Two-color quark matter at nonzero temperature and density

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Hefei, November 3, 2011

Outline

1. Motivation
2. QCD-like theories
3. Thermodynamics of two-color QCD
4. Center-symmetric effective theory for two-color QCD
5. Outlook & challenges

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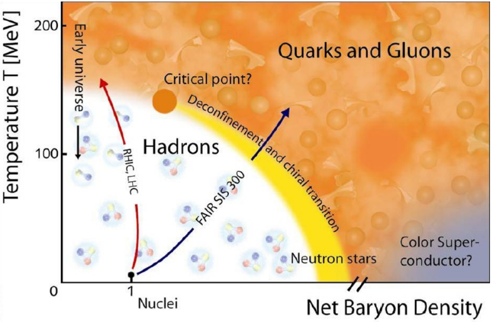
QCD phase diagram: What we (would like to) know

Low density and
high temperature:

- Lattice simulations.
- Heavy ion collisions.



http://www.gsi.de

High density and low temperature:

- Lattice simulation not feasible due to sign problem.
- Experimental data inconclusive.

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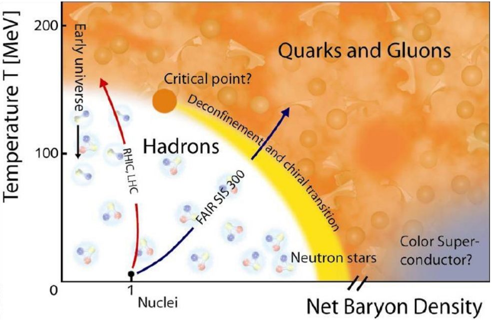

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High density and low temperature:

- Lattice simulation not feasible due to sign problem.
- Experimental data inconclusive.
- Are effective models all we can do?

Theorist's dream

Theory	Analytic approach
QCD	Solve exactly

Theorist's nightmare

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Freedman, McLerran, PRD 16 (1977) 1130, 1169
Kurkela, Romatschke, Vuorinen,
PRD 81 (2010) 105021

Perturbation theory

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Theory	Analytic approach	Lattice simulation
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↓		↓
QC_2D, aQCD, …		No sign problem

Theorist's dream

Theory

QCD

$\downarrow$

QC_2D, aQCD, …

Analytic approach

Solve exactly

$\dashrightarrow$

???

Lattice simulation

Sign problem

$\downarrow$

No sign problem

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Theory	Analytic approach	Lattice simulation
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↓	↓ Kanazawa, Wettig, Yamamoto, JHEP 08 (2009) 003	↓
QC_2D, aQCD, …	Low-energy EFT	No sign problem

Theorist's dream

Theory	Analytic approach	Lattice simulation
QCD	Solve exactly	Sign problem
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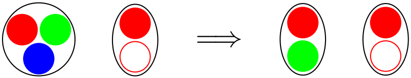
QC_2D, aQCD, …	Effective models	No sign problem

TB, Fukushima, Hidaka, PRD 80 (2009) 074035
Andersen, TB, PRD 81 (2010) 096004
Zhang, TB, Rischke, JHEP 1006 (2010) 064

Why QC_2D or aQCD (or similar)?

- No sign problem $\Longrightarrow$ lattice simulation feasible.
- Use the results to discriminate between the models.
- Baryons in QCD-like theories are bosons!



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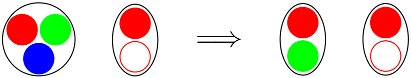

Ideal for model building!

- No annoying three-body physics at low density.
- Gauge-invariant order parameter at high density.
- Decent chance of describing low & high density matter with a single model: a dream of nuclear (astro)physicists!

Why models?

First a bit of pessimism.

- How reliable are the results?
  - Check model dependence: bad news.
  - Check approximation dependence.

- Don't go too far, it is not worth of the effort.

Why models?

There are some good news too.

