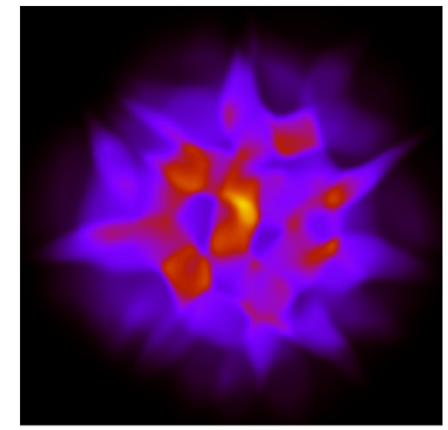
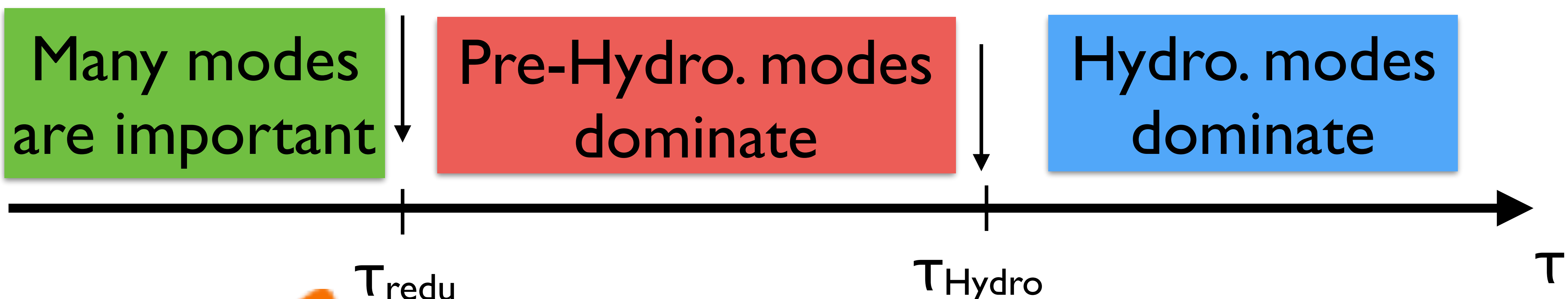
"Prehistory" of hydrodynamic modes in rapidly-expanding quark-gluon plasma

? ? ? ?





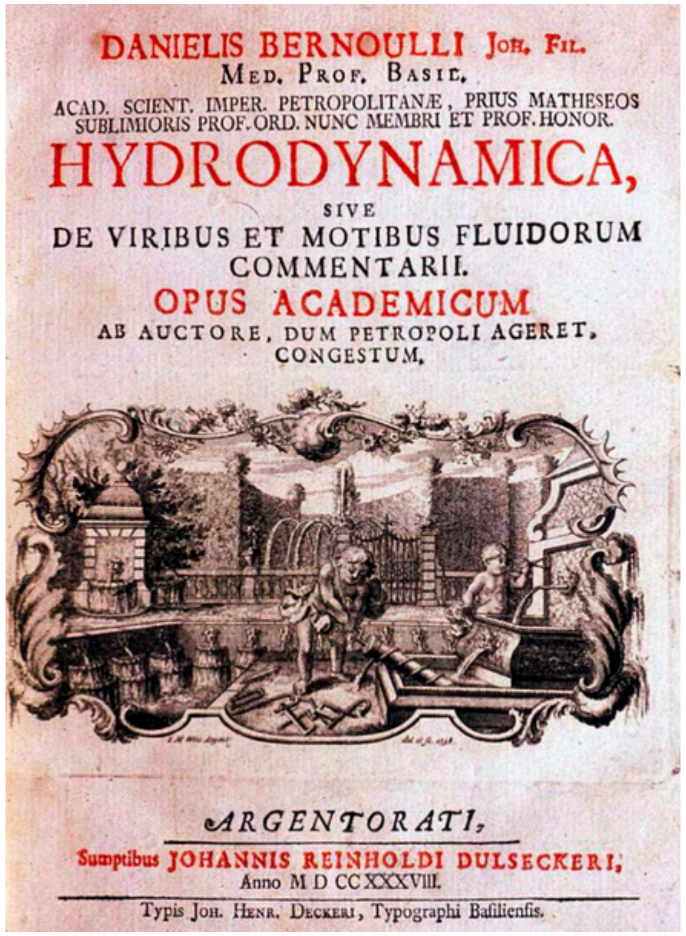
Yi Yin

with Jasmine Brewer (MIT) and Li Yan (Fudan U.), to be submitted

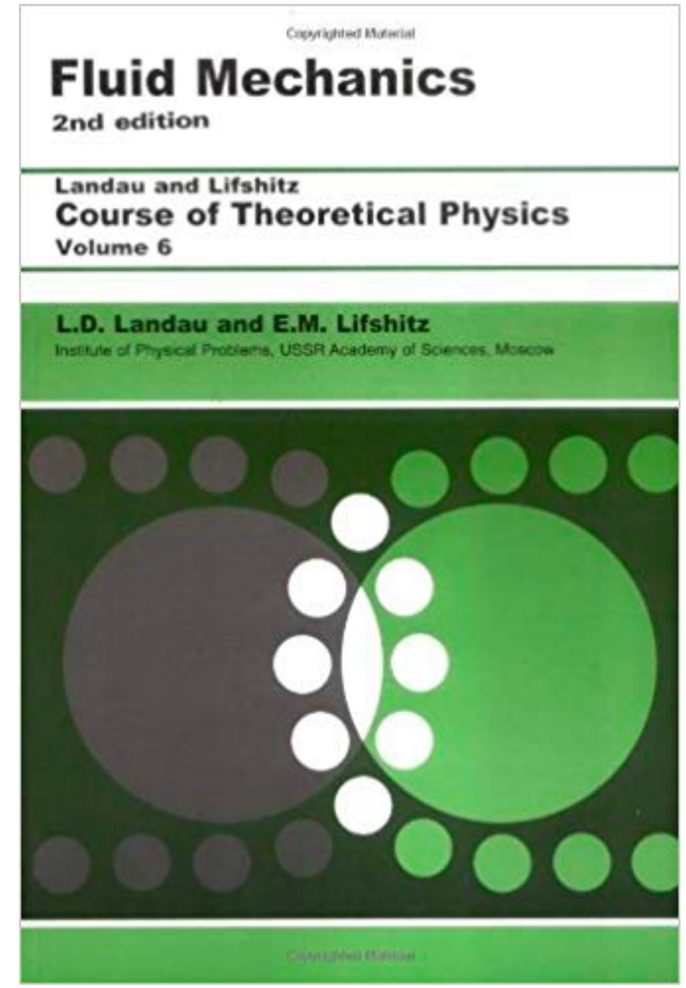
Brewer, Grad. of MIT

ICTS, USTC, Hefei, China, Sep.19, 2019

The development of hydrodynamics has a long history



Bernoulli, "Hydrodynamica", 1738

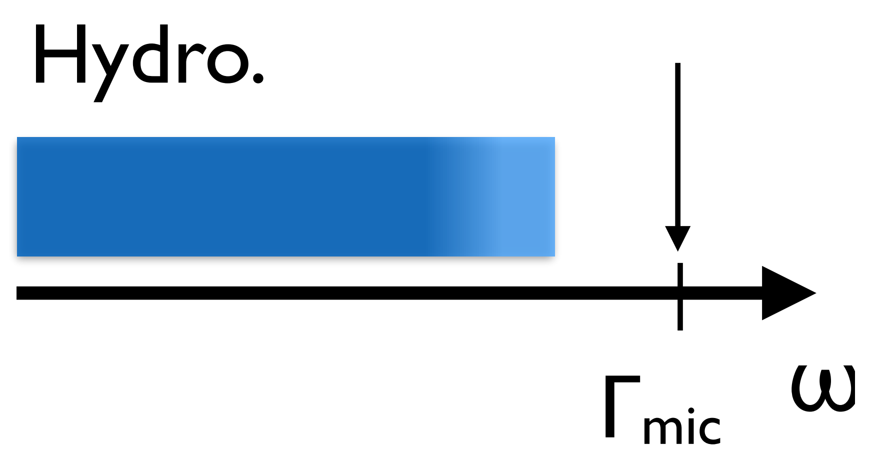


Landau-Lifshitz, "Fluid mechanics", 1950s

The modern view of hydro. : a low-energy effective theory

Hydro. can be considered as an effective theory of many-body interacting systems in long time and long wavelength limit.

Hydro. d.o.f.: conserved densities, e.g, energy density ε and momentum density (related to flow velocity u^μ) which evolves slowly near thermal equilibrium.



Small parameter: gradient times mean free path and/or frequency of hydro. modes times mean free time.

Hydro. equation: conservation laws together with the constitutive relation obtained by gradient expansion.

$$\partial_\mu T^{\mu\nu} = 0 . \qquad T^{\mu\nu} = \epsilon\, u^\mu u^\nu + p(\epsilon)\left(g^{\mu\nu} + u^\mu u^\nu\right) + \mathcal{O}(\partial)$$

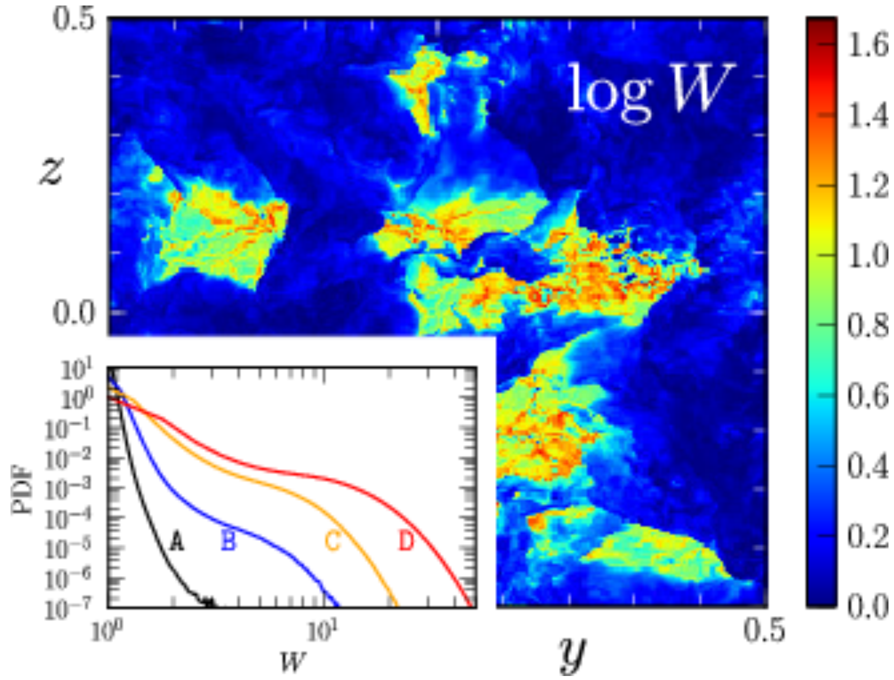
(Relativistic) Hydrodynamics: broad applications

