Off-equilibrium infrared structure of self-interacting scalar fields

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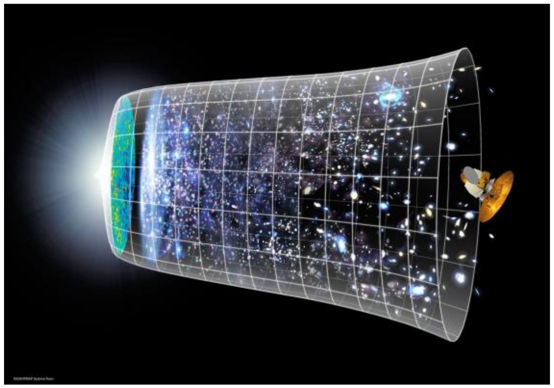
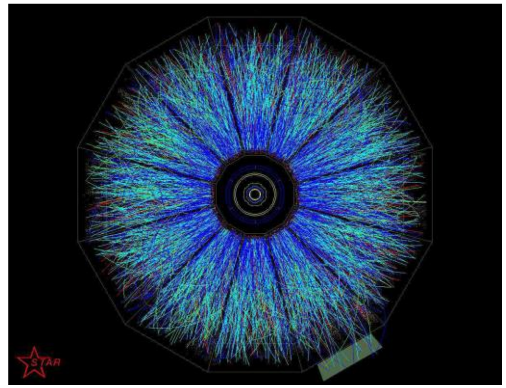
Outline

- **Thermalization: big bang to HIC, cold atoms to dark matter**
- **Non-thermal fixed points: ultra-violet & infrared fixed points, effective kinetic description, defects**
- **Formal Map: From relativistic real scalar to non-relativistic complex field**
- **Simulations of relativistic scalar fields in 2D**

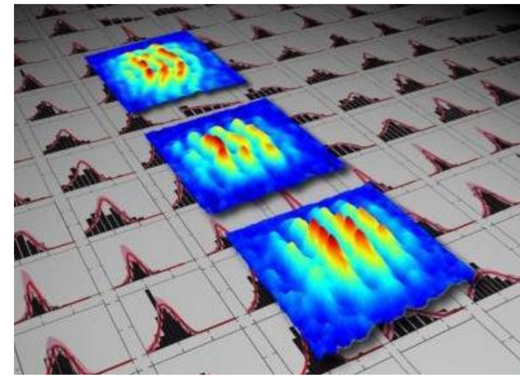
Physical systems far from equilibrium at different scale



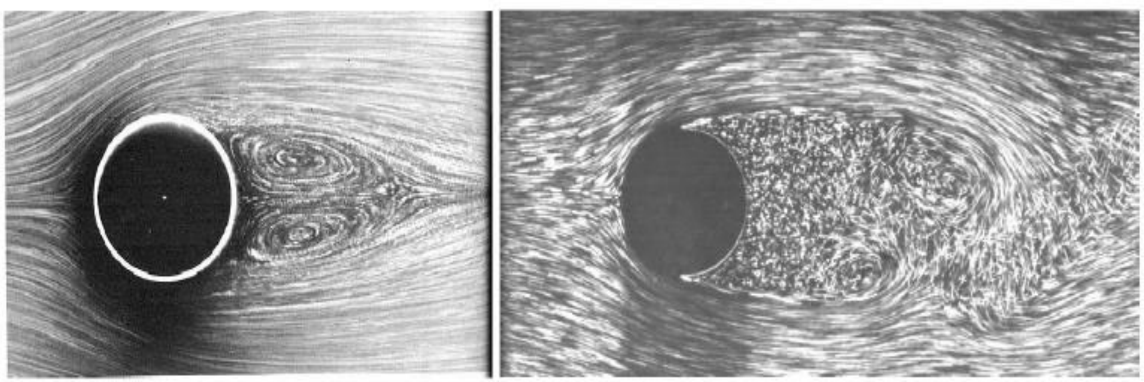
Inflationary cosmology



Heavy-ion collisions



Ultracold atoms



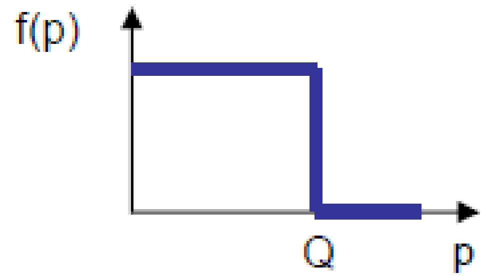
Classical and quantum fluids: Turbulence

Questions:

Do they have common properties?

Physical systems far from equilibrium

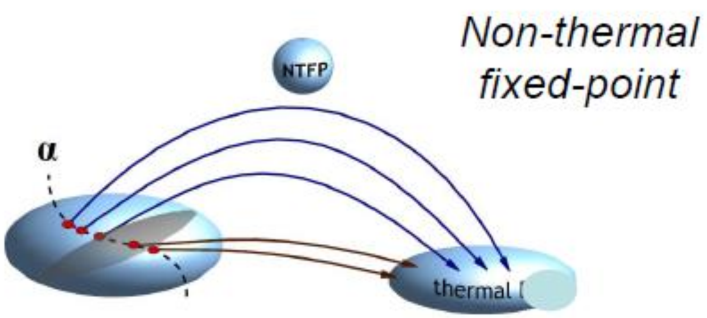
- **Heavy-ion collisions at early times with high energy and weak coupling g<<1**



- **Highly correlated systems (color glass condensate)**

$f(p \lesssim Q) \sim 1/g^2$ (or $\langle A\, A \rangle \sim Q^2/g^2$)

- **The system can take detour during thermalization**



Nowak, Scholle, Gasenzer

Non-thermal fixed-points & universality classes

- **At a non-thermal fixed point**
  → **Memory loss of the details of the initial conditions;**
  → **Self-similar evolution of distribution function f (critical slowing down)**
  → **With scaling behavior of typical scales** $f \sim t^{\alpha}$, $p \sim t^{-\beta}$

$$f(p,t) = t^{\alpha}\, f_S(t^{\beta} p)$$

  → **Possible: several fixed-points in different momentum regimes (inertial range)**
  → **Often connected to kinetic processes (e.g. turbulence) or topological defects**

