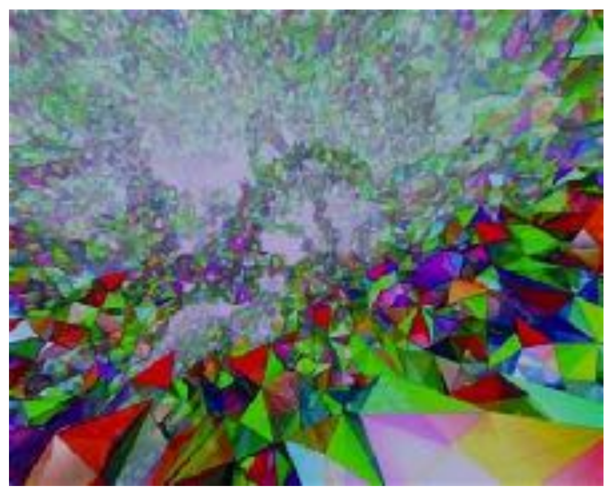
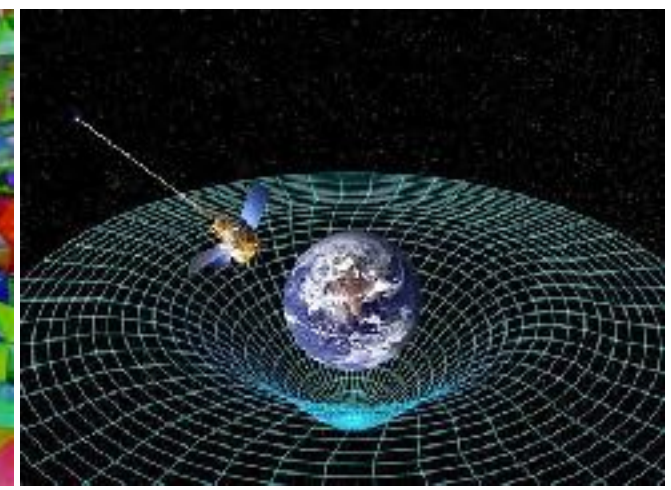
Spin Foam Model and Emergent Gravity

Muxin Han (韩慕辛)

（in collaboration with Zichang Huang (黄子罂) and Antonia Zipfel）

arXiv:1812.02110

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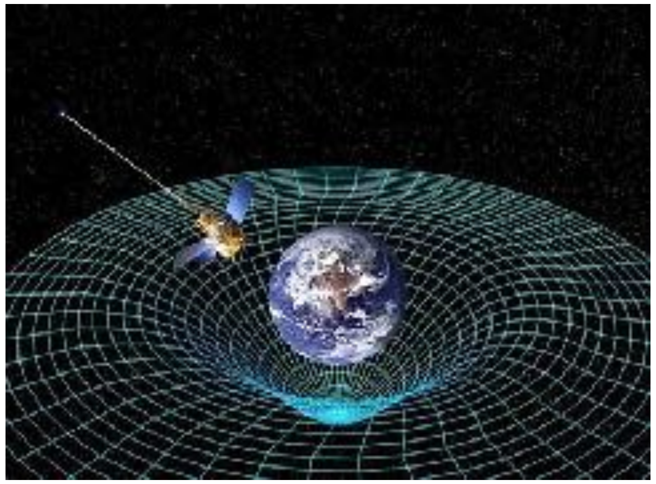
General Relativity: Gravity = Curved Spacetime Geometry



Quantum Gravity = Quantum Spacetime Geometry

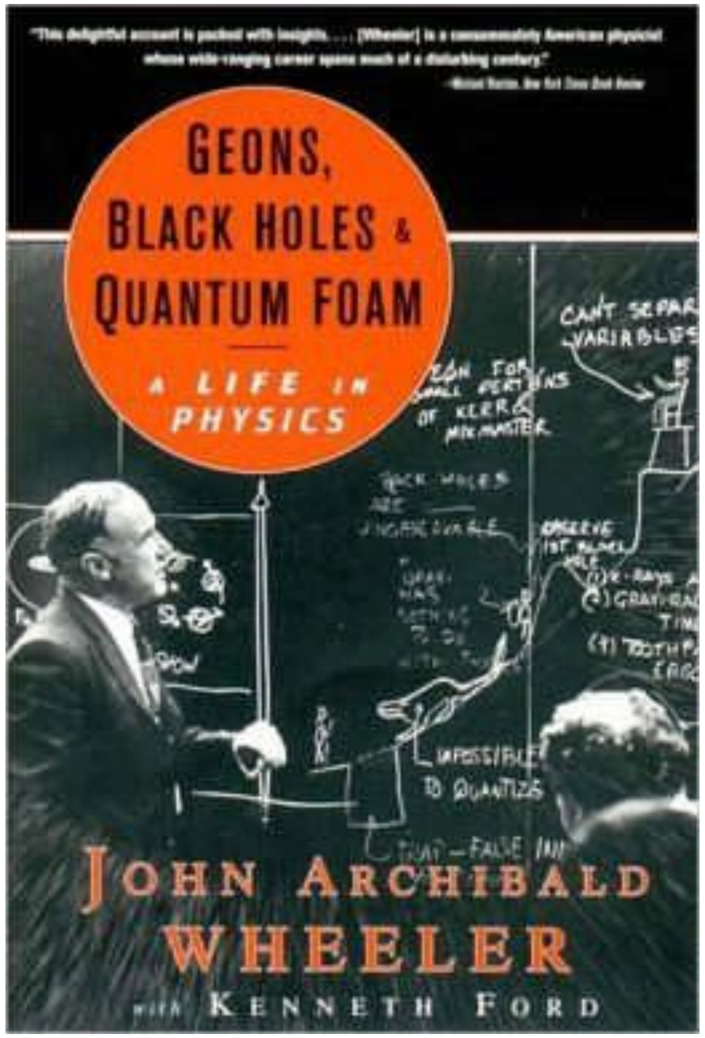
What is a Quantum Spacetime?

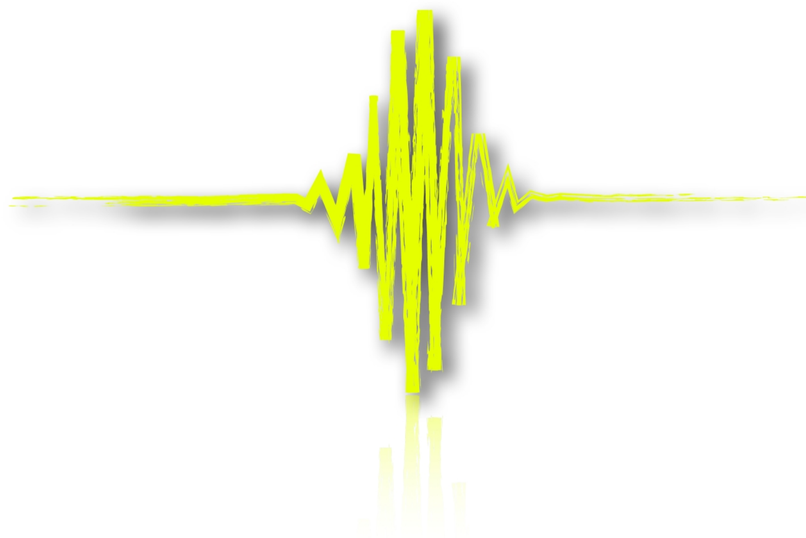
(In this talk, the spacetime dimension is 4 = 3+1)

Wheeler & Hawking's Quantum Foam

J.A. Wheeler, *Geometrodynamics and the issue of the final state*, in Relativity, groups and topology, C. DeWitt and B.S. DeWitt eds., Gordon and Breach, New York U.S.A. (1964).

S.W. Hawking, *Space-time foam*, Nucl. Phys. B 144 (1978) 349.

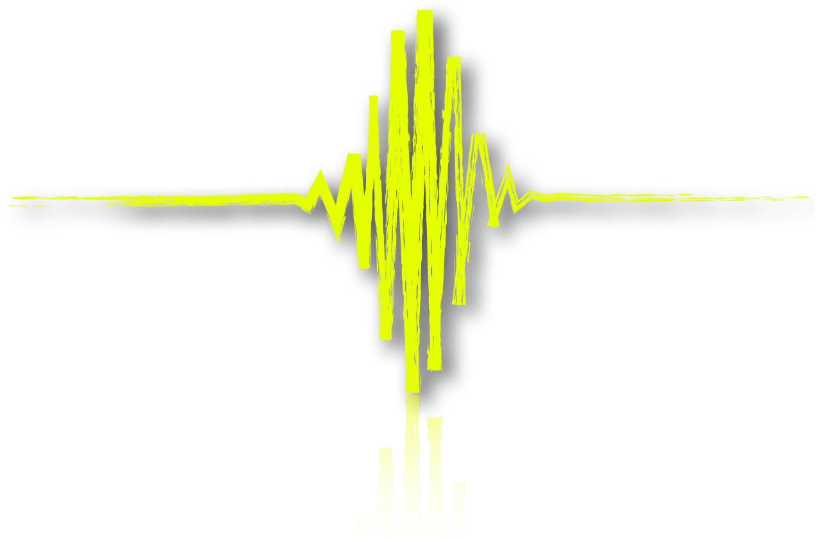




Quantum Mechanics:

If we localize too much a particle in spacetime, its energy and momentum grow.

$$\Delta t \, \Delta E \geq \hbar/2, \qquad \Delta x \, \Delta p \geq \hbar/2$$



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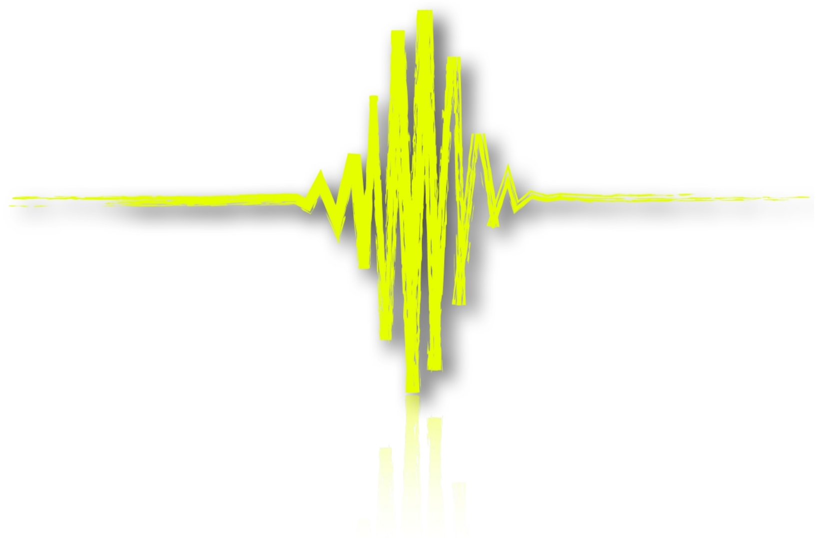
General Relativity:

If energy and momentum grow too much, it collapses and forms a black hole.

Singularities, infinities, difficulies



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General Relativity:

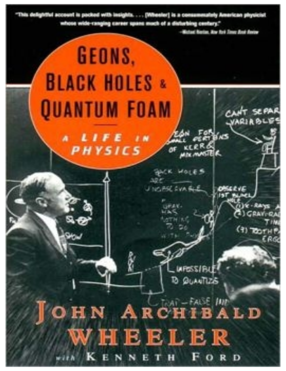
If energy and momentum grow too much, it collapses and forms a black hole.