Astrophysical scale:

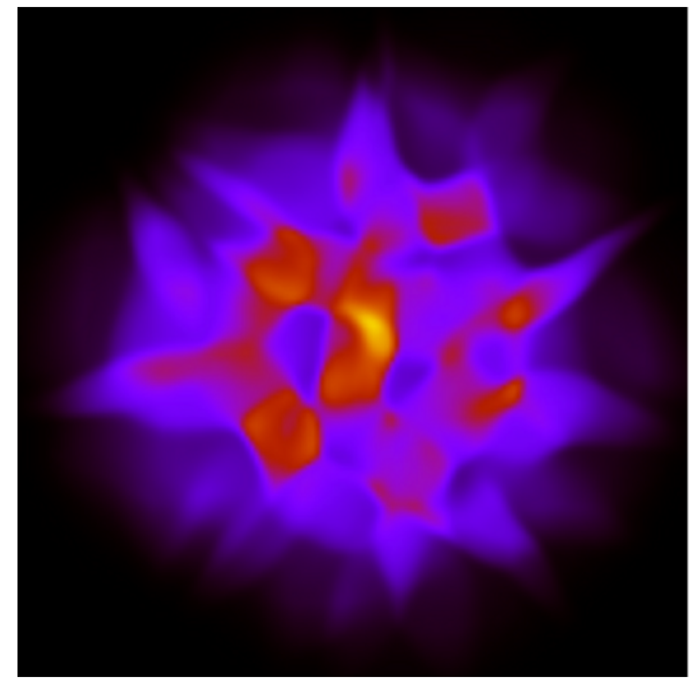
1 parsecs ~ 10^{17}m

Typical size of quark-gluon mater cerated in heavy-ion collisions:

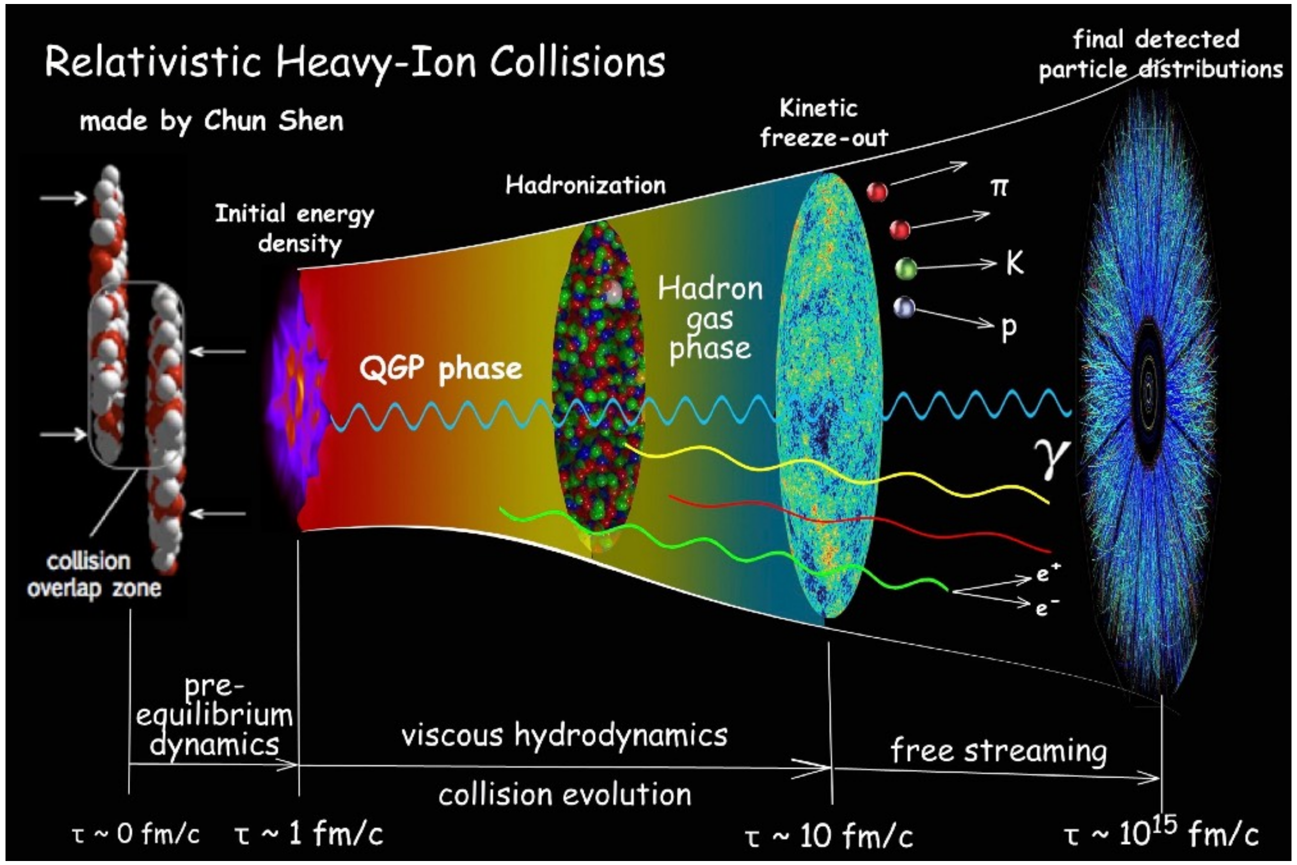
10 fm ~ 10^{-14} m

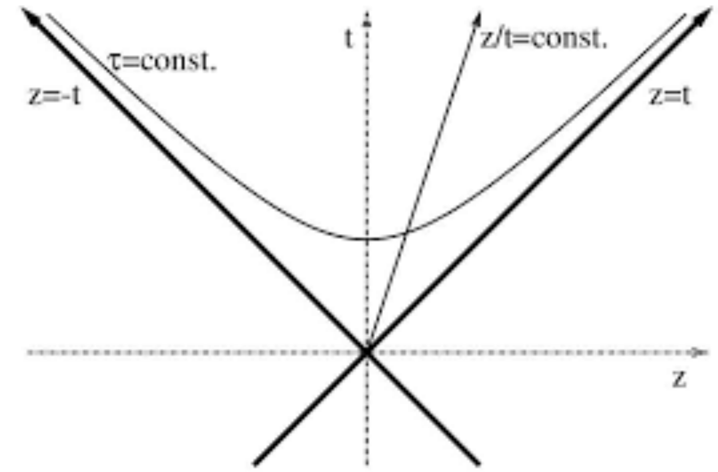


The turbulence of relativistic plasma from hydro. simulation(arXiv: 1209.2936v2)



Hydro. simulation for heavy-ion collisions (by Schenke)





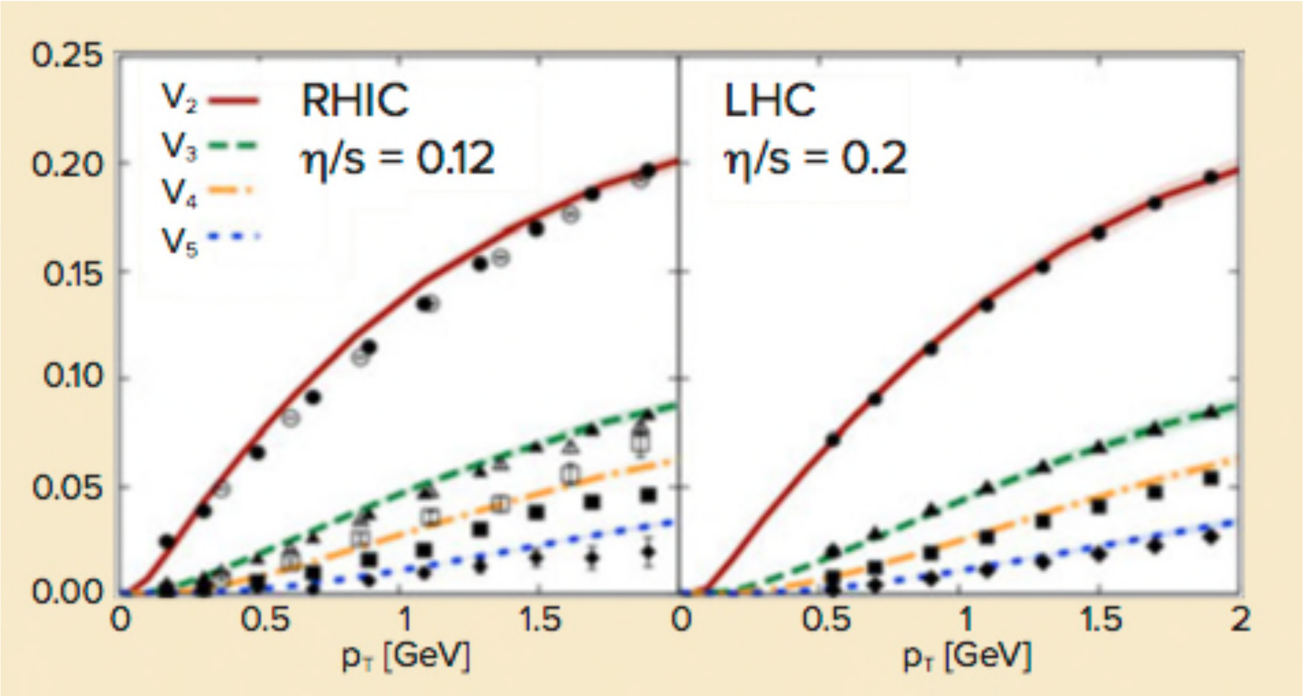
Bjorken time: $\tau = \sqrt{t^2 - z^2}$

Heavy-ion collisions (HIC) create extreme hot and dense quark-gluon matter.

