

# 常力驱动自旋孤子的交流振荡

赵立臣

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孟令正

博士生

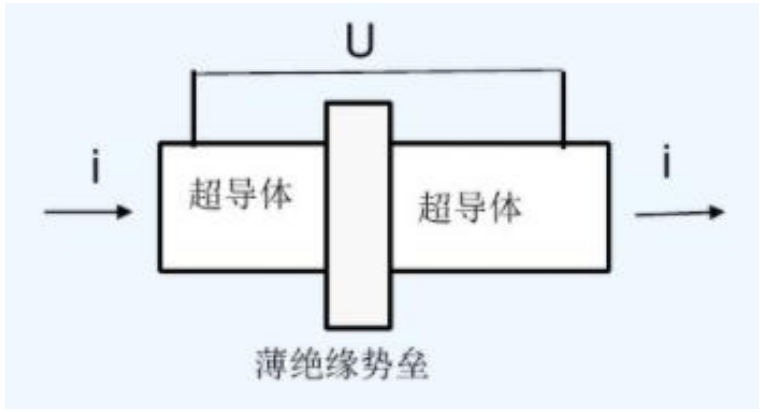
西北大学

# 报告结构

1. 直流驱动产生交流振荡的现象
2. 自旋孤子解
3. 自旋孤子的有效质量
4. 色散关系的应用
5. 总结与讨论

# 1. 直流驱动产生交流振荡的现象

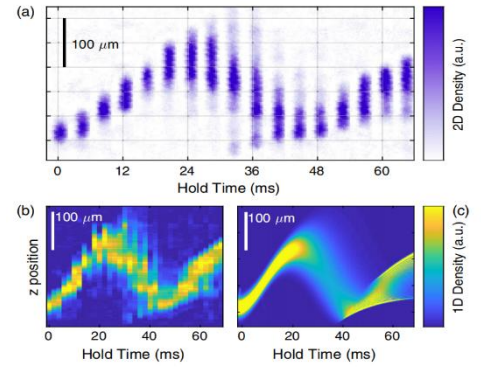
## 约瑟夫森振荡



现象：直流电压产生交流电  
 机制：量子相位相干  
 应用：磁强计、精密测量基本电荷  
 约瑟夫森电压标准等

B. Josephson, Phys. Lett. 1, 251 (1962).

## 布洛赫振荡



Z. A. Geiger, K. M. Fujiwara, et al., Phys. Rev. Lett. 120, 213201 (2018).

$$V = -V_L \cos^2(\pi z/d) \exp(-r^2/2\sigma^2) + m\omega_0^2 z^2/2.$$

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V(r, z)\Psi + \mathcal{F}z\Psi + g|\Psi|^2\Psi.$$

现象：常力驱动粒子产生周期振荡运动  
 机制：周期势场下能带结构导致的正负有效质量

应用：利用粒子的运动测量能带等

F. Bloch, Z. Phys. 52, 555 (1928).

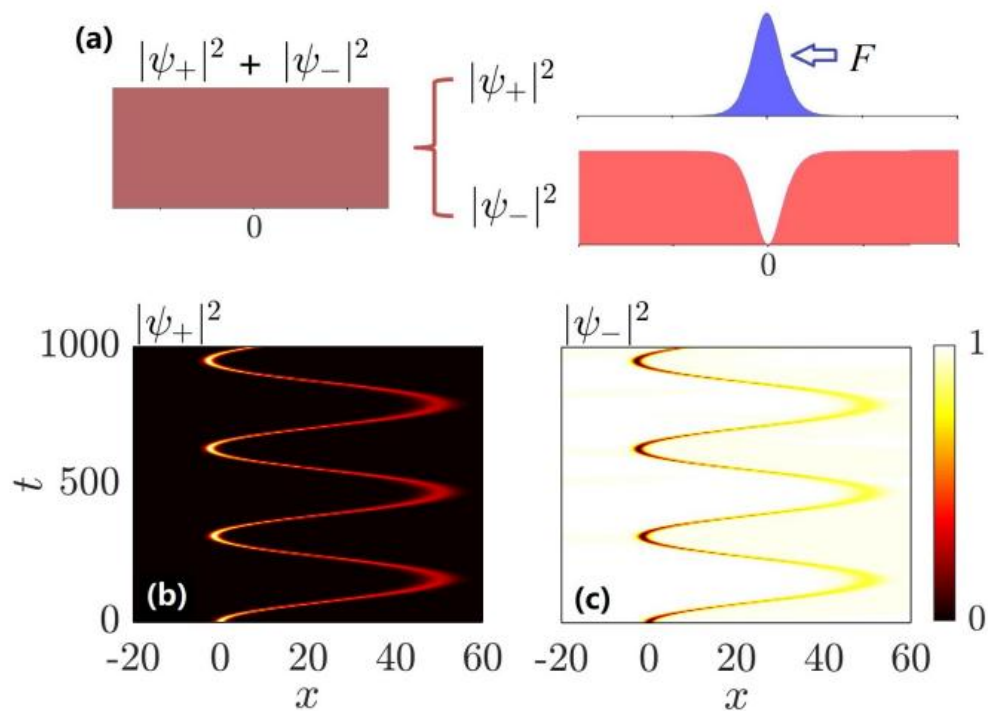
C. Zener, Proc. R. Soc. London, Ser. A 145, 523 (1934).

# 1. 直流驱动产生交流振荡的现象

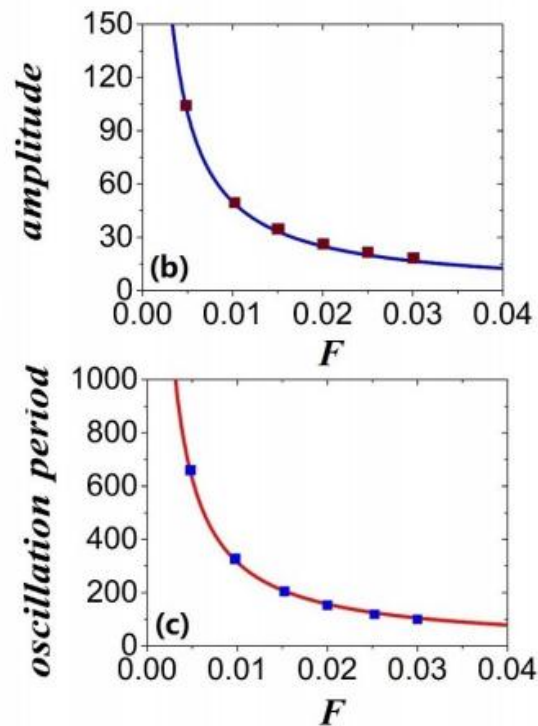
## 常力驱动自旋孤子的交流振荡

$$i \frac{\partial \psi_+}{\partial t} = -\frac{1}{2} \frac{\partial^2 \psi_+}{\partial x^2} + (g_1 |\psi_+|^2 + g_2 |\psi_-|^2) \psi_+, \quad \leftarrow -F x \psi_+$$
$$i \frac{\partial \psi_-}{\partial t} = -\frac{1}{2} \frac{\partial^2 \psi_-}{\partial x^2} + (g_2 |\psi_+|^2 + g_3 |\psi_-|^2) \psi_-.$$

