No. 1

Date

Brane Creation & Annihilation

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- Motivations
- Generating brane w/o background Flux
- " " w " "
- Discussion / Summary

⊙ Motivations

- Outside String/M Theory

short distance phenomenan

$$\Longleftrightarrow \quad \text{high energy}$$

$$\left(\begin{array}{l} \Delta x \, \Delta p \sim \hbar \\ \Delta t \, \Delta E \sim \hbar \end{array} \right)$$

- String/M Theory has a different story

⊗ Perturbative string says one cannot probe distance $< l_s = \sqrt{\alpha'}$

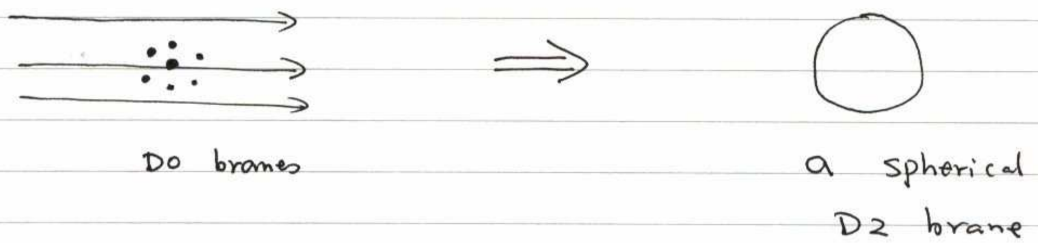
⊗ String/M Theory has a spacetime uncertainty relation

$\Rightarrow$

Particle (or string / or extended objects) size

$\propto$ its excess energy

⊗ Myers effect



dielectric effect

(a static process)

The presence of a background flux

⊗ Giant Gravitons (BPS)

$$AdS_m \otimes S^n \qquad (m,n) = \begin{cases} (4,7) \\ (5,5) \\ (7,4) \end{cases}$$

A zero-size graviton spinning along a S^1 in either AdS_m or S^n can grow its size.

Excess energy & the presence of a background flux.

So far, particle (extended objects) size grow

Appears to need:

ⓐ The presence of a background flux ✓

$\boxed{\text{and / or}}$

ⓑ internal or external spinning

The above $\overset{?}{\Longleftrightarrow}$ spacetime uncertainty relation

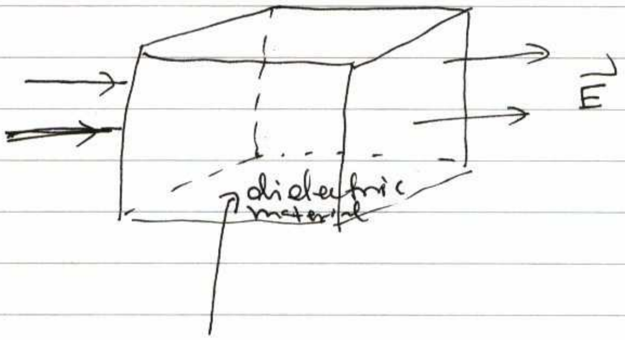
Questions

ⓐ Can we give a realization of spacetime uncertainty relation without the presence of a background flux ?

ⓑ Can we have a Myers effect without the presence of a background flux ?

In addition, we ~~like~~ to study the following:

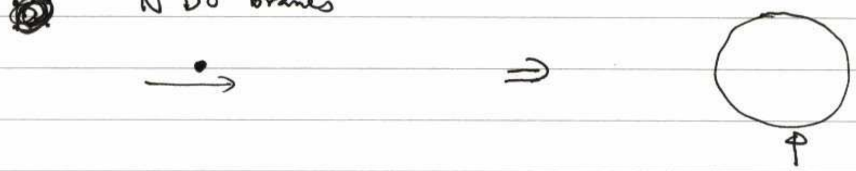
① Myers effect is analogous to



polarization

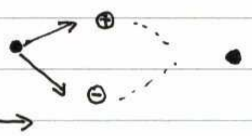
a spherical D2 brane

N D0 branes



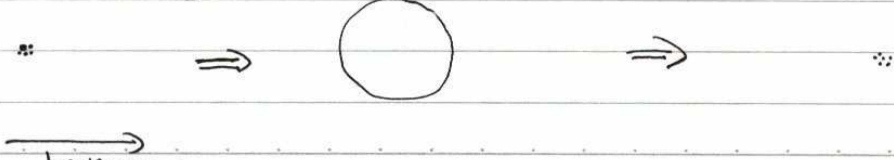
zero net
D2 brane
charge
but having a
dipole moment.

