Two Loop Effective Kähler Potential of Supersymmetric Models

Tino S. Nyawelo

The Abdus Salam
International Centre for Theoretical Physics, Trieste, Italy.

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with S. Groot Nibbelink
Shanghai Institute for Advanced Study (SIAS)
Shanghai, China.

What?

Study the renormalization of a Kähler potential to two loop order.

why?

The computation of the effective Kähler potential can be important for phenomenological applications:

- It encoded the wave function renormalization of the chiral multiplets
- The physical masses of the chiral multiplets

how?

Supergraph techniques

At two loop, the computations of self-energy energy of chiral multiplet involve over 100 diagrams, which is very hard to manage.

Plan

- Theoretical framework: a general $\mathcal{N} = 1$ supersymmetric model based on a Kähler manifold with some of its linear isometries gauged.

- Computation of the one loop Kähler potential.

- Two loop effective Kähler potential.

- Examples:

  1. Non–renormalizable Wess–Zumino model and its renormalizable limit.

  2. Super Quantum Electrodynamics constitutes our second example.

- Conclusions

$\mathcal{N} = 1$ gauge non-linear sigma model

The effective action for $(D = 4, \mathcal{N} = 1)$ a supersymmetric field theory up two derivatives is encoded in three functions of the chiral multiplets ϕ:

$$
\begin{aligned}
\text{Kähler potential}: &\quad K(\phi, \bar{\phi}) \quad \text{Real} \\
\text{superpotential}: &\quad W(\phi) \quad \text{Holomorphic} \\
\text{gauge kinetic}: &\quad f(\phi) \quad \text{Holomorphic}
\end{aligned}
$$

- The superpotential and the gauge kinetic function are constrained to be holomorphic.

  This lead to various non–renormalization theorems: [Grisaru et al.], [Seiberg]

- The Kähler potential is only required to be a real function, and therefore far less constrained. It receives corrections at all orders in perturbation theory

The effective action

We consider a general globally supersymmetric theory defined by a tree-level action, which can be divided into three parts:

- The Kähler term

Kähler potenial

Vector superfield

Fayet-Iliop

$$S_{\mathsf{K}} = \frac{1}{2}\int \mathrm{d}^8 z \Big\{ K(\bar{\phi},\, e^{2V}\phi) + K(\bar{\phi}\, e^{2V},\, \phi) + \xi \,\mathrm{tr}\, V \Big]$$

- the gauge kinetic part

gauge kinetic

superfield strengths

$$S_{\mathsf{G}} = \int \mathrm{d}^6 z \left[\frac{1}{4}\,\mathrm{tr}\, f_{IJ}(\phi)\mathcal{W}^{I\,\alpha}\mathcal{W}^J_\alpha + \text{h.c.}\right]$$

- the superpotential interactions:

superpotential

$$S_{\mathsf{W}} = \int \mathrm{d}^6 z \left[W(\phi) + \text{h.c.} \right]$$

- Some of the linear isometries, $\delta_\alpha \phi = i\alpha\,\phi = i\alpha^I T_I\,\phi$ are assumed to be gauged by the introduction of the non–Abelian gauge vector superfield $V = V^I T_I$.

- The Hermitean generators T_I of this group satisfy the algebra $[T_I, T_J] = c^K{}_{IJ}\,T_K$.

- Gauging is of course only possible if the Kähler potential and the superpotential are gauge invariant

$$K(\bar\phi\, e^{-i\alpha}, e^{i\alpha}\,\phi) - K(\bar\phi, \phi) = 0\,, \qquad W(e^{i\alpha}\,\phi) = W(\phi).$$

Quantum corrections

Quantizing the supersymmetric gauge theory involve several steps:

- Quantum corrections to the classical supersymmetric action can be computed by various techniques. split the suerfields (background field method) ϕ and V into:

$$V \;\rightarrow\; V \qquad \phi \;\rightarrow\; \phi + \Phi$$

- addition of a supersymmetric gauge fixing action

$$S_{\mathsf{GF}} \;=\; -\frac{1}{8}\int \mathrm{d}^8 z\, h_{IJ}(\phi)\bar{\Theta}^I\,\Theta^J, \quad \Theta^I = \frac{1}{\sqrt{2}}\bar{D}^2 V^I$$

real part of f_{IJ}

(This is for theories without spontaneous symmetries breaking)

- The corresponding supersymmetric Faddev–Pappov ghost $C, C', \bar{C}, \bar{C}'$:

$$S_{FP} = \frac{1}{\sqrt{2}} \int \mathrm{d}^6 z\, C'_I \delta_C \Theta^I + \frac{1}{\sqrt{2}} \int \mathrm{d}^6 \bar{z}\, \bar{C}'_I \delta_C \bar{\Theta}^I$$

where

$$\delta_\Lambda \Theta^I \rightarrow \; = \sqrt{2}\, \frac{\bar{D}^2}{-4} \Big\{ \bar{\Lambda}^I + [V, \Lambda^I - \bar{\Lambda}]^I \Big\} + \ldots, \quad \text{but} \quad \Lambda \rightarrow C$$

super gauge parameter

- The gauge fixing procedure is then implemented by the insertion of

$$\Delta_{FP} \Big| \delta(\Theta^I - F^I) \Big|^2 e^{iS_F} , \qquad S_F = \int d^8 z\, h_{IJ}\, \bar{F}^I F^J$$

