The Ideas and Structure of Loop Quantum Gravity

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ICTS, USTC, Mar. 24, 2017

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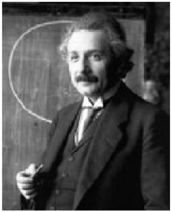
Outline

Intrinsic Contradiction in Fundamental Physics

From the beginning of last century to now, two fundamental theories of physics, QM and GR, have destroyed the coherent pictures of the physical world.



Classical Gravity - Quantum Matter Inconsistency

$$R_{\alpha\beta}[g] - \frac{1}{2}R[g]g_{\alpha\beta} = \kappa T_{\alpha\beta}[g].$$

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- In quantum field theory the energy-momentum tensor of matter field should be an operator-valued tensor $\hat{T}_{\alpha\beta}$.
- In QFT on curved spacetime one replaces $T_{\alpha\beta}[g]$ by the expectation value $< \hat{T}_{\alpha\beta}[g] >$ with respect to some quantum state of the matter on a fixed spacetime.
- However, in general it is difficult to obtain consistent solutions to the semiclassical equation [Flanagan and Wald, 1996].

Singularity in General Relativity

- Einstein's theory of General Relativity is considered as one of the most elegant theories in 20th century.
  Many experimental tests, especially in the solar system experiments [Will 2006], confirm the theory in classical domain.
- However, in 1960s Penrose and Hawking proved that singularities are inevitable in general spacetimes with certain reasonable conditions on energy and causality by the well-known singularity theorem.
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  Thus general relativity can not be valid unrestrictedly.
- One naturally expects that, in extra strong gravitational field domains near the singularities, the gravitational theory would probably be replaced by an unknown quantum theory of gravity.

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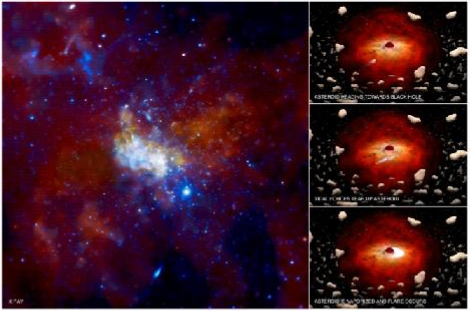
- It is expected that some quantum gravity theory, playing a fundamental role at Planck scale, would provide a natural cut-off to cure the UV singularity in quantum field theory.

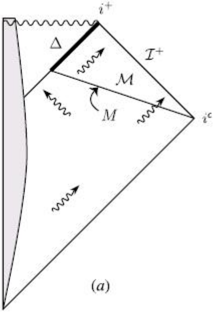
The Interpretation of Black Hole Thermodynamics

- Black hole entropy:

$$S_{BH} = \frac{k_B c^3 Ar_{BH}}{4G\hbar}.$$

This equation brings together the three pillars of fundamental physics.





Different Approaches to Quantum Gravity

- Quantum Field Theory + Extra Dimensions + Supersymmetry
  $\rightarrow$ String Theory (1968-): Green, Gross, Polchinski, Schwarz, Witten,...USTC...
  Perturbative, Background dependent
  Nonperturbative, Background independent $\rightarrow$? M theory

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- Other Alternatives:
  Noncommutative Geometry, Dynamical Triangulation, Causal Set, ...

Background independence

- GR-Notions of space, time and causality: Spacetime is dynamical;
  QM-Notions of matter and measurement: Dynamical entity is made up of quanta and in probabilistic superposition state.
- The application of perturbative quantization to GR fails due to its nonrenormalizability.

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  The separation of the gravitational field from background spacetime is in strident contradiction with the very lesson of GR.
- *The viewpoint of background independence:*
  GR's revolution: particle and fields are neither immersed in (external) space nor moving in (external) time, but live on one another.
  The quanta of the field cannot live in spacetime. They should build spacetime themselves.

Holonomies

- ⋆ LQG inherits the basic idea of Einstein that gravity is fundamentally spacetime geometry.
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- ⋆ Idea: combine the basic principles of GR and QM.
- ⋆ The choice of the algebra of field functions to be quantized: Not the positive and negative components of the field modes as in conventional QFT; but the holonomies of the gravitational connection and the electric flux.
- ⋆ The physical meaning of holonomies:
  Faraday - lines of force: the relevant variables do not refer to what happens at a point, but rather refer to the relation between different points connected by a line.

$$A(c) = \mathcal{P}\exp\Big(-\int_0^1 [A_a^i \dot{c}^a \tau_i]\ dt\Big).$$

Strategy

- Combining two fundamental principles: Background independence (GR) and Quantum mechanical property(QM).
- Minimal principle: QGR.
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⋆ Being conserve and of small ambition: No major additional physical hypothesis, no claim of being final theory of everything.

⋆ Radical and ambitious side: to merge the conceptual insight of GR into QM.

Classical connection dynamics of GR

- The phase space of GR can be embedded into that of Yang-Mills gauge theory by the following configuration and conjugate momentum variables:

$$
\begin{aligned}
A_a^i &:= \Gamma_a^i + \beta K_a^i, \\
\widetilde{P}_i^a &:= \frac{1}{2\kappa\beta}\widetilde{\eta}^{abc}\epsilon_{ijk}e_b^j e_c^k \;=\; \frac{1}{\kappa\beta}\sqrt{\det q}\, e_i^a.
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- The Hamiltonian density $\mathcal{H}_{tot}$ is a linear combination of first-class constraints:

$$\mathcal{H}_{tot} = \Lambda^i G_i + N^a V_a + NS,$$

where G_i is the standard Gaussian constraint of a $SU(2)$ Yang-Mills theory.

