Non-Gaussianities in Two-field Inflation

Tao Wang

Department of Physics
East China Normal University

Interdisciplinary Center for Theoretical Study, USTC, Hefei,
Jun 2011

Outline

Outline

Why inflation?

To blow up the universe:

- rapidly expanded e^{180} times in volume

To generate fluctuations:

- small: $\frac{\Delta T}{T} \sim 10^{-5}$
- nearly scale-invariant: $\frac{\Delta T}{T} \propto k^{-0.02}$

Implications for inflation in Einstein gravity:

- e-folds $N \sim 60 \sim \ln \frac{a_c}{a_*}$
- $\epsilon \sim \frac{\dot{H}}{H^2} \sim 10^{-2} \sim \left(\frac{M_{pl} V_{,\phi}}{V}\right)^2$ (a flat potential)
- $\frac{V}{\epsilon M_{pl}^4} \sim 10^{-10} \Rightarrow V^{1/4} \sim 10^{16}\mathrm{GeV}$ (a high energy density)

Why inflation?

To blow up the universe:

- rapidly expanded e^{180} times in volume

To generate fluctuations:

- small: $\frac{\Delta T}{T} \sim 10^{-5}$
- nearly scale-invariant: $\frac{\Delta T}{T} \propto k^{-0.02}$

Implications for inflation in Einstein gravity:

- e-folds $N \sim 60 \sim \ln \frac{a_c}{a_*}$
- $\epsilon \sim \frac{\dot{H}}{H^2} \sim 10^{-2} \sim \left(\frac{M_{pl} V_{,\phi}}{V}\right)^2$ (a flat potential)
- $\frac{V}{\epsilon M_{pl}^4} \sim 10^{-10} \Rightarrow V^{1/4} \sim 10^{16}\mathrm{GeV}$ (a high energy density)

Why inflation?

To blow up the universe:

- rapidly expanded e^{180} times in volume

To generate fluctuations:

- small: $\frac{\Delta T}{T} \sim 10^{-5}$
- nearly scale-invariant: $\frac{\Delta T}{T} \propto k^{-0.02}$

Implications for inflation in Einstein gravity:

- e-folds $N \sim 60 \sim \ln \frac{a_c}{a_*}$
- $\epsilon \sim \frac{\dot{H}}{H^2} \sim 10^{-2} \sim \left(\frac{M_{pl} V_{,\phi}}{V}\right)^2$ (a flat potential)
- $\frac{V}{\epsilon M_{pl}^4} \sim 10^{-10} \Rightarrow V^{1/4} \sim 10^{16}\mathrm{GeV}$ (a high energy density)

What is Gaussian (normal) distribution?

Probability density function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right].$$

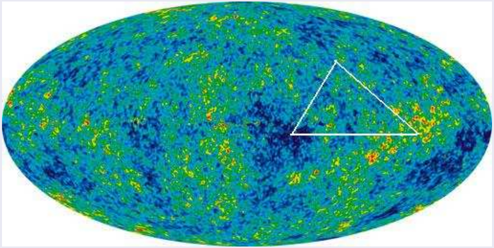
Central moment:

$$\mu_n = \int_{-\infty}^{+\infty} (x-\mu)^n f(x) dx.$$

n	0	1	2	3	4
μ_n	1	0	σ^2	0	$3\sigma^4$

Skewness: $\frac{\mu_3}{\sigma^3}$.
Kurtosis: $\frac{\mu_4}{\sigma^4} - 3$.

Non-Gaussianity of CMB Temprature Fluctuation



$$\frac{\Delta T}{T} = -\frac{\Phi}{3},$$

$$\Phi(\vec{x}) = \Phi_L(\vec{x}) + f_{NL}[\Phi_L^2(\vec{x}) - \langle\Phi_L^2(\vec{x})\rangle] + g_{NL}\Phi_L^3(\vec{x}) + \ldots,$$

$$\langle\zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle = (2\pi)^7\delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)\frac{3}{10}f_{NL}(\mathcal{P}_k^\zeta)^2\frac{\sum_i k_i^3}{\prod_i k_i^3}.$$

Outline

e-folds for Product Potential $W = U(\varphi)V(\chi)$

Background Equations

$$3H\dot{\varphi} = -U_{,\varphi}V, \quad 3He^{2b(\varphi)}\dot{\chi} = -UV_{,\chi}, \quad 3M_p^2H^2 = UV$$

e-folding Number $N = N(\varphi_*, \chi_*, \varphi_c, \chi_c)$

$$\begin{aligned}\int_*^c Hdt &= \frac{1}{M_p^2}\int_*^c \frac{3M_p^2H^2}{3H\dot{\varphi}}\dot{\varphi}dt \\ &= -\frac{1}{M_p^2}\int_*^c \frac{UV}{U_{,\varphi}V}d\varphi \\ &= -\frac{1}{M_p^2}\int_*^c \frac{U}{U_{,\varphi}}d\varphi\end{aligned}$$

e-folds for Product Potential $W = U(\varphi)V(\chi)$

Background Equations

$$3H\dot{\varphi} = -U_{,\varphi}V, \quad 3He^{2b(\varphi)}\dot{\chi} = -UV_{,\chi}, \quad 3M_p^2H^2 = UV$$

e-folding Number $N = N(\varphi_*, \chi_*, \varphi_c, \chi_c)$

$$\begin{aligned}
\int_*^c Hdt &= \frac{1}{M_p^2}\int_*^c \frac{3M_p^2H^2}{3H\dot{\varphi}}\dot{\varphi}dt \\
&= -\frac{1}{M_p^2}\int_*^c \frac{UV}{U_{,\varphi}V}d\varphi \\
&= -\frac{1}{M_p^2}\int_*^c \frac{U}{U_{,\varphi}}d\varphi
\end{aligned}$$

e-folds for Sum Potential $W = U(\varphi) + V(\chi)$, $b = 0$

Background Equations

$$3H\dot{\varphi} = -U_{,\varphi}, \quad 3H\dot{\chi} = -V_{,\chi}, \quad 3M_p^2H^2 = U + V$$

