Quantum chaos and transports

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USTC, 2019.4.25

1905.XXXXX, Wei Li, SL, Jiajie Mei

Outline

- Introduction to Quantum chaos
- Universal bounds on transport coefficients
- Bound on thermal diffusion in incoherent metal
- Magnetohydrodynamics
- Bound on thermal diffusion in magnetized plasma
- A special point and pole-skipping
- Summary&Outlook

Quantum chaos

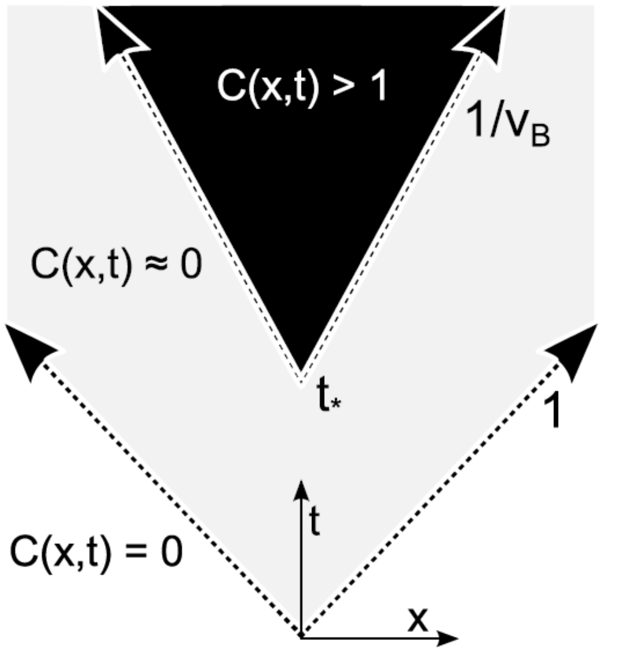
Out of time order correlator (OTOC)

$$C(t_w, |x-y|) = \mathrm{tr}\left\{\rho(\beta)[W_x(t_w), W_y]^\dagger [W_x(t_w), W_y]\right\} \sim e^{\lambda_L (t - t_* - \frac{|\vec{x}|}{v_B})}$$

t_*: scrambling time

λ_L: Lyapunov exponent

v_B: Butterfly velocity



Roberts, Stanford, Susskind, JHEP 2015

Maximally chaotic systems

Maldacena-Shenker-Stanford (MSS) conjecture

$$\lambda_L \leq 2\pi T$$

Maldacena, Shenker, Stanford JHEP 2016

Bound saturated by holographic models, strongly coupled SYK models

Universal bounds on transports

Kovtun, Son, Starinets	$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$		plasma
Hartnoll's reinterpretation	$\frac{\eta}{s} \sim \frac{DT}{c^2}$	$D \gtrsim \frac{c^2 \hbar}{T k_B}$	plasma
Hartnoll's proposal	$D \gtrsim \frac{v^2 \hbar}{T k_B}$		Incoherent metal

Hartnoll, Nature Physics 2015

Hydrodynamics in incoherent metal

consider neutral system, decoupled dynamics

Incoherent metal: energy density becomes diffusive

$$\partial_t \epsilon + \nabla \cdot j_E = 0$$
$$j_E = -\kappa \nabla T$$

$$D_E = \frac{\kappa}{c_\rho}$$

Hartnoll, Nature Physics 2015

Charge remains diffusive

$$\partial_t \rho + \nabla \cdot j = 0$$
$$j = -\sigma \nabla \mu$$

$$D_c = \frac{\sigma}{\chi}$$

$$D_{E/c} \gtrsim \frac{v^2 \hbar}{T k_B}$$

Which v should be used?