FP determinant

chiral superfield

Spontaneous symmetry breaking

For the general supersymmetric theories under consideration, two additional complications arise:

- Firstly, if the background ϕ spontaneously breaks some of the gauge symmetry, there will be mixing (at the quadratic level) between the vector V and the chiral $(\Phi,\ \bar{\Phi})$ multiplets.

  Therefore the gauage-fixing function Θ must be modified (if one wishes to work with diagonalized propagators)

$$\Theta^I \;=\; -\frac{\sqrt{2}}{4}\bar{D}^2\Big(V^I \;+\; (h^{-1})^{IJ}K^{\underline{a}}{}_a(T_J\phi)^a\,\frac{1}{\Box}\bar{\Phi}_{\underline{a}}\Big)\ .$$

  This is very similar to the 't Hooft R_ξ gauge fixing for spontaneously broken gauge theories.

- The second complication is that the Gaussian integral over S_F is not properly normalized. This can be implemented by the introduction of the Nielsen–Kallosh (NK) ghosts χ^I

$$S_{NK} = \int d^8z\, h_{IJ}(\phi)\,\bar{\chi}^I\chi^J$$

The full action of quantum theory

The full quantum action is given by:

$$S_{\mathsf{quantum}} = S_{\mathsf{K}}(\phi \to \phi + \Phi) + S_{\mathsf{W}}(\phi \to \phi + \Phi) + S_{\mathsf{G}}$$
$$+ S_{\mathsf{GF}} + S_{\mathsf{FP}} + S_{\mathsf{NK}}$$

Quantum bilinear and propagators

To obtain the functional dependence on the chiral multiplets of these one and two loop corrections, we expand the theory around: $\phi \to \phi + \Phi, V \to V$

- The zero-th order is just the original action for classical background superfields $S(\phi, V)$.
- The terms linear in quantum superfields do not contribute to the effective actions.
- The part bilinear in quantum superfields (is the relevant one for computations of one and two loops quantum corrections) are:

$$S^2 = S^2_V + S^2_{FP} + S^2_{\Phi} + S_{\mathsf{NK}}$$

- The quadratic actions of the vector

$$S_V^2 \;=\; -\int \mathrm{d}^8z\, V^I [\Delta_{VV}^{-1}]_{IJ} V^J \qquad \Delta_{VV} \;=\; [h\,\Box - M_V^2]^{-1}\,,$$

propagator

vector mass-matrix

- The quadratic Faddeev–Poppov ghost superfields are given

$$S_{FP}^2 = -\int \mathrm{d}^8z\; C'_I \left([\Delta^{-1}_{\bar{C}'C}]^I{}_J\, \bar{C}^J + \bar{C}'_I\, [\Delta^{-1}_{C\bar{C}'}]^I{}_J\, C^J \right)$$

ghost propagators

- Because of the gauge fixing Θ, the quadratic part of the chiral multiplet action has become more complicated

chiral propagator

propagator

propagator

$$S_\Phi^2 = \int \mathrm{d}^8z \left(\bar{\Phi}_{\bar{a}} [\Delta^{-1}_{\bar{\Phi}\Phi}]^{\bar{a}}{}_a\, \Phi^a + \Phi^a [\Delta^{-1}_{\Phi\Phi}]_{ab} \Phi^b + \bar{\Phi}_{\bar{a}} [\Delta^{-1}_{\bar{\Phi}\bar{\Phi}}]^{\bar{a}\bar{b}} \bar{\Phi}_{\bar{b}} \right)$$

- From the quadratic part of the quantum action we read off the propagators

$$\Delta_{C'\bar{C}} \quad = \quad [\Box - h^{-1} M_C^2]^{-1} \,, \qquad \Delta_{\bar{C}'C} \;=\; [\Box - h^{-1} {M_C^2}^T]^{-1}$$

with the Hermitean mass matrices for the ghost and vector multiplets

$$(M_C^2)_{IJ} \quad = \quad 2\,\bar{\phi} T_I G T_J \phi \,, \qquad M_V^2 \;=\; \frac{1}{2}\Big(M_C^2 + {M_C^2}^T\Big) \,,$$

- Finally, the chiral multiplet propagators

$$\begin{aligned}
\Delta_{\bar{\Phi}\Phi} \quad &= \quad [\Box - M^2]^{-1}\, G^{-1}, \\
\Delta_{\Phi\Phi} \quad &= \quad G^{-1}\,[\Box - M^2]^{-1}\,\bar{W}(G^{-1})^T \\
\Delta_{\bar{\Phi}\,\bar{\Phi}} \quad &= \quad (G^{-1})^T W G^{-1}\,[\Box - M^2]^{-1} \,.
\end{aligned}$$

