Integrable Open Spin Chains of Flavored ABJM Theory

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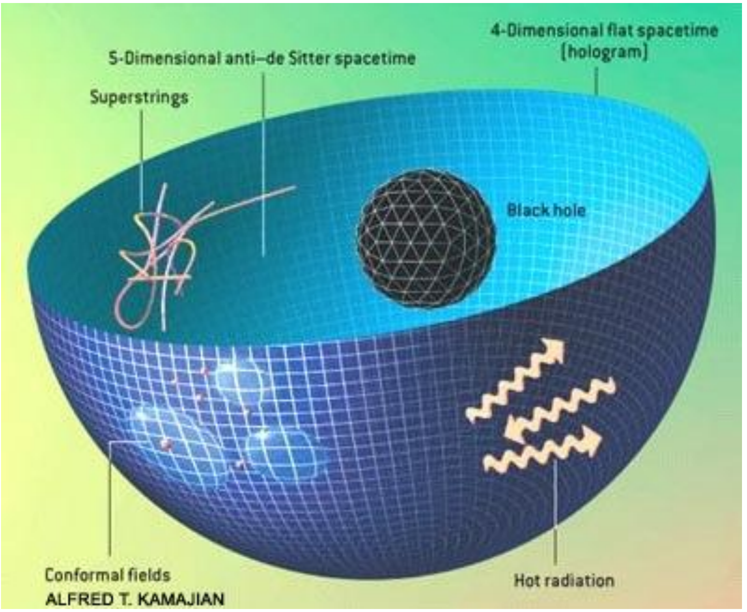
papers

- Based on
- **Nan Bai, Hui-Huang Chen, Song He, Wen-Li Yang, JW, Meng-Qi Zhu, 1704.05807, JHEP08(2017)001**
- **JW, to appear**

Motivations

- Gauge/gravity correspondence is one of the most important results in string theory the last two decades. *[Maldacena 97]*



Motivations

- As a kind of weak/strong duality, it allows us to study the strongly coupled gauge theories using weakly coupled gravity theories. (AdS/QCD, AdS/CMT)
- But this also makes the non-trivial check of this duality hard!
- Also it is still hard to compute quantities when coupling is intermediate(~O(1)).
- New tools in field theory: supersymmetric localization, integrable structure, bootstrap…

Motivations

- Rich integrable structures were found in both side of the holographic duality *between* 4d $\mathcal{N}$=4 SYM *and* type IIB string theory on AdS_5*S^5.

  [Minaham, Zarembo, 02] [Bena etal, 03]

  collection of reviews: [Beiserts etal, 10]

- Using this remarkable structure, people can compute non-trivial quantities like cusp anomalous dimension for a large range of the ‘t Hooft coupling in the planar limit.

Motivations

- Integrable structures were also found on both side of gauge-gravity duality *between* 3d $\mathcal{N}$=6 $U(N)_k*U(M)_{-k}$ Chern-Simons-matter theory (ABJ(M) theory) *and* type IIA string theory on AdS_4*CP^3.

  [Minaham, Zarembo, 08] [Bak, Rey, 08] …

- We want to search for theories with less supersymmetries while they still have such integrable structure.

Previous attempts

- Pre-ABJM: *Gaiotto and Yin (07)* searched for integrable structure in general *N*=3 CSM theories. They found a ***non***-integrable spin chain in certain sector.

- Post-ABJM: *Forcella and Schulgin (09)* studied *N*=3 theories based on $U(N)_k * U(N)_{-l}$ with unequal k and l. They had not found an integrable structure.

