Black Holes and Their Thermodynamics in Gravitational Theories without Lorentz Symmetry

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1.1 Thermodynamics of An Ordinary System

- In an ordinary thermodynamic system, thermal properties reflect the statistical mechanics of underlying microstates.
- The temperature T of the system is a measure of the average energy of its fundamental “quanta”.
- Its entropy S is a measure of the number of possible microscopic arrangements of those “quanta”, so normally $S \propto V$.

1.1 Thermodynamics of An Ordinary System (Cont.)

- The Four Laws of Thermodynamics:
  - *Zeroth law of thermodynamics:* If two systems are each in thermal equilibrium with a third, they are also in thermal equilibrium with each other.
  - *First law of thermodynamics:* The increase in internal energy of *a closed system* is equal to the difference of the heat supplied to the system and the work done by it,

$$\Delta U = Q - W,$$

U: Internal energy; Q: heat; W: work

1.1 Thermodynamics of An Ordinary System (Cont.)

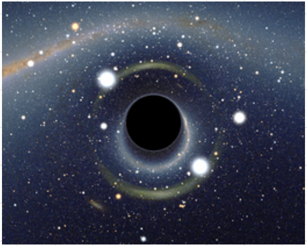
- The Four Laws Thermodynamics (Cont.):
  - *Second law of thermodynamics:* Heat cannot spontaneously flow from a colder location to a hotter location, which in terms of entropy is often expressed as,

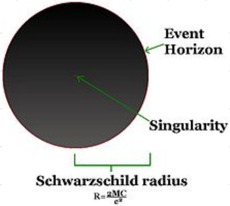
$$\Delta \mathrm{S} \geq 0.$$

  - *Third law of thermodynamics:* As a system approaches absolute zero the entropy of the system approaches a minimum value.

1.2 Black Holes & Thermodynamics

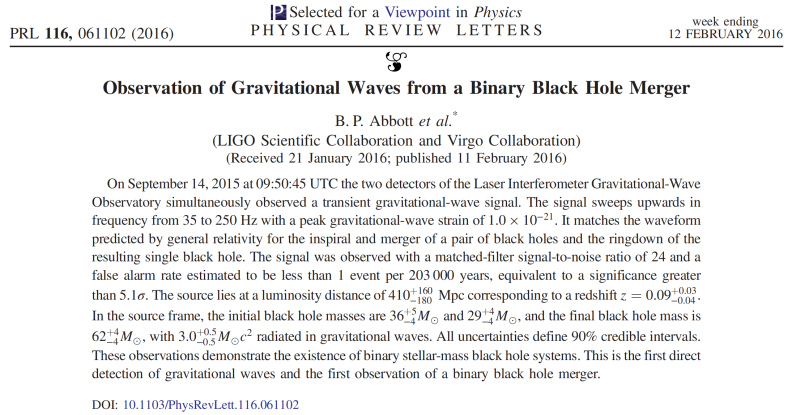
- *Black hole:* a region of spacetime exhibiting a strong gravitational attraction that no particle or light can escape from it.
- The boundary of the region from which no escape is possible is called the (event) horizon.





1.2 Black Holes & Thermodynamics (Cont.)

- Recent detections of gravitational waves by LIGO show that black holes indeed exist in our Universe.
- GW150914:



Selected for a Viewpoint in *Physics*
PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*[*]
(LIGO Scientific Collaboration and Virgo Collaboration)

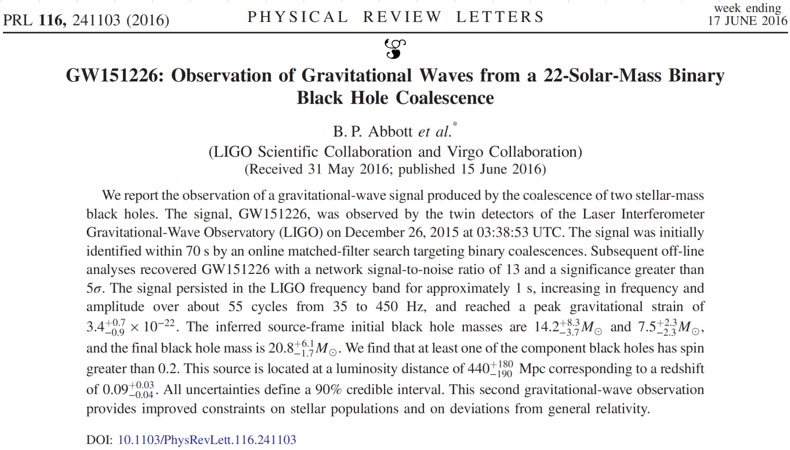
(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_\odot$ and $29^{+4}_{-4} M_\odot$, and the final black hole mass is $62^{+4}_{-4} M_\odot$, with $3.0^{+0.5}_{-0.5} M_\odot c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

DOI: 10.1103/PhysRevLett.116.061102

1.2 Black Holes & Thermodynamics (Cont.)

- GW151226:



PRL **116,** 241103 (2016) PHYSICAL REVIEW LETTERS week ending 17 JUNE 2016

GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence

B. P. Abbott *et al.*[*]

(LIGO Scientific Collaboration and Virgo Collaboration)

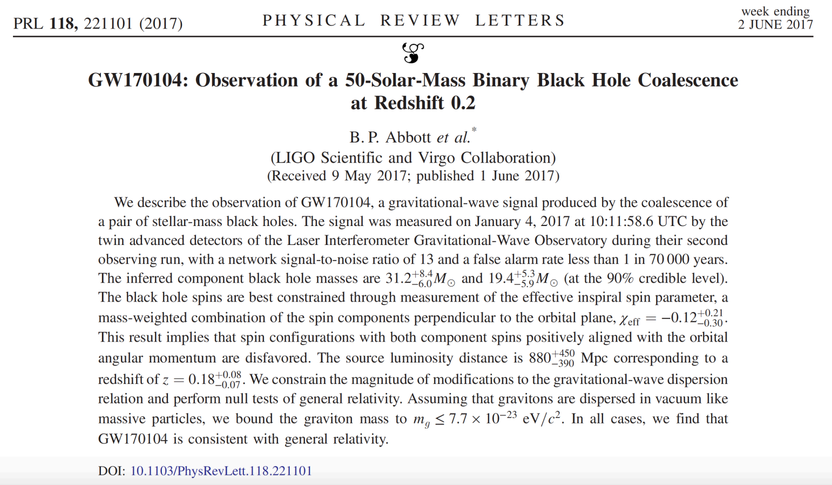
(Received 31 May 2016; published 15 June 2016)