Initially, the fireball created in HIC is highly non-equilibrium and an-isotropic, and expands rapidly along longitudinally direction.

Then, its subsequent evolution can be described by relativistic hydrodynamics.

Heavy-ion collisions creates QGP liquid



(Hadron spectrum vs hydro. simulations by McGill Group)

Hydro. provides a quantitative description of QGP evolution in heavy-ion collisions (HIC).

This means after some time scale, τ_{Hydro} , bulk evolution is dominant by conserved densities (e.g. energy density ε), i.e. hydrodynamic modes.

The discovery of QGP liquid in turn raises a number of deep and outstanding questions

RHIC Scientists Serve Up 'Perfect' Liquid

New state of matter more remarkable than predicted — raising many new questions

How does strongly coupled fluid emerge from the asymptotic free QCD as resolution scale increases?

How does the fluid-like behavior emerges from highly an-isotropic and non-equilibrium quark-gluon matter at early time? (this talk).

How does the properties of QGP liquid change as baryon density increases? (tomorrow's colloquium).

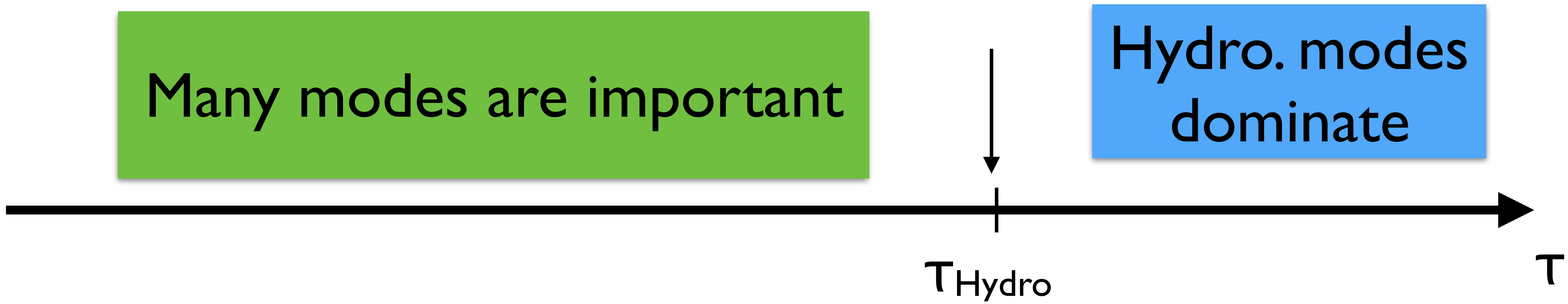
“Hydrodynamization” of QGP formed in heavy-ion collisions:

The “pre-history” of hydrodynamics has attracted much of the recent attention.

see Romatschke-Romatschke, 1712.05815; Florkowski et al, Rept. Prog. Phys, for recent review.

A significant part of the study is devoted to the problem of “hydrodynamization”: i.e. the process through which hydro. modes gain their dominance. (NB: the terminology “hydrodynamization” seems be invented by researchers in HIC community, though its physics is relevant in broad settings).

The common view of "hydrodynamization"



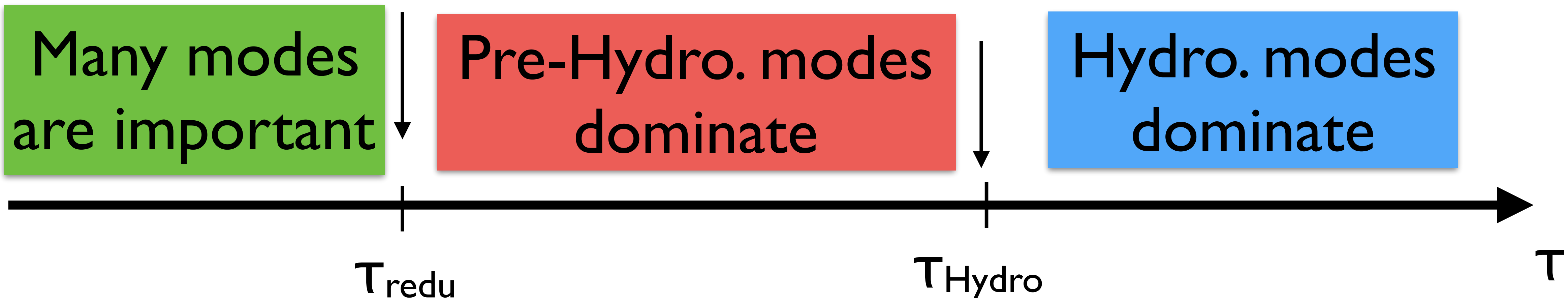
Conventional scenario: hydrodynamization as the decay of non-hydrodynamic modes.

For example, the decay of disturbances for fluids in a slow varying environment.

Note, this scenario does not rely on specifics of the underlying microscopic theory.

We propose a new hydrodynamization scenario in which the longitudinal expansion and some intrinsic hierarchy in time scale of QCD matter at high energy density regimes play an essential role.

A new scenario: "adiabatic hydrodynamization"

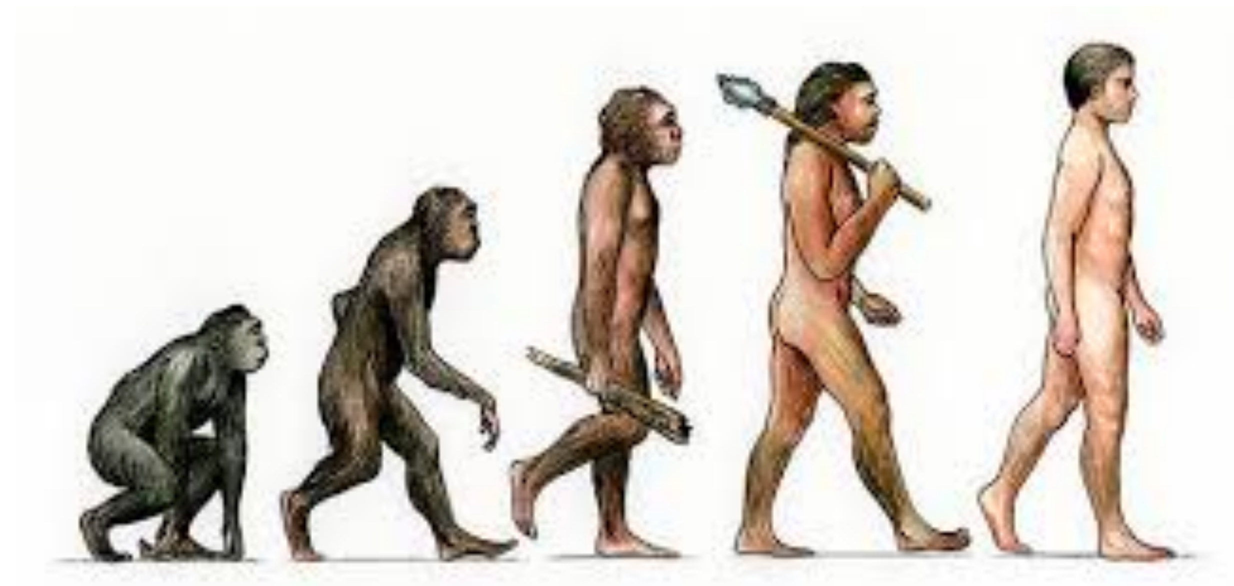


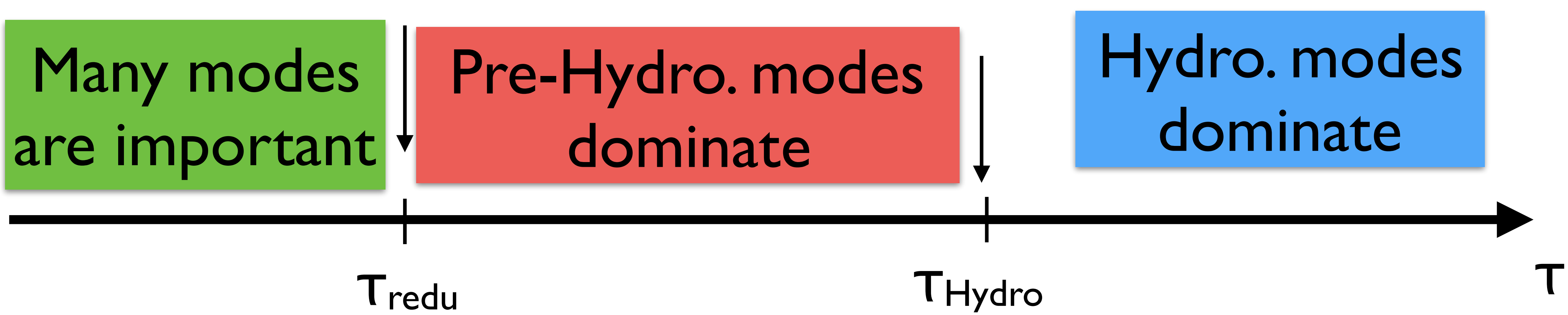
During adiabatic hydrodynamization, the evolution is governed by a set of slow modes that are equal in number to, but qualitatively distinct from, hydrodynamic modes. These modes change gradually as the system expands and evolve into hydrodynamic modes in the hydrodynamic limit.

τ_{redu} is an emergent scale that is parametrically shorter than τ_{Hydro}.

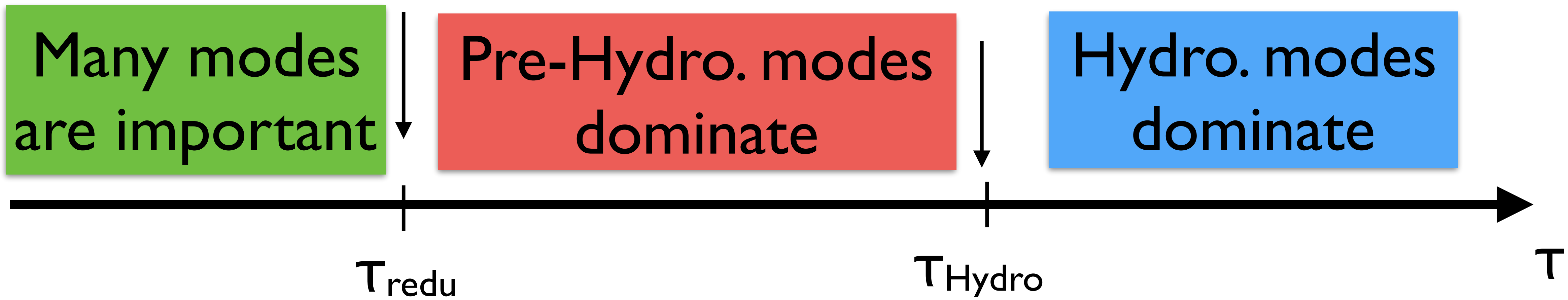
The pre-history of hydrodynamic modes is (almost) the history of pre-hydrodynamic modes!