无周期势场



自旋孤子振荡现象



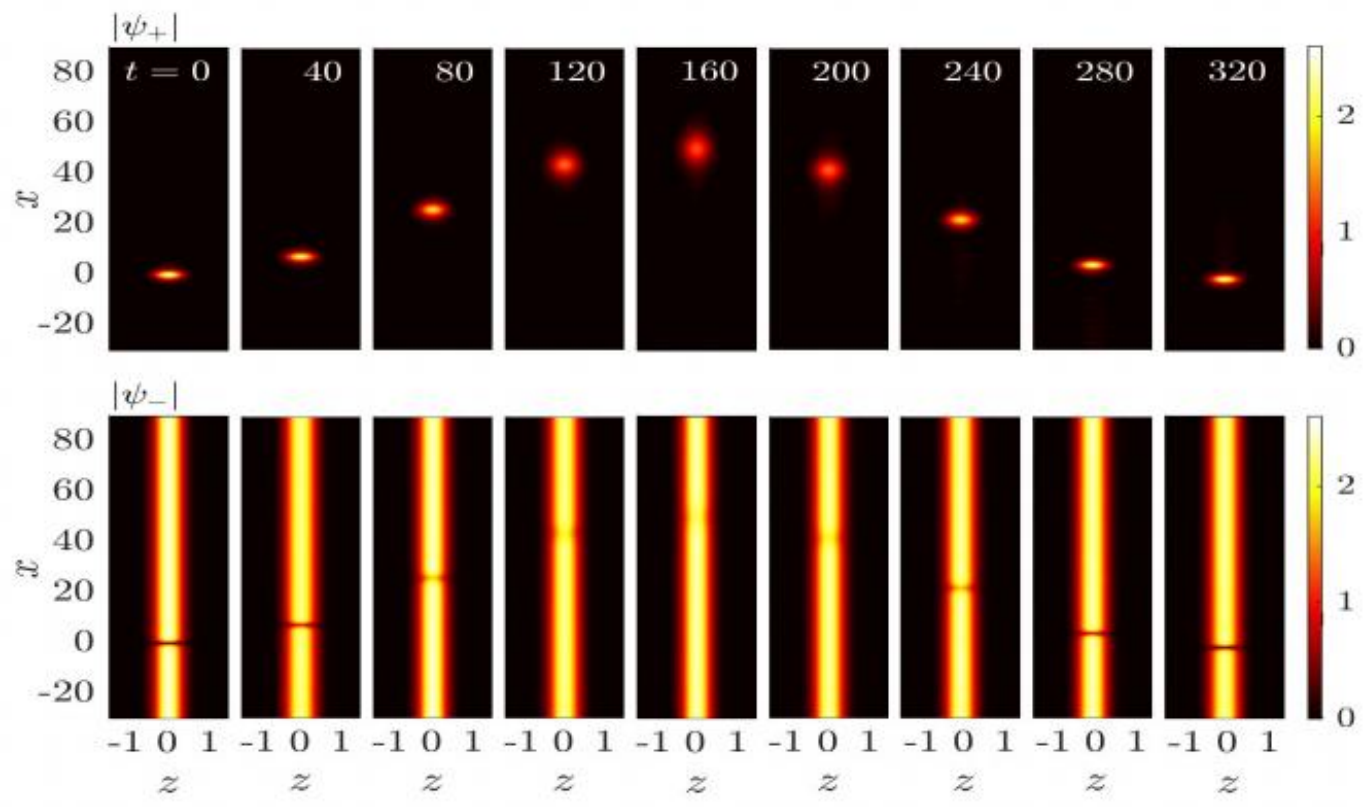
振荡规律

# 1. 直流驱动产生交流振荡的现象

## 实验观测的可能性

*K. M. Mertes, J. W. Merrill, et al.,  
Phys. Rev. Lett. 99, 190402 (2007).*

$$g_1 - g_2 = g_2 - g_3$$

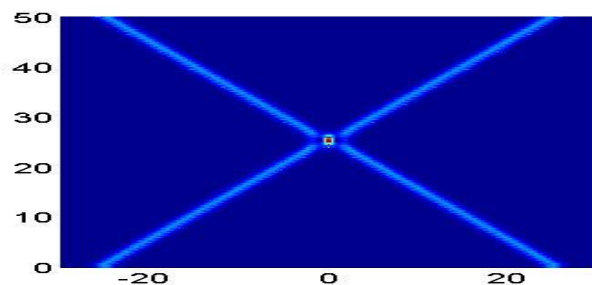
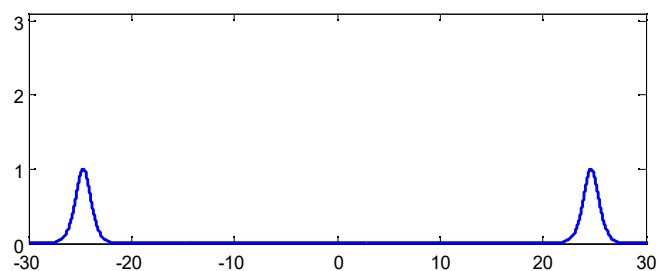
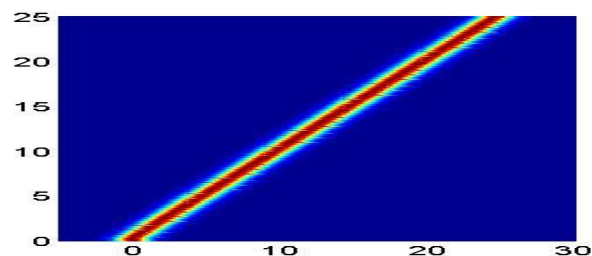
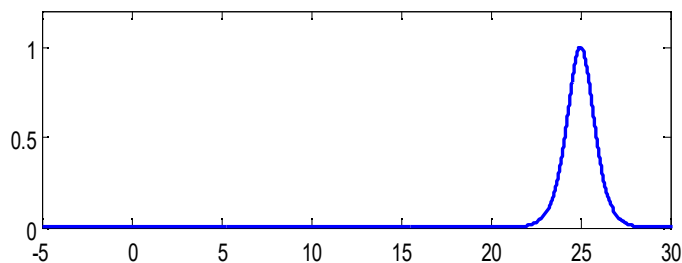


## 2. 自旋孤子解

### 孤子简介

孤子是具有稳定结构的局域波，且具有演化稳定性和碰撞不变形的特点。常见的孤子类型有：亮孤子、暗孤子、Kink、反暗孤子等。

孤子特性：稳定性和粒子性



存在于海洋、大气、光纤、等离子体、玻色凝聚等多种物理系统。

## 2. 自旋孤子解

### 为什么BEC中存在孤子激发?

量子多体系统

$$H = -\frac{\hbar^2}{2m} \sum_{i=1}^{N_a} \frac{\partial^2}{\partial x_i^2} + g \sum_{i<j} \delta(x_i - x_j) - \frac{\hbar^2}{2m} \sum_{\alpha=1}^{N_b} \frac{\partial^2}{\partial x_\alpha^2} + g \sum_{\alpha<\beta} \delta(x_\alpha - x_\beta) + \tilde{g} \sum_{i,\alpha} \delta(x_i - x_\alpha), \quad (1)$$

平均场近似

当原子数大于 $10^3$

已经被诸多实验支持

$$iq_{jz} + q_{jtt} + 2 \sum_{p=1}^2 |q_p|^2 q_j = 0, \quad j = 1, 2 \quad \text{G-P 方程}$$

带有非线性的单粒子模型



# 2. 自旋孤子解

## 物质波孤子的实验观测

暗孤子:

*Burger S, et al, Phys. Rev. Lett. 83, 5198 (1999);*  
*Denschlag J, et al, Science 287, 97 (2000).*

亮孤子:

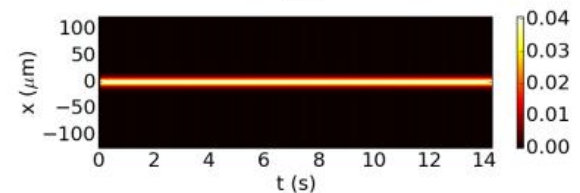
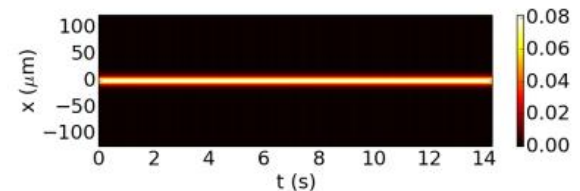
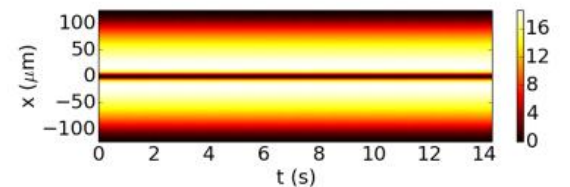
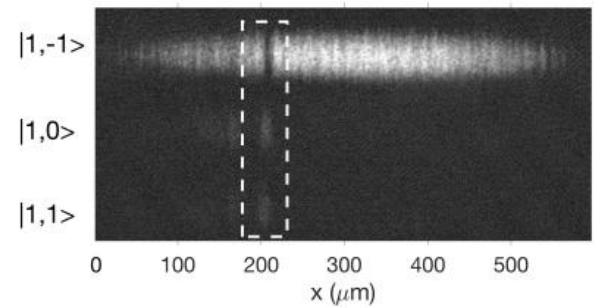
*Khaykovich L, et al. Science 296, 1290 (2002).*

暗-亮孤子:

*C. Becker, et al., Nature Phys. 4, 496 (2008);*  
*C. Hamner, et al., Phys. Rev. Lett. 106, 065302 (2011);*  
*P. G. Kevrekidis et al., Rev. Phys. 1, 140 (2016);*  
*A. Farolfi, et al, Phys. Rev. Lett. 125, 030401 (2020).*

暗-亮-亮孤子

*T.M. Bersano, et al., Phys. Rev. Lett. 120, 063202 (2018);*  
*X. Chai, et al, Phys. Rev. Lett. 125, 030402 (2020).*

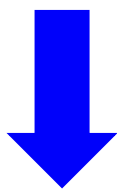


## 2. 自旋孤子解

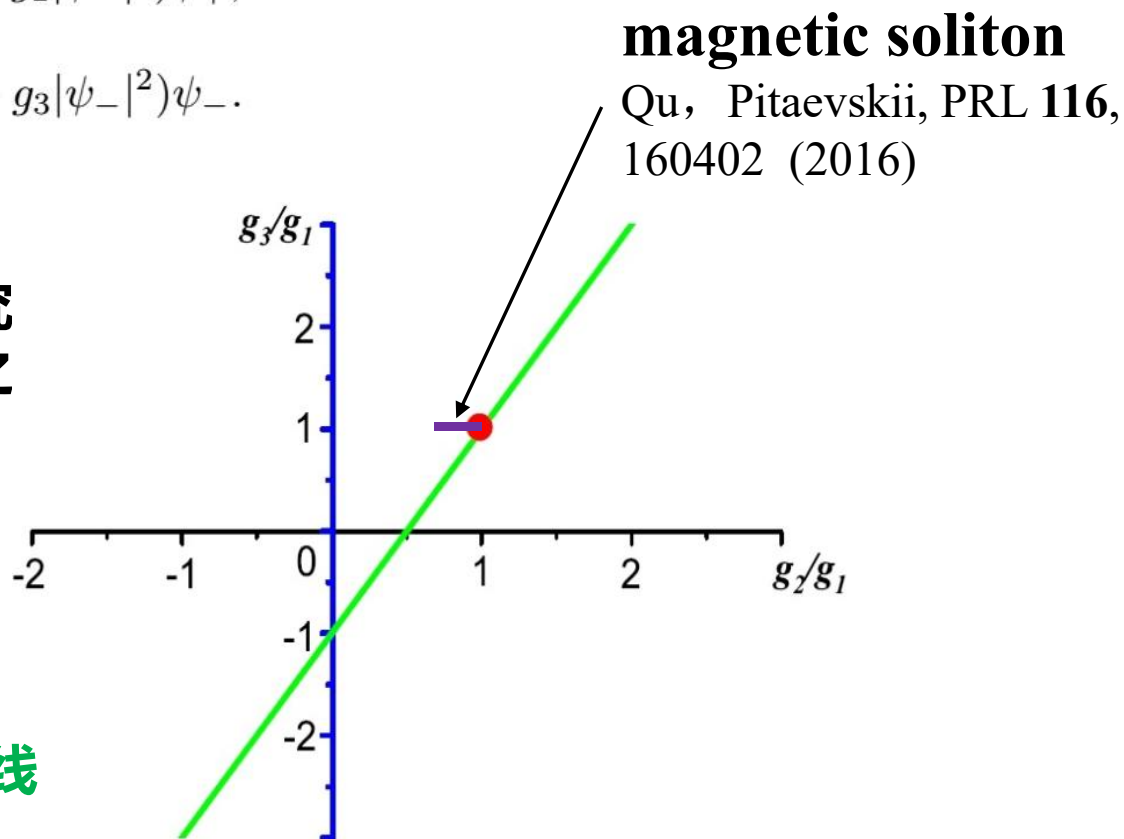
### 两组分玻色凝聚中的孤子解

$$i\frac{\partial\psi_+}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi_+}{\partial x^2} + (g_1|\psi_+|^2 + g_2|\psi_-|^2)\psi_+,$$
$$i\frac{\partial\psi_-}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi_-}{\partial x^2} + (g_2|\psi_+|^2 + g_3|\psi_-|^2)\psi_-.$$

之前大多数研究都集中研究左图的红点上，组分密度之和总是反暗孤子或暗孤子



孤子精确解可存在于绿线



*G. C. Katsimiga, J. Stockhofe, et al, Dark-bright soliton interactions beyond the integrable limit, Phys. Rev. A 95, 013621 (2017)*

# 2. 自旋孤子解

PRL **116**, 160402 (2016)

PHYSICAL REVIEW LETTERS

week ending  
22 APRIL 2016



## Magnetic Solitons in a Binary Bose-Einstein Condensate

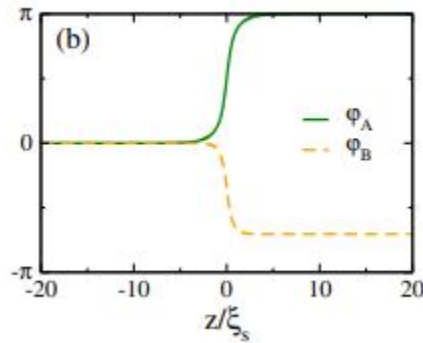
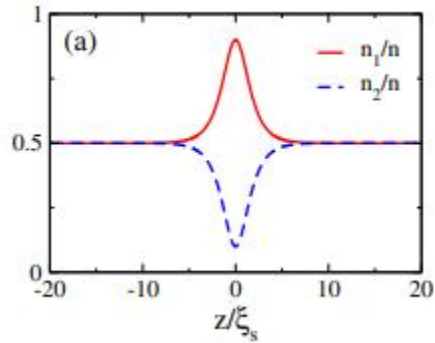
Chunlei Qu,<sup>1</sup> Lev P. Pitaevskii,<sup>1,2</sup> and Sandro Stringari<sup>1</sup>

<sup>1</sup>INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, 38123 Povo, Italy

<sup>2</sup>Kapitza Institute for Physical Problems RAS, Kosygina 2, 119334 Moscow, Russia

