Rapidity Divergence in QCD and Application to Collider Physics

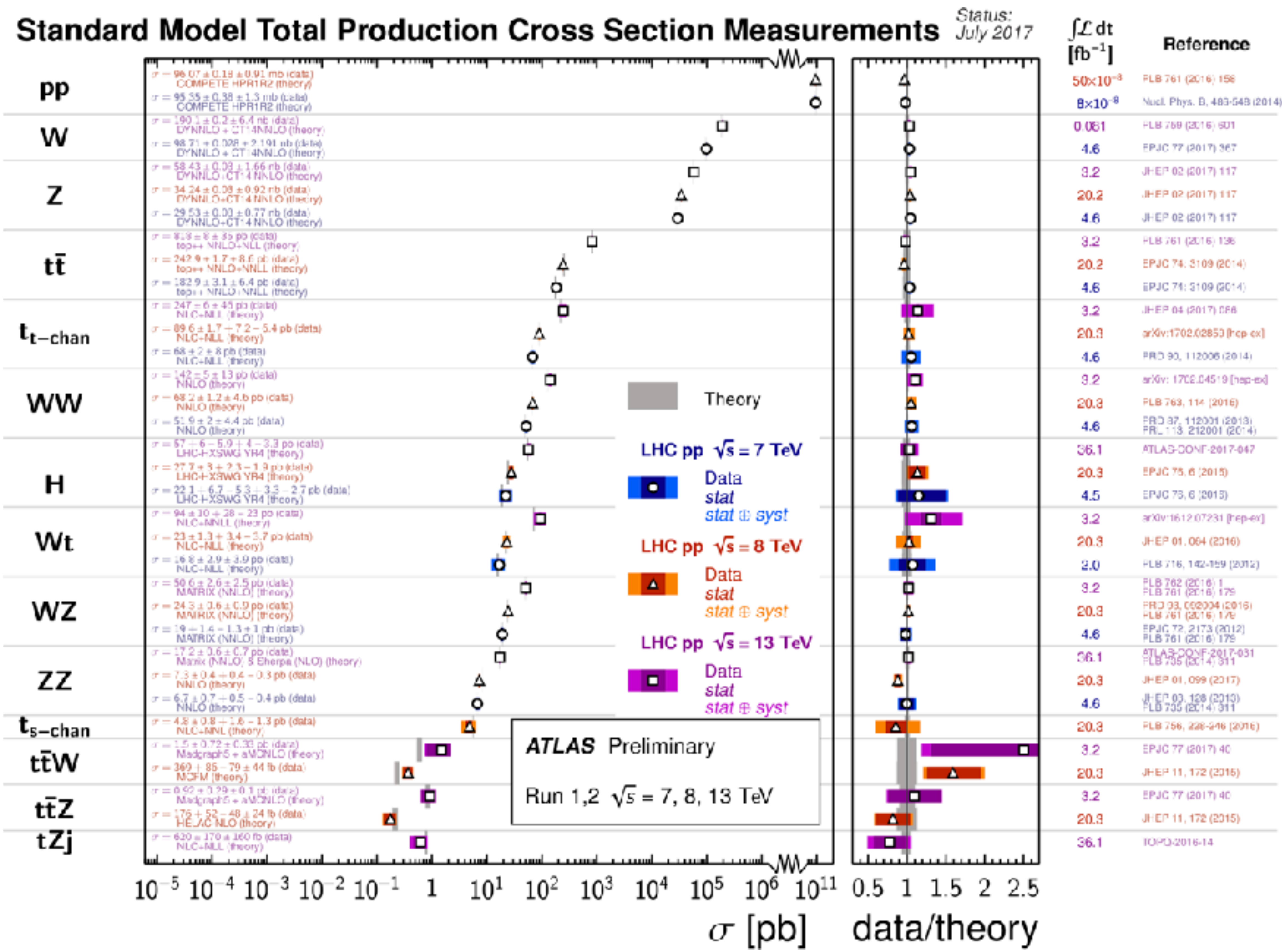
朱华星 (Hua Xing Zhu)

浙江大学

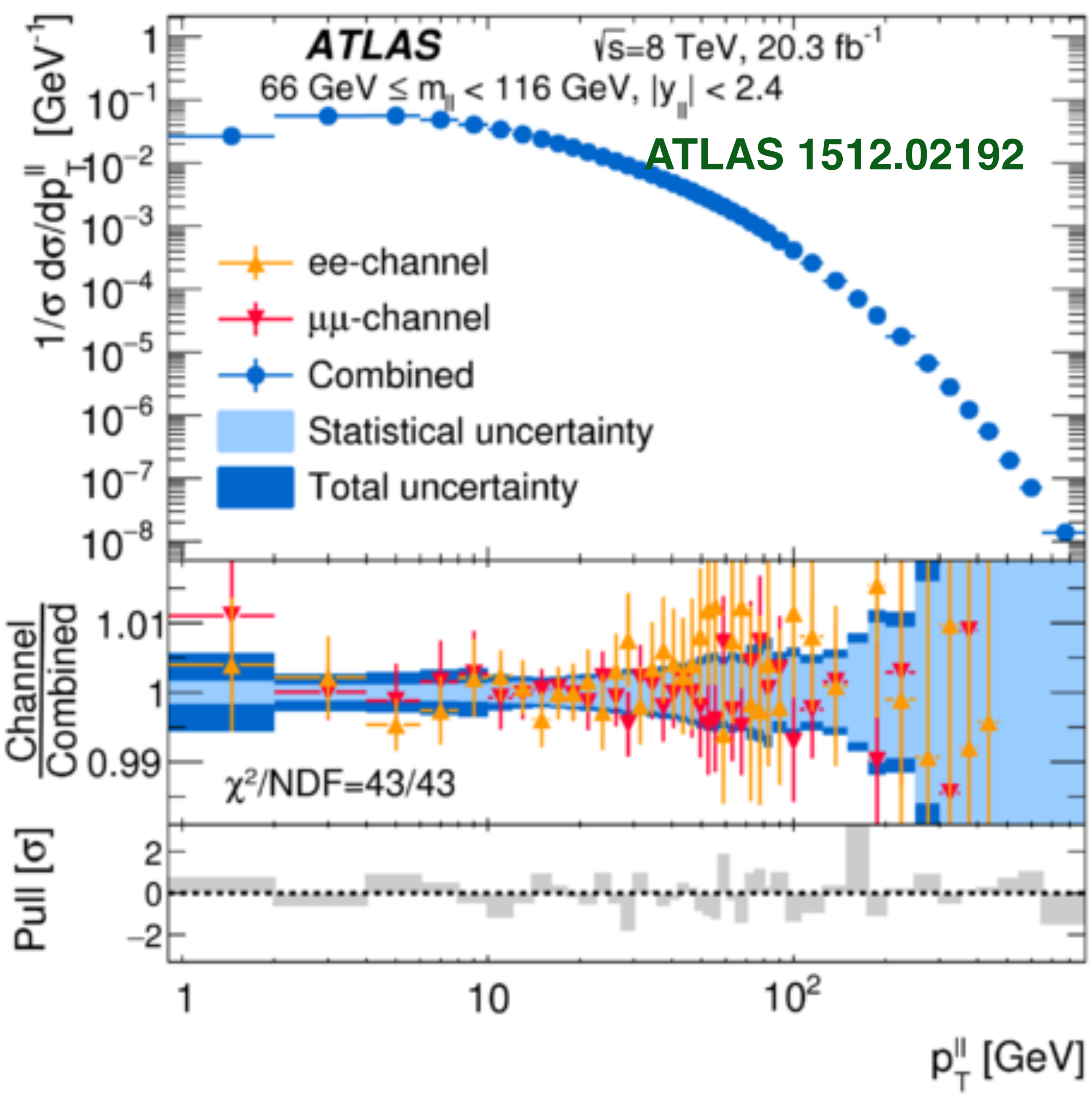
科大交叉中心seminar

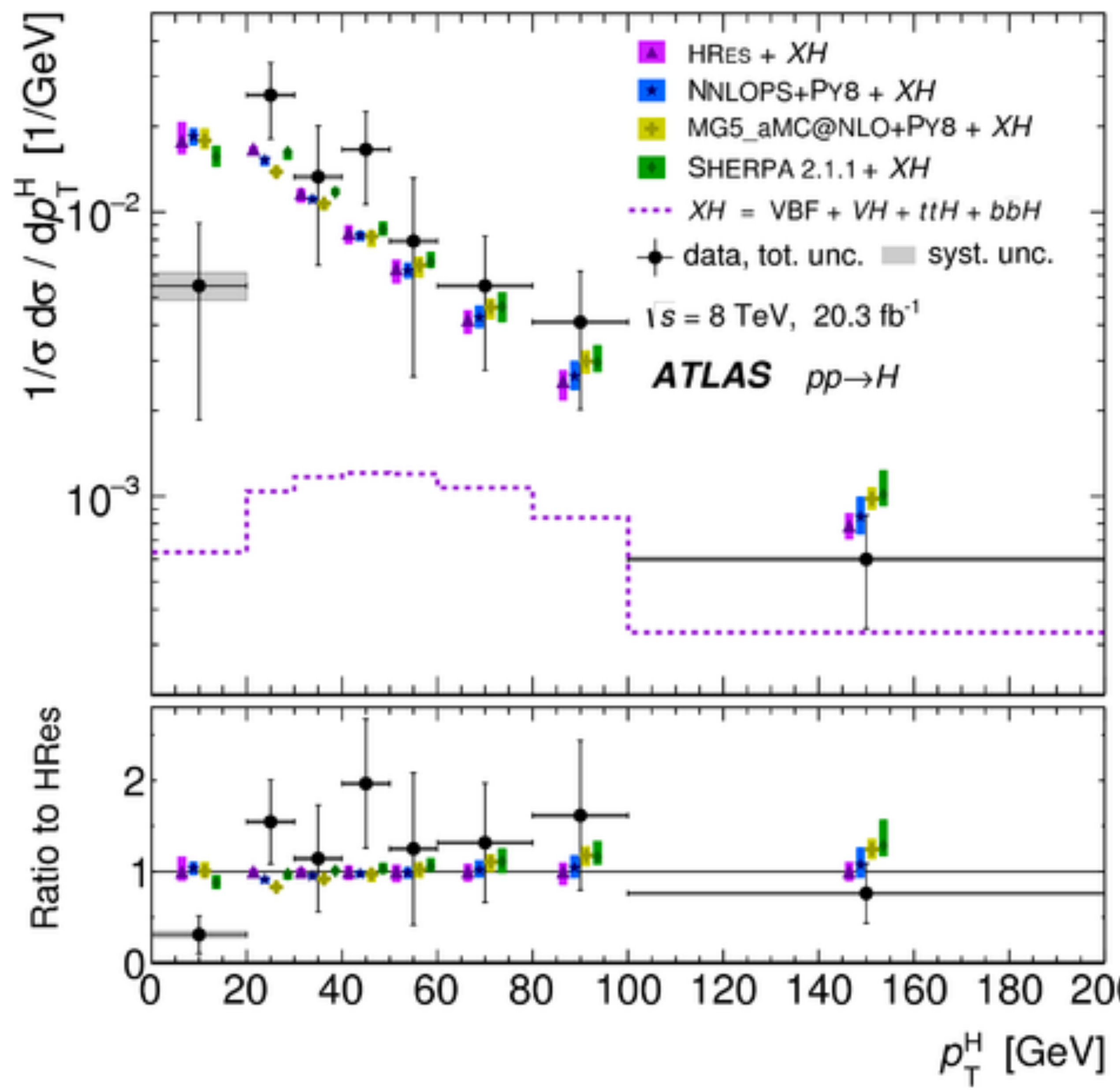
2017年9月14日

Fantastic Standard Model measurement from LHC



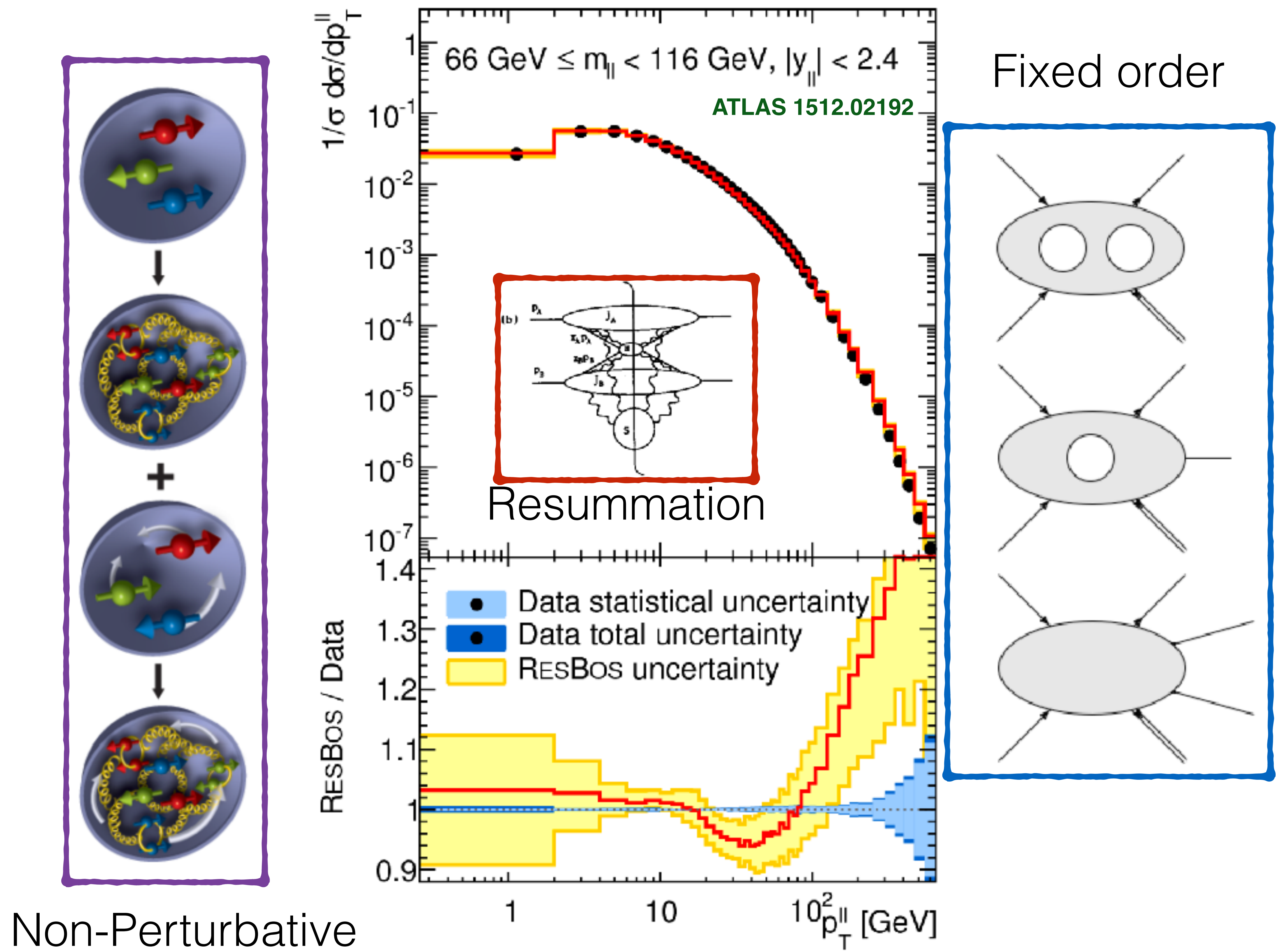
Motivation





✦ Percent level accuracy at LHC for Drell-Yan production!

✦ Differential distribution for Higgs production for probing new physics!



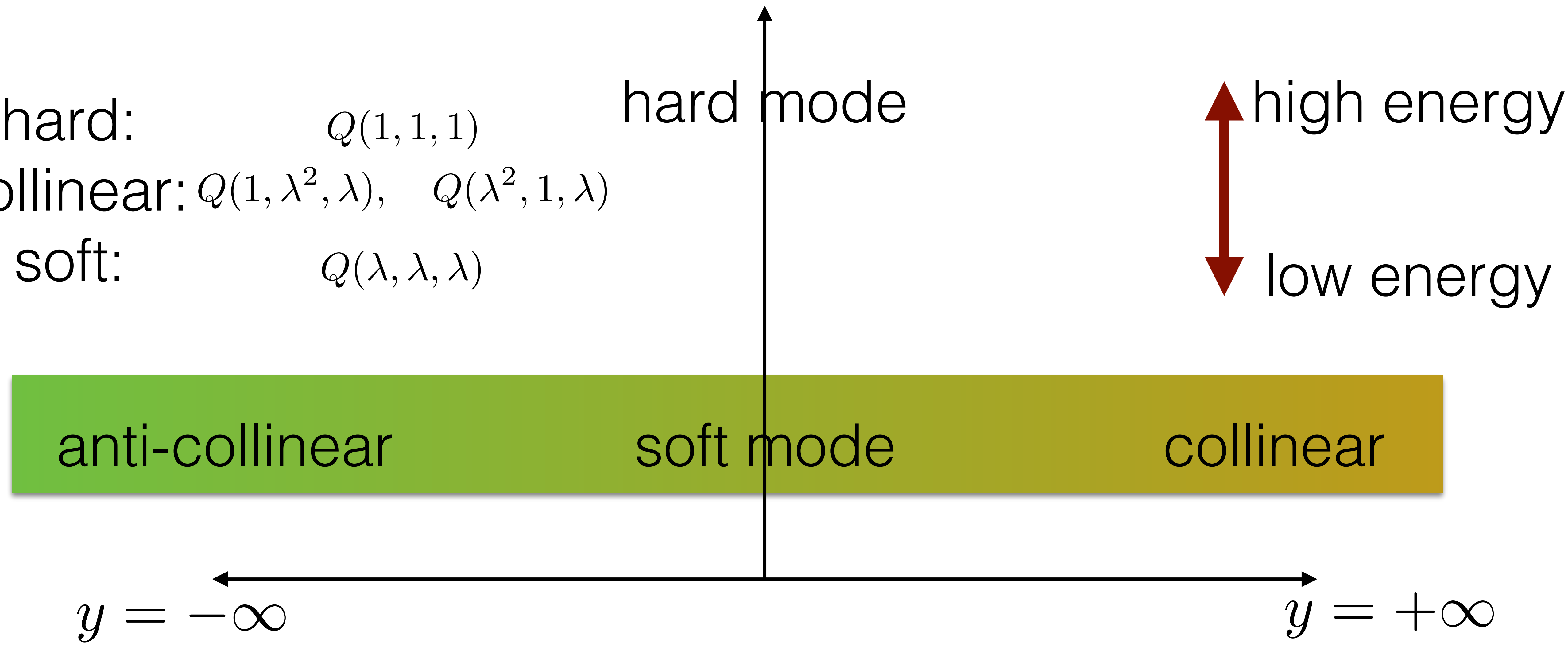
66 GeV ≤ m_ll < 116 GeV, |y_ll| < 2.4
ATLAS 1512.02192
1/σ dσ/dp_T^ll
ResBos / Data
Data statistical uncertainty
Data total uncertainty
ResBos uncertainty
p_T^ll [GeV]
Fixed order
Resummation
Non-Perturbative

