厚膜上费米子的质量谱和共振态

中国科技大学交叉学科理论研究中心

2009.10.22

刘玉孝

(兰州大学理论物理研究所)

Outline

- Introduction
- The thick brane model
  - AdS thick brane
  - dS thick brane
  - flat double brane
- Fermions on thick branes
  - Fermions on AdS thick brane
  - Fermions on dS thick brane
  - Fermions on flat double brane
- Conclusion

Introduction

- KK theory (Kaluza and Klein 1920's).
- Domain walls (Rubakov and Shaposhnikov etc. 1980's)
- Branes in String/M theory (J.X. Lu etc. 1995)
- Large extra dimensions (Arkani-Hamed etc. 1998)
- Warped extra dimensions (RS braneworld: Randall and Sundrum 1999)

Introduction

- Our universe is a braneworld embedded in a higher dimensional space-time
- The suggestion can provide new insights for solving gauge hierarchy problem and the cosmological constant problem
- Gravity is free to propagate in all dimensions
- While all the matter fields are confined to a 3–brane

Introduction

- **The localization problem of fermions on branes is interesting and important**

For example, both **the fermion mass hierarchy problem** and **the family problem** can be solved in a brane model with two extra dimensions [Z.-Q. Guo and B.-Q. Ma, JHEP**09**(2009)091]:

$$m_u = 0.64 \text{ MeV}, \quad m_c = 584.87 \text{ MeV}, \quad m_t = 136.48 \text{ GeV},$$
$$m_d = 2 \text{ MeV}, \quad m_s = 36.36 \text{ MeV}, \quad m_b = 2.278 \text{ GeV}.$$
$$m_e = 0.511 \text{ MeV}, \quad m_\mu = 105.229 \text{ MeV}, \quad m_\tau = 1849.15 \text{ MeV},$$
$$m_1 = 0.0019 \text{ eV}, \quad m_2 = 0.013 \text{ eV}, \quad m_3 = 0.05 \text{ eV}.$$

Experimental information on charged lepton masses is rather accurate [C. Amsler et al. [Particle Data Group], PLB**667**(2008)1]:

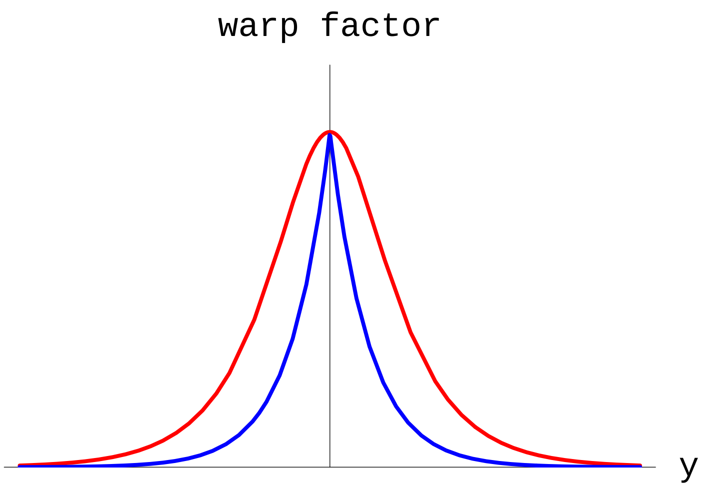
$$m_e = 0.510998902 \pm 0.000000021 \text{ MeV}, \ m_\mu = 105.658357 \pm 0.000005 \text{ MeV}, \ m_\tau = 1777.03^{+0.30}_{-0.26} \text{ MeV},$$

Introduction

For 5D problems, the line-element is usually assumed as

$$ds^2 = \mathrm{e}^{2A(y)}\hat{g}_{\mu\nu}(x)dx^\mu dx^\nu + dy^2, \quad (1)$$

where y is the coordinate of the extra dimension, $\mathrm{e}^{2A(y)}$ is warp factor.



Typical behavior of the warp factor and the energy density in **thin and thick** brane solutions.

Why thick? In more realistic models the thickness of the brane should be taken into account. In this scenario thick branes are realized by scalar field.

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The thick brane model

The action in 5D

$$S = \int d^4x dz \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\mathrm{g}^{MN}\nabla_M\phi\nabla_N\phi - V(\phi)\right], \qquad (2)$$

The line-element is assumed as

$$ds^2 = \mathrm{e}^{2A(y)}\hat{g}_{\mu\nu}(x)dx^\mu dx^\nu + dy^2 = e^{2A(z)}\left(\hat{g}_{\mu\nu}(x)dx^\mu dx^\nu + dz^2\right),$$

where e^{2A} is the warp factor. We suppose that $\hat{g}_{\mu\nu}$ is some general 4-dimensional metric such that $\hat{G}_{\mu\nu} = \Lambda\hat{g}_{\mu\nu}$. $\Lambda > 0$, $\Lambda < 0$ and $\Lambda = 0$ correspond to dS, AdS and Minkowski brane cosmologies, respectively.

The thick brane model

Assume $\phi = \phi(z)$. The field equations

$$\phi'^2 = 3(A'^2 - A'' - \Lambda), \quad (3)$$

$$V(\phi) = \frac{3}{2}\mathrm{e}^{-2A}(-3A'^2 - A'' + 3\Lambda), \quad (4)$$

$$\frac{dV(\phi)}{d\phi} = e^{-2A}(3A'\phi' + \phi''). \quad (5)$$

AdS thick brane

An AdS thick brane solution was found in [JHEP04(2007)062]:

$$e^{A(z)} = \frac{\sqrt{a}(a+b)\sec^2\left(\sqrt{a(a+b)}\,cz\right)}{\sqrt{a+(a+b)\tan^2\left(\sqrt{a(a+b)}\,cz\right)}}, \quad (6)$$

$$\phi = \sqrt{3}\arctan\left(\sqrt{\frac{a+b}{a}}\tan\left(\sqrt{a(a+b)}\,cz\right)\right)+\sqrt{3}acz, \quad (7)$$

where $-z_0 < z < z_0$ with z_0 defined by $z_0 = \frac{\pi}{2c}\sqrt{\frac{1}{a(a+b)}}$, and $\Lambda = -3c^2(5a^2+4ab) < 0$.

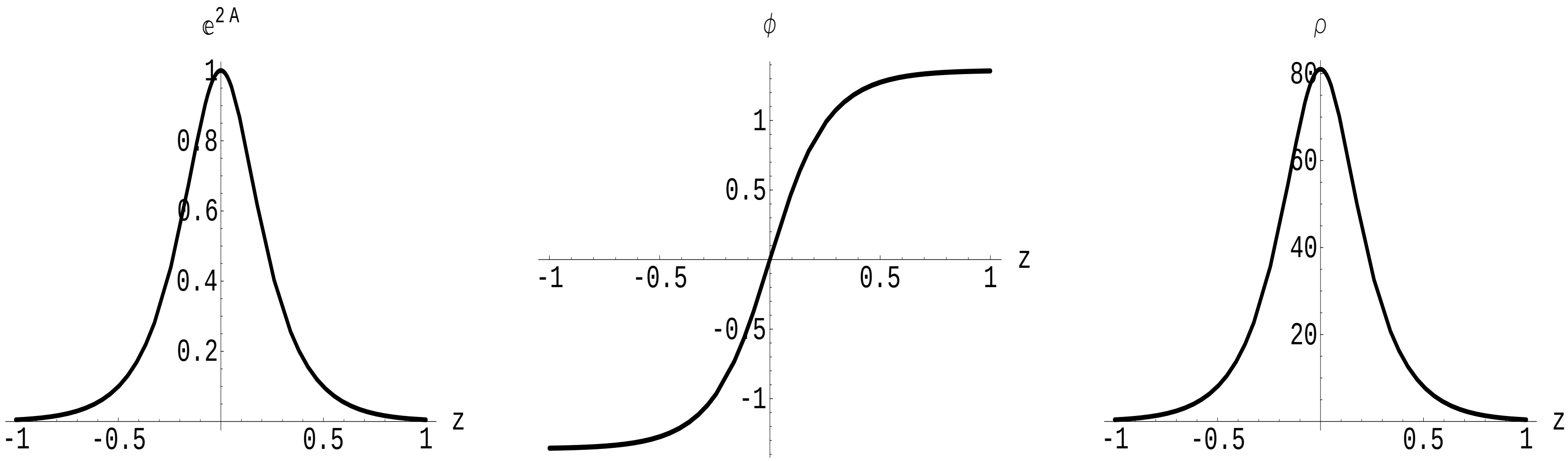
dS thick brane

A dS thick brane in 5D for the potential

$$V(\phi) = V_0\Big(\cos\phi/\phi_0\Big)^{2(1-\delta)} \tag{8}$$

was found in [JMP31(1990)2683,PRD60(1999)065011,PRD66(2002)024024]:

$$\begin{aligned} e^{2A} &= \cosh^{-2\delta}\left(\frac{\sqrt{\Lambda}}{\delta}z\right), \\ \phi &= \phi_0 \arctan\Big(\sinh\frac{\sqrt{\Lambda}}{\delta}z\Big). \end{aligned} \quad \left(0<\delta\le\frac{1}{2}, \Lambda>0\right) \tag{9}$$



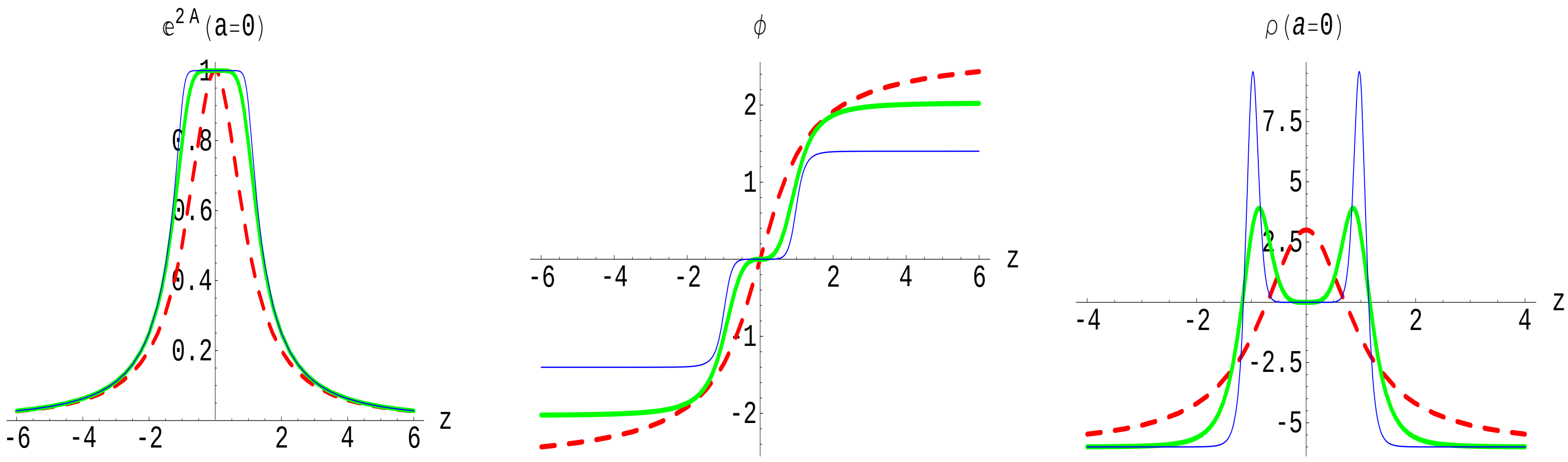
Flat double brane

A flat double brane solution for the potential

$$V(\phi) = \frac{3}{2}\lambda^2 \sin^{2-\frac{2}{s}}(\phi/\phi_0)\cos^2(\phi/\phi_0)\left[2s - 1 - 4\tan^2(\phi/\phi_0)\right], \quad (10)$$

is [PRD65(2002)084013],PRD67(2003)105003

$$\mathrm{e}^{2A} = \left[1 + (\lambda z)^{2s}\right]^{-1/s}, \qquad \phi = \phi_0 \arctan(\lambda z)^s. \qquad (11)$$



$s = 1, 3, 7$ for red, green, and blue lines, respectively.

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Fermions on thick branes

The Dirac action

$$S_{\frac{1}{2}} = \int d^5x\sqrt{-g}\left\{\bar{\Psi}\Gamma^M(\partial_M + \omega_M)\Psi - \eta\bar{\Psi}F(\phi)\Psi\right\},$$

where $\{\Gamma^M, \Gamma^N\} = 2g^{MN}$,

$$\Gamma^M = e^M{}_{\bar{M}}\Gamma^{\bar{M}} = \mathrm{e}^{-A}(\hat{e}^\mu{}_{\bar{\nu}}\gamma^{\bar{\nu}}, \gamma^5) = \mathrm{e}^{-A}(\gamma^\mu, \gamma^5). \quad (12)$$

The non-vanishing components of ω_M are

$$\omega_\mu = \frac{1}{2}(\partial_z A)\gamma_\mu\gamma_5 + \hat{\omega}_\mu. \quad (13)$$

Fermions on thick branes

The Dirac equation is

$$\left\{\gamma^\mu(\partial_\mu + \hat{\omega}_\mu) + \gamma^5 \left(\partial_z + 2\partial_z A\right) - \eta \, \mathrm{e}^A F(\phi)\right\} \Psi = 0, \qquad (14)$$

where $\gamma^\mu(\partial_\mu + \hat{\omega}_\mu)$ is the Dirac operator on the brane.

Making the chiral decomposition

$$\Psi(x,z) = \mathrm{e}^{-2A} \left(\sum_n \psi_{Ln}(x) f_{Ln}(z) + \sum_n \psi_{Rn}(x) f_{Rn}(z) \right), \qquad (15)$$

where $\psi_{Ln} = -\gamma^5 \psi_{Ln}$ and $\psi_{Rn} = \gamma^5 \psi_{Rn}$ are the left-handed and right-handed components of a 4D Dirac field,

Fermions on thick branes

and imposing the orthonormality condition

$$\int_{-z_0}^{z_0} f_{Lm} f_{Ln} dz = \int_{-z_0}^{z_0} f_{Rm} f_{Rn} dz = \delta_{mn}, \int_{-z_0}^{z_0} f_{Lm} f_{Rn} dz = 0,$$

we get the effective action

$$S_{1/2} = \sum_n \int d^4x \sqrt{-\hat{g}} \ \left[\bar{\psi}_n \gamma^\mu (\partial_\mu + \hat{\omega}_\mu) \psi_n - \mu_n \bar{\psi}_n \psi_n\right], \qquad (16)$$

and the Schrödinger equation for left and right chiral fermions

$$\left[-\partial_z^2 + V_{L,R}(z)\right] f_n(z) = \mu_n^2 f_n(z), \qquad (17)$$

where

$$\begin{aligned} V_L(z) &= \left(\eta \, \mathrm{e}^A F(\phi)\right)^2 - \partial_z \left(\eta \, \mathrm{e}^A F(\phi)\right), \\ V_R(z) &= \left(\eta \, \mathrm{e}^A F(\phi)\right)^2 + \partial_z \left(\eta \, \mathrm{e}^A F(\phi)\right). \end{aligned} \qquad (18)$$

Fermions on thick branes

- **In order to localize left or right chiral fermions:**
  - **Need scalar-fermion coupling** $\eta\bar{\Psi}F(\phi)\Psi$
  - $V_L(z)$ **or** $V_R(z)$ **should have a minimum at the location of the brane**
- **The spectra are determined by** V_∞**.**
  - **1.** $V_{L,R} \to 0$**. Continuous, gapless.**
  - **2.** $V_{L,R} \to C > 0$**. Discrete** $+$ **continuous.**
  - **3.** $V_{L,R} \to \infty$**. Discrete.**