We report the observation of a gravitational-wave signal produced by the coalescence of two stellar-mass black holes. The signal, GW151226, was observed by the twin detectors of the Laser Interferometer Gravitational-Wave Observatory (LIGO) on December 26, 2015 at 03:38:53 UTC. The signal was initially identified within 70 s by an online matched-filter search targeting binary coalescences. Subsequent off-line analyses recovered GW151226 with a network signal-to-noise ratio of 13 and a significance greater than 5σ. The signal persisted in the LIGO frequency band for approximately 1 s, increasing in frequency and amplitude over about 55 cycles from 35 to 450 Hz, and reached a peak gravitational strain of $3.4^{+0.7}_{-0.9} \times 10^{-22}$. The inferred source-frame initial black hole masses are $14.2^{+8.3}_{-3.7} M_{\odot}$ and $7.5^{+2.3}_{-2.3} M_{\odot}$, and the final black hole mass is $20.8^{+6.1}_{-1.7} M_{\odot}$. We find that at least one of the component black holes has spin greater than 0.2. This source is located at a luminosity distance of 440^{+180}_{-190} Mpc corresponding to a redshift of $0.09^{+0.03}_{-0.04}$. All uncertainties define a 90% credible interval. This second gravitational-wave observation provides improved constraints on stellar populations and on deviations from general relativity.

DOI: 10.1103/PhysRevLett.116.241103

1.2 Black Holes & Thermodynamics (Cont.)

- GW170104:



PRL **118,** 221101 (2017) PHYSICAL REVIEW LETTERS week ending 2 JUNE 2017

GW170104: Observation of a 50-Solar-Mass Binary Black Hole Coalescence at Redshift 0.2

B. P. Abbott *et al.*[*]
(LIGO Scientific and Virgo Collaboration)

(Received 9 May 2017; published 1 June 2017)

We describe the observation of GW170104, a gravitational-wave signal produced by the coalescence of a pair of stellar-mass black holes. The signal was measured on January 4, 2017 at 10:11:58.6 UTC by the twin advanced detectors of the Laser Interferometer Gravitational-Wave Observatory during their second observing run, with a network signal-to-noise ratio of 13 and a false alarm rate less than 1 in 70 000 years. The inferred component black hole masses are $31.2^{+8.4}_{-6.0} M_\odot$ and $19.4^{+5.3}_{-5.9} M_\odot$ (at the 90% credible level). The black hole spins are best constrained through measurement of the effective inspiral spin parameter, a mass-weighted combination of the spin components perpendicular to the orbital plane, $\chi_{\mathrm{eff}} = -0.12^{+0.21}_{-0.30}$. This result implies that spin configurations with both component spins positively aligned with the orbital angular momentum are disfavored. The source luminosity distance is 880^{+450}_{-390} Mpc corresponding to a redshift of $z = 0.18^{+0.08}_{-0.07}$. We constrain the magnitude of modifications to the gravitational-wave dispersion relation and perform null tests of general relativity. Assuming that gravitons are dispersed in vacuum like massive particles, we bound the graviton mass to $m_g \leq 7.7 \times 10^{-23}\ \mathrm{eV}/c^2$. In all cases, we find that GW170104 is consistent with general relativity.

DOI: 10.1103/PhysRevLett.118.221101

1.2 Black Holes & Thermodynamics (Cont.)

- In many ways a black hole acts like an ideal black body, as it reflects no light.
- Classically, a black hole has zero temperature,

  $T_{\mathrm{Classical}} = 0.$

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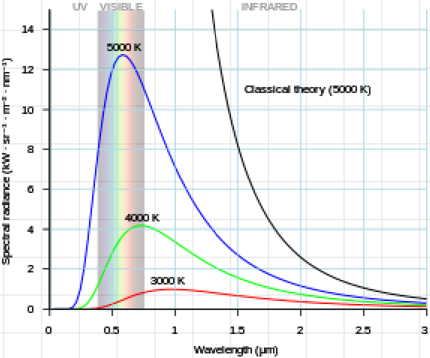

1.2 Black Holes & Thermodynamics (Cont.)

- Bekenstein [1] first proposed that a black hole has an entropy S that is proportional to its area A, [2]

$$S_{\mathrm{bh}} = \eta \left(\frac{k_B A}{\hbar G}\right), \qquad \eta = ???$$

k_B: Boltzmann constant
A: horizon area of the black hole
G: Newtonian constant
η: a constant.

---

[1] J.D. Bekenstein, Lett. Nuovo Cimento 4 (1972) 737; Phys. Rev. D9 (1973) 2333; D9 (1974) 3292.

[2] For an interesting historic review, see D.N. Page, New J. Phys. 7 (2005) 203.

1.2 Black Holes and Their Thermodynamics (Cont.)

- Dimensional Analysis: The entropy and area have the following dimensions,

$$[S_{bh}] = \frac{L^2 M}{T^2 K}, \quad [A] = L^2,$$

L: length, M: mass, T: time, K: temperature.

