Holographic DC conductivities from the open string metric

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Outline

What is Gauge/gravity duality?

- A holographic duality between a weakly-coupled theory of gravity in certain spacetime and a strongly-coupled field theory living on the boundary of that spacetime.
- A powerful new tool for investigating dynamics of strongly-coupled field theories in the dual gravity side.
- A new window towards understanding real-world physics: QCD, CMT, etc.
- Two complementary approaches: bottom-up and top-down.

Two complementary approaches:

Bottom-up

- Toy-models coming from simple gravity theory;
- Basic ingredients: $g_{\mu\nu}, A_\mu, \psi$ and/or dilaton ϕ;
- Advantage(s): simplicity and universality;
- Disadvantage(s): the dual field theory is unclear.

Top-down

- Configurations originated from string/M theory;
- Exact solutions of SUGRA or Dp/Dq-branes;
- Advantage(s): good understanding on the field theory;
- Disadvantage(s): complexity.

Three main approaches

Retarded Green's function method(Son, Starinets '02)

- General, resulting in many transport coefficients;
- The bulk retarded Green's function encodes a retarded correlator of its dual (field theory) operator;
- Kubo's formula $\Rightarrow$ transport coefficients.

The membrane paradigm (Iqbal, Liu '08)

- Hydrodynamic behavior of boundary field theory $\Leftrightarrow$ those at the stretched horizon of the black hole;
- Transport coefficients $\Leftrightarrow$ quantities at the horizon;
- This elegantly explains universalities of transport coefficients.

Three main approaches Cont'd

The real action method (Karch, O'Bannon '07)

- DC conductivity only, not applicable to other transport coefficients;
- Probe D-brane systems only;
- Non-linear current (electric field dependent conductivity).

These properties stem from the DBI action

$$S_{\mathrm{DBI}} = -T_p \int d^{p+1}\xi \sqrt{P[G] + \mathcal{F}}$$

by requiring that the on-shell action should be real.

Using open string metric

- The open string membrane paradigm with external electromagnetic fields, by K.Y.Kim, J.P.Shock and J.Tarrio, arXiv: 1103.4581[hep-th]
- a membrane paradigm method based on open string metric;
- DC conductivity of a D3/D7 system;
- We will see more generalizations in the current work.

Why open strings?

- When background Kalb-Ramond fields or world-volume gauge fields on a probe D-brane are turned on, the fluctuations of open strings on the probe D-brane do not feel simply the background geometry that they are probing;
- The open string metric (OSM) describes precisely the effective geometry felt by open strings in the presence of external fields.
- We may understand the dynamics of these fluctuating fields in terms of the OSM. In some sense, the background gauge fields are *geometrized*.

The definitions

- DBI+WZ

$$\mathcal{L} = \sqrt{-\mathrm{det} P[G] + \mathcal{F}} + P[C] \wedge \mathcal{F},$$

P[ ]-pull-back, $\mathcal{F} = \tilde{F} + \tilde{f}$, f-fluctuations. Quantities with tildes-those multiplied by $2\pi\alpha'$.

- Define the OSM as follows

$$\begin{aligned} \gamma_{mn} &\equiv P[G] + \tilde{F}, \\ \gamma^{mn} &= (\gamma_{mn})^{-1} = s^{mn} + \theta^{mn}, \end{aligned} \tag{1}$$

The definitions Cont'd

- s^{mn}-the symmetric part, θ^{mn}-the anti-symmetric part.
- The OSM s_{mn} is defined as

$$s_{mn} = g_{mn} - (\tilde{F} g^{-1} \tilde{F})_{mn}, \tag{2}$$

- Notice that $s_{mn} s^{np} = \delta_m^p$.

An overview

- The membrane paradigm (Iqbal, Liu, '08) is based on linear response theory $\Rightarrow$ linear conductivity-the conductivity is independent of the electric field.
- Non-linear conductivity which depends on the electric field $\Leftrightarrow$ the real-action condition.
- Example: D3/D5 described by a one-dimensional action

$$S \sim -\int_{r_H}^{\infty} dr g_{\Omega\Omega}^2 \sqrt{-g_{tt} g_{rr} g_{xx}^2} \sqrt{\frac{\xi}{\chi}}, \tag{3}$$

An overview Cont'd

where the general background

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{xx}\sum_{i=1}^{d} dx_i^2 + g_{\Omega\Omega}d\Omega_n^2,$$

and $d = n = 2$.

$$\xi = -g_{tt}g_{xx}^2 - g_{xx}\tilde{E}^2, \quad \chi = -g_{tt}g_{xx}^2 g_{\Omega\Omega}^2 - g_{xx}J_x^2, \tag{4}$$

Legendre transformed action

$$S_{\rm LT} \sim -\int_{r_H}^{\infty} dr \frac{\sqrt{g_{rr}}}{\sqrt{-g_{tt}g_{xx}}}\sqrt{\xi\chi}, \tag{5}$$