e-folding Number $N = N(\varphi_*, \chi_*, \varphi_c, \chi_c)$

$$\begin{aligned}
\int_*^c Hdt &= \frac{1}{M_p^2}\int_*^c \frac{3M_p^2H^2}{3H}dt \\
&= \frac{1}{M_p^2}\int_*^c \frac{U}{3H}dt + \frac{1}{M_p^2}\int_*^c \frac{V}{3H}dt \\
&= \frac{1}{M_p^2}\int_*^c \frac{U}{3H\dot{\varphi}}\dot{\varphi}dt + \frac{1}{M_p^2}\int_*^c \frac{V}{3H\dot{\chi}}\dot{\chi}dt \\
&= -\frac{1}{M_p^2}\int_*^c \frac{U}{U_{,\varphi}}d\varphi - \frac{1}{M_p^2}\int_*^c \frac{V}{V_{,\chi}}d\chi
\end{aligned}$$

e-folds for Sum Potential $W = U(\varphi) + V(\chi)$, $b = 0$

Background Equations

$$3H\dot{\varphi} = -U_{,\varphi}, \quad 3H\dot{\chi} = -V_{,\chi}, \quad 3M_p^2H^2 = U + V$$

e-folding Number $N = N(\varphi_*, \chi_*, \varphi_c, \chi_c)$

$$\begin{aligned}
\int_*^c H dt &= \frac{1}{M_p^2}\int_*^c \frac{3M_p^2H^2}{3H} dt \\
&= \frac{1}{M_p^2}\int_*^c \frac{U}{3H} dt + \frac{1}{M_p^2}\int_*^c \frac{V}{3H} dt \\
&= \frac{1}{M_p^2}\int_*^c \frac{U}{3H\dot{\varphi}}\dot{\varphi} dt + \frac{1}{M_p^2}\int_*^c \frac{V}{3H\dot{\chi}}\dot{\chi} dt \\
&= -\frac{1}{M_p^2}\int_*^c \frac{U}{U_{,\varphi}} d\varphi - \frac{1}{M_p^2}\int_*^c \frac{V}{V_{,\chi}} d\chi
\end{aligned}$$

e-folds for More General Potential $W(\varphi,\chi)$

Background Equations

$$3H\dot{\varphi} = -W_{,\varphi}, \quad 3He^{2b(\varphi)}\dot{\chi} = -W_{,\chi}, \quad 3M_p^2H^2 = W$$

e-folding Number $N = N(\varphi_*, \chi_*, \varphi_c, \chi_c)$?

$W = \lambda w^\alpha$, $w = U(\varphi) + V(\chi)$, $b = 0$

$$N = -\frac{1}{\alpha M_p^2}\left(\int_*^c \frac{U}{U_{,\varphi}}d\varphi + \int_*^c \frac{V}{V_{,\chi}}d\chi\right)$$

$d\ln W/dw = (pw + qw^{\nu+1})^{-1}$, $w = U(\varphi)V(\chi)$, $e^{2b} = U^{-\nu}$

$$N = -\int_*^c \frac{pU}{M_p^2U_{,\varphi}}d\varphi - \int_*^c \frac{qV^{\nu+1}}{M_p^2V_{,\chi}}d\chi$$

e-folds for More General Potential $W(\varphi, \chi)$

Background Equations

$$3H\dot{\varphi} = -W_{,\varphi}, \quad 3He^{2b(\varphi)}\dot{\chi} = -W_{,\chi}, \quad 3M_p^2H^2 = W$$

e-folding Number $N = N(\varphi_*, \chi_*, \varphi_c, \chi_c)$?

$$\int_*^c H dt = ?$$

$W = \lambda w^\alpha$, $w = U(\varphi) + V(\chi)$, $b = 0$

$$N = -\frac{1}{\alpha M_p^2}\left(\int_*^c \frac{U}{U_{,\varphi}} d\varphi + \int_*^c \frac{V}{V_{,\chi}} d\chi\right)$$

$d \ln W/dw = (pw + qw^{\nu+1})^{-1}$, $w = U(\varphi)V(\chi)$, $e^{2b} = U^{-\nu}$

e-folds for More General Potential $W(\varphi, \chi)$

Background Equations

$$3H\dot{\varphi} = -W_{,\varphi}, \quad 3He^{2b(\varphi)}\dot{\chi} = -W_{,\chi}, \quad 3M_p^2 H^2 = W$$

e-folding Number $N = N(\varphi_*, \chi_*, \varphi_c, \chi_c)$?

TW, Phys. Rev. D **82**, 123515 (2010) arXiv:1008.3198 [astro-ph.CO].

$W = \lambda w^\alpha$, $w = U(\varphi) + V(\chi)$, $b = 0$

$$N = -\frac{1}{\alpha M_p^2}\left(\int_*^c \frac{U}{U_{,\varphi}} d\varphi + \int_*^c \frac{V}{V_{,\chi}} d\chi\right)$$

$d \ln W / dw = (pw + qw^{\nu+1})^{-1}$, $w = U(\varphi)V(\chi)$, $e^{2b} = U^{-\nu}$

$$N = \int^c \frac{pU}{\ldots} d\varphi \ldots \int^c \frac{qV^{\nu+1}}{\ldots} d\chi$$

e-folds for More General Potential $W(\varphi, \chi)$

Background Equations

$$3H\dot{\varphi} = -W_{,\varphi}, \quad 3He^{2b(\varphi)}\dot{\chi} = -W_{,\chi}, \quad 3M_p^2H^2 = W$$

e-folding Number $N = N(\varphi_*, \chi_*, \varphi_c, \chi_c)$?

