The renormalization scale setting problem in QCD

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中国科技大学 2013.12.26

QCD理论基本问题之一 - 重整化能标设定

1) 夸克模型 – 1964年提– 1969年诺贝尔物理奖
核心：强子基本结构
未解决：夸克间如何相互作用形成强子 ？

<境界>
根本性
普适性

2) 夸克间（强）相互作用理论 –1973提- 2004年诺贝尔物理奖
核心：渐近自由，耦合常数展开微扰计算
未解决：微扰高阶发散如何解决？高阶圈动量积分发散

3) 强相互作用重整化 – 1969年用于QED也可适用于QCD- 99年诺贝尔物理奖
核心：强相互作用理论可重整, 理论预言为物理有限值
未解决： 仅知其小于1, 但每个物理过程的强相互作用强度(能标)究竟为多大？

有过尝试，但QCD理论发展40年,至今没有很好解决方案
我们坚信必有根本解决方案，但它在那？

机缘巧合
接触到该课题

Recent papers on PMC
(最大共形原理)

Idea and initial application

- Brodsky and Wu, Phys.Rev.D85,034038(2012)
- Brodsky and Wu, Phys.Rev.D85,114040(2012)
- Brodsky and Wu, Phys.Rev.D86,014021(2012)
- Brodsky and Wu, Phys.Rev.D86,054018(2012)

- Brodsky and Wu, **Phys.Rev.Lett.**109,042002(2012)
- Matin, Brodsky and Wu, **Phys.Rev.Lett.**110,192001(2013)
- Wu, Brodsky and Matin, **Prog.Part.Nucl.Phys.**72,44(2013) (Invited Review)

Features and applications

- Wang, Wu and etal., 1301.2992 (NPB876, 731(2013))
- Brodsky, Matin and Wu, 1304.4631 (PRD accepted)
- Zheng, Wu and etal., 1308.2381 (JHEP10, 117(2013))
- Wang, Wu and etal., 1308.6364 (NPB under review)
- Wang, Wu and etal., 1311.5108 (PRD under review)
- Chen, Wu and etal., 1311.2735 (PRD accepted)

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Feature Story Archive

Taking Some Guesswork Out of High-Energy Physics

August 6, 2012

by Lori Ann White

SLAC theorist Stan Brodsky and his collaborator Xing-Gang Wu of Chongqing University have just made the lives of high-energy particle theorists the world over a bit easier. They've demonstrated a way to literally take some of the guesswork out of predictions from quantum chromodynamics (QCD). QCD is the theory explaining the behavior of quarks, which in groups of three forr[...] and gluons, which carry the [...] the quarks together.

In the realm of QCD, interac[...] not enough to know which q[...]

SLAC theoretical physicist Stan Brodsky, right, and his

CP³ Origins

ABOUT RESEARCH NE[...]

NEWSLETTERS OUTREACH

CP³'s Matin Mojaza second in Danish Research Project of the Year Award

December 2, 2013

Matin Mojaza, together with colleagues Stanley J. Brodsky from Stanford University and Xing-Gang Wu from Chongqing University, win second place in the competition for Danish Research Project of the Year. They share the honor with with a research project on the cause of migraine.

Mojaza, Brodsky and Wu have developed a mathematical technique that can help theoretical physicists predict the result of experiments in which quarks – the constituents of nuclei – collide.

SLAC NATIONAL ACCELERATOR LABORATORY

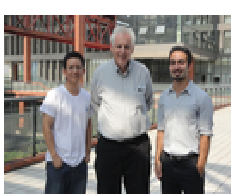
ABOUT | RESEARCH

« News Feature Archive

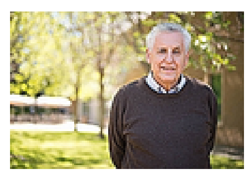
SLAC Theorist Helps Sharpen Tests of Fundamental Theory in High Energy Experiments

By Lori Ann White

August 7, 2013



Particle theorists attempt to put the quantum re[...] However, the world of subatomic particles ope[...] our familiar everyday world. Quantum uncertai[...] gluons, $E=mc^2$ is not a slogan on a t-shirt, it's t[...] a particular particle to exist, it, and others, will [...] lies under the physicists' calculational lenses.