- **Classification: universality classes far from equilibrium**
  → **Non-thermal fixed-points, described by their dynamical exponents α, β and the scaling function $f_S(x)$, may be classified in universality classes**

Dual cascade in scalar theory

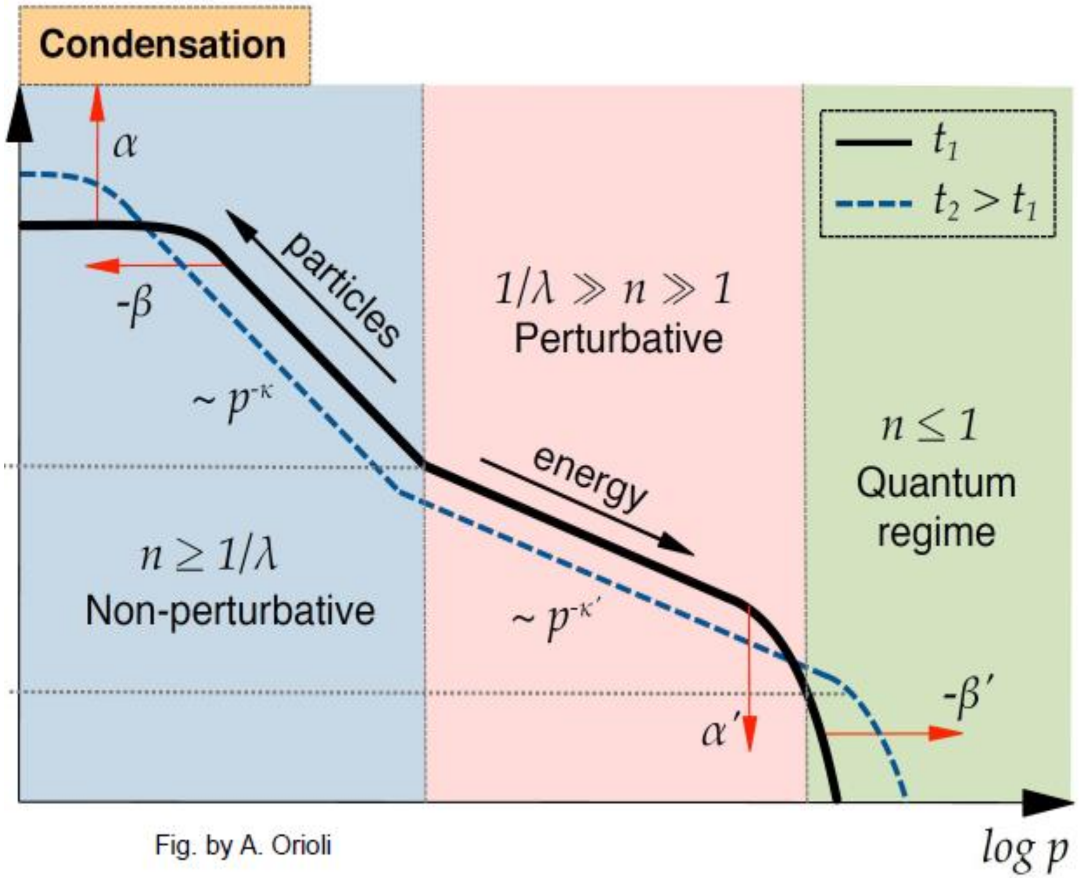


Fig. by A. Orioli

Self-similarity

$$f(p,t) = t^{\alpha} f_S(t^{\beta} p)$$

Perturbative

Micha & Tkachev,

PRL(2003), PRD(2004)

Non-perturbative

Berges, Rothkopf & Schmidt,

PRL (2008)

Nowak, Sexty & Gasenzer

PRB(2011)

Bose condensation

Berges & Sexty, PRL(2012)

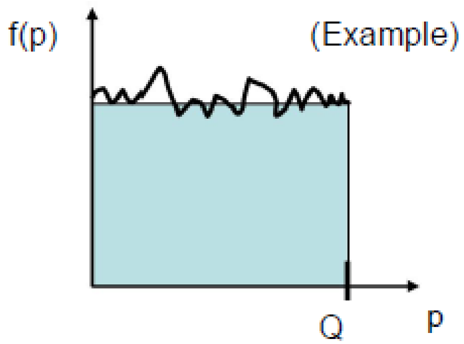
Classical-statistical lattice simulations

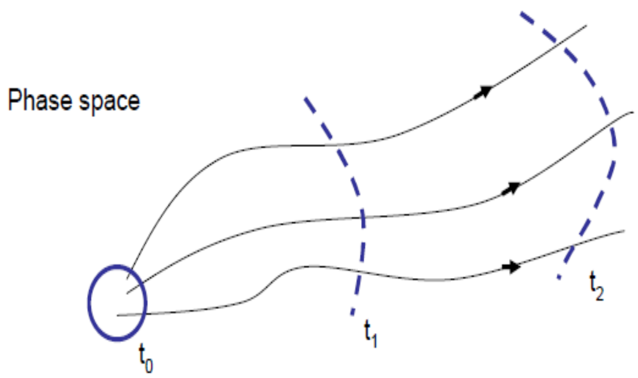
- **Start with fluctuating initial conditions at initial time t_0**
- **Solve initial value problem on the lattice. Example: self-interacting scalars:**

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2 - \frac{1}{4!}\lambda \phi^4$$

$$(\partial_\mu \partial^\mu + m^2)\phi + \frac{1}{6}\lambda\phi^3 = 0$$

- **Evolve observable classically and average over set of initial conditions at time of interest t**





Classical-statistical lattice simulations

- **Initial condition**

Gaussian distributed complex random numbers

$$\phi(\mathbf{x},t_0)=\phi(t_0)+\int[d^3\mathbf{p}]\sqrt{f(\mathbf{p},t_0)+\frac{1}{2}}\left[\xi(\mathbf{p},t_0)e^{i\mathbf{p}\cdot\mathbf{x}}+\xi^*(\mathbf{p},t_0)e^{-i\mathbf{p}\cdot\mathbf{x}}\right]$$