$W = \lambda w^\alpha$, $w = U(\varphi) + V(\chi)$, $b = 0$

$$N = -\frac{1}{\alpha M_p^2}\left(\int_*^c \frac{U}{U_{,\varphi}}d\varphi + \int_*^c \frac{V}{V_{,\chi}}d\chi\right)$$

$d\ln W/dw = (pw + qw^{\nu+1})^{-1}$, $w = U(\varphi)V(\chi)$, $e^{2b} = U^{-\nu}$

$$N = -\int_*^c \frac{pU}{M_p^2U_{,\varphi}}d\varphi - \int_*^c \frac{qV^{\nu+1}}{M_p^2V_{,\chi}}d\chi$$

Outline

From Inflation to Observation

$\delta\varphi$ quantum fluctuation of inflaton

$\zeta = -\frac{H}{\dot{\varphi}}\delta\varphi$

↓

ζ curvature perturbation

$\Phi = \frac{3}{5}\zeta$

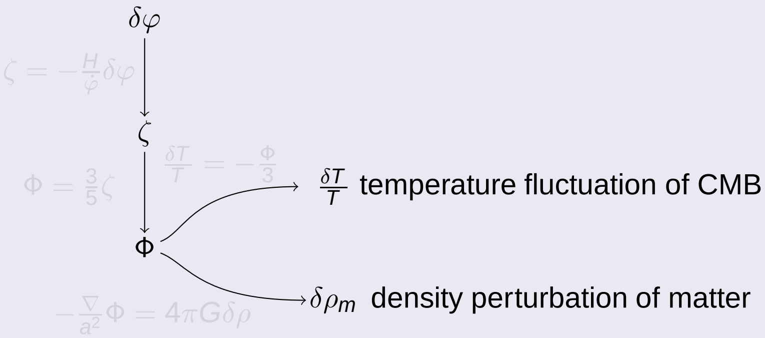
↓

Φ Bardeen potential

$\frac{\delta T}{T} = -\frac{\Phi}{3}$ → $\frac{\delta T}{T}$

$-\frac{\nabla}{a^2}\Phi = 4\pi G\delta\rho$ → $\delta\rho_m$

From Inflation to Observation



From Inflation to Observation

$$\delta\varphi$$

$$\zeta = -\frac{H}{\dot{\varphi}}\delta\varphi$$

$$\zeta$$

$$\Phi = \frac{3}{5}\zeta$$

$$\frac{\delta T}{T} = -\frac{\Phi}{3}$$

$$\Phi$$

$$-\frac{\nabla}{a^2}\Phi = 4\pi G\delta\rho$$

$$\frac{\delta T}{T} \dashrightarrow$$

PLANCK

$$\delta\rho_m \dashrightarrow$$

LAMOST

From Inflation to Observation

$$\delta\varphi$$

$$\zeta = -\frac{H}{\dot{\varphi}}\delta\varphi$$

$$\zeta$$

$$\Phi = \frac{3}{5}\zeta$$

$$\frac{\delta T}{T} = -\frac{\Phi}{3}$$

$$\Phi$$

$$\frac{\delta T}{T}$$

$$-\frac{\nabla}{a^2}\Phi = 4\pi G\delta\rho$$

$$\delta\rho_m$$

From Inflation to Observation

$\delta\varphi$

$\zeta = -\frac{H}{\dot{\varphi}}\delta\varphi$ ↓

ζ

$\Phi = \frac{3}{5}\zeta$ ↓ $\frac{\delta T}{T} = -\frac{\Phi}{3}$ → $\frac{\delta T}{T}$

Φ

$-\frac{\nabla}{a^2}\Phi = 4\pi G\delta\rho$ → $\delta\rho_m$

Gaussian initial condition: $\delta\varphi$

↓ linear evolution

Gaussian signals: $\frac{\delta T}{T}$, $\delta\rho_m$

Curvature Perturbation from Inflation

$\delta\varphi$ quantum fluctuation of inflaton

$\downarrow$

ζ curvature perturbation

Curvature Perturbation from Inflation

$\delta\varphi$ → ζ

Gaussian $\delta\varphi$ → (linear evolution) → Gaussian ζ

Curvature Perturbation from Inflation

$$\delta\varphi_k$$

$$\downarrow$$

$$\zeta_k = -\frac{H}{\dot\varphi_k}\delta\varphi_k$$

$$\downarrow$$

$$\zeta_k$$

inside the Hubble horizon: $k > aH$

across the Hubble horizon: $k = aH$

outside the Hubble horizon: $k < aH$

Non-Gaussianity from Inflation

$\delta\varphi_k$ — $k > aH$: Gaussian

$\downarrow$

$\zeta_k = -\frac{H}{\dot{\varphi}_k}\delta\varphi_k$ — $k = aH$: Gaussian

$\downarrow$

ζ_k — $k < aH$: Gaussian or non-Gaussian?

Non-Gaussianity from Inflation

$$\delta\varphi_k \qquad k > aH\text{: Gaussian}$$

$$\downarrow$$

$$\zeta_k = -\frac{H}{\dot\varphi_k}\delta\varphi_k \qquad k = aH\text{: Gaussian}$$

$$\downarrow$$

$$\zeta_k \qquad k < aH\text{: conserved or non-conserved?}$$

Outline

Focus:
Non-Gaussianity from Conventional Two-field Slow-roll Inflation in Einstein Gravity

$$\int d^4x\sqrt{-g}\left[\frac{1}{2}M_p^2R-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-\frac{1}{2}e^{2b(\varphi)}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-W(\varphi,\chi)\right]$$

- Two inflatons: φ, χ.
- Conventional kinetic terms.
- Minimally coupled to Einstein gravity.
- Slow-roll: $\dot{H}/H^2 \ll 1$, $\ddot{\varphi}/H\dot{\varphi} \ll 1$, $\ddot{\chi}/H\dot{\chi} \ll 1$.
- Sign different from WMAP: $\zeta(\vec{x}) = \zeta_L(\vec{x}) - \frac{3}{5}f_{NL}\zeta_L^2(\vec{x})$.