Rapidity divergence/Lightcone divergence

hard: $Q(1,1,1)$

collinear: $Q(1,\lambda^2,\lambda)$, $Q(\lambda^2,1,\lambda)$

soft: $Q(\lambda,\lambda,\lambda)$



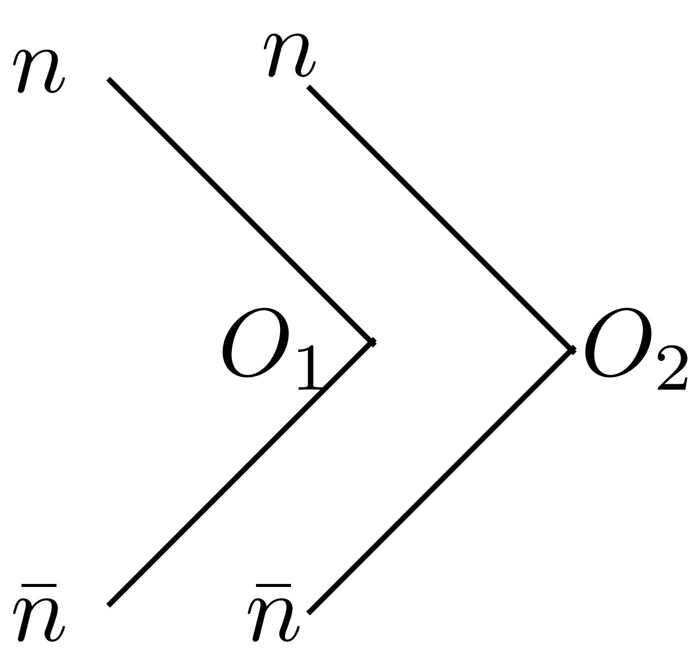
✦ Non-decoupling of UV from IR in an EFT ($SCET_{II}$)

Uniform degree of transcendentality in a class of Wilson loop expectation value in N=4 SYM

degree of transcendentality:

$$[\pi] = 1 \qquad [\zeta_n] = n \qquad [\zeta_m \zeta_n] = m + n$$

$$\ln x = \int^x \frac{dt}{t} \qquad \mathrm{Li}_2(x) = \int^x \frac{dt_1}{t_1} \int^{t_1} \frac{dt_2}{1-t_2} \qquad [\mathrm{Li}_n(x)] = n$$



$$a = \frac{g^2}{16\pi^2} N_c$$

$$\begin{aligned} G^{\mathcal{N}=4} =& D_0 \\ &+ a(16 D_2 + 8\zeta_2 D_0) \\ &+ a^2(128 D_4 - 160\zeta_2 D_2 + 312\zeta_3 D_1 - 2\zeta_4 D_0) \\ &+ a^3 \Big[512 D_6 - 3584\zeta_2 D_4 + 11584\zeta_3 D_3 - 4928\zeta_4 D_2 \\ &+ \left(-\frac{23200}{3}\zeta_2\zeta_3 + 11904\zeta_5\right) D_1 + \left(\frac{13216}{3}\zeta_3^2 - \frac{8012}{3}\zeta_6\right) \Big] \end{aligned}$$

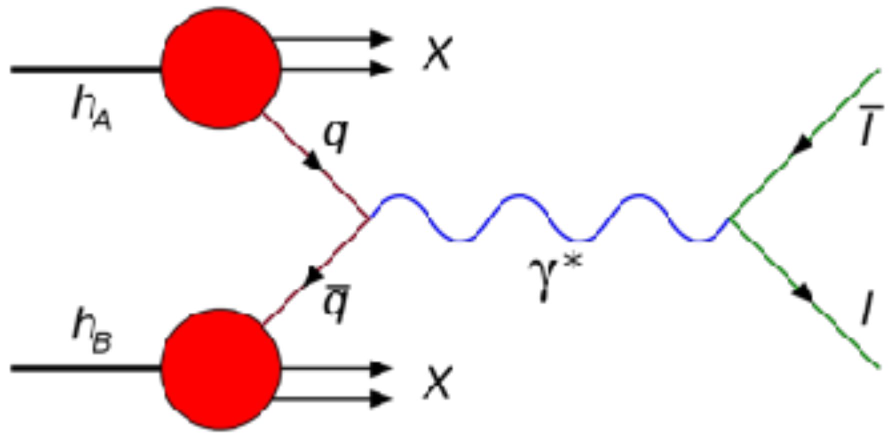
$$O_2 - O_1 = (\Delta t, 0, 0, 0)$$

Y. Li, von Manteuffel, Scharbinger, HXZ, 2014

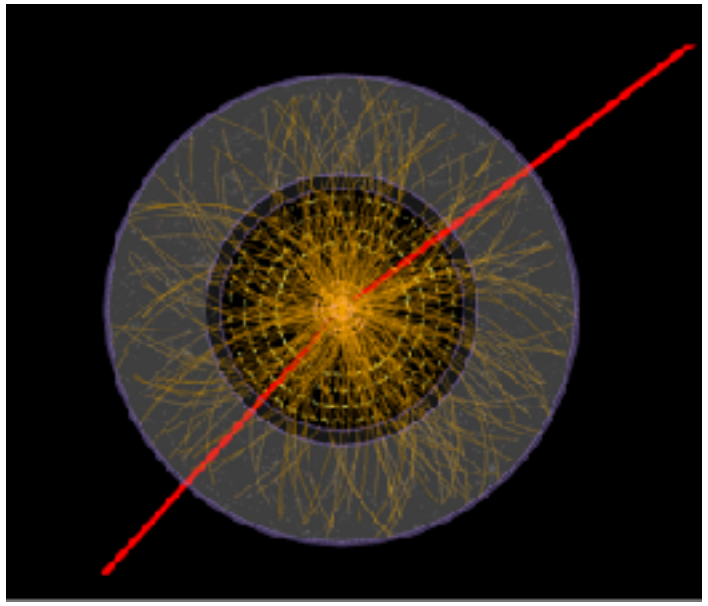
Outline

- Review factorization formalism for small pT resummation
- New regulator for rapidity divergence in TMD observable
- Three-loop anomalous dimension for rapidity evolution
- Phenomenological application to Higgs production

Transverse momentum of color neutral system



ATLAS event: 242090708



- Definition of the observable (Drell-Yan case):

$$\vec{q}_\perp = \vec{p}_{l^+,\perp} + \vec{p}_{l^-,\perp}$$

$$\frac{d\sigma}{d\vec{q}_\perp^{\,2}\,dY} = \sum_{i,j}\int_0^1 dx_a\,dx_b\,f_{i/h_1}(x_a,\mu_f)f_{j/h_2}(x_b,\mu_f)\frac{d\hat{\sigma}}{d\vec{q}_\perp^{\,2}\,dY}(\hat{s},\hat{t},\hat{u},Q^2)$$

- Analytical results at NLO available for long time **DY: Ellis, Martinelli, Petronzio, 1983; Higgs: Glosser, Schmidt, 2002**

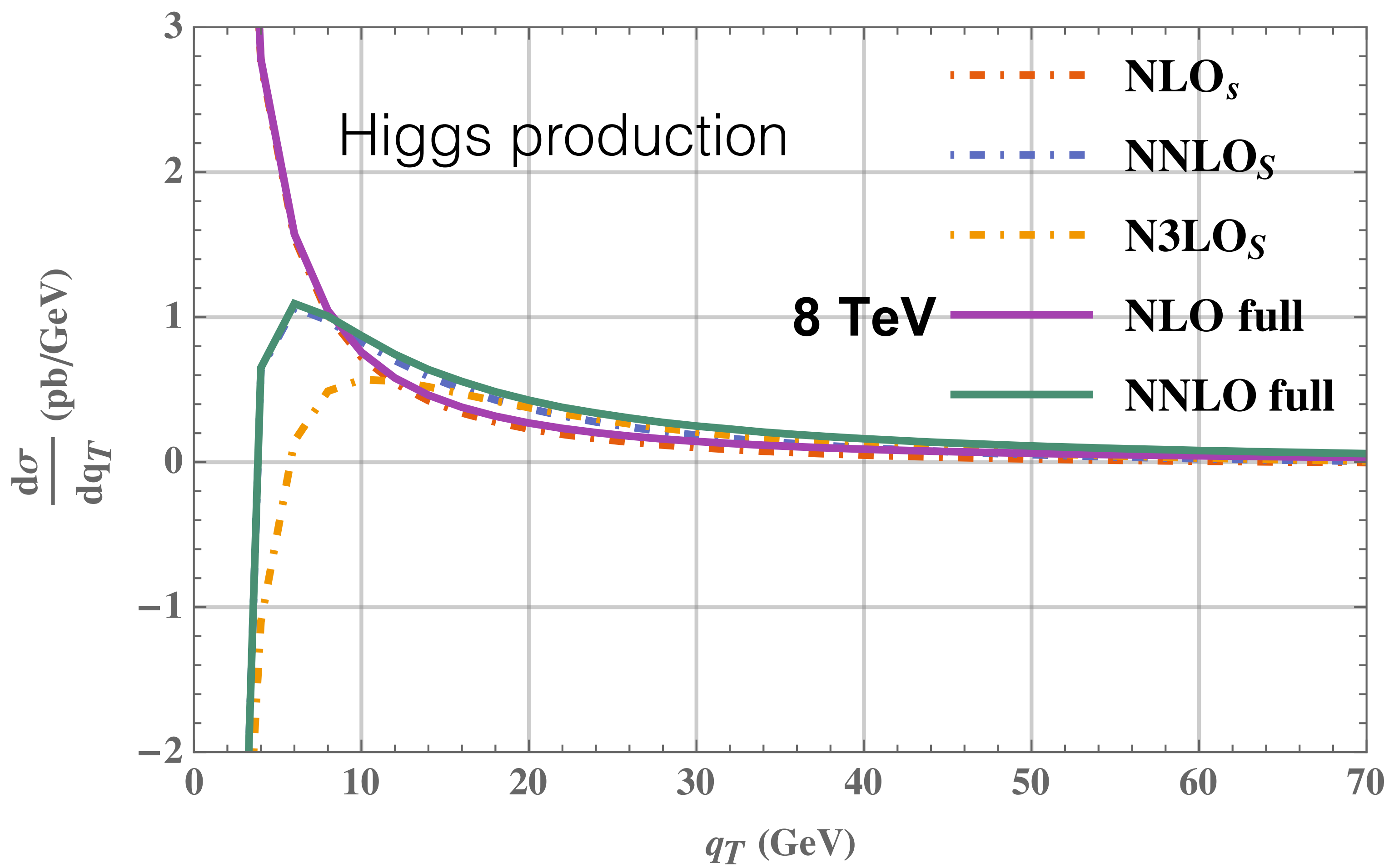
- Recent development: numerical calculation of V/H + jet at NNLO at large pT **Antenna subtraction: Gehrmann et al; N-jettiness, Boughezal et al; STRIPPER subtraction: Boughezal et al;**

- Progress towards analytical NNLO V/H+jet **Dulat et al.**

Break down of fixed order P.T. at small pT

✦ **Fixed order perturbation theory exhibits large logs at small pT**

$$\frac{d\sigma}{dp_T} = \alpha_s\left(c_{1,1}\frac{\ln p_T}{p_T} + c_{1,0}\frac{1}{p_T} + \ldots\right)$$
$$\alpha_s^2\left(c_{2,3}\frac{\ln^3 p_T}{p_T} + c_{2,2}\frac{\ln^2 p_T}{p_T} + \ldots\right)$$
$$\alpha_s^3\left(c_{3,5}\frac{\ln^5 p_T}{p_T} + c_{3,4}\frac{\ln^4 p_T}{p_T} + \ldots\right)$$



✦ **Impact parameter space**

$$\sigma(\vec{b}_\perp) = \int d^2\vec{q}_\perp \frac{d\sigma}{d^2\vec{q}_\perp}\exp(-i\vec{b}_\perp\cdot\vec{q}_\perp) \qquad \frac{\ln^{k-1}\vec{q}_\perp^{\,2}}{\vec{q}_\perp^{\,2}} \sim \ln^k \vec{b}_\perp^{\,2} \qquad \text{small } \vec{q}_\perp \Leftrightarrow \text{ large } \vec{b}_\perp$$

✦ **Cross section in b-space exponentiated to all orders**

$$\sigma(\vec{b}_\perp) \sim \exp\left(A(\alpha_s)\ln^2\vec{b}_\perp^{\,2} + B(\alpha_s)\ln\vec{b}_\perp^{\,2}\right) + \text{non-singular terms}$$

Collins, Soper, Sterman, 1985 …

✦ Hard, collinear, and soft modes dominate singular Xsec

$$p^\mu = (p^+, p^-, p_\perp) = (p_0 + p_z, p_0 - p_z, p_\perp)$$

$$n^\mu = (2,0,0) \quad \bar{n}^\mu = (0,2,0)$$

power counting parameter $\lambda \sim \frac{q_\perp}{Q}$

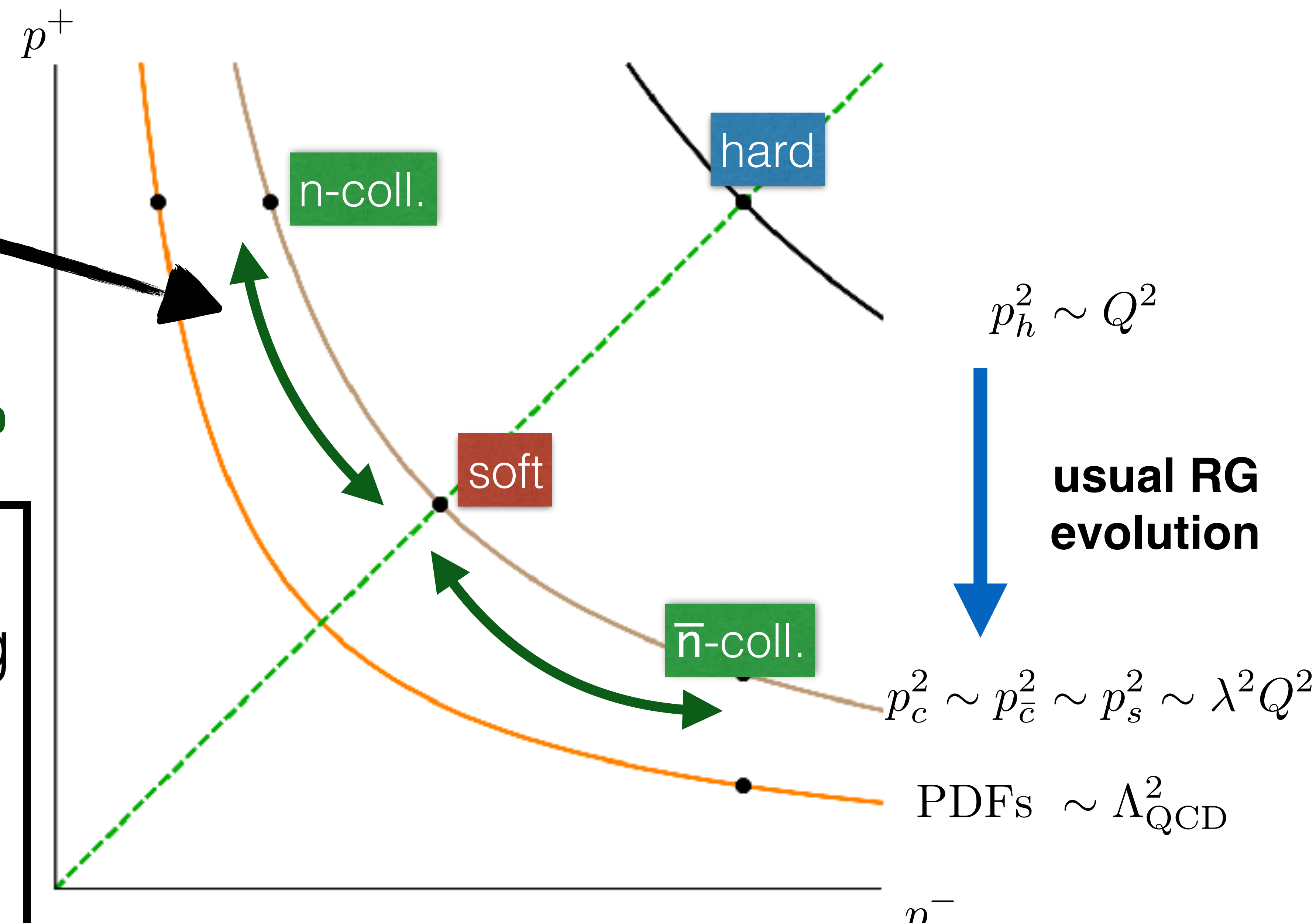
hard: $Q(1,1,1)$

collinear: $Q(1,\lambda^2,\lambda), \quad Q(\lambda^2,1,\lambda)$

soft: $Q(\lambda,\lambda,\lambda)$

Rapidity evolution between soft and collinear/anti-collinear modes

Collins-Soper equation, 82
Rapidity RG, Chiu et al, 2011,12
Collinear anomaly, Becher et al, 2010
…



Large logs are resumed by solving usual RG equation and rapidity RG equation

pT resummation in Effective theory

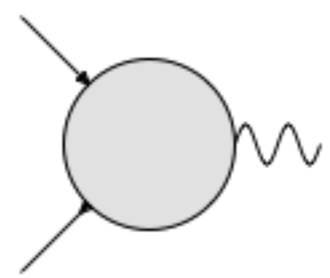
✦ pT resummation in the SCET rapidity RG formalism

