量子势、真空暴涨与宇宙波函数

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合肥

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报告大纲

- 标准宇宙学模型中的问题
- 德布罗意-玻姆量子轨道理论
- 真空暴涨
- 宇宙波函数的物理解释
- 总结和讨论

标准宇宙学模型：大爆炸+暴涨

- 1915年，爱因斯坦发表引力场方程（1917年提出了静态宇宙模型）
- 1922、1927年，费里德曼、莱美卓分别提出宇宙膨胀假说
- 1929年，哈勃通过天文观测，发现了宇宙膨胀（哈勃定律）
- 1946年，伽莫夫提出大爆炸理论
- 1965年，彭齐亚斯和威尔逊发现CMB（1978年诺贝尔奖）
- 1979-1982年，斯塔若宾斯基、古斯 、林德等先后提出暴涨理论，解决大爆炸理论存在的难题

暴涨宇宙

1. A. A. Starobinsky, “A new type of isotropic cosmological models without singularity”, Physics Letters B 91, 99 (1980).

2. A. H. Guth, "The inflationary universe: A possible solution to the horizon and flatness problems," Phys. Rev. D 23, 347 (1981) .

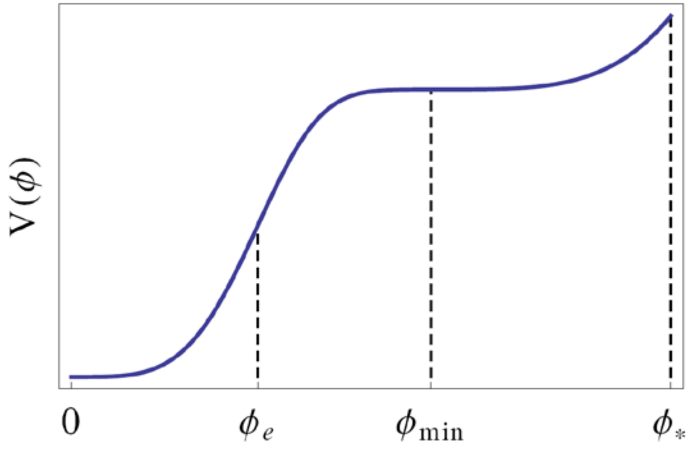
3. A. D. Linde, "A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems," Phys. Lett. B **108**, 389 (1982).

4. A. Vilenkin, "The birth of inflationary universes," Phys. Rev. D **27**, 2848 (1983).

5. S.W. Hawking,“The Quantum State of the Universe”, Nucl. Phys. B 239, 257 (1984)

标准模型：慢滚暴涨

$$S = \int d^4x \sqrt{|g|}\left(\frac{1}{2}R + \frac{1}{2}\dot{\phi}^2 - V(\phi)\right)$$



标量场的性质不清、来源明确。

真空能否暴涨？

真空作用量

$$S = \frac{1}{16\pi G}\int R\sqrt{-g}d^4x.$$

没有标量场或者宇宙常数项推动，经典无加速膨胀解。

宇宙从真空来？

396 NATURE VOL. 246 DECEMBER 14 1973

Is the Universe a Vacuum Fluctuation?

EDWARD P. TRYON

Department of Physics and Astronomy, Hunter College of the City University of New York, New York, New York 10021

"In any big bang model, one must deal with the problem of 'creation'".

这个问题一直没有准确答案。

一个问题，两种表述

- 宇宙能否从真空“产生”？
- 真空能否暴涨？

○

答案是肯定的！

德布罗意-玻姆量子轨道理论

de Broglie-Bohm quantum trajectory theory

- de Broglie and Bohm suggested a hidden variable theory for quantum mechanics, or called Bohmian mechanics, or quantum trajectory theory.

- It is mathematically equivalent with the traditional quantum mechanics of Copenhagen.

- They have different physical explanation for the world.

P. R. Holland, *The quantum theory of motion*, (Cambridge university press, 1993).

德布罗意-玻姆量子轨道理论

A wave function ψ can be expressed as

$$\psi(x,t) = R(x,t)e^{iS(x,t)/\hbar},$$

where R and S are real functions. Inserting $\psi(x,t)$ into the time-dependent Schrödinger equation, we obtain two equations by separating the time-dependent Schrödinger equation into real and imaginary parts. The real part gives

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + \underline{Q = 0}$$

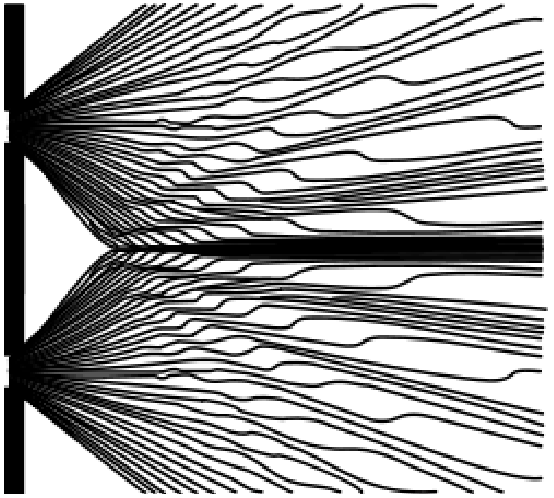
and the imaginary part has the form

$$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0$$

where $Q(x,t) = -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}$, $\rho(x,t) = R^2(x,t)$ and $v = \nabla S(x,t)/m$.