- Assuming that

$$S_{bh} = C_0 \, A,$$

we find that

$$[C_0] = \frac{[S_{bh}]}{[A]} = \frac{M}{T^2 K}.$$

1.2 Black Holes and Their Thermodynamics (Cont.)

- In general relativity, thermodynamics and quantum mechanics, there are four fundamental constants,

$$(c,\ G,\ k_B,\ \hbar)\,,$$

with the units,

$$[c]=\frac{L}{T},\quad [G]=\frac{L^3}{T^2M},\quad [k_B]=\frac{L^2M}{T^2K},\quad [\hbar]=\frac{L^2M}{T}.$$

- Assuming that

$$C_0=\eta c^{\alpha}G^{\beta}k_B^{\gamma}\hbar^{\delta},$$

we find that

$$[C_0]=\frac{L^{\alpha+3\beta+2\gamma+2\delta}M^{-\beta+\gamma+\delta}}{T^{\alpha+2\beta+2\gamma+\delta}K^{\gamma}}=\frac{M}{T^2K}.$$

1.2 Black Holes and Their Thermodynamics (Cont.)

- Thus, we have,

$$\begin{aligned}&\alpha + 3\beta + 2\gamma + 2\delta = 0,\\&-\beta + \gamma + \delta = 1,\\&\alpha + 2\beta + 2\gamma + \delta = 2,\\&\gamma = 1,\end{aligned}$$

which have the unique solution,

$$\alpha = 3,\ \beta = -1,\ \gamma = 1,\ \delta = -1,$$

that is,

$$\mathrm{C_0} = \eta \frac{\mathrm{c^3 k_B}}{\mathrm{G}\hbar} \quad \Rightarrow \quad \mathrm{S_{bh}} = \eta \cdot \frac{\mathrm{c^3 k_B}}{\hbar \mathrm{G}} \cdot \mathrm{A}.$$

1.2 Black Holes & Thermodynamics (Cont.)

- Using quantum field theory in curved spacetime, Hawking [3] found that the entropy of the black hole is indeed proportional to its area, with

$$\eta = \frac{1}{4} \quad \Rightarrow \quad S_{BH} = \frac{k_B A}{4\hbar G}.$$

- The Bekenstein-Hawking entropy S_{BH} has been derived from several different theories of quantum gravity, including string/M-Theory and loop quantum gravity, and now is considered as one of the milestones, and often taken as *a consistency checking for a theory of quantum gravity.*

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[3] S. W. Hawking, Nature 248 (1974) 30; Commun. Math. Phys. 43 (1975) 199.

1.2 Black Holes & Thermodynamics (Cont.)

- Hawking also found that black holes are not completely black, and instead emit radiation, with the same spectrum as a black body of a temperature,

$$\mathrm{T_H} = \frac{\hbar}{8\pi \mathrm{k_B G M}},$$

$\hbar$: Planck's constant, M: mass of the black hole

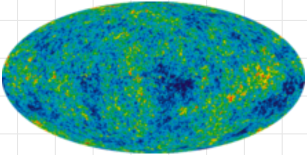
- This temperature is on the order of billionths of a kelvin for black holes of stellar mass,

$$\mathrm{T_H} \simeq 10^{-6}\ \mathrm{K}\Big|_{\mathrm{M}\simeq \mathrm{M}_\odot},$$

making it essentially impossible to observe.

1.2 Black Holes & Thermodynamics (Cont.)

- Note that the temperature of cosmic microwave background (CMB) is [4],



$$\mathrm{T_{CMB}} = 2.72548 \pm 0.00057\ \mathrm{K},$$

$$\frac{\mathrm{T_{CMB}}}{\mathrm{T_H}} \simeq 10^6,$$

which is sixth-order higher than that of a black hole of stellar mass.

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[4] D. J. Fixsen, ApJ 707 (2009) 916 [arXiv:0911.1955].

1.2 Black Holes & Thermodynamics (Cont.)

- Even before Bekenstein proposed that a black has entropy, Hawking found that: *the area of an event horizon can never decrease* [5],

$$\delta A \geq 0,$$

which is the reminiscent of the second law of thermodynamics.

- The correspondence was strengthened with the discovery of analogs of other three laws of thermodynamics [6].

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[5] S. W. Hawking, Commun. Math. Phys. 25 (1972) 152.

[6] J. M. Bardeen, B. Carter and S. W. Hawking, Commun. Math. Phys. 31 (1973) 161.

1.2 Black Holes & Thermodynamics (Cont.)

- For a stationary asymptotically flat black hole in 4D, uniquely characterized by a mass M, an angular momentum J, and a charge Q, the four laws are:
  - The surface gravity κ is constant over the event horizon.
  - For two stationary black holes differing only by small variations in the parameters M, J, Q,

$$\delta M = \frac{\kappa}{8\pi G}\delta A + \Omega J + \Phi\delta Q$$

    Ω: the angular velocity, Φ the electric potential, all defined at the horizon.
  - The area of the event horizon of a black hole never decreases,

$$\delta A \geq 0.$$

  - It is impossible by any procedure to reduce the surface gravity κ to zero in a finite number of steps.

1.2 Black Holes & Thermodynamics (Cont.)

- These laws are clearly formally analogous to the four laws of thermodynamics, if we make the identification,

$$(\kappa, \mathrm{A}) = \left(\frac{2\pi \mathrm{k_B T_H}}{\hbar}, 4\hbar \mathrm{G S_{BH}} \right)$$

- Note that, despite the formal analogy with the laws of thermodynamics, Bardeen, Carter and Hawking argued that κ cannot be a true temperature and A cannot be a true entropy. That is because that heat always flows from hot to cold. But, if one places a (classical) black hole in contact with a heat reservoir of any temperature, energy will flow into the black hole, and never out. Thus, a (classical) black hole must have an absolute zero temperature.

1.2 Black Holes & Thermodynamics (Cont.)

- The final confirmation of Bekenstein's proposal that a black hole has entropy had to wait for Hawking to discover: a black hole quantum mechanically radiates particles, and the spectrum of the radiation is that of a black body.

- While the four laws were originally formulated for 4D "electrovac" spacetimes, they can be extended to high dimensions, more charges and angular momenta, and to other "black" objects such as black strings, rings, and branes [7].

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[7] A. C. Wall, JHEP, 0906 (2009) 021.