photons



background

spherical
N D0 branes a D2 brane N D0 branes



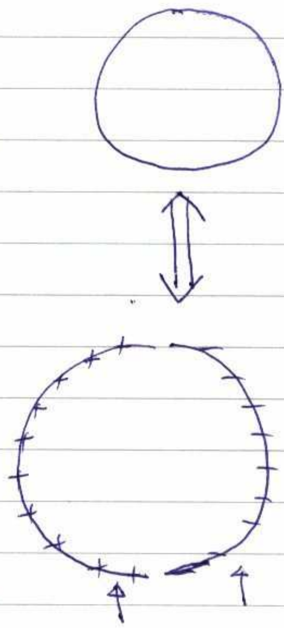
background

② Brane - Anti - Brane Creation & Condensation

a spherical D2 brane
with zero net
D2 brane charge
but carrying
a dipole moment

N D0 branes $\longrightarrow$

under certain
conditions, e.g.,
a background flux
or excess energy



D2 brane anti D2 brane
pair

Time evolution of the above

$\Longleftrightarrow$ Dynamical Tachyon $\Big\{$ creation / condensation

/ cosmology

⊙ Generating a brane w/o a Background flux (see: Chen & Lu hep-th/0406045
Chen, Lu & Roy, to appear)

The Matrix Theory of N D0 brane is

$$L = \frac{T_0 \lambda^2}{2} \mathrm{Tr}(\dot{\Phi}^i)^2 + \frac{T_0 \lambda^2}{4} \mathrm{Tr}[\Phi^i, \Phi^j]^2$$

Φ^i, $N \times N$ matrix

$i = 1, \ldots, 9$

$$\dot{\Phi}^i = \frac{d\Phi^i}{dt}$$

$$T_0 = \frac{2\pi}{g(2\pi l_s)} = \frac{1}{g\sqrt{\alpha'}}$$

$\lambda = 2\pi\alpha'$, Tr: trace

Hamiltonian

$$H = \frac{T_0 \lambda^2}{2} \mathrm{Tr}(\dot{\Phi}^i)^2 - \frac{T_0 \lambda^2}{4} \mathrm{Tr}[\Phi^i, \Phi^j]^2$$

Potential

$$V = -\frac{T_0 \lambda^2}{4} \mathrm{Tr} [\Phi^i, \Phi^j]^2$$

EOM:

$$\ddot{\Phi}^i + [[\Phi^i, \Phi^j], \Phi^j] = 0$$

Trivial static solution

$$\dot{\Phi}^i = \ddot{\Phi}^i = 0 \Rightarrow [\Phi^i, \Phi^j] = 0$$

i.e.,

$$\lambda \Phi^i = \begin{pmatrix} x_1^i & & 0 \\ & \ddots & \\ 0 & & x_N^i \end{pmatrix} \quad \text{(diagonal)}$$

representing the location of each D0 brane

$$U(N) \to U(1)^N$$

Non-trivial Solution must be

Time dependent Φ in a non-trivial representation of $U(N)$

or its non-trivial subgroup

Consider a subgroup $SU(2)$ and choose Φ^i $(i=1,2,3)$ in a $N\times N$ representation of $SU(2)$ ~~which/says~~ ~~not reducible or irreducible, but non-trivial, i.e.,~~

$$\sum_{i=1}^{3}(J^i)^2 = C\,\mathbb{1}_{N\times N} \qquad (C\neq 0)$$

With J^i the three generators of $SU(2)$ in this representation. C is the quadratic Casimir. For example, $C = N^2-1$ for the $N\times N$ irreducible rep.