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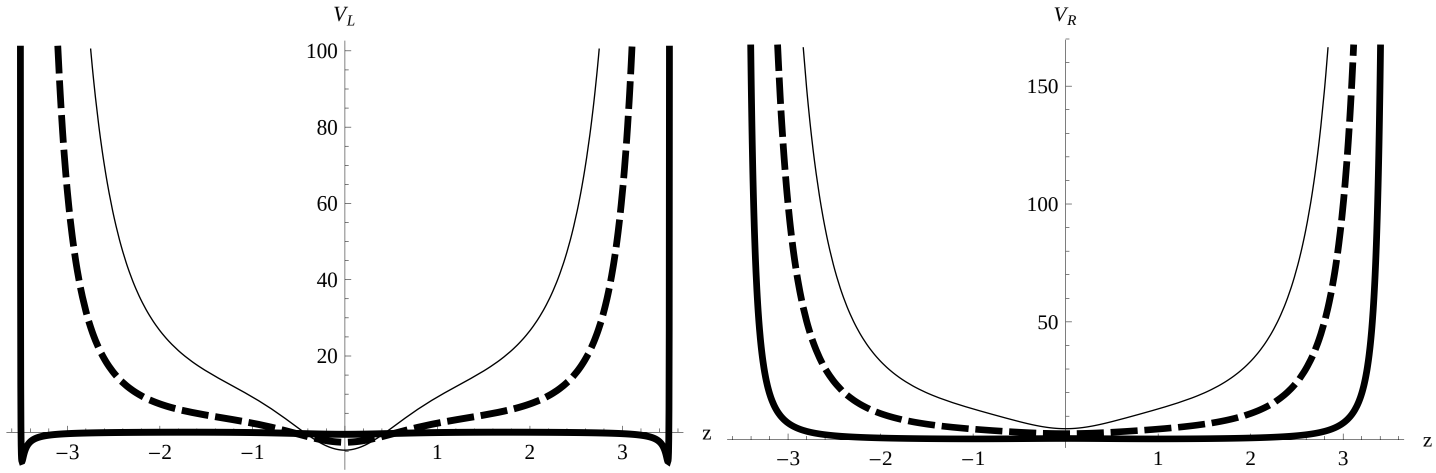
Fermions on AdS brane

[Y.-X. Liu, H. Guo et al, arXiv:0907.4424.]

$$V_L(z) = \frac{\sqrt{3a}(a+b)\eta\sec^2(\varrho cz)}{\sqrt{a+(a+b)\tan^2(\varrho cz)}}\Bigg\{-ac-\frac{\varrho c}{a+b\sin^2(\varrho cz)}\Big[\varrho + \Big(acz+\arctan\big[\frac{\varrho}{a}\tan(\varrho cz)\big]\Big)\times\big(a-b\cos(2\varrho cz)\tan(\varrho cz)\big)\Big]\Bigg\} + \frac{3\varrho^4\eta^2\left[acz+\arctan\left(\frac{\varrho}{a}\tan(\varrho cz)\right)\right]^2}{a\cos^2(\varrho cz)\big(a+b\sin^2(\varrho cz)\big)}, \quad (19)$$

$$V_R(z) = V_L(z)|_{\eta\to-\eta}, \quad (20)$$

The case $\eta > \eta_0$. $\eta_0 = \frac{2c}{\sqrt{3}\pi(1+\sqrt{a/(a+b)}\,)}$.

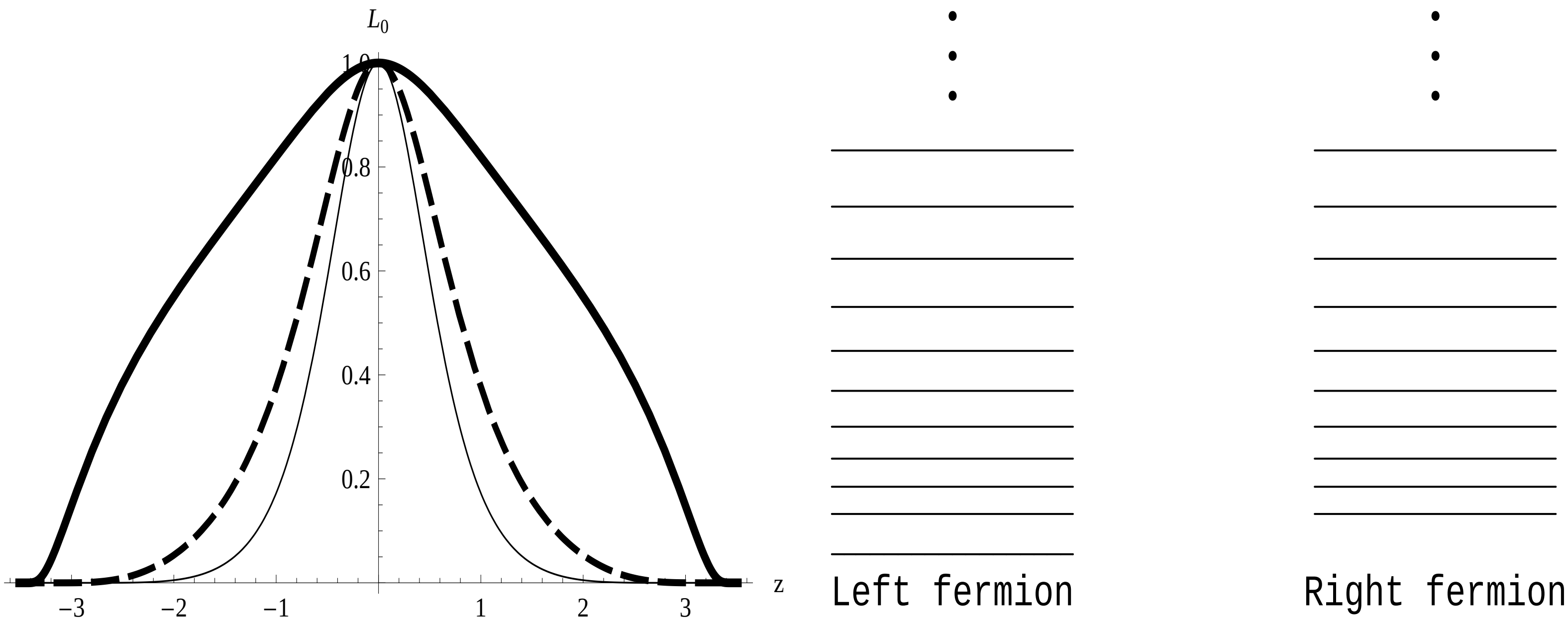


$\eta = 0.255$ (thick lines), $\eta = 1.254$ (dashed lines), $\eta = 2.254$ (thin lines).