1.2 Black Holes & Thermodynamics (Cont.)

- The first law, in particular, holds for arbitrary isolated horizons, and for much more general gravitational actions, for which the entropy can be understood as a Noether charge [8].
- Studies of black hole thermodynamics have led to profound understandings of gravity, including the reconstruction of general relativity (GR) as the thermodynamic limit of a more fundamental theory of gravity, and the AdS/CFT correspondence [9].

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[8] A. Ashtekar, C. Beetle and S. Fairhurst, CQG16 (1999) L1; R. M. Wald, PRD48 (1993) 3427.

[9] T. Jacobson, PRL75 (1995) 1260; G. ’t Hooft, arXiv:gr-qc/9310026; L. Susskind, JMP36, 6377 (1995); J.M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998).

1.2 Black Holes & Thermodynamics (Cont.)

- However, they also lead to various interesting puzzles. One of them is *the Information Loss Paradox*: Consider a black hole initially formed by the collapse of matter in a pure quantum state, which then evaporates completely into Hawking radiation. If the radiation is genuinely thermal, this would represent a transition from a pure state to a mixed state, a process that violates unitarity of evolution and is forbidden in ordinary quantum mechanics.

1.2 Black Holes & Thermodynamics (Cont.)

- If, on the other hand, the Hawking radiation is somehow a pure state, this would appear to require correlations between “early” and “late” Hawking particles that have never been in causal contact.

- This problem recently was further sharpened, and argued that one must sacrifice some cherished principle [10]:
  - the equivalence principle
  - low energy effective field theory, or
  - the nonexistence of high-entropy “remnants” at the end of black hole evaporation.

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[10] A. Almheiri, D. Marolf, J. Polchinski and J. Sully, JHEP02 (2013) 062 [arXiv:1207.3123].

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2.1 Lorentz Symmetry Breaking

- The invariance under the Lorentz symmetry group is a cornerstone of modern physics, and all the experiments carried out so far are consistent with it [11].

- Nevertheless, there are various reasons to construct gravitational theories with broken Lorentz invariance (LI). In particular, our understanding of space-times at Plank scale is still highly limited, and the renomalizability and unitarity of gravity often lead to the violation of LI [12].

---

[11] A. Kostelecky and N. Russell, Rev. Mod. Phys. 83 11 (2011) [arXiv:0801.0287v7, January 2014 Edition].

[12] C. Kiefer, Quantum Gravity (Oxford Science Publications, Oxford University Press, 2007).

2.1 Lorentz Symmetry Breaking (Cont.)

- An example is the Hořava theory [13], in which the LI is broken via the anisotropic scaling between time and space in the ultraviolet (UV),

$$t \rightarrow b^{-z}t, \quad x^i \rightarrow b^{-1}x^i, \; (i = 1, 2, 3), \qquad (1)$$

z: the dynamical critical exponent.

- This is a reminiscent of Lifshitz scalars in condensed matter physics [14], hence the theory is often referred to as the Hořava-Lifshitz (HL) quantum gravity at a Lifshitz fixed point.

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[13] P. Horava, PRD79 (2009) 084008.

[14] E.M. Lifshitz, Zh. Eksp. Toer. Fiz. 11 (1941) 255; 11 (1941) 269.

2.1 Lorentz Symmetry Breaking (Cont.)

- The anisotropic scaling (1) provides a crucial mechanism: The gravitational action can be constructed in such a way that only higher-dimensional spatial (but not time) derivative operators are included, so that the UV behavior of the theory is dramatically improved. In particular, for $z \geq 3$ it becomes power-counting renormalizable.

- The exclusion of high-dimensional time derivative operators, on the other hand, prevents the ghost instability[15], whereby the unitarity of the theory is assured.

---

[15] M. Ostrogradsky, Mem. Acad. St. Petersbourg, VI4 (1850) 385-517.

2.1 Lorentz Symmetry Breaking (Cont.)

- It must be emphasized that, the breaking of LI can have significant effects on the low-energy physics through the interactions between gravity and matter, no matter how high the scale of symmetry breaking is [16].

- Recently, Pospelov and Tamarit proposed a mechanism of SUSY breaking by coupling a Lorentz-invariant supersymmetric matter sector to non-supersymmetric gravitational interactions with Lifshitz scaling, and showed that it can lead to a consistent HL gravity [17].

---

[16] J. Collins, et al, PRL93 (2014) 191301.

[17] M. Pospelov and C. Tamarit, JHEP 01 (2014) 048.

2.1 Lorentz Symmetry Breaking (Cont.)

- It is remarkable to note that, despite of the stringent observational constraints of the violation of the LI, the nonrelativistic general covariant HL gravity constructed in [18] is consistent with all the solar system tests.
- Recently, it was embedded in string theory via the nonrelativistic AdS/CFT correspondence [19].
- Using the gravity/gauge duality, it corresponds to Newton-Carton theory, and a precise dictionary relating all fields was built [20].

---

[18] T. Zhu, Q. Wu, AW, F.-W. Shu, PRDD84 (2011) 101502(R); K. Lin, S. Mukohyama, AW, T. Zhu, PRD89 (2014) 084022.

[19] S. Janiszewski, A. Karch, PRL 110 (2013) 081601; JHEP 02 (2013) 123.

[20] J. Hartong, N.A. Obers, JHEP07 (2015) 155.

2.1 Lorentz Symmetry Breaking (Cont.)

- Another version of the HL gravity, the health extension [21], is also self-consistent and passes all the solar system, astrophysical and cosmological tests.

- In fact, in the IR the theory can be identified with the hypersurface-orthogonal Einstein-aether theory in a particular gauge, whereby the consistence of the theory with observations can be deduced.

---

[21] D. Blas, O. Pujolas, and S. Sibiryakov, JHEP 1104 (2011) 018.

2.1 Lorentz Symmetry Breaking (Cont.)

- Another example that violates LI is the Einstein-aether theory [22], in which the breaking is realized by a timelike vector field, while the gravitational action is still generally covariant.