Now set

$$\Phi^i = g(t)\,J^i \qquad (i=1,2,3)$$

$$\Phi^l = \text{trivial} \qquad (l=4,\dots 9)$$

$$[J^i, J^j] = i\,\epsilon^{ijk}J^k \qquad (i,j,k=1,2,3)$$

Then the non-trivial EOM

$$\boxed{\ddot{g} + 2g^3 = 0}$$

Integrate once $\Rightarrow$

$$\dot{g}^2 + g^4 = W = E^2 > 0$$

with W the integration constant

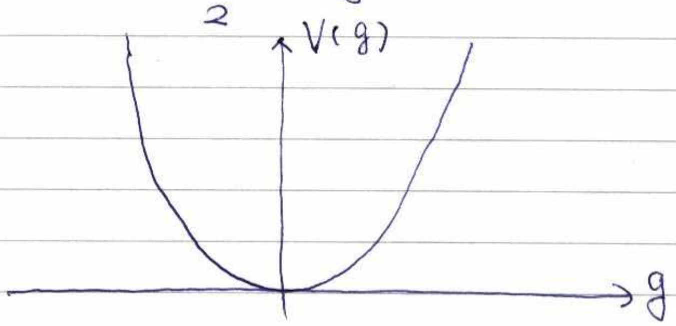
Now

$$H = \frac{T_0 \lambda^2 N C}{2} \left(\dot{g}^2 + g^4 \right)$$

$$= \frac{T_0 \lambda^2 N C}{2} W$$

$$\boxed{H \propto W}$$

$$V = \frac{T_0 \lambda^2 N C}{2} g^4$$



The solution

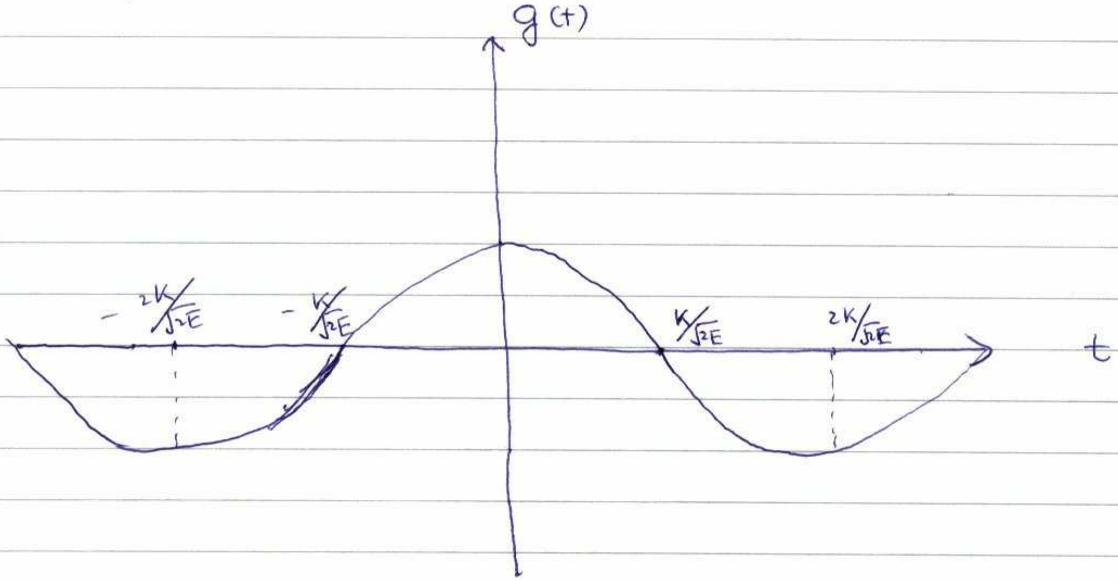
$$g(t) = \sqrt{E}\ \mathrm{cn}\left[\sqrt{2E}\ (t - t_0)\right]$$

with $\mathrm{cn}(x)$ the Jacobian elliptic function. Like $\cos x$, $\mathrm{cn}(x)$ is a periodic function with a period $4K$ with K the complete elliptic integral

$$K = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2\theta}}$$

For the present case $k^2 = 1/2$.