Initial distribution

- **Distribution function**

$$f(\mathbf{p},t)+\frac{1}{2}=\sqrt{F(\mathbf{p})F''(\mathbf{p})-[F'(\mathbf{p})]^2}$$

$$\begin{aligned}F(\mathbf{p}) &= \left\langle|\phi(\mathbf{p})|^2\right\rangle\\ F'(\mathbf{p}) &= \mathrm{Re}\,\langle\phi(\mathbf{p})\partial_t\phi^*(\mathbf{p})\rangle\\ F''(\mathbf{p}) &= \left\langle|\partial_t\phi(\mathbf{p})|^2\right\rangle\end{aligned}$$

- **Range of validity: classicality condition** $f(p)\gg 1$

- **Accurate low-energy description of quantum field theory!**

[Aarts & Berges, PRL 88, 041603 (2002); Jeon, PRC 72, 014907 (2005)]

Relativistic vs non-relativistic fields

Relativistic real scalar

(heavy ion collision)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4$$

$$(\partial_\mu\partial^\mu + m^2)\phi + \frac{1}{6}\lambda\phi^3 = 0$$

Non-relativistic complex scalar

(cold bosonic atoms)

$$\begin{aligned} \mathcal{H} &= -\frac{1}{2m}\psi^*\nabla^2\psi + V|\psi|^2 + \frac{1}{2}g|\psi|^4 \\ \mathcal{L} &= \frac{1}{2}i\left(\psi^*\dot{\psi} - \psi\dot{\psi}^*\right) - \mathcal{H} \\ i\frac{\partial\psi}{\partial t} &= \left(-\frac{1}{2m}\nabla^2 + V + g|\psi|^2\right)\psi \end{aligned}$$

Low-momentum universality class in scalar field theories

What is the connection between relativistic real scalar and non-relativistic complex scalar field theory

Formal Map: from Φ to Ψ

- **Hamiltonian density, scalar field and its canonical momentum**

$$\mathcal{H} = \frac{1}{2}[\pi^2 + (\nabla\phi)^2 + m^2\phi^2] + \frac{1}{24}\lambda\phi^4$$

$$\phi(\mathbf{x}) = \int [d^3\mathbf{k}] \frac{1}{\sqrt{2E_k}} \left(a_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + a^*_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}}\right)$$

$$\pi(\mathbf{x}) = -i\int [d^3\mathbf{k}] \sqrt{\frac{E_k}{2}} \left(a_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} - a^*_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}}\right)$$

all complex numbers

- **Hamiltonian in a_k and a^*_k**

EOM in Poisson Brackets

$$i\frac{da_{\mathbf{p}}}{dt} = \{a_{\mathbf{p}}, H\}_a, \qquad i\frac{da^*_{\mathbf{p}}}{dt} = \{a^*_{\mathbf{p}}, H\}_a$$

$$\begin{aligned} H(a,a^*) = & \int d^3x\mathcal{H} = \int [d^3\mathbf{k}] E_k a_{\mathbf{k}} a^*_{\mathbf{k}} \\ & + \frac{1}{24}\lambda \int [d^3\mathbf{k}][d^3\mathbf{k}_1][d^3\mathbf{k}_2][d^3\mathbf{k}_3] \frac{(2\pi)^3}{\sqrt{16E_kE_{k1}E_{k2}E_{k3}}} \\ & [a_{\mathbf{k}}a_{\mathbf{k}1}a_{\mathbf{k}2}a_{\mathbf{k}3}\delta(\mathbf{k}+\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) + 4a_{\mathbf{k}}a_{\mathbf{k}1}a_{\mathbf{k}2}a^*_{\mathbf{k}3}\delta(\mathbf{k}+\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3) \\ & + 6a_{\mathbf{k}}a_{\mathbf{k}1}a^*_{\mathbf{k}2}a^*_{\mathbf{k}3}\delta(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3) + 4a_{\mathbf{k}}a^*_{\mathbf{k}1}a^*_{\mathbf{k}2}a^*_{\mathbf{k}3}\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3) \\ & + a^*_{\mathbf{k}}a^*_{\mathbf{k}1}a^*_{\mathbf{k}2}a^*_{\mathbf{k}3}\delta(-\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)] \end{aligned}$$

Resonant terms: equal number of a_k and a^*_k

EOM and Poisson brackets

- **EOM in Poisson brackets**

$$i\frac{da_{\mathbf{p}}}{dt} = \{a_{\mathbf{p}}, H\}_a\,, \qquad i\frac{da^*_{\mathbf{p}}}{dt} = \{a^*_{\mathbf{p}}, H\}_a$$

- **Poisson brackets in the a_p basis**

$$\{F(\mathbf{p}), G(\mathbf{p}_1)\}_a = \int [d^3\mathbf{k}] \left[\frac{\partial F(\mathbf{p})}{\partial a_{\mathbf{k}}}\frac{\partial G(\mathbf{p}_1)}{\partial a^*_{\mathbf{k}}} - \frac{\partial F(\mathbf{p})}{\partial a^*_{\mathbf{k}}}\frac{\partial G(\mathbf{p}_1)}{\partial a_{\mathbf{k}}}\right]$$

$$\begin{aligned}\{a_{\mathbf{p}}, a^*_{\mathbf{p}1}\}_a &= (2\pi)^3\delta(\mathbf{p}-\mathbf{p}_1)\\ \{a_{\mathbf{p}}, a_{\mathbf{p}1}\}_a &= \{a^*_{\mathbf{p}}, a^*_{\mathbf{p}1}\}_a = 0\end{aligned}$$

- **Generally we can use any canonical variables** $(a_{\mathbf{k}}, a^*_{\mathbf{k}}) \rightarrow (b_{\mathbf{p}}, b^*_{\mathbf{p}})$ **,**

if $(b_{\mathbf{p}}, b^*_{\mathbf{p}})$ **satisfy the same Poisson brackets in the a_p-basis**

$$\begin{aligned}\{b_{\mathbf{p}}, b^*_{\mathbf{p}1}\}_a &= (2\pi)^3\delta(\mathbf{p}-\mathbf{p}_1)\\ \{b_{\mathbf{p}}, b_{\mathbf{p}1}\}_a &= \{b^*_{\mathbf{p}}, b^*_{\mathbf{p}1}\}_a = 0\end{aligned}$$