- The superpotential M_W , Goldstone M_G and total mass matrices M

$$\begin{aligned} M_W^2 &= G^{-1}\bar{W}(G^{-1})^T W \ , \quad (M_G^2)^a{}_b = 2\,(T_I\phi)^a\,(h^{-1})^{IJ}\,(\bar{\phi}T_J G)_b \ , \\ M^2 &= M_W^2 + M_G^2 \ , \end{aligned}$$

- Because of the super gauge invariance of the superpotential ,the superpotential matrix W has zero modes $T_I\phi$:

$$W_{ab}\,(T_I\phi)^b \ = \ 0, \quad M_W^2\,M_G^2 \ = \ M_G^2\,M_W^2 \ = \ 0.$$

- The background ϕ generically leads to spontaneous symmetry breaking and massive vector multiplets.
  - Massive vector multiplet consists of V^I: Goldstone mode chiral superfields the massive Faddeev–Poppov ghosts.
  - Moreover, in this gauge the chiral Goldstone multiplets and the ghost multiplets have the same mass eigenvalues

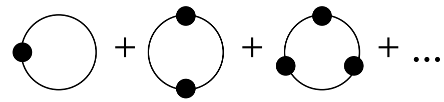
$$\text{tr}(M_G^2)^p \ = \ \text{Tr}(h^{-1}M_C^2)^p \ = \ \text{Tr}(h^{-1}M_C^{2\,T})^p \, .$$

- Our graphical representation notation for these propagators are:

$\Delta_{\bar{\Phi}\Phi}$ $\Delta_{\Phi\Phi}$ Δ_{VV} Δ_{CC}

One loop effective Kähler potential

The one loop calculation of the effective Kähler potential involves the computation of one loop vacuum bubble graphs with multiple insertions of two–point interaction terms



- To evaluate these bubbles in general, we consider a generic vector of commuting superfields Ψ with quadratic action

$$S = \frac{1}{2}\int \mathrm{d}^8 z\, \Psi^T \Big[\Delta^{-1} + \mathcal{M} + J^T\Big]\Psi$$

propagator

2–point interaction

sources

- The sum of the connected bubble graphs reads

$$i\Gamma_{1\mathrm{L}} = e^{\frac{1}{2}\int \mathrm{d}^8 z\, \frac{\delta}{i\delta J}\mathcal{M}(\frac{\delta}{i\delta J})^T}\, e^{-\frac{i}{2}\int \mathrm{d}^8 z\, J^T \Delta J} = \sum_{n\geq 1} i\Gamma_{(n)}$$

- We apply this to the various quadratic terms, the full one loop Kähler potential is given in a coordinate representation by

$$\begin{aligned} i\Gamma_{1L} \quad &= \quad \int (\mathrm{d}^4 x)_{12} \mathrm{d}^4\theta \left[\mathsf{Tr} \ln h + \mathsf{Tr} \ln \left(1\!\!1 - \frac{h^{-1} M_C^2}{\Box} \right) - \mathsf{tr} \ln G \right. \\ & \left. \quad - \frac{1}{2} \mathsf{tr} \ln \left(1\!\!1 - \frac{M_W^2}{\Box} \right) \right]_1 \delta^4_{12} \frac{1}{\Box_1} \delta^4_{12} \ . \end{aligned}$$

- The origins of the various terms are as follow:
  - The first term is due to the Nielsen–Kallosh ghosts.
  - The second term is the combined effective action of the Faddeev–Poppov ghosts and the Goldstone chiral multiplets.
  - The last two terms are due bubbles that contain chiral multiplets.
- As it stands this expression $i\Gamma_{1L}$ is ill–defined and requires regularization.

- Mainly because computational convenience at the two loop level, we choose to use dimensional reduction:
  - Wick rotation, . . . , Fourier transform to momentum space and evaluate the momentum integral in $D = 4 - 2\epsilon$ dimensions

$$\int_p = \mu^{2\epsilon} \int \mathrm{d}^D p/(2\pi)^D$$

- At the one loop level we encounter three different types of integrals.
  - The first integral reads

$$J(m^2) = \int \frac{\mathrm{d^D p}}{(2\pi)^D \mu^{D-4}} \frac{1}{p^2 + m^2} = -\frac{m^2}{16\pi^2} \left[\frac{1}{\epsilon} + 1 - \ln\frac{m^2}{\bar{\mu}^2} + \mathcal{O}\epsilon\right]..$$

    Here we have introduced the $\overline{MS}$ scale $\bar{\mu}^2 = 4\pi e^{-\gamma}\mu^2$ with the Euler constant γ