The profile



$(t_0 = 0)$

Three $\Phi^i(t) = g(t)\, J^i$ give rise to a fuzzy sphere with

$$R^2(t) = \frac{\lambda^2}{N} \mathrm{Tr}(\Phi^i)^2 = \lambda^2 g^2(t)\, C$$

$$H \propto W = E^2$$

$$H > 0 \Rightarrow E > 0 \Rightarrow \langle R^2(t) \rangle \neq 0$$

Excess energy

$E = 0 \Leftrightarrow H = 0$
trivial & zero size

$\Downarrow$

Non-zero Excess Energy $\Longleftrightarrow$ finite size

Dynamical Myers Effect
Effective Polarization of D0 branes

$\Downarrow$

A concrete realization of spacetime uncertainty relation

It can be shown if the $N \times N$ irreducible representation is taken ($C = N^2 - 1$), the above fuzzy 2-sphere is actually a spherical D2 brane with worldvolume magnetic flux $F_{\theta\varphi} = \frac{N}{2} \sin\theta$ when N is large and with ~~the~~ following background

$$ds^2 = -dt^2 + dR^2_{(t)} + R^2_{(t)}(d\theta^2 + \sin^2\theta \, d\varphi^2) + \sum_{l=4}^{9} dx^{l\,2}$$

see Chen, Lu & Roy for detail

- Generating a higher dimensional brane from lower dimensional ones with a background flux.

$$F_{tijk} = \begin{cases} -2f \, \epsilon_{ijk} & \text{for } i, j, k = 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

$$L = \frac{T_0 \lambda^2}{2} \mathrm{Tr}(\dot{\Phi}^i)^2 + \frac{T_0 \lambda^2}{4} \mathrm{Tr}[\Phi^i, \Phi^j]^2 + \frac{i \lambda^2 T_0}{3} \mathrm{Tr}\, \Phi^i \Phi^j \Phi^k F_{tijk}$$

The relevant EOM along 1, 2, 3 directions

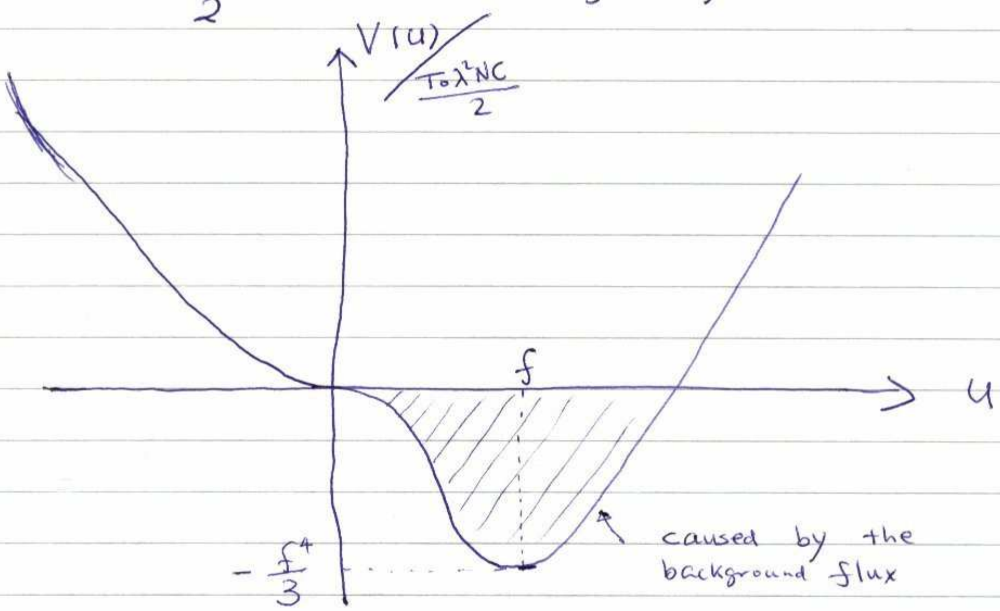
$$\ddot{\Phi}^i + \underbrace{[[\Phi^i, \Phi^j], \Phi^j] + 2if\,\Phi^j\Phi^k\epsilon_{ijk}}_{\text{Myers static case}} = 0$$