Quantum kinematics of LQG

- Classical configurations:
  $\mathcal{A} = \{A|_\Sigma,\ smooth,\ suitable\ boundary\ condition\}$.

Strategy: build the infinite dimensional integration theory from the finite one.

- The kinematical Hilbert space: $\mathcal{H}_{kin} = L^2(\overline{\mathcal{A}},\ d\mu^0)$.

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- The kinematical Hilbert space: $\mathcal{H}_{kin} = L^2(\overline{\mathcal{A}},\ d\mu^0)$.
    - Quantum connections on a finite graph α: $\overline{A}_\alpha \in \overline{\mathcal{A}}_\alpha$ can be identified with the holonomies of A_α along the edges of α.
    - *Diff*(Σ) has a natural induced action on $\overline{\mathcal{A}}$, and μ^0 is invariant under this action.
    - Uniqueness Theorem [LOST, 2005]:
      There is a unique gauge and diffeomorphism invariant representation of the holonomy-flux $*-$algebra, given by μ^0.

Well-defined quantum kinematics of LQG

- Solving the Gaussian constraint $\longrightarrow$
  Gauge invariant Hilbert space: $\mathcal{H}^G = \oplus_{\alpha,\mathbf{j}}\mathcal{H}'_{\alpha,\mathbf{j},\mathbf{l}=0} \oplus \mathbf{C}$.
- The gauge invariant spin-network basis T_s, $s = (\gamma(s), \mathbf{j}_s, \mathbf{i}_s)$ in $\mathcal{H}^G$:

$$T_{s=(\gamma,\mathbf{j},\mathbf{i})} = \bigotimes_{v\in V(\gamma)} i_v \bigotimes_{e\in E(\gamma)} \pi_{j_e}(A(e)), \quad (j_e \neq 0)$$

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- Solving the diffeomorphism constraint by group averaging $\longrightarrow$ The diffeomorphism invariant Hilbert space $\mathcal{H}_{Diff}$

Spin networks of LQG

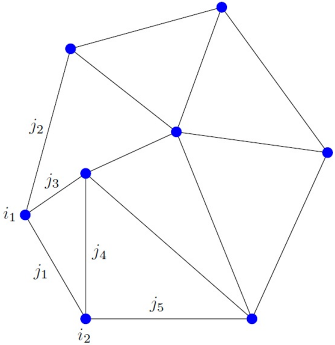

Figure: Dona and Speziale, arXiv:1007.0402.

Geometric operators

- Area operator [Rovelli and Smolin, 1995; Ashtekar and Lewandowski, 1997]
  Given a closed 2-surface or a surface S with boundary, its area can be well defined as a self-adjoint operator $\hat{A}_S$ on $\mathcal{H}_{kin}$:

$$\hat{A}_S \psi_\gamma = 4\pi\beta\ell_p^2 \sum_{v \in V(\gamma\cap S)} \sqrt{(\hat{J}^{(S,v)}_{i(u)} - \hat{J}^{(S,v)}_{i(d)})(\hat{J}^{(S,v)}_{j(u)} - \hat{J}^{(S,v)}_{j(d)})\delta^{ij}}\ \psi_\gamma.$$

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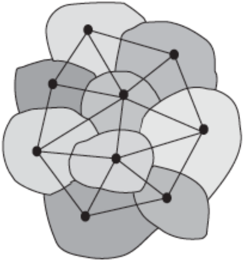
  The typical eigenvalues of $\hat{A}_S$ are given by finite sums,

$$a_S = 8\pi\beta\ell_p^2 \sum_I \sqrt{j_I(j_I+1)},$$

  where j_I are arbitrary half-integers labeling the irreducible representations on the edges of spin networks.

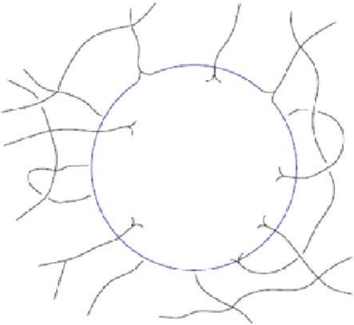
Physical meaning of spin networks

$|\Gamma, j_l, v_n\rangle$



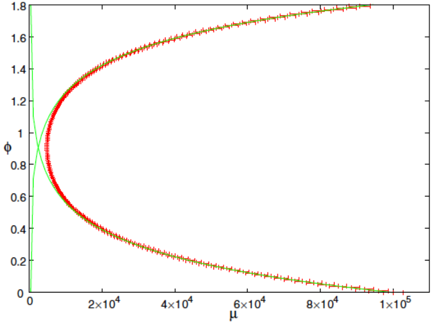
Black hole entropy

- Black hole entropy calculation in LQG [Ashtekar, Baez, Krasnov, 2000; Wang, YM, Zhao, 2014]



Big bang singularity

- Big bang singularity resolution in LQC [Ashtekar, Powlowski, Singh, 2006; Ding, YM, Yang, 2009]



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