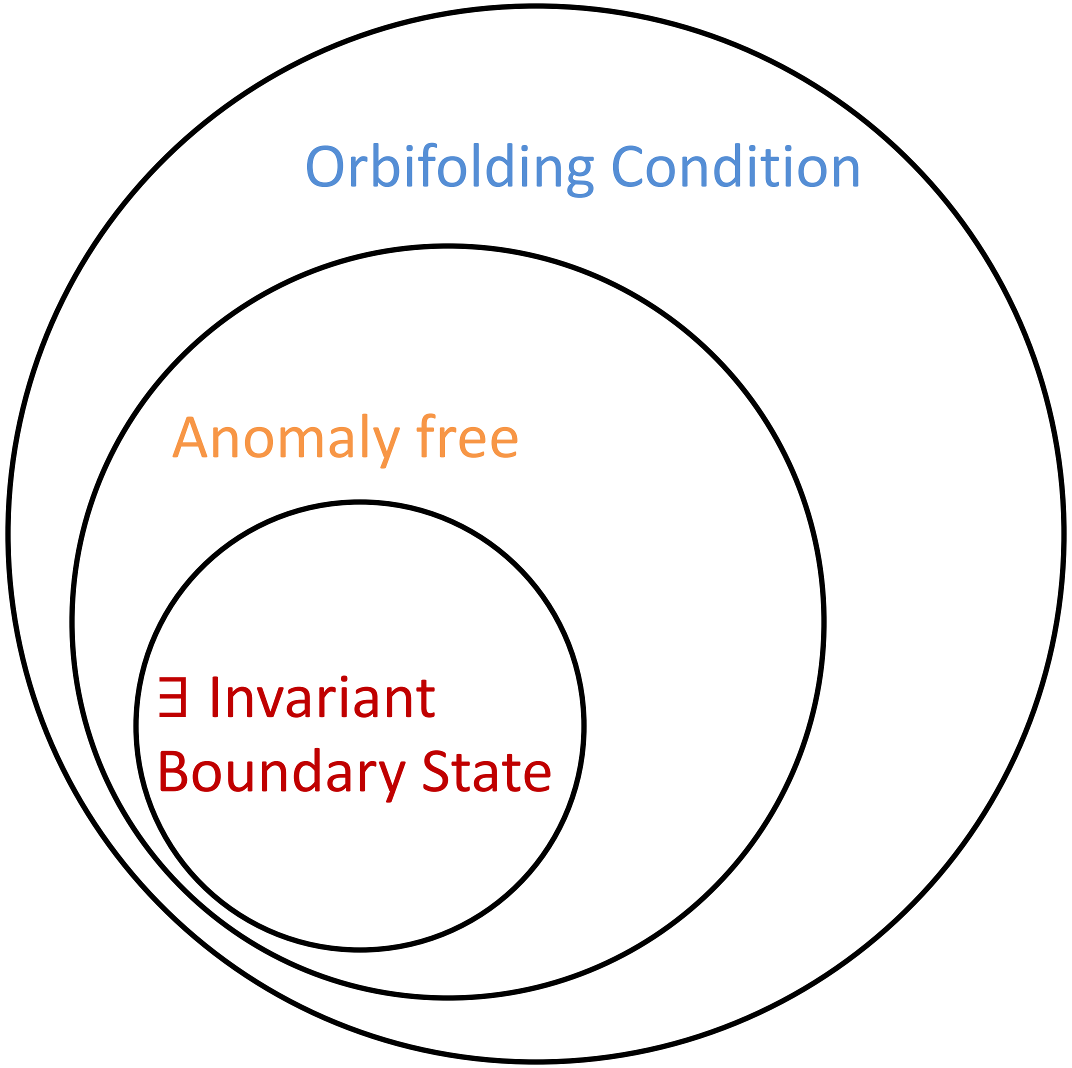
$$h : |\hat{\mu}\rangle_c \mapsto |A\hat{\mu}\rangle_c$$

- Symmetry invariant boundary state is given by

$$|A\hat{\mu}\rangle_c = |\hat{\mu}\rangle_c$$

∃ invariant boundary state=G is anomaly decoupled

- Invariant boundary state can be used to detect anomaly [Han-Tiwari-Hsieh-Ryu]
- G is anomaly decoupled if G is anomaly (Type I consistency) free and also free of any mixing with other symmetries G'
- For the center symmetry of WZW, we demonstrate the equivalence : G-invariant boundary state = G anomaly decoupled.
- We conjecture this is true for any diagonal RCFT