Take

$$\Phi^i(t) = u(t)\,J^i$$

Now

$$\begin{cases} H = \dfrac{T_0\lambda^2 NC}{2}\left[\dot{u}^2 + u^4 - \dfrac{4f}{3}u^3\right] \\[2ex] V = \dfrac{T_0\lambda^2 NC}{2}\left(u^4 - \dfrac{4f}{3}u^3\right) \end{cases}$$



EOM:

$$\ddot{u} + 2u^3 - 2fu^2 = 0$$

$\Rightarrow$

$$\dot{u}^2 + u^4 - \frac{4f}{3}u^3 = W \leftarrow \text{integration constant}$$

$$H \;\cancel{\propto}\; \frac{T_0 \lambda^2 N C}{2} W$$

$$H \propto W$$

$$\frac{2V}{T_0 \lambda^2 N C} \geq -\frac{f^4}{3} \Rightarrow \qquad \text{(see figure)}$$

$$\boxed{W \geq -\frac{f^4}{3}}$$

Again the three Φ^i give rise to a fuzzy 2-sphere with

$$R^2(t) = \frac{\lambda^2}{N} \mathrm{Tr}(\Phi^i)^2 = \lambda^2 C\, u^2(t)$$

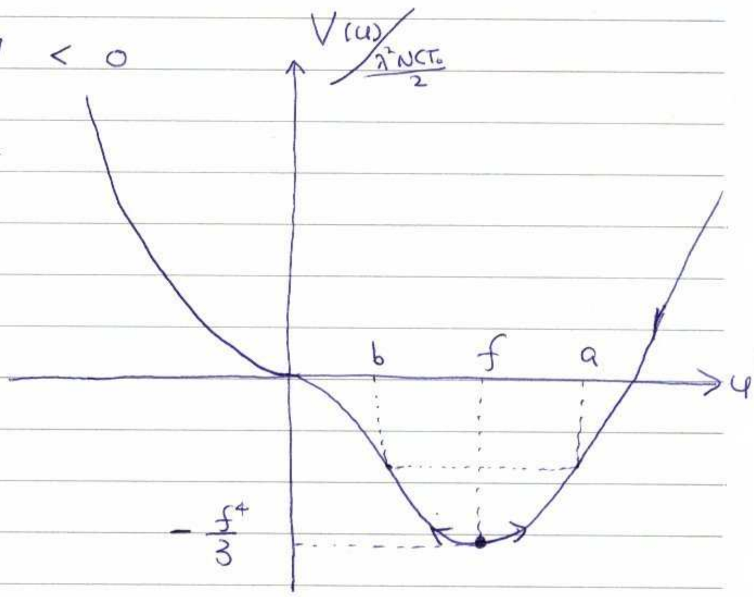
The actual solution depends on the range of W value.

We have three distinct branches for the solution

Case I. $-\frac{f^4}{3} \leq W < 0$

($W = -\frac{f^4}{3} \Rightarrow$ Myers static solution)

$$\boxed{\begin{array}{c} b \leq u(t) \leq a \\ b > 0 \end{array}}$$



$$W = -E^2$$

$$P(u) = u^4 - \frac{4f}{3}u^3 - W$$

has four roots

$$\begin{cases} a = \frac{f}{3}\left[1 + y^{1/2} + \left(3 + \frac{2}{y^{1/2}} - y\right)^{1/2}\right] \\ b = \frac{f}{3}\left[1 + y^{1/2} - \left(3 + \frac{2}{y^{1/2}} - y\right)^{1/2}\right] \\ c = \frac{f}{3}\left[1 - y^{1/2} - i\left(y + \frac{2}{y^{1/2}} - 3\right)^{1/2}\right] \\ \bar{c} = \frac{f}{3}\left[1 - y^{1/2} + i\left(y + \frac{2}{y^{1/2}} - 3\right)^{1/2}\right] \end{cases}$$