Fermions on AdS brane

The case $\eta > \eta_0$: the zero mode of left-handed fermions

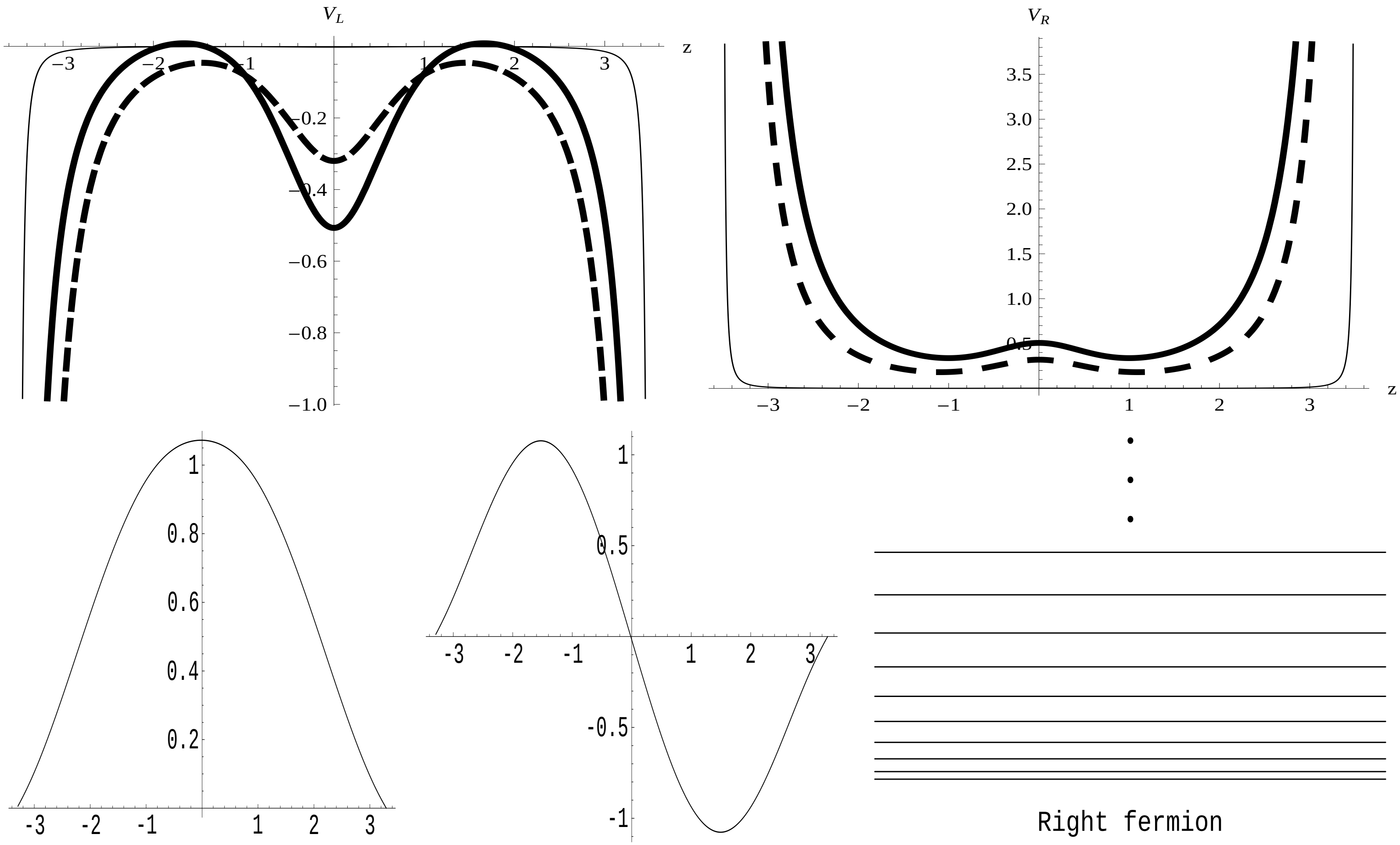
$$L_0 \equiv f_{L0}(z) \propto \exp\left(-\eta \int_0^z dz' \mathrm{e}^{A(z')}\phi(z')\right). \tag{21}$$



Zero mode mass spectra.

Fermions on AdS brane

The case $\eta < \eta_0$: No bound f_{Ln}.



f_{R1} f_{R2} μ^2_{Rn}

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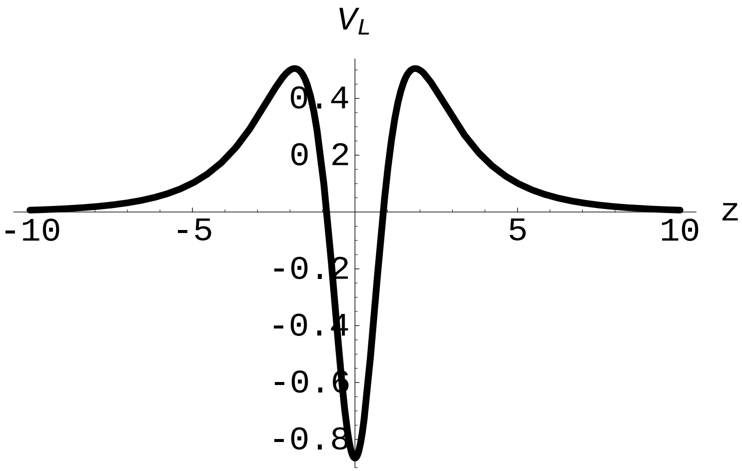
Fermions on dS brane

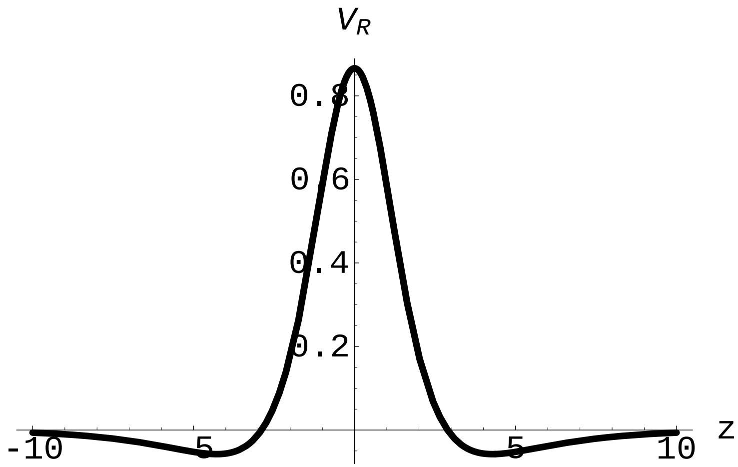
[Y.-X. Liu et al, PRD80(2009)065019, arXiv:0904.1785.]

$$V_L = \Lambda\phi_0^2 \cosh^{-2\delta}\left(\frac{\sqrt{\Lambda}z}{\delta}\right)\arctan^2\sinh\left(\frac{\sqrt{\Lambda}z}{\delta}\right)$$
$$+\frac{\eta\sqrt{\Lambda}\phi_0}{\delta}\cosh^{-1-\delta}\left(\frac{\sqrt{\Lambda}z}{\delta}\right)\left[\delta\sinh\left(\frac{\sqrt{\Lambda}z}{\delta}\right)\arctan\sinh\left(\frac{\sqrt{\Lambda}z}{\delta}\right)-1\right],$$
$$V_R = V_L^S(z)|_{\eta\to-\eta}.$$

The zero mode f_{L0} is bound but not normalizable.