Chiu, Jain, Neill, Rothstein, 2012

$$\frac{d\sigma_{\rm DY}}{dQ^2 dY d^2\vec{q}_\perp} = \sigma_0 \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{q}_\perp} H(Q,\mu_h) B_q(x_A, Q, \vec{b}_\perp, \mu_b, \nu_b) B_{\bar{q}}(x_B, Q, \vec{b}_\perp, \mu_b, \nu_b) S_\perp(\vec{b}_\perp, \mu_s, \nu_s)$$

$$\cdot \exp\left\{ -\int_{b_0^2/\vec{b}_\perp^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\left(\Gamma_{\rm cusp}\left[\alpha_s(\bar{\mu})\right] + \frac{d\gamma^r\left[\alpha_s(\bar{\mu})\right]}{d\ln\bar{\mu}^2} \right) \ln\frac{Q^2}{\bar{\mu}^2} + \left(\gamma^V\left[\alpha_s(\bar{\mu})\right] - \gamma^r\left[\alpha_s(\bar{\mu})\right] \right) \right] \right\}$$

✦ **Hard function H: quark/gluon form factor**



✦ **Beam function B: quark/gluon correlator (unrenormlized)**

$$W_n(x) = \mathrm{P}\exp\left(ig\int_{-\infty}^{0} ds\, \bar{n}\cdot A(x+s\bar{n})\right)$$

$$B_{q/N}(z, Q, \vec{b}_\perp) = \int dx^+ e^{izP^- x^+/2} \left\langle P \middle| (\bar{\psi}_n W_n)(x^+, 0, \vec{b}_\perp) \frac{\bar{n}_\mu\gamma^\mu}{2} (W_n^\dagger \psi_n)(0) \middle| P \right\rangle$$

✦ **Soft function S: VEV. of light-like Wilson loop (unrenormalized)**

$$S_\perp = \frac{\mathrm{Tr}}{C} \langle 0 | T\{S_{\bar{n}}^\dagger S_n(0,0,0)\} \overline{T}\{S_n^\dagger S_{\bar{n}}(0,0,\vec{b}_\perp)|0\rangle$$

$$S_n(x) = \mathrm{P}\exp\left(ig\int_{-\infty}^{0} ds\, n\cdot A(x+sn)\right)$$

Anomalous dimension for resummation

✦ Resummation formulae in the SCET formalism at canonical scale

$$\frac{d\sigma_{\rm DY}}{dQ^2 dY d^2\vec{q}_\perp} = \sigma_0 \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{q}_\perp} H(Q,\mu_h) B_q(x_A, Q, \vec{b}_\perp, \mu_b, \nu_b) B_{\bar{q}}(x_B, Q, \vec{b}_\perp, \mu_b, \nu_b) S_\perp(\vec{b}_\perp, \mu_s, \nu_s)$$

$$\cdot \exp\left\{ -\int_{b_0^2/\vec{b}_\perp^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\left(\Gamma_{\rm cusp}\left[\alpha_s(\bar{\mu})\right] + \frac{d\gamma^r\left[\alpha_s(\bar{\mu})\right]}{d\ln\bar{\mu}^2} \right) \ln\frac{Q^2}{\bar{\mu}^2} + \left(\gamma^V\left[\alpha_s(\bar{\mu})\right] - \gamma^r\left[\alpha_s(\bar{\mu})\right] \right) \right] \right\}$$

$A\ln^2\vec{b}_\perp^2$ double log.

$B\ln\vec{b}_\perp^2$ single log.

cusp anomalous dim.
three-loop

form factor
normal anomalous dim.
three-loop

rapidity anomalous dim.

previously only uncertainty source at N3LL

pT distribution as a precision probe of N.P. QCD

$$\frac{d\sigma_{\rm DY}}{dQ^2 dY d^2\vec{q}_\perp} = \sigma_0 \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{q}_\perp} H(Q,\mu_h) B_q(x_A, Q, \vec{b}_\perp, \mu_b, \nu_b) B_{\bar{q}}(x_B, Q, \vec{b}_\perp, \mu_b, \nu_b) S_\perp(\vec{b}_\perp, \mu_s, \nu_s)$$

$$\cdot \exp\left\{ -\int_{b_0^2/\vec{b}_\perp^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\left(\Gamma_{\rm cusp}[\alpha_s(\bar{\mu})] + \frac{d\gamma^r[\alpha_s(\bar{\mu})]}{d\ln\bar{\mu}^2} \right) \ln\frac{Q^2}{\bar{\mu}^2} + \left(\gamma^V[\alpha_s(\bar{\mu})] - \gamma^r[\alpha_s(\bar{\mu})] \right) \right] \right\}$$

$\cdot e^{-S_{\rm np}}$ (non-perturbative modification at large impact parameter)

- ❖ **b* prescription:** $b^* = \dfrac{b_\perp}{\sqrt{1 + b_\perp^2/b_{\rm max}^2}}$
- ❖ **Commonly used N.P. model:** $S_{\rm N.P.} = \exp\left[-\left(g_1 + g_2 \ln\frac{Q}{2Q_0} + g_1 g_3 \ln(100 x_A x_B) \right) b_\perp^2 \right]$

✦ **Different functional form for global fit**

Landry, Brock, Nadolsky, Yuan, 2002;
Konychev, Nadolsky, 2005;
Qiu, Zhang, 2001;
Echevarria, Idilbi, Schafer, Scimemi, 2011;
Sun, Isaacson, Yuan, Yuan, 2014;
…

- ✦ **Quadratic form at small b** — **Korchemsky, Sterman, 94; Scimemi, Vladimirov, 16**
- ✦ **No first principle prediction at large b**
  - ✦ **quadratic: original CSS parameterization**
  - ✦ **linear: Tafat, 2002**
  - ✦ **constant: Collins, Rogers, 2014**
  - ✦ **Logarithmic: Collins, Soper, 82; SIYY, 2014**
- ✦ **Need truly non-perturbative prediction. Lattice? integrability?**

$$\frac{d\sigma_{\rm DY}}{dQ^2 dY d^2\vec{q}_\perp} = \sigma_0 \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{q}_\perp} H(Q,\mu_h) B_q(x_A, Q, \vec{b}_\perp, \mu_b, \nu_b) B_{\bar{q}}(x_B, Q, \vec{b}_\perp, \mu_b, \nu_b) S_\perp(\vec{b}_\perp, \mu_s, \nu_s)$$

$$\cdot \exp\left\{ -\int_{b_0^2/\vec{b}_\perp^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\left(\Gamma_{\rm cusp}\left[\alpha_s(\bar{\mu})\right] + \frac{d\gamma^r\left[\alpha_s(\bar{\mu})\right]}{d\ln\bar{\mu}^2} \right) \ln\frac{Q^2}{\bar{\mu}^2} + \left(\gamma^V\left[\alpha_s(\bar{\mu})\right] - \gamma^r\left[\alpha_s(\bar{\mu})\right] \right) \right] \right\}$$

$$B \ln \vec{b}_\perp^2$$

$$B = \alpha_s B_1 + \alpha_s^2 B_2 + \cdots$$

$$\gamma_r = \gamma_0^r B_1 + \alpha_s^2 \gamma_1^r + \cdots$$

- **Process dependent. Two loops known:**
  - **DY: Davies, Stirling, 1984**
  - **Higgs: de Florian, Grazzini, 2000**

- **Obey Casimir scaling to the known perturbative order. Two loops:**
  - **Gehrmann, Lubbert, L.L.Yang (2012,2014)**
  - **Echevarria, Scimemi, Vladimirov (2015)**
  - **Luebbert, Oredsson, Stahlhofen (2016)**

Three-loop knowledge of rapidity anomalous dimension important for reduce perturbative uncertainty, and may shed light on non-perturbative large b behavior

- Hard function (form factor) free from rapidity evolution

- Consistency relation between Beam and soft function

$$\nu \frac{d}{d\nu}\Big[BBS_\perp\Big] = 0 \qquad \nu \text{ rapidity evolution scale}$$

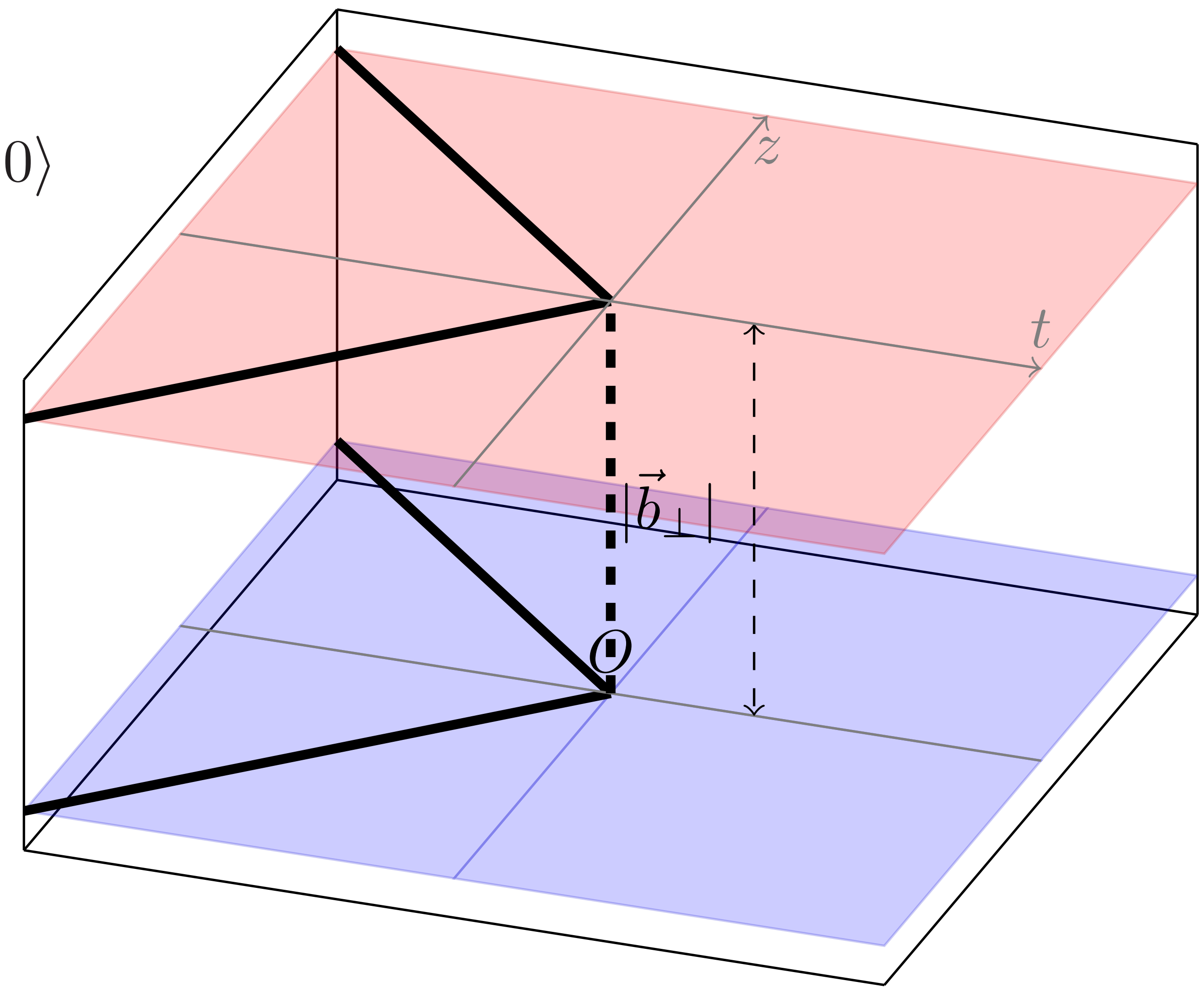
- Can compute either Beam function or soft function to obtain rapidity anomalous dimension

- The calculation would be simplest using soft function - vev. of light-like Wilson loop.

- Problem: light-cone singularity not regularized by dimensional regularization (problem also presented in the beam function)

(un-regulated) Rapidity singularity

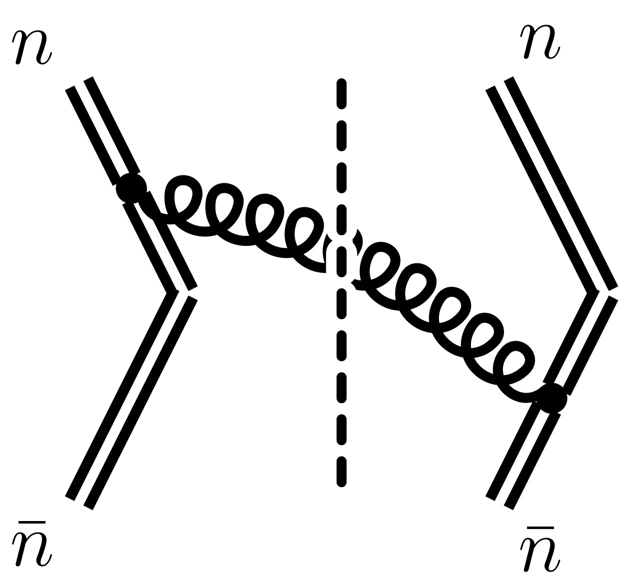
$$S_\perp = \frac{\mathrm{Tr}}{C}\langle 0|T\{S_{\bar{n}}^\dagger S_n(0,0,0)\}\overline{T}\{S_n^\dagger S_{\bar{n}}(0,0,\vec{b}_\perp)|0\rangle$$



$S_{n,\bar{n}}$ light-like Wilson line to -∞

invariant under arbitrary z boost

one-loop example:



$$\sim \int dx_a\, dx_b D_+(x_{ab}^2)$$

$$\sim \int_0^\infty dt_1 \int_0^\infty dt_2 \frac{1}{(t_1 t_2 + \vec{b}_\perp^2)^{1-\epsilon}}$$

rapidity divergence In momentum space: $\int_0^\infty \frac{dk^+}{k^+}$

- **Several rapidity regulators have been proposed**
  - **Tilting the Wilson line off light cone:** Ji, Ma, Yuan (2004); Collins (2011)
  - **analytic regulator:** Becher, Neubert (2009); Becher, Bell (2011); **two-loop calculation:** Gehrmann, Lubbert, Yang (2012,2014)

$$\int d^d k \to \int d^d k \left(\frac{\nu}{k^+}\right)^\alpha$$

  - **delta regulator (mass regulator):** Echevarria, Idilbi and Scimemi (2011); **two-loop calculation:** Echevarria, Scimemi, Vladimirov (2015)

$$\frac{1}{k^+ + i\varepsilon} \to \frac{1}{k^+ + \delta}$$

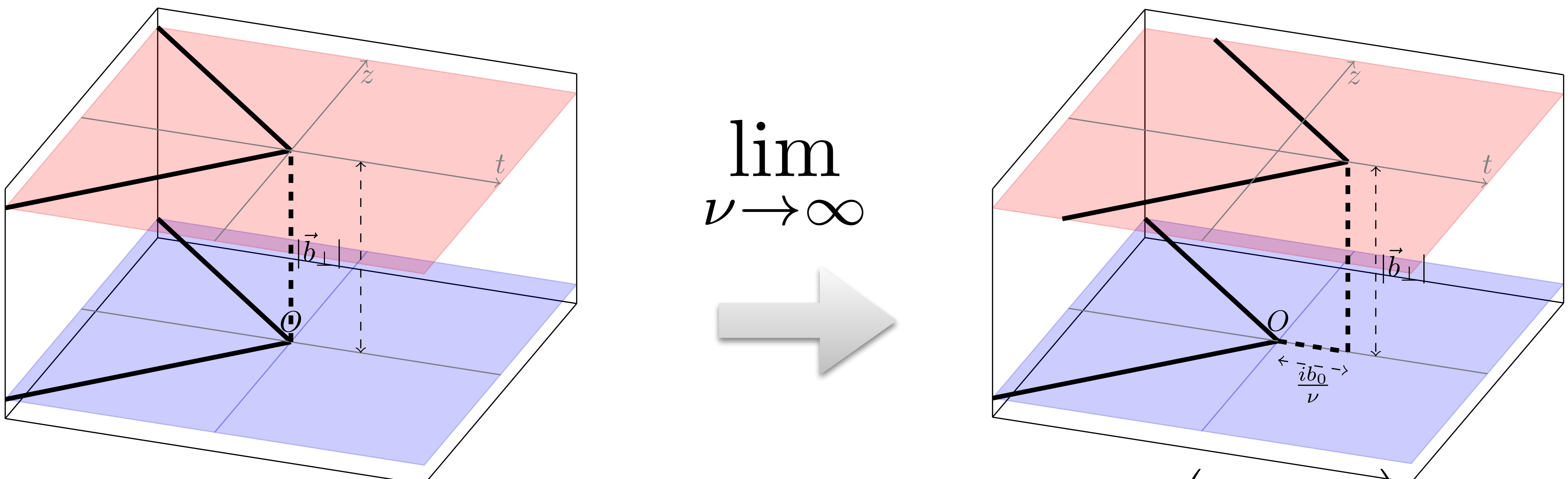
  - **rapidity renormalization group:** Chiu, Jain, Neill, Rothstein (2011,2012); **two-loop calculation:** Luebbert, Oredsson, Stahlhofen (2016)

$$\int d^d k \to \int d^d k \left(\frac{\nu}{|k_z|}\right)^\eta$$

A new regulator for rapidity divergence

1604.00392, Y. Li, Neill, HXZ

- The regulator: an infinitesimal shift to in Euclidean time



- In momentum space: $$\prod_i \int \frac{d^{d-1}k_i}{2k_i^0(2\pi)^3} \Rightarrow \left(\prod_i \int \frac{d^{d-1}k_i}{2k_i^0(2\pi)^3}\right) \exp\left(-\sum_j \frac{b^0 k_j^0}{\nu}\right)$$
- Manifestly preserve gauge symmetry and Non-Abelian exponentiation theorem.
- Logarithmic like singularity log(v). Don’t need O(v) terms
- Have operator definition. Possible to put on Lattice