$$y = 1 + \frac{3}{2}x^{1/3}\left[\left(1 - \sqrt{1-x}\right)^{1/3} + \left(1 + \sqrt{1-x}\right)^{1/3}\right]$$

$$x = 3E^2/f^4$$

$$u(t) = \frac{(aB + bA) - (aB - bA)\,cn\left[(t-t_0)/g\right]}{(A+B) + (A-B)\,cn\left[(t-t_0)/g\right]}$$

$$g = \frac{1}{\sqrt{AB}}\,, \qquad A^2 = (a-b_1)^2 + a_1^2$$

$$B^2 = (b-b_1)^2 + a_1^2$$

Aagain $u(t)$ is periodic and $b \leq u(t) \leq a$

Note $u(t) \neq 0$ for all t, therefore

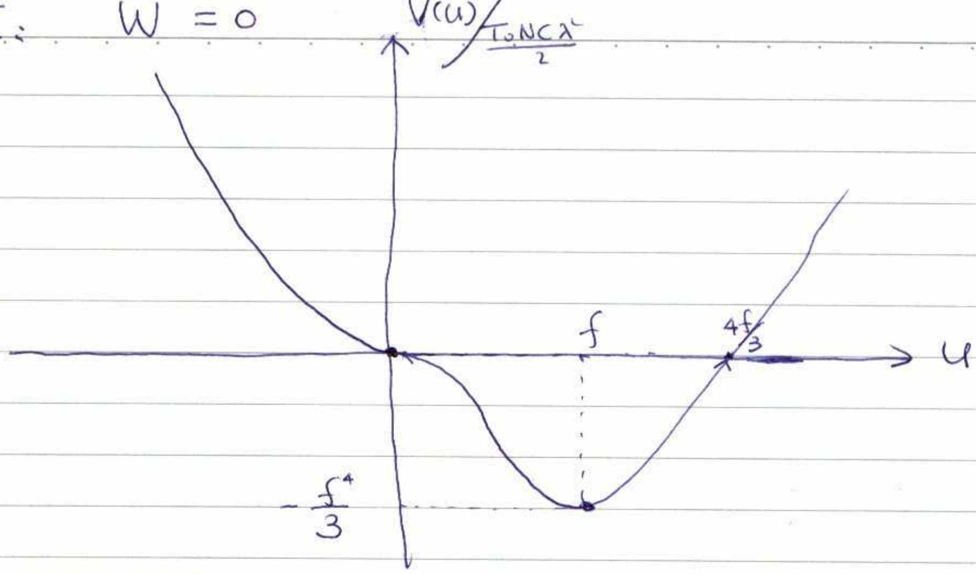
$R^2(t)$ cannot be zero at any instant

$\Rightarrow$ creating a finite size fussy sphere (spherical D2 brane n a semi-spherical D2 — anti semi-spherical D2 brane pair).