双缝实验

初始位置
为隐变量

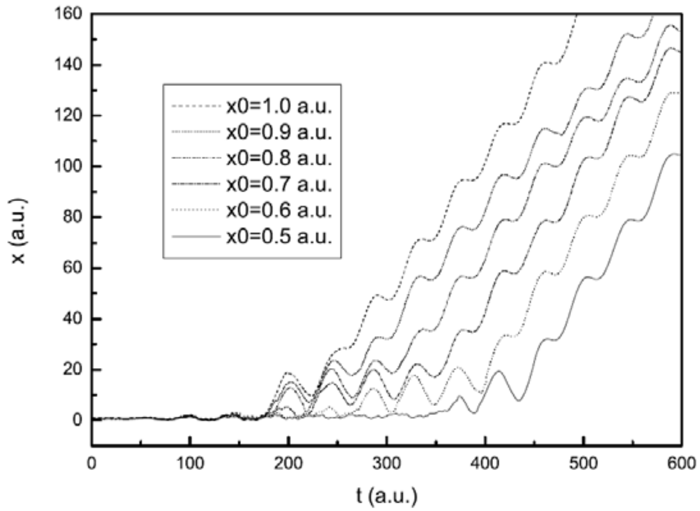


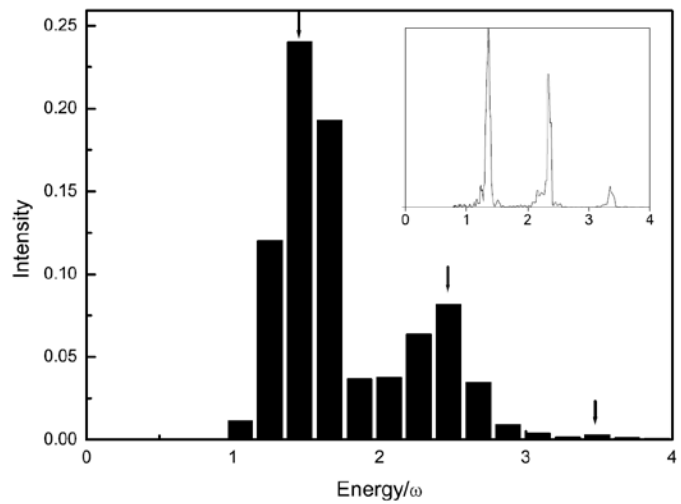
$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0$$