Action

$$\int d^4x\sqrt{-g}\left[\frac{1}{2}M_p^2R-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-\frac{1}{2}e^{2b(\varphi)}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-W(\varphi,\chi)\right]$$

Slow-roll Parameters

$$\epsilon_\varphi=\frac{M_p^2}{2}\left(\frac{W_{,\varphi}}{W}\right)^2,\qquad \epsilon_\chi=\frac{M_p^2}{2}\left(\frac{W_{,\chi}}{W}\right)^2e^{-2b},$$

$$\epsilon=-\frac{\dot{H}}{H^2}=\epsilon_\varphi+\epsilon_\chi,\qquad \epsilon_b=8M_p^2b_{,\varphi}^2,$$

$$\eta_{\varphi\varphi}=\frac{M_p^2W_{,\varphi\varphi}}{W},\qquad \eta_{\chi\chi}=\frac{M_p^2W_{,\chi\chi}}{W}e^{-2b},$$

$$\eta_{\varphi\chi}=\frac{M_p^2W_{,\varphi\chi}}{W}e^{-b}.$$

Action

$$\int d^4x\sqrt{-g}\left[\frac{1}{2}M_p^2R-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-\frac{1}{2}e^{2b(\varphi)}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-W(\varphi,\chi)\right]$$

Background Equations

$$3H\dot{\varphi}=-W_{,\varphi},\quad 3He^{2b}\dot{\chi}=-W_{,\chi},\quad 3M_p^2H^2=W.$$

Non-linear parameter in Two-field Inflation

$$-\frac{6}{5}f_{\rm NL}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=-\frac{6}{5}f^{(3)}_{\rm NL}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)-\frac{6}{5}f^{(4)}_{\rm NL}$$

First Term: Dependent of Momenta

Suppressed by tensor-to-scalar ratio, too small to be observed.

Second Term: Independent of Momenta (Local)

$$-\frac{6}{5}f^{(4)}_{\rm NL}=\frac{N^2_{,\varphi_*}N_{,\varphi_*\varphi_*}+2e^{-2b_*}N_{,\varphi_*}N_{,\chi_*}N_{,\varphi_*\chi_*}+e^{-4b_*}N^2_{,\chi_*}N_{,\chi_*\chi_*}}{\left(N^2_{,\varphi_*}+e^{-2b_*}N^2_{,\chi_*}\right)^2}$$

e-folding Number

$N=\ln\frac{a_c}{a_*}=\int_*^c Hdt=N(\varphi_*,\chi_*)$

Non-linear parameter in Two-field Inflation

$$-\frac{6}{5}f_{\rm NL}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=-\frac{6}{5}f_{\rm NL}^{(3)}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)-\frac{6}{5}f_{\rm NL}^{(4)}$$

First Term: Dependent of Momenta

Suppressed by tensor-to-scalar ratio, too small to be observed.

Second Term: Independent of Momenta (Local)

$$-\frac{6}{5}f_{\rm NL}^{(4)}=\frac{N_{,\varphi_*}^2N_{,\varphi_*\varphi_*}+2e^{-2b_*}N_{,\varphi_*}N_{,\chi_*}N_{,\varphi_*\chi_*}+e^{-4b_*}N_{,\chi_*}^2N_{,\chi_*\chi_*}}{\left(N_{,\varphi_*}^2+e^{-2b_*}N_{,\chi_*}^2\right)^2}$$

e-folding Number

$N=\ln\frac{a_c}{a_*}=\int_*^c Hdt=N(\varphi_*,\chi_*)$

Non-linear parameter in Two-field Inflation

$$-\frac{6}{5}f_{\rm NL}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = -\frac{6}{5}f^{(3)}_{\rm NL}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) - \frac{6}{5}f^{(4)}_{\rm NL}$$

First Term: Dependent of Momenta

Suppressed by tensor-to-scalar ratio, too small to be observed.

Second Term: Independent of Momenta (Local)

$$-\frac{6}{5}f^{(4)}_{\rm NL} = \frac{N^2_{,\varphi_*}N_{,\varphi_*\varphi_*} + 2e^{-2b_*}N_{,\varphi_*}N_{,\chi_*}N_{,\varphi_*\chi_*} + e^{-4b_*}N^2_{,\chi_*}N_{,\chi_*\chi_*}}{\left(N^2_{,\varphi_*} + e^{-2b_*}N^2_{,\chi_*}\right)^2}$$

e-folding Number

$$N = \ln\frac{a_c}{a_*} = \int_*^c H dt = N(\varphi_*,\chi_*)$$

Main Scheme

Second Term: Independent of Momenta (Local)

$$-\frac{6}{5}f_{\rm NL}^{(4)} = \frac{N_{,\varphi_*}^2 N_{,\varphi_*\varphi_*} + 2e^{-2b_*}N_{,\varphi_*}N_{,\chi_*}N_{,\varphi_*\chi_*} + e^{-4b_*}N_{,\chi_*}^2 N_{,\chi_*\chi_*}}{\left(N_{,\varphi_*}^2 + e^{-2b_*}N_{,\chi_*}^2\right)^2}$$

Main Scheme

Second Term: Independent of Momenta (Local)

$$-\frac{6}{5}f_{\rm NL}^{(4)} = \frac{N_{,\varphi_*}^2 N_{,\varphi_*\varphi_*} + 2e^{-2b_*}N_{,\varphi_*}N_{,\chi_*}N_{,\varphi_*\chi_*} + e^{-4b_*}N_{,\chi_*}^2 N_{,\chi_*\chi_*}}{\left(N_{,\varphi_*}^2 + e^{-2b_*}N_{,\chi_*}^2\right)^2}$$

e-folding Number

$$N = \ln\frac{a_c}{a_*} = \int_*^c H dt = N(\varphi_*, \chi_*)$$

Second Term: Independent of Momenta (Local)

$$-\frac{6}{5}f_{\rm NL}^{(4)} = \frac{N_{,\varphi_*}^2 N_{,\varphi_*\varphi_*} + 2e^{-2b_*}N_{,\varphi_*}N_{,\chi_*}N_{,\varphi_*\chi_*} + e^{-4b_*}N_{,\chi_*}^2 N_{,\chi_*\chi_*}}{\left(N_{,\varphi_*}^2 + e^{-2b_*}N_{,\chi_*}^2\right)^2}$$

e-folding Number

$N = \ln \frac{a_c}{a_*} = \int_*^c H dt = N(\varphi_*, \chi_*)$

Really Calculable?