SLAC particle theorist Stan Brodsky (Matt Beardsley/SLAC)

Tags

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Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

Review

The renormalization scale-setting problem in QCD

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D.V. Shirkov

PRL 109, 042002 (2012) PHYSICAL REVIEW LETTERS week ending 27 JULY 2012

Eliminating the Renormalization Scale Ambiguity for Top-Pair Production Using the Principle of Maximum Conformality

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192001 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

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PRL编辑推荐论文

`` I believe the importance of this research is close to the importance of the fundamental work from t'Hooft /Veltman concerning renormalization issues." –one referee's comments on our paper.

OUTLINE

1. Why PMC ? 微扰论思想当前的缺陷

I) Importance of scale-setting; II) General arguments for solution

2. What is PMC ? Features of PMC.

3. Recent progresses and applications : Top, Higgs, …

4. Summary and Outlook PMC, a final solution ?

微扰论基本思想-渐近自由使得我们可做微扰计算

Why PMC ?

Any pQCD calculable quantity ρ can be expanded in perturbative series

$$\frac{\partial\rho}{\partial\mu_R} \equiv 0$$

$$\rho = \underbrace{r_0\alpha_s(\mu_R)}_{LO}\left[1 + r_1(\mu_R)\frac{\alpha_s(\mu_R)}{\pi} + r_2(\mu_R)\frac{\alpha_s^2(\mu_R)}{\pi^2} + \cdots\right]$$

ρ stands for physical observable, Up to infinite order, there is no scheme- and scale- dependence: any choice of scheme/scale should result in same prediction.

因此，无限阶情况下不存在能标设定问题

固定阶计算

权宜之计
四十年的约定俗成真的对吗？

What's conventional scale setting

(核心：能标并不是什么问题；消除误差关键在于能完成高阶运算)

⇒**Guess** a renormalization scale Q, to be typical momentum transfer, or the one to eliminate the large log terms

=> **Keep it fixed to the end** during the calculation

=> **Vary** in a certain range, e.g. [Q/2, 2 Q]; or take several typical momentum transfer to discuss its uncertainty

At any finite order, the use of different scales and schemes may lead to quite different theoretical predictions, which **may be quite large.**

One point

能够完成高阶计算，当然至关重要，因为由此，我们可以同时确定：

I）**非共形项贡献**；确定高阶的贡献究竟为多大

II）**共形项贡献**；可用于确定前面每阶的相互作用强度究竟为多大

两者应当是同等重要。通常的能标设定方案不能解决第二部分且，不能消除renormalon项，[Q/2,2Q]只能获得与藕合常数相关的部分高阶信息，不能得到高阶非共形项的信息-这部分信息只能完成高阶计算后才能获得

特别是，

How to get more reliable pQCD estimation ?

我们真的需要更高阶/更复杂的理论计算才能得到 ?

Puzzles under Conventional Scale Setting

I) Its estimation is **scheme-dependent** at fixed order.

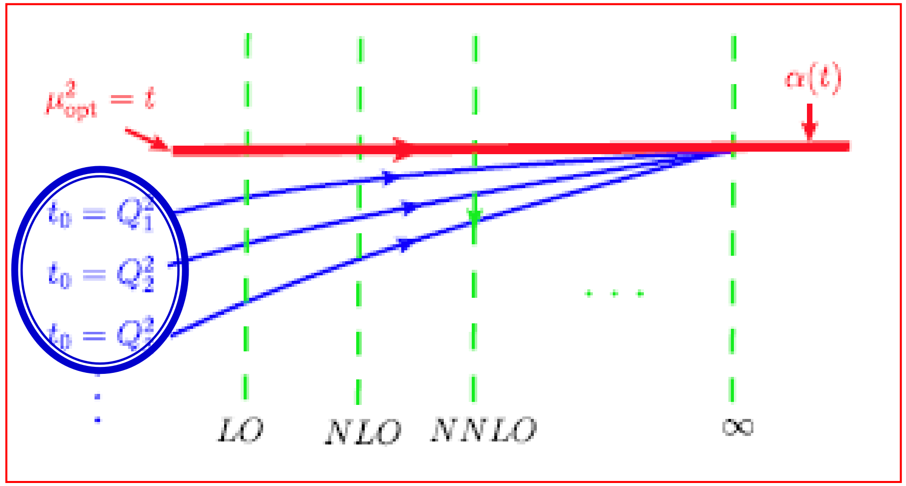
II) Its estimation **strongly depends on the choice of Q**. Why just a factor of ½ or 2 and not 10 or 20 ? Which is the favorable momentum transfer ? (准确与否取决于物理感觉)

III) The **convergence** of the pQCD series is problematic. Especially, when there are large **renormalon** terms. (事实上，很多人将提高pQCD收敛性作为选择有效能标的最重要依据)。

Even if it agrees with the data, it is only guess work !