~~$\Rightarrow$ Tachyon evolution & partial condensation & creation~~

Zero excess energy

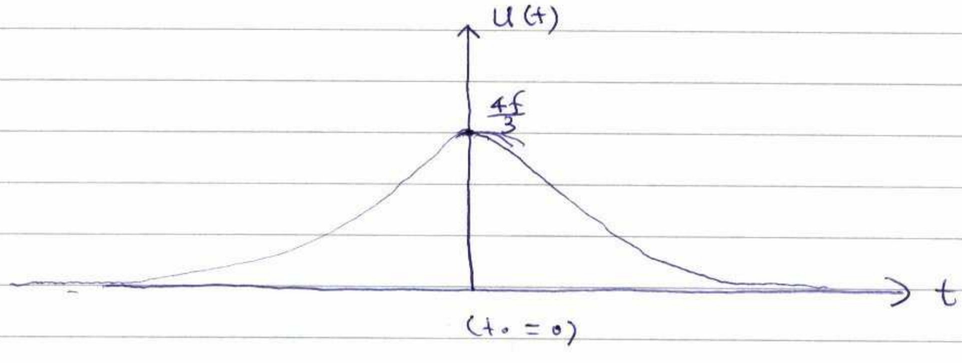
Case II: $W = 0$



$$\boxed{0 \leq u(t) \leq \frac{4f}{3}}$$

The solution can be solved in terms of elementary function as

$$u(t) = \frac{12 f}{9 + 4f^2 (t - t_0)^2}$$



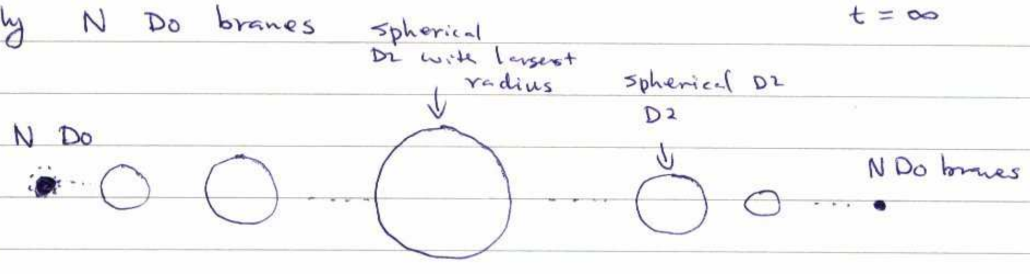
$$R^2(t) = \lambda^2 c \; u^2(t)$$

Physics picture

$\longrightarrow t$

$t = -\infty$
Only N D0 branes

$t = \infty$



$t = t_0$

The spherical D2 brane $\Leftrightarrow$ semi-spherical D2
$\oplus$ Anti semi-spherical D2
pair

$\Leftrightarrow$ the creation of tachyon
and condensation

~~Here the tachyon can be created from BPS D0 branes at $t = -\infty$ and condensed out completely at $t = \infty$.~~

This process is analogous to the photon $\rightarrow$

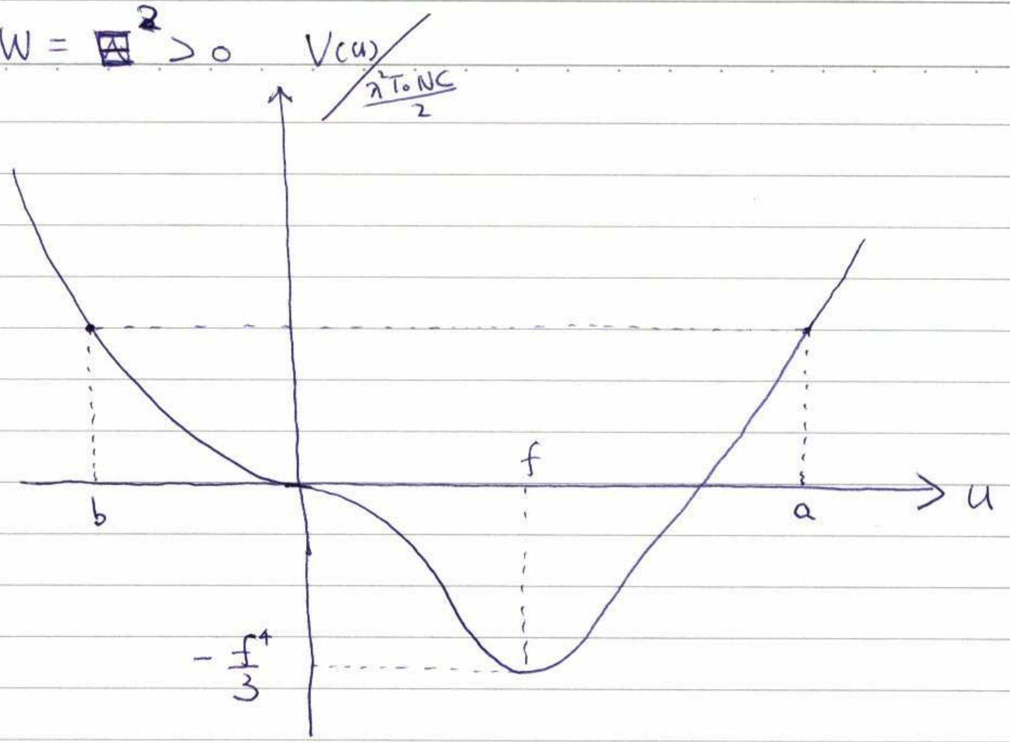
electron + positron pair $\rightarrow$ the pair annihilating back
to a photon.