非局域隐
变量理论

P. R. Holland, *The quantum theory of motion*, (Cambridge university press, 1993), pp.176-190.

原子阈上电离谱

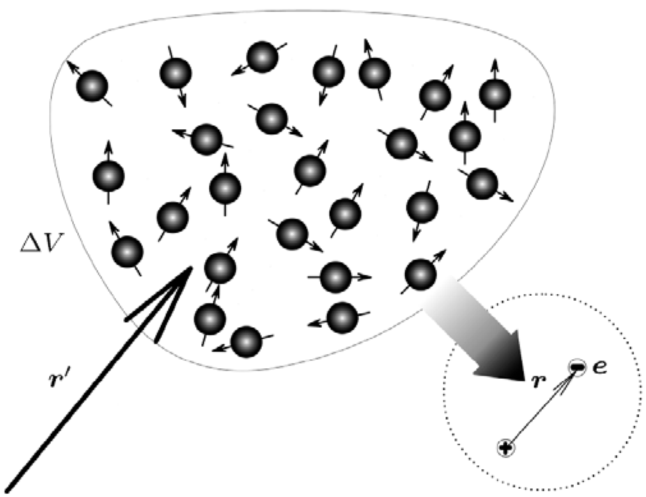


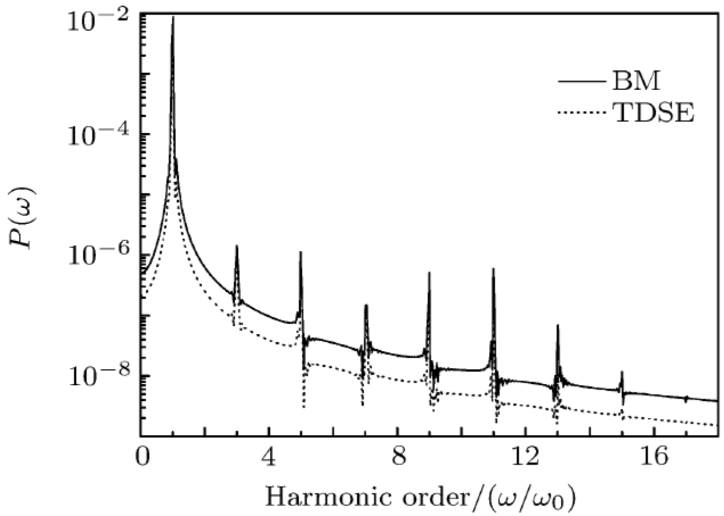


$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0$$

X.Y. Lai, Q.Y. Cai and M.S. Zhan Eur. Phys. J. D **53**, 393–396 (2009)

高次谐波谱

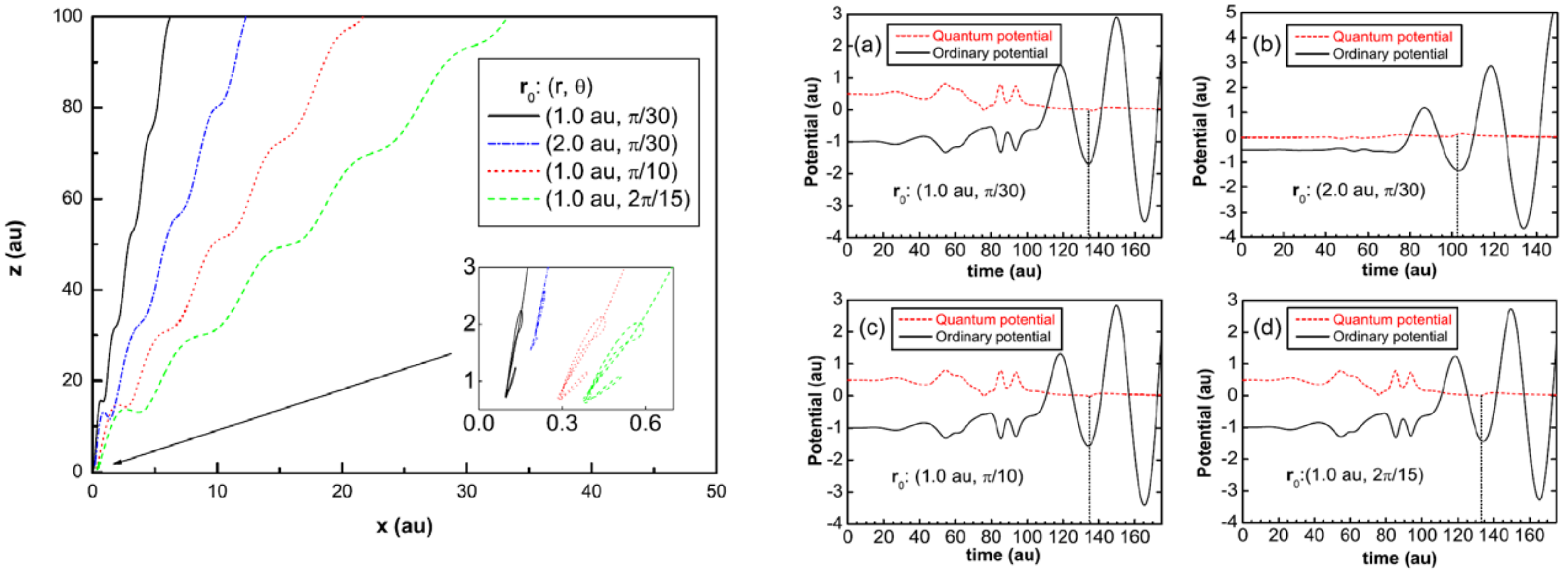




$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0$$

X.Y. Lai, Q.Y. Cai and M.S. Zhan **Chin. Phys. B Vol. 19, No. 2 (2010) 020302**

量子经典过渡



$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0$$

X.Y. Lai, Q.Y. Cai and M.S. Zhan *New Journal of Physics* **11** (2009) 113035

真空暴涨

惠勒-德威特方程(WDWE)

1967年DeWitt提出了一种对广义相对论正则量子化的方法。

$$[\frac{1}{2}\gamma^{-1/2}G_{ijkl}\pi^{ij}\pi^{kl} - \gamma^{1/2(3)}(R - 2\Lambda)]\Psi = 0$$

$$\hat{\mathcal{H}}\Psi \quad = \quad 0$$

宇宙波函数满足WDWE。

B. S. DeWitt, Phys. Rev. 160,1113 (1967)

真空哈密顿量

小超空间(minisuperspace)模型：均匀、各向同性的球对称度规：

$$ds^2 = \sigma^2[N^2(t)dt^2 - a^2(t)d\Omega_3^2]$$

爱因斯坦-希尔伯特作用量:

$$S = \frac{c^3}{16\pi G}\int \mathcal{R}\sqrt{-g}d^4x,$$

拉格朗日量及 $a(t)$ 的共轭动量为:

$$\mathcal{L} = \frac{Nc^4}{2G}\left(ka - \frac{a\dot{a}^2}{N^2c^2}\right) \qquad p_a = \frac{\partial \mathcal{L}}{\partial \dot{a}} = -\frac{c^2 a\dot{a}}{NG}$$