Canonical transformation

- **Poisson brackets are invariant under the change of canonical basis, for example**

$$i\frac{\partial b_{\mathbf{p}}}{\partial t} = \{b_{\mathbf{p}}, H\}_{\underline{a}} = \{b_{\mathbf{p}}, H\}_{\underline{b}}$$

- **Canonical variable defined by time shift through H_{aux} (b_p,b^*_p):**

$$\begin{aligned} b_{\mathbf{p}}(z) &= b_{\mathbf{p}} + z\left.\frac{db_{\mathbf{p}}(z)}{dz}\right|_{z=0} + \frac{1}{2}z^2\left.\frac{d^2b_{\mathbf{p}}(z)}{d^2z}\right|_{z=0} + \frac{1}{6}z^3\left.\frac{d^3b_{\mathbf{p}}(z)}{d^3z}\right|_{z=0} + \cdots \\ &= b_{\mathbf{p}} - iz\{b_{\mathbf{p}}, H_{\mathrm{aux}}\}_b + \frac{1}{2}(-iz)^2\{\{b_{\mathbf{p}}, H_{\mathrm{aux}}\}_b, H_{\mathrm{aux}}\}_b \\ &\quad + \frac{1}{6}(-iz)^3\{\{\{b_{\mathbf{p}}, H_{\mathrm{aux}}\}_b, H_{\mathrm{aux}}\}_b, H_{\mathrm{aux}}\}_b + \cdots \end{aligned}$$

$$a_{\mathbf{p}} \equiv b_{\mathbf{p}}(z)$$

$$b_{\mathbf{p}} \equiv b_{\mathbf{p}}(0) \qquad b^*_{\mathbf{p}} \equiv b^*_{\mathbf{p}}(0)$$

z is auxiliary time

- **One can prove that $b_{\mathbf{p}}(z), b^*_{\mathbf{p}}(z)$ satisfy canonical relations in b_p basis**

$$\begin{aligned} \{a_{\mathbf{p}}, a^*_{\mathbf{p}1}\}_b &= (2\pi)^3\delta(\mathbf{p}-\mathbf{p}_1) \\ \{a_{\mathbf{p}}, a_{\mathbf{p}1}\}_b &= \{a^*_{\mathbf{p}}, a^*_{\mathbf{p}1}\}_b = 0 \end{aligned}$$

New canonical variable through H_{aux}

- **Auxiliary Hamiltonian**

$$
\begin{aligned}
H_{\text{aux}}(b,b^*) \quad = \quad & \int [d^3\mathbf{k}_1][d^3\mathbf{k}_2][d^3\mathbf{k}_3][d^3\mathbf{k}_4] \\
& \times \left\{ \frac{1}{24}[B_1(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) b_{\mathbf{k}1} b_{\mathbf{k}2} b_{\mathbf{k}3} b_{\mathbf{k}4} + \text{c.c.}] \right. \\
& \times \delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4) \\
& + \frac{1}{6}[B_2(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3;\mathbf{k}_4) b_{\mathbf{k}1} b_{\mathbf{k}2} b_{\mathbf{k}3} b^*_{\mathbf{k}4} + \text{c.c.}] \\
& \left. \times \delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3-\mathbf{k}_4) \right\}
\end{aligned}
$$

- **B_1 and B_2 will be determined by removing non-resonant terms in the original Hamiltonian**

Canonical transformation: derive H_{GP}

- **New canonical variable by time shift through H_{aux}**

initial auxiliary time z=0

$$b_{\mathbf{p}} \equiv b_{\mathbf{p}}(0)$$
$$b^*_{\mathbf{p}} \equiv b^*_{\mathbf{p}}(0)$$

$$H_{\rm aux}(b, b^*)$$

at auxiliary time z

$$a_{\mathbf{p}} \equiv b_{\mathbf{p}}(z)$$
$$a^*_{\mathbf{p}} \equiv b^*_{\mathbf{p}}(z)$$

$$\begin{aligned} a_{\rm p} &= b_{\rm p} - iz\{b_{\rm p}, H_{\rm aux}\}_b \\ &\quad + \frac{1}{2}(-iz)^2\{\{b_{\rm p}, H_{\rm aux}\}_b, H_{\rm aux}\}_b \\ &\quad + \cdots \end{aligned}$$

substitute

$$H(a, a^*)$$

$$H'(b, b^*)$$

$$\begin{aligned} H'(b,b^*) &= H(b,b^*) + (-iz)\{H(b,b^*), H_{\rm aux}\}_b \\ &\quad + \frac{1}{2}(-iz)^2\{\{H(b,b^*), H_{\rm aux}\}_b, H_{\rm aux}\}_b \\ &\quad + \frac{1}{6}(-iz)^3\{\{\{H(b,b^*), H_{\rm aux}\}_b, H_{\rm aux}\}_b, H_{\rm aux}\}_b \\ &\quad + \cdots \end{aligned}$$

$$H_{\rm GP}(b, b^*)$$

with only resonant terms

vanishing of non-resonant terms fix B_1 and B_2 in H_{aux}

Gross-Pitaevskii Hamiltonian

$$H_{\mathrm{GP}}(b,b^*) \approx \int [d^3\mathbf{p}] E_p b_{\mathbf{p}} b^*_{\mathbf{p}} + \frac{1}{16}\lambda \int \prod_{i=1}^{4} [d^3\mathbf{k}_i] \frac{1}{\sqrt{E_{k1}E_{k2}E_{k3}E_{k4}}} \times b_{\mathbf{k}1} b_{\mathbf{k}2} b^*_{\mathbf{k}3} b^*_{\mathbf{k}4} (2\pi)^3 \delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4)$$

$$E_k \approx m + \frac{\mathbf{k}^2}{2m}$$

$$\psi(\mathbf{x}) = \int [d^3\mathbf{k}] b_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$H_{\mathrm{GP}}(\psi,\psi^*) \approx m\int d^3\mathbf{x} |\psi(t,\mathbf{x})|^2 + \int d^3\mathbf{x} \psi^*(t,\mathbf{x}) \left(-\frac{1}{2m}\nabla^2\right) \psi(t,\mathbf{x}) + \frac{\lambda}{16m^2} \int d^3\mathbf{x} |\psi(t,\mathbf{x})|^4$$