- Model calculations are usually economical.
- May be used for a first rough calculation.
- Help to identify interesting problems.
  (Much literature on color superconductivity.)
- Can test ideas used in other approaches.
  Andersen, Kyllingstad, Splittorff, JHEP 01 (2010) 055

Outline

What QCD-like theories do I have in mind?

Pseudoreal theories

- "QC_2D" = two-color QCD with fundamental quarks.
- (Almost) the same structure is shared by all QCD-like theories with quarks in a pseudoreal representation.

Real theories

- "aQCD" = QCD with adjoint quarks.
- (Almost) the same structure is shared by all QCD-like theories with quarks in a real representation.

What makes them interesting? (pseudoreal)

Lattice simulation.

- Determinant of Dirac operator real even at nonzero chemical potential.
- Even number of flavors with equal chemical potentials $\Longrightarrow$ no sign problem.

Spectrum and the phase diagram.

- Baryons are bosons antisymmetric in color.
- Nonzero density realized by BEC of diquarks rather than a Fermi sea of nucleons.
- Global symmetry of theory with N_f massless quarks is not $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$, but rather $SU(2N_f)$!

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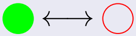
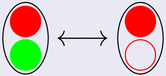
More about the flavor symmetry (pseudoreal)

The flavor symmetry ...

- q and $\bar{q}$ have the same color transformation properties.
- Exchange  does not affect color symmetry.
- Trade $q_{\mathrm{R}} \to q_{\mathrm{L}}^{\mathcal{C}} \Longrightarrow$ the theory effectively has $2N_f$ flavors of Weyl fermions, thus $\mathrm{SU}(2N_f)$.

... and its consequences.

-  is a symmetry of the theory.
- Multiplets of states contain both mesons and diquarks.
- There are diquark NG bosons of $\mathrm{SU}(2N_f) \to \mathrm{Sp}(2N_f)$.
- $N_f = 2$: five NG bosons $\pi^0, \pi^{\pm}, \Delta, \Delta^*$.
- Dense matter in reach of chiral perturbation theory!

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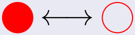
- Exchange  does not affect color symmetry.
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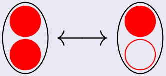
-  is a symmetry of the theory.
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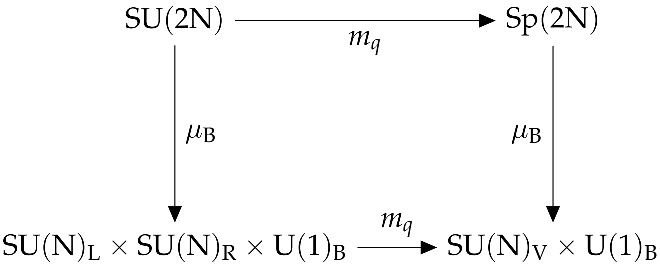
Symmetry-breaking patterns (pseudoreal)



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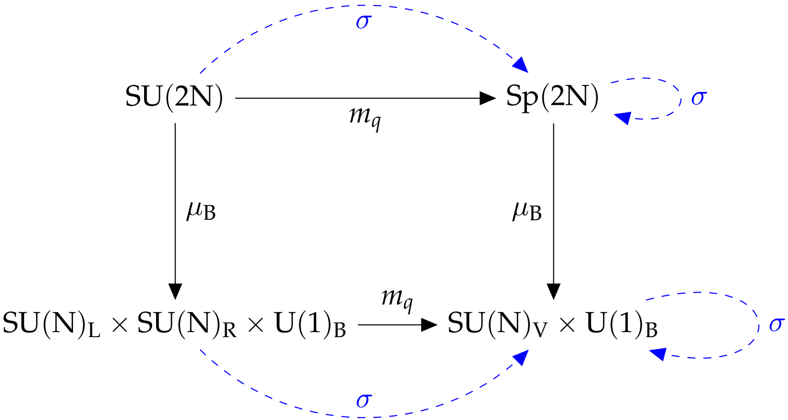