An overview Cont'd

- ξ becomes negative near the horizon.
  singular shell–the location where the sign of ξ flips.
- by introducing and adjusting J_x to flip the sign of χ at the singular shell, we may keep the action real.

$$\begin{aligned} \chi(r_s) &= 0 \Leftrightarrow \\ J_x &= \sqrt{-g_{tt}g_{xx}g_{\Omega\Omega}^2}|_{r=r_s} = g_{\Omega\Omega}(r_s)\tilde{E} \equiv \sigma(r_s)\tilde{E}, \quad (6) \end{aligned}$$

  where r_s is determined by $\xi(r_s) = 0$.
- Notice that $r_s = r_s(r_H, \tilde{E})$, so the current is nonlinear in $\tilde{E}$.

The real-action condition and OSM

- If we introduce only $\tilde{E}$ then the geometry of s_{mn} becomes singular at the singular shell. It can be seen from the Ricci scalar $\mathcal{R}$ near r_s ($\xi \to 0$)

$$\mathcal{R} \sim \frac{(g_{xx}g'_{tt} + g_{tt}g'_{xx})^2}{2\xi^2 g_{rr}}, \tag{7}$$

- To make the geometry regular we can introduce the current J_x then it changes OSM and yields

$$\mathcal{R} \sim \frac{\chi(g_{xx}g'_{tt} + g_{tt}g'_{xx})^2}{2\xi^2 g_{tt}g_{rr}g_{xx}g^2_{\Omega\Omega}}, \tag{8}$$

The real-action condition and OSM Cont'd

- The regularity of the Ricci scalar yields the same result as the real-action method.
- Steps to compute non-linear DC conductivity:
  - compute the linear conductivity using OSM and membrane paradigm;
  - compute the singular shell position r_s from $\xi(r_s) = 0$ with finite $\tilde{E}$;
  - apply the same formula obtained in step 1 at $r = r_s$.

Minkowski embedding

- We cannot apply the real-action method since there is no singular shell on the world volume;
- From the OSM point of view, the geometry is regular everywhere and there seems to be no reason to introduce the current;
- We still require regularity on the gauge field configuration;
- This was proposed in arXiv: 1003.4965[hep-th] (Bergman, Jokela, Lifshytz and Lippert).

General assumptions

- Consider probe Dq-branes sharing t, x, y field theory space. The induced metric and gauge field

$$\begin{aligned} ds^2 &= g_{tt}dt^2 + \sum_{i=1}^{2} g_{ii}dx_i^2 + g_{rr}dr^2 + ds_{(I)}^2, \\ 2\pi\alpha' A &= \tilde{A}_t dt + \tilde{B}xdy + 2\pi\alpha' a, \end{aligned} \tag{9}$$

  $ds_{(I)}^2$-the metric of the internal space, $I = q - 3$.
- There may be nontrivial background RR fields and fluxes through the internal space in concrete examples.

the DBI term

- Assume the matrix $\gamma = g + \tilde{F}$ is a direct sum of the submatrix in the bulk spacetime $m = t, 1, 2, r$ and the internal space $\alpha = 4, \cdots, q+1$. $\det\gamma = \det\gamma_{ab}\det\gamma_{\alpha\beta}$, where

$$\det\gamma_{\alpha\beta} \sim \Theta(r) \times \text{a function of } \xi^{\alpha}.$$

- The DBI action becomes

$$\begin{aligned} S_{\text{DBI}} &= -N_f T_{Dq} V_{(I)} \int dt d\vec{x} dr e^{-\phi} \sqrt{\Theta} \sqrt{-\det\gamma_{mn}}, \\ &\equiv \mathcal{N} \int dt d\vec{x} dr \mathcal{L}_{\text{DBI}} \end{aligned} \tag{10}$$

the DBI term Cont'd

- The normalization constant

$$\mathcal{N} = N_f T_{Dq} V_{(I)}, \quad \mathcal{N}' \equiv (2\pi\alpha')^2 \mathcal{N}, \tag{11}$$

$\mathcal{N}'$ is defined for later convenience.

- The leading order Lagrangian

$$\mathcal{L}^{(0)}_{\mathrm{DBI}} = -e^{-\phi}\sqrt{\Theta\kappa}\sqrt{-g_{tt}g_{rr} - \tilde{A}_t'^2}, \tag{12}$$

$$\kappa \equiv \det\gamma_{ij} = \tilde{B}^2 + g_{xx}g_{yy}, \quad i,j = 1,2. \tag{13}$$

the DBI term Cont'd

- The conserved quantity

$$\hat{J}_t \equiv \frac{\partial \mathcal{L}}{\partial \tilde{A}'_t} = \frac{e^{-\phi}\tilde{A}'_t \Theta \kappa}{\sqrt{-(g_{tt}g_{rr} + \tilde{A}'^2_t)\Theta\kappa}}, \tag{14}$$

$$\tilde{A}'_t = \sqrt{-\frac{\hat{J}^2_t g_{tt} g_{rr}}{\hat{J}^2_t + e^{-2\phi}\Theta\kappa}}, \tag{15}$$