One way out is to use the experimental data, which inversely greatly depresses the predictive power of pQCD !

选择的能标可能对应真值
也可能离得很远
但没有判据



As a conventional wisdom: one may think that by finishing more and more higher order calculations, such scale uncertainty can be reduced to a large degree.

The question for such naïve scale-setting is:

I) **One may want to obtain accurate estimation as much as possible with known loop results**. This method can not answer this via a systematic way.

II) We still do not know definitely **which scale provides the central value**.

III) We do not know **whether the relative importance of known LO, NLO**, …, is correct or just the fakes for wrong choice of scales.

Thus, the renormalization program inserts a non-physical scale at which the UV pole is removed, and **this artificial scale is a serious and dangerous source of theoretical systematic uncertainties**.

Then, how to solve the problem ?

The key point is to find a universal way to set the right behavior of running coupling for any process.

如何确定跑动藕合常数的准确行为是关键
【跑动行为以及确定能标】

The suggestion of asymptotic free theory (pQCD)
=> only results in coupling < 1; but do no know its accurate value

While, as a further step, PMC or PMS or others = try to determine
=> a definite value for running coupling (or determining its scale)

Extended Renormalization Group Equations

类比

Naïve truncated series to know α_s at different scale --- not reliable !

$$\alpha_S(P)=\alpha_R(Q)+\boxed{f(P,Q)}\alpha_R^2(Q)$$

A way out for scale-dependence

Conventional RGE

$$\frac{d}{d\ln\mu^2}\left(\frac{\alpha_s^R(\mu)}{4\pi}\right)=-\sum_{i=0}^{\infty}\beta_i^R\left(\frac{\alpha_s^R(\mu)}{4\pi}\right)^{i+2}$$

Why it is better and useful ?

The scale is changed along the evolution trajectory with a continuous fashion, thus avoiding the presence of dissimilar scales and large expansion coefficients

universal

$\boxed{\beta_0,\ \beta_1}$

$$a^R=\beta_1\alpha_s^R/(4\pi\beta_0)$$

$$\tau=\frac{\beta_0^2}{\beta_1}\ln\mu^2$$

$$\frac{da^R}{d\tau}=-(a^R)^2\left[1+a^R+c_2^R(a^R)^2+c_3^R(a^R)^3+\cdots\right]$$

Simpler form

Each scheme leads to different c_i^R, and vice versa

Can we discuss the uncertainty of c_i^R in a consistent way as that of the scale ?

核心:找到类似的偏微分方程

Extended RGE !

S.J. Brodsky and H.J. Lu, Phys.Rev. D**51**, 3652(1995).
G. Grunberg, Phys.Rev. D**46**, 2228(1992).

Equivalent to usual RGE

universal coupling constant $a(\tau, \{c_i\})$

$$a^R(\tau_R) = a(\tau_R, \{c_i^R\})$$

$$\beta(a, \{c_i\}) = \frac{\partial a}{\partial \tau} = -a^2\left[1 + a + c_2 a^2 + c_3 a^3 + \cdots\right]$$

Scale equation

and

$$\beta_n(a, \{c_i\}) = \frac{\partial a}{\partial c_n} = -\beta(a, \{c_i\}) \int_0^a \frac{x^{n+2} dx}{\beta^2(x, \{c_i\})}$$

Scheme equations

Useful for a reliable error analysis on higher order

Solution for the scale-equation up to the four-loop level

$$\left(\frac{\beta_0^2}{\beta_1}\ln\frac{\mu^2}{\mu_0^2}\right) = \int_{a(\tau_0,\{c_i\})}^{a(\tau,\{c_i\})} \frac{da}{\beta(a,\{c_i\})}$$

where $\tau_0 = (\beta_0^2/\beta_1)\ln\mu_0^2$ with μ_0 stands for an initial scale. Up to $\mathcal{O}(a^3)$, it leads to

Convenient way $\Longrightarrow$ $\boxed{L = (\beta_0^2/\beta_1)\ln(\mu^2/\Lambda^2)}$

对初始行为的依赖
吸收进Λ_{QCD}-实验确定

$$\boxed{L = C} + \frac{1}{a} + \ln a + (c_2 - 1)\,a + \frac{c_3 - 2c_2 + 1}{2}a^2 + \mathcal{O}(a^3)$$

Λ the **asymptotic scale parameter**, its value is correlated with the integration parameter *C*.