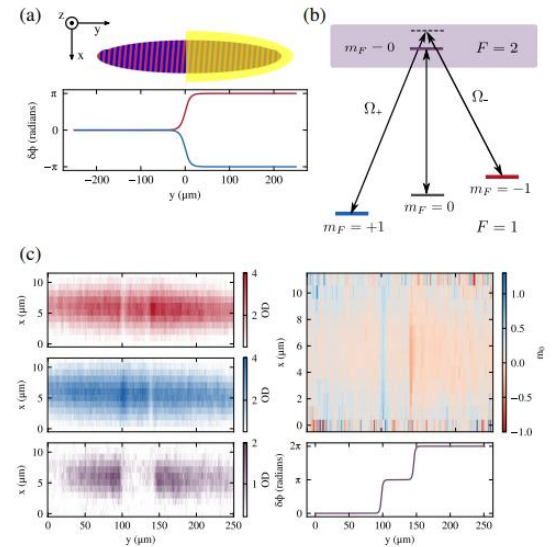
(Received 19 February 2016; published 20 April 2016)

$$\mathcal{E} = \sum_j \left[ \frac{\hbar^2}{2m} |\nabla \psi_j|^2 + V_{\text{ext}} |\psi_j|^2 + \sum_l \frac{g_{jl}}{2} |\psi_j|^2 |\psi_l|^2 \right]$$



assume  $g_{11} = g_{22} = g$      $\delta g \equiv g - g_{12} \ll g$ ,

$$\epsilon = n \hbar c_s \sqrt{1 - V^2/c_s^2}$$



A. Farolfi, et al, Phys. Rev. Lett. **125**, 030401 (2020).

X. Chai, et al, Phys. Rev. Lett. **125**, 030402 (2020).

# 2. 自旋孤子解

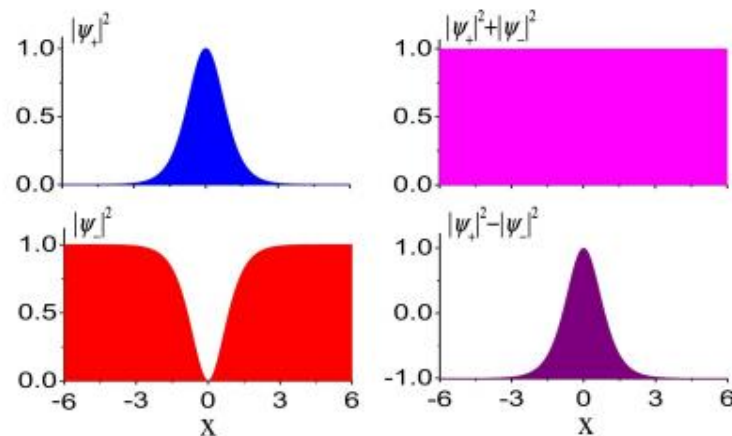
## 自旋孤子的概念

已有方法失效  
的不可积模型

约束条件  
解耦变换

可解  
模型

$$S_{x,y,z} = (\psi_+^*, \psi_-^*) \sigma_{x,y,z} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$



自旋孤子密度分布特征

存在条件

$$\frac{g_3 - g_2}{g_1 - g_2} = -1$$

$$|\psi_+|^2 + |\psi_-|^2 = s$$

$$\psi_+(x, t) = \sqrt{\frac{c_s^2 - v^2}{g_2 - g_1}} \operatorname{sech}[\sqrt{c_s^2 - v^2}(x - vt)] e^{\frac{1}{2}i[-g_1 st - g_2 st + 2v(x - vt)]}, \quad (1)$$

$$\psi_-(x, t) = \sqrt{s} \left( \sqrt{1 - \frac{v^2}{c_s^2}} \tanh[\sqrt{c_s^2 - v^2}(x - vt)] + \frac{iv}{c_s} \right) e^{-i(-g_1 + 2g_2)st}, \quad (2)$$

where  $c_s = \sqrt{(g_2 - g_1)s}$  denotes the the speed of sound

### 3. 自旋孤子的有效质量

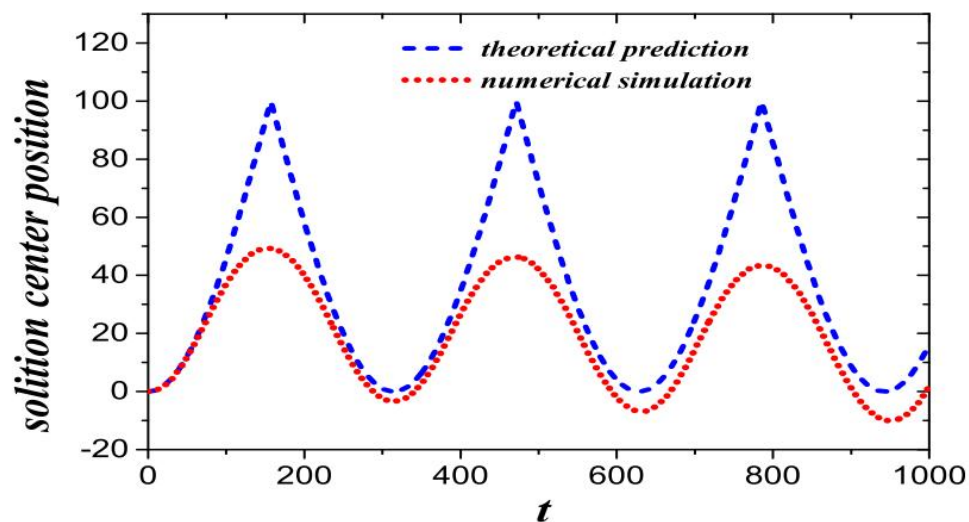
精确解得到的色散关系

$$E_s = \sqrt{c_s^2 - v^2}$$

每个组分的粒子数

$$N_+ = \int_{-\infty}^{\infty} |\psi_+|^2 dx = \frac{2\sqrt{c_s^2 - v^2}}{c_s^2}$$

$$N_- = \int_{-\infty}^{\infty} (|\psi_-|^2 - 1) dx = -\frac{2\sqrt{c_s^2 - v^2}}{c_s^2}$$



精确解描述的每个组分粒子数依赖速度，使得自旋孤子解不能描述加速过程。

# 3. 自旋孤子的有效质量

## 保模假设下的Lagrangian变分

孤子粒子数守恒

$$\psi_+ = f(T) \operatorname{sech}\left[\frac{X - b(T)}{w(T)}\right] e^{i\phi_0(T) + i\phi_1(T)(X - b(T))}, \quad (3)$$

自旋孤子特点保持

$$\psi_- = \left\{ i\sqrt{1 - f(T)^2} + f(T) \tanh\left[\frac{X - b(T)}{w(T)}\right] \right\} e^{i\theta_0(T)} \quad (4)$$

where  $X = c_s x$  and  $T = c_s^2 t$

$$\begin{aligned} L(T) &= \int_{-\infty}^{+\infty} \left\{ c_s \left[ \frac{i}{2} (\psi_+^* \partial_T \psi_+ - \psi_+ \partial_T \psi_+^*) + \frac{i}{2} (\psi_-^* \partial_T \psi_- - \psi_- \partial_T \psi_-^*) \left( 1 - \frac{1}{|\psi_-|^2} \right) - \frac{1}{2} |\partial_X \psi_+|^2 - \frac{1}{2} |\partial_X \psi_-|^2 \right] \right. \\ &\quad \left. - \frac{1}{c_s} \left[ \frac{g_1}{2} |\psi_+|^4 + \frac{g_3}{2} (|\psi_-|^2 - 1)^2 + g_2 |\psi_+|^2 (|\psi_-|^2 - 1) \right] + \frac{1}{c_s^2} F X |\psi_+|^2 \right\} dX. \\ &= c_s \left\{ 2f(T)^2 w(T) [\phi_1(T) b'(T) - \phi_0'(T)] - \frac{f(T)^2}{w(T)} [1 + \phi_1(T)^2 w(T)^2] + 2f(T)^2 w(T) \theta_0' \right. \\ &\quad \left. + 2b'(T) \arcsin[f(T)] - 2b'(T) f(T) \sqrt{1 - f(T)^2} \right\} + \frac{1}{c_s^2} 2F f(T)^2 w(T) b(T), \end{aligned}$$

Y. S. Kivshar and W. Królikowski, Lagrangian approach for dark solitons, *Opt. Commun.* **114**, 353 (1995).



# 3. 自旋孤子的有效质量

## 暗孤子激发能的推导

PHYSICAL REVIEW LETTERS **128**, 125301 (2022)