- The sub-leading action

$$S^{(2)}_{\rm DBI} = -\mathcal{N}' \int dt d\vec{x} dr \left[\frac{\sqrt{-s}}{4g_4^2} s^{mp} s^{nq} f_{mn} f_{pq} + \frac{1}{8}\epsilon^{mnpq} f_{mn} f_{pq} Q \right], \tag{16}$$

the DBI term Cont'd

- The effective coupling

$$g_4^2 = \frac{\sqrt{-s}}{e^{-\phi}\sqrt{-\mathrm{det}\gamma_{mn}}\sqrt{\Theta}}.$$

- The non-vanishing components of θ

$$\theta^{tr} = -\frac{e^{\phi}\hat{J}_t}{\sqrt{-\mathrm{det}\gamma_{mn}}\sqrt{\Theta}}, \quad \theta^{xy} = -\frac{\tilde{B}}{\kappa}, \tag{17}$$

- The OSM (recall (2))

$$s_{mn}dx^m dx^n = g_{tt}\mathcal{G}^2 dt^2 + g_{rr}\mathcal{G}^2 dr^2 + \frac{\kappa}{g_{yy}}dx^2 + \frac{\kappa}{g_{xx}}dy^2, \tag{18}$$

the DBI term Cont'd

$$\mathcal{G}^2 = \frac{e^{-2\phi}\Theta\kappa}{\hat{J}_t^2 + e^{-2\phi}\Theta\kappa},$$

$$Q = -\frac{1}{8}e^{-\phi}\sqrt{-\det\gamma_{mn}}\sqrt{\Theta}\epsilon_{mnpq}\theta^{mn}\theta^{pq} = -\frac{\tilde{B}\hat{J}_t}{\kappa},$$

with $\epsilon_{txyr} = 1$.

- The effects of density $\hat{J}_t$ and magnetic field $\tilde{B}$ are geometrized through $\mathcal{G}$ and κ.

the WZ term

- The relevant WZ term

$$S_{\rm WZ}=\frac{1}{2}N_f T_{Dq}(2\pi\alpha')^2\int P[C_{q-3}]\wedge F\wedge F\,, \qquad (19)$$

The leading order action

$$S^{(0)}_{\rm WZ}=\mathcal{N}\int dtd\vec{x}drC_{q-3}\tilde{F_{0r}}\tilde{F_{12}}=\mathcal{N}\int dtd\vec{x}dr\,\mathcal{L}^{(0)}_{\rm WZ}, \qquad (20)$$

- The conserved quantity

$$\hat{J}_t\equiv\frac{\partial(\mathcal{L}^{(0)}_{\rm DBI}+\mathcal{L}^{(0)}_{\rm WZ})}{\partial\tilde{A}'_t}=\bar{J}_t(r)-C_{q-3}(r)\tilde{B}\,, \qquad (21)$$

the WZ term Cont'd

- Notice that $\hat{J}_t$ is a constant but $\bar{J}_t$ is a function of r.
- We may simply replace $\hat{J}_t$ in previous expressions by

$$\hat{J}_t \to \bar{J}_t(r) = \hat{J}_t + C_{q-3}(r)\tilde{B}.$$

- Two different contributions to $\hat{J}_t$: topological charge ($C_{q-3}(r)$) and strings. The latter–a delta-function source at the IR end of the probe brane (say r_0). Its existence is manifested by the boundary condition of nonzero $A_t'(r_0)$.
- Vanishing string source (Minkowski embedding), we require $A_t'(r_0) = 0 \Rightarrow \hat{J}_t = -C_{q-3}(r_0)\tilde{B}$.

the WZ term Cont'd

- The above expression shows a typical property of a quantum Hall state.
- At sub-leading order, the quadratic fluctuations read

$$S^{(2)}_{\rm WZ} = \mathcal{N}' \int d^4\xi C_{q-3}\epsilon^{ji} f_{jt} f_{ri} + \cdots , \tag{22}$$

where we have explicitly shown only the terms which are relevant to the DC conductivity.

The membrane paradigm

- The canonical momentum

$$\mathcal{J}^i(r) = -\frac{\mathcal{N}'}{g_4^2}\sqrt{-s}f^{ri} - \mathcal{N}'Q\epsilon^{ji}f_{jt} + \mathcal{N}'C_{q-3}\epsilon^{ji}f_{jt}\,, \qquad (23)$$

- The current and conductivity tensor (Iqbal and Liu, '08)

$$j^i(k^\mu) \equiv \mathcal{J}^i(r\to\infty)(k^\mu) \equiv \sigma^{ij}(k^\mu)f_{jt} = \sigma^{ij}(k^\mu)\mathcal{E}_j\,, \qquad (24)$$

- In the limit $k^\mu \to 0$, $\mathcal{J}^i$ and f_{rt} are constants in r. We may evaluate it at any IR radial position (say r_0).