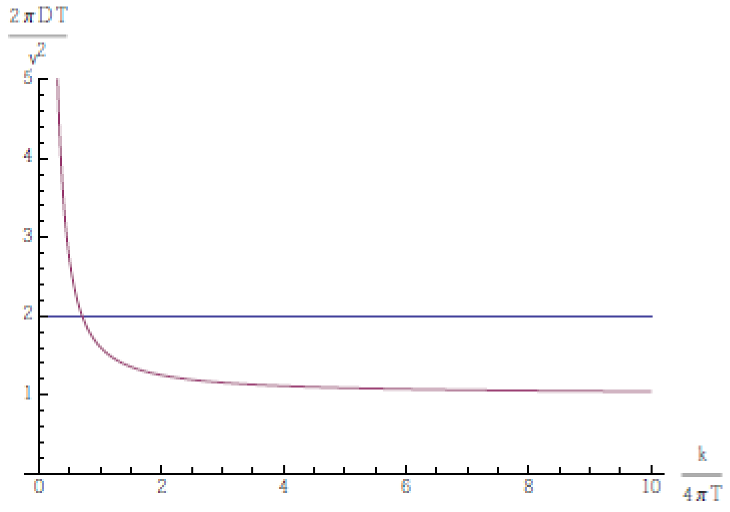
Blake's bound with butterfly velocity

$$D_c = C\frac{v_B^2}{2\pi T}$$ charge diffusion

$$D_E = E\frac{v_B^2}{2\pi T}$$ thermal diffusion

Blake PRL 2016

The bounds relate transport coefficients to chaotic quantities!



k: strength of
momentum dissipation

Blake JHEP 2016

$$C = 2, E \geq 1$$

Transport bound for thermal diffusion only

Two Examples

holography with curvature correction: C can be arbitrary small, E remains finite

Baggioli, Gouteraux, Kiritsis, Li, JHEP 2017

field theory of critical Fermi surface: C not universal, E universal

$$v_B \approx 4.10 \frac{NT^{1/3}}{e^{4/3}} \frac{v_F^{5/3}}{\gamma^{1/3}}$$

$$\lambda_L \approx 2.48\, T$$

$$D^E \approx 0.42 \frac{v_B^2}{\lambda_L}.$$

Patel, Sachdev, Proc. Nat. Acad. Sci. 2017

Magnetohydrodynamics (MHD)

Hydrodynamics with external magnetic field $B \sim O(1),\, E \sim O(\partial)$

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P} \Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

$$J^\mu = \mathcal{N} u^\mu + \mathcal{J}^\mu$$

$$\nabla_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \,,$$

$$\nabla_\mu J^\mu = 0 \,.$$

Hernandez, Kovtun, JHEP 2017
Grozdanov, Hofman, Iqbal, PRD 2017
Huang, Sedrakian, Rischke, Annal Phys 2011
Finazzo, Critelli, Rougemont, Noronha, PRD 2016

Gapless modes of MHD

$k \parallel B$:

$$\omega = \pm k v_s - i \frac{\Gamma_{s,\parallel}}{2} k^2 \,,$$

$$\omega = -i D_{\parallel} k^2 \,,$$

propagation of energy density

$k \perp B$:

$$\omega = -i D_{\perp} k^2 \,,$$

$$\omega = -i \frac{\eta_{\parallel} k^2}{w_0} \,,$$

$$\omega = -i \frac{\eta_{\perp} k^4}{B_0^2 \chi_{33}} \,.$$

diffusion of energy density due to Lorentz force, analogous to momentum dissipation in incoherent metal

Thermal diffusion constant from MHD

Perturbed neutral plasma

$$J_y = i\omega M u_x - \sigma_\perp B_0 u_x - ik\delta M$$
$$\delta T^{tt} = c_V \delta T$$
$$\delta T^{xx} = \delta p - \delta M B_0$$
$$\delta T^{tx} = (\epsilon + p) u_x$$

B along z direction

M: magnetization

$$\omega = -i D_E k^2$$

$$D_E = \frac{(\epsilon + p)^2}{B_0^2 c_V T \sigma_\perp}$$

Holographic neutral magnetized plasma

$$S = \int d^5x\sqrt{-g}\left(R + 12 - \frac{1}{4}F_{MN}F^{MN}\right)$$

$$ds^2 = -U dt^2 + \frac{dr^2}{U} + e^{2V}\left((dx^1)^2 + (dx^2)^2\right) + e^{2W}dy^2,$$

$$F = B dx^1 \wedge dx^2.$$

D'Hoker, Kraus, JHEP 2009

UV fixed point $\quad U = e^{2V} = e^{2W} = r^2$

IR fixed point $\quad ds^2 = -3(r^2 - r_+^2)dt^2 + \frac{dr^2}{3(r^2 - r_+^2)} + \frac{B}{\sqrt{3}}\left((dx^1)^2 + (dx^2)^2\right) + 3r^2 dy^2$