  - The second integral is

$$L(m^2) = \int \frac{\mathrm{d^D p}}{(2\pi)^D \mu^{D-4}} \frac{1}{p^2} \ln\left(1 + \frac{m^2}{p^2}\right) = \frac{m^2}{16\pi^2} \left[\frac{1}{\epsilon} + 2 - \ln\frac{m^2}{\bar{\mu}^2}\right]$$

  - Finally the integral

$$S(m^2) = \int \frac{\mathrm{d^D p}}{(2\pi)^D \mu^{D-4}} \frac{1}{(p^2 + m^2)^2} = \frac{1}{16\pi^2} \left[\frac{1}{\epsilon} - \ln\frac{m^2}{\bar{\mu}^2}\right].$$

- Using these integrals, and dropping the $1/\epsilon$ poles, we find that the effective one loop Kähler potential is given by

$$K_{1L} \quad = \quad -\frac{1}{16\pi^2}\,\text{Tr}\, h^{-1}M_C^2\Big(2 - \ln\frac{h^{-1}M_C^2}{\bar{\mu}^2}\Big) + \frac{1}{32\pi^2}\,\text{tr}\, M_W^2\Big(2 - \ln\frac{M_W^2}{\bar{\mu}^2}\Big).$$

- One–loop corrections to the Kähler potential have been computed by many Authors (in supersymmetric Landau gauge) [Grisaru, de Wit, Buchbinder, . . .] [Brignole]

Their results for the effective one loop Kähler potential read

$$\Delta K_{1L} \quad = \quad -\frac{1}{16\pi^2}\,\text{Tr}\, M_V^2\Big(2 - \ln\frac{M_V^2}{\bar{\mu}^2}\Big) + \frac{1}{32\pi^2}\,\text{tr}\, M_W^2\Big(2 - \ln\frac{M_W^2}{\bar{\mu}^2}\Big) \qquad \text{[Brignole]}.$$

- In the Abelian case, their result agree with our one loop effective Kähler potential result:

$$M_C^2 = M_V^2 \quad \Rightarrow \Delta K_{1L} = K_{1L}$$

- In the non–Abelian case the mass matrices M_C^2 and M_V^2 are not equal anymore, and our results slightly deviate from their results

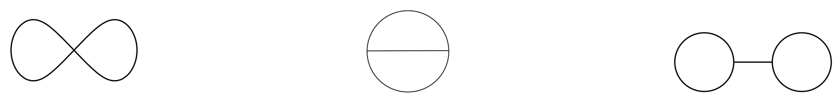
$$(M_C^2)_{IJ} = 2\,\bar{\phi} T_I G T_J \phi, \quad M_V^2 = \frac{1}{2}\Big(M_C^2 + {M_C^2}^T\Big).$$

(This might be an artifact of the use of different gauge fixing procedures)

Two loop effective Kähler potential

- At the two loop level there are three different topologies of the supergraphs that may contribute to the Kähler potential.

  They have the topologies of an "8" (figure a) and "$\ominus$" (figure b), a "double tadpole" (figure c), respectively.



a b c

"Double tadpole" supergraphs

- Most computations of the effective (Kähler) potential are restricted to only those connected graphs that are 1–P–I.

  – The argument for this restriction is that all 1–P–I contain one or more tadpole subgraphs, which are generically absent by symmetry arguments.

  – For example, a ϕ^4 theory has the symmetry $\phi \rightarrow -\phi$ which forbids tadpoles to arise.

- Because we are dealing with rather generic supersymmetric models in arbitrary backgrounds, we reconsider the issue of one–particle–reducible graphs.

The connecting line can represent either

- We can divide these diagrams into two classes depending on whether the connecting line is a chiral or a vector multiplet.
  - In the case that the connecting line is a chiral superfield, one can show by some partial integrations of D^2 or $\bar{D}^2$ that these diagrams contain too little D^2 or $\bar{D}^2$, and therefore vanish.
  - This leaves us with double tadpole graphs with a vector multiplet as a connecting line.
- Because a vector multiplet is not chiral, no D^2 or $\bar{D}^2$ appear on the connecting line.

  This implies that these graphs are non–vanishing iff the sum of Fayet–Illiopoulos tadpole graphs is non–zero. [Weinberg's 3rd vo]

  [Nilles et al.]
- Let us briefly review the arguments which are applicable in our case:
  - If the vector multiplet is non–Abelian no tadpole is possible because the tadpole graph is never gauge invariant

– For a $U(1)$ vector superfield V a tadpole is possible. The, induced ξ at the one loop level

$$\xi_{1L} = \mathrm{tr} T_a \int \mathrm{d}^4\mathrm{p}/\mathrm{p}^2.$$

Since in this work we use dimensional reduction throughout, this integral vanishes.