The membrane paradigm Cont'd

- For a black hole embedding we evaluate it at the stretched horizon and make use of a regularity condition at the horizon,

$$f_{rj} = \sqrt{\frac{s_{rr}}{-s_{tt}}} f_{tj} .$$

- The Ohm's law $j^i = \sigma^{ij} \mathcal{E}_j$ leads to

$$\sigma^{ij} = \mathcal{N}' \left[\frac{1}{g_4^2} \sqrt{\frac{s}{s_{tt} s_{rr}}} s^{ij} - Q \epsilon^{ij} - C_{q-3} \epsilon^{ij} \right] , \qquad (25)$$

- This is a conductivity which is electric field independent (a linear conductivity).

The membrane paradigm Cont'd

- For the nonlinear conductivity, we first determine the position of the singular shell r_s,

$$\det\gamma_{\mu\nu}(r_s) = [\tilde{B}^2 g_{tt} + \tilde{E_x}^2 g_{yy} + \tilde{E_y}^2 g_{xx} + g_{tt} g_{xx} g_{yy}]_{r\to r_s} = 0, \tag{26}$$

then we evaluate (25) at $r = r_s$.

- For the Minkowski embedding, the regularity of the gauge fields at r_0 requires $f_{rt}(r_0) = \tilde{A}_t'(r_0) = 0$, therefore

$$\sigma^{ij} = -\mathcal{N}' C_{q-3}(r_0)\epsilon^{ij} = \mathcal{N}' \frac{\hat{J}_t}{\tilde{B}}\epsilon^{ij}, \tag{27}$$

General assumptions

- The logic of 3+1 dimensions is the same as that of 2+1 dimensions.
- The induced metric and gauge fields

$$
\begin{aligned}
ds^2 &= g_{tt}dt^2 + \sum_{i=1}^{3} g_{ii}dx_i^2 + g_{rr}dr^2 + ds^2_{(l)}, \\
\tilde{A} &= \tilde{A}_t(r)dt + \tilde{B}_y zdx + \tilde{B}_z xdy + \tilde{B}_x ydz + \tilde{a}, \quad (28)
\end{aligned}
$$

  where the field theory directions are t, x, y, z and $l = q - 4$.
- We keep all the components of the magnetic field for generality.

The DBI and WZ terms

- The leading order Lagrangian is the same

$$\mathcal{L}_{\mathrm{DBI}}^{(0)} = -e^{-\phi}\sqrt{\Theta\kappa}\sqrt{-g_{tt}g_{rr} - \tilde{A}_t'^2}, \tag{29}$$

$$\kappa \equiv \det\gamma_{ij} = g_{xx}g_{yy}g_{zz} + \sum_{i=1}^{3} g_{ii}^2 \tilde{B}_i^{\,2}, \tag{30}$$

with $i, j = x, y, z$.

- The expression for $\tilde{A}_t'$ is the same (with different κ).

The DBI and WZ terms Cont'd

- The OSM

$$s_{mn}dx^m dx^n = g_{tt}\mathcal{G}^2 dt^2 + g_{rr}\mathcal{G}^2 dr^2 + \frac{\kappa\delta^{ij}g_{ij} - B_i B_j g_{ii} g_{jj}}{g_{xx}g_{yy}g_{zz}} dx^i dx^j, \tag{31}$$

where $\mathcal{G}^2$ is the same as that of the 2+1 dimensional case.

- The non-vanishing components of θ^{mn}

$$\theta^{tr} = -\frac{e^{\phi}\hat{J}_t}{\sqrt{-\det\gamma_{mn}}\sqrt{\Theta}}, \quad \theta^{ij} = -\frac{1}{\kappa}\epsilon^{ijk}\tilde{B}_k g_{kk}, \tag{32}$$

- The sub-leading action

The DBI and WZ terms Cont'd

$$S^{(2)}_{\mathrm{DBI}} = -\mathcal{N}' \int dtd\vec{x}dr \left[\frac{\sqrt{-s}}{4g_5^2} s^{mp} s^{nq} f_{mn} f_{pq} + \frac{1}{8} \epsilon^{mnpql} f_{mn} f_{pq} Q_l \right], \tag{33}$$

$$\begin{aligned} g_5^2 &= \frac{\sqrt{-s}}{e^{-\phi}\sqrt{-\det\gamma_{mn}}\sqrt{\Theta}}, \quad \epsilon_{txyzr} = 1 \\ Q_l &= -\frac{1}{8} e^{-\phi} \sqrt{-\det\gamma_{mn}} \sqrt{\Theta} \epsilon_{mnpql} \theta^{mn} \theta^{pq} = \frac{\tilde{B}_l g_{ll} \hat{J}_t}{\kappa}. \end{aligned} \tag{34}$$

- The WZ term can be considered in a similar way as the 2+1 dimensional case, so we omit it.