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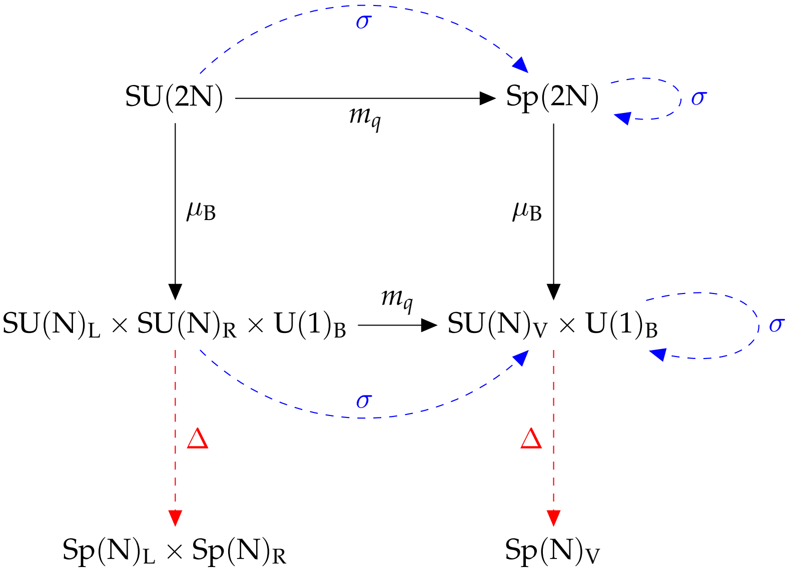
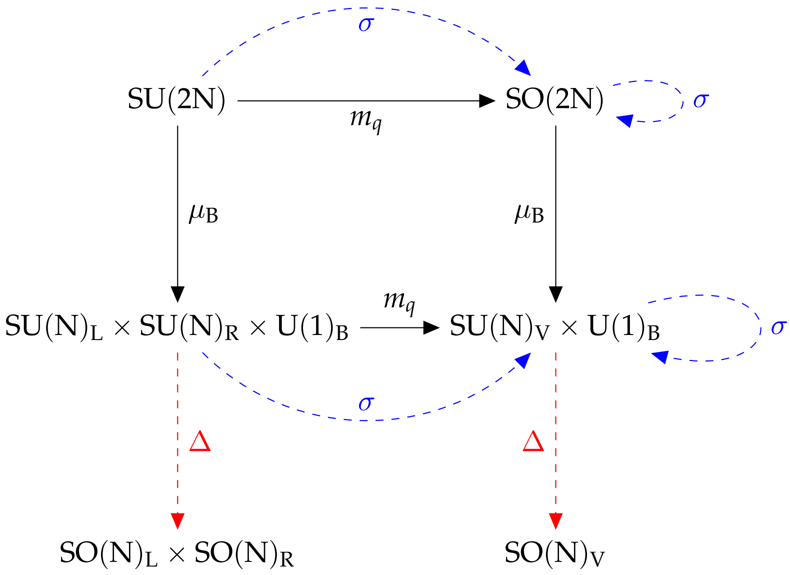


Symmetry-breaking patterns (real)



Outline

Model setup

- We use the (P)NJL model in the mean-field approximation:

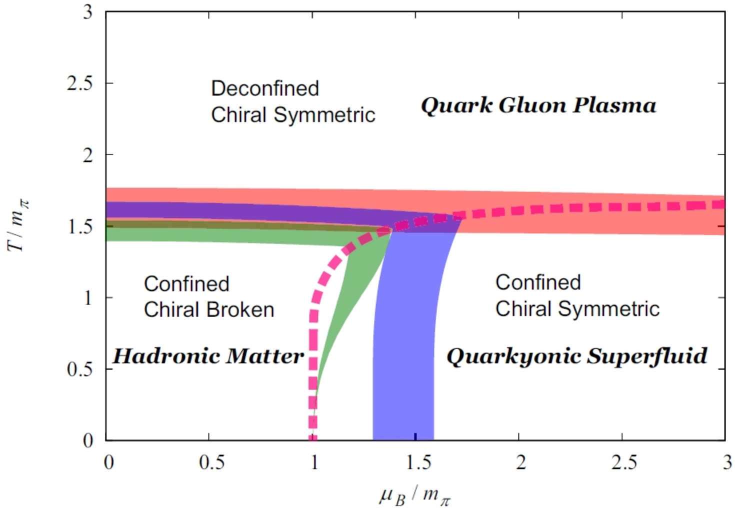
$$\mathcal{L} = \overline{\psi}(\mathrm{i}\not{D} - m_0)\psi + G\left[(\overline{\psi}\psi)^2 + (\overline{\psi}\mathrm{i}\gamma_5\vec{\tau}\psi)^2 + |\overline{\psi^{\mathcal{C}}}\gamma_5\sigma_2\tau_2\psi|^2\right]$$

- Diquark and meson couplings same thanks to SU(4).
- Input physical quantities:

$$T_c = 270\text{ MeV} \qquad \sigma_s = (425\text{ MeV})^2$$
$$\langle\overline{\psi}_u\psi_u\rangle = (-218\text{ MeV})^3 \qquad f_\pi = 75.4\text{ MeV} \qquad m_\pi = 140\text{ MeV}$$

- Alternative: O(6) linear sigma model at tree level.
- Improves on LO χPT by including finite m_σ effects.

Phase diagram



Ratti, Weise, PRD 70 (2004) 054013; Sun, He, Zhuang, PRD 75 (2007) 096004

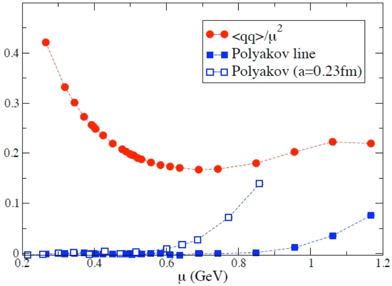
TB, Fukushima, Hidaka, PRD 80 (2009) 074035

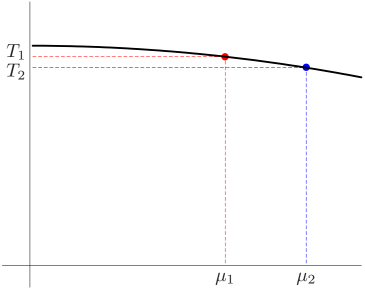
Deconfinement in dense matter

- The insensitivity of deconfinement temperature to μ_{B} is an obvious artifact of the PNJL model.

Deconfinement in dense matter

- The insensitivity of deconfinement temperature to μ_B is an obvious artifact of the PNJL model.
- However, can it still be right?

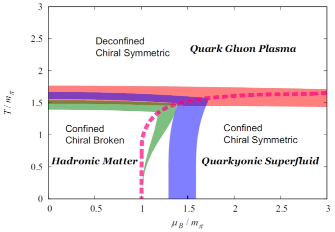


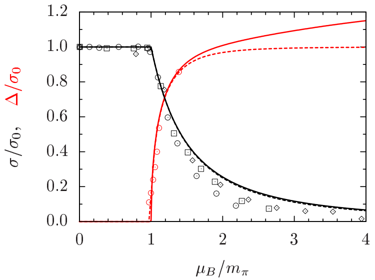


Hands, Kim, Skullerud, PRD 81 (2010) 091502

Chiral regime

- Vacuum: $\langle\overline{\psi}\psi\rangle \neq 0$.
- $\mu_B > m_\pi$: diquark condensation.
- $\langle\psi\psi\rangle \neq 0$.