- This theory is consistent with all the solar system tests and binary pulsar observations [23].

- But, unlike the HL theory, the Einstein-aether theory is still considered as the low-energy effective theory, and is not UV complete.

---

[22] T. Jacobson and Mattingly, PRD64, 024028 (2001); T. Jacobson, Proc. Sci. QG-PH, 020 (2007).

[23] K. Yagi, etal, PRL 112 (2014) 161101.

2.2 Black Holes in Theories with Lorentz Symmetry Breaking

- Once the LI is broken, particles can have different speeds. In the HL theory, because of the presence of high-order spatial operators, the dispersion relation generically becomes nonlinear[24],

$$E^2 = c_p^2 p^2 \left(1 + \alpha_1 \left(\frac{p}{M_*}\right)^2 + \alpha_2 \left(\frac{p}{M_*}\right)^4\right), \qquad (2)$$

E, p: the energy and momentum of the particle

c_p, α_i: coefficients, depending on the species of the particle

M_*: the energy scale of the HL theory

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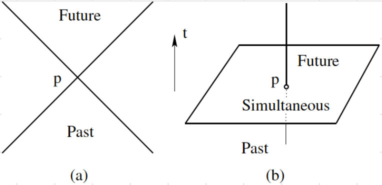
[24] AW, R. Maartens, PRD 81 (2010) 024009; AW, PRD 82 (2010) 124063.

2.2 Black Holes (Cont.)



(a) (b)

- Then, both phase and group velocities of the particle become unbounded as p increases. The causal structure is quite different from that in theories with Lorentz symmetry, in which light- cones play a central role [Fig. (a)]. In fact, it is more like the case in Newtonian theory [Fig. (b)] [25].
- This suggests that black holes may be only low-energy phenomena [26], and are foundmentally absent in the UV.

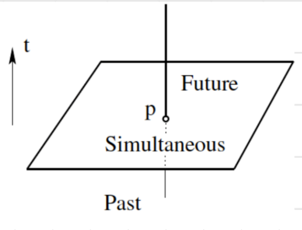
---

[25] J. Greenwald, J. Lenells, J. X. Lu, V. H. Satheeshkumar, AW, PRD 84, 084040 (2011).

[26] AW, PRL 110 (2013) 091101[arXiv:1212.1876].

2.2 Black Holes (Cont.)

- Lately, a potential breakthrough was made by the discovery that there still exist absolute causal boundaries, the so-called *universal horizons*, in theories with broken LI [27].
- The main idea is as follows. In a given spacetime, a globally timelike scalar field, the so-called khronon [26], might exist. Then, similar to the Newtonian theory, this khronon field defines a global absolute time, and all particles are assumed to move along the increasing direction of the khronon, so the causality is well defined.



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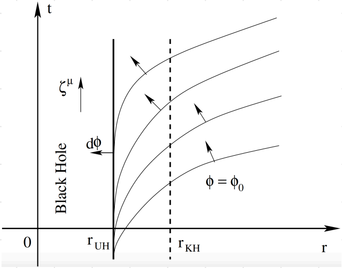
[27] D. Blas and S. Sibiryakov, PRD 84 (2011) 124043.

2.2 Black Holes (Cont.)

- In a spacetime, there may exist a surface as shown in the figure, denoted by the vertical solid line. Given that all particles move along the increasing direction of the khronon, from the figure it is clear that a particle must cross this surface and move inward, once it arrives at it. This is an one-way membrane, and particles even with infinitely large speed cannot escape from it, once they are trapped inside it. So, it acts as an absolute horizon to all particles (with any speed)

  — *the universal horizon*.



ϕ: the khronon field

2.2 Black Holes (Cont.)

- From the above figure we can see that the location of the universal horizon is defined by

$$\mathrm{dt}\cdot\mathrm{d}\phi = 0 \qquad \Rightarrow \qquad \zeta\cdot\mathrm{u} = 0, \tag{3}$$

$$\mathrm{u}_\mu \equiv \frac{\phi_{,\mu}}{\sqrt{|\phi_{,\alpha}\phi^{,\alpha}|}}, \qquad \zeta^\mu = \delta^\mu_\mathrm{t}.$$

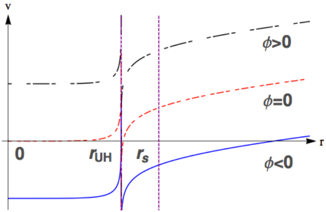
- Since u_μ is well-defined in the whole space-time and always timelike, then Eq.(3) is possible only inside the killing horizon, in which ζ^μ becomes spacelike,

$$\zeta\cdot\zeta = \begin{cases} < 0\ (\text{timelike}), & \mathrm{r} > \mathrm{r}_{\mathrm{KH}}, \\ = 0\ (\text{null}), & \mathrm{r} = \mathrm{r}_{\mathrm{KH}}, \\ > 0\ (\text{spacelike}), & \mathrm{r} < \mathrm{r}_{\mathrm{KH}}. \end{cases}$$

2.2 Black Holes (Cont.)

- In the static spacetimes, it was found that the surface gravity κ defined above is identical with the peeling behavior of the khronon at the universal horizon [28],

$$\kappa = \kappa_{\text{peeling}} \equiv \frac{1}{2}\frac{d}{dr}\left(\frac{s^r}{s^v}\right)\Big|_{UH}$$



s^λ: a spacelike unit vector, always orthogonal to u_μ,

$$s \cdot u = 0.$$

v: the Eddington-Finkelstein coordinate.

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[28] K. Lin, O. Goldoni, M. F. da Silva & AW, PRD 91 (2015) 024047.

2.2 Black Holes (Cont.)

Summary:

- The khronon defines globally an absolute time, and the trajectory of a particle is always along the increasing direction of ϕ. Thus, once it crosses the universal horizon, the particle must move toward the singularity $\mathrm{r} = 0$ and reaches it within a finite proper time.

- Only particles with infinitely large velocities can move around on the universal horizon, quite similar to the motion of light rays on the Killing horizon.