A cartoonish summary: pre-hydro. modes c.f. pre-existing type of human.





Outline for the remainder of this talk



1. The kinetic theory and the identification of pre-hydrodynamic modes.

2. Longitudinal expansion and the emergence of τ_{redu} .

3. The demonstration of adiabatic hydrodynamization.

Summary and outlook.

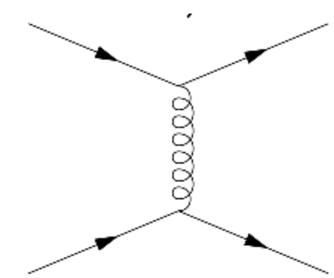
The kinetic theory and the identification of pre-hyrodynamic modes

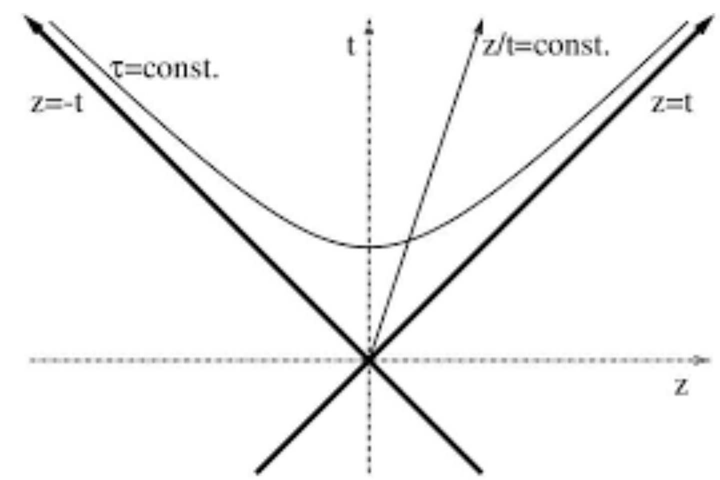
The microscopic model: kinetic theory

We use kinetic equation to describe the evolution of single particle distribution of gluons (quarks).

$$\partial_\tau f(\overrightarrow{p},\tau) = -\frac{p_z}{\tau}\partial_{p_z} f(\overrightarrow{p},\tau) - \hat{C}[f]$$

$$\partial_\tau p_z = -p_z/\tau$$





Simplification: assuming boost-invariance and homogeneity in transverse plane.

The kinetic theory model captures the physics of the transition from free-streaming quark-gluon gas to quark-gluon liquid.

Limitation: assuming weak coupling throughout the evolution.

See Florkowski et al, Rept. Prog. Phys for references on studies based on holography.

Angle distribution:

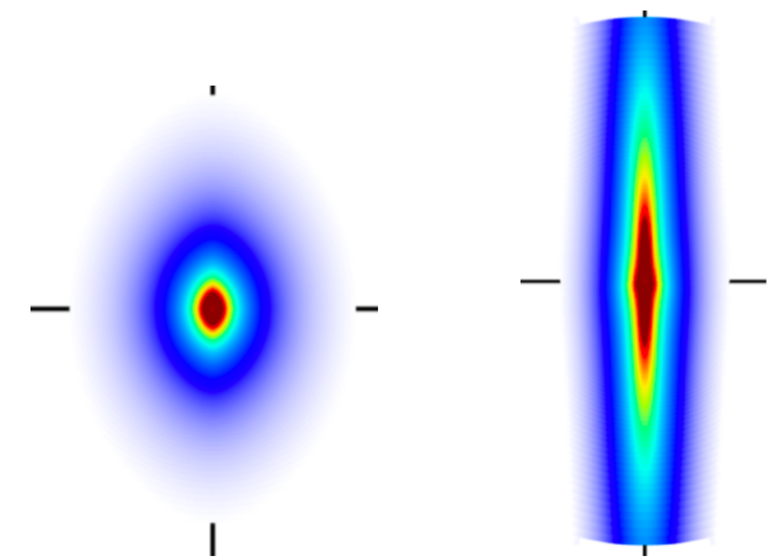
We introduce the angle distribution ($\mathrm{ctan}\theta = p_z/p$) function

$$F_\epsilon(\cos\theta) \equiv \frac{1}{2\pi^2}\int_0^\infty dp\, p^3 f(p,\theta,\tau)\,, \qquad \epsilon = \int_{-1}^{1} d\cos\theta F_\epsilon(\cos\theta)$$

Consider the multipole expansion of F_ε

$$F_\epsilon(\cos\theta) = \epsilon(\tau) + \sum_{n=1} \frac{4n+1}{2} L_n(\tau) P_{2n}(\cos\theta)$$

$$\psi = \left(\epsilon, L_1, L_2, \ldots, \right)$$



$L_1/\varepsilon \ll 1$ $\qquad$ $L_1/\varepsilon \sim 1$

(Figs from Kurkela et al, PRC 19')

By studying the evolution of ψ, we could equivalently understand the evolution of angle distribution F_ε .

The evolution equation for ψ

$$\partial_y f(\overrightarrow{p},\tau) = -\,\partial_{p_z} f(\overrightarrow{p},\tau) - \tau\,\hat{C}[f]$$

$$y \equiv \log(\tau/\tau_I)$$

a more convenient temporal variable

$$\times \int dpp^3$$

$$\partial_y F = \left(-4\cos^2\theta + \sin^2\theta\cos\theta\frac{1}{\partial\cos\theta}\right) F - \text{collisions}$$

Multipole expansion

$$\partial_y \psi = -\,H_F\,\psi - \text{ collisions}\,.$$

$$H_F = \begin{pmatrix} 4/3 & 2/3 & \ldots\ldots \\ 8/15 & 38/21 & \ldots\ldots \\ \ldots & \ldots & \ldots\ldots \end{pmatrix}$$

In what follows: we will consider the class of collision integrals that equation for ψ can be recast into the form:

$$\partial_y \psi = -\,\big(H_F + H_{Coll}(y)\big)\;\psi = -\,H(y)\,\psi\,.$$

The identification of pre-hydrodynamic mode

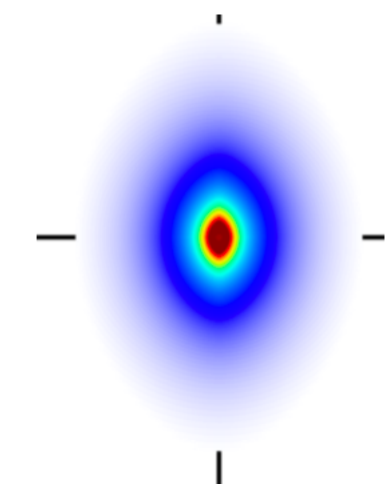
We consider the instantaneous eigenvalue of non-Hermitian matrix H(y) ordered by the real part of the eigenvalue

$$H(y)\phi_n(y) = E_n(y)\phi_n(y) \qquad \text{Re}E_0(y) < \text{Re}E_1(y) \leq \dots$$

In long time limit, the instantaneous ground state (g.s.) mode $\phi_0(y)$ will approach hydrodynamic mode:

$$f(y \to \infty) \to f_{\text{eq}}; \qquad \hat{C}[f_{eq}] = 0$$

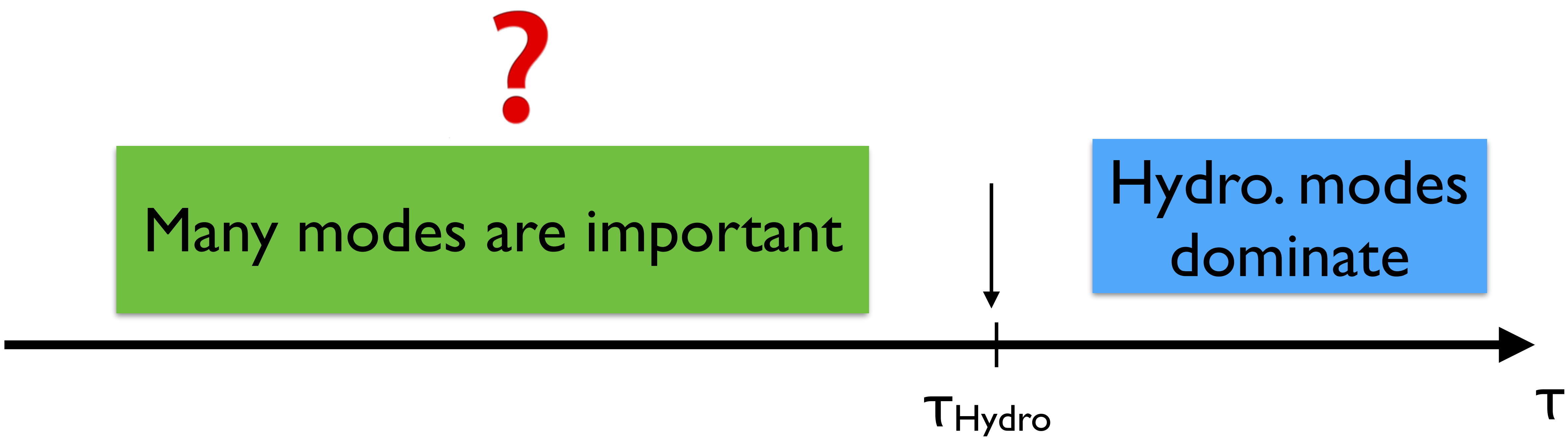
$$\phi_0(y \to \infty) \to \phi_H = (\epsilon, 0, \dots)$$



We identify instantaneous g.s. modes as slow modes during the non-equilibrium evolution of the system and refer them as pre-hydrodynamic modes (ancestors of hydrodynamic modes):

For each given y, pre-hydrodynamics has the slowest damping rate.