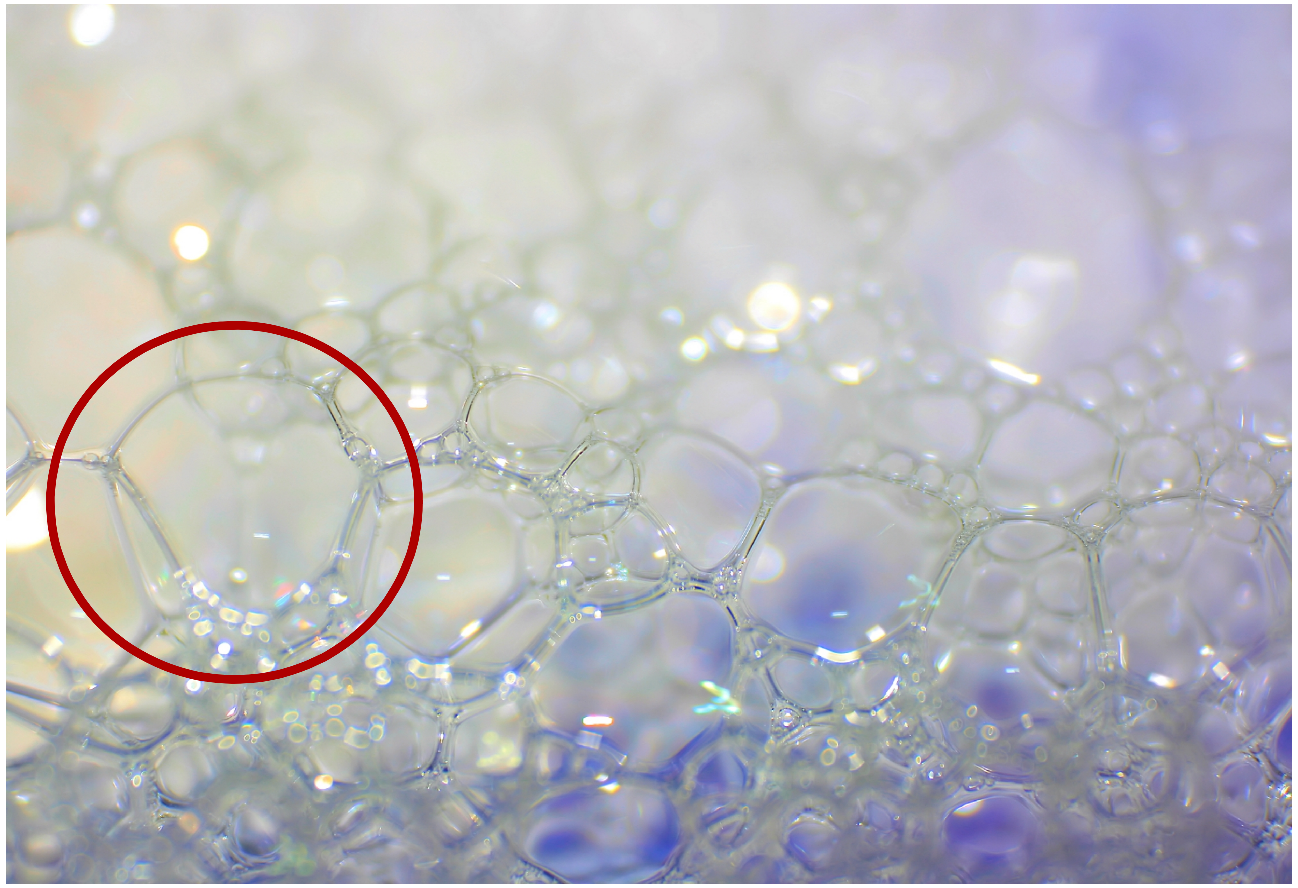
Singularities, infinities, difficulies

Resolution:

There is a minimal scale (Planck scale ℓ_p) in spacetime against localization.

Quantum Spacetime is Foam-like

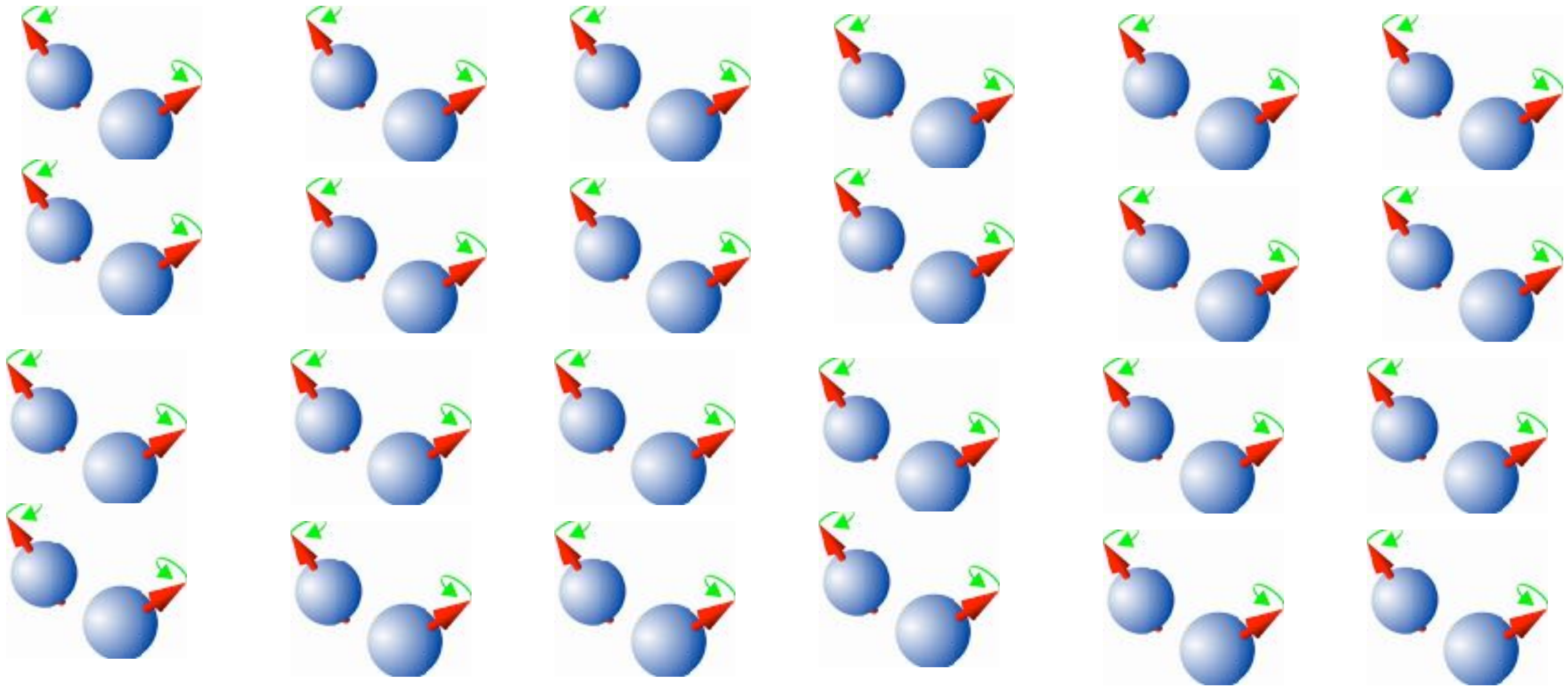




A minimal cell of quantum spacetime

A Complementary Point of View: Emergent Gravity

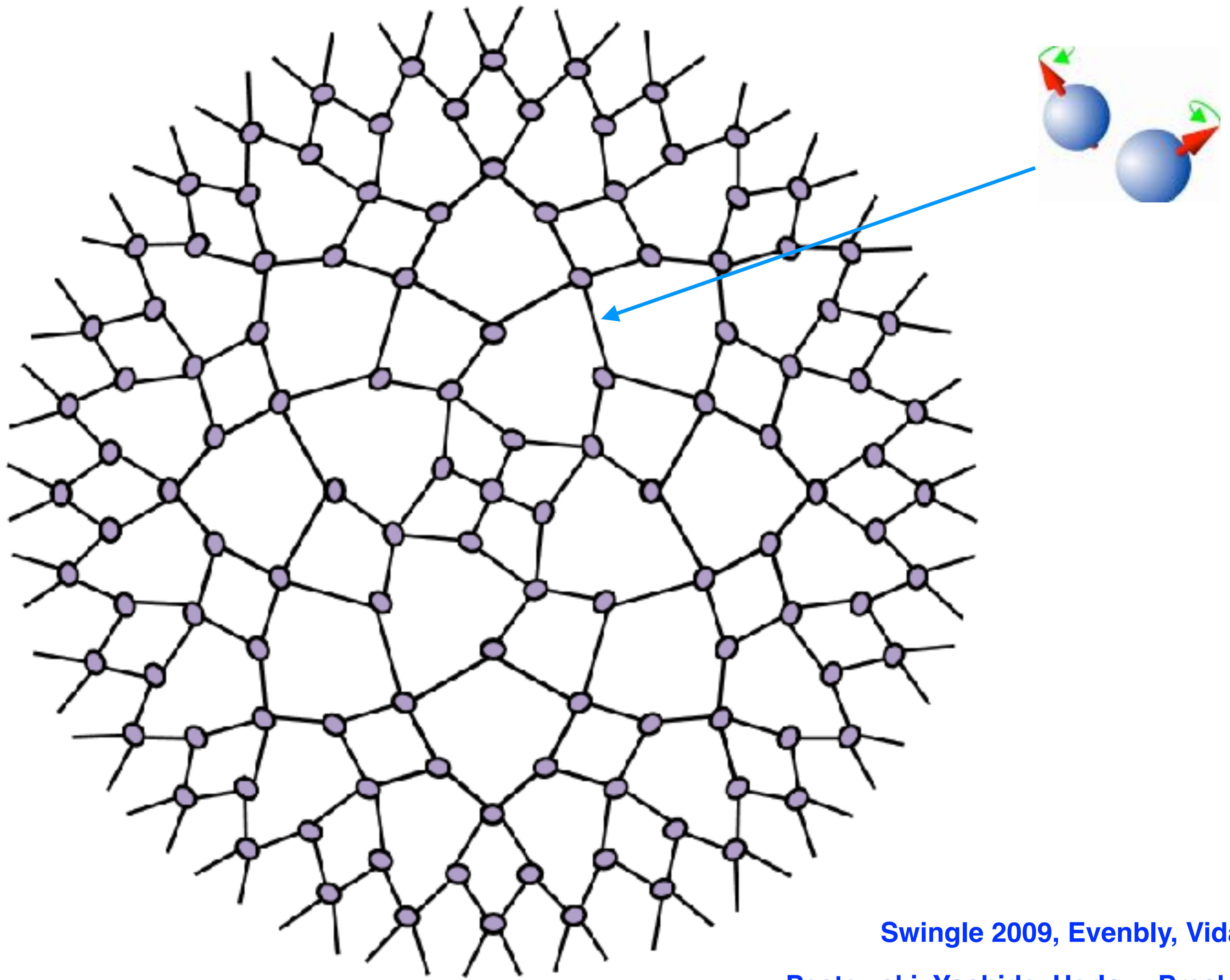
Quantum Gravity as an Ocean of Entangled Qubits



Xiao-Gang Wen, Zhenghan Wang 2018
Zheng-Cheng Gu, Xiao-Gang Wen 2009

A Complementary Point of View: Emergent Gravity

Quantum Spacetime is a Tensor Network



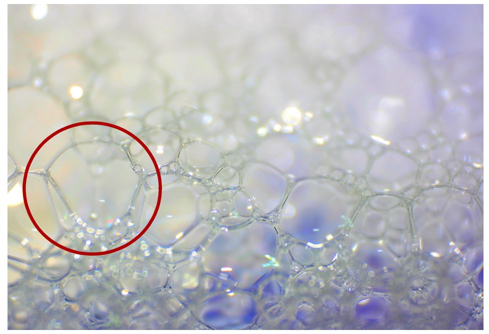
Swingle 2009, Evenbly, Vidal, 2014

Pastawski, Yoshida, Harlow, Preskill, 2015

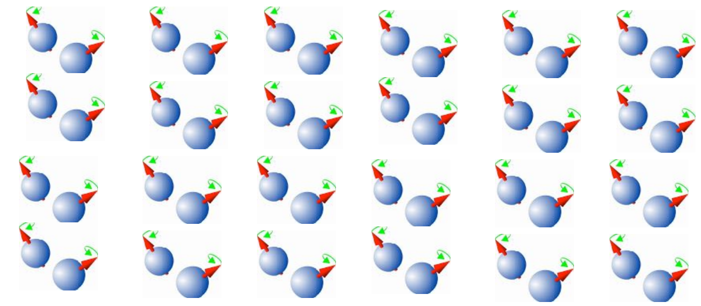
Hayden, Nezami, Qi, Thomas, Water, Yang 2016

Qi, Yang 2018

Question of Emergent Gravity (Semiclassical Consistency of QG Model)



Foam-like spacetime



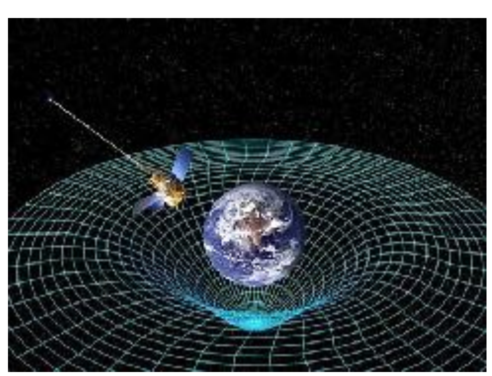
Ocean of entangled qubits

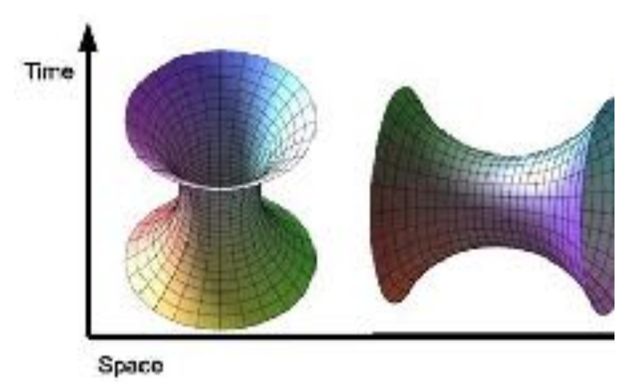
Quantum, Discrete, and Algebraic (fundamental)

Classical Gravity:

Smooth Spacetime Geometry

(emergent low energy excitations)



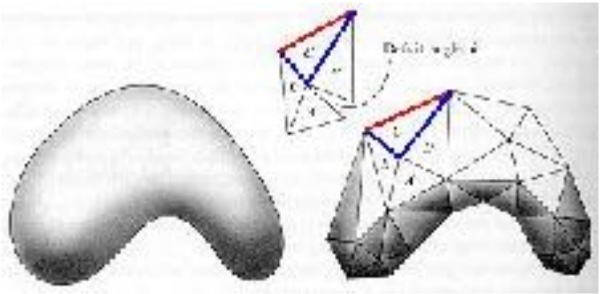


Spin Foam Model (SFM) and Emergent Gravity

- **The definition of SFM as a State-Sum Model and a Tensor Network**

$$Z(\mathcal{K}) = \sum_{\vec{J},\vec{I}} \prod_f A_f(J_f) \prod_\sigma A_\sigma(\vec{J},\vec{I})$$