Relation to other soft function: threshold

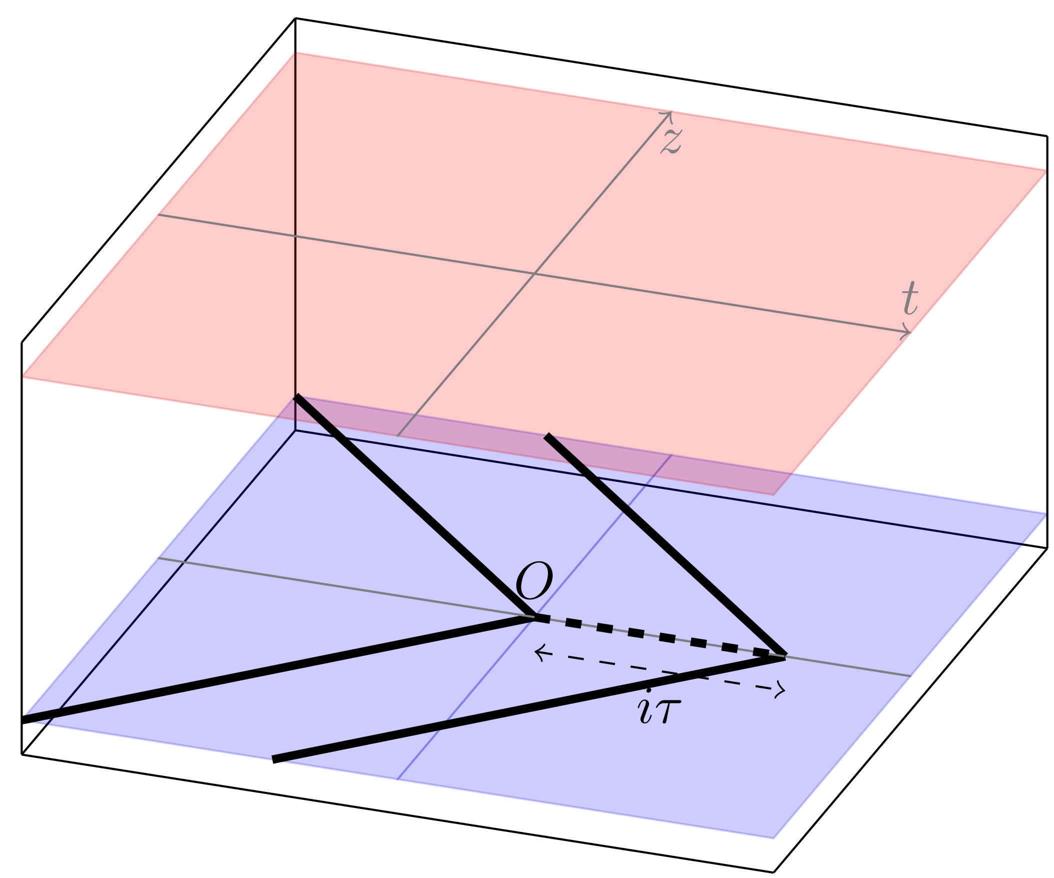
- Light-like Wilson loop separated in Euclidean time only

$$S_{\text{thr.}} = \frac{\text{Tr}}{C} \langle 0 | T\{S^\dagger_{\bar{n}} S_n(0,0,0)\} \overline{T}\{S^\dagger_n S_{\bar{n}}(i\tau, i\tau, 0) | 0 \rangle$$

$$\sigma = \tau \int \frac{dx}{x} \frac{dz}{z} f_1(x) f_2(\tau/x/z) \hat{\sigma}(z)$$

$$\hat{\sigma}(z) \sim \delta(1-z) + \alpha_s \left[\frac{\ln(1-z)}{1-z} \right]_+ + \cdots$$

$$1 - z = 1 - \frac{Q^2}{\hat{s}} \simeq 2 \frac{k_s^0}{Q} + \ldots$$



- Useful for resummation of large logarithms of (1-z) in partonic cross section of Drell-Yan and Higgs production

Korchemsky, Marchesini, 1993
Becher, Neubert, Xu, 2007

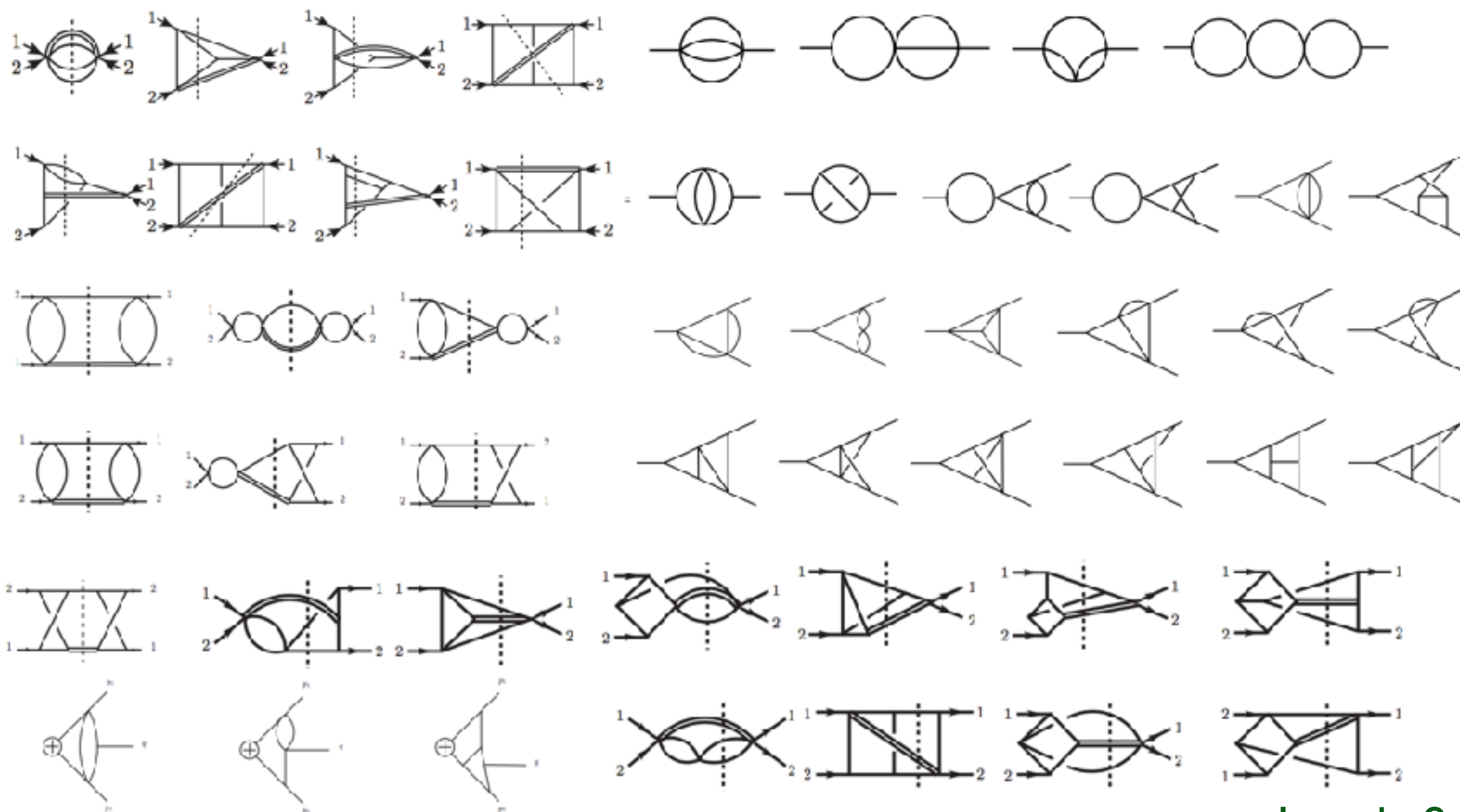


Image by C. Duhr

- ✦ All three-loop integrals for threshold soft function known

Anastasiou et al, 2015; Y. Li et al, 2014

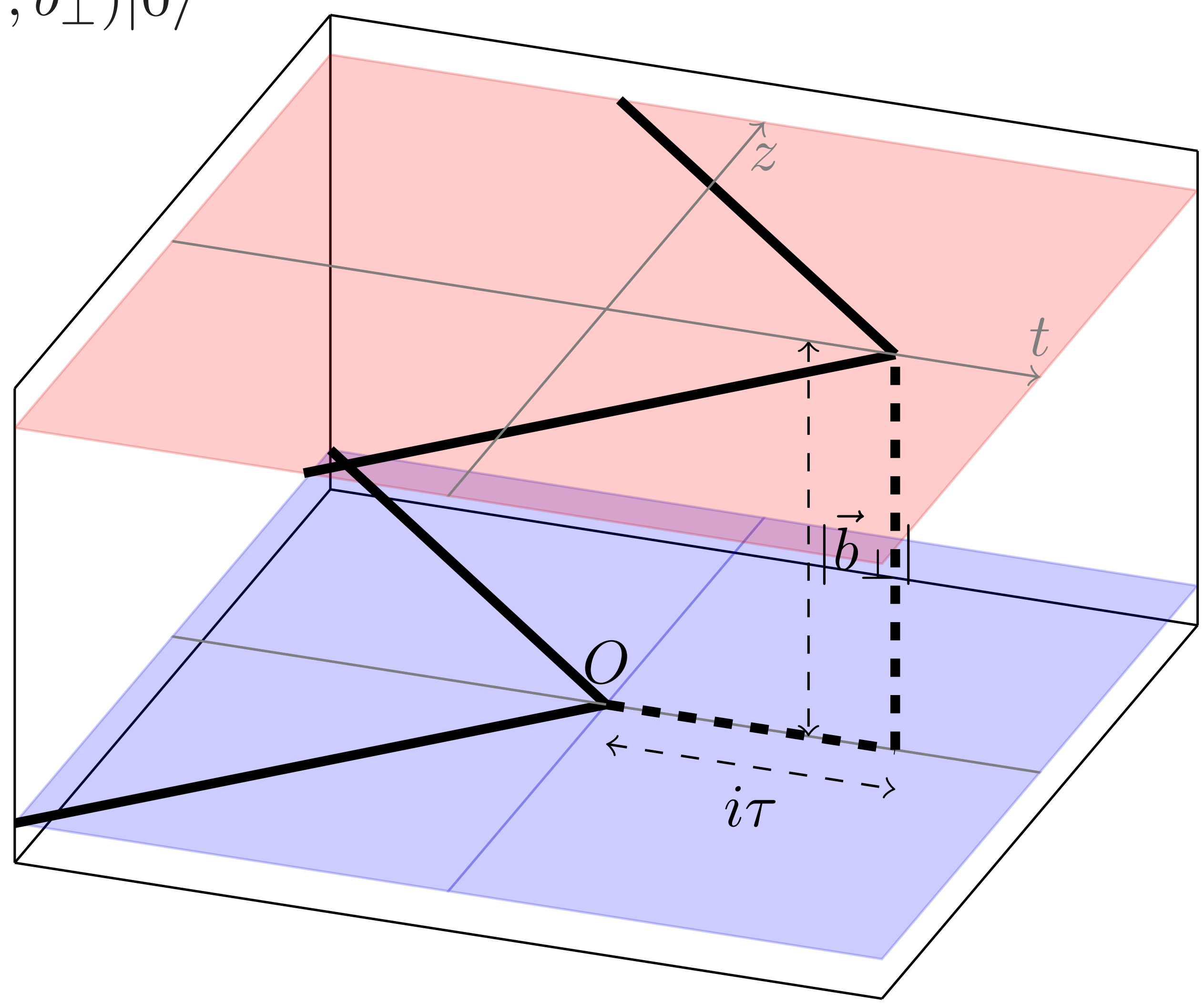
- ✦ Building block for Higgs production at N3LO

Anastasiou et al, 2015

Relation to other soft function: fully differential

- Light-like Wilson loop separated both in time and transverse spatial direction **Laenen, Sterman, Vogelsang, 2000; Mantry, Petriello, 2009**

$$S_{\mathrm{F.D.}} = \frac{\mathrm{Tr}}{C} \langle 0 | T\{S_{\bar{n}}^\dagger S_n(0,0,0)\} \overline{T}\{S_n^\dagger S_{\bar{n}}(i\tau, i\tau, \vec{b}_\perp)|0\rangle$$



- Fully differential soft function free from rapidity divergence

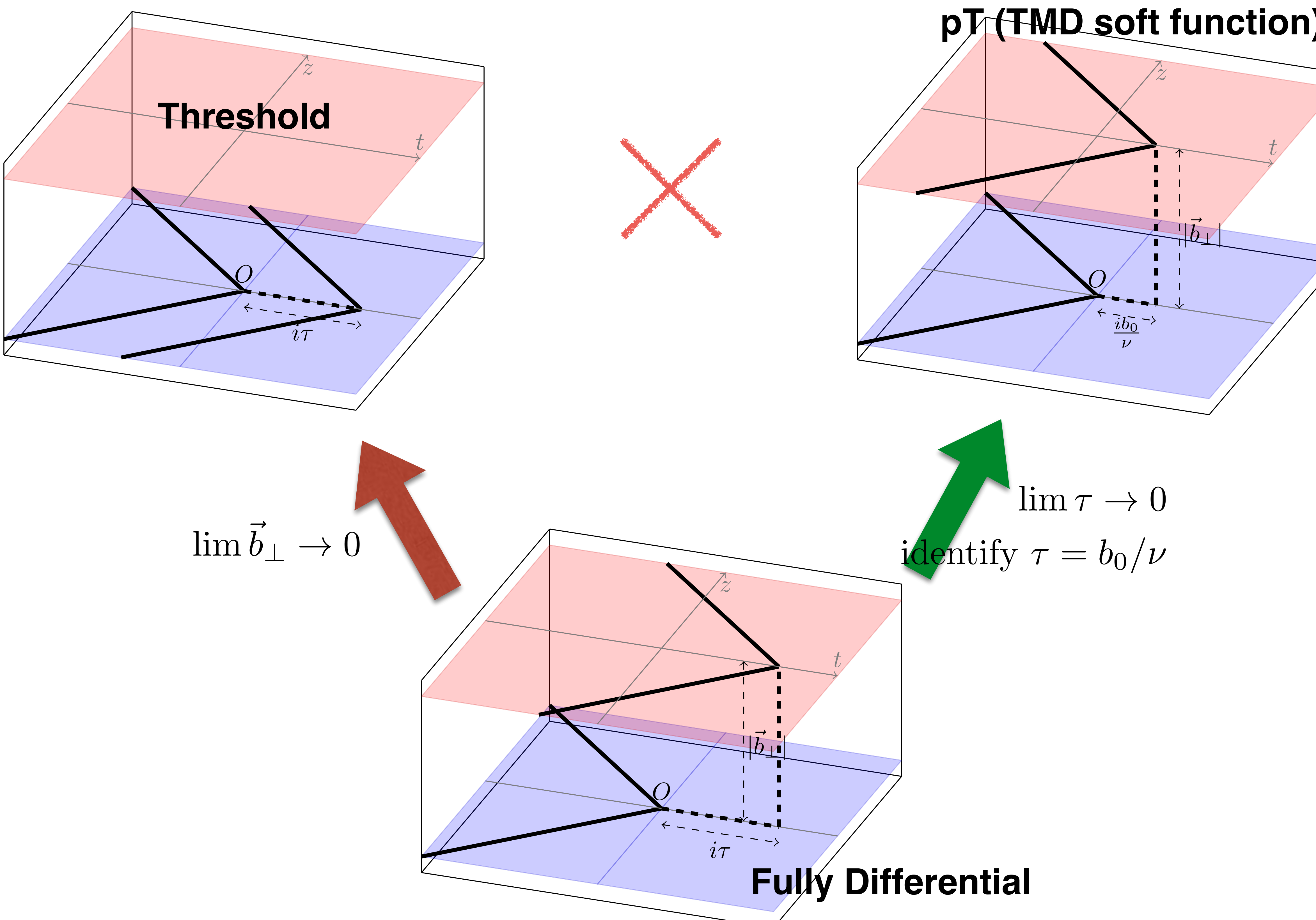
- Useful for joint resummation **H.-n Li,98; Laenen, Sterman, Vogelsang, 2000; Lustermans, Waalewijn, Zeune, 05**

- Non-trivial dependence on dimensionless ratio

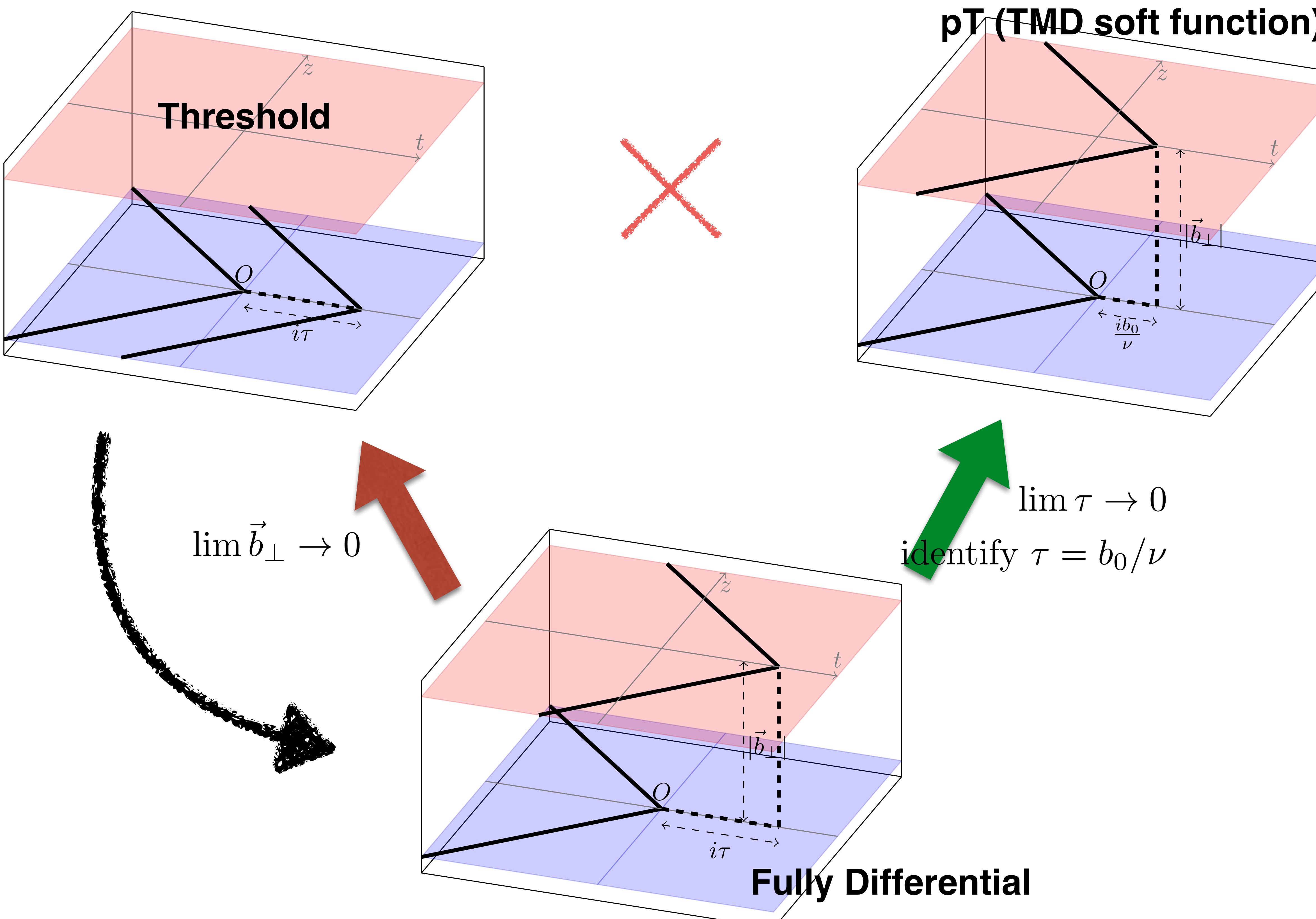
$$x = \frac{\vec{b}_\perp^2}{(i\tau)^2}$$