Supergraphs of the "8" topology

- There is in fact only one "8" supergraph that results from the vertex

$$\Delta S^4 \supset \int \mathrm{d}^8 z \, \frac{1}{4} \, K_{ab}{}^{\underline{a}\,\underline{b}} \, \Phi^a \Phi^b \, \bar{\Phi}_{\underline{a}} \bar{\Phi}_{\underline{b}} \ ,$$

- Using standard supergraphs techniques we find that the supergraph, becomes the following scalar integral

$$i\Gamma_{2L}^{\text{“8”}} = -\frac{i}{2} \int (\mathrm{d}^4 \mathrm{x})_{123} \mathrm{d}^4\theta \, \mathrm{K}_{1\,\mathrm{ab}}{}^{\underline{\mathrm{a}}\,\underline{\mathrm{b}}} \, \delta^4_{21} (\Delta_{\Phi\bar{\Phi}})^{\mathrm{a}}_{2\,\underline{\mathrm{a}}} \delta^4_{21} \, \delta^4_{31} (\Delta_{\Phi\bar{\Phi}})^{\mathrm{b}}_{3\,\underline{\mathrm{b}}} \delta^4_{31}$$

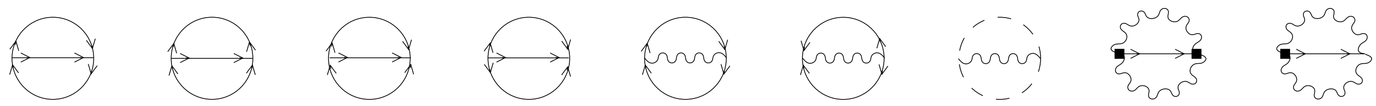
- By doing a Fourier transform to momentum, we find that the "8" supergraph can be compactly expressed as

$$i\Gamma_{2L}^{\text{“8”}} = \frac{i}{2} \int \mathrm{d}^8 z \, K^{\underline{a}\,\underline{b}}{}_{ab} \, \bar{J}^{a}_{\underline{a}}{}^{b}{}_{\underline{b}}(M^2, M^2) \ ,$$

Notice that, this expression is not covariant. This signals that this result is not complete.

Supergraphs of the "⊖" topology

- The non–vanishing supergraphs of the "⊖" topology that can be obtained from the interaction ΔS^3 are:



A B C D E F G H I

- Diagrams "8", A B, C, $\overline{\text{C}}$ and D combined to form curvature $R^{\underline{a}}{}_{a}{}^{\underline{b}}{}_{b}$ and and covariant derivatives of the superpotential $W^{;\underline{a}\,\underline{b}\,\underline{c}}$.

$$R^{\underline{a}}{}_{a}{}^{\underline{b}}{}_{b} \quad = \quad K^{\underline{a}\,\underline{b}}{}_{ab} - K^{\underline{a}\,\underline{b}}{}_{c}\, G^{-1c}{}_{\underline{c}}\, K_{ab}{}^{\underline{c}},$$

$$W_{;abc} \quad = \quad W_{abc} - \Gamma_{ab}^{\;d}\, W_{dc} - \Gamma_{bc}^{\;d}\, W_{da} - \Gamma_{ca}^{\;d}\, W_{db}$$

Summary results for the effective Kähler potential at two loops

The full two loop corrections to the Kähler potential is naturally divided into two parts:

$$K_{2L} \;=\; K_{2L}^{\text{universal}} \;+\; K_{2L}^{\text{gauge kinetic}} \;.$$

- $K_{2L}^{\text{universal}}$ is the part that is only present for constant gauge kinetic functions and takes the form

$$K_{2L}^{\text{universal}} = \frac{1}{2} R^{\underline{a}}{}_{a}{}^{\underline{b}}{}_{b}\, \bar{J}^{a}{}_{\underline{a}}{}^{b}{}_{\underline{b}}(M^2, M^2) + \frac{1}{6} \bar{W}^{;\underline{a}\,\underline{b}\,\underline{c}} W_{;abc}\, \bar{I}^{a}{}_{\underline{a}}{}^{b}{}_{\underline{b}}{}^{c}{}_{\underline{c}}(M^2, M^2, M^2)$$

$$+\frac{1}{2} h_{LP}\, c^{P}{}_{IN}\, h_{JQ}\, c^{Q}{}_{KM} \left\{ \bar{I}^{IJKLMN}(M_C^2, M_C^2, M_V^2) \right.$$

$$\left. -\bar{I}^{IJKLMN}(M_C^2, {M_C^2}^T, M_V^2) \right\}$$

$$-(GT_I\phi)^{\underline{a}}{}_{;a}\, (\bar{\phi}T_J G)_{b}{}^{;\underline{b}}\, \bar{I}^{a}{}_{\underline{a}}{}^{b}{}_{\underline{b}}{}^{IJ}(M^2, M^2, M_V^2).$$

- This result is manifestly covariant under diffeomorphisms that preserve the Kähler structure.

- The combination of the diagrams “8” and A-D have been computed for a single ungauged chiral multiplet [Buchbinder, Petrov]

  (However, there seemed to be some differences with our results, in particular that result is not covariant.)