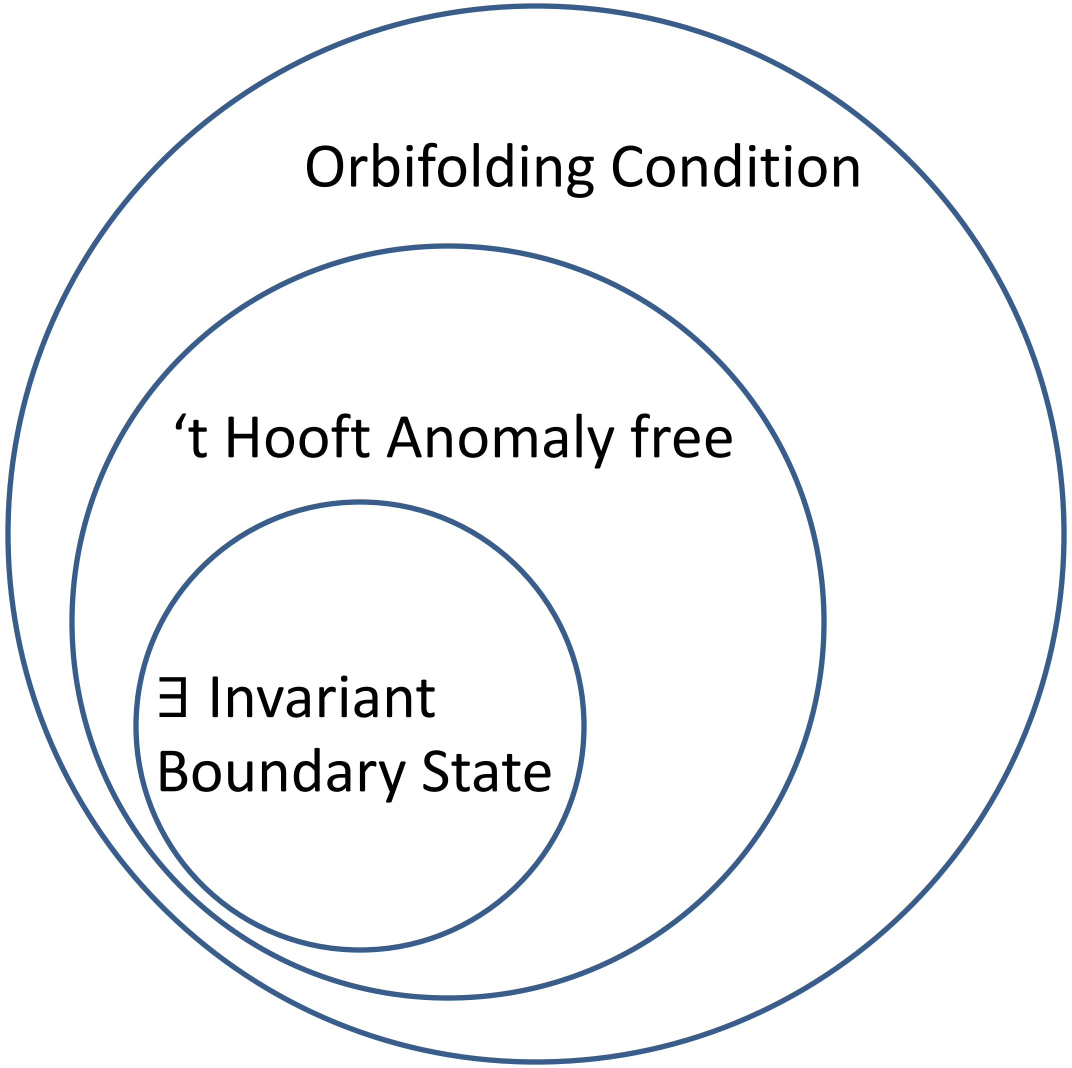


Orbifolding Condition
Anomaly free
∃ Invariant
Boundary State

Conclusion

- We find a general way to detect ‘t Hooft anomaly based on twisted torus partition function
- In particular a new anomaly between G and the outer automorphism->such theory can not be trivially gapped
- gapless approach to detect bulk topological phase
- Generalizations? (arbitrary CFT, non-abelian G, higher dimensions, higher form symmetry)

Thank You!


Orbifolding Condition
‘t Hooft Anomaly free
∃ Invariant
Boundary State