- Numerically calculable for any potential.
- Analytically calculable for very special potential.

Second Term: Independent of Momenta (Local)

$$-\frac{6}{5}f_{\rm NL}^{(4)}=\frac{N_{,\varphi_*}^2N_{,\varphi_*\varphi_*}+2e^{-2b_*}N_{,\varphi_*}N_{,\chi_*}N_{,\varphi_*\chi_*}+e^{-4b_*}N_{,\chi_*}^2N_{,\chi_*\chi_*}}{\left(N_{,\varphi_*}^2+e^{-2b_*}N_{,\chi_*}^2\right)^2}$$

e-folding Number

$N=\ln\frac{a_c}{a_*}=\int_*^c Hdt=N(\varphi_*,\chi_*)$

Background Equations

$3H\dot{\varphi}=-W_{,\varphi},\quad 3He^{2b}\dot{\chi}=-W_{,\chi},\quad 3M_p^2H^2=W.$

Initial values: $\varphi_*,\ \chi_*$.

End condition: $\epsilon=-\dot{H}/H^2=\epsilon^c$.

Action

$$\int d^4x\sqrt{-g}\left[\frac{1}{2}M_p^2 R-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-\frac{1}{2}e^{2b(\varphi)}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-W(\varphi,\chi)\right]$$

Sum Type Potential

$W=U(\varphi)+V(\chi)$, $b=0$.

F. Vernizzi and D. Wands, JCAP **0605**, 019 (2006) [arXiv:astro-ph/0603799].

Product Type Potential

$W=U(\varphi)V(\chi)$.

K. Y. Choi, L. M. H. Hall and C. van de Bruck, JCAP **0702**, 029 (2007) [arXiv:astro-ph/0701247].

Action

$$\int d^4x\sqrt{-g}\left[\frac{1}{2}M_p^2R-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-\frac{1}{2}e^{2b(\varphi)}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-W(\varphi,\chi)\right]$$

Sum Type Potential

$W=U(\varphi)+V(\chi)$, $b=0$.

F. Vernizzi and D. Wands, JCAP **0605**, 019 (2006) [arXiv:astro-ph/0603799].

Product Type Potential

$W=U(\varphi)V(\chi)$.

K. Y. Choi, L. M. H. Hall and C. van de Bruck, JCAP **0702**, 029 (2007) [arXiv:astro-ph/0701247].

e-folds for Sum Potential $W = U(\varphi) + V(\chi)$, $b = 0$

Background Equations

$$3H\dot{\varphi} = -U_{,\varphi}, \quad 3H\dot{\chi} = -V_{,\chi}, \quad 3M_p^2 H^2 = U + V$$

e-folding Number $N = N(\varphi_*, \chi_*, \varphi_c, \chi_c)$

$$\begin{aligned}
\int_*^c H dt &= \frac{1}{M_p^2}\int_*^c \frac{3M_p^2 H^2}{3H} dt \\
&= \frac{1}{M_p^2}\int_*^c \frac{U}{3H} dt + \frac{1}{M_p^2}\int_*^c \frac{V}{3H} dt \\
&= \frac{1}{M_p^2}\int_*^c \frac{U}{3H\dot{\varphi}} \dot{\varphi} dt + \frac{1}{M_p^2}\int_*^c \frac{V}{3H\dot{\chi}} \dot{\chi} dt \\
&= -\frac{1}{M_p^2}\int_*^c \frac{U}{U_{,\varphi}} d\varphi - \frac{1}{M_p^2}\int_*^c \frac{V}{V_{,\chi}} d\chi
\end{aligned}$$

e-folds for Sum Potential $W = U(\varphi) + V(\chi)$, $b = 0$

Background Equations

$$3H\dot{\varphi} = -U_{,\varphi}, \quad 3H\dot{\chi} = -V_{,\chi}, \quad 3M_p^2 H^2 = U + V$$

e-folding Number $N = N(\varphi_*, \chi_*, \varphi_c, \chi_c)$

$$\begin{aligned}
\int_*^c H dt &= \frac{1}{M_p^2}\int_*^c \frac{3M_p^2 H^2}{3H} dt \\
&= \frac{1}{M_p^2}\int_*^c \frac{U}{3H} dt + \frac{1}{M_p^2}\int_*^c \frac{V}{3H} dt \\
&= \frac{1}{M_p^2}\int_*^c \frac{U}{3H\dot{\varphi}} \dot{\varphi} dt + \frac{1}{M_p^2}\int_*^c \frac{V}{3H\dot{\chi}} \dot{\chi} dt \\
&= -\frac{1}{M_p^2}\int_*^c \frac{U}{U_{,\varphi}} d\varphi - \frac{1}{M_p^2}\int_*^c \frac{V}{V_{,\chi}} d\chi
\end{aligned}$$

e-folds for Sum Potential $W = U(\varphi) + V(\chi)$, $b = 0$

e-folding Number

$$
\begin{aligned}
N &= -\frac{1}{M_p^2}\int_*^c \frac{U}{U_{,\varphi}} d\varphi - \frac{1}{M_p^2}\int_*^c \frac{V}{V_{,\chi}} d\chi \\
&= N(\varphi_*, \chi_*, \varphi_c, \chi_c).
\end{aligned}
$$

Second Term: Independent of Momenta (Local)

$$-\frac{6}{5}f_{\rm NL}^{(4)} = \frac{N_{,\varphi_*}^2 N_{,\varphi_*\varphi_*} + 2e^{-2b_*}N_{,\varphi_*}N_{,\chi_*}N_{,\varphi_*\chi_*} + e^{-4b_*}N_{,\chi_*}^2 N_{,\chi_*\chi_*}}{\left(N_{,\varphi_*}^2 + e^{-2b_*}N_{,\chi_*}^2\right)^2}$$

e-folding Number

$$N = \ln\frac{a_c}{a_*} = \int_*^c H dt = N(\varphi_*, \chi_*)$$

e-folds for Sum Potential $W = U(\varphi) + V(\chi)$, $b = 0$

e-folding Number

$$\begin{aligned} N &= -\frac{1}{M_p^2}\int_*^c \frac{U}{U_{,\varphi}}d\varphi - \frac{1}{M_p^2}\int_*^c \frac{V}{V_{,\chi}}d\chi \\ &= N(\varphi_*, \chi_*, \varphi_c, \chi_c). \\ \varphi_c &= \varphi_c(\varphi_*, \chi_*), \\ \chi_c &= \chi_c(\varphi_*, \chi_*). \end{aligned}$$

Background Equations

$3H\dot{\varphi} = -W_{,\varphi}$, $\quad 3He^{2b}\dot{\chi} = -W_{,\chi}$, $\quad 3M_p^2H^2 = W$.