- Known to two loops **Y. Li, Mantry, Petriello, 2011**

Relation between different soft functions



An almost triangular relations



Fully Differential soft function in N=4 SYM

$$S_{\text{F.D.}} = \exp\left\{\sum_{i=0}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^{i+1}\left[\frac{\Gamma_i^{\text{cusp}}}{2}L_\tau^2 - \gamma_i^s L_\tau + c_{i+1}^{\text{F.D.}}(x)\right]\right\}$$

$$L_\tau = \ln\frac{\tau^2}{b_0^2\mu^2}$$

$$x = \frac{\vec{b}_\perp^{\,2}}{(i\tau)^2}$$

- The μ dependent part fixed by RG equation

$$c_1^{\text{F.D.}} = 4N_c H_{0,1}(x) + c_{1,\mathcal{N}=4}^s$$

$$c_2^{\text{F.D.}} = N_c^2\Big[-8\zeta_2 H_{0,1}(x) - 8H_{0,0,0,1}(x) - 8H_{0,1,0,1}(x) - 16H_{0,0,1,1}(x) - 16H_{0,1,1,1}(x)\Big] + c_{2,\mathcal{N}=4}^s$$

- Maximal transcendental weight at each order

- HPLs with 0 first entry, 1 last entry. Suggest a simple ansatz on three loops

- Constraint from single logarithmic rapidity divergence at each order

$x \to -\infty$

- Based on the pattern observed at one and two loops, we will make an ansatz at three loops
- The ansatz is linear combination of Harmonic Polylogarithms with rational or Zeta-value coefficients
- We will do the calculation in N=4 Supersymmetric Yang-Mills theory first

$$
\begin{aligned}
Nc^3 \Big(& \frac{1}{90}\pi^4 c_{23} H_{0,1}[x] + c_{21}\,\mathrm{Zeta}[3]\, H_{0,0,1}[x] + c_{22}\,\mathrm{Zeta}[3]\, H_{0,1,1}[x] + \\
& \frac{1}{6}\pi^2 c_{17} H_{0,0,0,1}[x] + \frac{1}{6}\pi^2 c_{18} H_{0,0,1,1}[x] + \frac{1}{6}\pi^2 c_{19} H_{0,1,0,1}[x] + \\
& \frac{1}{6}\pi^2 c_{20} H_{0,1,1,1}[x] + c_1 H_{0,0,0,0,0,1}[x] + c_2 H_{0,0,0,0,1,1}[x] + \\
& c_3 H_{0,0,0,1,0,1}[x] + c_4 H_{0,0,0,1,1,1}[x] + c_5 H_{0,0,1,0,0,1}[x] + c_6 H_{0,0,1,0,1,1}[x] + \\
& c_7 H_{0,0,1,1,0,1}[x] + c_8 H_{0,0,1,1,1,1}[x] + c_9 H_{0,1,0,0,0,1}[x] + c_{10} H_{0,1,0,0,1,1}[x] + \\
& c_{11} H_{0,1,0,1,0,1}[x] + c_{12} H_{0,1,0,1,1,1}[x] + c_{13} H_{0,1,1,0,0,1}[x] + \\
& c_{14} H_{0,1,1,0,1,1}[x] + c_{15} H_{0,1,1,1,0,1}[x] + c_{16} H_{0,1,1,1,1,1}[x] \Big)
\end{aligned}
$$

Need to determine the coefficients c_i

Constraint from single logarithmic rapidity divergence

fixed by RG

one-loop ansatz: $S(\vec{b}_\perp, \tau, \mu) = C_a \left[2\ln^2(\mu^2\tau^2) + 2\zeta_2 + r_1 H_2\right]$

Using $\lim_{\tau\to 0} H_2(x) = \lim_{\tau\to 0} \mathrm{Li}_2\left(-\frac{\vec{b}_\perp^{\,2}}{b_0^2\tau^2}\right) = -\frac{1}{2}\ln^2\left(\frac{\vec{b}_\perp^{\,2}}{b_0^2\tau^2}\right) - \zeta_2 + \mathcal{O}(\tau)$

$$\Rightarrow \quad r_1 = 4$$

One-loop "almost" for free!

- ✦ 12 undetermined coefficients after applying constraints
- ✦ Need further data to fully fix the ansatz
- ✦ The only available three loop data: threshold soft function (b=0)

Using threshold soft function as boundary data

- Expanding around the zero-impact parameter limit (b=0)

$$S_{\text{F.D.}} = \frac{\text{Tr}}{C}\langle 0|T\{S_{\bar n}^\dagger S_n(0,0,0)\}\overline{T}\{S_n^\dagger S_{\bar n}(i\tau, i\tau, \vec{b}_\perp)|0\rangle$$

$$= \frac{\text{Tr}}{C}\langle 0|T\{S_{\bar n}^\dagger S_n(0,0,0)\}\int d^{d_\perp}y_\perp \delta^{(d_\perp)}(y_\perp)\sum_{n=0}^{\infty}\frac{1}{n!}\left(b_\perp^\mu \cdot \frac{\partial}{\partial y_\perp^\mu}\right)^n \overline{T}\{S_n^\dagger S_{\bar n}(i\tau, i\tau, \vec{y}_\perp)|0\rangle$$

- Implement the expansion in momentum space

$$-i\frac{\partial}{\partial y_\perp^\mu} \to k_\perp^\mu = \sum_{i \in \text{on-shell parton}} k_{i,\perp}^\mu$$

- Rotational invariance in the transverse plane

$$(-i\vec{b}_\perp \cdot \vec{k}_\perp)^{2m} = f(2m)(\vec{b}_\perp^2)^m (k^+k^- - k^2)^m \; ; \quad f(2m) = (-1)^m \frac{1\cdot 3\cdot 5\ldots(2m-1)}{d_\perp \cdot (d_\perp+2)\cdot(d_\perp+4)\ldots(d_\perp+2m-2)}$$

- IBP reduction to known 3-loop integral. Obtain data up to

$$x^{17} = \left(\frac{\vec{b}_\perp^2}{(i\tau)^2}\right)^{17}$$

F.D. soft function at three loops in N=4 SYM

Y. Li, HXZ, 1604.01404

$$S_{\rm F.D.} = \exp\left\{\sum_{i=0}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^{i+1}\left[\frac{\Gamma_i^{\rm cusp}}{2}L_\tau^2 - \gamma_i^s L_\tau + c_{i+1}^{\rm F.D.}(x)\right]\right\}$$

one and two loops

$$c_1^{\rm F.D.} = 4N_c H_{0,1}(x) + c^s_{1,\mathcal{N}=4}$$

$$c_2^{\rm F.D.} = N_c^2\Big[-8\zeta_2 H_{0,1}(x) - 8H_{0,0,0,1}(x) - 8H_{0,1,0,1}(x) - 16H_{0,0,1,1}(x) - 16H_{0,1,1,1}(x)\Big] + c^s_{2,\mathcal{N}=4}$$

three-loop scale independent part

$$c^s_{3,\mathcal{N}=4} + N_c^3\Big(16\zeta_2 H_4 + 48\zeta_2 H_{2,2} + 64\zeta_2 H_{3,1} + 96\zeta_2 H_{2,1,1} + 120\zeta_4 H_2 + 48H_6 + 24H_{2,4} + 40H_{3,3}$$
$$+ 72H_{4,2} + 128H_{5,1} + 16H_{2,1,3} + 56H_{2,2,2} + 80H_{2,3,1} + 80H_{3,1,2} + 144H_{3,2,1} + 224H_{4,1,1}$$
$$+ 64H_{2,1,1,2} + 96H_{2,1,2,1} + 160H_{2,2,1,1} + 256H_{3,1,1,1} + 192H_{2,1,1,1,1}\Big)$$

- Uniform and maximal degree of transcendentality

  Anomalous dimension, form factor, momentum space Wilson loop

- Coefficients are integers

- Alternating/uniform sign and each loop order

  also see cusp anomalous dimension, Henn, Huber, 2013

QCD = ([N=4]) + (QCD - [N=4])

- N=4 SYM Also "predict" maximal transcendental part of QCD
  Kotikov, Lipatov, Velizhanin, 2003

- Knowing the maximal transcendental part significantly reduce the undetermined coefficient to be fixed

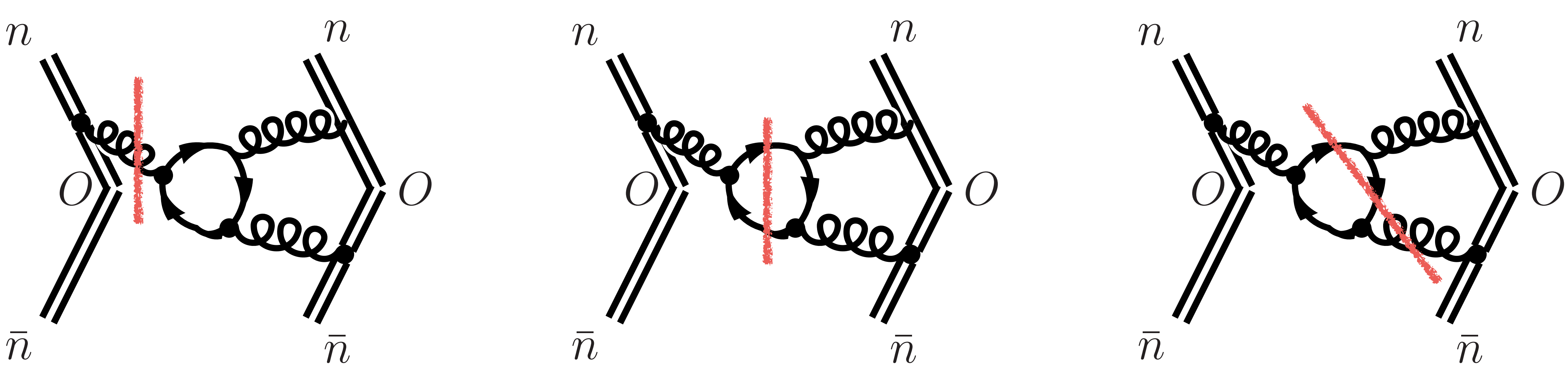
- QCD from deconstructing N=4 SYM

# of terms	transcendental weight	N=4 SYM		QCD	pure gluon	fermion	scalar
16	6		=				
8	5	$\emptyset$					
4	4	$\emptyset$					
2	3	$\emptyset$					
0	2	$\emptyset$					

[N=4 SYM] = 1 gluon + 4 majorana fermion + 3 complex scalar

Directly integrating Nf matter part

$$S_{\rm F.D.} = \frac{\rm Tr}{C}\sum_X \langle 0|T\{S^\dagger_{\bar n} S_n(0)\}X|\rangle e^{-\tau k_X^0 + ib_\perp\cdot k_{X,\perp}}\langle X|\overline{T}\{S^\dagger_n S_{\bar n}(0)|0\rangle$$



Example of the most complicated diagrams

- New functions appear in the double cut and triple cut contribution

$$H_1(x) - \frac{H_1(x)}{x} \qquad H_{11}(x) - \frac{H_{11}(x)}{x} \qquad \frac{H_{01}(x)}{x} \qquad \zeta_2 H_1(x) - H_{101}(x)$$

- Cancel in the sum of different cuts. Only one additional term survive in the final result

The QCD results to three loops

RG equation

$$\frac{d\ln S_{\mathrm{F.D.}}(\vec{b}_\perp,\tau,\mu)}{d\ln\mu^2} = \Gamma_{\mathrm{cusp}}\left[\alpha_s(\mu)\right]\ln\frac{\tau^2\mu^2}{b_0^2} - \gamma_s\left[\alpha_s(\mu)\right]$$

$$4C_aH_2$$

one loop

$$C_AC_a\left(-8\zeta_2H_2+\frac{268}{9}H_2+\frac{44}{3}H_3-8H_4-\frac{44}{3}H_{2,1}-8H_{2,2}-16H_{3,1}-16H_{2,1,1}\right)$$
$$+C_an_f\left(-\frac{40}{9}H_2-\frac{8}{3}H_3+\frac{8}{3}H_{2,1}\right)$$

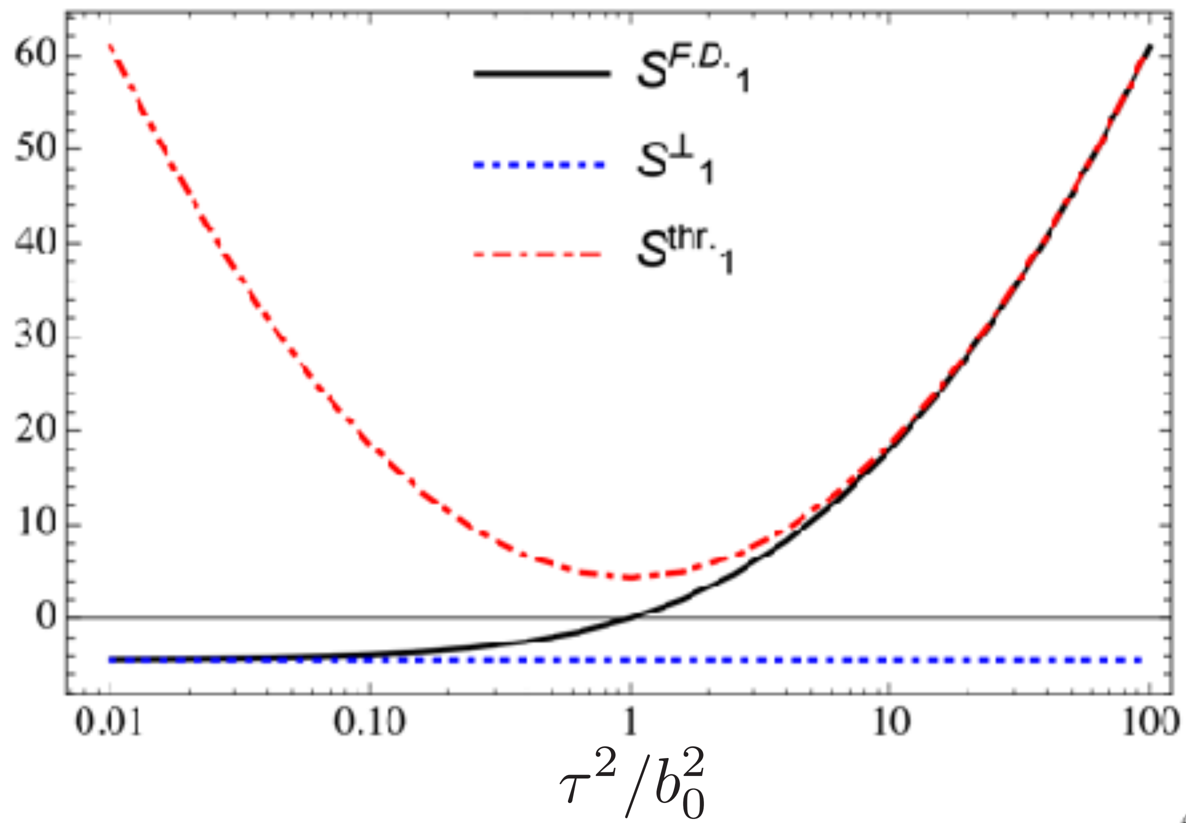
two loop

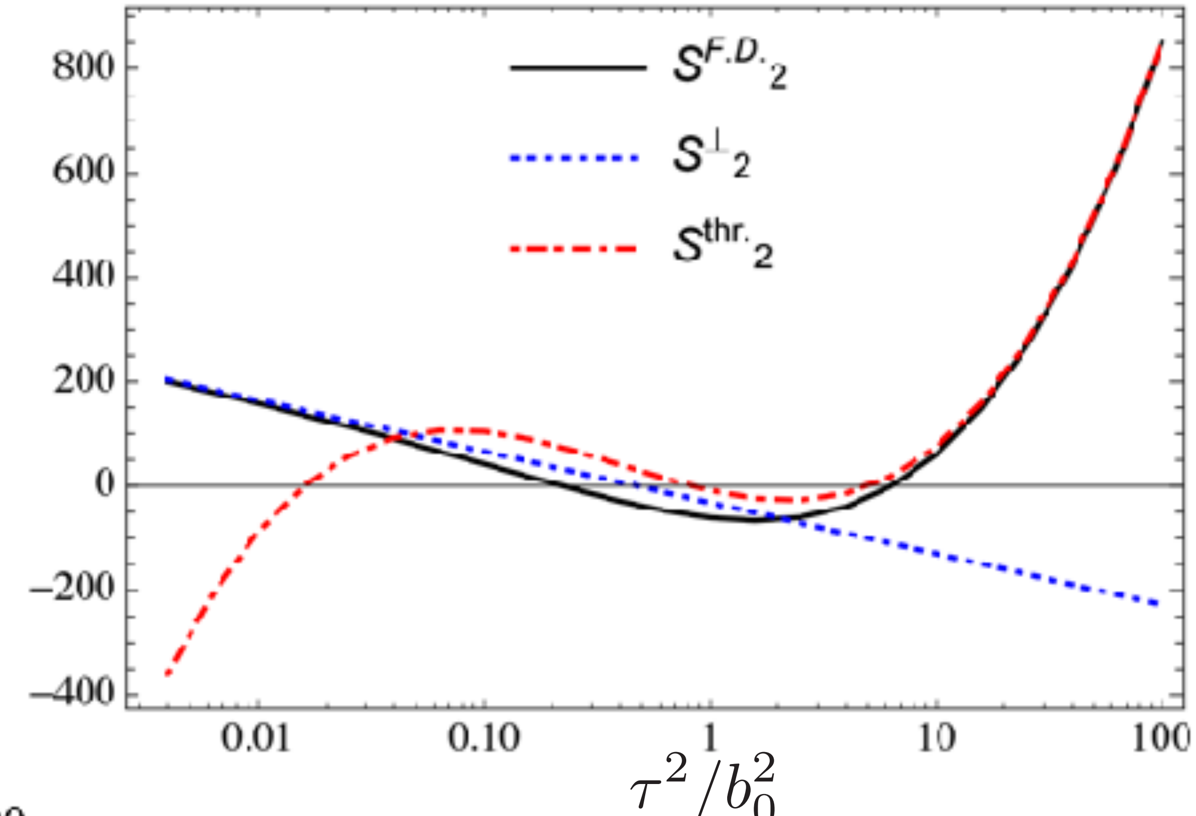
N=4SYM (maximal transcendental part)