~~One can calculate the mass parameter for the fluctuation around this configuration $\Rightarrow$~~

~~$t = t_0$ $\quad m^2(t) < 0$ $\quad$ (tachyon mode)~~

~~$t = \pm\infty$ $\quad m^2(t) = 0$ $\quad$ (BPS mode)~~

Case III: $W = [illegible]^2 > 0$



$$b \leq u(t) \leq a$$

$$\boxed{b < 0}$$

So $u(t)$ can be zero at certain instants, actually twice for each cycle.

Characteristically, this solution is no different from the case without the presence of the background flux. This feature holds true in general. This indicates that different vacua have their effect only up to a certain energy. Above that, the characteristics of the underlying dynamics remains essentially the same even though the detail may be different.

The solution is given by the same expressions as in case (I) but now

$$y = 1 - \frac{3}{2} x^{1/3} \left[(\sqrt{1+x} + 1)^{1/3} - (\sqrt{1+x} - 1)^{1/3} \right]$$

$$x = 3E^2/\xi^4 \qquad (0 \le y \le 1)$$

Aagin

$$H \propto W = E^2 > 0$$

⊙ Discussion / Summary

As mentioned ~~above~~ alone above, the actual form of the solution depends on the excess energy W. Two of three branes (Case I & III) cannot be expressed in terms of elementary functions and the remaining one can. This feature remains true for all the examples studied (see Chen & Lu hep-th/0405265, 0406045; Chen, Lu & Roy, to appear)

and seem to remain universally true.

Also, there seems always a case like case II, the solution can be expressed in terms of elementary function and appear to serve as a critical one.

Solution-wise, there appears a transition between case I & III with case II as a critical one. The parameter governning this is the excess energy.

There also appears that ~~case III~~ the characteristics of Case III remains essentially the same, independent of the background chosen.

Case I is dictated by the background chosen.

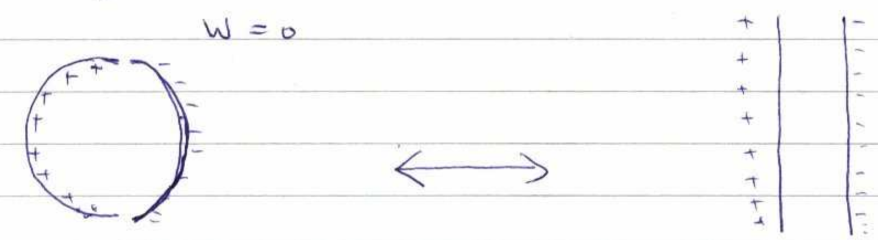
The generated spherical D2 brane $\Longleftrightarrow$

a semi-spherical D2 — anti semi-spherical D2,

We can draw analogous

① $-\frac{f^4}{3} < W < 0$ $\boxed{W < 0}$

② $W = 0$



③ $W > 0$

Conclusion

To have a finite size (or generating a higher dimensional brane from lower dimensional ones), "excess" energy is a must which can be provided either by motion or by a background.

This is consistent with spacetime uncertainty relation.