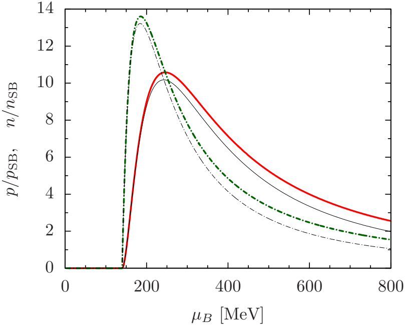


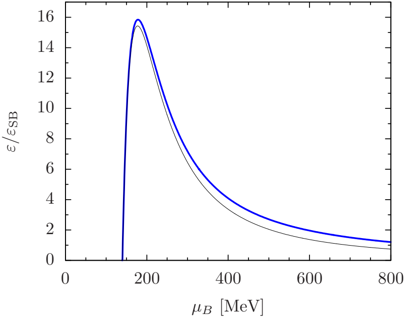


- LO χPT at finite μ_B:
  Kogut et al., NPB 582 (2000) 477
- Lattice simulation (staggered adjoint quarks):
  Hands et al., EPJC 17 (2000) 285; EPJC 22 (2001) 451
- NJL model calculation:
  Ratti, Weise, PRD 70 (2004) 054013

Thermodynamics of BEC transition

$$m_\sigma / m_\pi = \infty \qquad (\chi\text{PT})$$



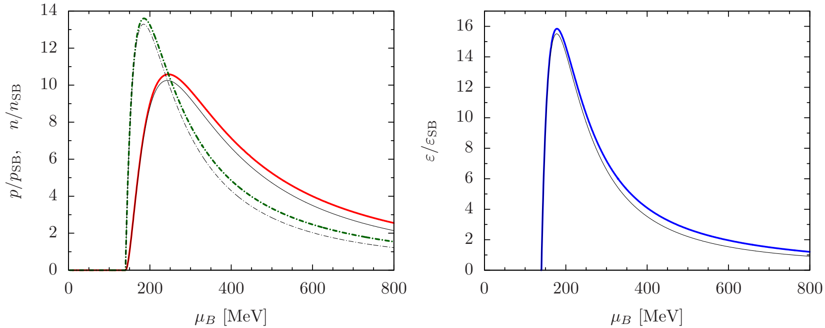


- NJL and linear sigma models give identical results.
- Explanation: condensate contribution dominates!
- Do the models stand comparison with lattice?

Andersen, TB, PRD 81 (2010) 096004

Thermodynamics of BEC transition

$$m_\sigma / m_\pi = 20 \qquad (\ell\sigma\text{m})$$



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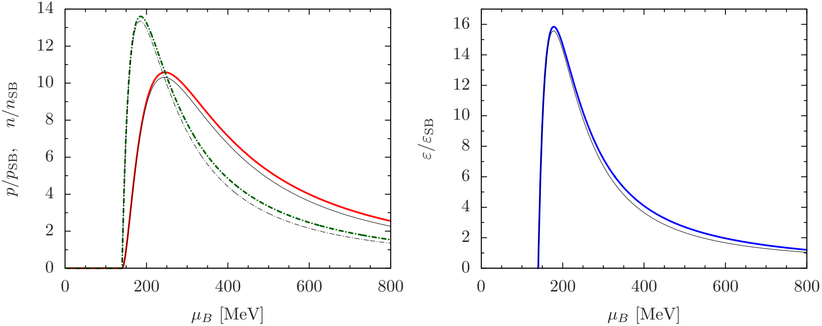
Andersen, TB, PRD 81 (2010) 096004

Thermodynamics of BEC transition

$$m_\sigma / m_\pi = 15 \qquad (\ell\sigma\mathrm{m})$$



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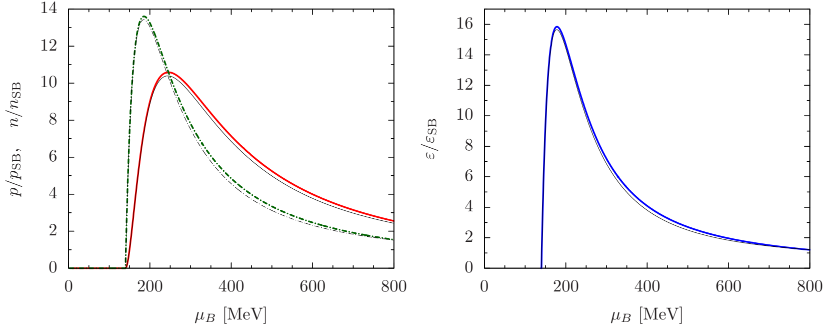
Andersen, TB, PRD 81 (2010) 096004