### Propagating Ferrodark Solitons in a Superfluid: Exact Solutions and Anomalous Dynamics

Xiaoquan Yu<sup>1,2,\*</sup> and P. B. Blakie<sup>2</sup>

*Excitation energy and inertial mass.*—The excitation energy of FDSs can be obtained by evaluating the difference of grand canonical energies  $\delta K = K_{\text{FDS}} - K_g$ , where  $K_{\text{FDS}} = \int dx(\mathcal{H}[\psi] - \mu n)$ ,  $K_g = \int dx(\mathcal{H}[\psi_g] - \mu n_b)$ ,  $\psi_g$  is the ground state wave function, and  $\mu = (g_n + g_s)n_b + q/2$  is the chemical potential. For type-I FDSs, we obtain

化学势不变，粒子数变化

Energy of dark soliton  
= Grand canonical energy of the system in the presence of soliton  
- Grand canonical energy of the system in the absence of soliton

$$E' = E - \mu \int |\Psi_0|^2 d\vec{r}$$

$$E_{\text{soliton}} = \left[ \int_{-\infty}^{\infty} \left[ \frac{\hbar^2}{2m} \left| \frac{d\Psi}{dz} \right|^2 + \frac{g}{2} |\Psi_0|^4 \right] dz - \mu \int |\Psi_0|^2 dz \right] - \left[ \int_{-\infty}^{\infty} \left( \frac{g}{2} n^2 \right) dz - \mu \int n dz \right]$$

$$= \int_{-\infty}^{\infty} \left[ \frac{\hbar^2}{2m} \left| \frac{d\Psi}{dz} \right|^2 + \underbrace{\left[ \frac{g}{2} |\Psi_0|^4 - \mu |\Psi_0|^2 - \frac{g}{2} n^2 + \mu n \right]}_{E_{\text{non}}} \right] dz$$

$\mu = ng$

Busch T and Anglin J R *Phys. Rev. Lett.* **84** 2298 (2000)

Muryshev A, Shlyapnikov G V, Ertmer W, et al., *Phys. Rev. Lett.* **89** 110401 (2002)

Konotop V V and Pitaevskii L P *Phys. Rev. Lett.* **93**, 240403 (2004)

# 3. 自旋孤子的有效质量

## 暗孤子激发能的再推导

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} + |\psi|^2\psi,$$

平均场能量为:

$$\psi_d = \{\sqrt{\mu - v^2} \tanh[\sqrt{\mu - v^2}(x - vt)] + iv\}e^{-i\mu t}, \quad H = \int_{-\infty}^{\infty} \psi^* \left(-\frac{1}{2}\frac{\partial^2}{\partial x^2}\right)\psi + \frac{1}{2}|\psi|^4 dx$$

$$\begin{aligned} E_d &= H[\psi_d] - H[\psi_g] \\ &= -\frac{2}{3}(\mu + 2v^2)\sqrt{\mu - v^2}, \end{aligned}$$

粒子数不变，化学势变化

$$E_d(L) = H_L[\psi_d] - H_L[\psi_{gL}]$$

有限的区间范围  $[-L, L]$

$$N_L[\psi_d] = N_L[\psi_{gL}]$$

$$\begin{aligned} E_d &= \lim_{L \rightarrow \infty} E_d(L) = \frac{4}{3}(\mu - v^2)^{\frac{3}{2}} \\ &= \int_{-\infty}^{\infty} \psi_d^* \left(-\frac{1}{2}\frac{\partial^2}{\partial x^2}\right)\psi_d - \frac{1}{2}(|\psi_d|^2 - \mu)^2 dx \end{aligned}$$



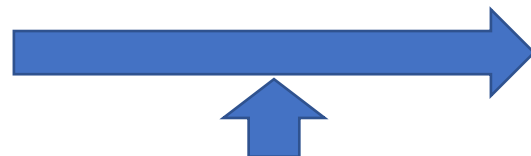
# 3. 自旋孤子的有效质量

修正因子

$$\left(1 - \frac{1}{|\psi_-|^2}\right)$$

的再推导

变分法



不可积系统的暗孤子

可积系统的暗孤子解

$$\psi_d = \{iv + \sqrt{a^2 - v^2} \tanh[\sqrt{a^2 - v^2}(x - vt)]\} \exp(-ia^2t),$$

$$\psi_p = a_p \exp[i\phi(x) - ia_p^2t],$$

$$I_s = \int_{-L}^{+L} \left[ \frac{i}{2}(\psi_d \partial_x \psi_d^* - \psi_d^* \partial_x \psi_d) - \frac{i}{2}(\psi_p \partial_x \psi_p^* - \psi_p^* \partial_x \psi_p) \right] dx$$

$$L \rightarrow \infty, \text{ then } a_p^2 = a^2, I_s = -2v\sqrt{a^2 - v^2} - a^2[\phi(+\infty) - \phi(-\infty)]$$

$$I_s = \int_{-\infty}^{+\infty} \left[ \frac{i}{2}(\psi \partial_x \psi^* - \psi^* \partial_x \psi) \left(1 - \frac{a^2}{|\psi|^2}\right) \right] dx,$$

Noether's theorem

$$I_s = - \int_{-\infty}^{+\infty} \left[ \partial_x \psi \frac{\partial \mathcal{L}}{\partial(\partial_t \psi)} + \partial_x \psi^* \frac{\partial \mathcal{L}}{\partial(\partial_t \psi^*)} \right] dx$$

$$H = \int_{-\infty}^{+\infty} \left[ \frac{1}{2}|\psi|^2 + \frac{1}{2}(|\psi|^2 - a^2)^2 \right] dx$$

$$L = \int_{-\infty}^{+\infty} \left[ \frac{\partial \mathcal{L}}{\partial(\partial_t \psi)} \partial_t \psi + \frac{\partial \mathcal{L}}{\partial(\partial_t \psi^*)} \partial_t \psi^* \right] dx - H$$

$$L = \int_{-\infty}^{+\infty} \left[ \frac{i}{2}(\psi^* \partial_t \psi - \psi \partial_t \psi^*) \left(1 - \frac{a^2}{|\psi|^2}\right) - \frac{1}{2}|\partial_x \psi|^2 - \frac{1}{2}(|\psi|^2 - a^2)^2 \right] dx,$$

### 3. 自旋孤子的有效质量

#### 保模假设下的Lagrangian变分结果

$$f(T) = \sin\left(\frac{\sqrt{c_s^2 - v_0^2}}{c_s^4} FT + \delta\right),$$

$$E_k = \int_{-\infty}^{+\infty} \left[ \psi_+^* \left(-\frac{1}{2} \partial_x^2\right) \psi_+ + \psi_-^* \left(-\frac{1}{2} \partial_x^2\right) \psi_- \right] dx$$

$$b(T) = \pm \frac{c_s^5}{2F(c_s^2 - v_0^2)} \sin^2\left(\frac{\sqrt{c_s^2 - v_0^2}}{c_s^4} FT + \delta\right),$$

$$= \frac{c_s^2}{\sqrt{c_s^2 - v_0^2}} \sin^2\left(\frac{\sqrt{c_s^2 - v_0^2}}{c_s^4} FT + \delta\right).$$

$$\phi_1(T) = \pm \frac{c_s}{2\sqrt{c_s^2 - v_0^2}} \sin\left(2\frac{\sqrt{c_s^2 - v_0^2}}{c_s^4} FT + 2\delta\right),$$

$$E_{\text{inter}} = \int_{-\infty}^{+\infty} \left[ \frac{g_1}{2} |\psi_+|^4 + \frac{g_3}{2} (|\psi_-|^2 - 1)^2 + g_2 |\psi_+|^2 (|\psi_-|^2 - 1) \right] dx = 0.$$

where  $\delta = \arcsin[\sqrt{1 - v_0^2/c_s^2}]$ .

$$E_s = \frac{c_s^2}{2\sqrt{c_s^2 - v_0^2}} \pm \sqrt{\left(\frac{c_s^2}{2\sqrt{c_s^2 - v_0^2}}\right)^2 - v^2}.$$