Three roads in 4d case

- Beta-/gamma-deformation *[Roiban, 03][Berenstein, Cherkis, 04][Beisert, Roiban, 05]*

- Orbifold *[Wang, Y.-S. Wu, 03][Ideguchi, 04][Beisert, Roiban, 05]*

- Adding flavors (+ orentifolding)

  D7's+O7 *[Chen, Wang, Y.-S. Wu, 04]*2*

  D7's*[Erler, Mann, 05]*

Three roads in 3d

- Beta-/gamma-deformed planar ABJM theories

  two loop, scalar sector: *[He, JW, 13]*

  all-loop and full sector (asymptotic Bethe ansatz), Y-system for finite size effects: *[Chen, JW, 16]*

  Finite-size effects for Dyonic Gaint magnons: *[Chen, JW, 16]*

- Planar orbifold ABJM theories: from two-loop to all-loop *[Bai, Chen, Ding, Li, JW, 16]*
- Planar flavored ABJM theory: two-loop scalar sector *[This talk]*

Double scaling limit

- Special double scaling limit of gamma-deformed $\mathcal{N}$=4 SYM or ABJM leads to integrable theories with only scalars and fermions.
- *[Gurdogan, Kazakov, 15] [Caetano, Gurdogan, Kazakov, 16]*
- Related to fishnet graphs *[Zamolodchikov, 80].*

- *[Mamroud, Torrents, 17][Chichenin etal, 17]*

  [Basso, Dixon, 17][Gromov etal, 17]

ABJM theory

- Aharony-Bergman-Jafferis-Maldacena theory is a three-dimensional Chern-Simons-matter theory with $\mathcal{N}$=6 supersymmetries. The gauge group is U(N)*U(N) with Chern-Simons levels k and –k.
- The matter fields are scalars, Y^I and fermions Ψ_I in the bi-fundamental representation of the gauge group. Here I=1, …, 4 and the

  R-symmetry group is SU(4).

Properties of ABJM theory

- 1/k is the coupling constant.
- The theory has a large N (planar or ‘t Hooft) limit:

$$N \to \infty, k \to \infty, \lambda \equiv \frac{N}{k} \text{ fixed}$$

- This theory is found to be the LEFT for N M2-branes putting on the tip of the orbifold $\mathbf{C}^4/\mathbf{Z}_k$.

The gravity dual

- When $N >> k^5$, this theory is dual to M-theory on $AdS_4 * S^7/Z_k$.
- While when $k << N << k^5$, a better description is in terms of type IIA string theory on $AdS_4 * CP^3$.

Adding flavors

- One can add flavor which is (anti-)fundamental representation of either U(N). *[Hohenegger, Kirsch, 09][Gaiotto, Jafferis, 09][Hikida, Li, Takayanagi, 09]*

- This is dual to adding D6 branes to IIA theory on $AdS_4 * CP^3$.

- The maximal supersymmetry one can get is 3d $\mathcal{N}=3$.