Initial values: φ_*, χ_*.

End condition: $\epsilon = -\dot{H}/H^2 = \epsilon^c$.

e-folds for Sum Potential $W = U(\varphi) + V(\chi)$, $b = 0$

e-folding Number

$$
\begin{aligned}
N &= -\frac{1}{M_p^2}\int_*^c \frac{U}{U_{,\varphi}} d\varphi - \frac{1}{M_p^2}\int_*^c \frac{V}{V_{,\chi}} d\chi \\
&= N(\varphi_*, \chi_*, \varphi_c, \chi_c). \\
\varphi_c &= \varphi_c(\varphi_*, \chi_*), \\
\chi_c &= \chi_c(\varphi_*, \chi_*).
\end{aligned}
$$

Background Equations

$3H\dot{\varphi} = -W_{,\varphi}$, $\quad 3He^{2b}\dot{\chi} = -W_{,\chi}$, $\quad 3M_p^2 H^2 = W$.

Initial values: φ_*, χ_*.

End condition: $\epsilon = -\dot{H}/H^2 = \epsilon^c$.

Action

$$\int d^4x\sqrt{-g}\left[\frac{1}{2}M_p^2 R-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-\frac{1}{2}e^{2b(\varphi)}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-W(\varphi,\chi)\right]$$

Sum Type Potential

$W=U(\varphi)+V(\chi)$, $b=0$.

F. Vernizzi and D. Wands, JCAP **0605**, 019 (2006) [arXiv:astro-ph/0603799].

Product Type Potential

$W=U(\varphi)V(\chi)$.

K. Y. Choi, L. M. H. Hall and C. van de Bruck, JCAP **0702**, 029 (2007) [arXiv:astro-ph/0701247].

More General Potential

Action

$$\int d^4x\sqrt{-g}\left[\frac{1}{2}M_p^2R-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-\frac{1}{2}e^{2b(\varphi)}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-W(\varphi,\chi)\right]$$

TW, Phys. Rev. D **82**, 123515 (2010) arXiv:1008.3198 [astro-ph.CO].

$W=\lambda w^\alpha$, $w=U(\varphi)+V(\chi)$, $b=0$

$$N=-\frac{1}{\alpha M_p^2}\left(\int_*^c\frac{U}{U_{,\varphi}}d\varphi+\int_*^c\frac{V}{V_{,\chi}}d\chi\right)$$

$d\ln W/dw=(pw+qw^{\nu+1})^{-1}$, $w=U(\varphi)V(\chi)$, $e^{2b}=U^{-\nu}$

$$N=-\int_*^c\frac{pU}{M_p^2U_{,\varphi}}d\varphi-\int_*^c\frac{qV^{\nu+1}}{M_p^2V_{,\chi}}d\chi$$

Action

$$\int d^4x\sqrt{-g}\left[\frac{1}{2}M_p^2R-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-\frac{1}{2}e^{2b(\varphi)}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-W(\varphi,\chi)\right]$$

TW, Phys. Rev. D **82**, 123515 (2010) arXiv:1008.3198 [astro-ph.CO].

$W=\lambda w^\alpha$, $w=U(\varphi)+V(\chi)$, $b=0$

$$N=-\frac{1}{\alpha M_p^2}\left(\int_*^c\frac{U}{U_{,\varphi}}d\varphi+\int_*^c\frac{V}{V_{,\chi}}d\chi\right)$$

$d\ln W/dw=(pw+qw^{\nu+1})^{-1}$, $w=U(\varphi)V(\chi)$, $e^{2b}=U^{-\nu}$

$$N=-\int_*^c\frac{pU}{M_p^2U_{,\varphi}}d\varphi-\int_*^c\frac{qV^{\nu+1}}{M_p^2V_{,\chi}}d\chi$$

More General Potential

$W=\lambda w^{\alpha}$, $w=U(\varphi)+V(\chi)$, $b=0$

Analytic Expression for Non-linear Parameter

$$
\begin{aligned}
-\frac{6}{5}f_{\rm NL}^{(4)} = & \frac{2}{\alpha}\left(\frac{u^2}{\epsilon_\varphi^*}+\frac{v^2}{\epsilon_\chi^*}\right)^{-2}\left\{\frac{u^2}{\epsilon_\varphi^*}\left[\left(1-\frac{\eta_{\varphi\varphi}^*}{2\epsilon_\varphi^*}\right)\alpha u+v\right]\right. \\
& \left.+\frac{v^2}{\epsilon_\chi^*}\left[\left(1-\frac{\eta_{\chi\chi}^*}{2\epsilon_\chi^*}\right)\alpha v+u\right]+\left(\frac{u}{\epsilon_\varphi^*}+\frac{v}{\epsilon_\chi^*}\right)^2\alpha^2\mathcal{A}\right\}.
\end{aligned}
$$

$$
\mathcal{A}=-\frac{w^{c2}}{w^{*2}}\frac{\epsilon_\varphi^c\epsilon_\chi^c}{\alpha^2\epsilon^c}\left[1+\frac{4(\alpha-1)\epsilon_\varphi^c\epsilon_\chi^c}{\epsilon^{c2}}-\frac{\alpha(\epsilon_\chi^c\eta_{\varphi\varphi}^c+\epsilon_\varphi^c\eta_{\chi\chi}^c)}{\epsilon^{c2}}\right],
$$

$$
u=\frac{U^*+\alpha Z^c}{w^*},\quad v=\frac{V^*-\alpha Z^c}{w^*},\quad Z=\frac{U_{,\varphi}^2V-UV_{,\chi}^2}{\alpha(U_{,\varphi}^2+V_{,\chi}^2)}.
$$