- When the gauge kinetic function is not constant we find the additional contributions

$$
\begin{aligned}
K_{2L}^{\text{gauge kinetic}} \quad = \quad & \frac{1}{8} f_{IK\,a}\, \bar{f}_{JL}{}^{\underline{a}} \Big\{ 2\, h^{-1KL}\, \bar{J}^{a}{}_{\underline{a}}{}^{IJ}(M^2, M_V^2) - G^{-1a}{}_{\underline{a}}\, \bar{J}^{IJKL}(M_V^2, M_V^2) \\
& + (T_M\phi)^a\, (\bar{\phi}T_N)_{\underline{a}}\, \bar{I}^{IJKLMN}(M_V^2, M_V^2, M_C^2) \Big\} \\
& + \frac{1}{8} \Big\{ f_{IK\,b} (G^{-1}\bar{W})^{b\underline{a}}\, \bar{f}_{JL}{}^{\underline{b}} (G^{-1^T} W)_{\underline{b}a} - f_{MK\,a}\, \bar{f}_{NL}{}^{\underline{a}} \Big(\delta^M{}_I (h^{-1} M_V^2)^N{}_J \\
& \quad + \ \delta^N{}_J (h^{-1} M_V^2)^M{}_I \Big) \Big\}\, \bar{I}^{aIJKL}_{\underline{a}}(M^2, M_V^2, M_V^2) \\
& + \frac{1}{2} \Big(f_{IK\,a}\, (M_C^2)_{JL}{}^{;\underline{a}} \ + \ \bar{f}_{IK}{}^{\underline{a}}\, (M_C^2)_{JL\,;a} \Big) \bar{I}^{aIJKL}_{\underline{a}}(M^2, M_V^2, M_V^2) \ .
\end{aligned}
$$

- The terms that are proportional to the product of tensors f and $\bar{f}$ arise from diagram H.

- The last line is the effect of diagram I and it's Hermitian conjugate.

Simple applications

We illustrate our general formulae for the effective Kähler potential at one and two loops, by applying them to some simple supersymmetric models.

The (non–)renormalizable Wess–Zumino model

We consider a single chiral multiplet ϕ described by a Kähler potential $K = K(\bar{\phi}, \phi)$ and a superpotential $W(\phi)$.

- The metric, connection and curvature read

$$G = K^{\underline{1}}{}_{1} \, , \qquad \Gamma = G^{-1} K^{\underline{1}}{}_{11} \, , \qquad R = K^{\underline{1}\,\underline{1}}{}_{11} - \bar{\Gamma}\, G\, \Gamma \, ,$$

- The triple covariant derivative of the superpotential and the superpotential mass are given by

$$W_{;111} = W_{111} - 3\,\Gamma\, W_{11} \, , \qquad M_W^2 = G^{-2}\, |W_{11}|^2 \, .$$

- The one and two loop corrections to the effective Kähler potential read

$$K_{1L} = \frac{1}{16\pi^2} \frac{1}{2} M_W^2 \Big(2 - \ln \frac{M_W^2}{\bar{\mu}^2} \Big)$$

$$K_{2L} = \frac{1}{2} R G^{-2}\, \bar{J} + \frac{1}{6} |W_{;111}|^2 G^{-3}\, \bar{I} \, ,$$

with the short hand notations

$$\bar{J} \;=\; \frac{1}{(16\pi^2)^2}\,(M_W^2)^2\Big(1-\ln\frac{M_W^2}{\bar{\mu}^2}\Big)^2\,,$$

$$\bar{I} \;=\; \frac{1}{(16\pi^2)^2}\,\frac{3}{2}\,M_W^2\Big[-5+4\,\ln\frac{M_W^2}{\bar{\mu}^2}-\ln^2\frac{M_W^2}{\bar{\mu}^2}+12\,\kappa(\bar{x})\Big]\,.$$

- Reduction to the renormalizable Wess–Zumino model:

$$K \;=\; \bar{\phi}\phi, \qquad W(\phi) \;=\; \frac{1}{2}\,m\,\phi^2+\frac{1}{3!}\,\lambda\,\phi^3.$$

- Hence the expressions for the one and two loop Kähler potentials further simplify to

$$K_{1L} \;=\; \frac{1}{16\pi^2}\,\frac{1}{2}\,M_W^2\Big(2\;-\;\ln\frac{M_W^2}{\bar{\mu}^2}\Big)\,,$$

$$K_{2L} \;=\; \frac{1}{(16\pi^2)^2}\,\frac{1}{4}\,|\lambda|^2\,M_W^2\Big\{-5+4\,\ln\frac{M_W^2}{\bar{\mu}^2}-\ln^2\frac{M_W^2}{\bar{\mu}^2}+12\,\kappa(\bar{x})\Big\}\,,$$

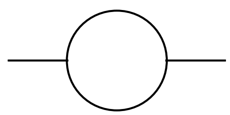
with the mass $M_W^2 \;=\; |m+\lambda\,\phi|^2\,.$

A consistency check

- The effective Kähler potential can be used to determine the wave function renormalization at one loop by taking the second mixed derivative of it.

$$\Sigma_{\text{eff. Kähler pot.}} = \frac{\partial^2 K_{1L}}{\partial\phi\,\partial\bar{\phi}} = -\frac{|\lambda|^2}{32\pi^2}\ln\frac{|m+\lambda\,\phi|^2}{\bar{\mu}^2}\,,$$

- This wave function renormalization can also be computed directly from the one loop self energy supergraph



$$\Sigma_{\text{self energy}} = -\frac{|\lambda|^2}{32\pi^2}\ln\frac{|m+\lambda\,\phi|^2}{\bar{\mu}^2}\,,$$

which agrees with our one loop effective Kähler potential result.