The membrane paradigm

- The canonical momentum

$$\mathcal{J}^i(r) = -\frac{\mathcal{N}'}{g_5^2}\sqrt{-s}f^{ri} - \mathcal{N}'\epsilon^{jik}f_{jt}Q_k\,, \tag{35}$$

- The conductivity

$$\begin{aligned}\sigma^{ii} &= \frac{\mathcal{N}'}{g_5^2}\sqrt{\frac{s}{s_{tt}s_{rr}}}\frac{1}{s_{ii}}\Big|_{r\to r_s}\,,\\ \sigma^{ij} &= -\mathcal{N}'Q_k\epsilon^{kij}\Big|_{r\to r_s}\,,\end{aligned} \tag{36}$$

- For an off-diagonal metric in time and space

The membrane paradigm Cont'd

$$ds^2 = s_{tt}dt^2 + s_{rr}dr^2 + s_{xx}dx^2 + 2s_{tx}dtdx + s_{yy}dy^2 + s_{zz}dz^2,$$

The conductivity (Kim, Shock and Tarrio, '11)

$$\begin{aligned} \sigma^{ii} &= \frac{\mathcal{N}'}{g_5^2}\frac{\sqrt{-s}}{\sqrt{s_{rr}}\sqrt{-s_{tt}s_{xx}+s_{tx}^2}}\frac{\sqrt{s_{xx}}}{s_{ii}}\Big|_{r\to r_s}, \\ \sigma^{ij} &= -\mathcal{N}'Q_k\epsilon^{kij}\Big|_{r\to r_s}, \end{aligned} \tag{37}$$

which agree with the previous results when $s_{tx} = 0$.

- Advantages: matrix calculations done by Mathmatica, without particular combinations.

D3-D7' model

- The model was proposed in arXiv:1003.4965[hep-th] (Bergman, Jokela, Lifshytz and Lippert).
- The configuration

	0	1	2	3	4	5	6	7	8	9
D3	•	•	•	•						
D7	•	•	•		•	•	•	•	•	

- The configuration is non-supersymmetric and unstable.

D3-D7' model Cont'd

- To ensure the stability, we assume that D7-brane wraps $S^2 \times S^2$ inside S^5 and we introduce the following magnetic fluxes through S^2's

$$\tilde{F} = \frac{1}{2}(f_1 d\Omega_2^{(1)} + f_2 d\Omega_2^{(2)}), \quad f_i = 2\pi\alpha' n_i, \tag{38}$$

$d\Omega_2^{(i)} = \sin\theta_i \wedge d\phi_i, n_i$ are integers.

- The gauge field

$$\tilde{A} = \tilde{A}_t dt + \tilde{B} x dy,$$

- Assuming that the scalars $z(= x_3)$ and $\psi(= x_9)$ are functions of r only,

D3-D7' model Cont'd

the induced metric and the RR 4-form

$$\begin{aligned} ds^2_{D7} &= r^2(-f(r)dt^2 + dx^2 + dy^2) + (\frac{1}{r^2 f(r)} + r^2 z'^2(r) \\ &\quad + \psi'^2(r))dr^2 + \cos^2\psi(d\Omega_2^{(1)})^2 + \sin^2\psi(d\Omega_2^{(2)})^2, \\ C_4 &= r^4 dt\wedge dx\wedge dy\wedge dr + \frac{1}{2}c(r)d\Omega_2^{(1)}\wedge d\Omega_2^{(2)}, \end{aligned} \tag{39}$$

where $f(r) = 1 - r_H^4/r^4$ and

$$c(r) = \frac{1}{8\pi^2}\int_{S^2\times S^2} C_4 = \psi(r) - \frac{1}{4}\sin 4\psi(r) - \psi(\infty) + \frac{1}{4}\sin 4\psi(\infty). \tag{40}$$

D3-D7' model Cont'd

- The leading DBI and WZ terms

$$\begin{aligned}\mathcal{L}^{(0)}_{\rm DBI} &= -\sqrt{\Theta\kappa}\sqrt{1+r^4fz'^2+r^2f\psi'^2-\tilde{A}_t'^2}\,,\\ \mathcal{L}^{(0)}_{\rm WZ} &= f_1f_2r^4z'-2c(r)\tilde{B}\tilde{A}_t{}'\,,\end{aligned} \tag{41}$$

$$\Theta=\left(\cos^4\psi+\frac{1}{4}f_1^2\right)\left(\sin^4\psi+\frac{1}{4}f_2^2\right),\quad \kappa=\tilde{B}^2+r^4, \tag{42}$$

- For black hole embedding, the conductivity reads

D3-D7’ model Cont’d

$$\sigma^{xx} = \frac{\mathcal{N}' r_s^2}{\tilde{B}^2 + r_s^4} \times \sqrt{\bar{J}_t^{\,2} + \left(\cos^4\psi + \frac{1}{4} f_1^2\right)\left(\sin^4\psi + \frac{1}{4} f_2^2\right)(\tilde{B}^2 + r_s^4)},$$

$$\sigma^{xy} = -\mathcal{N}' \left(\frac{\tilde{B}\bar{J}_t(r_s)}{\tilde{B}^2 + r_s^4} + \frac{c(r_s)}{2} \right), \tag{43}$$

where $\bar{J}_t = \hat{J}_t + c(\psi)/2 \times \tilde{B}$.