The problem of hydrodynamization within the current model



How does ψ evolve into ϕ_H for system described by $\partial_y \psi = -H(y)\,\psi\,.$

The dominance of pre-hydro mode at early time limit (or collision-less limit)

Consider the evolution of ψ at very early time $\tau \ll \tau_{Coll}$

$$\partial_y \psi = -\left(H_F + H_{Coll}\right)\,\psi \approx -H_F \psi\,.$$

We expanding in terms of eigenfunctions of matrix H_F:

$$\psi(\tau) = b_0(\tau)\phi_0^F + \sum_{n=1} b_n(\tau)\phi_n^F$$

Then,

$$\frac{b_n(\tau)}{b_0(\tau)} \sim \frac{e^{-E_n^F y}}{e^{-E_0^F y}} \sim \left(\frac{\tau}{\tau_I}\right)^{-(E_n^F - E_0^F)}, \qquad (E_n^F - E_0^F) \sim \mathcal{O}(1)$$

There must exist an emergent scale τ_{Redu} around which the evolution is dominant by ϕ_F :

$$\lim_{\tau/\tau_I \to \infty} \lim_{\tau_{\rm Coll}/\tau \to 0} \psi(\tau) \sim \phi_0^F\,.$$

The separation of scale for weakly coupled QGP in high energy density regime

So far, we have assumed the separation of scale between the initial time and typical collision time (or mean free time), i.e,

$$\tau_I \ll \tau_{\rm Coll}$$

In many cases, τ_I is a matter of choice, but for weakly coupled QGP in high energy density limit, τ_I is set by an emergent scale, i.e. saturation scale Q_s (typical momentum of gluon produced after a collision which is much larger than Λ_{QCD}):

$$\tau_I \sim Q_s^{-1}$$

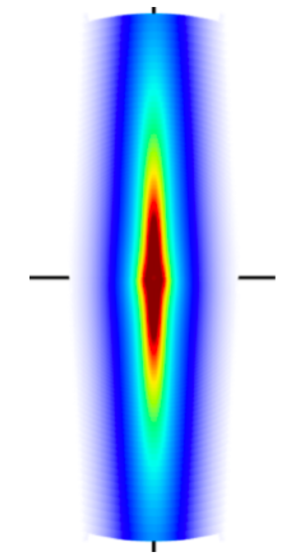
see e.g. McLerran-Venugopalan, PRD 94'; Blaizot-Mueller, Nucl.Phys.B 87'

Meanwhile, the collision time is inversely proportional to the cross-section of scattering process, therefore we see the parametrically separation between τ_I and $\tau_{\rm Coll}$.

$$\tau_I \sim Q_s^{-1} \ll \tau_{\rm coll} \sim \alpha_s^{-x} Q_s^{-1} , \qquad x > 0$$

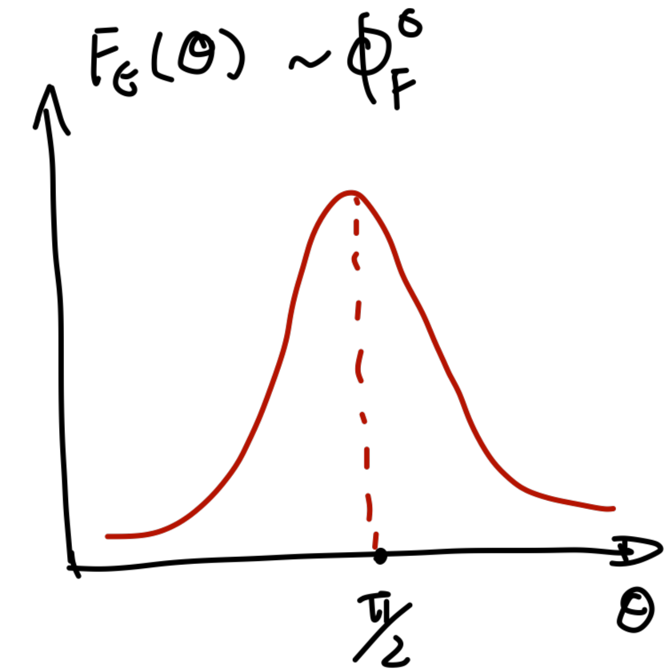
Longitudinal expansion that drives the dominance of ϕ_F.

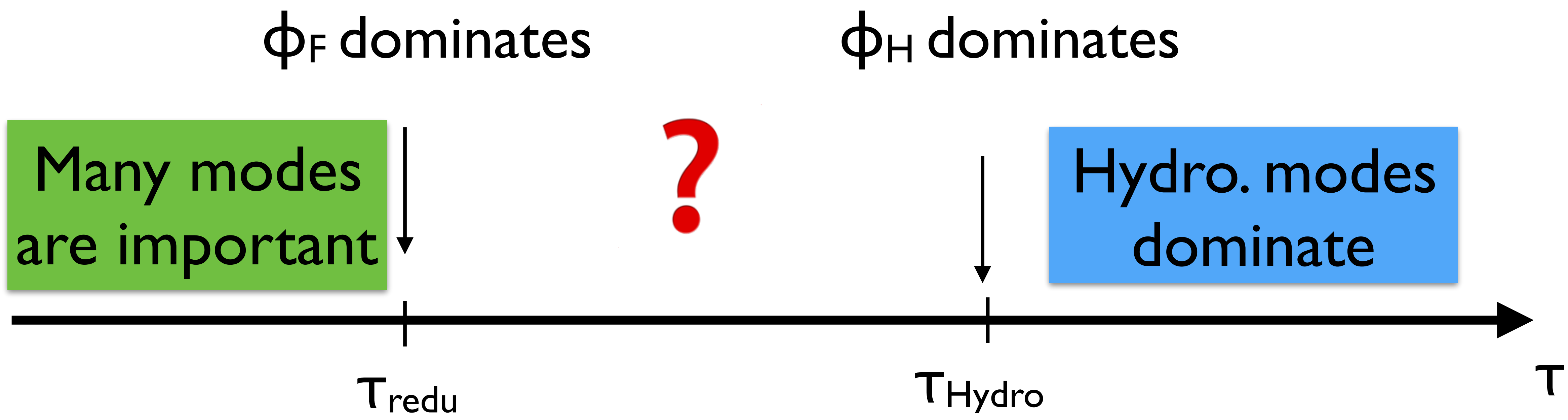
At free-streaming stage, distribution function f at given time τ can be related by the initial distribution by a boost along longitudinal direction. Consequently, the resulting f would become highly an-isotropic for $\tau \gg \tau_I$, regardless of the details of the initial condition.



c.f. Peter Arnold, 0708.0812.

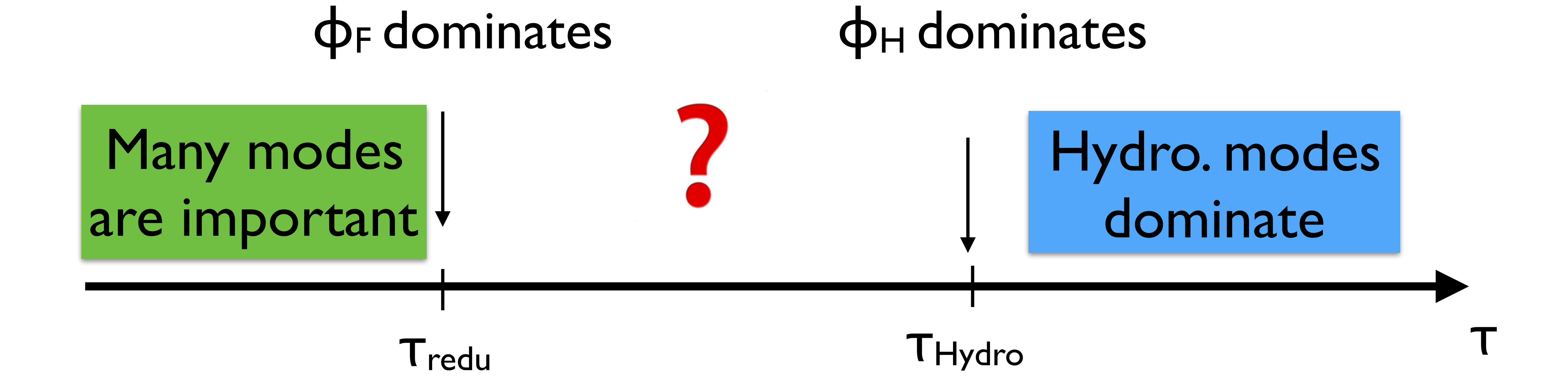
By explicit calculation, one finds that ϕ_F represents angle distribution sharply peak at $\theta=\pi/2$, meaning the characteristic p_z is much smaller than $p_\perp$.





Message 1: the reduction of d.o.f. happens at very early time for expanding QGP. This is due to not-trivial properties of QCD which describes the interaction among gluons (and quarks) and the non-trivial way that QGP is created in heavy-ion collision.

Adiabatic hydrodynamization



So far, we have seen the dominance of ground state mode at early times.

Next, how does ψ evolve from ϕ_F to ϕ_H ?

We shall consider and demonstrate the scenario of adiabatic evolution, i.e.

$$\psi(y) \sim \phi_0(y) \,.$$

Pre-hydro. modes and bulk properties of the pre-equilibrium evolution

The dominance of the pre-hydrodynamic mode suggests that the bulk properties of the pre-equilibrium medium can be related to this mode and its eigenvalue.

In particular: consider the most important quantity characterizing the bulk evolution of a boost-invariant plasma undergoing Bjorken expansion, i.e., *the percentage rate of change of the energy density ε*

Since ε is the zeroth component of ψ, we then have the following non-trivial relation under adiabatic hydrodynamization:

$$(-\partial_y \epsilon / \epsilon) \approx E_0(y)$$

Likewise, the ratio $p_L(y)/\varepsilon(y)$ can also be related to the zeroth and first component of $\varphi_0\,(y)$, i.e. the "direction" of instantaneous g.s. under adiabatic hydrodynamization.