Super Quantum Electrodynamics

The theory of Super Quantum Electrodynamics consists of two oppositely charged chiral multiplets ϕ_+ and ϕ_- under a $U(1)$ gauge symmetry of which V is the vector superfield.

- The Kähler potential and superpotential for this model have the well known form

$$K = \bar{\phi}_+ e^{2V} \phi_+ + \bar{\phi}_- e^{-2V} \phi_- \,, \qquad W = m\, \phi_+ \phi_- .$$

  where m is the mass of the super electron.

- The gauge kinetic action reads

$$S_G = \frac{1}{4g^2} \int \mathrm{d}^6 z\, \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{h.c.} \,,$$

  where $g^{-2} = h = f$ is the inverse gauge coupling.

- The one and two loop corrections to the effective Kähler potential are given by the following expressions:

  – At the one loop level we find

$$K_{1L} = -\frac{1}{16\pi^2} g^2 M_V^2 \Big(2 - \ln \frac{g^2 M_V^2}{\bar{\mu}^2} \Big) + \text{constant}.$$

– The two loop result takes the form

$$K_{2L} = -\Big\{ \bar{I}(m_+^2, m_+^2, g^2 M_V^2) + \bar{I}(m_-^2, m_-^2, g^2 M_V^2) \Big\} \Big(\frac{\bar{\phi}\sigma_3 \phi}{\bar{\phi}\phi} \Big)^2$$

$$-2\, \bar{I}(m_+^2, m_-^2, g^2 M_V^2) \Big| \frac{\phi^T \sigma_1 \phi}{\bar{\phi}\phi} \Big|^2 .$$

with the mass eigenvalues $m_+^2 \;=\; |m|^2 + g^2 M_V^2$ and $m_-^2 \;=\; |m|^2$.

Conclusions

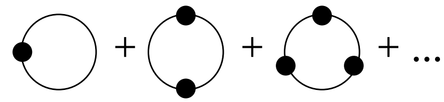
We perform a supergraph computation of the effective Kähler potential at one and two loops for (Non-)Renormalizable $\mathcal{N} = 1$ Supersymmetric Models.

- As long as no non-abelian gauge interaction are taken into account, our one–loop results are consistent with some existing literature concerning the computations of the Kähler potential [Grisaru, de Wit, Buchbinder, . . .]
  - In the non-abelian case, our results slightly deviate from these reference (This might be an artifact of the use of different gauge fixing procedures.)
- When we restrict to the ungauged case, we obtain the same terms at two loops as [Buchbinder,Petrov], (but with different coefficients) such that the result contains the curvature tensor and covariant derivatives of the superpotential.)
  - The result of the two loop Kähler potential looks surprisingly simple as long as the gauge kinetic function is strictly constant.
- Apart from the possible phenomenological applications, our results at the two loop level might be interesting for various applications in $\mathcal{N} = 2$ theories.
  - In theories with extend supersymmetry the Kähler and super–potential are obtained from a single holomorphic prepotential

– Since our results are obtained for generic $\mathcal{N} = 1$ supersymmetric theories they can be applied in particular to $\mathcal{N} = 2$ theories, and can lead to important cross checks on the validity of the constraints that come from the $\mathcal{N} = 2$ structure.

One loop effective Kähler potential

The one loop calculation of the effective Kähler potential involves the computation of one loop vacuum bubble graphs with multiple insertions of two–point interaction terms



- To evaluate these bubbles in general, we consider a generic vector of commuting superfields Ψ with quadratic action

$$S \;=\; \frac{1}{2}\int \mathrm{d}^8z\,\Psi^T\Big[\Delta^{-1} + \mathcal{M}\Big]\Psi + \text{tems involving sources}$$

propagator

2–point interaction

- To understand the notation, consider quadratic action for Φ

$$\begin{aligned} S_2 &= S_0 + S_J + \Delta S_2, \quad S_J = \int \mathrm{d}^6z\, J_a\Phi^a + \text{h.c.} \\ S_0 &= \int \mathrm{d}^8z\,\bar{\Phi}_a\delta^a{}_b\Phi^b, \quad \Delta S_2 = \int \mathrm{d}^8z\,\bar{\Phi}_a L^a{}_b\Phi^b \end{aligned}$$

with $L^a{}_b = G^a{}_b - \delta^a{}_b$.