哈密顿量:

$$\mathcal{H} = -\frac{1}{2}\left(\frac{Gp_a^2}{c^2 a} + \frac{c^4 ka}{G}\right)$$

真空的WDWE

由 $\mathcal{H}\Psi=0$ 以及 $p_a^2=-\hbar^2a^{-p}\frac{\partial}{\partial a}(a^p\frac{\partial}{\partial a})$,

$$\left(\frac{\hbar^2}{m_p}\frac{1}{a^p}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a}-\frac{E_p}{l_p^2}ka^2\right)\psi(a)=0.$$

- 方程中a为宇宙尺度因子（唯一参数）.
- k=1，封闭宇宙；k=0，平坦宇宙；k=-1，开放宇宙.
- 常数p：算符次序因子(operator ordering factor).

方程不含时，怎么办?

量子哈密顿-雅克比方程

波函数是复数，因此总可以写成

$$\psi(a) = R(a)\exp(iS(a)/\hbar)$$

其中R和S是实函数。

带入到方程

$$\left(\frac{\hbar^2}{m_p}\frac{1}{a^p}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a} - \frac{E_p}{l_p^2}ka^2\right)\psi(a) = 0$$

可以得到

$$\frac{\hbar}{m_p}\left(S'' + 2\frac{R'S'}{R} + \frac{p}{a}S'\right) = 0,$$

$$\boxed{\frac{(S')^2}{m_p} + V + Q = 0}$$

其中$V(a)$是经典势ka^2，$Q(a)$是量子势：

$$Q(a) = -\frac{\hbar^2}{m_p}\left(\frac{R''}{R} + \frac{p}{a}\frac{R'}{R}\right).$$

作用量量子化

由

$$\psi(a) = U + iW = R(a)\exp(iS(a)/\hbar)$$

$$S = \frac{c^3}{16\pi G}\int R\sqrt{-g}d^4x$$

可以得到

$$R^2 = U^2 + W^2,$$

以及

$$S = \hbar\tan^{-1}(W/U).$$

作用量量子化

利用诱导关系

$$\frac{\partial \mathcal{L}}{\partial \dot{a}} = \frac{-c^2}{G}a\dot{a} = \frac{\partial S}{\partial a},$$

可得

$$\dot{a} = -\frac{G}{c^2 a}\frac{\partial S}{\partial a}.$$

封闭宇宙（k=1）

$$\left(\frac{1}{a^p}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a}-ka^2\right)\psi(a)=0.$$

$$\psi(a)=a^{(1-p)/2}\left[ic_1 I_\nu\left(\frac{a^2}{2}\right)-c_2K_\nu\left(\frac{a^2}{2}\right)\right],\qquad \nu=|1-p|/4$$

$$S=\tan^{-1}\left[-\frac{c_1}{c_2}\frac{I_\nu\left(\frac{a^2}{2}\right)}{K_\nu\left(\frac{a^2}{2}\right)}\right]$$

$$\dot{a}=-\frac{1}{a}\frac{\partial S}{\partial a}.$$

For small arguments $0<x\ll\sqrt{\nu+1}$,

$$I_\nu(x)\sim\frac{1}{\Gamma(\nu+1)}\left(\frac{x}{2}\right)^\nu\qquad K_\nu(x)\sim\frac{\Gamma(\nu)}{2}\left(\frac{2}{x}\right)^\nu,\quad \nu\neq 0.$$

$$S(a\ll 1)\approx-\frac{2c_1}{c_{2\Gamma(\nu)\Gamma(\nu+1)}}\left(\frac{a^2}{4}\right)^{2\nu},\qquad \nu\neq 0.$$

$$a(t)=\begin{cases}\left[\frac{(3-4\nu)\lambda(\nu)}{3}(t+t_0)\right]^{\frac{1}{3-4\nu}} & \nu\neq 0,\frac{3}{4}\\ e^{\lambda(3/4)(t+t_0)}, & \nu=\frac{3}{4},\end{cases}$$

where $\lambda(\nu)=6c_1/(4^{2\nu}c_2\Gamma(\nu)\Gamma(\nu+1))$.