- A particle inside this surface cannot get out of it, no matter how large its velocity would be.

2.3 A Concrete Example (Cont.)

- Note that the Killing and universal horizons, as well as the surface gravity, are all defined in covariant form, so they are gauge-invariant.
- Restricting ourselves to the Schwarzschild space-time, which is also a solution of the HL theory [29],

$$\mathrm{ds}^2 = -\left(1 - \frac{\mathrm{r_s}}{\mathrm{r}}\right)\mathrm{dv}^2 + 2\mathrm{dvdr} + \mathrm{r}^2\mathrm{d}\Omega,$$

we find that the universal horizon is located at

$$\mathrm{r_{UH}} = \frac{3}{4}\mathrm{r_s}.$$

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[29] J. Greenwald, J. Lenells, J. X. Lu, V. H. Satheeshkumar, AW, PRD 84 (2011) 084040; AW, IJMPD 26 (2017) 1730014.

2.3 A Concrete Example (Cont.)

- Introducing a spacelike cooridate ψ vis the relation,

$$\psi \equiv -\left(\mathrm{v} + \int \frac{\mathrm{s_r}}{\mathrm{s_v}} \mathrm{dr}\right),$$

for which the normal vector of the hypersurface $\psi =$ Constant is s_μ,

- we find that

$$\begin{aligned} \mathrm{ds}^2 &= -\frac{(\mathrm{r}-\mathrm{r_{UH}})^2}{\mathrm{r}^4}\left(\mathrm{r}^2 + \frac{\mathrm{r_s}}{2}\mathrm{r} + \frac{3\mathrm{r_s^2}}{16}\right)\mathrm{d}\phi^2 \\ &\quad + \left(\frac{\frac{3^{3/4}}{4}\mathrm{r_s}}{\mathrm{r}}\right)^4 \mathrm{d}\psi^2 + \mathrm{r}^2\mathrm{d}\Omega^2, \end{aligned}$$

which is free of coordinate singularity at the Killing horizon, $\mathrm{r} = \mathrm{r_s}$, but singular at the universal horizon, $\mathrm{r} = \mathrm{r_{UH}}$.

2.3 A Concrete Example (Cont.)

- Note that the singular behavior at the universal horizon seems quite different from that at the Killing horizon found in the Schwarzschild coordinates. In particular, here $g_{\psi\psi}$ is finite and nonzero, while $g_{\phi\phi}$ becomes zero, but remains positive in both limits.

2.3 A Concrete Example (Cont.)

- In terms of (v, r), the function ϕ is given by,

$$
\begin{aligned}
\phi &= \mathrm{v} - \mathrm{r} - \mathrm{r_s} \ln \left|1 - \frac{\mathrm{r}}{\mathrm{r_s}}\right| + \varphi(\mathrm{r}), \\
\varphi(\mathrm{r}) &\equiv \varphi_0 + \frac{\mathrm{r_s}\epsilon_{\mathrm{UH}}}{8\sqrt{3}} \\
&\times \left\{ 9\sqrt{2} \ln \left| \frac{16\mathrm{r} + 6\mathrm{r_s} + 3\sqrt{2}\sqrt{16\mathrm{r}^2 + 8\mathrm{r_s}\mathrm{r} + 3\mathrm{r_s^2}}}{4(\mathrm{r} - \mathrm{r_{UH}})} \right| \right. \\
&\left. + 8\sqrt{3} \ln \left| \frac{20\mathrm{r} + 7\mathrm{r_s} + 3\sqrt{3}\sqrt{16\mathrm{r}^2 + 8\mathrm{r_s}\mathrm{r} + 3\mathrm{r_s^2}}}{\mathrm{r} - \mathrm{r_s}} \right| \right\},
\end{aligned}
$$

φ_0: a constant, and $\epsilon_{\mathrm{UH}} \equiv \mathrm{sign}(\mathrm{r} - \mathrm{r_{UH}})$.

2.3 A Concrete Example (Cont.)

- The function ψ is given by,

$$\begin{aligned}
\psi(\mathrm{r}) \quad = \quad & \psi_2 - (\mathrm{v} - \mathrm{r}) + \mathrm{r_s} \ln \left| 1 - \frac{\mathrm{r}}{\mathrm{r_s}} \right| \\
& - \frac{\epsilon_{\mathrm{UH}}}{108\mathrm{r_s^2}} \left\{ \mathrm{sqrt}48\mathrm{r}^2 + 24\mathrm{r_s r} + 9\mathrm{r_s^2} \left(16\mathrm{r}^2 + 8\mathrm{r_s r} + 15\mathrm{r_s^2}\right) \right. \\
& \quad + 60\sqrt{3}\mathrm{r_s^3} \ln \left(4\mathrm{r} + \mathrm{r_s} + \sqrt{16\mathrm{r}^2 + 8\mathrm{r_s r} + 3\mathrm{r_s^2}} \right) \\
& \quad + 108\mathrm{r_s^3} \ln \left[\frac{|\mathrm{r} - \mathrm{r_s}|}{20\mathrm{r} + 7\mathrm{r_s} + 3\sqrt{48\mathrm{r}^2 + 24\mathrm{r_s r} + 9\mathrm{r_s^2}}} \right] \\
& \qquad \qquad \left. - \psi_1 \right\},
\end{aligned}$$

$\psi_{1,2}$: constants.

2.3 A Concrete Example (Cont.)

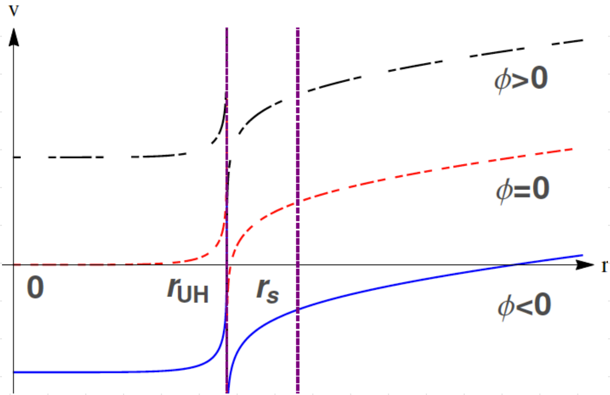

Figure: The surfaces of $\phi(v, r) = \text{Constant}$ in the (v, r)-plane for the Schwarzschild solution. It is peering across the universal horizon $r = r_{UH}$, but smoothly crossing the Killing horizon $r = r_s$.