Relaxation time approximation (RTA)

(other studies of RTA model: Denicol& Noronha, 1608.07869; Heller et al, PRD 18'; Blaizot-Li, PLB 18'; Heller& Svensson, PRD 18';…)

We consider collision integral under relation time approximation (RTA):

$$\hat{C}\,\left[f(\overrightarrow{p},\tau)\right] = -\frac{\left(f(\overrightarrow{p},\tau) - f_{\rm eq}(p,\tau)\right)}{\tau_{\rm Coll}}$$

Then we have the matrix equation

$$\partial_y \psi = -\,H_{RTA}(y)\,\psi\,.$$

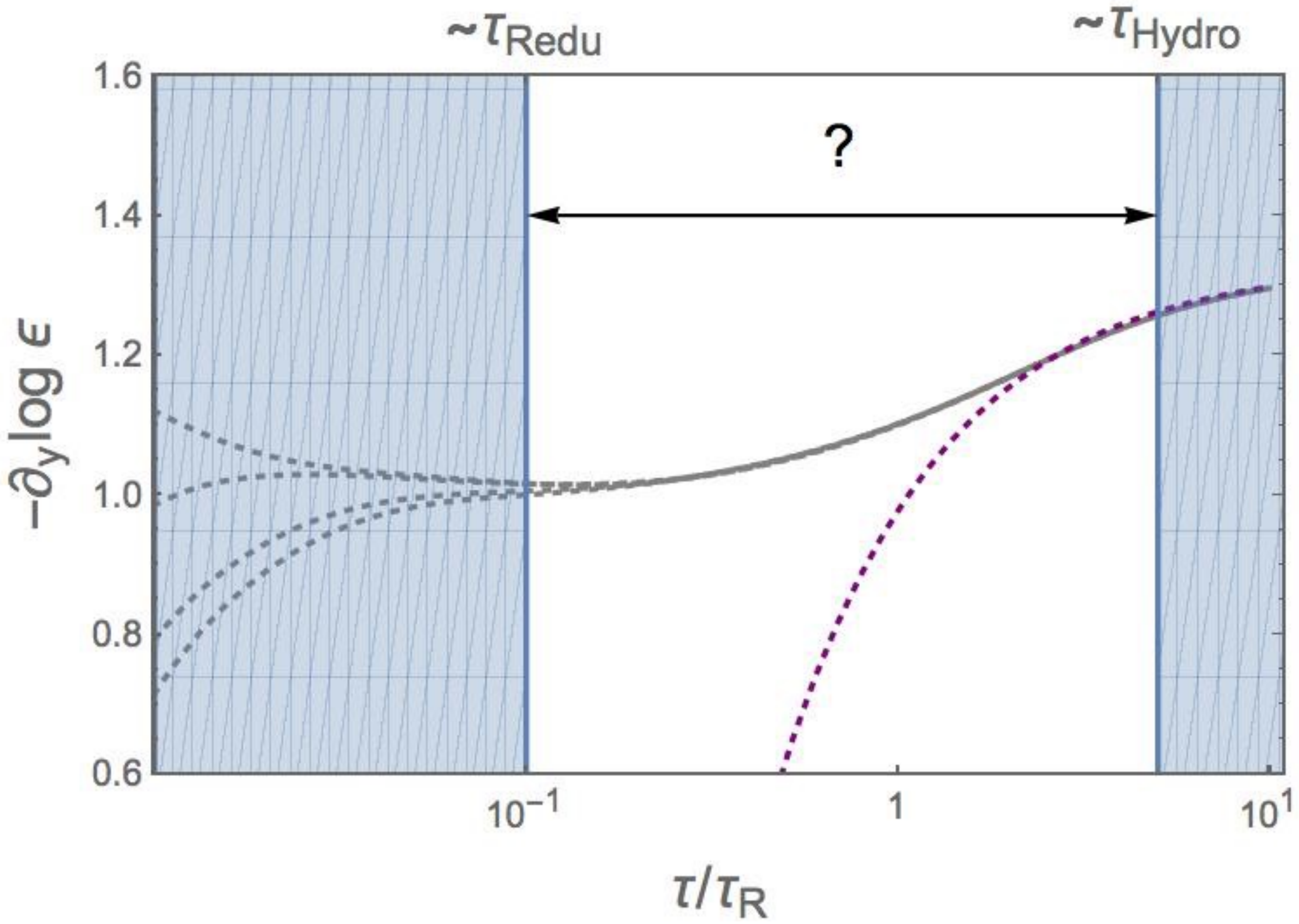
$$H_{RTA}(y) = H_F + \left(\frac{\tau}{\tau_{\rm Coll}}\right) H_1$$

$$H_1 = \begin{pmatrix} 0 & 0 & \cdots\cdots \\ 0 & 1 & \cdots\cdots \\ \cdots & \cdots & \cdots\cdots \end{pmatrix}$$

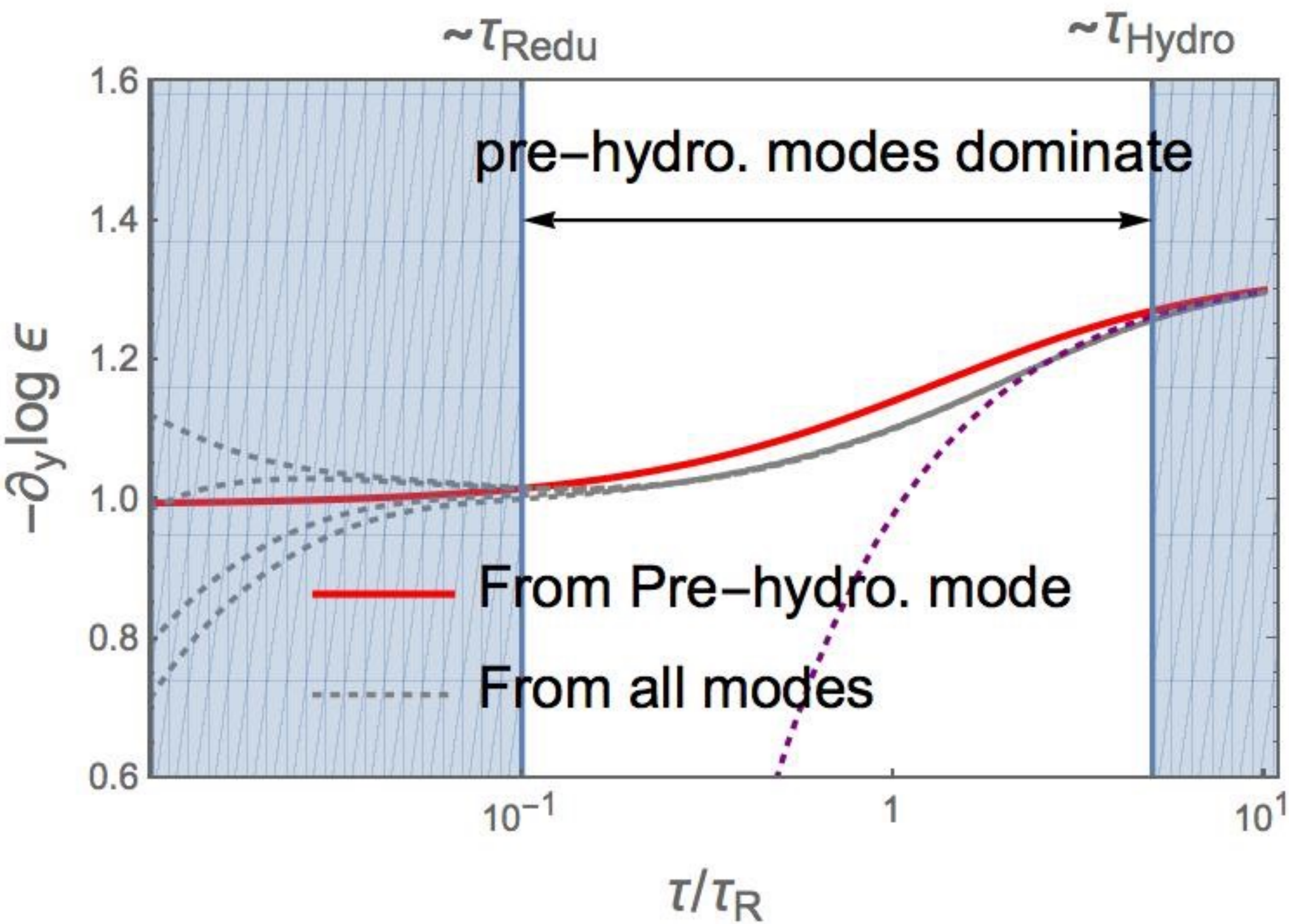
We now ready to test the previous relation explicitly.