Express b_p as function of a_p

- **Change the basis from b_p to a_p for all Poisson brackets**

$$\begin{aligned} b_{\mathbf{p}} &= a_{\mathbf{p}} + iz\{b_{\mathbf{p}}, H_{\mathrm{aux}}(b)\}_a \\ &\quad -\frac{1}{2}(iz)^2\{\{b_{\mathbf{p}}, H_{\mathrm{aux}}(b)\}_a, H_{\mathrm{aux}}(b)\}_a + \cdots \end{aligned}$$

- **Perturbation to solve b_p as a function of a_p**

$$\begin{aligned} b_{\mathbf{p}}^{(0)} &= a_{\mathbf{p}} \\ b_{\mathbf{p}}^{(1)} &= iz\ \{b_{\mathbf{p}}, H_{\mathrm{aux}}(b)\}_a\big|_{b\to a} \\ b_{\mathbf{p}}^{(2)} &= -\frac{1}{2}(iz)^2\ \{\{b_{\mathbf{p}}, H_{\mathrm{aux}}(b)\}_a, H_{\mathrm{aux}}(b)\}_a\big|_{b\to a} \\ &\quad + iz\ \{b_{\mathbf{p}}, H_{\mathrm{aux}}(b)\}_a\big|_{b\to a+b^{(1)}} \\ \cdots &\quad \cdots \quad \cdots \end{aligned}$$

The order parameter:

$\lambda\phi^2/m^2$

Express (Ψ, Ψ*) as function of (ϕ,π)

- **Transform to space-time,**

$$\psi(\mathbf{x}) = \int [d^3\mathbf{k}] b_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}$$

- **Express (Ψ, Ψ*) as function of (ϕ,π). In the large mass limit, the zeroth order**

$$\begin{aligned}
\psi_{(0)}(\mathbf{x}) &= \int \frac{d^3k}{(2\pi)^3} a_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} \\
&= \sqrt{\frac{m}{2}} \left(1 - \frac{1}{4m^2}\nabla_x^2\right) \phi(x) + i\frac{1}{\sqrt{2m}} \left(1 + \frac{1}{4m^2}\nabla_x^2\right) \pi(x) \\
&\equiv \Phi + \Pi - \frac{1}{4}\Phi'' + \frac{1}{4}\Pi''
\end{aligned}$$

$$\Phi \equiv \sqrt{\frac{m}{2}}\phi(\mathbf{x}) \qquad \Pi \equiv i\frac{1}{\sqrt{2m}}\pi(\mathbf{x}) \qquad O'' \equiv \frac{1}{m^2}\nabla_{\mathbf{x}}^2 O$$

Express (Ψ, Ψ*) as function of (Φ,π)

- **The 1st order contribution**

$$
\begin{aligned}
\psi_{(1)}(\mathrm{x}) \quad = \quad & -\frac{\lambda}{16m^3}\left[-\frac{5}{6}\Phi^3-\frac{7}{4}\Phi^2\Phi''-\frac{27}{8}\Phi\Phi'\Phi'+\frac{5}{2}\Phi^2\Pi\right. \\
& +\frac{11}{8}\Pi\Phi'\Phi'+\frac{3}{2}\Phi\Pi\Phi''+\frac{11}{4}\Phi\Phi'\Pi'+3\Phi^2\Pi'' \\
& +\frac{3}{2}\Phi\Pi^2+\frac{5}{4}\Pi^2\Phi''+\frac{5}{4}\Pi\Phi'\Pi'+\frac{5}{8}\Phi\Pi'\Pi' \\
& \left.+2\Phi\Pi\Pi''-\frac{1}{2}\Pi^3-\frac{21}{8}\Pi\Pi'\Pi'-2\Pi^2\Pi''\right]
\end{aligned}
$$

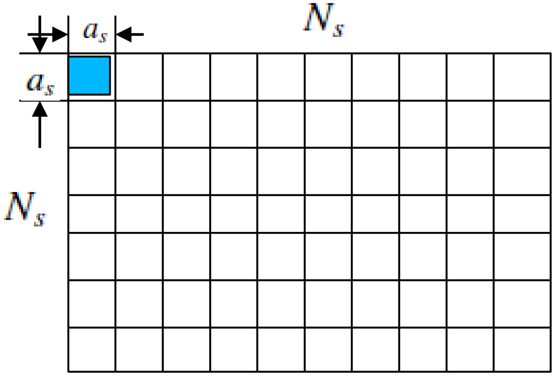
- **An important application is in the study of axion as candidate for dark matter. Our approach has advantage over previous approaches by (1) A. Guth's group; (2) E. Braaten's group.**

Comparison with previous approaches

- **Previous approaches:**
- **(1) Guth's group employ a different method from our work to remove nonresonant terms in the effective Lagrangian by exploiting its U(1) symmetry corresponding to the conservation of particle number. The nonresonant terms carry powers of the phase factor and are therefore dropped from the effective action.**
- **(2) Braaten's group construct an effective field theory for nonrelativistic complex scalar fields by including all terms that satisfy Gallilean invariance, with the coefficients of the EFT determined by matching to the underlying relativistic scalar field theory.**
- **Our approach: a rigorous map between relativistic real scalar field to non-relativistic complex scalar is constructed based on classical canonical transformation. The mapped relation can be derived order by order. It is a non-relativistic effective field theory (NREFT)!**

Numerical simulation of massive relativistic scalar fields in (2+1)D

- **Solve classical EOM for ϕ in real time lattice simulation**



$$\partial_t \phi(t,\mathbf{x}) = \pi(t,\mathbf{x}) \ ,$$

$$\partial_t \pi(t,\mathbf{x}) = \partial_i \partial^i \phi(t,\mathbf{x}) - m^2 \phi(t,\mathbf{x}) - \frac{\lambda}{6}\phi(t,\mathbf{x})^3$$