- For Minkowski embedding,

D3-D7' model Cont'd

$$\sigma^{xx} = 0, \quad \sigma^{xy} = -\frac{1}{2}\mathcal{N}'c(r_0) = \mathcal{N}'\frac{\hat{J}_t}{\tilde{B}}, \tag{44}$$

- The results obtained by OSM method agree to arXiv: 1003.4965, where the real-action method was used.
- Black hole embedding-metal phase, Minkowski embedding-fractional QHE phase.

D2-D8' model

- Holographic integer QHE model proposed in arXiv:1101.3329[hep-th] by Jokela, Jarvinen and Lippert.
- The configuration

$$\begin{array}{ccccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \text{D2} & \bullet & \bullet & \bullet & & & & & & & \\ \text{D8} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \end{array} \tag{45}$$

- Here the D8 brane wraps $S^2 \times S^3$ inside S^6 and this configuration is also nonsupersymmetric and unstable.
- We have to introduce the following magnetic field on the internal S^2 to ensure the stability

D2-D8' model Cont'd

$$\tilde{F} = \tilde{h} \sin\theta d\theta \wedge d\phi, \quad (46)$$

- The world volume gauge field

$$\tilde{A} = \tilde{A}_t dt + \tilde{B} x dy.$$

- With an assumption that the scalar $\psi (= x_9)$ is a function of only the radial coordinate r, the pull-back of the metric and the RR 4-from field

$$\begin{aligned} ds^2_{D8} &= r^{\frac{5}{2}}(-f(r)dt^2 + dx^2 + dy^2) + r^{-\frac{5}{2}}(\frac{1}{f(r)} + r^2\psi'^2) \\ &+ r^{-\frac{1}{2}} \sin^2\psi d\Omega_2^2 + r^{-\frac{1}{2}} \cos^2\psi d\Omega_3^2, \end{aligned} \quad (47)$$

D2-D8' Cont'd

$$\begin{aligned} C_5 &= c(\psi(r))d\Omega_2 \wedge d\Omega_3, \\ c(\psi(r)) &= \frac{5}{8}\left(\sin\psi - \frac{1}{6}\sin 3\psi - \frac{1}{10}\sin 5\psi\right) \end{aligned} \tag{48}$$

$f(r) = 1 - r_H^5/r^5, e^{-\phi} = r^{-\frac{5}{4}}$.

- The leading order DBI and WZ terms

$$\begin{aligned} \mathcal{L}^{(0)}_{\rm DBI} &= -\sqrt{\Theta\kappa}\sqrt{1 + r^2 f\psi'^2 - \tilde{A}_t{}'^2}, \\ \mathcal{L}^{(0)}_{\rm WZ} &= -c(r)\tilde{A}_t{}'\tilde{B}, \end{aligned} \tag{49}$$

D2-D8' Cont'd

$$\Theta = (r^{-1}\sin^4\psi + \tilde{h}^2)r^{-\frac{3}{2}}\cos^6\psi, \quad \kappa = \tilde{B}^2 + r^5.$$

- Conductivity for black hole embedding,

$$\begin{aligned} \sigma^{xx} &= \frac{\mathcal{N}' r_s^{\frac{5}{2}}}{\tilde{B}^2 + r_s^5}\sqrt{\bar{J}_t^{\,2} + \cos^6\psi_s(\tilde{h}^2 r_s + \sin^4\psi_s)(\tilde{B}^2 + r_s^5)}, \\ \sigma^{xy} &= \mathcal{N}'\left(\frac{\hat{J}_t\tilde{B}}{\tilde{B}^2 + r_s^5} + c(r_s)\right), \end{aligned} \tag{50}$$

where $\bar{J}_t = \hat{J}_t + c(\psi)\tilde{B}$.

D2-D8 Cont'd

- For Minkowski embedding,

$$\sigma^{xx} = 0, \quad \sigma^{xy} = -\mathcal{N}' c(r_0) = \mathcal{N}' \frac{\tilde{J}_t}{\tilde{B}} = \frac{N}{2\pi}. \tag{51}$$

- The above results agree to those obtained in arXiv:1101.3329[hep-th], where the real-action method was adopted.
- It was argued that the black hole embedding described the metal phase while the Minkowski embedding described integer QHE state with the filling fraction $\mathcal{N}' = 3N/(4\pi)$.