$$(-\partial_y \epsilon/\epsilon) = (-\partial_y \log \epsilon) \approx E_0(y)$$

Change rate of ε obtained from numerical solution of RTA kinetic equation

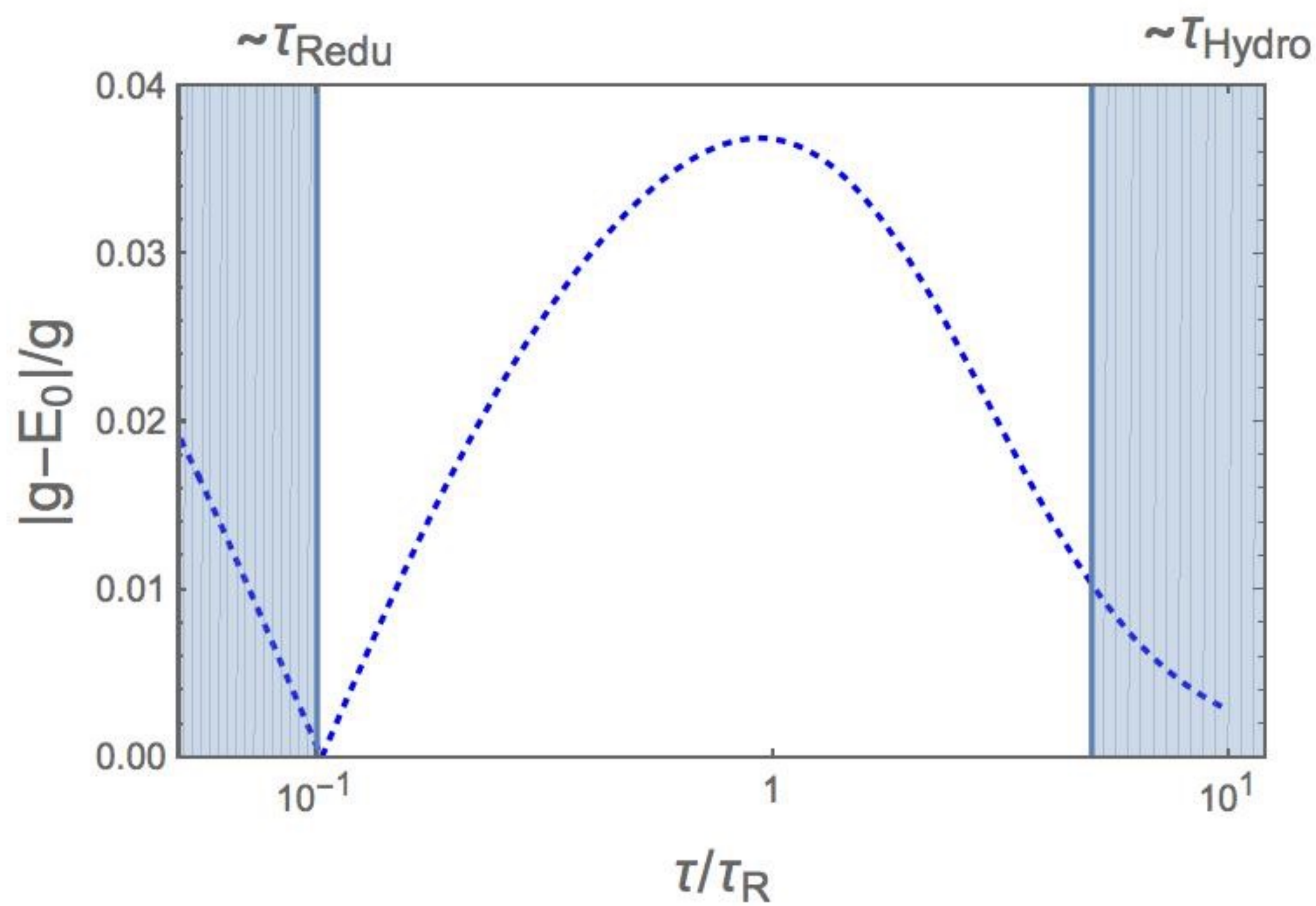


For RTA model, "pre-hydro. mode" dominates



J. Brewer, Li Yan and YY, to be submitted

The contribution from excited modes is suppressed during pre-hydrodynamic evolution



In fact, we have identified a small parameter that controls the adiabaticity. By expanding in this parameter, the contributions from the excited modes can also be accounted for quantitatively by generalizing adiabatic perturbation theory in QM.

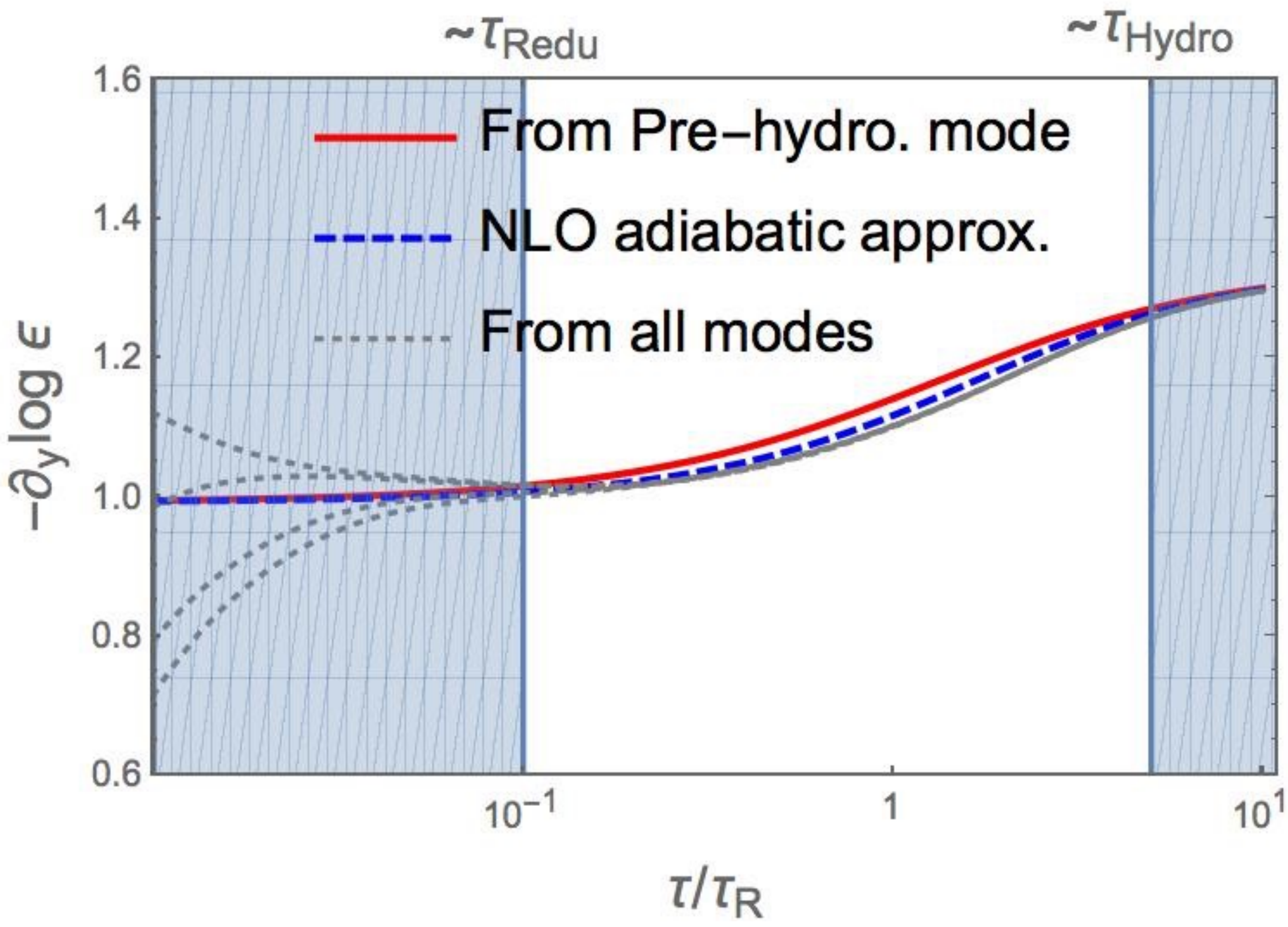
Adiabatic perturbation theory: from Landau-Zener problem to quenching through a quantum critical point

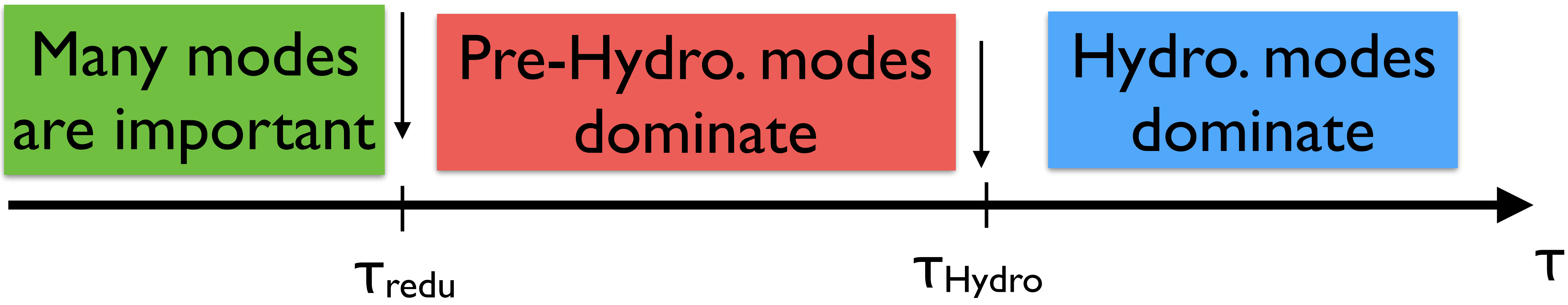
C. De Grandi and A. Polkovnikov
Department of Physics, Boston University, 590 Commonwealth Avenue, Boston, MA 02215, USA

We discuss the application of the adiabatic perturbation theory to analyze the dynamics in various systems in the limit of slow parametric changes of the Hamiltonian. We first consider a two-level system and give an elementary derivation of the asymptotics of the transition probability when the tuning parameter slowly changes in the finite range. Then we apply this perturbation theory to many-particle systems with low energy spectrum characterized by quasiparticle excitations. Within this approach we derive the scaling of various quantities such as the density of generated defects, entropy and energy. We discuss the applications of this approach to a specific situation where the system crosses a quantum critical point. We also show the connection between adiabatic and sudden quenches near a quantum phase transitions and discuss the effects of quasiparticle statistics on slow and sudden quenches at finite temperatures.

A review of modern formulation of adiabatic pTheory : De Grandi, A. Polkovnikov, 0910.2236

Systematic improvement with NLO adiabatic perturbation theory





So, we have illustrated the adiabatic hydrodynamization within RTA kinetic theory.

We now examine the criterion for adiabaticity.

The criterion for adiabaticity in Quantum mechanics

Considering a prototype QM time-dependent Hamiltonian:

$$H_{\mathrm{QM}}(t) = H_{\mathrm{QM},0} + \lambda_{\mathrm{QM}}(t) H_{\mathrm{QM},1}$$

The transition rate between instantaneous g.s. and excited states is proportional to:

$$\text{transition rate} \propto \langle n, t \,|\, \lambda_{\mathrm{QM}} H_{\mathrm{QM},1} \,|\, 0,1 \rangle \times \frac{\partial_t \lambda_{\mathrm{QM}}}{\Delta E(t)}$$

Adiabaticity means such transition is suppressed if

Either change rate is slow (slowly-quenching adiabaticity): $\dfrac{\partial_t \lambda_{\mathrm{QM}}}{\Delta E(t)} \ll 1$

or time-dependent part of the Hamiltonian is small in magnitude (fast-quenching adiabaticity):

$$\langle n, t \,|\, \lambda_{\mathrm{QM}} H_{\mathrm{QM},1} \,|\, 0,1 \rangle \ll 1$$