$$+C_aC_A^2\left[-\frac{1072}{9}\zeta_2H_2-176\zeta_3H_2-\frac{88}{3}\zeta_2H_3+88\zeta_2H_{2,1}\right.$$
$$+\frac{30790}{81}H_2+\frac{7120}{27}H_3-\frac{104}{9}H_4-\frac{440}{3}H_5-\frac{8}{3}\left(H_{1,1}-\frac{H_{1,1}}{x}\right)-\frac{7120}{27}H_{2,1}-\frac{1072}{9}H_{2,2}-\frac{88}{3}H_{2,3}$$
$$\left.-\frac{3112}{9}H_{3,1}-88H_{3,2}-\frac{352}{3}H_{4,1}-\frac{392}{3}H_{2,1,1}+\frac{88}{3}H_{2,1,2}+\frac{352}{3}H_{2,2,1}+\frac{352}{3}H_{3,1,1}+352H_{2,1,1,1}\right]$$
$$+C_aC_An_f\left[\frac{160}{9}\zeta_2H_2+\frac{16}{3}\zeta_2H_3-16\zeta_2H_{2,1}-\frac{7988}{81}H_2-\frac{2312}{27}H_3-\frac{64}{3}H_4+\frac{80}{3}H_5+\frac{8}{3}\left(H_{1,1}-\frac{H_{1,1}}{x}\right)\right.$$
$$+\frac{2312}{27}H_{2,1}+\frac{160}{9}H_{2,2}+\frac{16}{3}H_{2,3}+\frac{224}{3}H_{3,1}+16H_{3,2}+\frac{64}{3}H_{4,1}-\frac{32}{9}H_{2,1,1}-\frac{16}{3}H_{2,1,2}-\frac{64}{3}H_{2,2,1}$$
$$\left.-\frac{64}{3}H_{3,1,1}-64H_{2,1,1,1}\right]+C_an_f^2\left(\frac{400}{81}H_2+\frac{160}{27}H_3+\frac{32}{9}H_4-\frac{160}{27}H_{2,1}-\frac{32}{9}H_{3,1}+\frac{32}{9}H_{2,1,1}\right)$$
$$+C_aC_Fn_f\left(32\zeta_3H_2-\frac{110}{3}H_2-8H_3+8H_{2,1}\right) \quad (8)$$

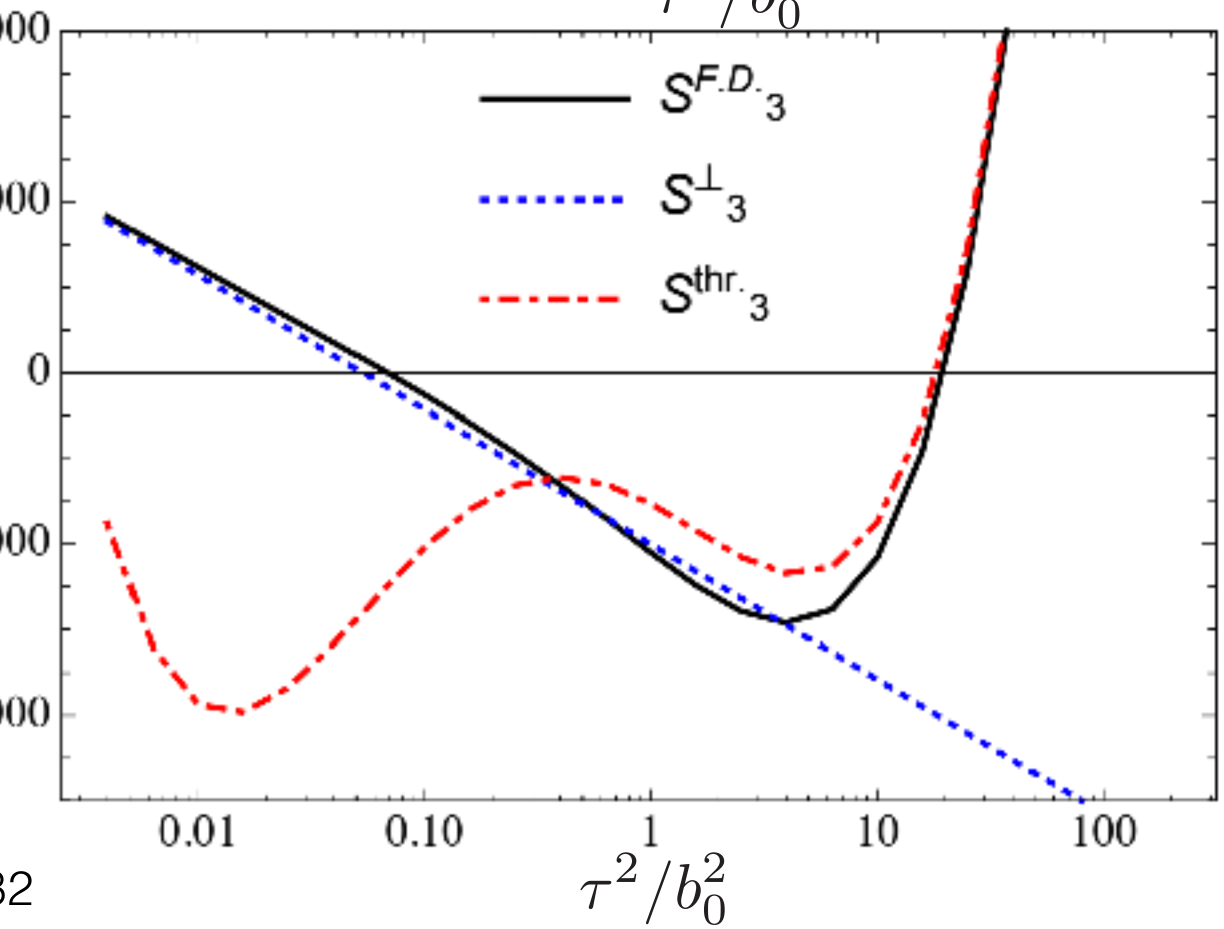
Three loop

Transition between different soft functions

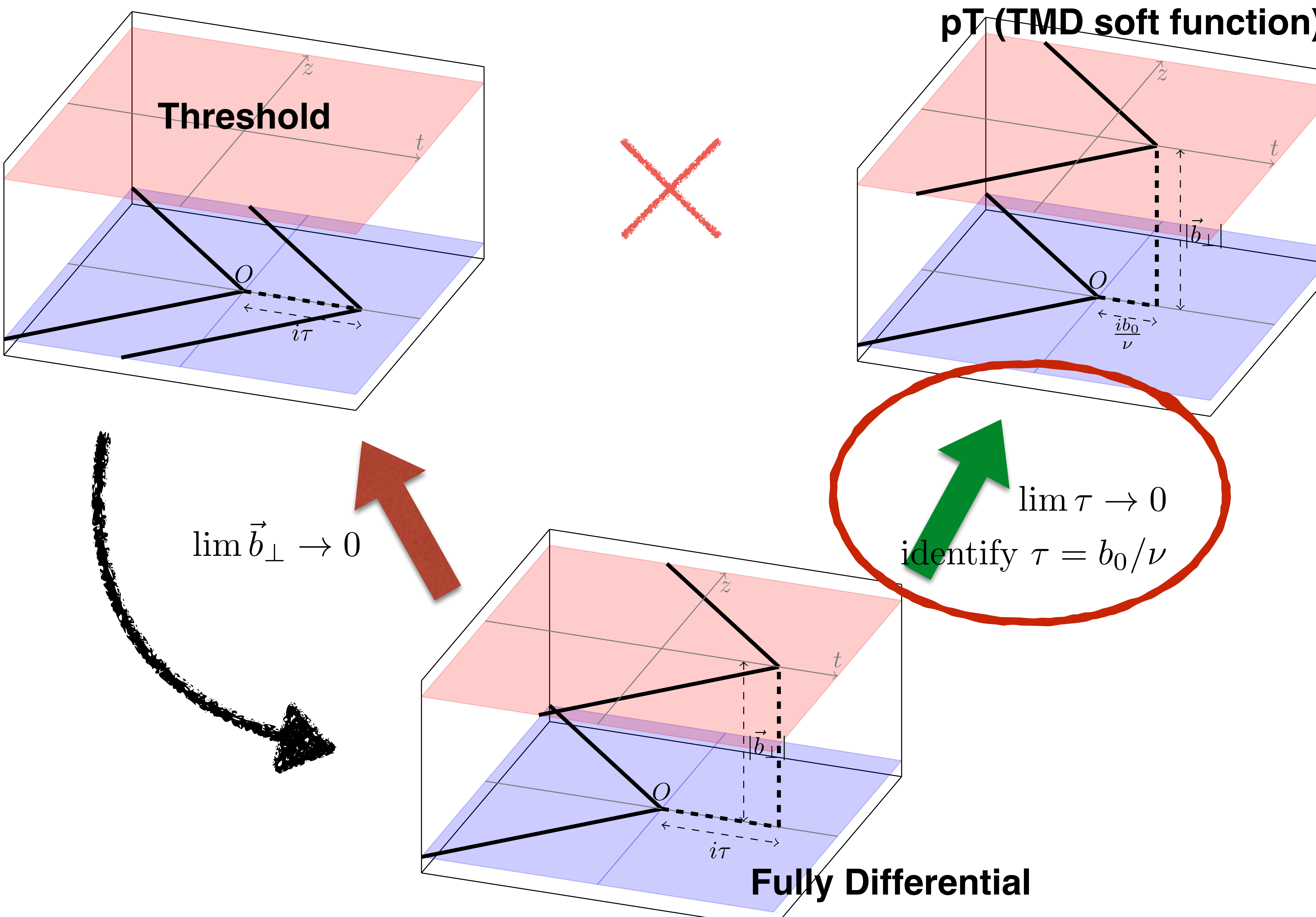




- ✦ At each order in αs, the fully differential soft function interpolate between threshold and TMD soft function smoothly



An almost triangular relations



Rapidity anomalous dimension @ 3 loop

The rapidity renormalization group

Chiu, Jain, Neill, Rothstein, 2012

$$\frac{\mathrm{d}\ln S_\perp(\vec{b}_\perp,\mu,\nu)}{\mathrm{d}\ln\nu^2} = \int_{\mu^2}^{b_0^2/\vec{b}_\perp^2}\frac{\mathrm{d}\bar{\mu}^2}{\bar{\mu}^2}\Gamma_{\mathrm{cusp}}[\alpha_s(\bar{\mu})] + \gamma_r[\alpha_s(b_0/|\vec{b}_\perp|)]$$

$$\frac{d\sigma_{\mathrm{DY}}}{dQ^2dY\,d^2\vec{q}_\perp} = \sigma_0\int\frac{d^2\vec{b}_\perp}{(2\pi)^2}e^{i\vec{b}_\perp\cdot\vec{q}_\perp}H(Q,\mu_h)B_q(x_A,Q,\vec{b}_\perp,\mu_b,\nu_b)B_{\bar{q}}(x_B,Q,\vec{b}_\perp,\mu_b,\nu_b)S_\perp(\vec{b}_\perp,\mu_s,\nu_s)$$
$$\cdot\exp\left\{-\int_{b_0^2/\vec{b}_\perp^2}^{Q^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\left(\Gamma_{\mathrm{cusp}}[\alpha_s(\bar{\mu})]+\frac{d\gamma^r[\alpha_s(\bar{\mu})]}{d\ln\bar{\mu}^2}\right)\ln\frac{Q^2}{\bar{\mu}^2}+\left(\gamma^V[\alpha_s(\bar{\mu})]-\gamma^r[\alpha_s(\bar{\mu})]\right)\right]\right\}$$

$$\gamma_0^r = 0$$
$$\gamma_1^r = C_aC_A\left(28\zeta_3-\frac{808}{27}\right)+\frac{112C_an_f}{27}$$
$$\gamma_2^r = C_aC_A^2\left(-\frac{176}{3}\zeta_3\zeta_2+\frac{6392\zeta_2}{81}+\frac{12328\zeta_3}{27}+\frac{154\zeta_4}{3}-192\zeta_5-\frac{297029}{729}\right)+C_aC_An_f\left(-\frac{824\zeta_2}{81}-\frac{904\zeta_3}{27}+\frac{20\zeta_4}{3}+\frac{62626}{729}\right)+C_an_f^2\left(-\frac{32\zeta_3}{9}-\frac{1856}{729}\right)+C_aC_Fn_f\left(-\frac{304\zeta_3}{9}-16\zeta_4+\frac{1711}{27}\right) \quad (9)$$

New three loop results!

one and two loops known. Direct calculation:
Luebbert, Oredsson, Stahlhofen (2016)
also extractable from:

- Davies, Webber, Stirling (1985)
- Grazzini, de Florian (2000)
- Gehrmann, Lubbert, Yang (2012,2014)
- Echevarria, Scimemi, Vladimirov (2015)

Intriguing relation between rapidity anomalous dimension and threshold anomalous dimension

Control $\left[\frac{1}{1-z}\right]_+$ **of threshold logarithms**

constant term in threshold soft function

$$\gamma_0^r = \gamma_0^s$$

$$\gamma_1^r = \gamma_1^s - \beta_0 c_1^s$$

$$\gamma_2^r = \gamma_2^s - 2\beta_0 c_2^s - \beta_1 c_1^s + 2 C_a C_A \beta_0 \zeta_4$$

Control $\left[\frac{1}{P_T^2}\right]_*$ **of pT distribution**

Small pT cross section for Higgs production

- **There are many different ways to perform pT resummation for Higgs production. We follow** Neill, Rothstein, Vaidya (2015)

$$\frac{d^2\sigma}{d^2\vec{Q}_T} = \int x_a \int x_b\, \delta\Big(x_a x_b - \frac{m_H^2}{S}\Big)\sigma_0 \int \frac{d^2\vec{b}}{(2\pi)^2} e^{i\vec{b}\cdot\vec{Q}_T} W\big(x_a, x_b, m_H, \vec{b}, \mu, \nu\big) + \frac{d^2\sigma}{d^2\vec{Q}_T}\Big|_{\text{n.s.}}$$

$$W(x_a, x_b, m_H, \vec{b}, \mu, \nu) = \Big|C_V\big(m_t, m_H, \mu\big)\Big|^2 S(\vec{b}, \mu, \nu) B^{\alpha\beta}_{g/N_1}(x_a, Q, \vec{b}, \mu, \nu) B^{\alpha\beta}_{g/N_2}(x_b, Q, \vec{b}, \mu, \nu)$$

$$C_V(m_t, m_H, \mu) = C_V(m_t, m_H, \mu_H) \exp\left[\frac{1}{2}\int_{\mu_H^2}^{\mu^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left(\Gamma_{\text{cusp}}[\alpha_s(\bar{\mu})] \ln\frac{M_H^2}{\bar{\mu}^2} + \gamma^V[\alpha_s(\bar{\mu})]\right)\right]$$

$$B^{\alpha\beta}_{g/N}(x, \vec{b}, Q, \mu, \nu) = \frac{g_\perp^{\alpha\beta}}{d-2} B_{g/N}(x, b, Q, \mu, \nu) + \left(\frac{g_\perp^{\alpha\beta}}{d-2} + \frac{b^\alpha b^\beta}{b^2}\right) B'_{g/N}(x, b, Q, \mu, \nu)$$

$$B_{g/N}(x, b, Q, \mu, \nu) = \sum_j \int_x^1 \frac{dz}{z} I_{gj}(z, b, Q, \mu, \nu) f_{j/N}(x/z, \mu) + \dots$$

$$S_\perp(b, \mu, \nu) = S_\perp(b, \mu_s, \nu_s) \exp\Bigg[\int_{\mu_s^2}^{\mu^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\Big(\Gamma_{\text{cusp}}[\alpha_s(\bar{\mu})] \ln\frac{b^2\bar{\mu}^2}{b_0^2} + \gamma^s[\alpha_s(\bar{\mu})]\Big) + \ln\frac{\nu^2}{\nu_s^2}\Big(-\int_{b_0^2/b^2}^{\mu^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\Gamma_{\text{cusp}}[\alpha_s(\bar{\mu})] + \gamma^r[\alpha_s(b_0/b)]\Big)\Bigg]$$