AdS space in light-cone frame

Such a metric can be obtained by the transformation $x^+ = b(t+x), x^- = 1/(2b)(t-x)$.

$$\begin{aligned} ds^2 &= g_{++}dx^{+2} + 2g_{+-}dx^+dx^- + g_{--}dx^{-2} \\ &\quad + g_{yy}dy^2 + g_{zz}dz^2 + g_{rr}dr^2 \\ &\quad + R^2\cos^2\theta d\Omega_3^2 + R^2\sin^2\theta d\phi^2, \end{aligned} \quad (52)$$

$$\begin{aligned} g_{++} &= \frac{(1-f(r))r^2}{4b^2R^2},\ g_{+-} = -\frac{1+f(r)r^2}{2R^2},\ g_{--} = \frac{(1-f(r))b^2r^2}{R^2}, \\ g_{yy} &= g_{zz} = \frac{r^2}{R^2},\ g_{rr} = \frac{R^2}{r^2f(r)},\ f(r) = 1-\frac{r_H^4}{r^4}, \end{aligned} \quad (53)$$

where R is AdS_5 radius and b is the parameter related to the rapidity.

Probe D7-branes

- The world-volume gauge field

$$\tilde{A} = \tilde{h_+}(r)dx^+ + \tilde{h_-}(r)dx^- + \tilde{B_b}ydz,$$

- The DBI action

$$\begin{aligned} S_{D7} &= -N_f T_{D7} \int d^8\xi \sqrt{-\det(g_{D7} + 2\pi\alpha' F)} \\ &= \mathcal{N} \int d^5\xi \mathcal{L}, \end{aligned} \tag{54}$$

where $\mathcal{N} \equiv 2\pi^2 N_f T_{D7}$ and

Probe D7-brane Cont'd

$$\mathcal{L} = -\sqrt{(\tilde{B}^2 + g_{yy}g_{zz})g_{\Omega\Omega}^3}\sqrt{Gpm}, \quad (55)$$

$$\begin{aligned} Gpm &= -\breve{g}g_{rr}^{D7} - g_{--}g_{yy}\tilde{h_+}'^2 \\ &\quad -g_{++}g_{yy}\tilde{h_-}'^2 + 2g_{+-}g_{yy}\tilde{h_+}'\tilde{h_-}', \end{aligned} \quad (56)$$

$$g_{rr}^{D7} = g_{rr} + R^2\theta'(r)\,, \qquad g_{\Omega\Omega} = R^2\cos^2\theta\,, \qquad \breve{g} = g_{+-}^2 - g_{++}g_{--}\,. \quad (57)$$

- Two conserved currents conjugate to two cyclic coordinates $\tilde{h}_+$ and $\tilde{h}_-$:

The OSM

$$\hat{J}_+ = \frac{\partial \mathcal{L}}{\partial \tilde{h'_+}} = \frac{g_{zz} g_{\Omega\Omega}^3}{\mathcal{L}} \left(g_{--} g_{yy} \tilde{h'_+} - g_{+-} g_{yy} \tilde{h'_-} \right) ,$$
$$\hat{J}_- = \frac{\partial \mathcal{L}}{\partial \tilde{h'_-}} = \frac{g_{zz} g_{\Omega\Omega}^3}{\mathcal{L}} \left(g_{++} g_{yy} \tilde{h'_-} - g_{+-} g_{yy} \tilde{h'_+} \right) . \quad (58)$$

- The OSM

$$ds^2 = s_{++} dx^{+2} + 2 s_{+-} dx^+ dx^- + s_{--} dx^{-2} + s_{yy} dy^2 + s_{zz} dz^2 + s_{rr} dr^2 , \quad (59)$$

The OSM Cont'd

$$s_{++} = g_{++} + \frac{\breve{g}_1{}^2}{\chi}, \quad s_{+-} = g_{+-} + \frac{\breve{g}_1 \breve{g}_2}{\chi}, \quad s_{--} = g_{--} + \frac{\breve{g}_2{}^2}{\chi},$$
$$s_{yy} = g_{yy} + \frac{\tilde{B}}{g_{zz}}, \quad s_{zz} = g_{zz} + \frac{\tilde{B}}{g_{yy}}, \quad s_{rr} = \frac{\xi_1 g_{\Omega\Omega} g^{D7}_{rr}}{\chi}, \tag{60}$$

$$\chi = \frac{\xi_1 g_{--} g^3_{\Omega\Omega} - \breve{g}_2{}^2 + \breve{g} J^2_+}{g_{--}}, \quad \xi_1 = \breve{g}(\tilde{B}^2 + g_{yy} g_{zz}), \tag{61}$$

The OSM Cont'd

$$\begin{aligned} \breve{g} &= g_{+-}^2 - g_{--}g_{++}\,, \quad \breve{g}_1 = g_{+-}J_- + g_{++}J_+\,, \\ \breve{g}_2 &= g_{--}J_- + g_{+-}J_+\,. \end{aligned} \tag{62}$$