- **Large Spin Asymptotics and Emergent Geometry**

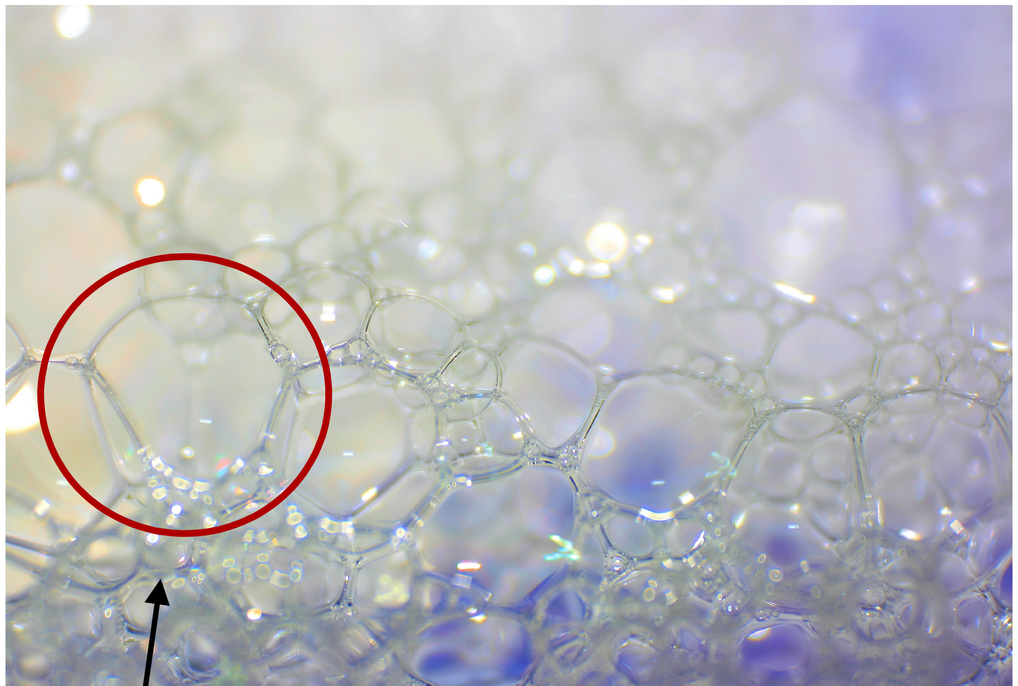


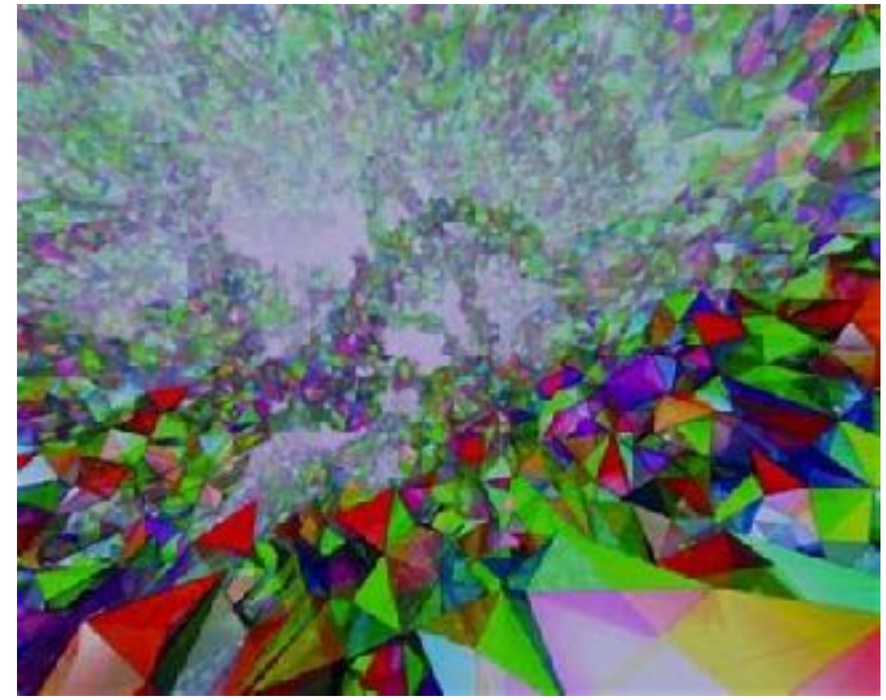
- **Continuum limit and Emergent (vacuum) Einstein Equation**

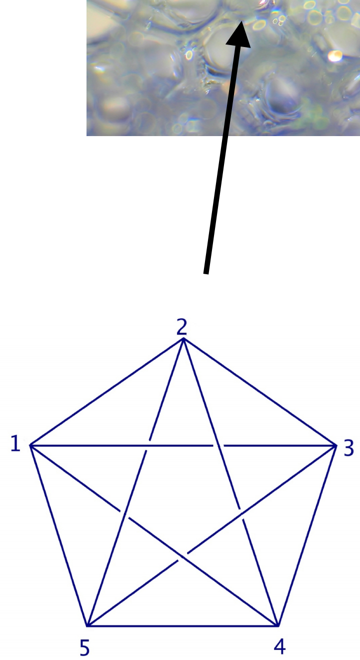
$$G_{\mu\nu} = 0$$

Spin Foam Model and Covariant Loop Quantum Gravity

Foam-like discrete spacetime: a triangulation of 4-manifold (simplicial complex)







4-simplex: minimal cell in 4d triangulation (10 triangles and 5 tetrahedra)

Spin Foam 4-simplex amplitude $A_\sigma(\vec{J}, \vec{I})$

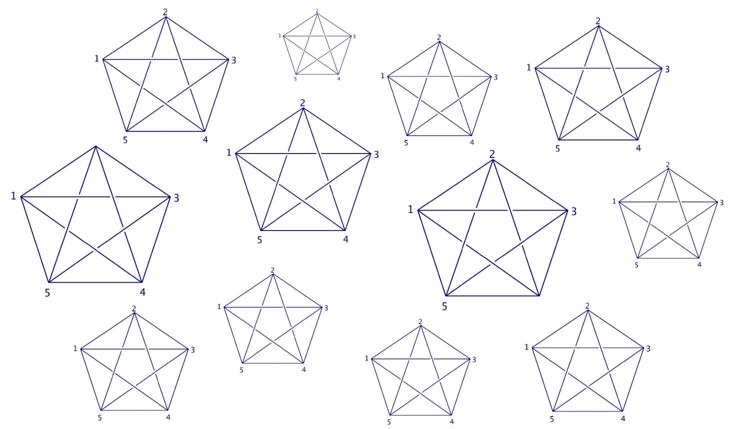
10 triangles f $\longleftrightarrow$ **10 SU(2) spins** $J_f \in \mathbb{Z}_+/2$

5 tetrahedra τ $\longleftrightarrow$ **5 SU(2) invariant tensors**

$$I_\tau \in \mathrm{Inv}_{SU(2)}[V_{j_1} \otimes \cdots \otimes V_{j_4}]$$

A triangulation of 4-manifold

$\mathcal{K}$



Spin Foam Model as a state-sum

4-simplex amplitude

$$Z(\mathcal{K}) = \sum_{\vec{J},\vec{I}} \prod_f A_f(J_f) \prod_\sigma A_\sigma(\vec{J},\vec{I})$$

face amplitude: $A_f = 2J_f + 1$

4-simplex amplitude:

Engle, Pereira, Rovelli, Livine, 2007

$$A_\sigma = \mathrm{tr}(i_1 \otimes \cdots \otimes i_5)$$

$$= \{\mathrm{SL}(2,\mathbb{C})\ 15\mathrm{j\ symbol}\} \times (\mathrm{fusion\ coefficients})$$

SL(2,C) invariant tensor:

SL(2,C) Wigner D-matrix

$$i = \int_{\mathrm{SL}(2,\mathbb{C})} \mathrm{d}g \ \otimes_{k=1}^{4} D^{(j_k,\gamma j_k)}_{(l'_k,m'_k),(j_k,m_k)}(g)\ I^{m_1\cdots m_4}$$

4-simplex amplitude is a linear map of 5 invariant tensors:

$$A_\sigma :\ \left(\mathrm{Inv}_{SU(2)}[V_{j_1} \otimes \cdots \otimes V_{j_4}]\right)^{\otimes 5} \to \mathbb{C}$$

Spin Foam Model as a Tensor Network

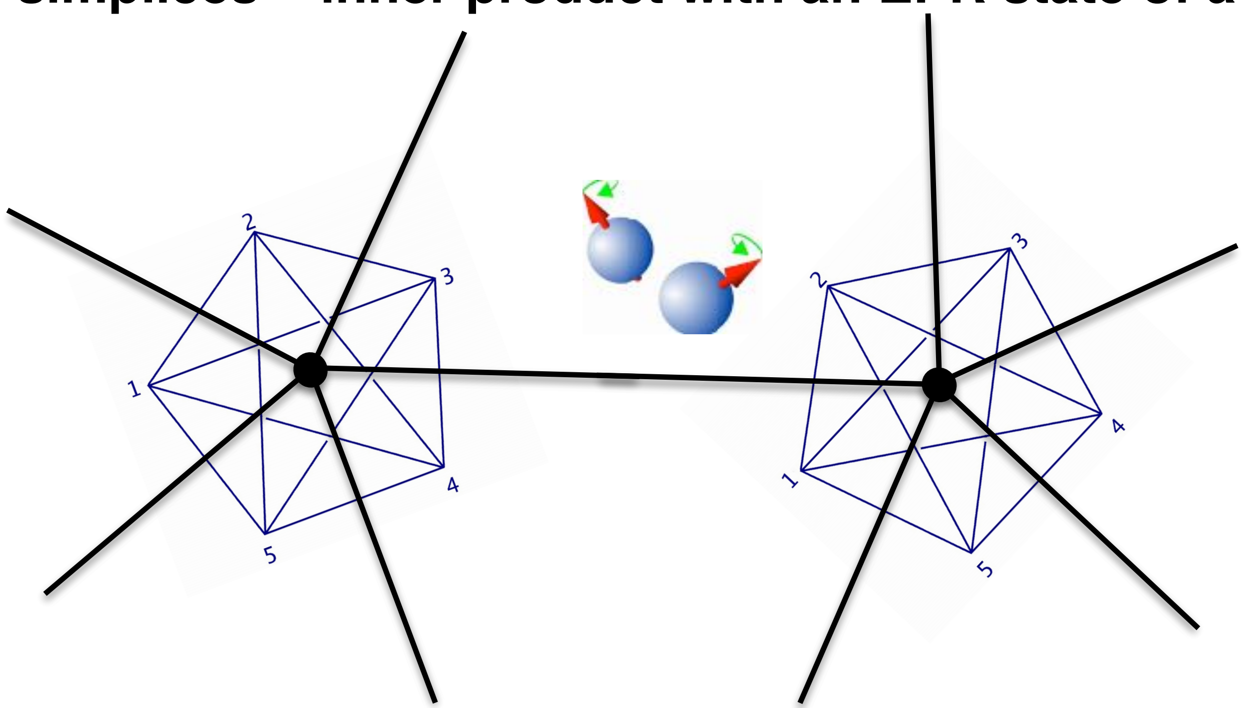
4-simplex amplitude is a linear map of 5 invariant tensors:

$$A_\sigma : \left(\mathrm{Inv}_{SU(2)}[V_{j_1} \otimes \cdots \otimes V_{j_4}]\right)^{\otimes 5} \to \mathbb{C}$$

4-simplex amplitude is a tensor state (of 5 invariant tensors)

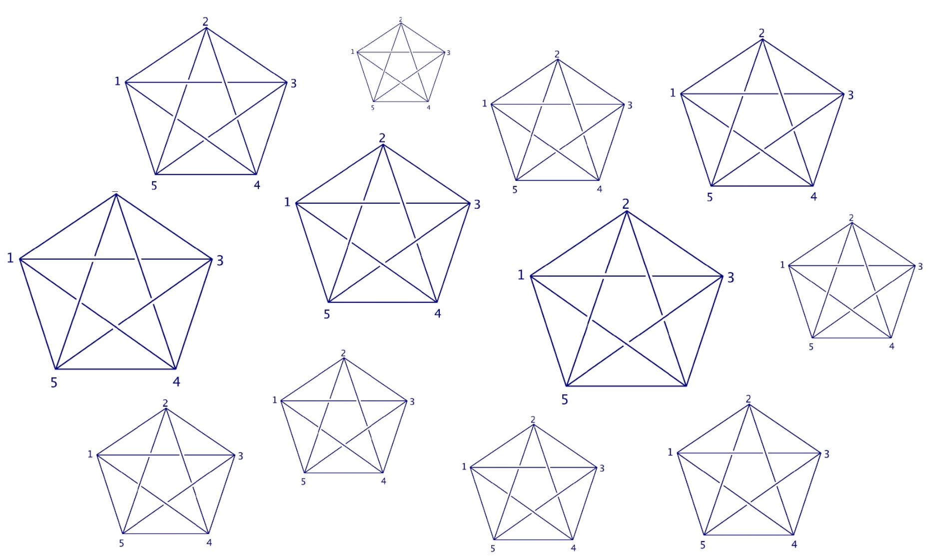
$$|A_\sigma\rangle \in \left(\mathrm{Inv}_{SU(2)}[V_{j_1} \otimes \cdots \otimes V_{j_4}]\right)^{\otimes 5}$$