- **With initial condition**

$$\phi_0(\mathbf{x}) = \frac{1}{(N_s a_s)^2}\sum_{\mathbf{p}} \frac{1}{\sqrt{2E_p}}[\alpha_{\mathbf{p}} e^{+i\mathbf{p}\cdot\mathbf{x}} + \alpha^*_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{x}}],$$

$$\pi_0(\mathbf{x}) = \frac{(-i)}{(N_s a_s)^2}\sum_{\mathbf{p}} \sqrt{\frac{E_p}{2}}[\alpha_{\mathbf{p}} e^{+i\mathbf{p}\cdot\mathbf{x}} - \alpha^*_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{x}}],$$

Sampled with Gaussian magnitude and uniform random-phase distribution

$$\langle \alpha_{\mathbf{p}} \alpha^*_{\mathbf{q}} \rangle = (N_s a_s)^2 \delta_{\mathbf{p},\mathbf{q}} f(t=0,p)$$

$$f(t=0,p) = \frac{6 n_0 Q}{\lambda}\theta(Q-p).$$

- **Single particle distribution**

$$f(t,p) = \frac{1}{(N_s a_s)^2}\sqrt{\langle|\tilde{\phi}(t,\mathbf{p})|^2\rangle\langle|\tilde{\pi}(t,\mathbf{p})|^2\rangle},$$

Fourier transform of ϕ(t,x) and π(t,x)

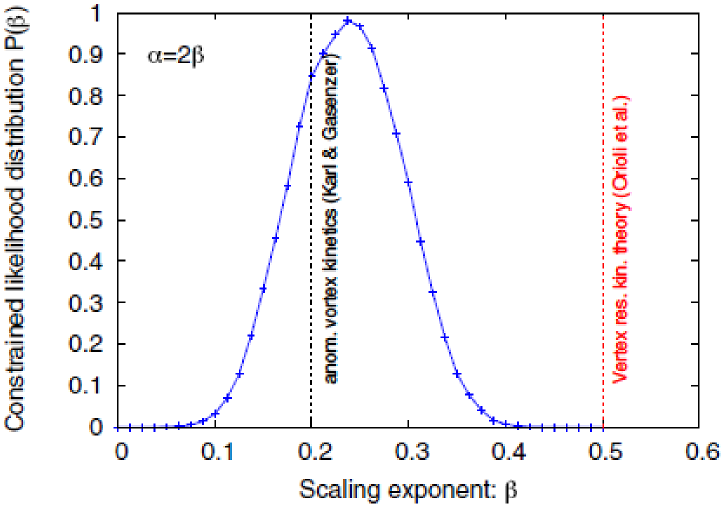
Numerical results

- **Scaling solution for ϕ :**

$$f(t,p) = t^{\alpha} f\left(t^{\beta}, p\right)$$

- **With exponents**

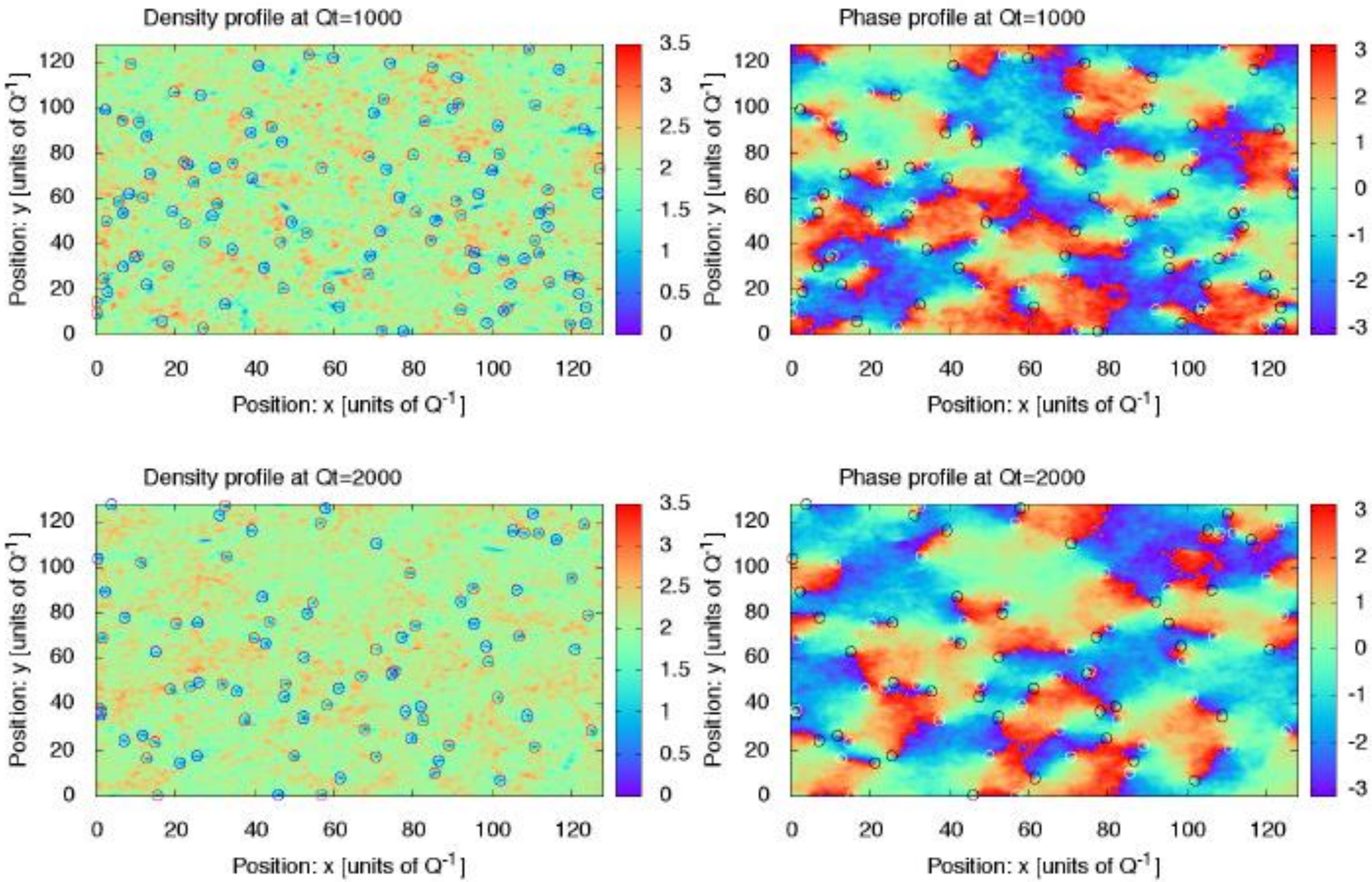
$$\beta = 0.24 \pm 0.08$$
$$\alpha = 2\beta$$