$$L = \mathcal{C} + \frac{1}{a} + \ln a + (c_2 - 1)\, a + \frac{c_3 - 2c_2 + 1}{2} a^2 + \mathcal{O}(a^3)$$

Scale-equation to be solved iteratively

- Setting $a = \frac{1}{L}$ to cancel the L^1-term. And we can find the coefficient L^0
- Setting $a = \frac{1}{L} + \frac{c_2}{L^2}$ to cancel the L^0-term. And we can find the coefficient for L^{-1}
- Setting $a = \frac{1}{L} + \frac{c_2}{L^2} + \frac{c_3}{L^3}$ to cancel the L^{-1}-term. And we can find the coefficient for L^{-2}
- Setting $a = \frac{1}{L} + \frac{c_2}{L^2} + \frac{c_3}{L^3} + \frac{c_4}{L^4}$ to cancel the L^{-2}-term. And the final renormalization equation is of accuracy $\mathcal{O}(1/L^3)$, which is rightly our present required accuracy.

Final four-loop formulae

$$a = \frac{1}{L} + \frac{1}{L^2}(\mathcal{C} - \ln L) + \frac{1}{L^3}\left[\mathcal{C}^2 + \mathcal{C} + c_2 - (2\mathcal{C} - \ln L + 1)\ln L - 1\right] + \frac{1}{L^4}\left\{\mathcal{C}\left(\mathcal{C}^2 + \frac{5}{2}\mathcal{C} + 3c_2 - 2\right) - \frac{1 - c_3}{2} - \left[3\mathcal{C}^2 + 5\mathcal{C} + 3c_2 - 2 - \left(3\mathcal{C} - \ln L + \frac{5}{2}\right)\ln L\right]\ln L\right\} + \mathcal{O}\left(\frac{1}{L^5}\right)$$

Only gives four-loop formulae for a particular MSbar C ←

K.G. Chetyrkin, B.A. Kniehl and M. Steinhauser, Phys.Rev.Lett. **79**, 2184(1997).

Several scale-setting have been suggested: BLM, PMS, FAC and PMC; which one is correct, in principle ?

各能标方案正确性判据

It depends on whether it satisfies the self-consistency conditions derived from the renormalization group invariance

PHYSICAL REVIEW D 86, 054018 (2012)

Self-consistency requirements of the renormalization group for setting the renormalization scale

First-belief

Existence of the [optimal] scale μ_r

No scale dependence

No scheme dependence

或者限制在理想的范围内

$$\frac{\partial a(\tau_R, \{c_i^R\})}{\partial \tau_S} \equiv 0 \quad \text{[scale invariance]}, \qquad (9)$$

$$\frac{\partial a(\tau_R, \{c_i^R\})}{\partial c_j^S} \equiv 0 \quad \text{[scheme invariance]}. \qquad (10)$$

简化描述

any physical observable can be used to define an effective coupling constant

G. Grunberg, Phys. Lett. 95B, 70 (1980).
G. Grunberg, Phys. Lett. 110B, 501 (1982).
G. Grunberg, Phys. Rev. D 29, 2315 (1984).

second

Reflexivity. Given an effective coupling $\alpha_s(\mu)$ specified at a renormalization scale μ, we can express it in terms of itself but specified at another renormalization scale μ',

$$\alpha_s(\mu) = \alpha_s(\mu') + f_1(\mu, \mu')\alpha_s^2(\mu') + \cdots, \quad (14)$$

where $f_1(\mu, \mu') \propto \ln(\mu^2/\mu'^2)$. When the scale μ' is chosen to be μ, the above equation reduces to a trivial identity.

$$\frac{\partial \alpha_s(\mu)}{\partial \ln\mu'^2} \propto \frac{(\ln\mu^2/\mu'^2)^n}{n!} \frac{\partial^{(n+1)} \alpha_s(\mu')}{\partial(\ln\mu'^2)^{(n+1)}}.$$

This shows, generally, the right-hand side of Eq. (14) depends on μ' at any fixed order.
Thus, to get a correct fixed-order estimate for $\alpha_s(\mu)$, a self-consistency scale setting must take the unique value $\mu' = \mu$ on the right-hand side of Eq. (14). If a scale setting satisfies such property, we say it is *reflexive*.