The criterion for adiabaticity in expanding QGP

Slowly-quenching adiabaticity applies to the late stage

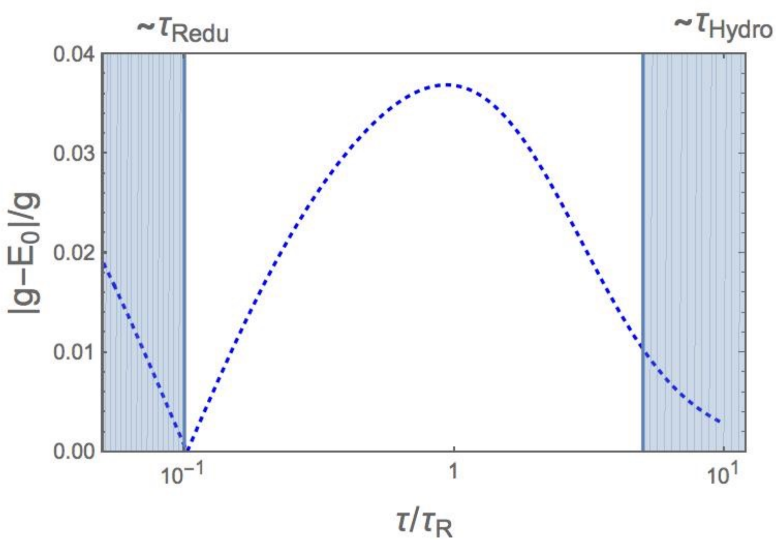
$$\frac{\partial_t \lambda_{\mathrm{QM}}}{\Delta E(t)} \ll 1 \quad \Rightarrow \quad \frac{1/\tau}{1/\tau_{\mathrm{Coll}}} = \frac{\tau_{\mathrm{Coll}}}{\tau} \ll 1$$

Fast-quenching adiabaticity applies to the early stage (since collision rate is very rare.)

$$H_F \gg H_{Coll}(y)$$

Despite of the difference in quench rate, expanding QGP satisfies the adiabaticity criterion when:

$$\tau \ll \tau_{\mathrm{Coll}}, \qquad \text{and} \qquad \tau \gg \tau_{\mathrm{Coll}}$$



How about the transition interval?

The stages of pre-equilibrium evolution of weakly coupled QGP has been delineated parametrically by Baier et al in 2001. Based on their analysis, we could discuss the applicability of adiabaticity:

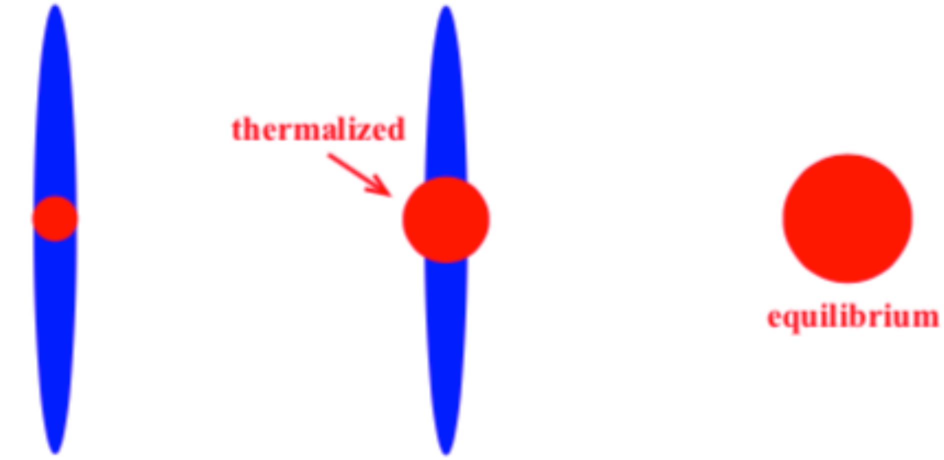


Figure adapted from Arnold, 0708.0812.

"Fast quench picture" applies to the period when ψ represents the angular distribution of hard gluons (with typical energy Q_s) that rarely collide with one another, i.e when

$$\tau_{\text{redu}} \leq \tau \leq \alpha_s^{-5/2} Q_S^{-1}$$

"Slow quench picture" applies to the period when ψ represents the angular distribution of soft gluons (with typical energy T) that are already in thermal equilibrium, i.e when

$$\tau \geq \alpha_s^{-13/5} Q_S^{-1}$$

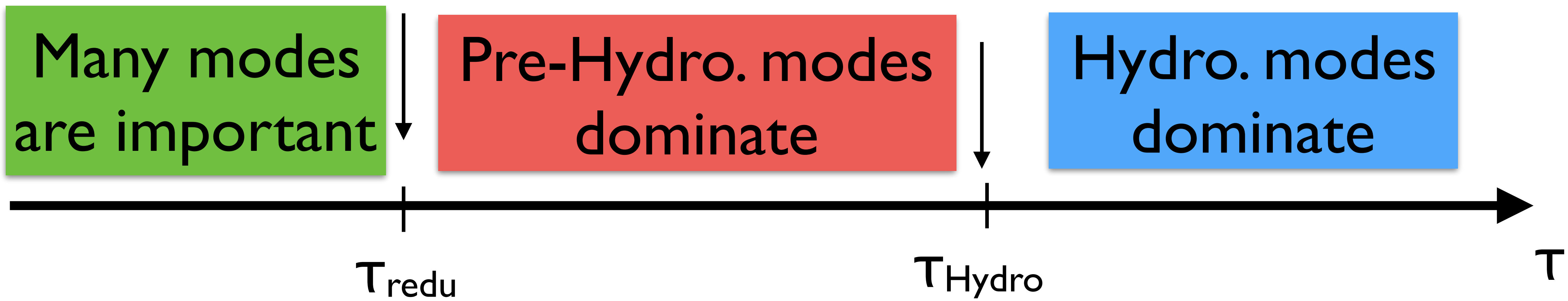
The transition interval when adiabaticity might be broken down is parametrically narrow for QCD in weakly coupled limit!

$$\alpha_s^{-5/2} Q_S^{-1} \leq \tau \leq \alpha_s^{-13/5} Q_S^{-1} \text{ , i.e. , } \alpha_s^{-2.5} Q_S^{-1} \leq \tau \leq \alpha_s^{-2.6} Q_S^{-1}$$

Message 11: The adiabaticity is maintained during the pre-hydro stage for expanding QGP under RTA model. This scenario might be relevant to heavy-ion collisions, again, due to the non-trivial way that QGP is thermalized.

Summary and outlook

Conclusion and outlook



We propose a new hydrodynamization scenario, "adiabatic hydrodynamization" (AH), for a weakly-coupled, longitudinally-expanding QGP. ➡More studies are necessary to test this scenario!

The concept of pre-hydrodynamic modes and AH is expected to be relevant more broadly, e.g, strongly coupled QGP-like theories, condensed matter systems.

A more systematically way to identify pre-hydrodynamic mode is needed, e.g., studying pole structure of off-equilibrium two-point functions?

Future: effective theory with pre-hydro. modes as relevant d.o.f. .

Back-up

"attractor"

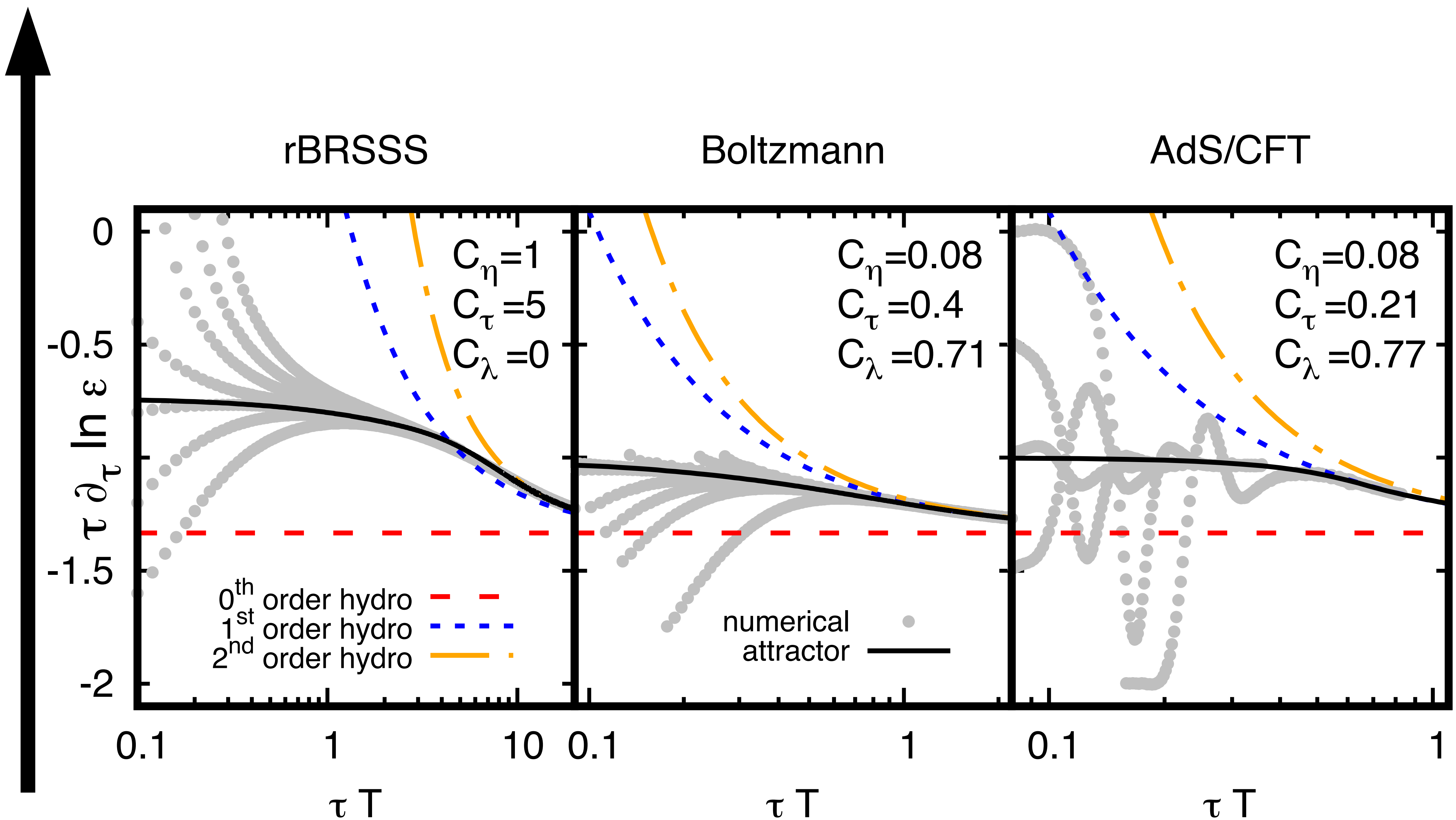


Figure adapted from Romatschke, 1704.08699, PRL 17'.