pT resummation for Higgs production at N3LL

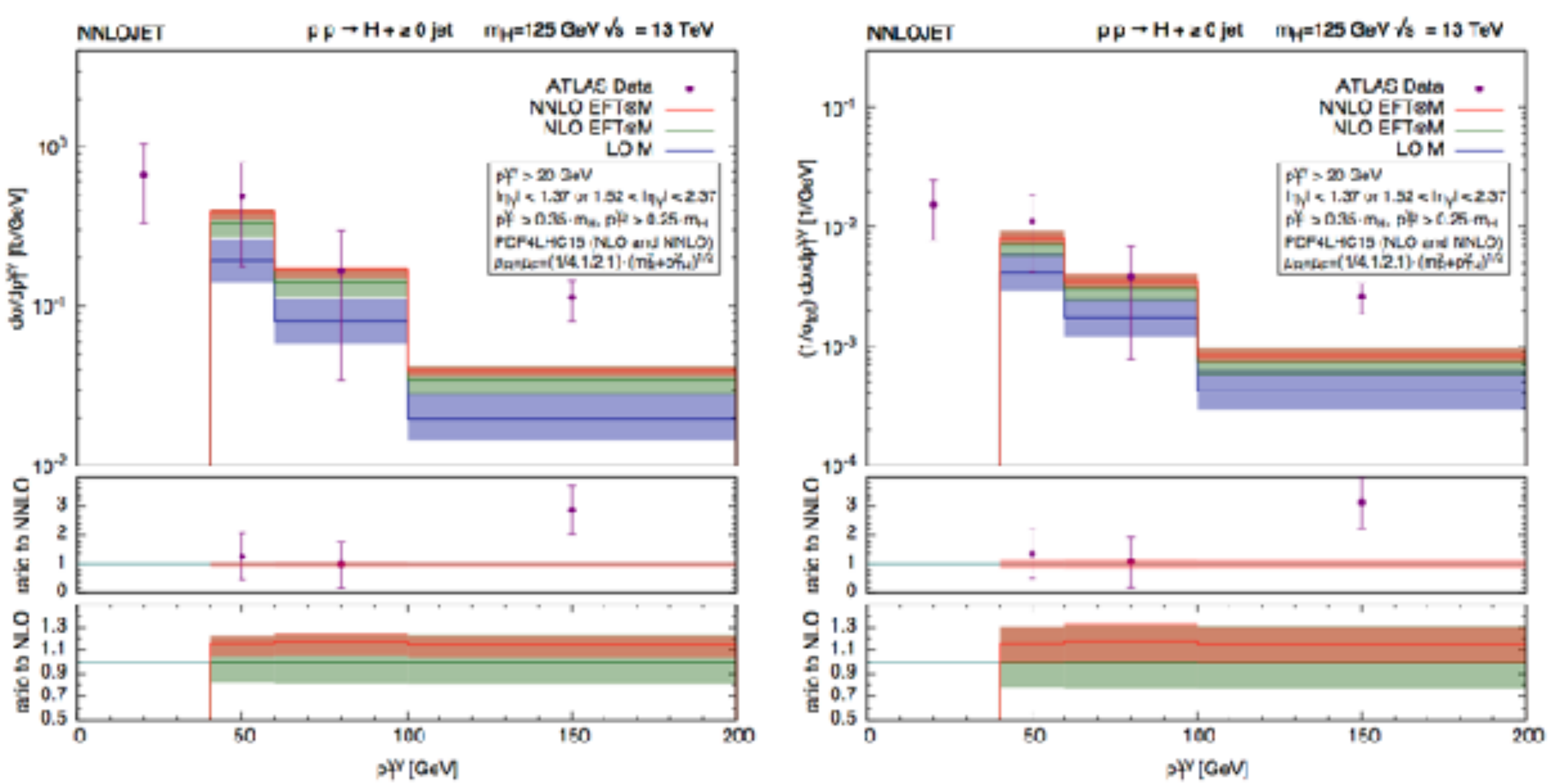
X. Chen, Gehrmann, Glover, Huss, Y. Li, Neill, Schulze, Stewart, HXZ, work in progress

- **Perturbative order of various ingredients:**
  - Two-loop hard function, beam function, soft function
  - Three-loop normal anomalous dimension
  - Three-loop splitting amplitude
  - Three-loop rapidity anomalous dimension (new)
  - Four-loop cusp anomalous dimension (Pade approximation)
  - Higgs + jet production at NNLO
- **Resummation performed in b space**
- **Simple b* scheme for non-perturbative effects** $b^* = \frac{b}{\sqrt{1 + b^2/b_{\mathrm{max}}^2}}$
- **Light quark mass effects included at fixed order**

Higgs + jet production at NNLO

X. Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier, 2016
see also Boughezal et al 2015; Melnikov et al 2015

LO	$gg \to Hg$, $qg \to Hq$, $q\bar{q} \to Hg$	tree level
NLO	$gg \to Hg$, $qg \to Hq$, $q\bar{q} \to Hg$	one loop
	$gg \to Hgg$, $gg \to Hq\bar{q}$, $qg \to Hqg$, $qq \to Hqq$, $q\bar{q} \to Hgg$, $q\bar{q} \to Hq\bar{q}$	tree level
NNLO	$gg \to Hg$, $qg \to Hq$, $q\bar{q} \to Hg$	two loop
	$gg \to Hgg$, $gg \to Hq\bar{q}$, $qg \to Hqg$, $qq \to Hqq$, $q\bar{q} \to Hgg$, $q\bar{q} \to Hq\bar{q}$	one loop
	$gg \to Hggg$, $gg \to Hq\bar{q}g$, $qg \to Hqgg$, $qg \to Hqq\bar{q}$, $qq \to Hqqg$, $q\bar{q} \to Hggg$, $qq \to Hqqg$	tree level

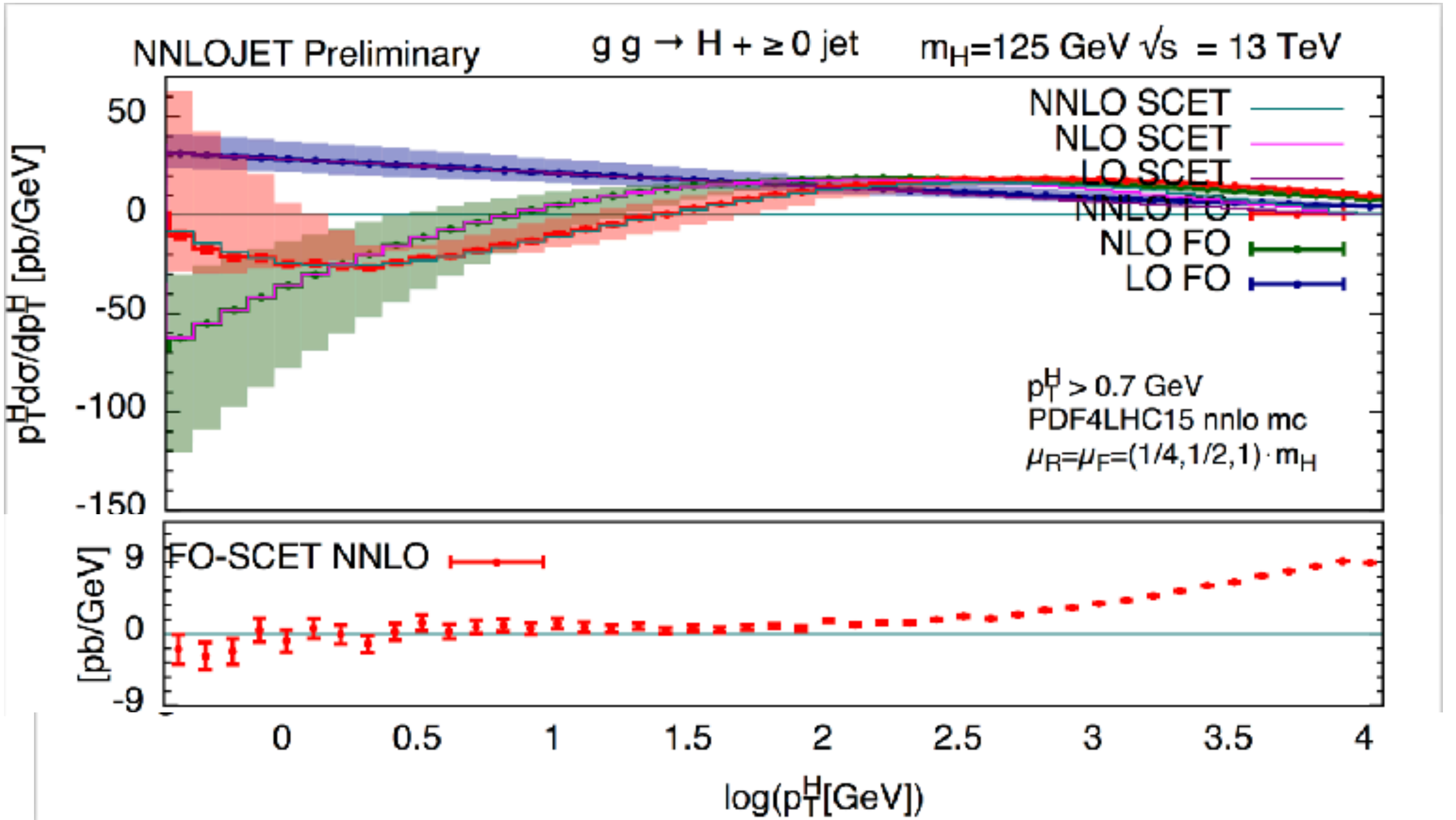


Comparison of SCET and fixed-order Pert. Theory

$$\frac{1}{\sigma_0}\frac{d\sigma}{dp_T} \sim \underbrace{\frac{1}{q_T}}_{\text{SCET}} + \mathcal{O}(q_T/Q^2) + \cdots$$

SCET

Fixed-order pert. theory

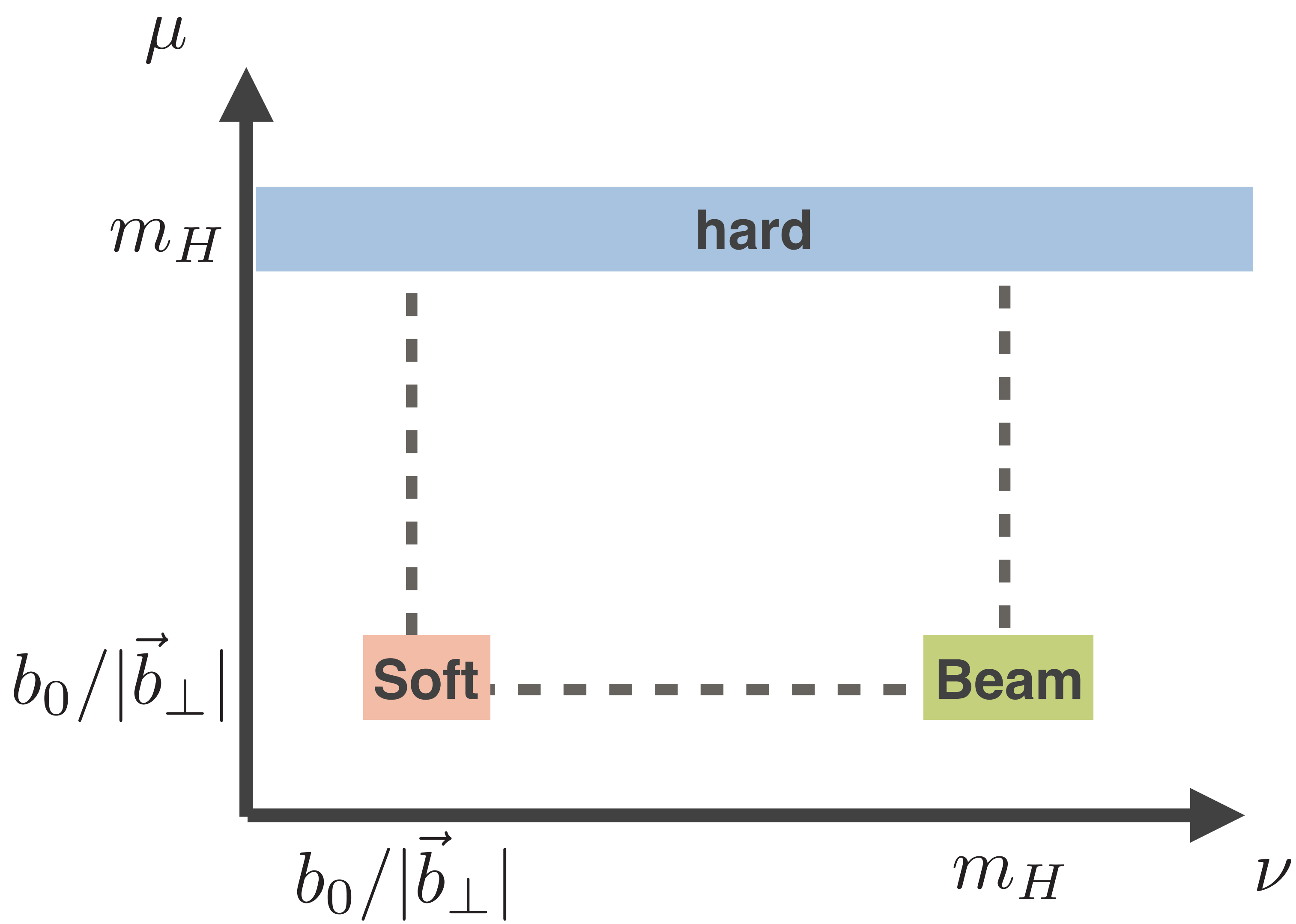


Scale setting in the resumed regime

- **Three independent scale variation:**
  - hard μ scale, beam and soft μ scale, soft ν (rapidity) scale

$$\mu_H \in [\frac{1}{4}m_H, m_H]$$

$$\mu_B = \mu_S \in [\frac{1}{2}\frac{b_0}{b_\perp}, 2\frac{b_0}{b_\perp}]$$

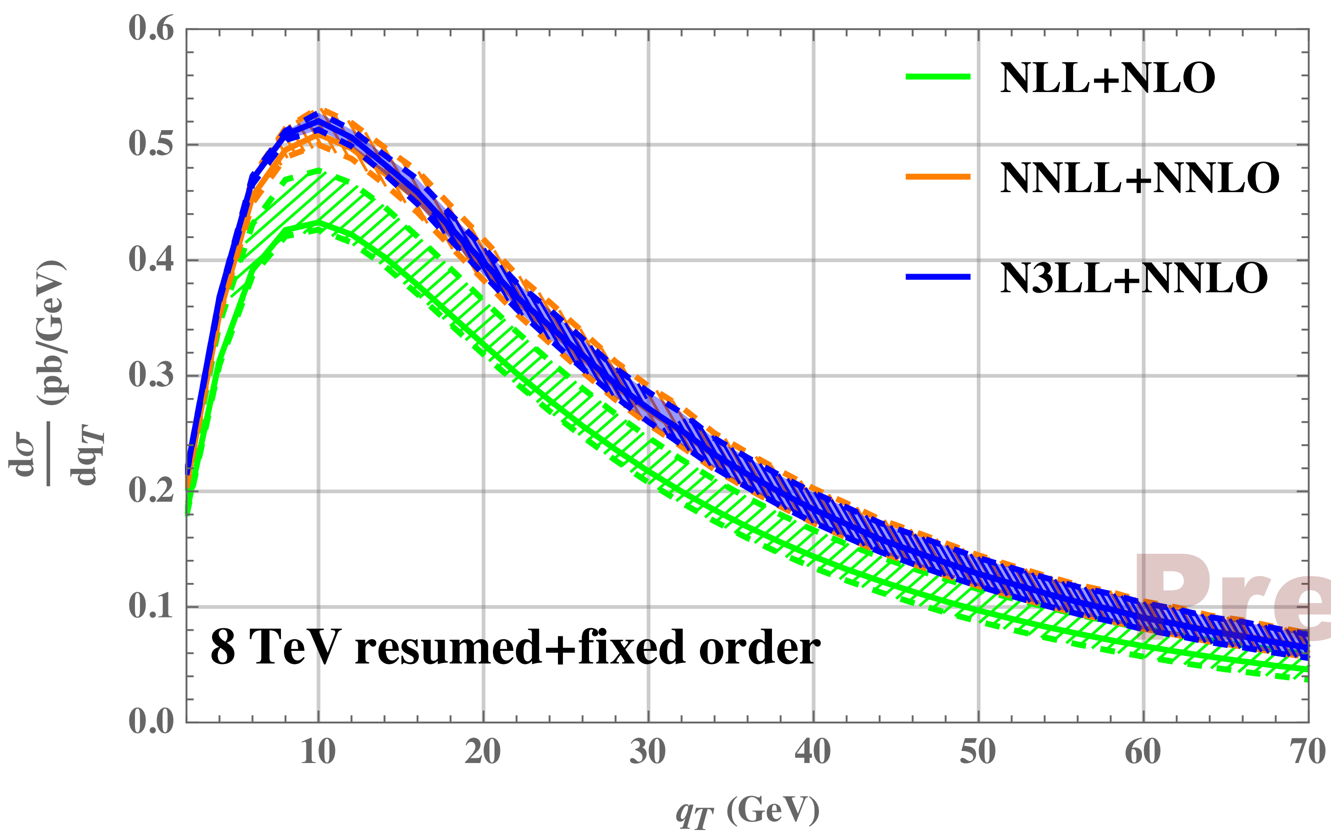


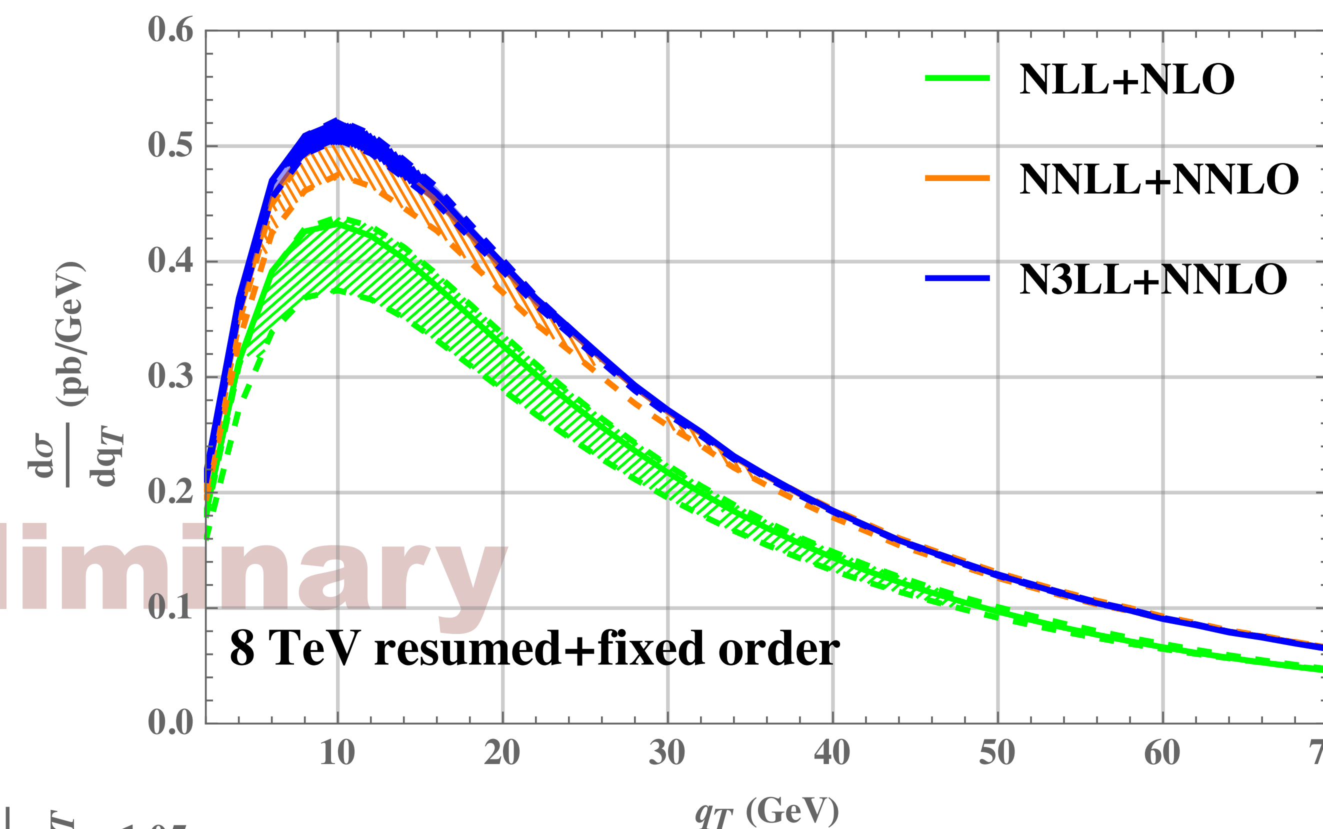
$$\nu_S \in [\frac{1}{2}\frac{b_0}{b_\perp}, 2\frac{b_0}{b_\perp}]$$