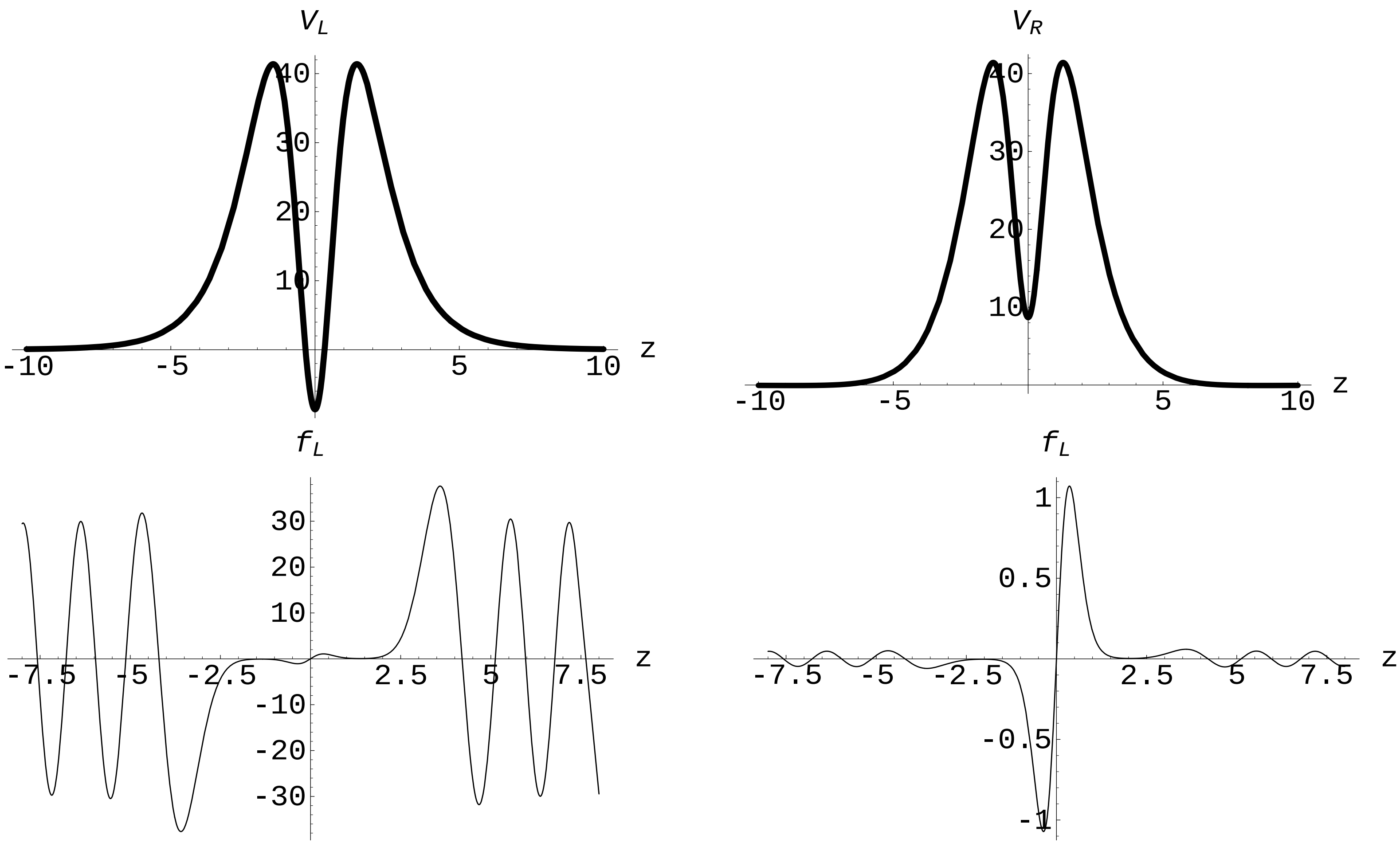
$\eta = 1$, no resonance states, $(\delta = \sqrt{\Lambda} = 1/2)$.





Fermions on dS brane

$\eta = 10$, have resonance states.

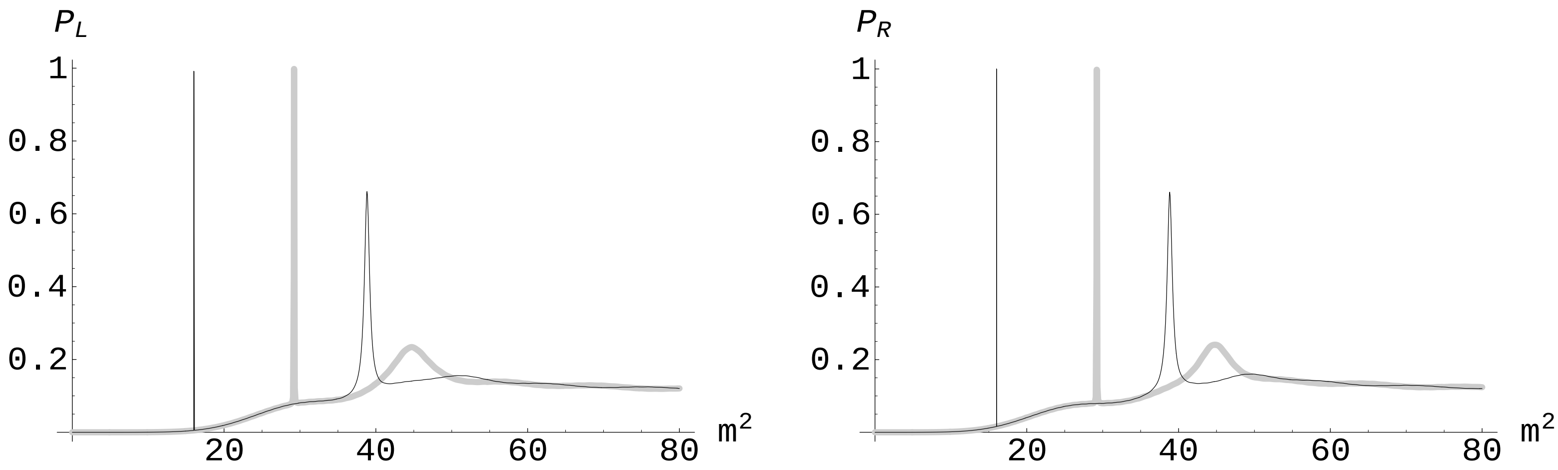


(a) $m^2 = 16$ (b) $m^2 = 16.01372$

Fermions on dS brane

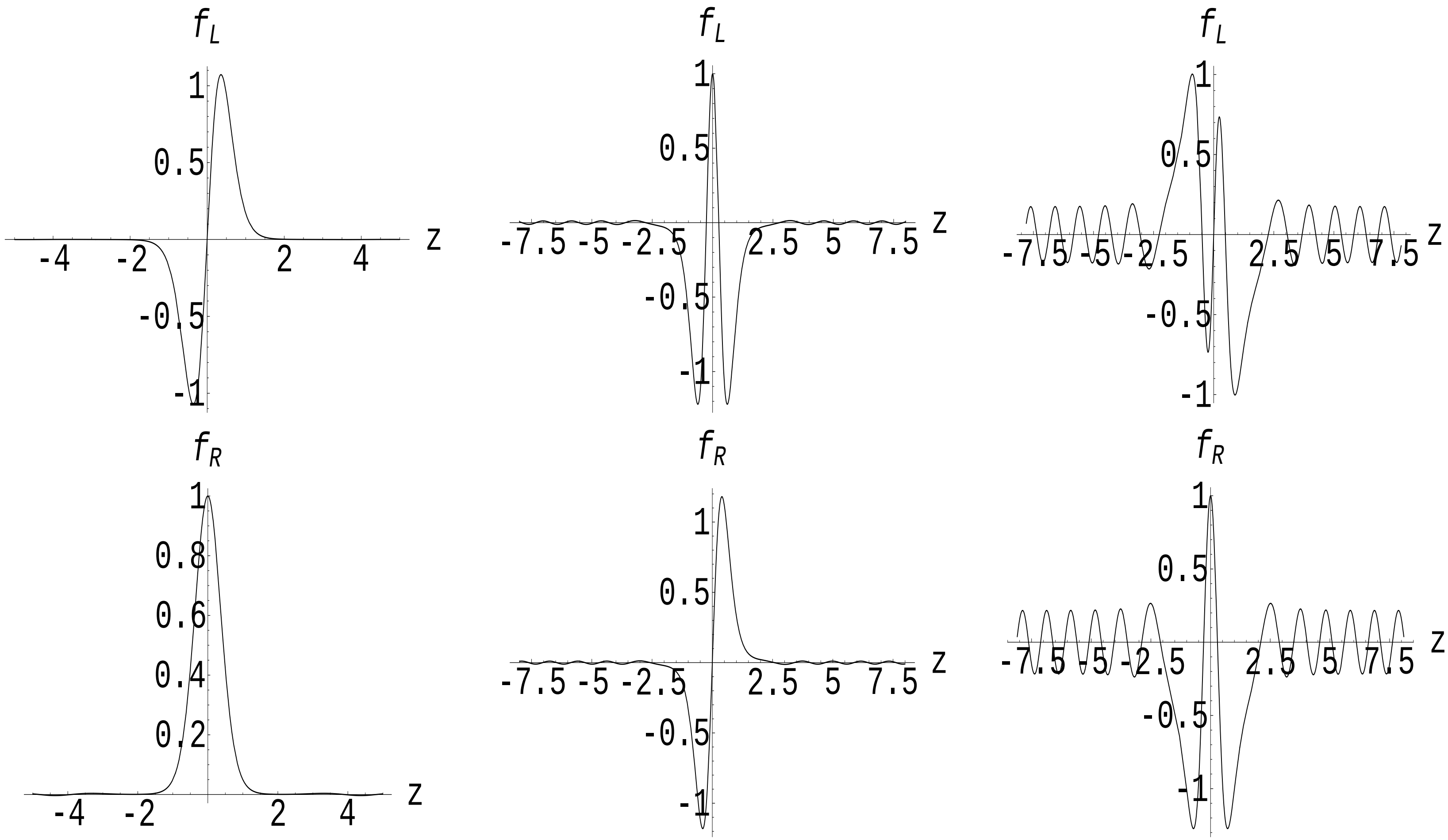
The relative probabilities are defined as follows:

$$P_{L,R}(m) = \frac{\int_{-z_b}^{z_b} |f_{L,R}(z)|^2 dz}{\int_{-z_{max}}^{z_{max}} |f_{L,R}(z)|^2 dz}, \tag{22}$$