$W=\lambda w^{\alpha}, w=U(\varphi)+V(\chi), b=0$

Analytic Expression for Non-linear Parameter

$$
\begin{aligned}
-\frac{6}{5}f_{\mathrm{NL}}^{(4)} = \; & \frac{2}{\alpha}\left(\frac{u^2}{\epsilon_\varphi^*}+\frac{v^2}{\epsilon_\chi^*}\right)^{-2}\left\{\frac{u^2}{\epsilon_\varphi^*}\left[\left(1-\frac{\eta_{\varphi\varphi}^*}{2\epsilon_\varphi^*}\right)\alpha u+v\right]\right. \\
& \left. +\frac{v^2}{\epsilon_\chi^*}\left[\left(1-\frac{\eta_{\chi\chi}^*}{2\epsilon_\chi^*}\right)\alpha v+u\right]+\left(\frac{u}{\epsilon_\varphi^*}+\frac{v}{\epsilon_\chi^*}\right)^2\alpha^2\mathcal{A}\right\}.
\end{aligned}
$$

$$
\mathcal{A}=-\frac{w^{c2}}{w^{*2}}\frac{\epsilon_\varphi^c\epsilon_\chi^c}{\alpha^2\epsilon^c}\left[1+\frac{4(\alpha-1)\epsilon_\varphi^c\epsilon_\chi^c}{\epsilon^{c2}}-\frac{\alpha(\epsilon_\chi^c\eta_{\varphi\varphi}^c+\epsilon_\varphi^c\eta_{\chi\chi}^c)}{\epsilon^{c2}}\right],
$$

$$
u=\frac{U^*+\alpha Z^c}{w^*},\quad v=\frac{V^*-\alpha Z^c}{w^*},\quad Z=\frac{U_{,\varphi}^2V-UV_{,\chi}^2}{\alpha(U_{,\varphi}^2+V_{,\chi}^2)}.
$$

Action

$$\int d^4x\sqrt{-g}\left[\frac{1}{2}M_p^2R-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-\frac{1}{2}e^{2b(\varphi)}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-W(\varphi,\chi)\right]$$

TW, Phys. Rev. D **82**, 123515 (2010) arXiv:1008.3198 [astro-ph.CO].

$W=\lambda w^\alpha$, $w=U(\varphi)+V(\chi)$, $b=0$

$$N=-\frac{1}{\alpha M_p^2}\left(\int_*^c\frac{U}{U_{,\varphi}}d\varphi+\int_*^c\frac{V}{V_{,\chi}}d\chi\right)$$

$d\ln W/dw=(pw+qw^{\nu+1})^{-1}$, $w=U(\varphi)V(\chi)$, $e^{2b}=U^{-\nu}$

$$N=-\int_*^c\frac{pU}{M_p^2U_{,\varphi}}d\varphi-\int_*^c\frac{qV^{\nu+1}}{M_p^2V_{,\chi}}d\chi$$

$W = \lambda w^\alpha$, $w = U(\varphi) + V(\chi)$, $b = 0$

Example

- Model: $\lambda = 1$, $\alpha = 2 \Rightarrow W = (\alpha\varphi^2 + \beta\chi^2)^2$, $b = 0$.
- Notation: $R = \alpha/\beta$.
- Assumptions: $N = 60$, $\epsilon_\varphi^c + \epsilon_\chi^c = 1$.

Action

$$\int d^4x\sqrt{-g}\left[\frac{1}{2}M_p^2R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}e^{2b(\varphi)}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - W(\varphi,\chi)\right]$$

Slow-roll Parameters

$$\epsilon_\varphi = \frac{M_p^2}{2}\left(\frac{W_{,\varphi}}{W}\right)^2, \qquad \epsilon_\chi = \frac{M_p^2}{2}\left(\frac{W_{,\chi}}{W}\right)^2 e^{-2b},$$

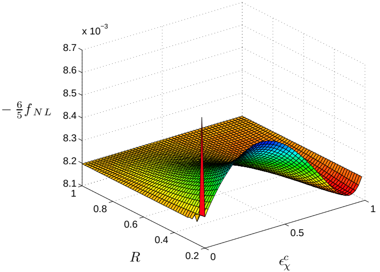
$$\epsilon = -\frac{\dot{H}}{H^2} = \epsilon_\varphi + \epsilon_\chi, \qquad \epsilon_b = 8M_p^2 b_{,\varphi}^2,$$

$$\eta_{\varphi\varphi} = \frac{M_p^2 W_{,\varphi\varphi}}{W}, \qquad \eta_{\chi\chi} = \frac{M_p^2 W_{,\chi\chi}}{W}e^{-2b},$$

$$\eta_{\varphi\chi} = \frac{M_p^2 W_{,\varphi\chi}}{W}e^{-b}.$$

Examples

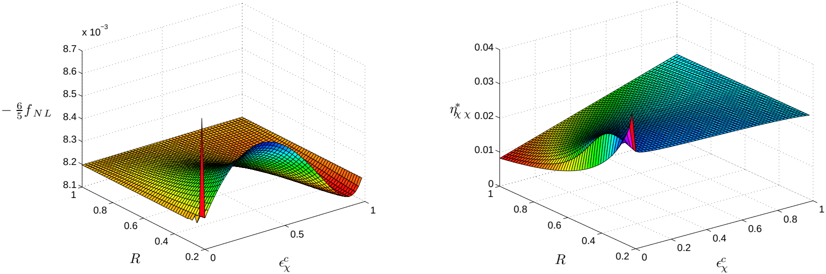
$W = \lambda w^{\alpha}$, $w = U(\varphi) + V(\chi)$, $b = 0$



Example

- Model: $W = \left(\alpha\varphi^2 + \beta\chi^2\right)^2$, $b = 0$.
- Notation: $R = \alpha/\beta$.
- Assumptions: $N = 60$, $\epsilon_\varphi^c + \epsilon_\chi^c = 1$.