Thermodynamics of BEC transition

$$m_\sigma / m_\pi = 12 \qquad (\ell\sigma\text{m})$$



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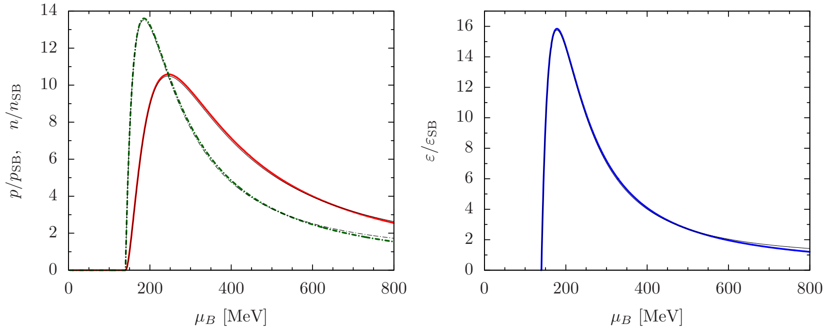
Andersen, TB, PRD 81 (2010) 096004

Thermodynamics of BEC transition

$$m_\sigma / m_\pi = 10 \qquad (\ell\sigma\text{m})$$



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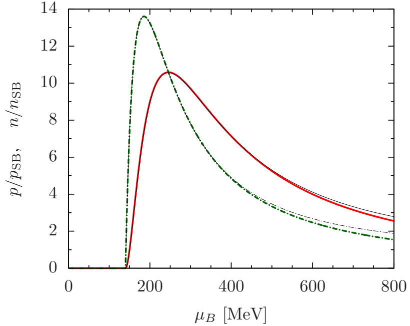
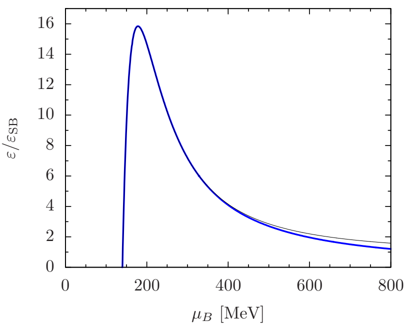
Andersen, TB, PRD 81 (2010) 096004

Thermodynamics of BEC transition

$$m_\sigma / m_\pi = 9 \qquad (\ell\sigma\text{m})$$





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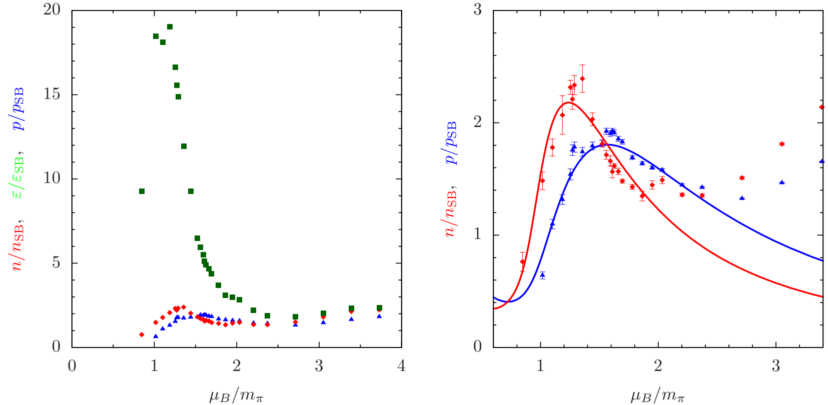
Andersen, TB, PRD 81 (2010) 096004