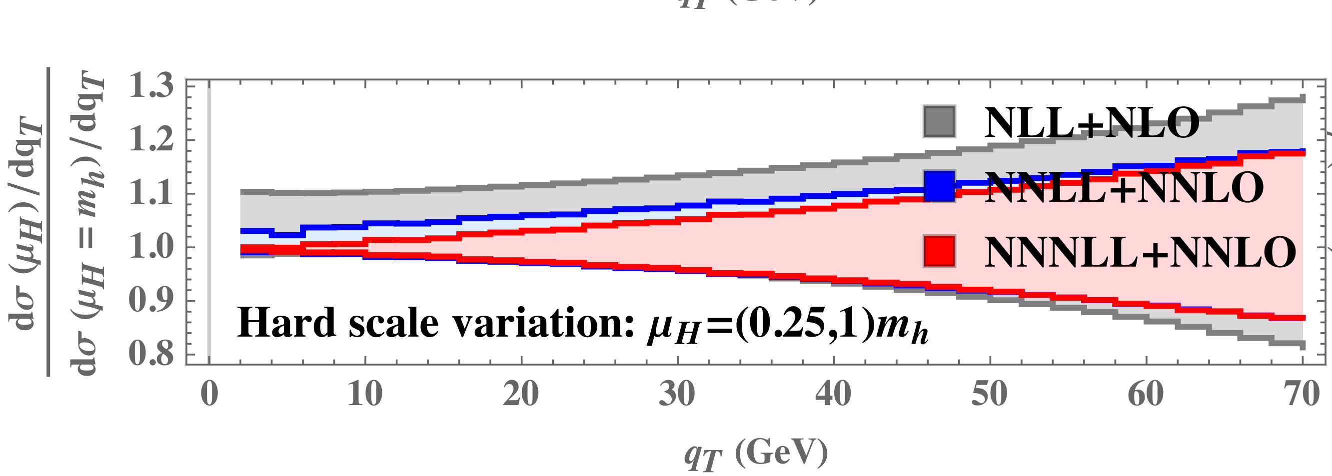
$$\nu_B = m_H$$

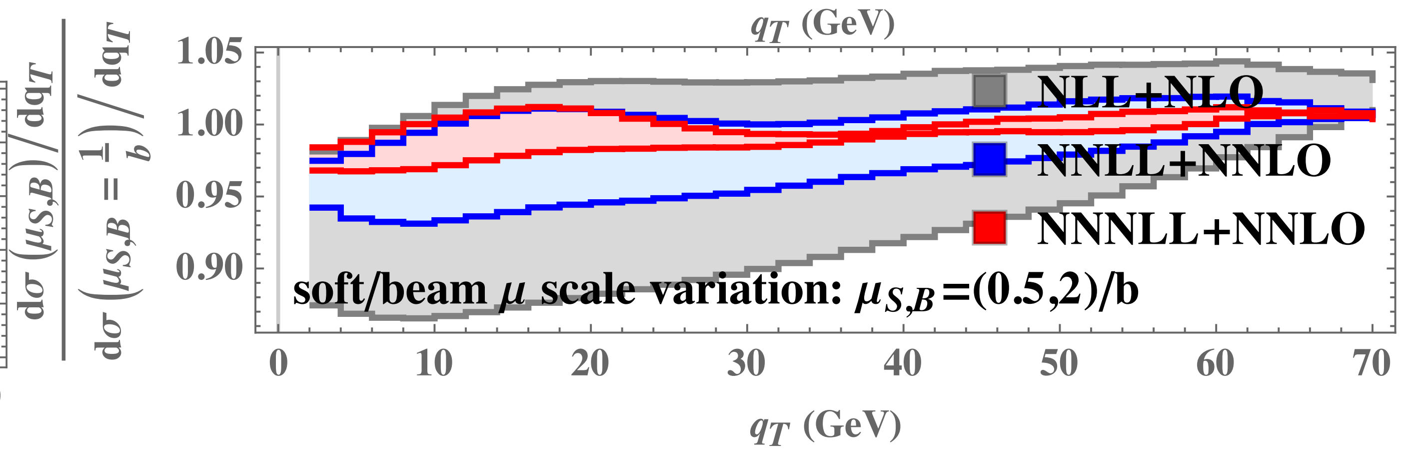
Hard and soft scale variation

- Order-by-order reduction of scale uncertainties shows good convergence of perturbative series

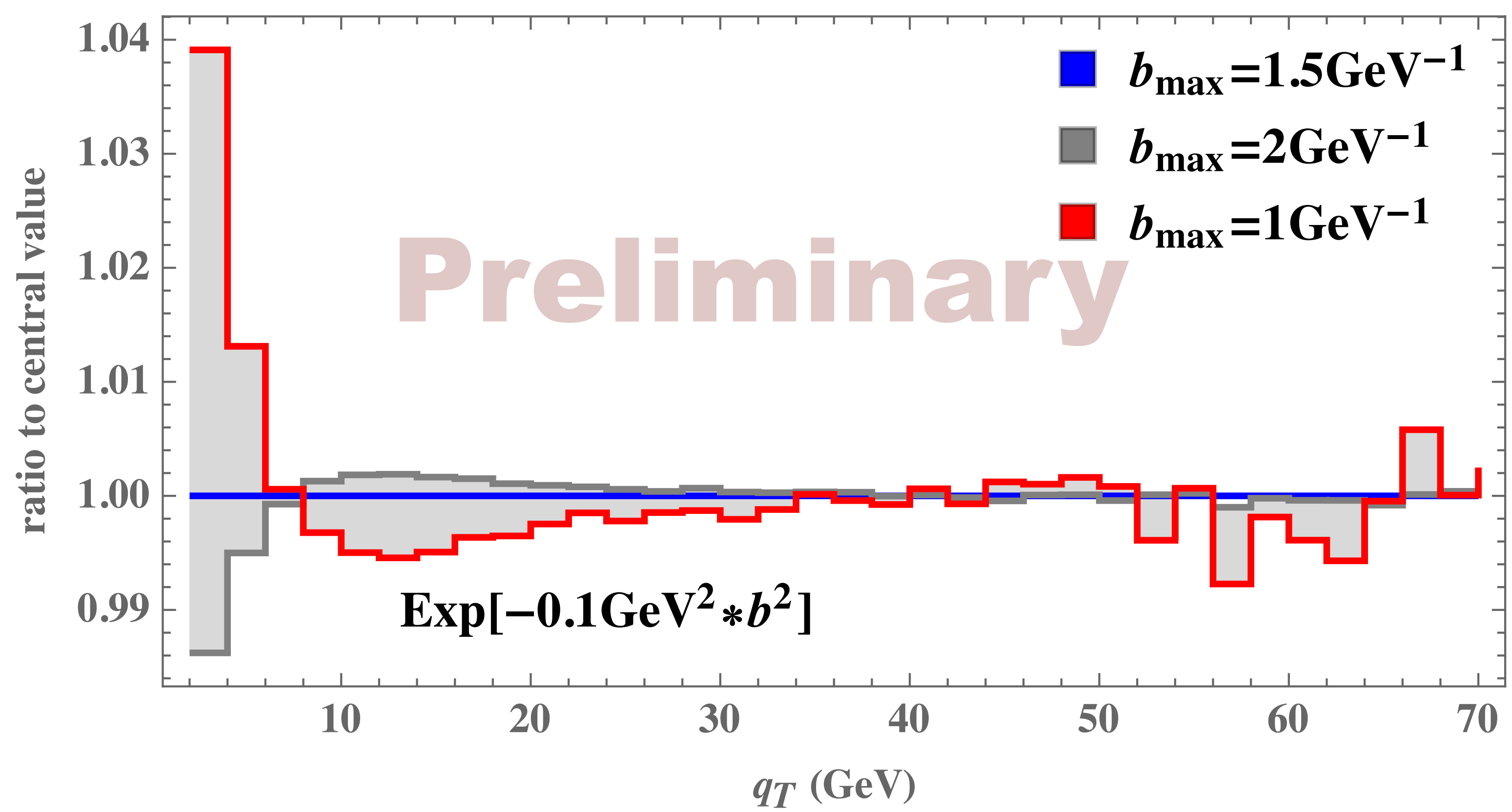


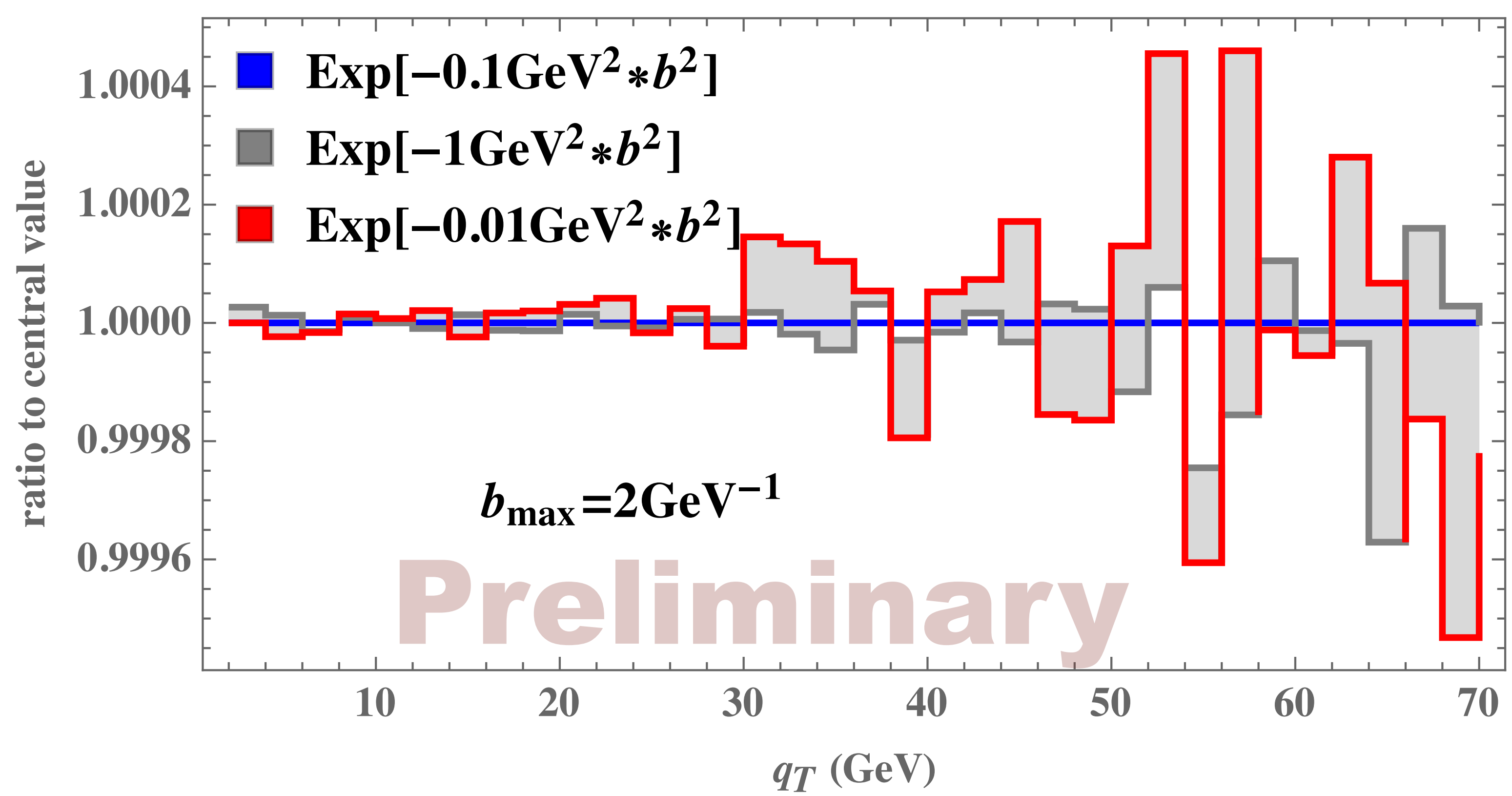




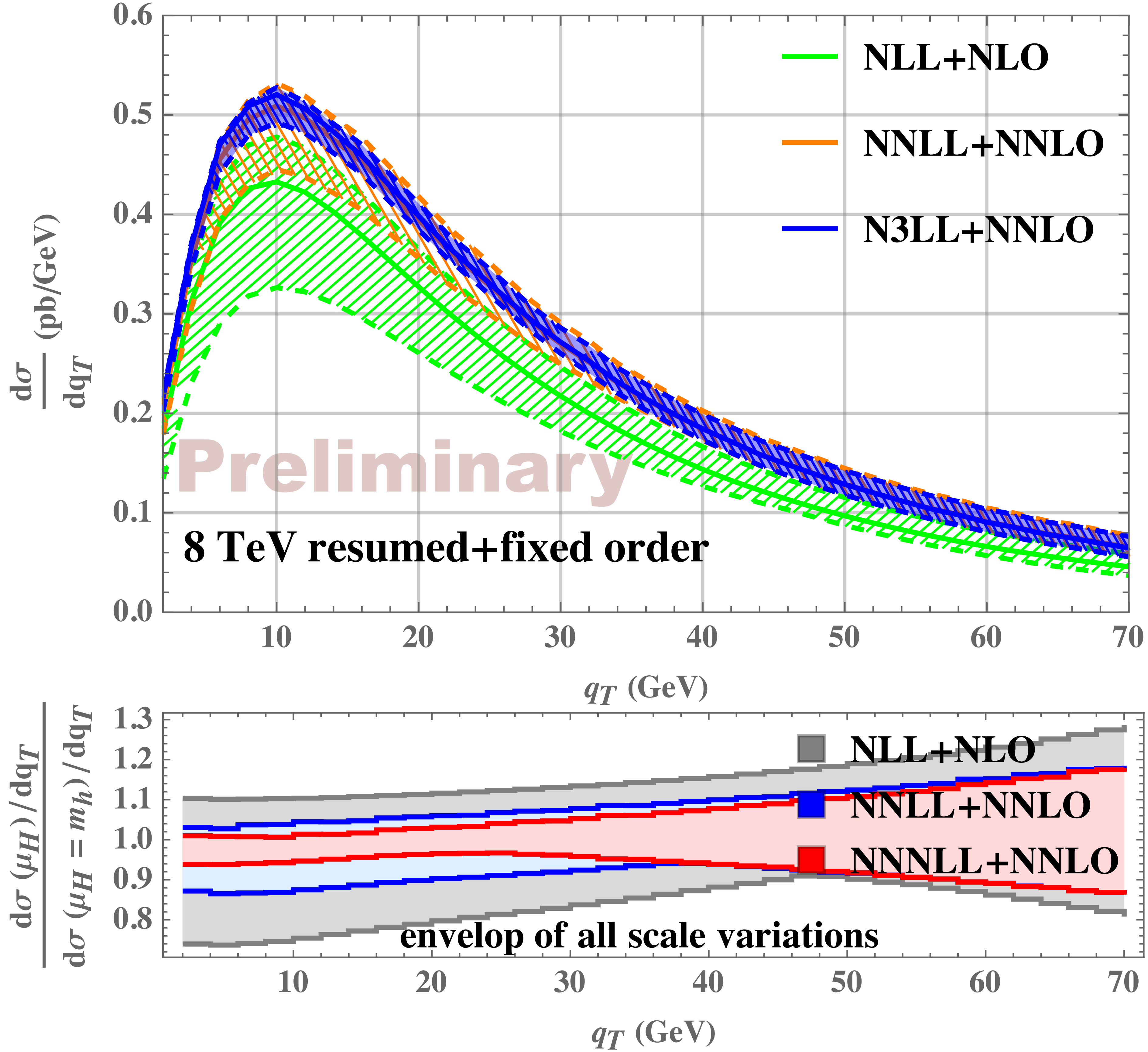


Non-Perturbative uncertainties





Total scale uncertainties



Conclusion

- **Introduce a new regulator for rapidity divergence in SCET description of transverse-momentum distribution.**
- **Analytic calculation of the resulting three-loop soft function through three-loops for the first time, extracting the rapidity anomalous dimension (also known as collinear anomaly d2)**
  - Lifting the rapidity regulator as an dynamical variable: double differential soft function
  - Compute the double differential soft function (the N=4 part) by making an ansatz, and then fixing the coefficient using expansion around b=0. Two different method for the remaining QCD part.
  - Intriguing relation between rapidity anomalous dimension and soft anomalous dimension.
- **N3LL pT resummation for Higgs production (except for four-loop cusp)**
  - Significant reduction of uncertainties. About 10% total uncertainties in the resumed region.