magnetic brane is a domain wall
interpolating UV and IR fixed points

Thermal diffusion constant from holography

$$\pi^{\mu\nu} = Kh^{\mu\nu} - K^{\mu\nu},$$

$$J^{\mu} = -N^{-1}(F_r^{\mu} - N_{\nu}F^{\nu\mu}).$$

Hamiltonian formulation of bulk EOM

$$H_{\mu} \equiv 2\nabla_{\nu}\pi^{\nu}_{\mu} - F_{\mu\nu}J^{\nu} = 0,$$

$$C = \nabla_{\mu}J^{\mu} = 0.$$

Constraint equation on the horizon alone fixes D_E

Donos, Gauntlett, Ziogas, JHEP 2017

$$\omega = -iD_E k^2$$

$$D_E = \frac{(\epsilon + p)^2}{B_0^2 c_V T \sigma_{\perp}}$$

Shock wave in black hole background

Black hole in Kruskal coordinate

$$ds^2 = \ell_{AdS}^2 \Big[- A(uv)dudv + B(uv)dx^i dx^i \Big]$$

$W_x(t_w)|TFD\rangle = e^{-iH_L t_w} W_x \, e^{iH_L t_w} |TFD\rangle$ dual to particle injection in gravity

boosted particle energy becomes comparable with background, backreaction needed

$e^{2\pi t_w/\beta} \sim O(N^2)$ scrambling time $t_w \sim t_* = \frac{\beta}{2\pi} \log N^2$

particle compressed near horizon $(\delta T_{uu})_{particle} \sim E_0 e^{\frac{2\pi}{\beta} t_w} \delta(u)\delta(\vec{x})$

backreacted background: shock wave

$$ds^2 = \ell_{AdS}^2 \Big[- A(uv)dudv + B(uv)dx^i dx^i + A(uv)\delta(u)h(x)du^2 \Big]$$

Roberts, Stanford, Susskind, JHEP 2015
Shenker, Stanford, JHEP 2014

Shock wave in magnetic brane background

$$ds^2 = A(uv)\mathrm{d}u\mathrm{d}v - A(uv)\delta(u)h(\vec{x})\mathrm{d}u^2 + V_x(uv)\Big((\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2\Big) + V_y(uv)\mathrm{d}y^2$$

$$\delta G_{uu} = (\delta T_{uu})_{particle}$$

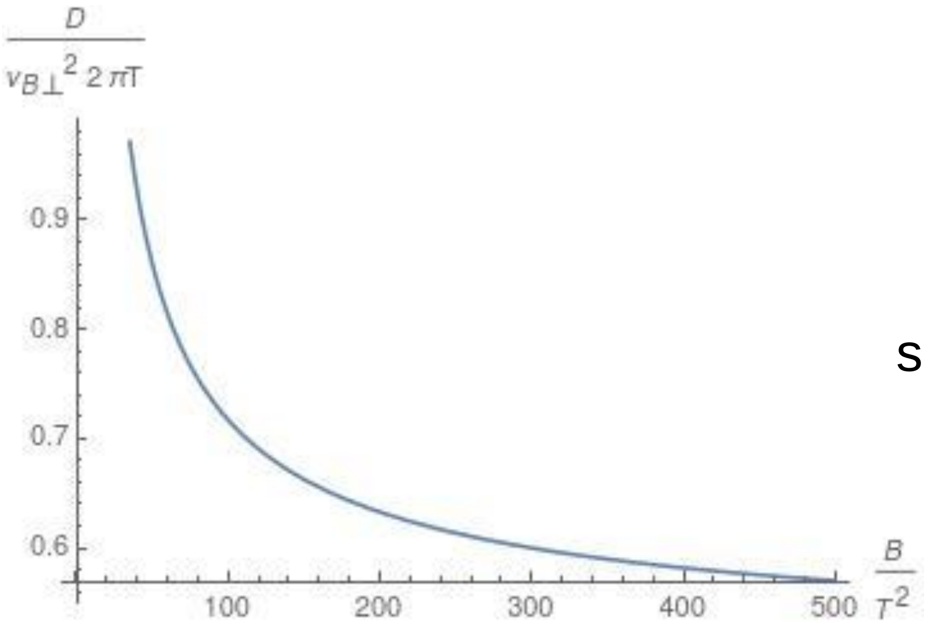
$$h(x_1, x_2, y) \sim \frac{E_0 e^{\frac{2\pi}{\beta}(t_w - t_*) - m|x|}}{|x|}$$

Butterfly velocity $\quad v_{x1}^2 = v_{x2}^2 = \frac{4\pi^2 T^2}{V_x(r_h)m^2} \qquad v_y^2 = \frac{4\pi^2 T^2}{V_y(r_h)m^2}$