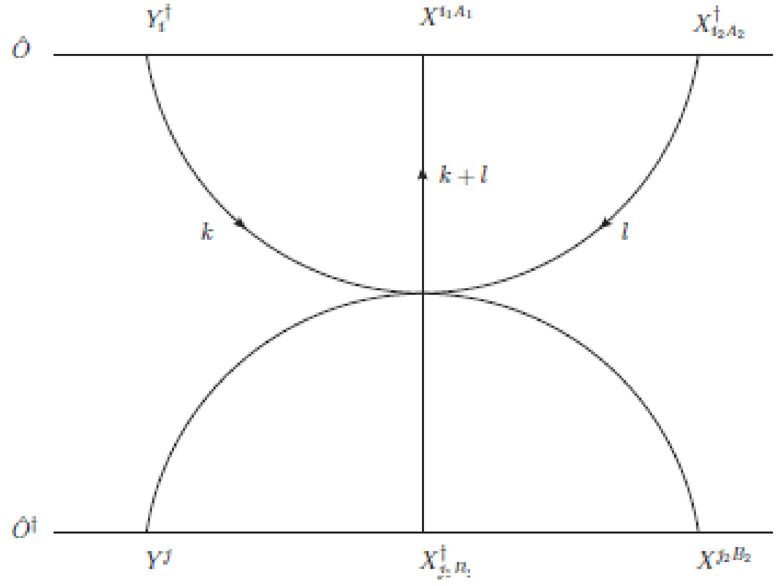
ADM

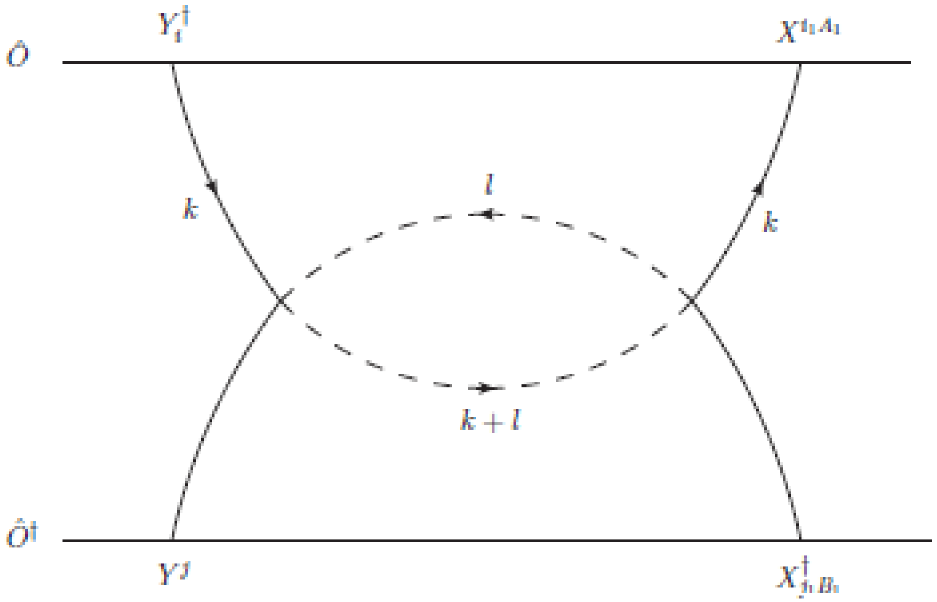
- We compute the anomalous dimension matrix(ADM) of

$$\hat{O} \;=\; Y_i^\dagger X^{i_1 A_1} X^\dagger_{i_2 A_2} \cdots X^{i_{2L-1} A_{2L-1}} X^\dagger_{i_{2L} A_{2L}} Y^{i'},$$

- We focus the ‘t Hooft limit, with k, N to infinity, λ=N/k fixed and N_f<<N. This is dual to probe limit of D6 brane in the gravity side.

Two loop diagrams





Two loop Hamiltonian

$$\mathcal{H} = \mathcal{H}_l + \mathcal{H}_r + \mathcal{H}_{bulk} + \alpha \mathbb{I},$$

$$(\mathcal{H}_l)^{j,i_1 A_1, j_2 B_2}_{i, j_1 B_1, i_2 A_2} = \lambda^2 \left[\left(\delta^{A_1}_{A_2} \delta^{B_2}_{B_1} - \delta^{A_1}_{B_1} \delta^{B_2}_{A_2} \right) \cdot \delta^{j_2}_{i} \delta^{i_1}_{j_1} \delta^{j}_{i_2} + \delta^{A_1}_{B_1} \delta^{B_2}_{A_2} \cdot \delta^{i_1}_{i} \delta^{j}_{j_1} \delta^{j_2}_{i_2} \right],$$

$$(\mathcal{H}_r)^{i_{2L-1} A_{2L-1}, j_{2L} B_{2L}, i'}_{j_{2L-1} B_{2L-1}, i_{2L} A_{2L}, j'} = \lambda^2 \Bigg[\left(\delta^{B_{2L}}_{B_{2L-1}} \delta^{A_{2L-1}}_{A_{2L}} - \delta^{B_{2L}}_{A_{2L}} \delta^{A_{2L-1}}_{B_{2L-1}} \right) \cdot \delta^{i_{2L-1}}_{j'} \delta^{j_{2L}}_{i_{2L}} \delta^{i'}_{j_{2L-1}} + \delta^{B_{2L}}_{A_{2L}} \delta^{A_{2L-1}}_{B_{2L-1}} \cdot \delta^{i_{2L-1}}_{j_{2L-1}} \delta^{i'}_{i_{2L}} \delta^{j_{2L}}_{j'} \Bigg],$$

$$\mathcal{H}_{bulk} = \lambda^2 \sum_{l=1}^{2L-2} \left(\mathbb{I}_{l,l+1} - \mathbb{P}_{l,l+2} + \frac{1}{2} \mathbb{P}_{l,l+2} \mathbb{K}_{l,l+1} + \frac{1}{2} \mathbb{K}_{l,l+1} \mathbb{P}_{l,l+2} \right),$$