2.3 A Concrete Example (Cont.)

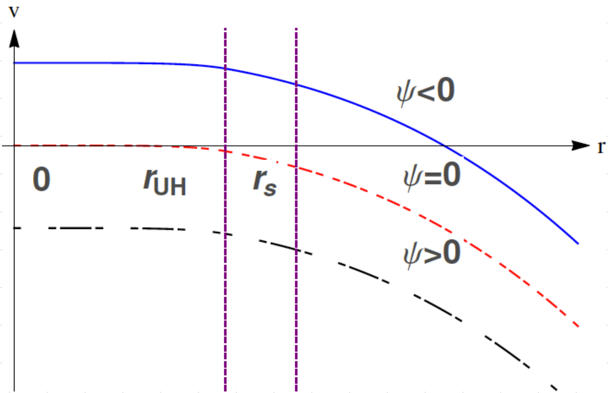

Figure: The surfaces of $\psi(v, r) = \text{Constant}$ in the $(v,\ r)$-plane for the Schwarzschild solution. It is smoothly crossing both of the universal and Killing horizons.

2.3 A Concrete Example (Cont.)

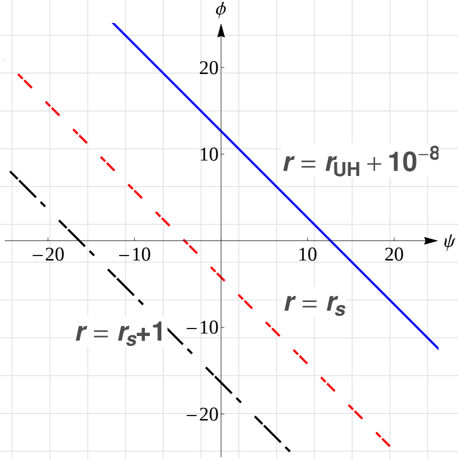

Figure: The surfaces of $r =$ Constant in the (ϕ, ψ)–plane for the Schwarzschild solution. The $r = r_{UH}$ surface corresponds to $\phi = +\infty$.

2.3 A Concrete Example (Cont.)

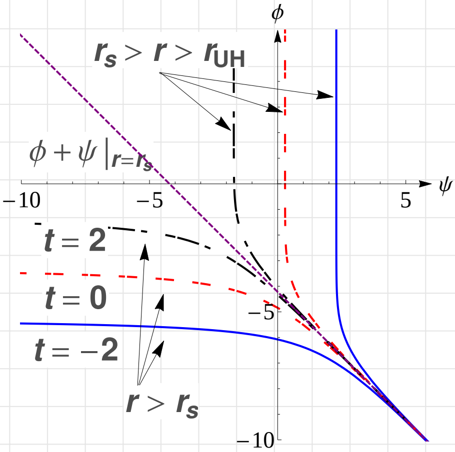

Figure: The surfaces of $t =$ Constant in the (ϕ, ψ)-plane for the Schwarzschild solution.

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3.1 The first law

- The first law of black hole mechanics,

$$\delta \mathrm{M}_{\mathrm{ADM}} = \frac{\kappa}{8\pi \mathrm{G}} \delta \mathrm{A}_{\mathrm{UH}}, \tag{4}$$

holds at the universal horizon for neutral black hole in the Einstein-aether theory [30].

- Recently we found two classes of exact solutions of the Einstein-aether-Maxwell theory, which represent charged black holes. It is surprising that the first law of black hole mechanics, Eq.(4), does not holds at the universal horizon for these charged black holes [31].

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[30] P. Berglund, J. Bhattacharyya, and D. Mattingly, PRD85 (2012) 124019.

[31] C.-K. Ding, AW, X.-W. Wang, PRD92 (2015) 084055; C.-K. Ding, C.-Q. Liu, AW, and J.L. Jing, PRD94 (2016) 124034.

3.2 Hawking Radiation

- The universal horizon radiates like a black body at a fixed temperature [32],

$$\mathrm{T_{UH}} = \frac{\kappa}{2\pi}, \tag{5}$$

but now with κ [33],

$$\kappa \equiv \frac{1}{2}\mathrm{u}^{\alpha}\mathrm{D}_{\alpha}\left(\mathrm{u}_{\lambda}\zeta^{\lambda}\right). \tag{6}$$

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[32] P. Berglund, et al, PRL 110 (2013) 071301

[33] B. Cropp, S. Liberati, A. Mohd, and M. Visser, PRD 89 (2014) 064061.

3.2 Hawking Radiation

- On the other hand, using the Hamilton-Jacobi method [34], we studied quantum tunneling of non-relativistic particles at universal horizons of the Einstein-Maxwell-aether black holes [35], after higher-order curvature corrections are taken into account [36],

$$E^2 = k^2 \sum_{n=0}^{z-1} a_n \left(\frac{k}{M_*}\right)^{2n}. \tag{7}$$

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[34] M. Agheben, M. Nadalini, L. Vanzo and S. Zerbini, JHEP 05 (2005) 014; C. Ding and J. Jing, Class. Quan. Grav. 25, 145015 (2008).

[35] C.-K. Ding, AW, X.-W. Wang, PRD92 (2015) 084055.

[36] C. Ding, AW, X. Wang and T. Zhu, Hawking radiation of charged Einstein-aether black holes at both Killing and universal horizons, Nucl. Phys. B913 (2016) 694 [arXiv:1512.01900].