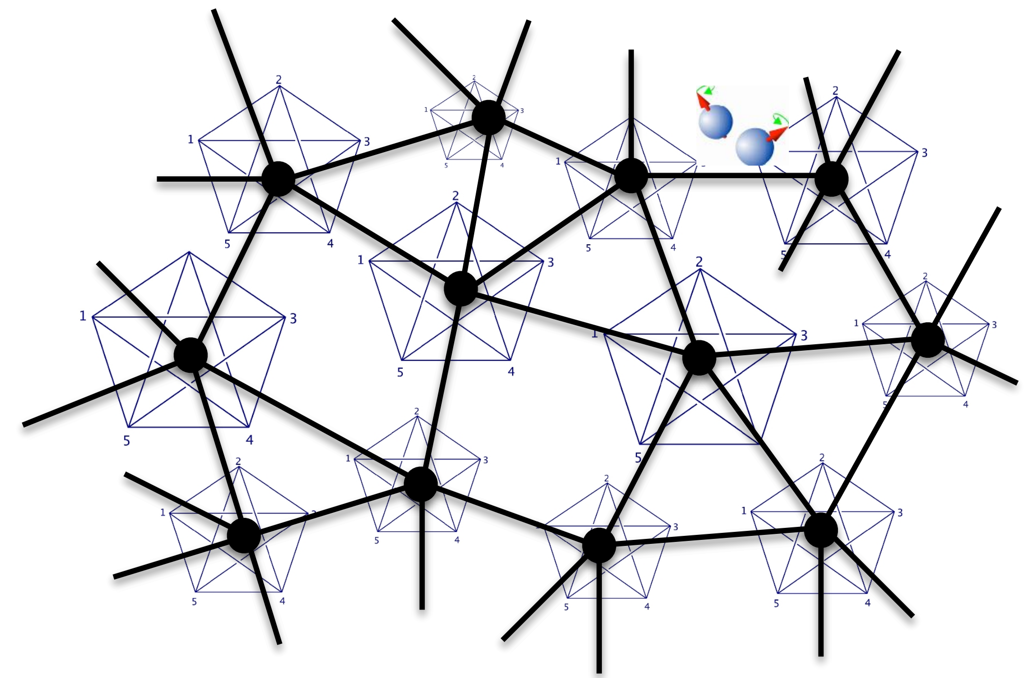
Gluing two 4-simplices = inner product with an EPR state of a pair of invariant tensors



$$\langle A_\sigma| \qquad \sum_I |I\rangle \otimes \langle I| \qquad |A_{\sigma'}\rangle$$

Spin Foam Model as a Tensor Network





spin sum (spin dependent) tensor network

$$Z(\mathcal{K}) = \sum_{\vec{J},\vec{I}} \prod_f A_f(J_f) \prod_\sigma A_\sigma(\vec{J},\vec{I})$$

$$Z(\mathcal{K}) = \sum_{\vec{J}} \prod_f A_f(J_f) \; \otimes_e \langle e| \otimes_\sigma |A_\sigma\rangle$$

$$|e\rangle = \sum_{I_e} |I_e\rangle \otimes |I_e\rangle$$

Spin Foam Model is an Ocean of Entangled Qubits (Qudits)

Spin Foam Model (SFM) and Emergent Gravity

- **The definition of SFM as a State-Sum Model and a Tensor Network**

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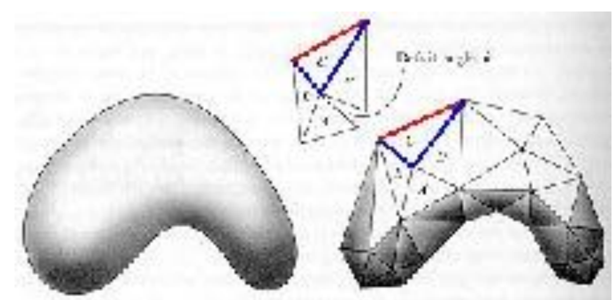
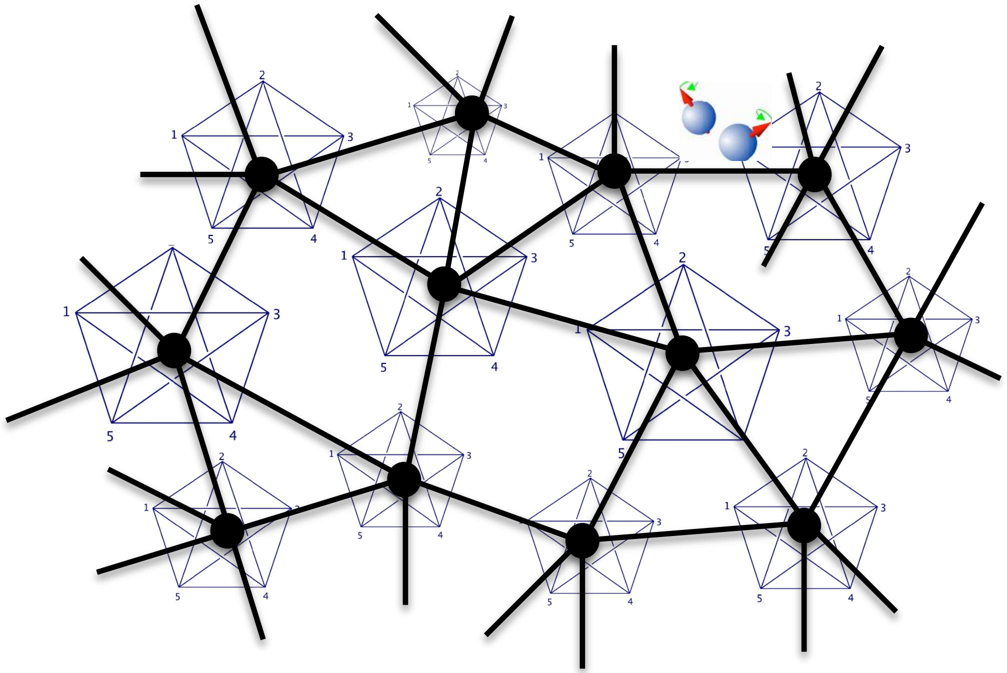
- **Large Spin Asymptotics and Emergent Geometry**



- **Continuum limit and Emergent (vacuum) Einstein Equation**

$$G_{\mu\nu} = 0$$

Integral Representation of the Tensor Network, Spin Foam Asymptotics



$$Z(\mathcal{K}) = \sum_{\vec{J}} \prod_f A_f(J_f) \ \otimes_e \langle e| \otimes_\sigma |A_\sigma\rangle$$

$$= \sum_{\vec{J}} \prod_f A_f(J_f) \int [\mathrm{d}X]\ e^{\sum_f J_f F_f[X]}$$

Integration variables X

$g_{ve} \in \mathrm{SL}(2,\mathbb{C})$ **"half edge" holonomy**

$z_{vf} \in \mathbb{CP}^1$ **spinors**

The regime where classical spacetime geometry emerges from the model: Large-J regime

- **LQG area spectrum:** $\mathbf{a}_f = 8\pi\gamma\ell_P^2 \sqrt{J_f(J_f+1)}$
- **Semiclassical regime:** $\mathbf{a}_f \gg \ell_P^2 \quad \Leftrightarrow \quad J_f \gg 1$
- **The "action" is linear to the spins J_f:** **large J_f** ➡ **stationary phase analysis**
- **Classical (discrete) spacetime geometries = solutions of EOM**
- **EOM: Geometrical interpretation of variables, geometrical reconstruction**

Freidel, Conrady 2008
Barrett et al, 2009
MH, Zhang, 2011

Euclidean EPRL/FK:

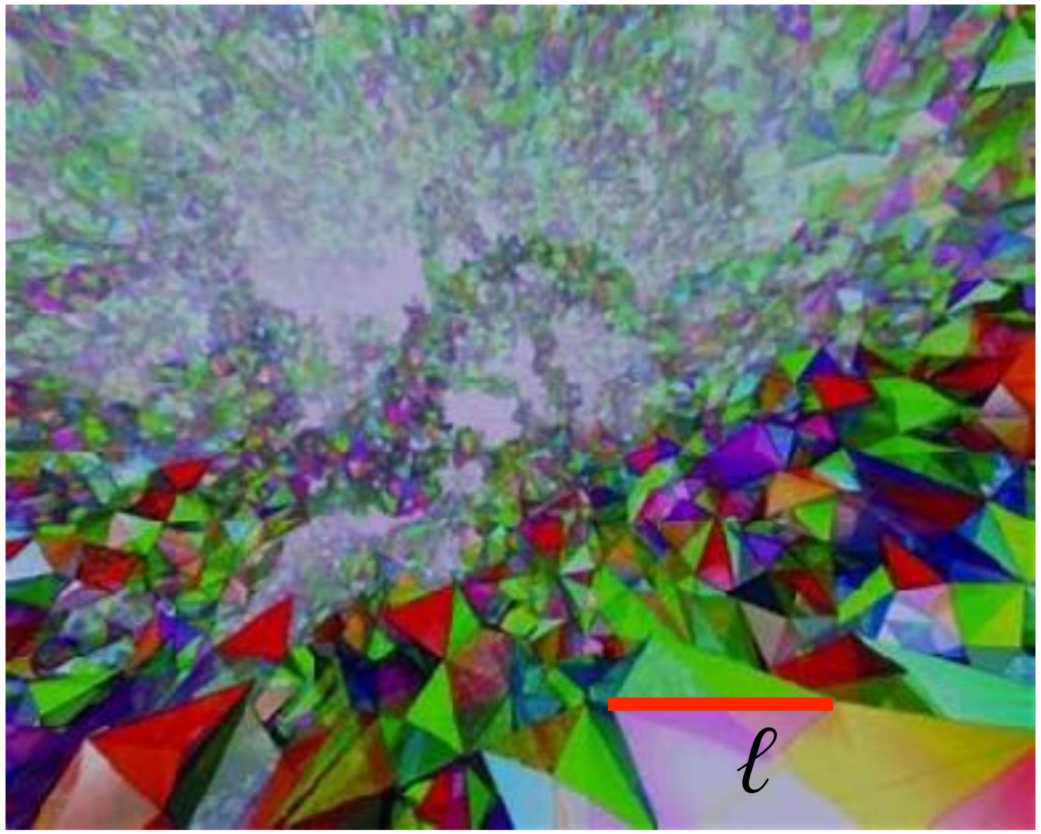
$$F_f[g^{\pm}_{ve}, \xi_{ef}] = \sum_{\pm} \sum_{v\in f} \frac{1 \pm \gamma}{2} j_f \ln \left\langle \xi_{ef} \middle| (g^{\pm}_{ve})^{-1} g^{\pm}_{ve'} \middle| \xi_{e'f} \right\rangle$$

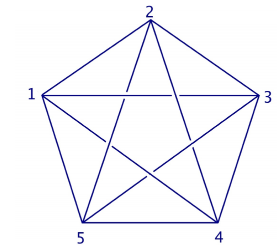
Lorentzian EPRL/FK:

$$\begin{aligned} F_f[g_{ve}, z_{vf}] &= \ln \prod_{e\subset\partial f} \frac{\left\langle g^{\dagger}_{ve} z_{vf}, g^{\dagger}_{v'e} z_{v'f} \right\rangle^2}{\left\langle g^{\dagger}_{ve} z_{vf}, g^{\dagger}_{ve} z_{vf} \right\rangle \left\langle g^{\dagger}_{v'e} z_{v'f}, g^{\dagger}_{v'e} z_{v'f} \right\rangle} \\ &+ i\gamma \ln \prod_{e\subset\partial f} \frac{\left\langle g^{\dagger}_{ve} z_{vf}, g^{\dagger}_{ve} z_{vf} \right\rangle}{\left\langle g^{\dagger}_{v'e} z_{v'f}, g^{\dagger}_{v'e} z_{v'f} \right\rangle}. \end{aligned} \tag{10}$$