# 3. 自旋孤子的有效质量

**有效质量**

$$M^* = 2 \frac{\partial E_s}{\partial (v^2)}$$

**有正负两支**

*Phys. Rev. Lett.* **106**, 185301(2011)

$$M^* = \pm \frac{M_0^*}{\sqrt{1 - \frac{v^2}{c_s^2}}}$$

类似于狭义相对论中  
质量-速度关系

$$M_0^* = \frac{2\sqrt{c_s^2 - v_0^2}}{c_s^2}$$

初始质量  
( $v_0 = 0$  静止质量)

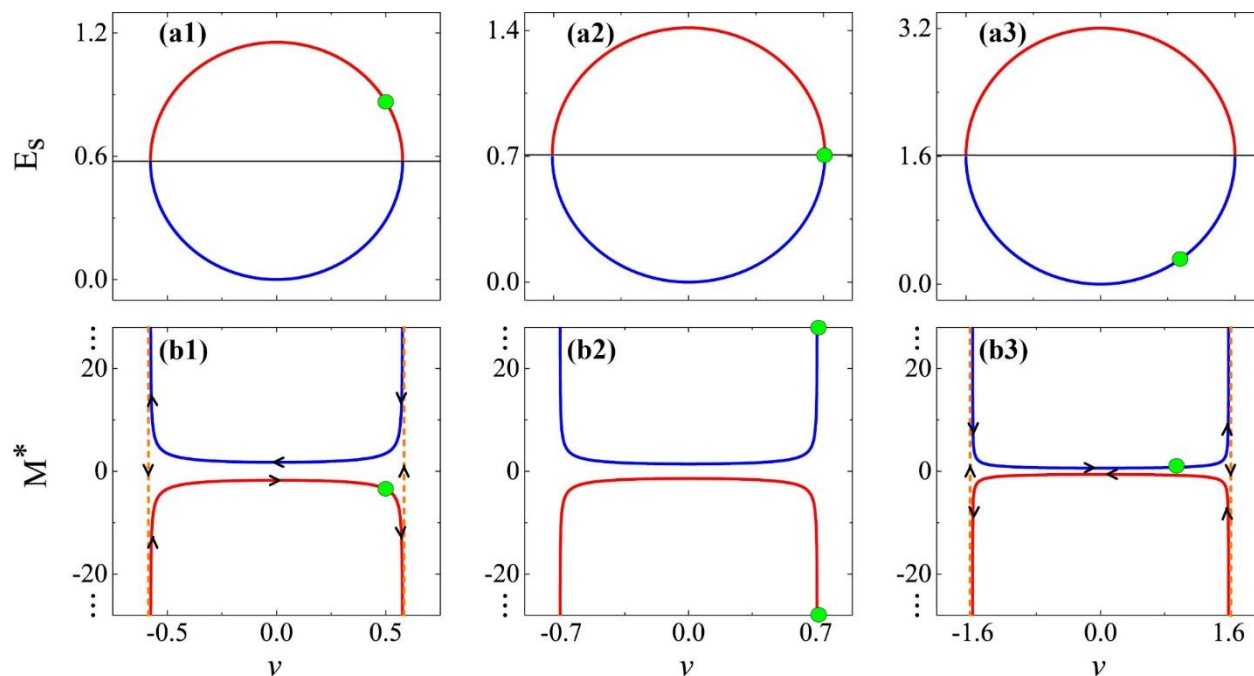
# 3. 自旋孤子的有效质量

质量转换速度

$$c_m = \sqrt{\frac{c_s^4}{4(c_s^2 - v_0^2)}}$$

初始质量临界速度

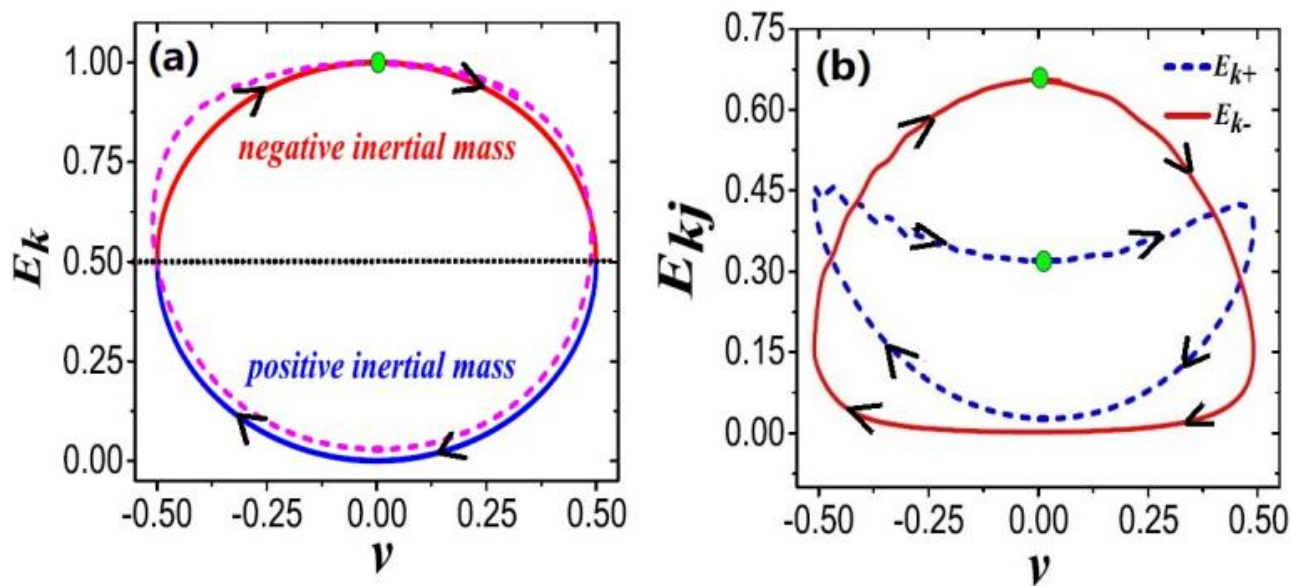
$$v_{0c} = \frac{c_s}{\sqrt{2}}$$



周期性表现出正、负质量

# 3. 自旋孤子的有效质量

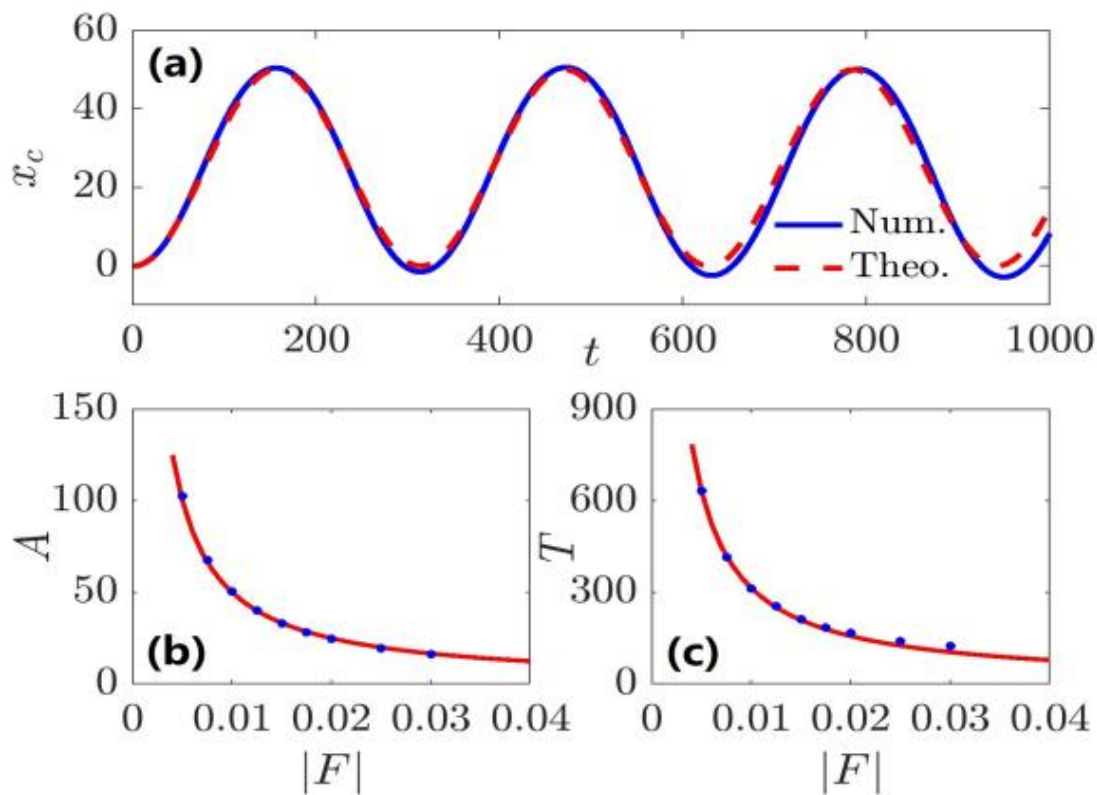
## 有效质量揭示振荡的机制



自旋孤子在常力驱动下发生交流振荡，  