$W = \lambda w^{\alpha}, w = U(\varphi) + V(\chi), b = 0$



Example

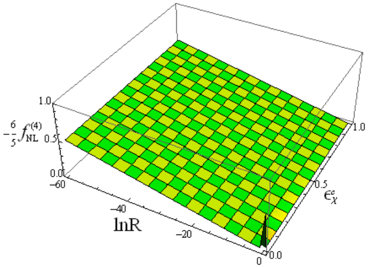
- Model: $W = (\alpha\varphi^2 + \beta\chi^2)^2$, $b = 0$.
- Notation: $R = \alpha/\beta$.
- Assumptions: $N = 60$, $\epsilon_\varphi^c + \epsilon_\chi^c = 1$.

$$d \ln W / dw = (pw + qw^{\nu+1})^{-1},\ w = U(\varphi)V(\chi),\ e^{2b} = U^{-\nu}$$

Example

- Model: $p = 0$, $\nu = q = -1 \Rightarrow W = \lambda e^{-\beta\varphi^2\chi^2}$, $e^{2b} = \alpha\varphi^2$.
- Notation: $R = (\epsilon_\chi^c \epsilon_\varphi^*)/(\epsilon_\varphi^c \epsilon_\chi^*)$.
- Assumptions: $N = 60$, $\epsilon_\varphi^c + \epsilon_\chi^c = 1$.

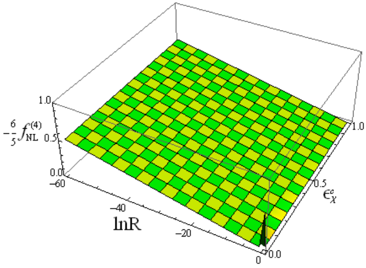
$d \ln W/dw = (pw + qw^{\nu+1})^{-1}$, $w = U(\varphi)V(\chi)$, $e^{2b} = U^{-\nu}$

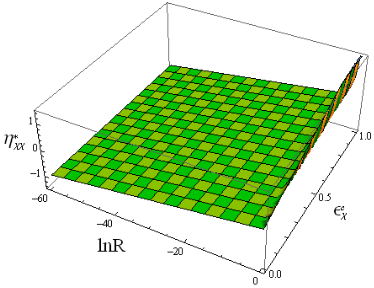


Example

- Model: $p = 0$, $\nu = q = -1 \Rightarrow W = \lambda e^{-\beta\varphi^2\chi^2}$, $e^{2b} = \alpha\varphi^2$.
- Notation: $R = (\epsilon_\chi^c \epsilon_\varphi^*)/(\epsilon_\varphi^c \epsilon_\chi^*)$.
- Assumptions: $N = 60$, $\epsilon_\varphi^c + \epsilon_\chi^c = 1$.

$d\ln W/dw = (pw + qw^{\nu+1})^{-1}$, $w = U(\varphi)V(\chi)$, $e^{2b} = U^{-\nu}$

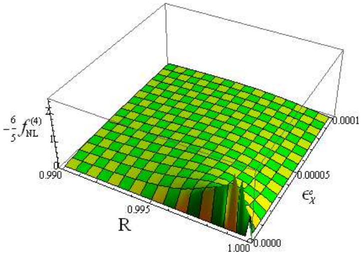




Example

- Model: $p = 0$, $\nu = q = -1 \Rightarrow W = \lambda e^{-\beta\varphi^2\chi^2}$, $e^{2b} = \alpha\varphi^2$.
- Notation: $R = (\epsilon^c_\chi \epsilon^*_\varphi)/(\epsilon^c_\varphi \epsilon^*_\chi)$.
- Assumptions: $N = 60$, $\epsilon^c_\varphi + \epsilon^c_\chi = 1$.

$d \ln W / dw = (pw + qw^{\nu+1})^{-1}$, $w = U(\varphi)V(\chi)$, $e^{2b} = U^{-\nu}$

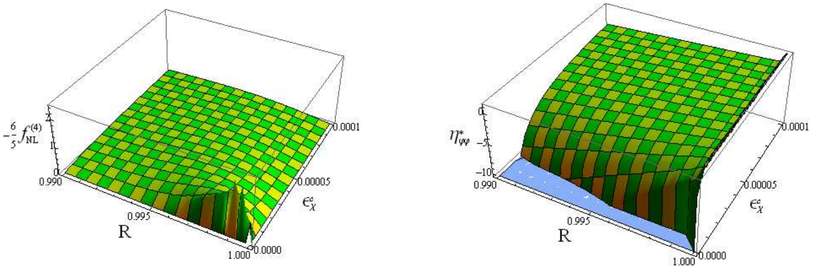


Example

- Model: $p = 0$, $\nu = q = -1 \Rightarrow W = \lambda e^{-\beta\varphi^2\chi^2}$, $e^{2b} = \alpha\varphi^2$.
- Notation: $R = (\epsilon^c_\chi \epsilon^*_\varphi)/(\epsilon^c_\varphi \epsilon^*_\chi)$.
- Assumptions: $N = 60$, $\epsilon^c_\varphi + \epsilon^c_\chi = 1$.

Examples

$d \ln W/dw = (pw + qw^{\nu+1})^{-1}$, $w = U(\varphi)V(\chi)$, $e^{2b} = U^{-\nu}$



Example

- Model: $p = 0$, $\nu = q = -1 \Rightarrow W = \lambda e^{-\beta\varphi^2\chi^2}$, $e^{2b} = \alpha\varphi^2$.
- Notation: $R = (\epsilon_\chi^c \epsilon_\varphi^*)/(\epsilon_\varphi^c \epsilon_\chi^*)$.
- Assumptions: $N = 60$, $\epsilon_\varphi^c + \epsilon_\chi^c = 1$.

Summary


Thank You...