……

The Discrete Geometry from Spin Foam Model: Regge Geometry



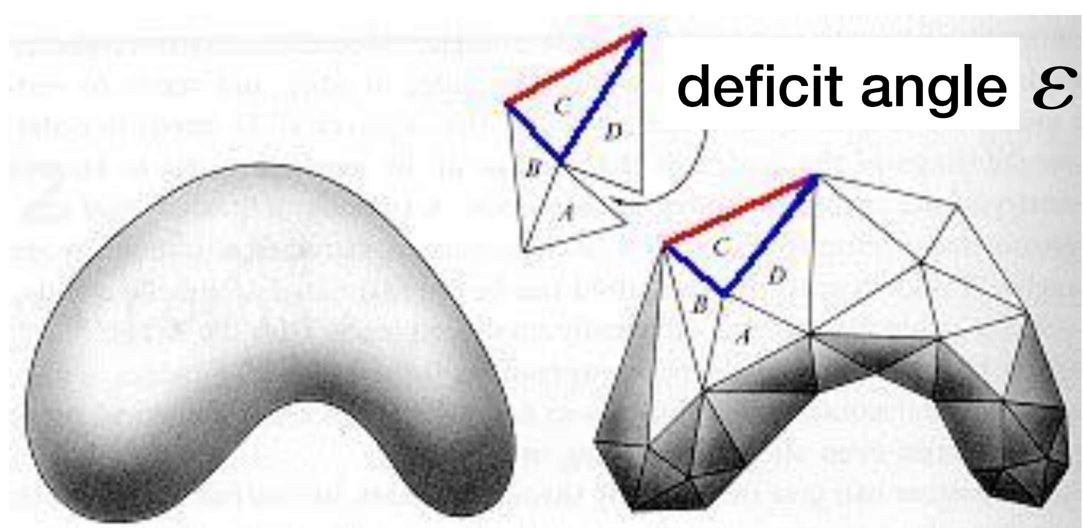


flat interior geometry

curved geometry are made by gluing geometrical 4-simplices of different shapes

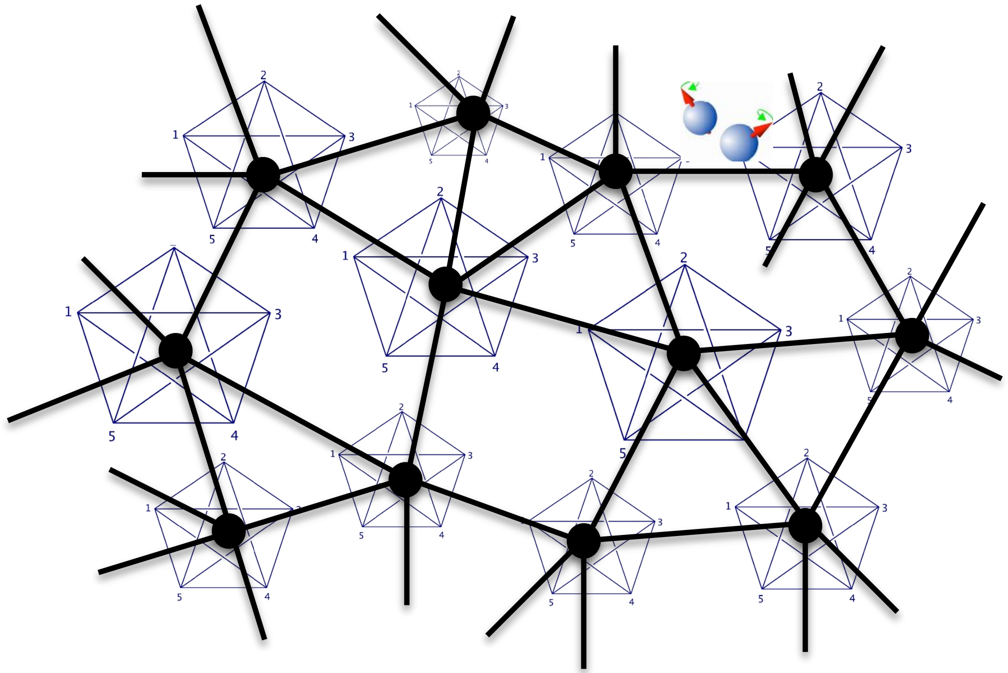
Geometries are charactered by discrete metric: ℓ **edge lengths**

discrete curvature: Deficit angle at co-dimension-2 hinges



By refining the triangulation, the discrete geometries converge to smooth geometries

Spin Foam Asymptotics



$$Z(\mathcal{K}) = \sum_{\vec{J}} \prod_f A_f(J_f) \otimes_e \langle e| \otimes_\sigma |A_\sigma\rangle$$

$$= \sum_{\vec{J}} \prod_f A_f(J_f) \int [\mathrm{d}X]\, e^{\sum_f J_f F_f[X]}$$

Large-J asymptotics of the integral: Evaluating the action at the critical point

$$\sum_f J_f F_f[X_c] = \frac{i}{\ell_P^2} \sum_f \mathbf{a}_f \varepsilon_f \qquad (\mathbf{a}_f \simeq \gamma J_f \ell_P^2)$$

MH, Zhang, 2011

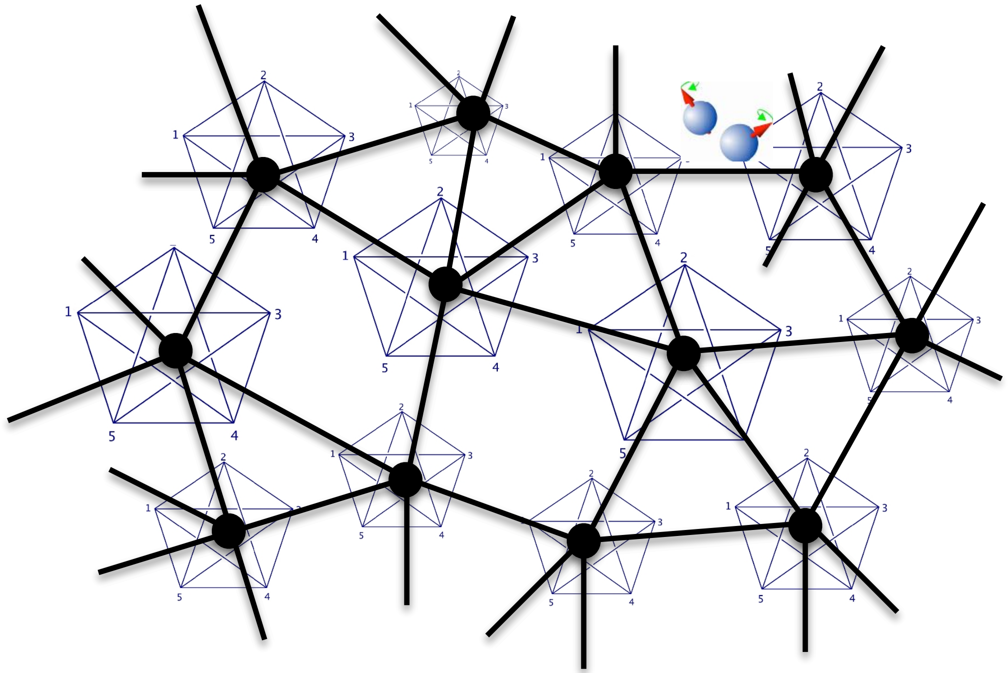
Regge action: Discrete version of Einstein-Hilbert action $\int \mathrm{d}^4x\, \sqrt{-g}\, R$

Regge, 1961
Friedberg, T.D. Lee 1984

Large-J asymptotics of the integral at "Regge critical points":

$$\int [\mathrm{d}X]\, e^{\sum_f J_f F_f[X]} \sim e^{\frac{i}{\ell_P^2} \sum_f \mathbf{a}_f \varepsilon_f}$$

How to Include the Sum of Spins



$$Z(\mathcal{K}) = \sum_{\vec{J}} \prod_f A_f(J_f) \otimes_e \langle e| \otimes_\sigma |A_\sigma\rangle$$

$$= \sum_{\vec{J}} \prod_f A_f(J_f) \int [\mathrm{d}X]\, e^{\sum_f J_f F_f[X]}$$

Fix a background and consider perturbations of all spinfoam variables

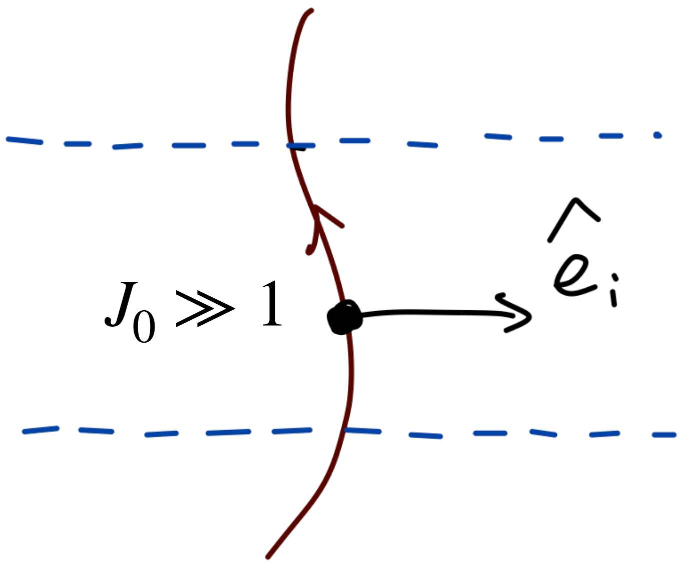
$$(J, X) = (J_0, X_0) + (\delta J, \delta X)$$

↑

A geometrical critical point

$J_0 \gg 1$ **The perturbation theory is in the large J regime**

How to Include the Sum of Spins



$\vec{J} = \vec{J}(\ell)$

$$\delta \vec{J} = \delta \vec{J}(\ell) + \sum_i t^i \, \hat{e}_i$$

ℓ : **edge lengths on the triangulation**

$\hat{e}_i$: **constant transverse basis**

$$Z(\mathcal{K}) = \sum_{\vec{J}} \prod_f A_f(J_f) \ \otimes_e \langle e| \otimes_\sigma |A_\sigma\rangle$$
$$= \sum_{\vec{J}} \prod_f A_f(J_f) \int [\mathrm{d}X] \ e^{\sum_f J_f F_f[X]}$$

Poisson resummation formula

$$\sum_{2J \in \mathbb{Z}} f(J) = \sum_{k \in \mathbb{Z}} 2 \int \mathrm{d}J \, f(J) \, e^{4\pi i k J}$$

Regularize the (transverse) spin sum

$$\int \mathrm{d}J \ = \int \mathrm{d}\ell \, \mathrm{d}t \, \mathcal{J}(\ell) \ \to \ \int \mathrm{d}\ell \ \mathcal{J}(\ell) \int_{-\infty}^{\infty} \mathrm{d}t \, e^{-\delta t^2}, \qquad \delta \ll 1$$

How to Include the Sum of Spins

Performing the Gaussian integral of t:

$$Z(\mathscr{K}) \sim \int [\mathrm{d}\ell \mathrm{d}X]\, e^{\langle \vec{J}(\ell), \vec{F}(X)\rangle} D_\delta(\ell, X), \qquad \langle \vec{J}(\ell), \vec{F}(X)\rangle = \sum_f J_f(\ell) F_f(X)$$

$$D_\delta \propto e^{\frac{1}{\delta}\sum_i \langle \hat{e}^i, \vec{F}(X)\rangle^2}$$

"Effective action"

$$S_{eff} = \sum_f J_f(\ell) F_f(X) + \frac{1}{\delta} \sum_i \langle \hat{e}^i, \vec{F}(X)\rangle^2$$

Regime: $J \gg \frac{1}{\delta} \gg 1$ **and assuming** δ **real**

$S_{eff} =$ **spinfoan action** $+$ **perturbative correction**

- $\delta_X S = Re(S) = 0 \qquad \rightarrow \qquad (J(\ell), X)$ **critical points**

  $$F_f(X) = i\gamma\varepsilon_f$$

- $\delta_\ell S = \sum_f \frac{\partial J_f}{\partial \ell} F_f(X) \quad = \quad i\gamma \sum_f \frac{\partial J_f}{\partial \ell} \varepsilon_f = 0$ **Regge EOM**

Effect of the Regulator

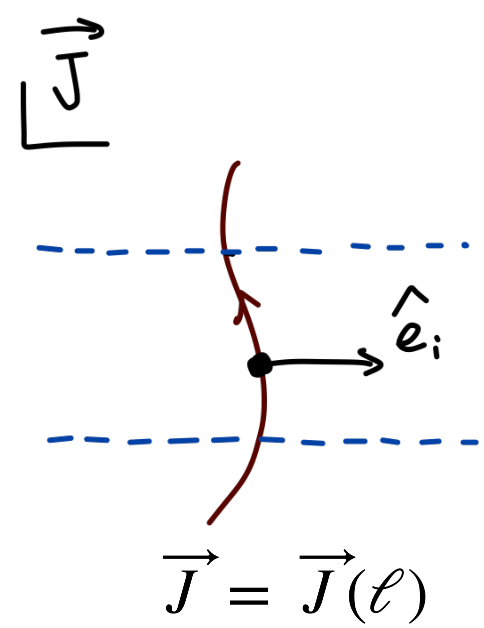
Performing the Gaussian integral of t:

$$Z(\mathscr{K}) \sim \int [\mathrm{d}\ell \mathrm{d}X]\, e^{\langle \vec{J}(\ell), \vec{F}(X)\rangle} D_\delta(\ell, X), \qquad \langle \vec{J}(\ell), \vec{F}(X)\rangle = \sum_f J_f(\ell) F_f(X) \equiv S$$

$$D_\delta \propto e^{\frac{1}{\delta}\sum_i \langle \hat{e}^i, \vec{F}(X)\rangle^2} = e^{-\frac{1}{\delta}\sum_i \langle \hat{e}^i, \gamma \vec{\varepsilon}\rangle^2}$$

Bound of deficit angles
(For non-suppressed contribution):

$$|\langle \hat{e}^i, \gamma \vec{\varepsilon}\rangle| \leq \delta^{1/2}$$



Regge EOM:

$$\langle \frac{\partial \vec{J}}{\partial \ell}, \gamma \vec{\varepsilon}\rangle = 0$$

$$\Rightarrow \quad |\gamma \vec{\varepsilon}| \leq \delta^{1/2}$$

If there wasn't the regulator, we would have the flatness.