Fermions on dS brane

Wavefunctions of the resonance states



$$n = 1, m^2 = 16.013742, \quad n = 2, m^2 = 29.22241, \quad n = 3, m^2 = 38.837314$$

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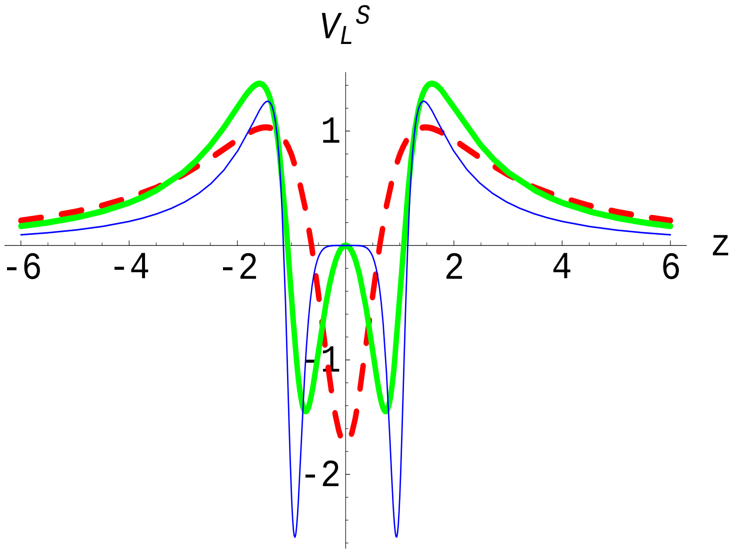
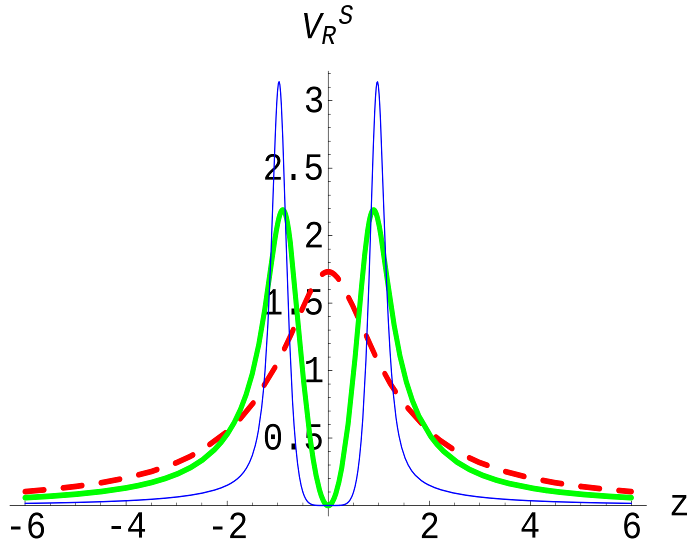
Fermions on flat double brane

Coupling 1: $F_1(\phi) = \phi$ [Y.-X. Liu et al, PRD80(2009)065020,arXiv:0907.0910.]

$$
\begin{aligned}
V_L(z) &= 3\eta^2 \frac{(2s-1)}{s^2} \frac{\arctan^2(\lambda^s z^s)}{[1+(\lambda z)^{2s}]^{\frac{1}{s}}} \\
&- \eta \frac{\sqrt{6s-3}}{s} \frac{(\lambda z)^s \left[s-(\lambda z)^s \arctan(\lambda^s z^s)\right]}{z[1+(\lambda z)^{2s}]^{1+\frac{1}{2s}}},
\end{aligned} \tag{23}
$$

The case $\eta = \lambda = 1$.





Fermions on flat double brane

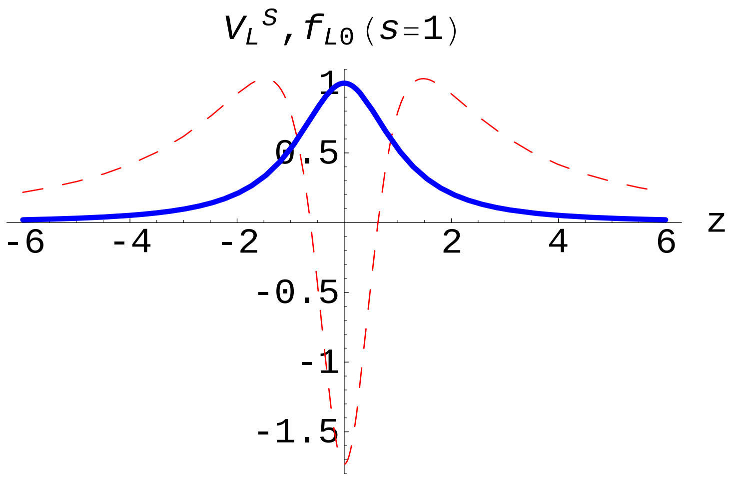
The zero mode

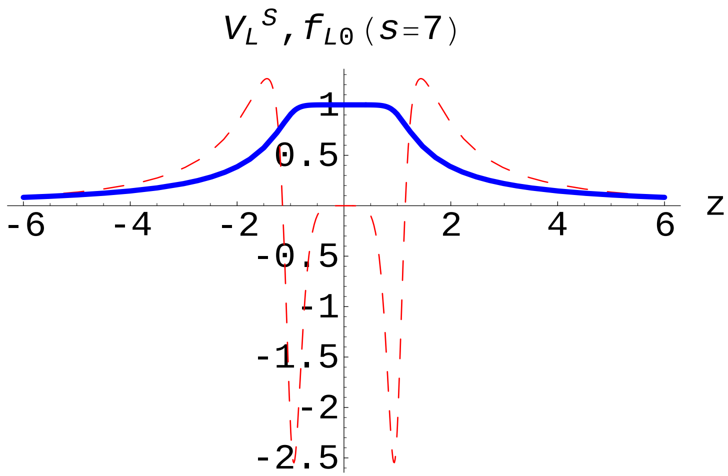
$$f_{L0}^2(z) \propto \exp\left(-2\eta \int^z d\bar{z} \mathrm{e}^{A(\bar{z})}\phi(\bar{z})\right) \rightarrow z^{-\eta/\eta_0}, \quad (24)$$

is normalized provided $\eta > \eta_0$, where

$$\eta_0 = \frac{s\lambda}{\sqrt{3(2s-1)}\,\pi}\left(1 + a\frac{\Gamma(1+1/2s)\Gamma(1/2s)}{\lambda\,\Gamma(1/s)}\right). \quad (25)$$