Symmetry. Given two different effective coupling constants $\alpha_{s1}(\mu_1)$ and $\alpha_{s2}(\mu_2)$ under two different renormalization schemes, we can expand any one of them in terms of the other:

$$\alpha_{s1}(\mu_1) = \alpha_{s2}(\mu_2) + r_{12}(\mu_1, \mu_2)\alpha_{s2}^2(\mu_2) + \cdots,$$
$$\alpha_{s2}(\mu_2) = \alpha_{s1}(\mu_1) + r_{21}(\mu_2, \mu_1)\alpha_{s1}^2(\mu_1) + \cdots.$$

third

After a general scale setting, we have

$$\alpha_{s1}(\mu_1) = \alpha_{s2}(\mu_2^*) + \tilde{r}_{12}(\mu_1, \mu_2^*)\alpha_{s2}^2(\mu_2^*) + \cdots,$$
$$\alpha_{s2}(\mu_2) = \alpha_{s1}(\mu_1^*) + \tilde{r}_{21}(\mu_2, \mu_1^*)\alpha_{s1}^2(\mu_1^*) + \cdots.$$

Setting $\mu_2^* = \lambda_{21}\mu_1$ and $\mu_1^* = \lambda_{12}\mu_2$, if

$$\lambda_{12}\lambda_{21} = 1,$$

we say that the scale setting is symmetric.

fourth

Transitivity. Given three effective coupling constants $\alpha_{s1}(\mu_1)$, $\alpha_{s2}(\mu_2)$, and $\alpha_{s3}(\mu_3)$ under three renormalization schemes, we can expand any one of them in terms of the other; i.e.,

$$\alpha_{s1}(\mu_1) = \alpha_{s2}(\mu_2) + r_{12}(\mu_1, \mu_2)\alpha_{s2}^2(\mu_2) + \cdots,$$
$$\alpha_{s2}(\mu_2) = \alpha_{s3}(\mu_3) + r_{23}(\mu_2, \mu_3)\alpha_{s3}^2(\mu_3) + \cdots,$$
$$\alpha_{s3}(\mu_3) = \alpha_{s1}(\mu_1) + r_{31}(\mu_3, \mu_1)\alpha_{s1}^2(\mu_1) + \cdots.$$

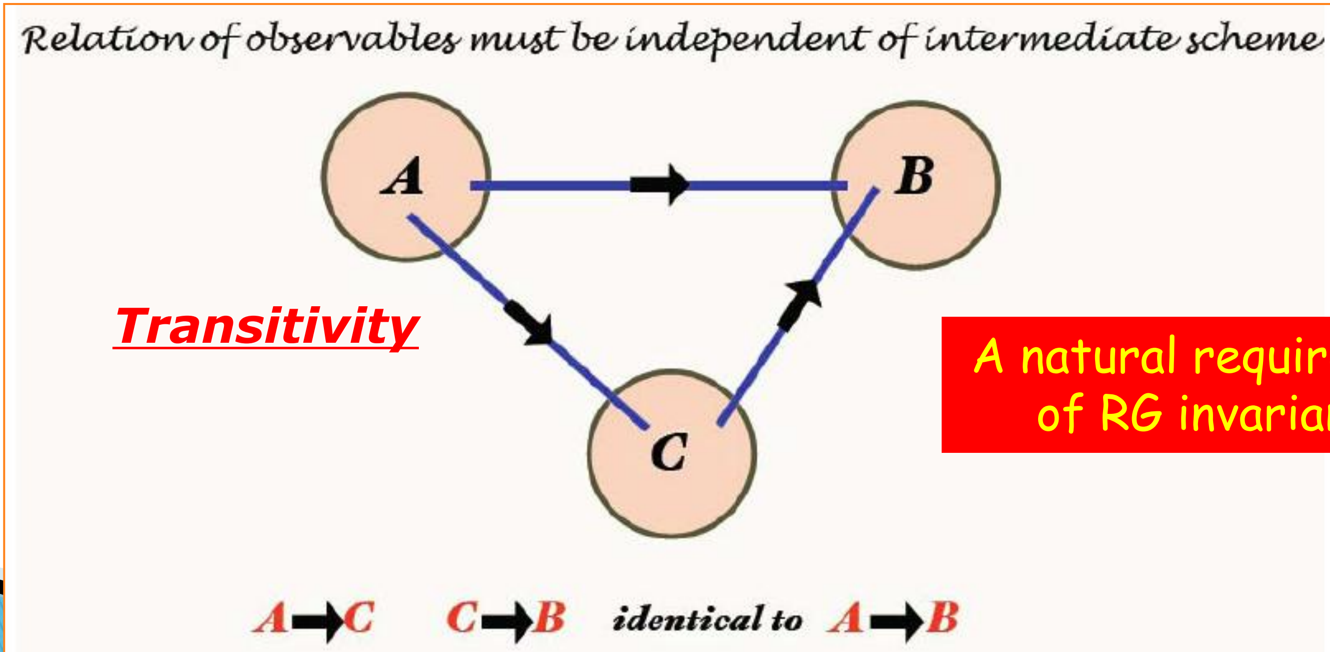
After a general scale setting, we obtain