暴涨的驱动力

量子势和经典势的表示形式 (p=-2, or 4)

$$\dot{a} = -\frac{1}{a}\frac{\partial S}{\partial a}.$$

$$Q(a \to 0) = -a^2 - \lambda(3/4)^2 a^4$$

$$V(a) = a^2$$

$$(S')^2 + V + Q = 0.$$

量子势+经典势

$$V+Q = -\lambda(3/4)^2 a^4$$

暴涨的动力

通常定义

$$H \equiv \dot{a}/a \text{ and } \Lambda = 3H^2$$

可以得到一个等效"宇宙常数"：

$$\Lambda \approx 3\lambda(3/4)^2.$$

开放宇宙（k=-1）

$$\left(\frac{1}{a^p}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a}-ka^2\right)\psi(a)=0.$$

$$\psi(a)=a^{(1-p)/2}\left[ic_1J_\nu\left(\frac{a^2}{2}\right)+c_2Y_\nu\left(\frac{a^2}{2}\right)\right]$$

$$a(t)=\begin{cases}\left[\frac{(3-4\nu)\bar{\lambda}(\nu)}{3}(t+t_0)\right]^{\frac{1}{3-4\nu}}, & \nu\neq 0,\frac{3}{4}\\ \underline{e^{\bar{\lambda}(3/4)(t+t_0)},} & \nu=\frac{3}{4},\end{cases}$$

$$Q(a\to 0)=a^2-\bar{\lambda}(3/4)^2a^4$$

$$v(a)=-a^2$$

平直宇宙（k=0）

$$\left(\frac{1}{a^p}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a}-ka^2\right)\psi(a)=0.$$

$$\psi(a)=ic_1\frac{a^{1-p}}{1-p}-c_2$$

$$a(t)=\begin{cases}\left[\frac{c_1}{c_2}(3-|1-p|)(t+t_0)\right]^{\frac{1}{3-|1-p|}}, & |1-p|\neq 0,3,\\ e^{c_1(t+t_0)/c_2}, & |1-p|=3.\end{cases}$$