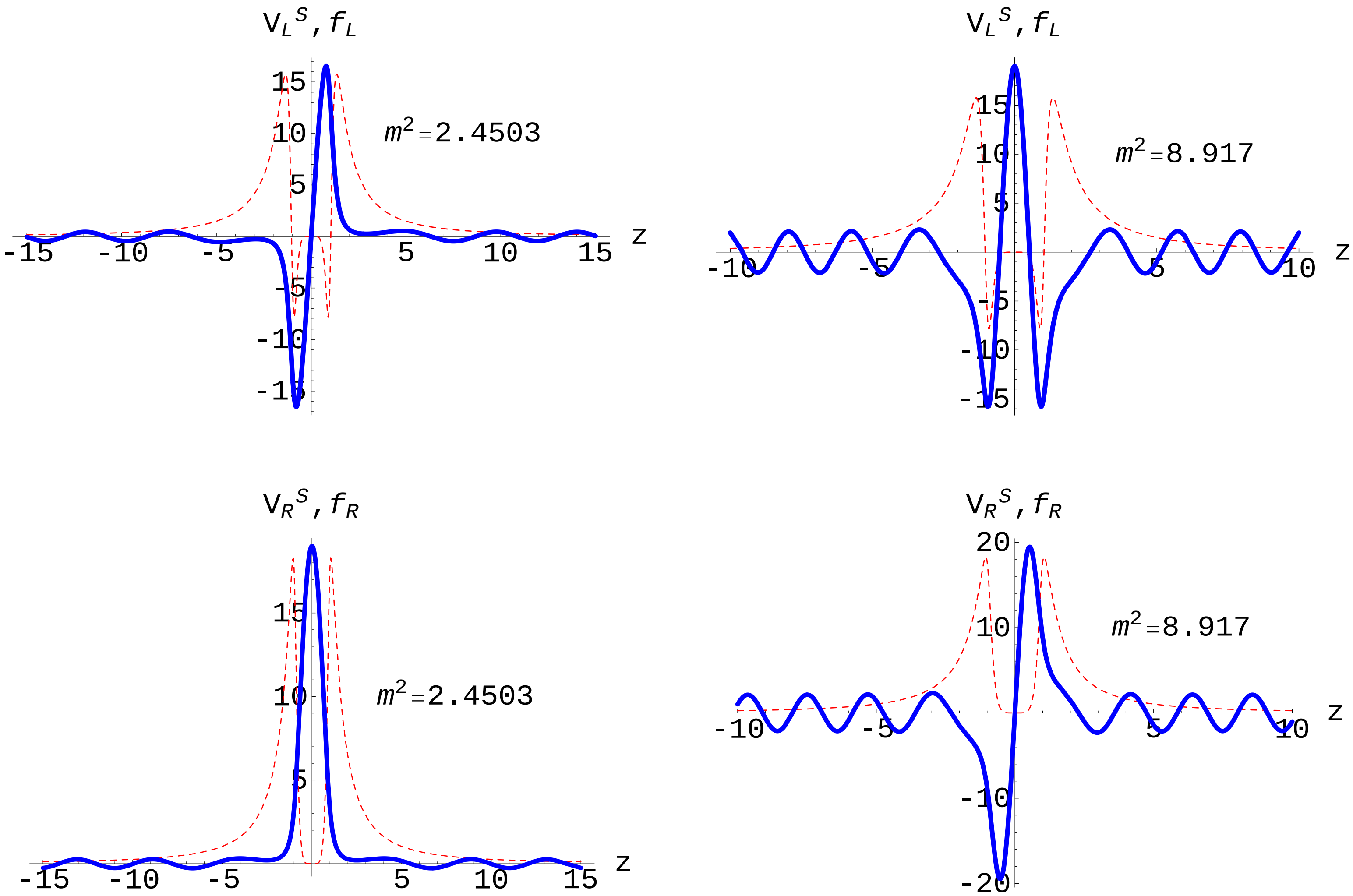
$\eta = \lambda = 1$, $s = 1$, $s = 7$.





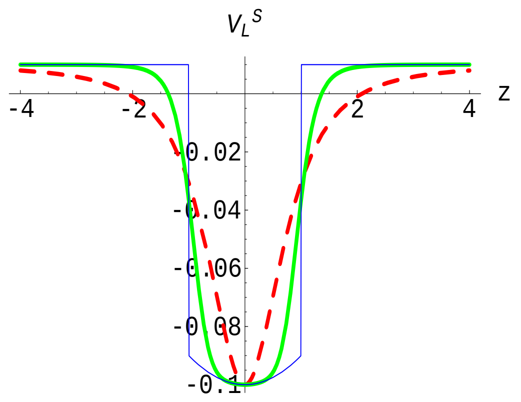
Fermions on flat double brane

Resonances $\eta = 4$, $\lambda = 1$, $s = 7$.

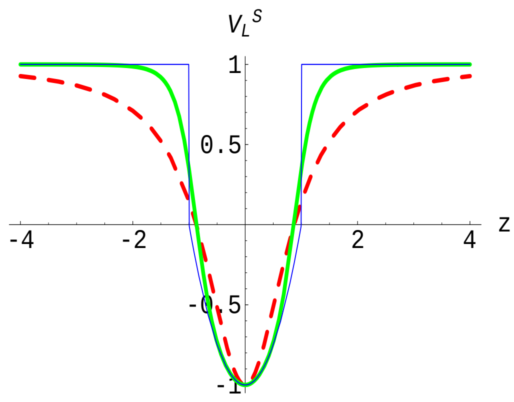


Fermions on flat double brane

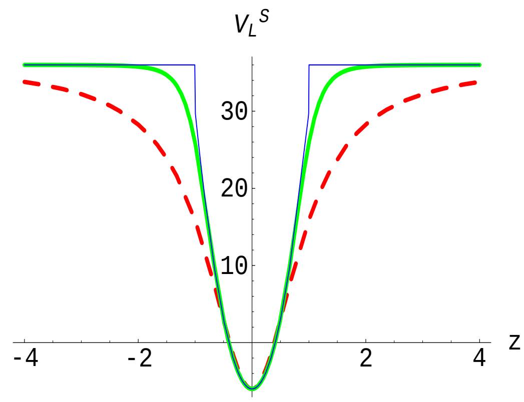
Coupling 2: $F_2(\phi) = \tan^{1/s}(\frac{\phi}{\phi_0})$, $\quad V_L(z) = \frac{\eta^2(\lambda z)^2}{[1+(\lambda z)^{2s}]^{\frac{1}{s}}} - \frac{\eta\lambda}{[1+(\lambda z)^{2s}]^{1+\frac{1}{2s}}}$ (26)