Thermodynamics of BEC transition II

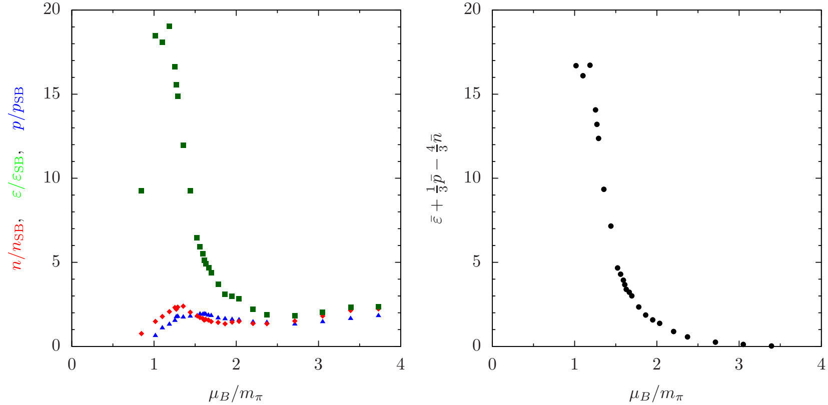
Hands, Kim, Skullerud, EPJC 48 (2006) 193; PRD 81 (2010) 091502



- Simulations at nonzero temperature and diquark source.
- Data for pressure and density can be reasonably explained using LO χPT with source term: dilute Bose gas.

Thermodynamics of BEC transition III

High peak in the energy density!



- Energy dominated by entropy/thermal component.
- Inclusion of thermal order parameter fluctuations needed, NLO χPT or beyond-mean-field NJL.

Outline

Basic ingredients

Dimensional reduction

4d (Euclidean) quantum field theory at high temperature reduces to a 3d theory of the zero Matsubara mode.

- Heavy modes: "hard mass" $\omega_n = 2\pi n T, n \neq 0$.
- Light modes: "soft mass" $\propto gT$ by loop corrections.
- Dimensionally reduced theory of QCD: EQCD.
- Degrees of freedom: $3d$ gauge field $\mathbf{A}_a$ + adjoint scalar A_a^0.

$$\mathcal{L}_{\text{EQCD}} = \frac{1}{4}(F_{ij}^a)^2 + \frac{1}{2}(D_i A_0^a)^2 + \frac{1}{2} m_E^2 (A_0^a)^2 + \frac{1}{8}\lambda_E (A_0^a A_0^a)^2$$

- The EFT determines physics on length scales $\propto 1/gT$.

  Braaten, Nieto, PRD 53 (1996) 3421

Basic ingredients (continued)

Center symmetry

Global Z_N symmetry of the Yang–Mills theory; its spontaneous breaking is associated with deconfinement phase transition.

- Second/first order transition for two/three colors.
- Order parameter: Polyakov loop.

$$\Omega(\mathbf{x}) = \mathrm{Tr}\left\{\mathcal{P}\exp\left[ig\int_0^\beta d\tau\, A_0(\tau,\mathbf{x})\right]\right\}$$

- EQCD breaks Z_N explicitly by expanding around one of the N degenerate minima.

Vuorinen, Yaffe, PRD 74 (2006) 025011
de Forcrand, Kurkela, Vuorinen, PRD 77 (2008) 125014
Zhang, TB, Vuorinen, in progress