源于加速过程中正负有效质量的周期性转换。

# 3. 自旋孤子的有效质量

基于正负质量转换的准粒子模型，成功解释振荡行为。



$$x_c = -\frac{c_s^2}{2F} \sin^2(Ft/c_s).$$

非线性耦合效应

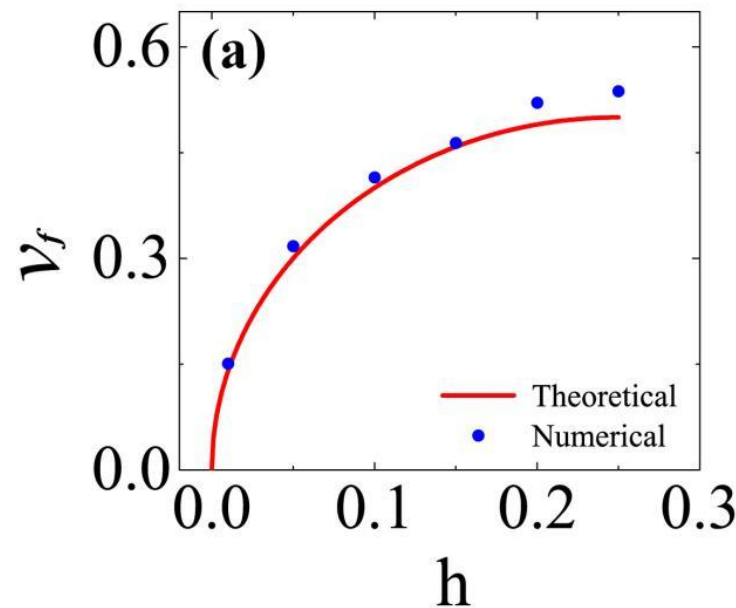
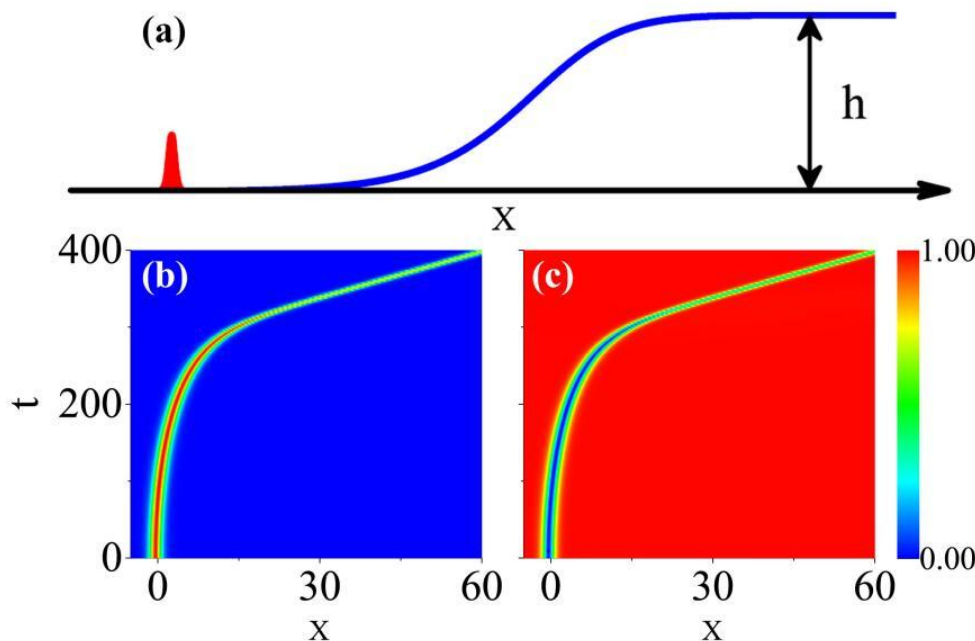
亮暗孤子的质量竞争

# 4. 色散关系的应用

$$E_s = \frac{c_s^2}{2\sqrt{c_s^2 - v_0^2}} \pm \sqrt{\left(\frac{c_s^2}{2\sqrt{c_s^2 - v_0^2}}\right)^2 - v^2}.$$

$$V_+(x) = \frac{h}{4} [\tanh(-\sqrt{1 - D^2}x) - 1]^2$$

色散关系适用于其它缓变势场

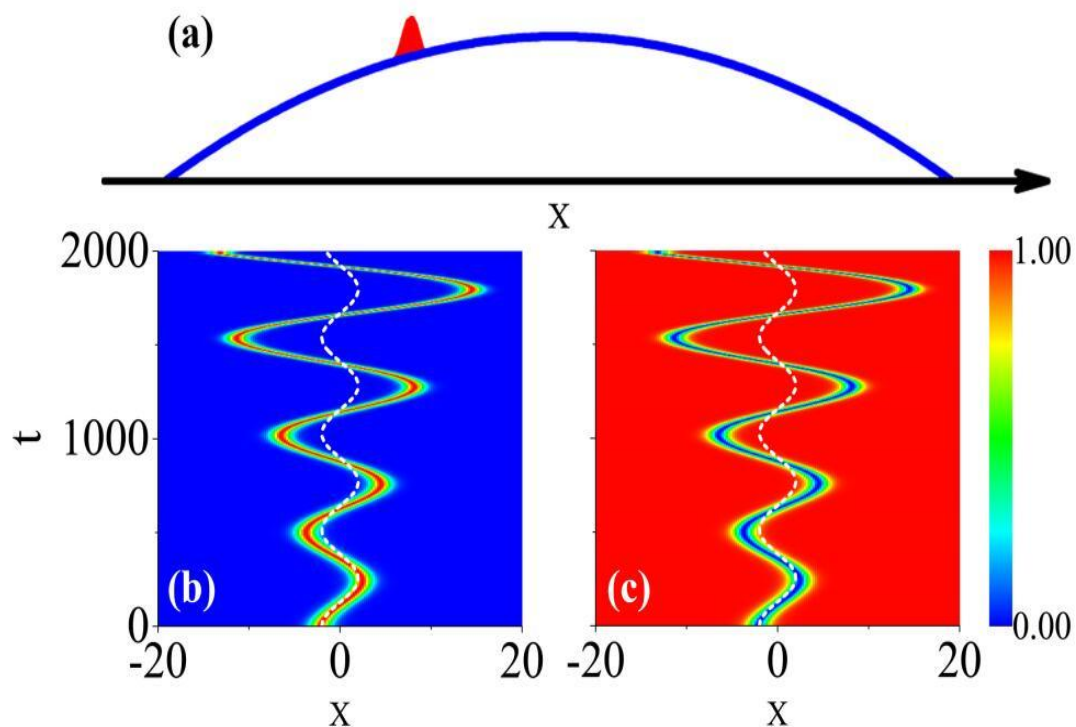


理论描述和数值模拟的比较

$$\frac{c_s}{2} + \sqrt{\frac{c_s^2}{4} - v^2} + h \int_{-\infty}^{\infty} |\psi_+|^2 dx = \sqrt{1 - v_0^2}$$