Two loop Hamiltonian

$$(\mathbb{I}_{l,l+1})^{iA,j'B'}_{jB,i'A'} = \delta^i_j \delta^{j'}_{i'} \delta^A_B \delta^{B'}_{A'}, \quad (\mathbb{P}_{l,l+2})^{iA,i'A'}_{jB,j'B'} = \delta^i_{j'} \delta^{i'}_j \delta^A_{B'} \delta^{A'}_B, \quad (\mathbb{K}_{l,l+1})^{iA,j'B'}_{jB,i'A'} = \delta^i_{i'} \delta^{j'}_j \delta^A_{A'} \delta^{B'}_B,$$

- α was fixed to $2\lambda^2$ (<=anomalous dimension of BPS operator is zero).

Multi-body problem

- Now we want to solve the spectrum of this multi-body Hamiltonian.
- Usually it is very hard to get the exact solution.
- However the problem will be simplified when the Hamiltonian is integrable.
- To check the integrability using coordinate Bethe ansatz, we need to compute the S-matrix of two-particle scattering and reflection matrix for one-(bulk)-particle state.

Vacuum and excitations

- Vacuum

$$|\Omega\rangle = |Y_2^\dagger X^{11} X_{22}^\dagger \cdots X^{11} X_{22}^\dagger Y^1\rangle.$$

- Excitations

$$\begin{aligned}
\text{bulk A type}: \quad & Y_2^\dagger(A_1B_2)\cdots(A_2B_2)\cdots(A_1B_2)Y^1, \\
& Y_2^\dagger(A_1B_2)\cdots(B_1^\dagger B_2)\cdots(A_1B_2)Y^1, \\
\text{bulk B type}: \quad & Y_2^\dagger(A_1B_2)\cdots(A_1A_2^\dagger)\cdots(A_1B_2)Y^1, \\
& Y_2^\dagger(A_1B_2)\cdots(A_1B_1)\cdots(A_1B_2)Y^1, \\
\text{boundary} \quad : \quad & Y_1^\dagger(A_1B_2)\cdots(A_1B_2)\cdots(A_1B_2)Y^1, \\
& Y_2^\dagger(A_1B_2)\cdots(A_1B_2)\cdots(A_1B_2)Y^2.
\end{aligned}$$

$$X^{11} = A_1, \quad X^{12} = A_2, \quad X^{21} = B_1^\dagger, \quad X^{22} = B_2^\dagger.$$

Bulk S-matrix

- The two-loop S-matrix has been computed in *[Ahn, Nepomechine, 09]*

$$u_i \equiv \frac{1}{2}\cot\frac{k_i}{2}, \quad u_{ij} \equiv u_i - u_j.$$

The non-zero elements of the bulk S-matrix is

$$S^{\phi\phi}_{\phi\phi}(k_2, k_1) = \frac{u_{21} + i}{u_{21} - i},$$

where ϕ is one of $A_2, B_1^\dagger, A_2^\dagger, B_1$;

Bulk S-matrix (II)

$$S^{A_2B_1^\dagger}_{A_2B_1^\dagger}(k_2,k_1) = S^{B_1^\dagger A_2}_{B_1^\dagger A_2}(k_2,k_1) = S^{A_2^\dagger B_1}_{A_2^\dagger B_1}(k_2,k_1) = S^{B_1A_2^\dagger}_{B_1A_2^\dagger}(k_2,k_1) = \frac{u_{21}}{u_{21}-i};$$

$$S^{B_1^\dagger A_2}_{A_2B_1^\dagger}(k_2,k_1) = S^{A_2B_1^\dagger}_{B_1^\dagger A_2}(k_2,k_1) = S^{B_1A_2^\dagger}_{A_2^\dagger B_1}(k_2,k_1) = S^{A_2^\dagger B_1}_{B_1A_2^\dagger}(k_2,k_1) = \frac{i}{u_{21}-i};$$