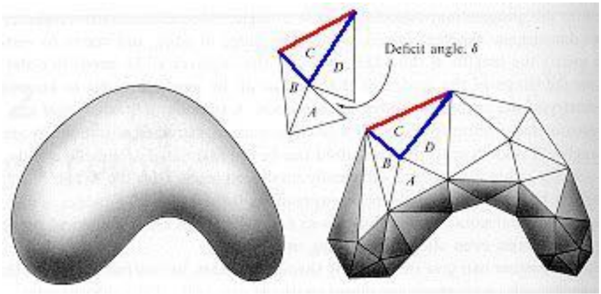
The regularization opens a window for arbitrary curved geometries by refining the lattice.

Spin Foam Model (SFM) and Emergent Gravity

- **The definition of SFM as a State-Sum Model and a Tensor Network**

$$Z(\mathcal{K}) = \sum_{\vec{J},\vec{I}} \prod_f A_f(J_f) \prod_\sigma A_\sigma(\vec{J},\vec{I})$$

- **Large Spin Asymptotics and Emergent Geometry**



- **Continuum limit and Emergent (vacuum) Einstein Equation**

$$G_{\mu\nu} = 0$$

Continuum limit

$\mathcal{K}_1$ → $Z(\mathcal{K}_1)$ → **discrete geometries on** $\mathcal{K}_1$

$\mathcal{K}_{1/2}$ → $Z(\mathcal{K}_{1/2})$ → **discrete geometries on** $\mathcal{K}_{1/2}$

$\cdots$

$\mathcal{K}_\mu$ → $Z(\mathcal{K}_\mu)$ → **discrete geometries on** $\mathcal{K}_\mu$

$\cdots$

continuum limit: $\mu \to 0$

Semiclassical continuum limit: **The flow of parameters:** $J(\mu),\ \alpha_f(\mu),\ \delta(\mu)$

satisfying:

$$\lim_{\mu\to 0} J(\mu) \to \infty \qquad \frac{1}{\lambda}\frac{d\lambda}{d\mu} < \frac{1}{C}\frac{dC}{d\mu}$$

← **quantum corrections in SFM**

$$\lim_{\mu\to 0} \mathbf{a}(\mu) \to 0$$

shrink the lattice spacing

$$\lim_{\mu\to 0} \delta(\mu) = 0 \qquad J(\mu) \gg \delta(\mu)^{-1} \gg 1$$

bound of deficit angle: $|\varepsilon_f(\mu)| \le \delta(\mu)^{1/2}$

MH, 2017

MH, Zichang Huang, Antonia Zipfel, 2018

Continuum limit

$\mathcal{K}_1$ $\quad$ $\mathcal{K}_{1/2}$ $\quad \cdots \quad$ $\mathcal{K}_\mu$ $\quad \cdots$ $\qquad$ **continuum limit:** $\mu \to 0$

$\downarrow$

$Z(\mathcal{K}_1)$ $\quad$ $Z(\mathcal{K}_{1/2})$ $\quad \cdots \quad$ $Z(\mathcal{K}_\mu)$ $\quad \cdots$

$\downarrow$

discrete geometries on $\mathcal{K}_1$ $\quad$ **discrete geometries on** $\mathcal{K}_{1/2}$ $\quad$ **discrete geometries on** $\mathcal{K}_\mu$ $\longrightarrow$ **Smooth geometries**

Semiclassical continuum limit: **The flow of parameters:** $J(\mu),\ \alpha_f(\mu),\ \delta(\mu)$

satisfying:

$$\lim_{\mu\to 0} J(\mu) \to \infty \qquad \frac{1}{\lambda}\frac{d\lambda}{d\mu} < \frac{1}{C}\frac{dC}{d\mu}$$

$\longleftarrow$ **quantum corrections in SFM**

$$\lim_{\mu\to 0} \mathbf{a}(\mu) \to 0$$

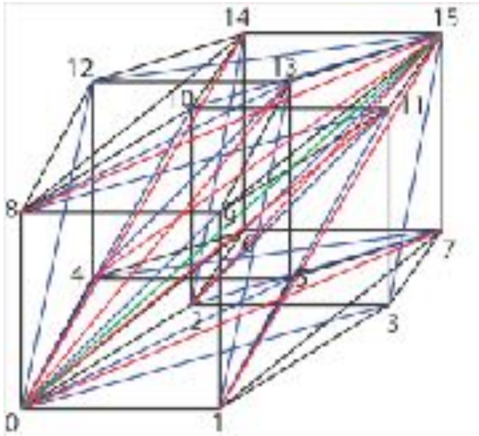
shrink the lattice spacing

$$\lim_{\mu\to 0} \delta(\mu) = 0 \qquad J(\mu) \gg \delta(\mu)^{-1} \gg 1$$

bound of deficit angle: $|\varepsilon_f(\mu)| \le \delta(\mu)^{1/2}$

MH, 2017

MH, Zichang Huang, Antonia Zipfel, 2018



Lattice Refinement:

Subdividing hypercubes followed by triangulation (Not Pachner moves for 4-simpices)

Spin foam continuum limit ($\mu \to 0$) V.S. Regge geometry continuum limit ($\ell \to 0$)

Define semiclassical continuum limit (SCL):

The flows of $J(\mu), \alpha(\mu), \delta(\mu)$ **such that spinfoam continuum limit contact with Regge**

- $\mu \to 0 \implies \delta(\mu) \to 0$ $\quad$ $J \gg \delta^{-1} \gg 1 \implies J(\mu) \to \infty$
  Combining large-j and continuum limit

- $\mathbf{a}(\mu) = \gamma J(\mu)\,(\mu^2 \ell_P^2)$ $\quad$ μ^2 **scales** ℓ_P^2 **to zero: semiclassical limit**
  such that $\mathbf{a}(\mu) \to 0$ **as** $\mu \to 0$

i.e. $\mu^2 \to 0$ **faster than** $J(\mu) \to \infty$ $\qquad$ $-\frac{2}{\mu} < \frac{1}{J}\frac{dJ}{d\mu} < 0$

- Asymptotic expansion on the refining sequence:

$$\left| Z(\mathcal{K}_\mu) \ - \ (\text{large-j approximation}) \right| \leq \left(\frac{2\pi}{\lambda(\mu)}\right)^{\frac{N}{2}} \frac{C(\mu)}{\lambda(\mu)}.$$

$\lambda(\mu)$: typical background spin

Semi-classically converge to Regge geometries for all μ if and only if quantum corrections C/λ are always small

$$\frac{C(\mu)}{\lambda(\mu)} \leq \frac{C(1)}{\lambda(1)} \quad \text{For all } \mu \to 0 \qquad \textbf{OR} \qquad \frac{1}{\lambda}\frac{d\lambda}{d\mu} < \frac{1}{C}\frac{dC}{d\mu}$$

- Theorem:

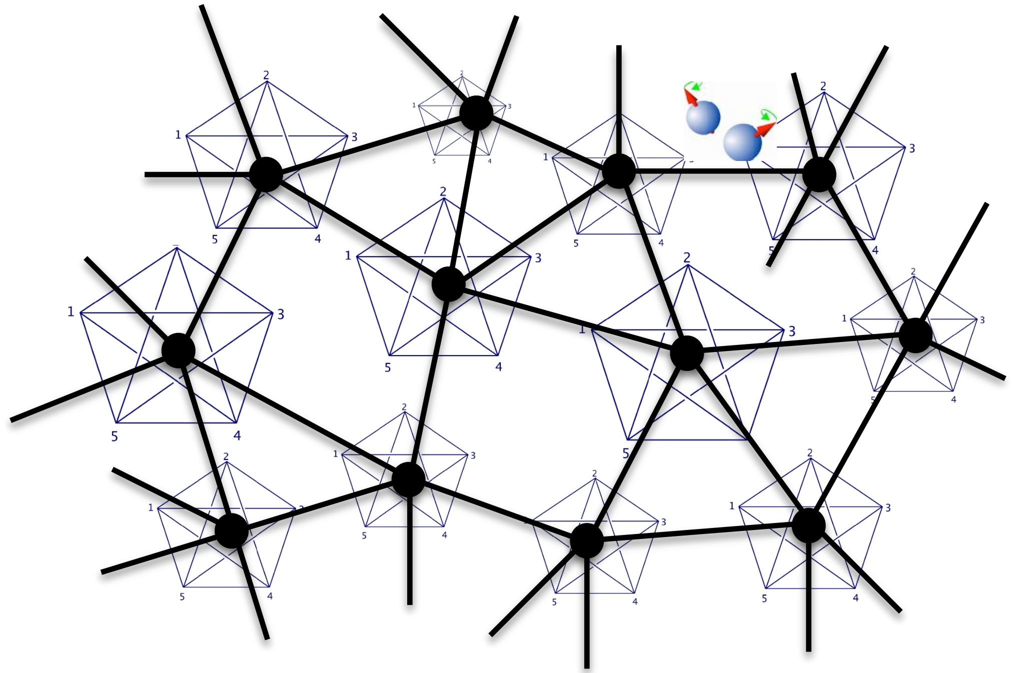
SCL is well defined because the flows satisfying the requirements always exist.

- Example:

$$\lambda(\mu) = \lambda(1)\mu^{-2+u}, \qquad 0 < u < \frac{2}{5}$$

$$a(\mu) = \mu^{u/2}\sqrt{\gamma\lambda(1)l_p^2}$$

$$\lambda(\mu)^{-1/2}\mu^{1-u/2} \ll \delta(\mu) \leq L^2\mu^{2u}$$



$$Z(\mathcal{K}_\mu) = \sum_{\vec{J}} \prod_f A_f(J_f) \otimes_e \langle e| \otimes_\sigma |A_\sigma\rangle$$
$$= \sum_{\vec{J}} \prod_f A_f(J_f) \int [\mathrm{d}X]\, e^{\sum_f J_f F_f[X]}$$

Under the semiclassical continuum limit:

- $$\int [\mathrm{d}X]\, e^{\sum_f J_f F_f[X]} \sim \exp\left(\frac{1}{\mu^2 \ell_P^2} \sum_f \mathbf{a}_f(\mu)\varepsilon_f(\mu)\right) = \exp\left(\frac{1}{\mu^2 \ell_P^2} \int \mathrm{d}^4x \sqrt{-g}\, R\, [1 + O(\mu)]\right)$$

$\mu \to 0$ **The low energy effective theory of spin foam model is Einstein gravity**

Formally: $Z(\mathcal{K}) \sim \int [\mathrm{d}\ell \mathrm{d}X]\, e^{\langle \vec{J}(\ell), \vec{F}(X)\rangle} D_\delta(\ell, X)$

- $\int \mathrm{d}\ell \sim \int Dg_{\mu\nu}$ **sum over geometries** $Z(\mathcal{K}_\mu) \sim \int Dg_{\mu\nu}\, e^{\frac{i}{\mu^2 \ell_P^2} \int \mathrm{d}^4x\, \sqrt{-g}\, R\, [1+\epsilon(\mu)]}$

- $\mu \to 0$ **equation of motion: Einstein equation** $G_{\mu\nu} = 0$

MH, 2013
MH, 2017

$$Z(\mathcal{K}_\mu) \sim \int Dg_{\mu\nu}\, e^{\frac{i}{\mu^2 \ell_P} \int \mathrm{d}^4x \, \sqrt{-g} \, R \, [1+\epsilon(\mu)]}$$ **is not rigorous because path integral is not well defined**

Rigorously, on each triangulation, we obtain the Regge equation and a bound of deficit angles

$$\sum_f \frac{\partial \mathbf{a}_f(\mu)}{\partial \ell(\mu)} \varepsilon_f(\mu) = 0 \qquad |\varepsilon_f(\mu)| \leq \delta(\mu)^{1/2}$$

SFM semiclassical continuum limit $\mu \to 0$ ➡ **continuum limit of Regge equation**

There is no general proof $\sum_f \frac{\partial \mathbf{a}_f(\mu)}{\partial \ell(\mu)} \varepsilon_f(\mu) = 0$ **converges to smooth Einstein equation due to non-linearity**

But there is no counter-example. All known examples of solutions demonstrate the convergence.