# 4. 色散关系的应用

## 负质量振荡子



$$V_+(x) = -\frac{1}{2}\omega^2 x^2$$



# 5. 总结与讨论

- ◆ 在更宽的参数区域给出**严格孤子解**
- ◆ 通过**色散关系**揭示自旋孤子**正负质量转换**特性
- ◆ 预言一种**直流驱动产生交流振荡**的现象
- ◆ 提出**实验中直接观测负质量效应**的方案

L.-C. Zhao, W-L Wang, Z.-Y. Yang, et al., Phys. Rev. A **101**, 043621 (2020)

L.-Z. Meng, S-W Guan, L-C Zhao, Phys. Rev. A **105**, 013303 (2022)

# 5. 总结与讨论

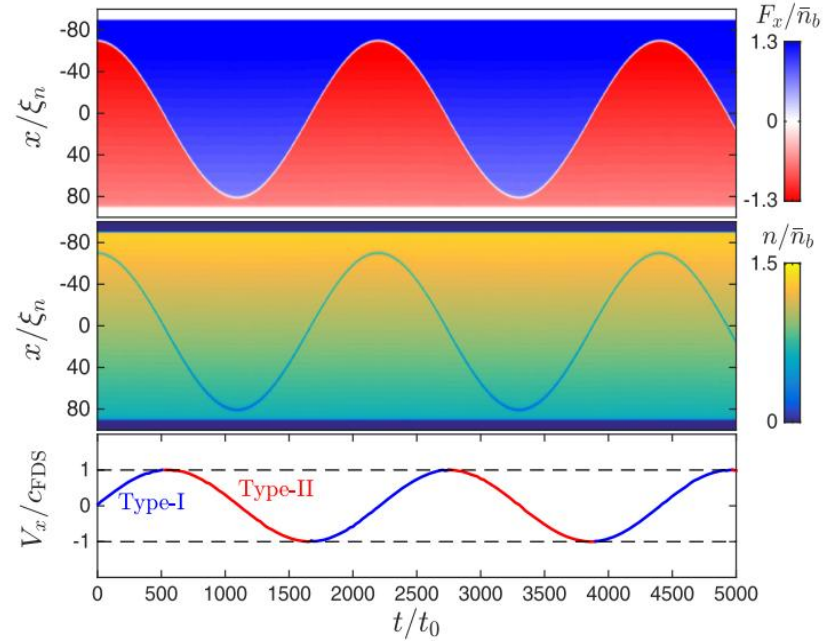
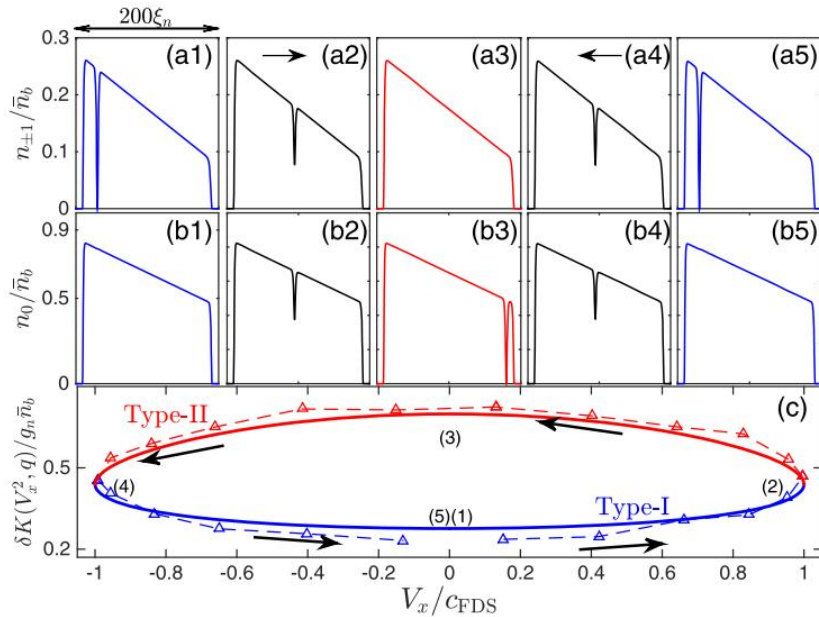
PHYSICAL REVIEW LETTERS **128**, 125301 (2022)

## Propagating Ferrodark Solitons in a Superfluid: Exact Solutions and Anomalous Dynamics

Xiaoquan Yu<sup>1,2,\*</sup> and P. B. Blakie<sup>2</sup>

*Spin-1 BECs.*—The Hamiltonian density of a spin-1 condensate reads

$$\mathcal{H} = \frac{\hbar^2 |\nabla\psi|^2}{2M} + \frac{g_n}{2} |\psi^\dagger \psi|^2 + \frac{g_s}{2} |\psi^\dagger \mathbf{S}\psi|^2 + q\psi^\dagger S_z^2 \psi, \quad (1)$$



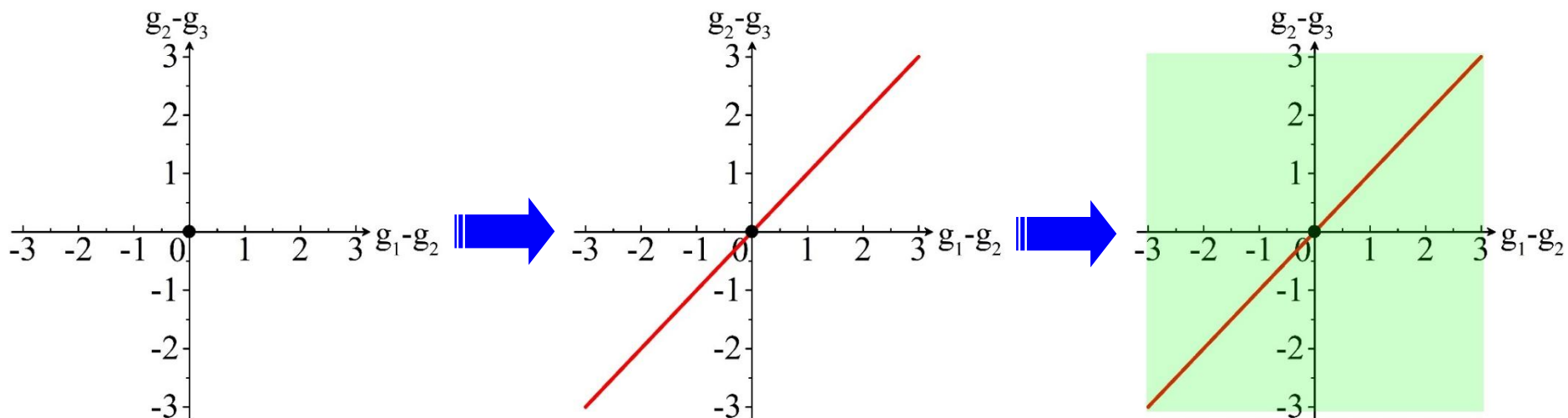
# 5. 总结与讨论

## 如何求解更一般不可积情形下的矢量孤子

**可积模型的**  
**精确矢量孤子解：**  
亮-亮；亮-暗；  
暗-亮；暗-暗。

**不可积模型的**  
**精确自旋孤子解**

**不可积模型的**  
**精确矢量孤子解：**  
亮-亮；亮-暗；  
暗-亮；暗-暗……



X. X. Liu, H. Pu, B. Xiong, W. M. Liu, and J.B. Gong, Phys. Rev. A **79**, 013423 (2009).

发展前面的变分法，我们近期求解了任意非线性系数N-组分耦合系统的精确孤子解

**谢谢大家!**