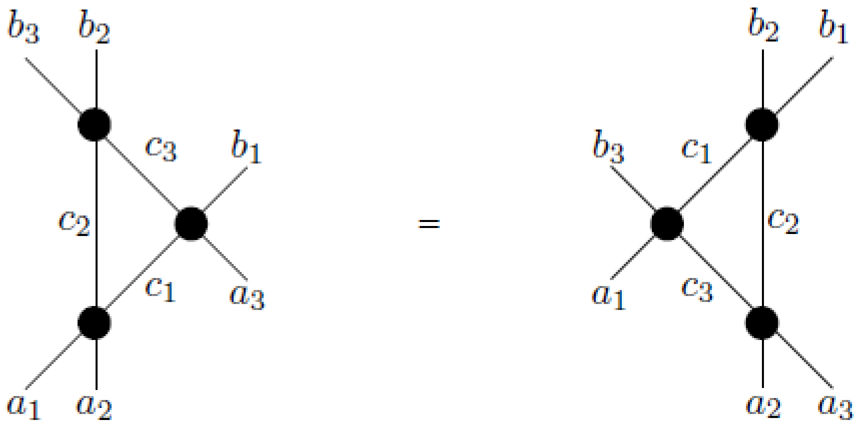
$$S^{A_2B_1}_{A_2B_1}(k_2,k_1) = S^{B_1A_2}_{B_1A_2}(k_2,k_1) = S^{A_2^\dagger B_1^\dagger}_{A_2^\dagger B_1^\dagger}(k_2,k_1) = S^{B_1^\dagger A_2^\dagger}_{B_1^\dagger A_2^\dagger}(k_2,k_1) = 1;$$

$$S^{A_2A_2^\dagger}_{A_2A_2^\dagger}(k_2,k_1) = S^{B_1^\dagger B_1}_{B_1^\dagger B_1}(k_2,k_1) = S^{A_2^\dagger A_2}_{A_2^\dagger A_2}(k_2,k_1) = S^{B_1B_1^\dagger}_{B_1B_1^\dagger}(k_2,k_1) = \frac{u_{12}}{u_{12}-i};$$

$$S^{A_2A_2^\dagger}_{B_1^\dagger B_1}(k_2,k_1) = S^{B_1^\dagger B_1}_{A_2A_2^\dagger}(k_2,k_1) = S^{A_2^\dagger A_2}_{B_1B_1^\dagger}(k_2,k_1) = S^{B_1B_1^\dagger}_{A_2^\dagger A_2}(k_2,k_1) = \frac{i}{u_{12}-i}.$$

YBE

- The S-matrix has been computed in *[Ahn, Nepomechine, 09]* and we confirm that it satisfies Yang-Baxter equation.



Action of left boundary Hamiltonian

$$\begin{aligned}
\mathcal{H}_l|1\rangle_{A_2} &= \lambda^2|1\rangle_{B_1}, \\
\mathcal{H}_l|1\rangle_{B_1^\dagger} &= \lambda^2|l\rangle_{Y_1^\dagger}, \\
\mathcal{H}_l|1\rangle_{A_2^\dagger} &= -\lambda^2|l\rangle_{Y_1^\dagger}, \\
\mathcal{H}_l|1\rangle_{B_1} &= \lambda^2|1\rangle_{A_2}, \\
\mathcal{H}_l|l\rangle_{Y_1^\dagger} &= -\lambda^2|1\rangle_{A_2^\dagger} + \lambda^2|l\rangle_{Y_1^\dagger} + \lambda^2|1\rangle_{B_1^\dagger}, \\
\mathcal{H}_l|x\rangle &= -\lambda^2|x\rangle, \quad x \neq 1,
\end{aligned}$$