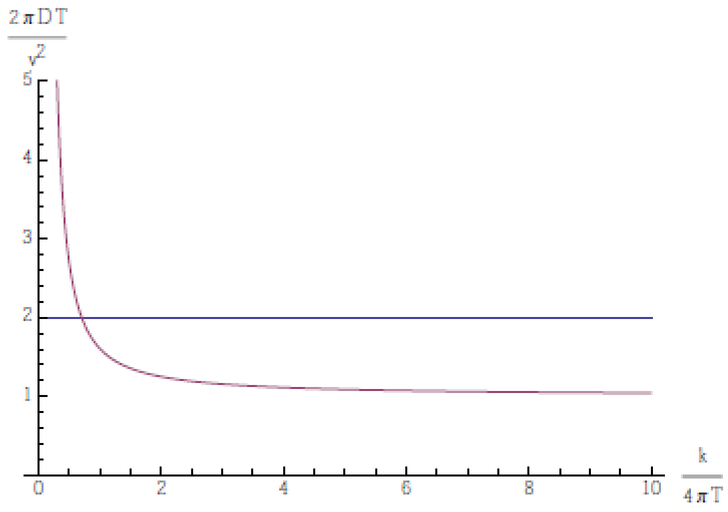
$$m^2 = \pi T\left(\frac{2V_x'(r_h)V_y(r_h) + V_y'(r_h)V_x(r_h)}{V_x(r_h)V_y(r_h)}\right)$$

only depends on horizon geometry

Bound on thermal diffusion constant



similar to



Large B limit can be obtained from BTZ background

$$v_{B\perp}^2 \to \frac{8\pi^2 T^2}{\sqrt{3}B}, D_\perp \to \frac{2\pi T}{\sqrt{3}B}$$

asymptotes $$\frac{D_\perp}{v_{B\perp}^2 2\pi \mathrm{T}} \to \frac{1}{2}$$

A special point

$$\omega = i\lambda\,, \qquad k = ik_0\,, \qquad k_0 = \frac{\lambda}{v_B}$$

$$\lambda = 2\pi T\,, \qquad k_0^2 = \frac{(2\pi T)^2}{v_B^2} = d\pi T h'(r_0)$$

vv-component of Einstein equation trivially satisfied

$$\left(-i\frac{d}{2}\omega h'(r_0) + k^2\right)\delta g_{vv}^{(0)} - i(2\pi T + i\omega)\left[\omega\delta g_{x^i x^i}^{(0)} + 2k\delta g_{vx}^{(0)}\right] = 0$$

corresponding energy density correlator undetermined at the special point!

$$G^R_{T_{00}T_{00}}(\omega, k) \propto \frac{\delta\omega - 4\lambda^2 q_z/v_B\delta k}{\delta\omega - 4\lambda^2 q_p/v_B\delta k}$$