- The first two terms can be written as

$$\tilde{S}_2 = S_0 + S_J \quad = \quad \int d^8z \left(\bar{\Phi}_a \delta^a{}_b \Phi^b + \frac{D^2}{-4\Box} J_a \Phi^a + \frac{\bar{D}^2}{-4\Box} \bar{J}^a \bar{\Phi}_a\right)$$

This (Gaussian) integral is of the form $I(y)$ and may be evaluated by shift of variables to give

$$I(y) = \int \, dx\, d\bar{x}\, e^{i(\frac{1}{2}x^T A\, x + x^T y)} = \text{cons.}\; e^{(-\frac{i}{2} y^T A^{-1} y)}.$$

Using this

$$\tilde{S}_2 = -\int \, d^8z\, J_a \frac{\delta^a{}_b}{\Box} \bar{J}^b$$

- Intoducing the notation:

$$J \quad \to \quad \begin{pmatrix} J_a \\ \bar{J}^a \end{pmatrix}, \quad \frac{\delta}{i\delta J} \to \begin{pmatrix} \Phi^a & \bar{\Phi}_a \end{pmatrix}, \quad \mathcal{M} = \begin{pmatrix} 0 & L^T \\ L & 0 \end{pmatrix},$$

$$\Delta \quad = \quad \begin{pmatrix} 0 & \delta^a{}_b \\ \delta_a{}^b & 0 \end{pmatrix} \frac{1}{\Box}$$

we write:

$$i\tilde{S}_2 = -\frac{i}{2} \int \, d^8z\, J^T \Delta J, \quad i\Delta S_2 = \frac{i}{2} \int \, d^8z\, \frac{\delta}{i\delta J} \mathcal{M} (\frac{\delta}{i\delta J})^T$$

- The connected bubble graphs is given:

$$i\Gamma = e^{i\Delta S_2}\, e^{i\tilde{S}_2} = \sum_{n\geq 1} i\Gamma_{(n)}$$

for example:

$$\begin{aligned} i\Gamma_1 &= \frac{i}{2}\Big[\int \mathrm{d}^8 z \frac{\delta}{i\delta J}\mathcal{M}(\frac{\delta}{i\delta J})^T\Big]_1 \Big[-\frac{1}{2}\int \mathrm{d}^8 z\, J^T \Delta J\Big]_2 \\ &= -\frac{1}{2}\int (\mathrm{d}^8 z)_{12}\, \mathrm{tr}\Big(\mathcal{M}_1 X_{21}\Delta_2 X_{21}\Big) \end{aligned}$$

with

$$X_{21} = \Big(\frac{\delta}{i\delta J}\Big)_1^T J_2^T$$

Superspace integrals

- The full superspace integral is

$$\int \mathrm{d}^8 z = \int \mathrm{d}^4 x \, \mathrm{d}^4\theta = \int \mathrm{d}^4 x \Big(-\frac{1}{4} D^2\Big)\Big(-\frac{1}{4}\bar{D}^2\Big)$$

with superspace covariant derivatives:

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^n_{\alpha\dot{\alpha}}\,\bar{\theta}^{\dot{\alpha}}\partial_n, \quad D^2 = D^\alpha D_\alpha,$$

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma^n_{\alpha\dot{\alpha}}\,\partial_n \quad \bar{D}^2 = D^{\dot{\alpha}}\,D_{\dot{\alpha}}$$

- The chiral subintegral is given by

$$\int \mathrm{d}^6 z = \int \mathrm{d}^4 x \, \mathrm{d}^2\theta = \int \mathrm{d}^4 x \Big(-\frac{1}{4} D^2\Big)$$

- The superspace covariant derivatives have the following properties:

$$D^2\bar{D}^2 D^2 = 16\Box D^2, \quad \bar{D}^2 D^2 \bar{D}^2 = 16\Box\bar{D}^2, \quad \bar{D}^2 D^2\phi = 16\Box\phi$$

- We can write a chiral integral as a full superspace integral:

$$\begin{aligned}\int \mathrm{d}^4x\,\mathrm{d}^2\,\theta\phi\cdot j &= \int \mathrm{d}^4x\,\phi\Big(\frac{\bar{D}^2 D^2\, j}{16\Box}\Big)\\ &= \int\,\mathrm{d}^2\,\theta\Big(-\frac{1}{4}\,\bar{D}^2\Big)\int \mathrm{d}^4x\phi\Big(-\frac{\bar{D}^2\, j}{4\Box}\Big)\\ &= -\int\,\mathrm{d}^4\,\theta\,\mathrm{d}^4x\phi\Big(\frac{\bar{D}^2\, j}{4\Box}\Big)\end{aligned}$$

- The general superspace δ–function is $\delta_{21} = \delta^4(x_2 - x_1)\,\delta^4(\theta_2 - \theta_1)$
-