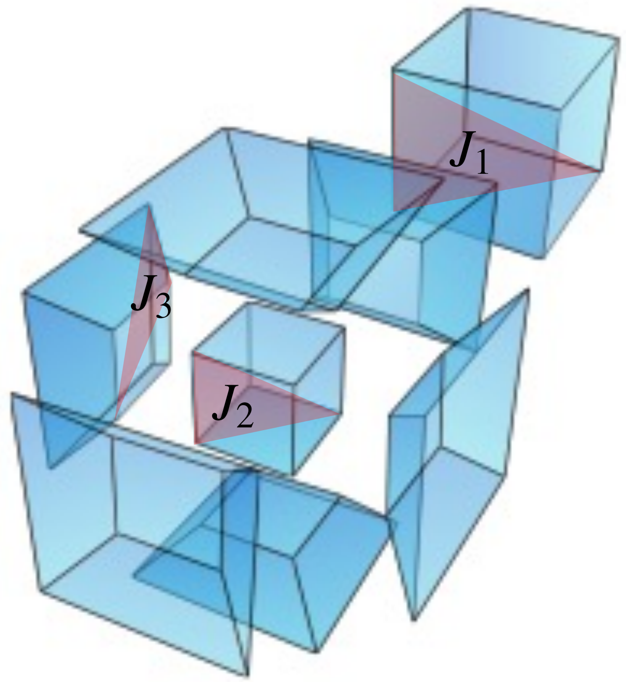
Every Regge convergence result ➡ **Convergence of spinfoam critical pts to smooth Einstein solution**

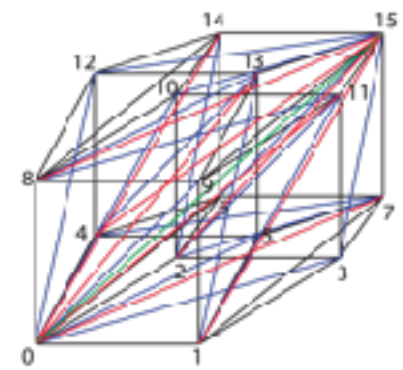
- **On the flat background, the low energy excitations SFM give linearized gravity (gravitons)**
- **Some highly curved Einstein solutions: e.g. cosmology solutions**

MH, Zichang Huang, Antonia Zipfel 2018
MH, Hongguang Liu, to appear

Symmetry reduction in spinfoam

Isotropic and homogeneous spin configurations $\bar{J}_f$





sum over all deviations away from symmetric configurations: $t_f = J_f - \bar{J}_f$

$$S_{eff} = \sum_f \bar{J}_f F_f(X) + \frac{1}{\delta}\sum_f F_f(X)^2 \;\rightarrow\; \sum_f \bar{J}_f \varepsilon_f + \frac{1}{\delta}\sum_f \varepsilon_f^2$$

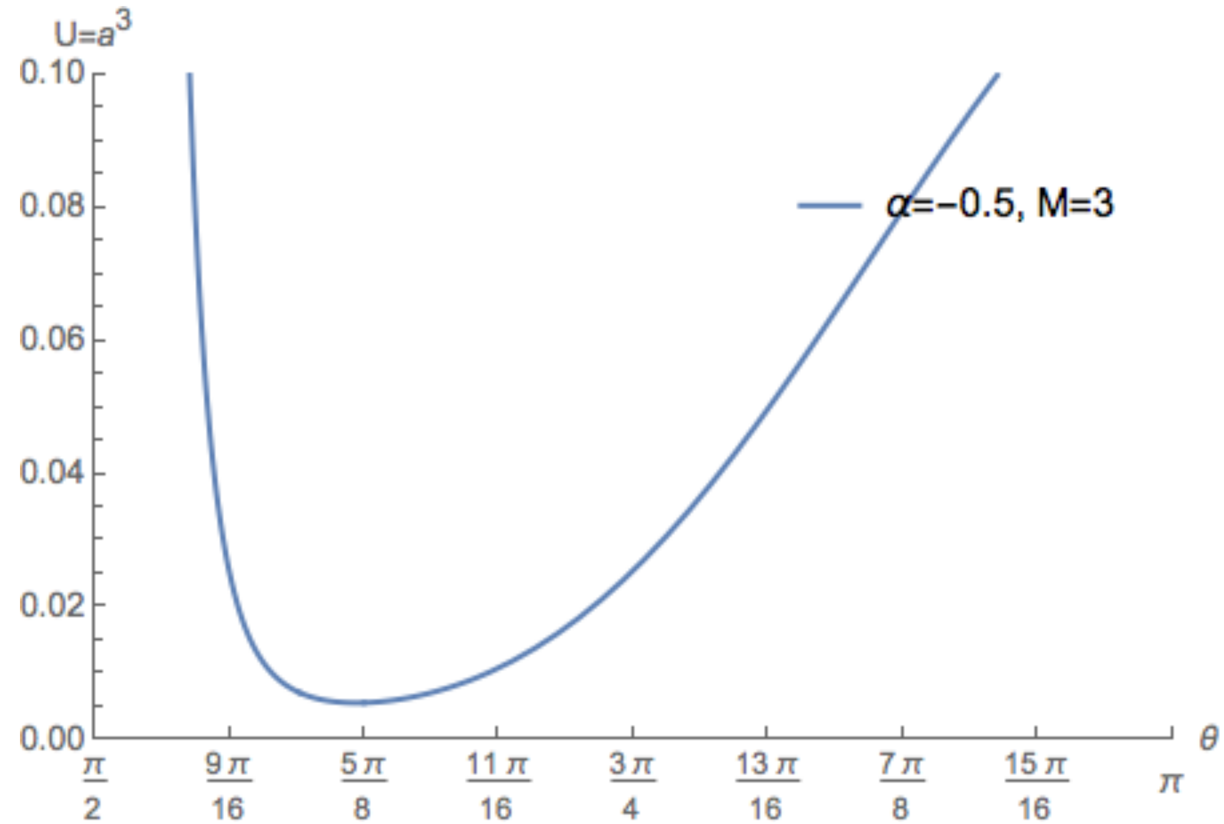
$$\delta \in is + 0^+$$

$$\bar{J} \gg 1/\delta \gg 1$$

couple to world-line matter

$$S_M = -M\int \mathrm{d}s \;\; = \; -M\sum_n H_n,$$

Evidence of singularity resolution:



$$a_n = \sqrt{\gamma J_n}\ell_P$$

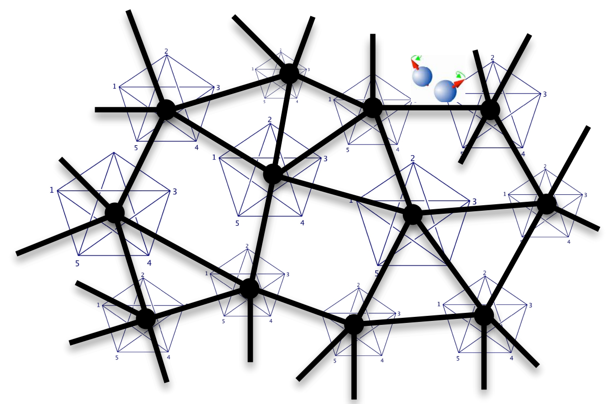
$$\alpha = \frac{\ell_P}{s\sqrt{J}}$$

$$\theta = \arccos \frac{J'^2}{J'^2 - 16J}$$

MH, Hongguang Liu, to appear

Conclusion

- The Spin Foam Model, as a model in Covariant Loop Quantum Gravity, can be understood as a Tensor Network model, or an ocean of entangled qubits.



- Semiclassical consistency of SFM = emergent gravity from SFM

- Indeed we show that under the SFM semiclassical continuum limit

$$Z(\mathcal{K}_\mu) \sim \int Dg_{\mu\nu}\, e^{\frac{i}{\mu^2 \ell_P^2} \int d^4x\, \sqrt{-g}\, R\, [1+\epsilon(\mu)]} \qquad \mu \to 0$$

- Convergence of SFM critical pts to smooth Einstein solutions

- We find SFM is a good candidate of quantum gravity model (emergent gravity model)

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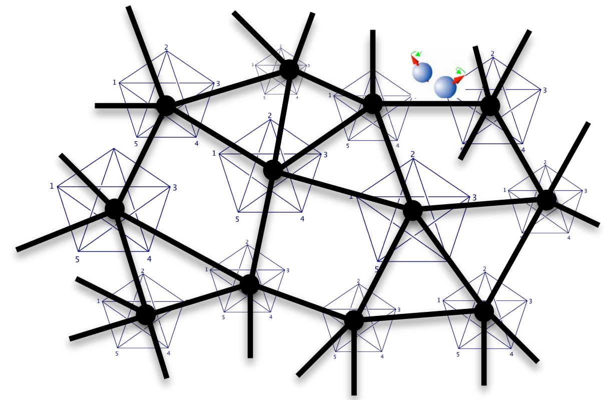

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Thanks for your attention !

backup slides

Singularity Resolution in Spinfoams

$$Z(\mathscr{K}) \sim \int [\mathrm{d}\ell\,\mathrm{d}X]\, e^{\langle \vec{J}(\ell), \vec{F}(X)\rangle} D_\delta(\ell, X), \qquad D_\delta \propto e^{\frac{1}{\delta}\sum_i \langle \hat{e}^i, \vec{F}(X)\rangle^2} = e^{-\frac{1}{\delta}\sum_i \langle \hat{e}^i, \gamma\vec{\varepsilon}\rangle^2}$$

Bound of deficit angles: $|\gamma\vec{\varepsilon}| \leq \delta^{1/2}, \qquad \varepsilon_f \simeq \dfrac{a^2}{\rho^2}$

a^2 **Typical lattice spacing**

ρ^2 **Typical curvature radius**

Combine $a^2 \simeq \gamma J \ell_P^2$ **and large J**

$$\ell_P \ll a \ll \rho$$

Condition for non-suppressed large J amplitudes

The condition breaks down near a curvature singularity: all large J amplitude are suppressed.

The singularity corresponds to small J amplitudes in spinfoam models (well-defined objects).

MH, Mingyi Zhang, 2016

Resolve the tension between large J and continuum limit (small area)

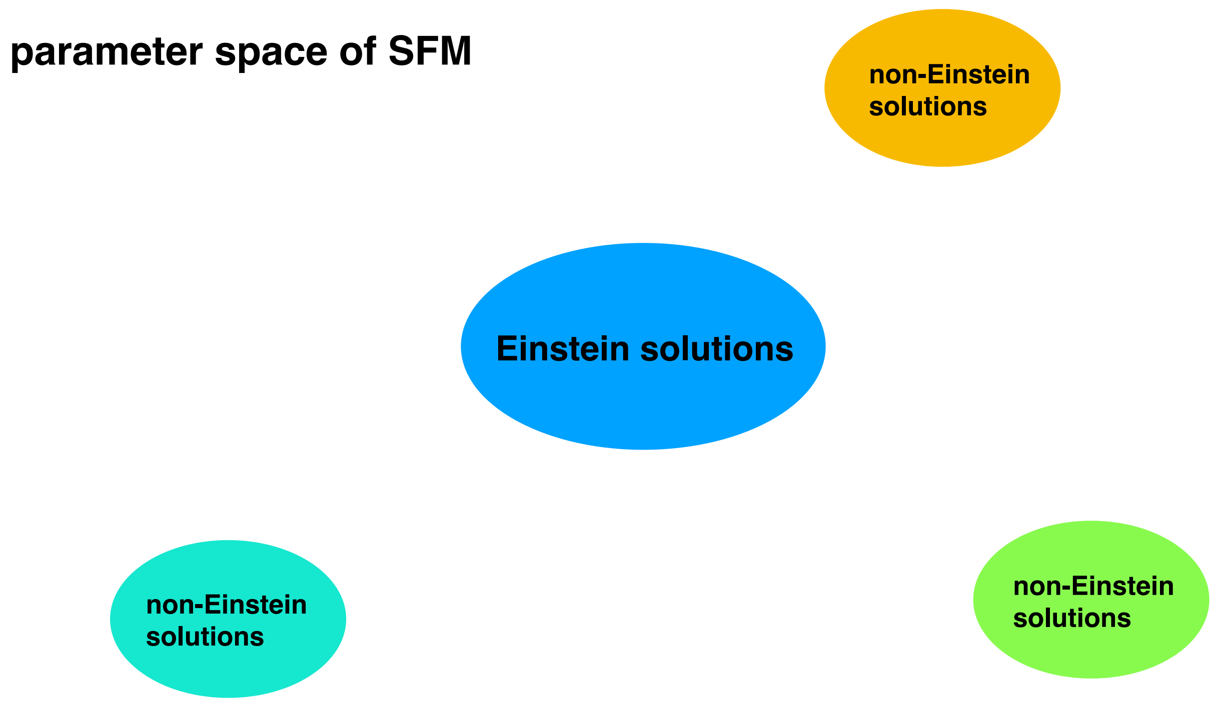
$$\mathbf{a}_f(\mu) \simeq \gamma J_f(\mu)\ell_P^2 = \alpha_f(\mu)\mu^{-2}$$

μ^{-1} **is a length unit**

such that $\lim_{\mu\to 0} \alpha_f(\mu) = 0$ **for the continuum limit of the geometry**

An open issue

- **There might exists disjoint "superselection sectors" in SFM other than Einstein solutions.**

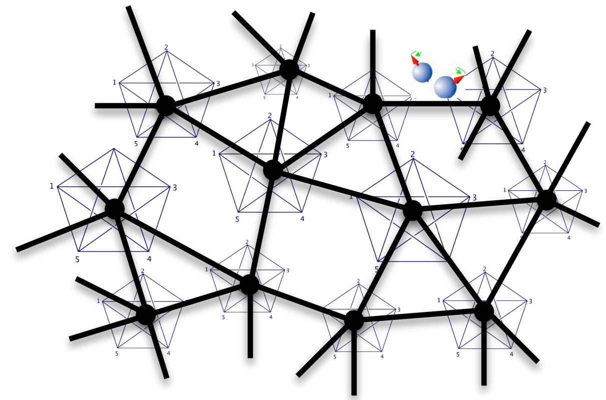


Different sectors are not connected by continuous deformation of SFM solutions.

$$\sum_{2J\in\mathbb{Z}} f(J) = \sum_{k\in\mathbb{Z}} 2\int \mathrm{d}J\, f(J)\, e^{4\pi i k J}$$

Conclusion

- The Spin Foam Model, as a model in Covariant Loop Quantum Gravity, can be understood as a Tensor Network model, or an ocean of entangled qubits.