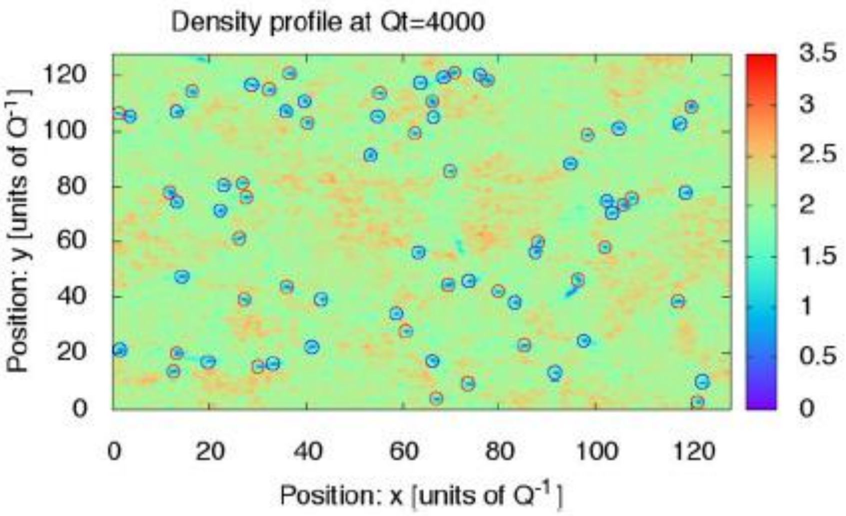
- **Non-relativistic complex scalar Ψ constructed from ϕ. We can draw density $|\Psi(x)|^2$ and phase θ(x)**

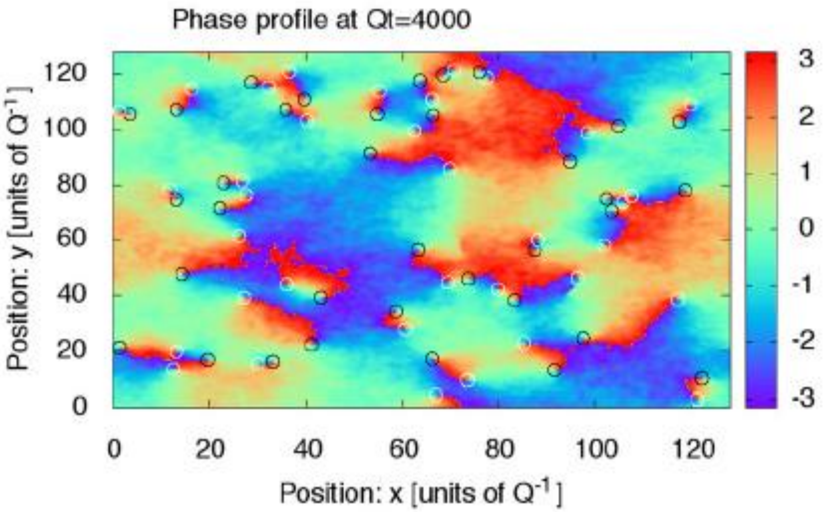
$$\psi(\mathbf{x}) = \sqrt{\rho(\mathbf{x})} e^{i\theta(\mathbf{x})}$$