3.2 Hawking Radiation

- Only relativistic particles are created at the Killing horizon, and the corresponding radiation is thermal with a temperature exactly the same as that found in GR.
- In contrary, only non-relativistic particles are created at the universal horizon and are radiated out to infinity with a thermal spectrum, but different species of particles, in general, experience different temperatures [37],

$$T_{\rm UH}^{z\geq 2} = \frac{2(z-1)}{z}\left(\frac{\kappa_{\rm UH}}{2\pi}\right) \tag{8}$$

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[37] C. Ding, AW, X. Wang and T. Zhu, Hawking radiation of charged Einstein-aether black holes at both Killing and universal horizons, Nucl. Phys. B913 (2016) 694 [arXiv:1512.01900].

3.2 Hawking Radiation

- Recently, more careful studies of ray trajectories showed that the surface gravity for particles with a non-relativistic dispersion relation is given by [38],

$$\kappa_{\mathrm{UH}}^{z\geq 2} = \left(\frac{2(z-1)}{z}\right)\kappa_{\mathrm{UH}}. \tag{9}$$

  The same results were also obtained in [39].

- It is remarkable to note that the standard relationship

$$T_{\mathrm{UH}}^{z\geq 2} = \frac{\kappa_{\mathrm{UH}}^{z\geq 2}}{2\pi}. \tag{10}$$

  still holds here.

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[38] C.-K. Ding and C.-Q. Liu, Dispersion relation and surface gravity of universal horizons, arXiv:1611.03153.

[39] B. Cropp, Strange Horizons: Understanding Causal Barriers Beyond General Relativity, arXiv:1611.00208.

3.2 Hawking Radiation

- Is this a coincidence? Without a deeper understanding of thermodynamics of universal horizons, it is difficult to say.
- But, whenever case like this raises, it is worthwhile of paying some special attention on it. In particular, does entropy of a universal horizon also depend on the dispersion relations of particles?

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4.1 Main Conclusions

- In contrast to our physical intuitions, absolute horizons exist even in theories with broken Lorentz symmetry, in which particles with arbitrary velocities are allowed.
- Once particles are trapped inside such horizons, they cannot escape from them, including particles with infinitely large velocities.
- The universal horizon radiates like a black body.

4.2 Open Questions

- The zeroth law:
  - In the spherically symmetric case, the surface gravity is always constant, so the first law always holds. But, in the non-spherically symmetric case it is not clear whether the surface gravity remains a constant or not. So far, most of the work has been done in the static case.
  - Recently, we showed that rotating universal horizons indeed exist not only in the IR but also in the UV [40].
  - However, since this is realized in (2+1)-dimensions, the surface gravity, like in the static case of (3+1)-dimensional spacetimes, is always constant.

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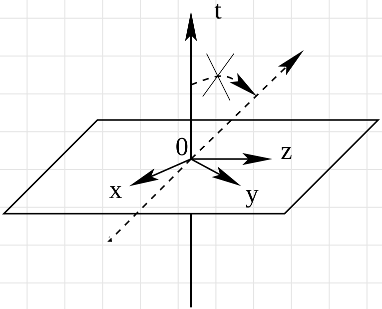
[40] K. Lin, V. H. Satheeshkumar, AW, Phys. Rev. D93 (2016) 124025.

4.2 Open Questions

- The zeroth law (Cont.):
  - In spacetimes with a preferred foliation, the zeroth law of black hole mechanics holds at universal horizons [41].



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[41] J. Bhattacharyya, M. Colombo, and T. P. Sotiriou, Causality and black holes in spacetimes with a preferred foliation, Class. Quant. Grav. 33 (2016) 235003

4.2 Open Questions (Const.)

- The first law:
  - The first law of black hole mechanics,

$$\delta M_{ADM} = \frac{\kappa}{8\pi G}\delta A_{UH},$$

    holds at the universal horizon for spherical and neutral black hole in the Einstein-aether theory [42].
  - However, it does not holds at the universal horizon for spherical charged black hole in the same theory [43].

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[42] P. Berglund, J. Bhattacharyya, and D. Mattingly, PRD85 (2012) 124019.

[43] C.-K. Ding, AW, X.-W. Wang, PRD92 (2015) 084055; C.-K. Ding, C.-Q. Liu, AW, and J.L. Jing, PRD94 (2016) 124034.

4.2 Open Questions (Cont.)

- The Second law:

  To recover the second law of black hole thermodynamics, Blas and Sibiryakov (BS) [44] considered the following two possibilities:

  - The missing entropy is accumulated somewhere inside the black hole. BS studied the stabilities of the universal horizons and found that they are linearly stable. But, they argued that after nonlinear effects are taken into account, these horizons will be turned into singularities with finite areas. One hopes that in the full HL gravity this singularity is resolved into a high-curvature region of finite width accessible to the instantaneous and fast high-energy modes. In this way the BH thermodynamics can be saved.

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[44] D. Blas and S. Sibiryakov, PRD84 (2011) 124043.

4.2 Open Questions (Cont.)

- Black holes have a large amount of static long hairs, which have tails that can be measured outside the horizon. After measuring them, an outer observer could decode the entropy that had fallen into the BH.
  - However, BS found that spherically symmetric hairs do not exist, and to have this scenario to work, one has to use non-spherically symmetric hairs.
  - Recently, we reconsidered this problem in the case where the velocity of the hkronon is infinitely large, and found that the BS conclusions hold even in this case [45].

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[45] K. Lin, S. Mukohyama, AW, T. Zhu, No static black hole hairs in gravitational theories with broken Lorentz invariance, PRD, in press (2017) [arXiv:1704.02990].

4.2 Open Questions (Cont.)

- The third law?
- Information Loss?
- Non-relativistic gravity/gauge correspondence [46]?
- Formation of universal horizons from gravitational collapse?
- ...?

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[46] F.-W. Shu, K. Lin, AW, Q. Wu, JHEP 04 (2014) 056; K. Lin, F.-W. Shu, AW, Q. Wu, PRD91 (2015) 044003.

Thank You!