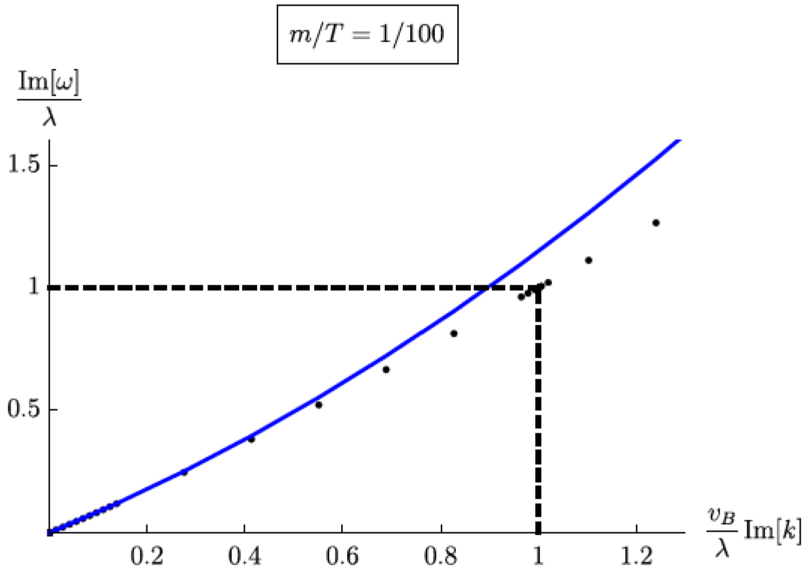
Pole-skipped at the special point

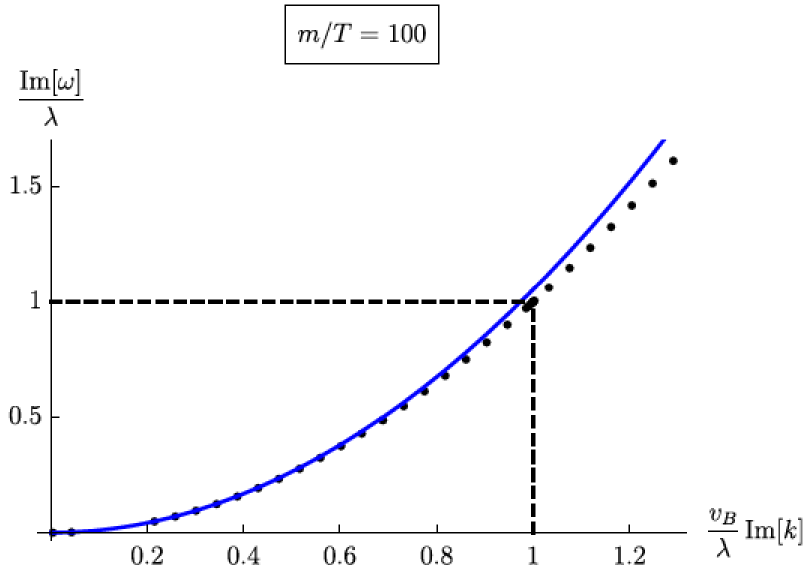
Blake, Davison, Grozdanov, Liu, JHEP 2018
Grozdanov, Schalm, Scopelliti, PRL 2018

Hydrodynamics at large $\omega \& k$?

$$S = \int d^4x \sqrt{-g} \left(\mathcal{R} + 6 - \frac{1}{2} \sum_{i=1}^{2} \partial_\mu \varphi_i \partial^\mu \varphi_i \right) \qquad \varphi_i = m x^i$$

m: strength of momentum dissipation





$$\omega(k) = \pm v_s k - i \frac{k^2}{8\pi T} + \ldots$$

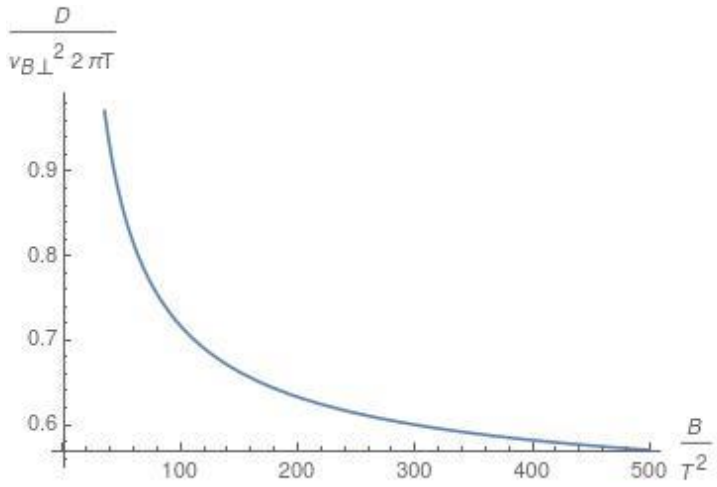
$$\omega(k) = -i D_E k^2 + \ldots$$

special point: $\omega = -\frac{i v_B^2}{\lambda} k^2$

$$\frac{m}{T} \to \infty, \qquad \frac{D_E}{v_B^2 \lambda} \to 1$$

Blake, Davison, Grozdanov, Liu, JHEP 2018

Coefficient in magnetized plasma diffusion



asymptotes $\frac{D_\perp}{v_{B\perp}^2 2\pi \mathrm{T}} \to \frac{1}{2}$

at certain B $\frac{D_\perp}{v_{B\perp}^2 2\pi \mathrm{T}} = 1$

IR: dimensional reduction to BTZ

IR geometry with hyperscaling violation

$$D_T = \frac{z}{2z-2} v_B^2 \tau_L$$

z: dynamical critical exponent

$$z \to \infty, \qquad \frac{D}{v_B^2 2\pi T} \to \frac{1}{2}$$

Blake, Davison, Sachdev, PRD 2017

Summary

- Magnetized plasma as another example of thermal diffusion bound
- Dimensional reduction leads to coefficient ½ rather than the standard 1
- Special point exists. Possible extension of hydrodynamics to large gradient at specific value of B?

Outlook

- Extension to charged magnetized plasma
- Quasi-normal mode confirmation of pole-skipping phenomenon
- Possible connection with resurgent theory?