$$Q(a\to 0)=-(c_1/c_2)^2a^4$$

$$V(a)=0$$

暴涨退出机制

封闭宇宙

$$S(a \gg 1) = -\tan^{-1}(c_1 e^{a^2}/c_2),$$

$$\dot{a} = 2c_1 e^{-a^2}/c_2 \to 0.$$

开放宇宙

$$S(a \gg 1) \sim -\tan^{-1}[c_1 \tan(a^2/2 + \pi/4 - \nu\pi/2)/c_2]$$

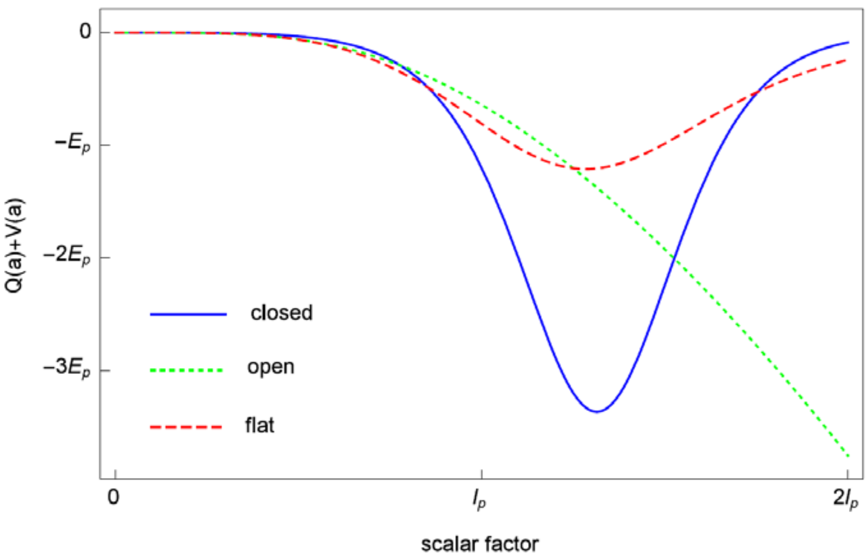
$$\dot{a}^2 = 1$$

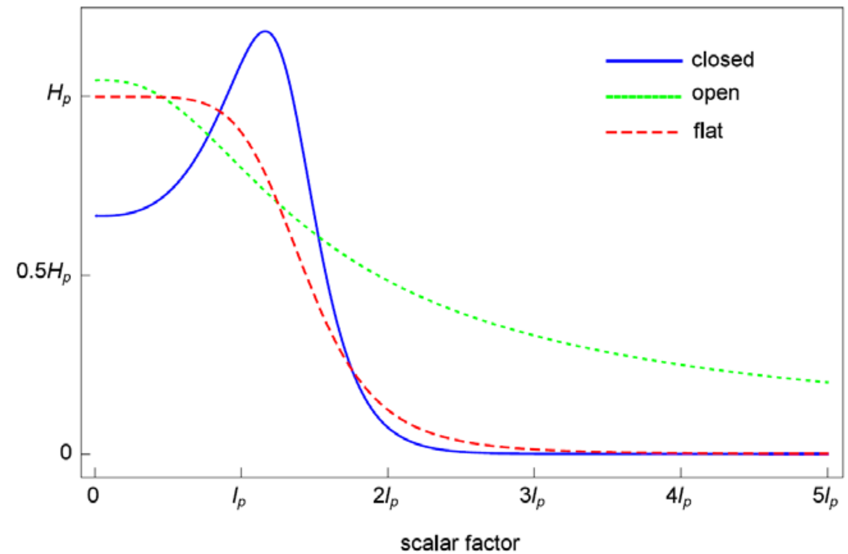
平直宇宙

$$\dot{a} = c_1 a^{-|1-p|-2}/c_2$$

$$\dot{a}^2 \to 0$$

量子效应推动宇宙迅速长大



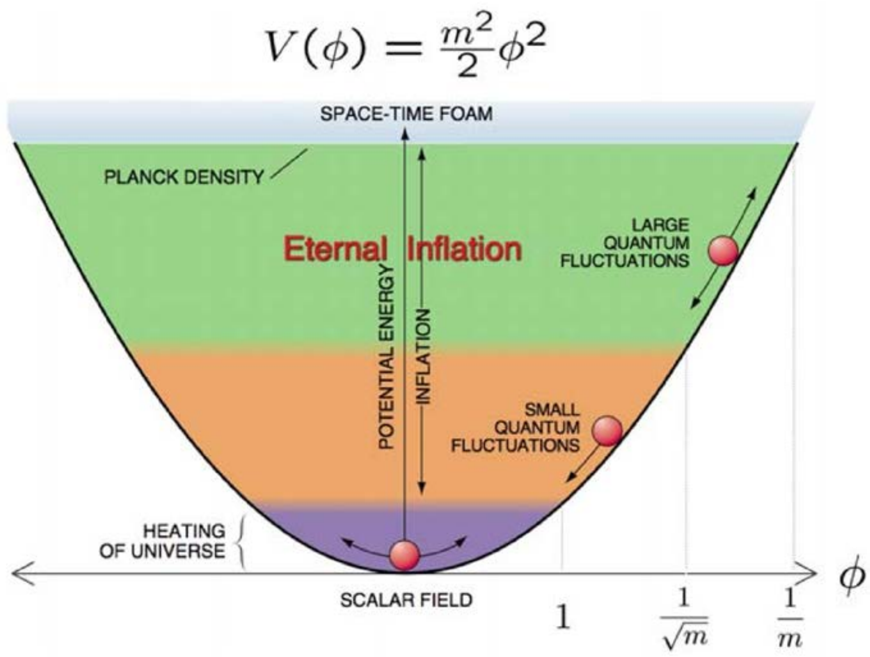


$$\left(\frac{\hbar^2}{m_p}\frac{1}{a^p}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a}-\frac{E_p}{l_p^2}ka^2\right)\psi(a)=0$$