Numerical results: density and phase of ψ

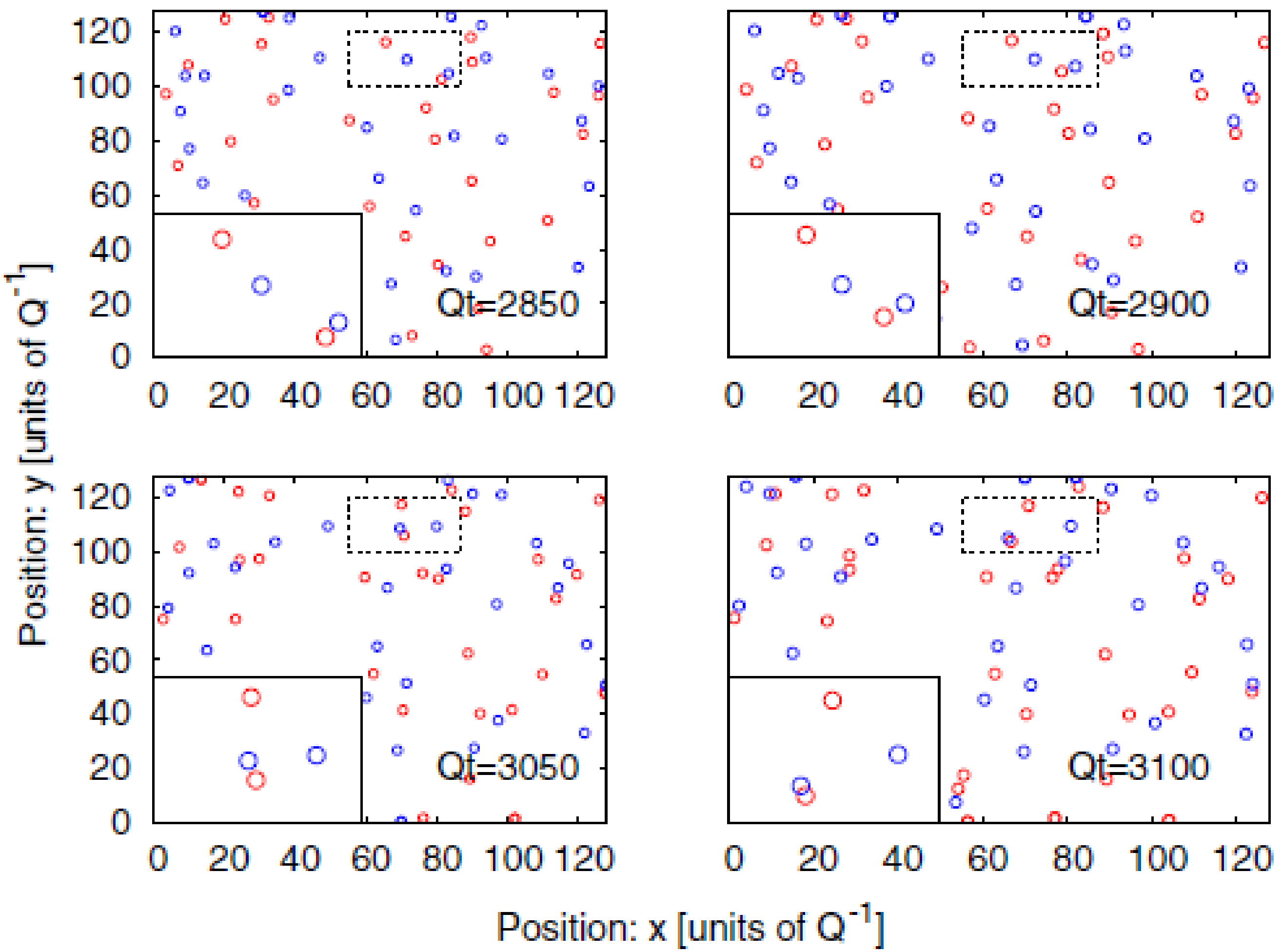


Numerical results: density and phase of ψ





Numerical results:
evolution of vortex and anti-vortex

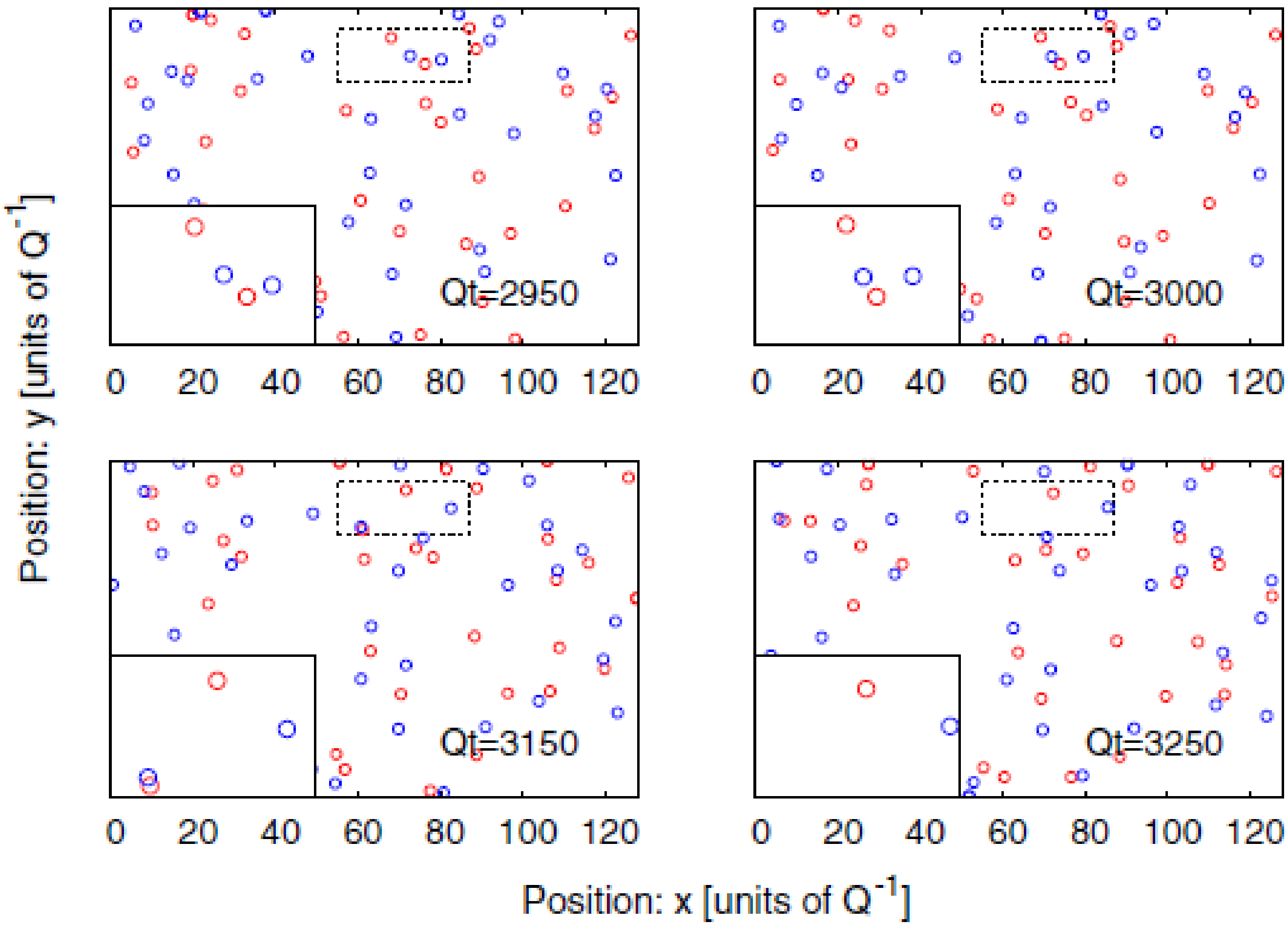


Non-equilibrium realization of Topological phase transition (KT transition):

Kosterlitz, Thouless,

Nobel Prize 2016

Numerical results: evolution of vertex and anti-vortex



Summary

- We demonstrated a formal map between the infrared structure of an N=1 relativistic self-interacting scalar field theory and the Gross-Pitaevskii (GP) theory for nonrelativistic fields, which is widely employed as a model theory describing the behavior of superfluids.
- This map is constructed by classical canonical transformation in a perturbation scheme. In this way, we build up a non-relativistic effective field theory (NREFT)!
- Many applications:

(1) axion as dark matter

(2) superfluids + superconductors

(3) turbulence

(4) polarization in fluids