Closed sector I

- Ansatz

$$|\psi_1(k)\rangle = \sum_{x=1}^{L} \left(f(x)|x\rangle_{A_2} + g(x)|x\rangle_{B_1} \right),$$

$$f(x) = Fe^{ikx} + \tilde{F}e^{-ikx},$$
$$g(x) = Ge^{ikx} + \tilde{G}e^{-ikx}.$$

- Eigenvalue equation

$$\mathcal{H}|\psi_1\rangle = E(k)|\psi_1\rangle$$

- Bulk part of the chain gives

$$E(k) = 2\lambda^2 - 2\lambda^2 \cos k.$$

Reflection

- Left boundary $$F = -e^{-ik}\tilde{G}, \quad G = -e^{-ik}\tilde{F}.$$
- Definition

$$\begin{pmatrix} F \\ G \end{pmatrix} \equiv K_l(k) \begin{pmatrix} \tilde{F} \\ \tilde{G} \end{pmatrix}.$$

$$K_l(k) = \begin{pmatrix} 0 & -e^{-ik} \\ -e^{-ik} & 0 \end{pmatrix}.$$

- The right boundary $$F = -e^{-2ikL-ik}\tilde{G}, \quad G = -e^{-2ikL-ik}\tilde{F}.$$

$$e^{2ikL} \begin{pmatrix} F \\ G \end{pmatrix} \equiv K_r(k) \begin{pmatrix} \tilde{F} \\ \tilde{G} \end{pmatrix}. \qquad K_r(k) = \begin{pmatrix} 0 & -e^{-ik} \\ -e^{-ik} & 0 \end{pmatrix}.$$

Closed sector II

$$|\psi_2(k)\rangle = \sum_{x=1}^{L} n(x)|x\rangle_{B_1^\dagger} + \sum_{x=1}^{L} h(x)|x\rangle_{A_2^\dagger} + \beta|l\rangle_{Y_1^\dagger} + \gamma|r\rangle_{Y^2},$$

$$K_l(k) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}. \qquad K_r(k) = \begin{pmatrix} 0 & -e^{-2ik} \\ -e^{-2ik} & 0 \end{pmatrix},$$

Reflection matrices

$$K_l(k) = \begin{pmatrix} 0 & 0 & 0 & -e^{-ik} \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ -e^{-ik} & 0 & 0 & 0 \end{pmatrix},$$

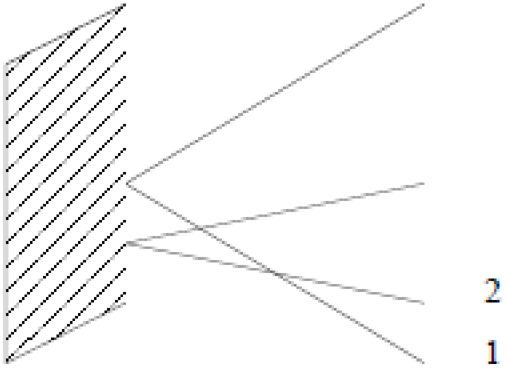
$$K_r(k) = \begin{pmatrix} 0 & 0 & 0 & -e^{-ik} \\ 0 & 0 & -e^{-2ik} & 0 \\ 0 & -e^{-2ik} & 0 & 0 \\ -e^{-ik} & 0 & 0 & 0 \end{pmatrix}.$$

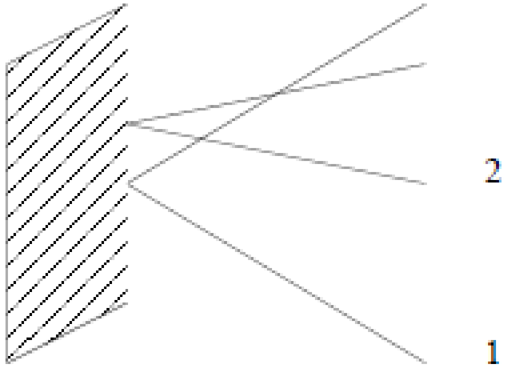
Boundary Yang-Baxter equation

$$[S(k_1,k_2)]_{l_1 l_2}^{m_1 m_2}\,[K_l(k_2)]_{j_2}^{l_2}\,[S(-k_2,k_1)]_{j_1 i_2}^{l_1 j_2}\,[K_l(k_1)]_{i_1}^{j_1}$$
$$= [K_l(k_1)]_{l_1}^{m_1}\,[S(-k_1,k_2)]_{j_1 l_2}^{l_1 m_2}\,[K_l(k_2)]_{j_2}^{l_2}\,[S(-k_2,-k_1)]_{i_1 i_2}^{j_1 j_2}\,,$$