$$\dot{a}=-\frac{G}{c^2a}\frac{\partial S}{\partial a}$$

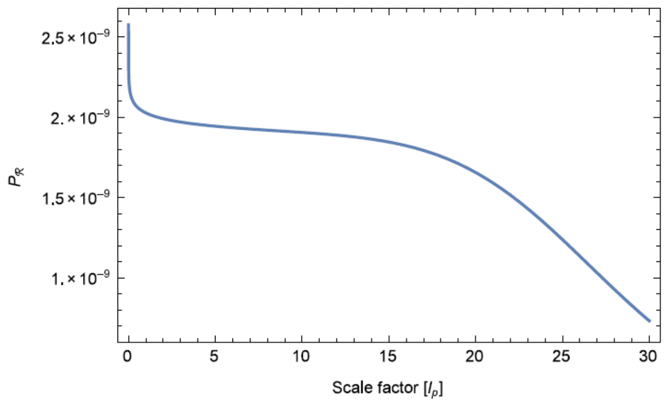
量子化导致真空暴涨！

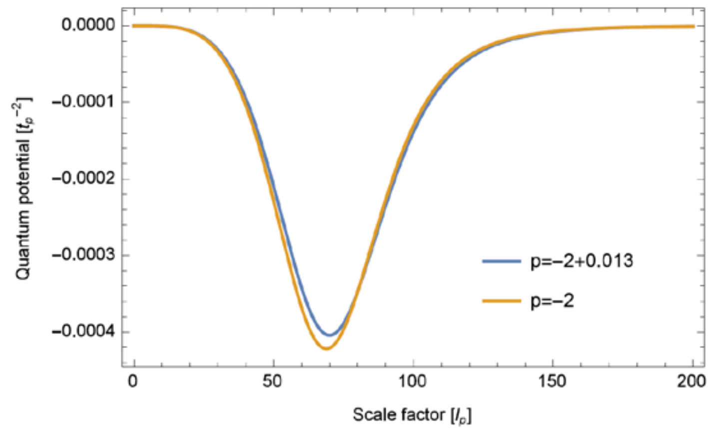
物质产生

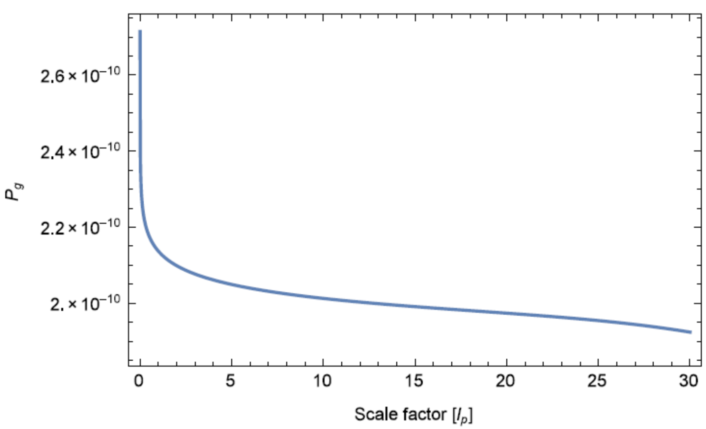


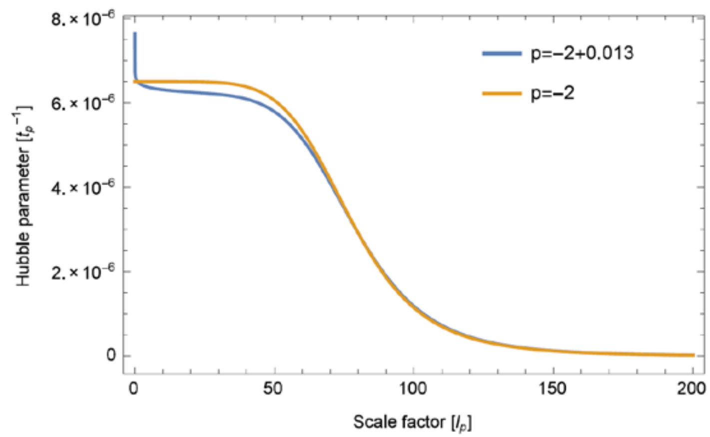
A. Linde, Lect. Notes Phys. 738,1–54 (2008)

是否符合天文观测？









初步计算 2.5*10^-27kg/m^3

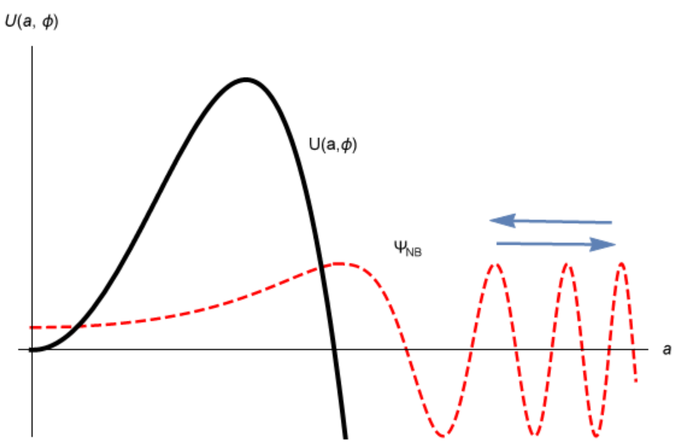
Planck 1.8*10^-27kg/m^3

宇宙波函数的物理解释

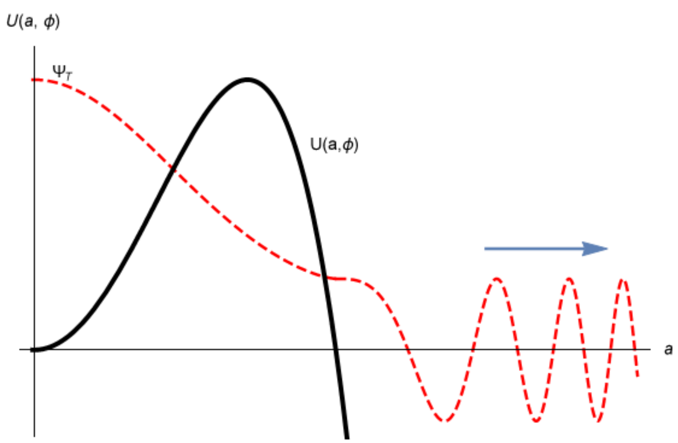
宇宙波函数

宇宙的状态由宇宙的波函数来描述，满足WDWE：

$$\left(\frac{1}{a^p}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a}-\frac{1}{a^2}\frac{\partial^2}{\partial\phi^2}-ka^2+a^4V(\phi)\right)\psi(a)=0$$



Hartle-Hawking: no boundary wave function



Vilenkin: tunneling wave function

A.Vilenkin, PLB 117,25 (1982).