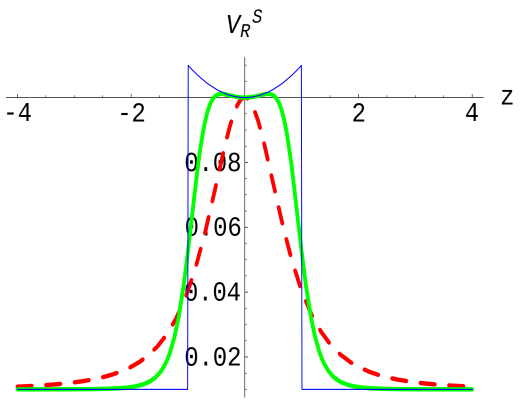
(a) $\eta = 0.1$



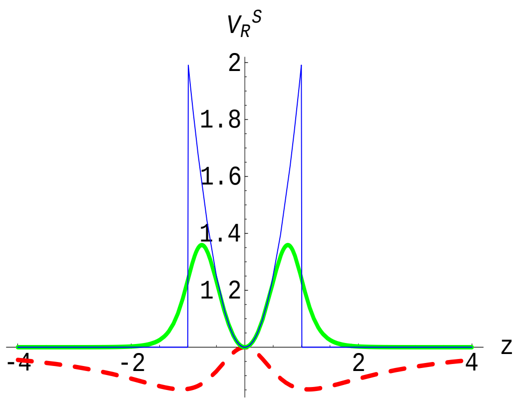
(b) $\eta = 1$



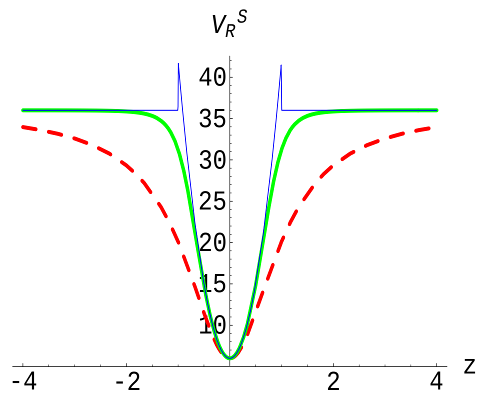
(c) $\eta = 6$



(d) $\eta = 0.1$



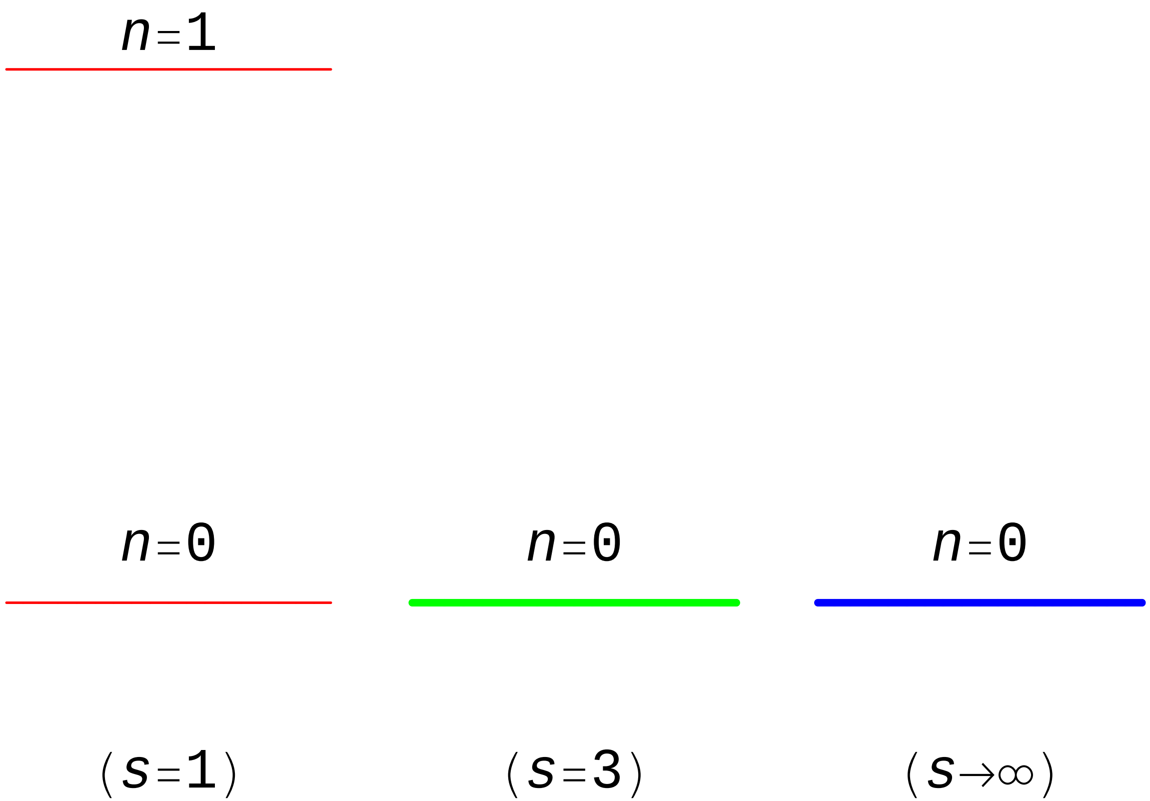
(e) $\eta = 1$



(f) $\eta = 6$

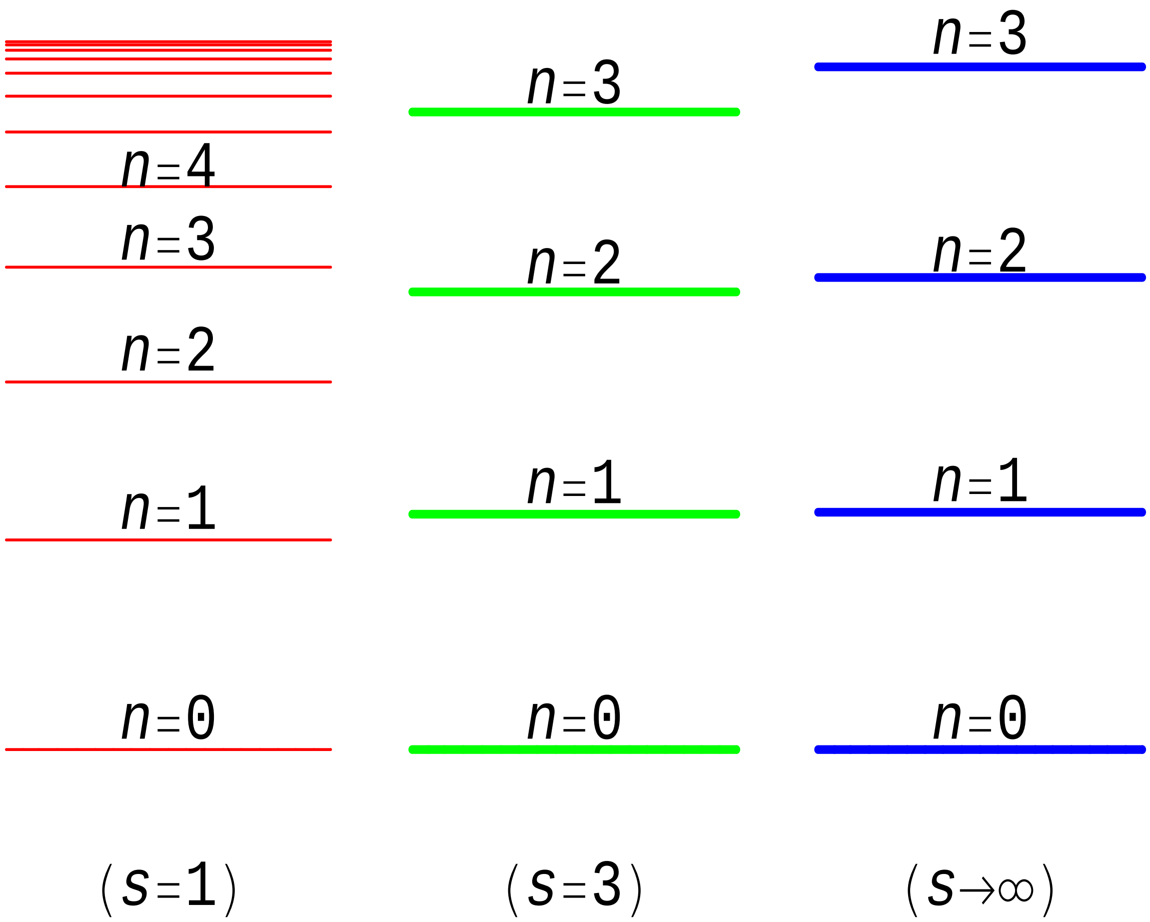
Fermions on flat double brane

Mass spectra m_{Ln}^2 with $\lambda = 1$, $\eta = 1$.



Fermions on flat double brane

Mass spectra m_{Ln}^2 with $\lambda = 1$, $\eta = 6$.



Conclusion

- Fermions on AdS thick brane ($\eta\bar{\Psi}\phi\Psi$):
  - Left fermion zero mode is normalizable ($\eta > \eta_0$)
  - Discrete spectra for left and right fermions ($\eta > \eta_0$)
  - Discrete spectra for right fermions ($0 < \eta < \eta_0$)
- Fermions on dS thick brane ($\eta\bar{\Psi}\phi\Psi$):
  - Left fermion zero mode is non-normalizable ($\eta > 0$)
  - Continuous gapless spectra
  - Resonances for large η

Conclusion

- Fermions on flat double thick brane ($\eta\bar{\Psi}\phi\Psi$):
  - Left fermion zero mode is normalizable ($\eta > \eta_0$)
  - Continuous gapless spectra
  - Resonances for large η
- Fermions on flat double thick brane ($\eta\bar{\Psi}\tan^{1/s}(\frac{\phi}{\phi_0})\Psi$):
  - Left fermion zero mode is normalizable ($\eta > 0$)
  - Discrete spectra for left and right fermions ($m^2 < \eta^2$)
  - Continuous spectra ($m^2 > \eta^2$)
  - No resonances

Thank you!