- Note that the background gauge field information is geometrized in $\chi, \chi_1, \breve{g}_1, \breve{g}_2$ and the effective coupling

$$g_5 = \sqrt{\frac{g_{yy}g_{zz}\chi\breve{g}}{\xi_1^2 g_{\Omega\Omega}^3}}\,. \tag{63}$$

The conductivity

- Consider an electric field along y direction, $F_{y+} = E_b$, and find the singular shell position by

$$\xi(r_s) = \det \gamma_{\mu\nu} = \tilde{E_b}^2 g_{--}(r_s) g_{zz}(r_s) - \xi_1(r_s) = 0\,, \qquad (64)$$

which yields

$$r_s = \left(2b^2 \tilde{E_b}^2 R^4 \left(t^4 - \mathcal{B}^2 + \sqrt{t^4 + (\mathcal{B}^2 + t^4)^2}\right)\right)^{1/4}\,, \qquad (65)$$

where

$$t = \frac{\pi R b T}{\sqrt{2b\tilde{E_b}}}\,, \qquad \mathcal{B} = \frac{\tilde{B}}{2b\tilde{E_b}}\,. \qquad (66)$$

The conductivity Cont'd

- Substituting the above data into (37), we obtain

$$\sigma^{yy} = \sigma_0 \frac{\sqrt{\mathcal{F}_- J^2 + t^4 \sqrt{\mathcal{F}_- \mathcal{F}_+}}}{\mathcal{F}_+}, \qquad \sigma^{yz} = \bar{\sigma}_0 \frac{\mathcal{B}}{\mathcal{F}_+}, \tag{67}$$

where

$$\mathcal{F}_\pm = \frac{\sqrt{(\mathcal{B}^2 + t^4)^2 + t^4} \pm \mathcal{B}^2 + t^4}{2}, \qquad J = \frac{\hat{J}^+}{R^3 b \cos^3\theta(r_s)(2b\tilde{E})^{3/2}}, \tag{68}$$

The conductivity Cont'd

and

$$\sigma_0 = \mathcal{N}' R^3 \sqrt{2b^3 \cos^6\theta(r_s)\tilde{E}_b}, \quad \bar{\sigma}_0 = \mathcal{N}' \frac{\hat{J}^+}{b\tilde{E}_b}. \qquad (69)$$

- This agrees to Kim, Kiritsis and Panagopoulos'10 obtained by real-action method.
- At $B = 0$

$$\mathcal{F}_+ = \mathcal{F}_- = t^2 \mathcal{A}, \qquad \mathcal{A} = \frac{t^2 + \sqrt{1+t^4}}{2}, \qquad (70)$$

and the Ohmic conductivity is simplified as

$$\sigma^{yy} = \sigma_0 \sqrt{\frac{J^2}{t^2 \mathcal{A}} + \frac{t^3}{\sqrt{\mathcal{A}}}}. \qquad (71)$$

The conductivity Cont'd

- At $B = 0$, in the regime $t \ll J^{1/3}$ and $1 \ll J$,

$$\sigma^{yy} \sim \frac{\hat{J}^+}{t\sqrt{t^2 + \sqrt{1+t^4}}} \sim \begin{cases} \hat{J}^+/t & t \ll 1 \\ \hat{J}^+/t^2 & t \gg 1 \end{cases}, \qquad (72)$$

  where t can be tuned by changing b at fixed $\tilde{E}_b$ and RT.
- At $t \ll 1$ we obtained the resistivity linear in temperature. Interpreting b as a doping parameter we see a typical cross over behavior of the strange metal.
- At $B \neq 0$, in the regime, $t \ll \sqrt{\mathcal{B}}$, $t \ll \frac{J}{\mathcal{B}}$, and $\mathcal{B} \gg 1$, the conductivity (67) is approximated as

The conductivity Cont'd

$$\sigma^{yy} \sim \frac{\hat{J}^+ t^2}{\mathcal{B}^2}, \qquad \sigma^{yz} \sim \frac{\hat{J}^+}{\mathcal{B}}, \quad \Rightarrow \quad \frac{\sigma^{yy}}{\sigma^{yz}} \sim \frac{t^2}{\mathcal{B}}, \qquad (73)$$

where the Ohmic conductivity is dominated by the first term.

- The temperature dependence ($\sim t^2$) of the inverse Hall angle is the typical property of the strange metal. Note that, if $\mathcal{B} \gg J$ ($t \ll 1$), the ohmic conductivity is $1/T$, and, if $J \gg \mathcal{B}$ ($t \ll \infty$), it is possible to cross over to $1/T^2$.

Summary and discussion

- We studied the holographic DC conductivities of various systems using the OSM method.
- We proposed a new method to compute the DC conductivity based on OSM. We showed that all results obtained by the OSM method agreed to the results obtained by the real-action method.
- OSM can be used to study other transport coefficients and effective temperature induced by the effective world volume horizon, contrary to the real-action method.

Thank you!