$$[S(-k_1,-k_2)]_{l_1 l_2}^{m_1 m_2}\,[K_r(-k_1)]_{j_1}^{l_1}\,[S(-k_2,k_1)]_{i_1 j_2}^{j_1 l_2}\,[K_r(-k_2)]_{i_2}^{j_2}$$
$$= [K_r(-k_2)]_{l_2}^{m_2}\,[S(-k_1,k_2)]_{l_1 j_2}^{m_1 l_2}\,[K_r(-k_1)]_{j_1}^{l_1}\,[S(k_2,k_1)]_{i_1 i_2}^{j_1 j_2}\,,$$

BYBE





Nonintegrability in the Veneziano limit

- Flavor-backreacted background was found in *[Conde, Ramallo, 11].* The backreacted of D6 branes are taken into account. This is dual to field theory in the Veneziano limit: N_f, N, k to infinity, N_f/N, N/k fixed. (unquenched flavors).

- Very recently, *[Giataganas, Zoubos, 17]* showed that the classical string motion in this background is chaotic! This gave strong evidence that the field theory in the Veneziano limit is nonintegrable.

More general integrable boundary interactions?

- Let us consider boundary interactions with general coefficients,

$$(\mathcal{H}_l)^{j,i_1A_1,j_2B_2}_{i,j_1B_1,i_2A_2} = (-a\delta^{A_1}_{B_1}\delta^{B_2}_{A_2} + b\delta^{A_1}_{A_2}\delta^{B_2}_{B_1})\cdot\delta^{j_2}_{i}\delta^{i_1}_{j_1}\delta^{j}_{i_2} + c\delta^{A_1}_{B_1}\delta^{B_2}_{A_2}\cdot\delta^{i_1}_{i}\delta^{j}_{j_1}\delta^{j_2}_{i_2},$$

$$(\mathcal{H}_r)^{i_{2L-1}A_{2L-1},j_{2L}B_{2L},i'}_{j_{2L-1}B_{2L-1},i_{2L}A_{2L},j'} = \left(-a'\delta^{B_{2L}}_{A_{2L}}\delta^{A_{2L-1}}_{B_{2L-1}} + b'\delta^{B_{2L}}_{B_{2L-1}}\delta^{A_{2L-1}}_{A_{2L}}\right)\cdot\delta^{i_{2L-1}}_{j'}\delta^{j_{2L}}_{i_{2L}}\delta^{i'}_{j_{2L-1}} + c'\delta^{B_{2L}}_{A_{2L}}\delta^{A_{2L-1}}_{B_{2L-1}}\cdot\delta^{i_{2L-1}}_{j_{2L-1}}\delta^{i'}_{i_{2L}}\delta^{j_{2L}}_{j'},$$

More general integrable boundary interactions?

- Among these boundary interactions, only two are integrable. *[JW, to appear]*

- 1. a=b=c=a'=b'=c'= λ^2, this is the one from flavored ABJM theory.

- 2. a=b=c=a'=b'=c'=0.

Conclusion and Discussions

- We proved that the planar flavored ABJM theory is integrable at two loop in the scalar sector.
- We used the coordinate Bethe ansatz, notice that the reflection matrix is non-diagonal!
- If we consider more general coefficients of the boundary interaction, only two cases are intergrable!

To do list

- Find the spectrum: using off-diagonal Bethe ansatz *[Cao, Shi, Wang, Yang]*.
- Generalization to full sector and/or high loop.
- Integrability of the theories with both D6’s and O6-planes added?
- Integrable open spin chain from Wilson loops or giant gravitons?

Thank you very much!