J. B. Hartle and S. W. Hawking, PRD 28,2960 (1983).

A.D. Linde, PRD 58,083514(1998).

宇宙波函数的概率解释

- Hawking，Vilenkin等人曾给出过波函数的解释：

$$\mathcal{P}(\mathcal{A}) \propto \int_{\mathcal{A}} |\psi|^2 d\mathcal{A}$$

根据这种解释，宇宙被产生的概率为：

$$P \sim e^{-2S} = \exp\left(\frac{3M_p^4}{8V(\phi)}\right).$$

S. W. Hawking, Nucl.Phys.B 239,257 (1984)
A. Vilenkin, PRD 39, 1116 (1989).

宇宙波函数-概率解释的困难

- 首先，对于宇宙来说，如何定义观测者和被观测对象？
- 其次，对于我们来说，宇宙的演化只发生一次，如何讨论宇宙处于某个状态的概率？

宇宙波函数的动力学解释

宇宙波函数写为：$\psi(a) = R(a)\exp(iS(a)/\hbar)$,

宇宙处于状态a的概率密度为：$\rho(a) \equiv |\psi(a)|^2 = R(a)^2$

从WDWE出发可以得到系统的守恒流：

$$j^a = \frac{i}{2}a^p(\psi^*\partial_a\psi - \psi\partial_a\psi^*) = -a^pR^2S'$$

$$\partial_a j^a = 0 \longrightarrow a^pR^2S' = c_0$$

根据诱导关系：

$$\frac{\partial S}{\partial a} = \frac{\partial \mathcal{L}}{\partial \dot{a}} = -a\dot{a}$$

可以得到

$$\rho(a) = -\frac{c_0}{a^{p+2}H(a)}.$$

算符次序因子p的约束

宇宙波函数的概率密度应有限

暴涨时期：
$$\rho(a \to 0) = \frac{-c_0}{a^{p+2} H_{In}} \Longrightarrow p + 2 \leq 0$$

暗能量时期：
$$\rho(a \to \infty) = \frac{-c_0}{a^{p+2} H_{DE}} \Longrightarrow p + 2 \geq 0$$

算符次序因子p的约束

$$p = -2$$

宇宙波函数的动力学解释

由 $p=-2$ ，得宇宙波函数概率密度为：

$$\rho(a)=-\frac{c_0}{H(a)}.$$

只与哈勃参数有关：反比于宇宙膨胀的速率。

$\int_{a_1}^{a_2}\psi^*(a)\psi(a)da$ is proportional to the time spent when the universe involves from the state a_1 to a_2.

量子经典对应

应用动力学解释可以得到宇宙很大时的演化规律：

$$a(t) \propto \begin{cases} (t+t_0)^{2/n}, & n \neq 0, \\ e^{t+t_0}, & n = 0. \end{cases}$$

根据爱因斯坦场方程得到的宇宙的演化规律：

dominator	density	evolution
radiation	$n=4$	$a(t) \propto (t+t_0)^{1/2}$
matter	$n=3$	$a(t) \propto (t+t_0)^{2/3}$
dark energy	$n=0$	$a(t) \propto e^t$

宇宙波函数的动力学解释满足量子经典对应。

总结与讨论

The problem of "creation"

396 NATURE VOL. 246 DECEMBER 14 1973

Is the Universe a Vacuum Fluctuation?

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PHYSICAL REVIEW D **89,** 083510 (2014)

Spontaneous creation of the universe from nothing

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量子势推动宇宙迅速长大、物质产生，暴涨自然退出。

到底是“谁”推动宇宙暴涨？

是“假设存在的标量场”？　　还是“量子势”？

去年Kavli奖：

1978、2011年宇宙学先后获得诺贝尔物理学奖

诺贝尔奖的风向标？

宇宙波函数的物理解释

1. 宇宙波函数物理解释-动力学解释

2. 满足量子经典对应

3. 宇宙是非局域的吗?

（发生了什么？退相干？何时发生？）

Publications

1. Dongshan He, Dongfeng Gao and QYC,
   Spontaneous creation of the universe from nothing.
   Phys. Rev. D **89**, 083510 (2014).

2. Dongshan He and QYC,
   Inflation of small true vacuum bubble by quantization of Einstein-Hilbert action.
   Sci. Chin. **58**, 079801 (2015).

3. Dongshan He, Dongfeng Gao and QYC,
   Dynamical interpretation of the wavefunction of the universe.
   Phys. Lett. B **748**, 361-365 (2015).

谢谢大家！