- Semiclassical consistency of SFM = emergent gravity from SFM

- Indeed we show that under the SFM semiclassical continuum limit (IR limit)

$$Z(\mathcal{K}_\mu) \sim \int Dg_{\mu\nu}\, e^{\frac{i}{\mu^2 \ell_P^2} \int d^4x \sqrt{-g}\, R\, [1+\epsilon(\mu)]} \qquad \mu \to 0$$

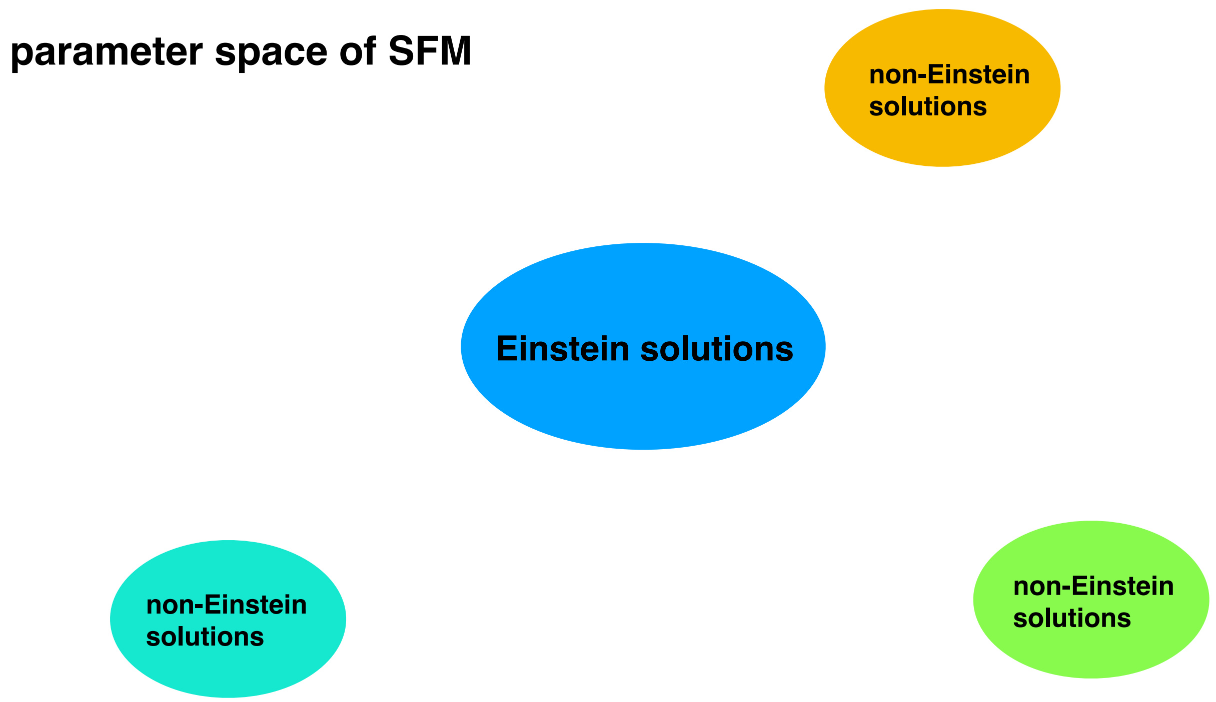
- Convergence of solutions in the limit: spin-2 gravitons and Kasner universe (solutions of Einstein equation)

- We find SFM is a good candidate of quantum gravity model (emergent gravity model)

- To do: more Einstein solutions, quantum corrections, etc……

An open issue

- There might exists disjoint “superselection sectors” in SFM other than Einstein solutions.

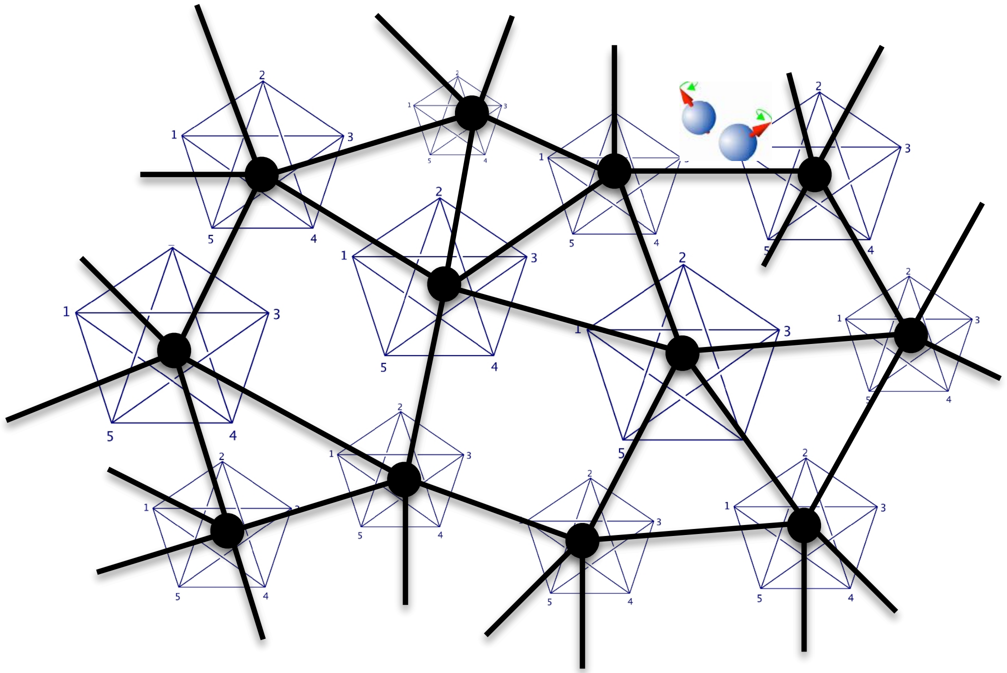


Different sectors are not connected by continuous deformation of SFM solutions.

The end

Thanks for your attention !

Integral Representation of the Tensor Network, Spin Foam Asymptotics



$$Z(\mathcal{K}) = \sum_{\vec{J}} \prod_f A_f(J_f) \ \otimes_e \langle e| \otimes_\sigma |A_\sigma\rangle$$

$$= \sum_{\vec{J}} \prod_f A_f(J_f) \int [\mathrm{d}X] \ e^{\sum_f J_f F_f[X]}$$

Integration variables X

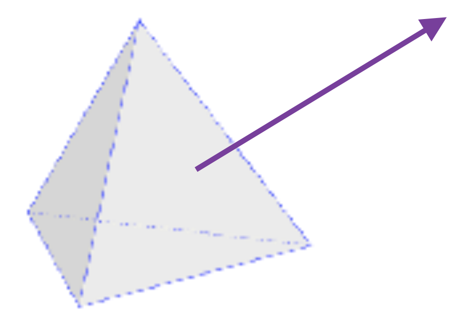
$g_{ve} \in \mathrm{SL}(2,\mathbb{C})$ **"half edge" holonomy**

$z_{vf} \in \mathbb{CP}^1$ **spinors at vertices**

- **The equations of motion (critical equations) from the action tell that the integration variables have the geometrical interpretation.**

$g_{ve} \in \mathrm{SL}(2,\mathbb{C})$ **relates to the spin connection parallel transport**

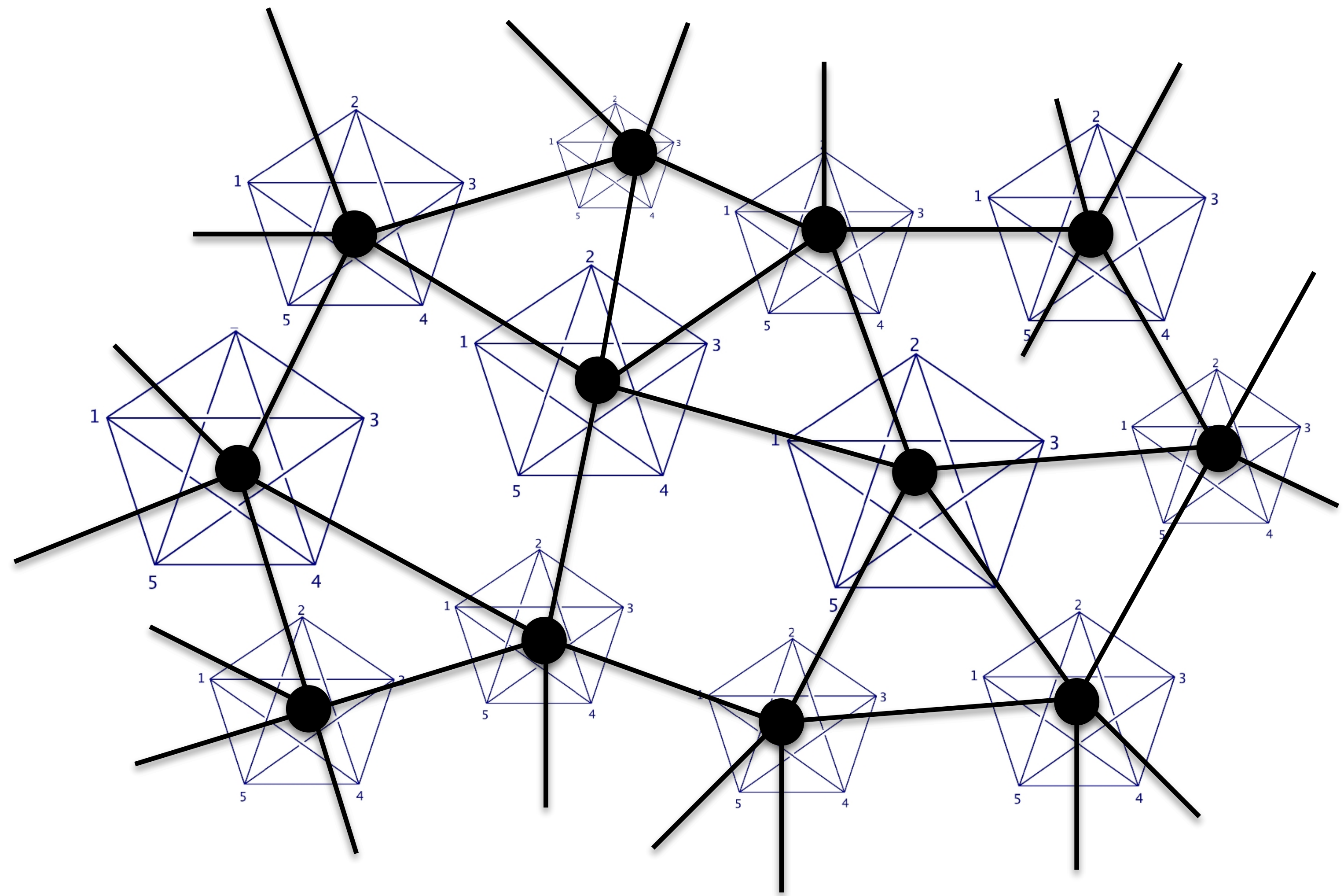
$z_{vf} \in \mathbb{CP}^1$ **relates to the tetrahedron faces normals (tetrahedron geometry)**



- **Critical equation: geometrical tetrahedra are parallel transported and glued.**

- **The integration variables satisfying the critical equations can reconstruct a 4d discrete geometry on the triangulation.**

Freidel, Conrady 2008
Barrett et al, 2009
MH, Zhang, 2011



Mathematically Rigorous Story

$$Z(\mathcal{K}_\mu) \sim \int Dg_{\mu\nu}\, e^{\frac{i}{\mu^2 \ell_P} \int d^4x\, \sqrt{-g}\, R\, [1+\epsilon(\mu)]}$$ is not rigorous because path integral is not well defined

Rigorously, on each triangulation, we obtain a Regge equation from variating Regge action

$$\sum_f \frac{\partial \alpha_f(\mu)}{\partial \ell} \varepsilon_f(\mu) = 0$$ discrete Einstein equation Regge, 1961

bound of deficit angle: $|\varepsilon_f(\mu)| \leq \delta(\mu)^{1/2}$ from SFM

SFM semiclassical continuum limit $\mu \to 0$ → continuum limit of Regge equation

There is no general proof $\sum_f \frac{\partial \alpha_f(\mu)}{\partial \ell} \varepsilon_f(\mu) = 0$ converges to smooth Einstein equation due to non-linearity

But there is no counter-example. All known examples of solutions demonstrate the convergence.

The situation is similar to the Numerical Relativity, where one can obtain arbitrary spacetimes with discrete data.

Some mathematically rigorous results by using the convergence of Regge solutions (case by case study):

- On the flat background, the only low energy excitations SFM are gravitational waves (spin-2 gravitons)
- The highly curved Einstein solutions: e.g. Kasner universe

MH, Zichang Huang, Antonia Zipfel, to appear
MH, Hongguang Liu, to appear