Construction of the theory

- Basic degree of freedom: coarse-grained Polyakov loop $\mathcal{Z}$.
- $\mathcal{Z}$ acts as an adjoint scalar.
- Center symmetry transformation: $\mathcal{Z} \to \pm\mathcal{Z}$.
- $\mathcal{Z}$ is unitary up to a real scale factor:

$$\mathcal{Z} = \frac{1}{2}(\Sigma + \mathrm{i}\sigma_a \Pi_a)$$

- Superrenormalizable 3d gauge theory of $\mathcal{Z}$:

$$\mathcal{L} = \frac{1}{g_3^2}\left\{\frac{1}{2}\,\mathrm{Tr}\,F_{ij}^2 + \mathrm{Tr}\left(\mathcal{D}_i\mathcal{Z}^\dagger\mathcal{D}_i\mathcal{Z}\right) + V(\mathcal{Z})\right\}$$

$$V(\mathcal{Z}) = b_1\Sigma^2 + b_2\Pi_a^2 + c_1\Sigma^4 + c_2(\Pi_a^2)^2 + c_3\Sigma^2\Pi_a^2 + d_1\Sigma^3 + d_2\Sigma\Pi_a^2$$

Scales and degrees of freedom

Scales

- Scale T: "amplitude mode" $|\mathcal{Z}(\mathbf{x})|$; to be integrated out.
- Scale gT: electric gluons; phases of $\mathcal{Z}(\mathbf{x})$.
- Scale g^2T: magnetic gluons; 3d gauge potential $A_i(\mathbf{x})$.

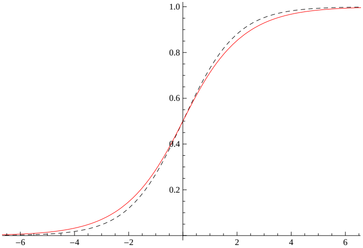
Couplings

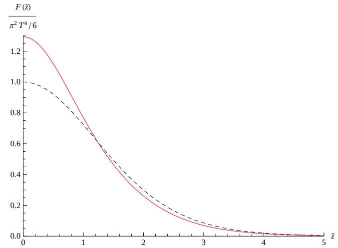
- How to ensure the hierarchy of scales: use global "$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$" symmetry.
- Preserved by "hard" couplings h_i of order T.
- Broken to $\mathrm{SU}(2)_\mathrm{V} \times \mathrm{Z}_2$ by "soft couplings" s_i.

$$b_1 = \frac{1}{2}h_1, \quad b_2 = \frac{1}{2}(h_1 + g_3^2 s_1), \quad d_1 = \frac{1}{2}g_3^2 s_4, \quad d_2 = \frac{1}{2}g_3^2 s_5$$
$$c_1 = \frac{1}{4}h_2 + g_3^2 s_3, \quad c_2 = \frac{1}{4}(h_2 + g_3^2 s_2), \quad c_3 = \frac{1}{2}h_2$$

Perturbative matching

- Match to Z_2-symmetric one-loop Weiss potential of QCD.
- Reduces to Taylor coefficients (EQCD) and period.
- Domain wall tension predicted at 8% from YM value.



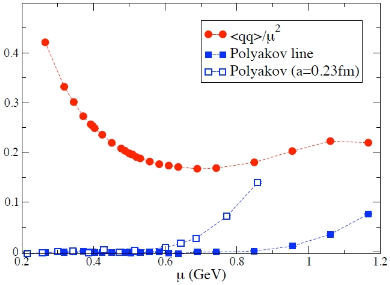


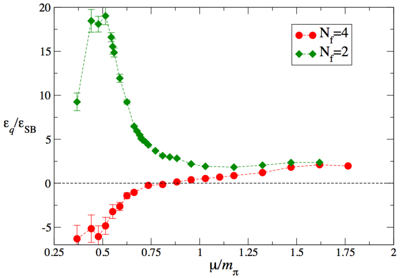
- Explicit Z_2 breaking by quarks: use bubble solution.
- Remaining parameter(s) (to be) fixed nonperturbatively.
- Predictions for QC_2D thermodynamics as a function of N_f, quark masses and chemical potentials (in progress).

Outline

Outlook & challenges

If you are interested in (deconfinement in) cold dense matter,
understand available lattice data on dense two-color QCD first!





Hands, Kenny, Kim, Skullerud, 1101.4961 [hep-lat]

Ideal playground for understanding dense matter:

- Simplified modeling due to two-body physics for baryons.
- (Some) lattice data available and more to come.