$$\alpha_{s1}(\mu_1) = \alpha_{s2}(\mu_2^*) + \tilde{r}_{12}(\mu_1, \mu_2^*)\alpha_{s2}^2(\mu_2^*) + \cdots,$$
$$\alpha_{s2}(\mu_2) = \alpha_{s3}(\mu_3^*) + \tilde{r}_{23}(\mu_2, \mu_3^*)\alpha_{s3}^2(\mu_3^*) + \cdots,$$
$$\alpha_{s3}(\mu_3) = \alpha_{s1}(\mu_1^*) + \tilde{r}_{13}(\mu_3, \mu_1^*)\alpha_{s1}^2(\mu_1^*) + \cdots.$$

Setting $\mu_2^* = \lambda_{21}\mu_1$, $\mu_3^* = \lambda_{32}\mu_2$, and $\mu_1^* = \lambda_{13}\mu_3$, if

$$\lambda_{13}\lambda_{32}\lambda_{21} = 1, \tag{21}$$

we say that the scale setting is *transitive*.



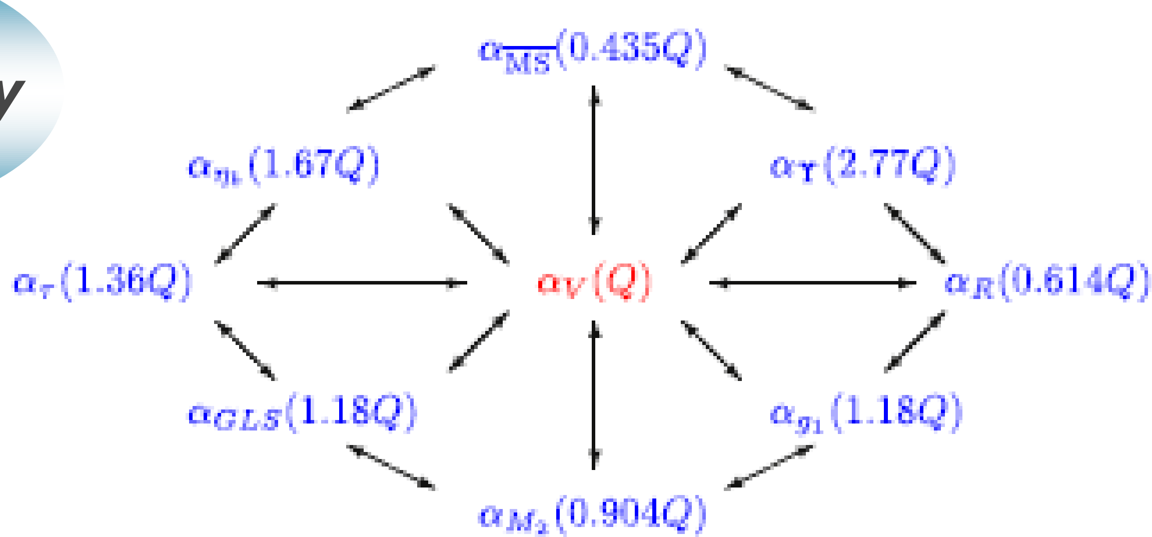
Transitivity

A natural requirement of RG invariance

Commensurate relation among different α_s

S.J. Brodsky and H.J. Lu, Phys.Rev. D**51**, 3652(1995);

Transitivity



保证在不同的方案下理论预言不变

Comparison of different scale-settings

不实用且
准确性不高

Any observable <=> an effective coupling constant (idea useful)

Fastest Apparent Convergence (FAC)

G. Grunberg, Phys. Lett. 95B, 70 (1980).
G. Grunberg, Phys. Lett. 110B, 501 (1982).
G. Grunberg, Phys. Rev. D 29, 2315 (1984).

$\rho_N = \mathcal{C}_0 \bar{\alpha}_s^p(\mu^{\rm FAC})$

How about directly cut off all higher-order-terms ?

有效方法
但非本质

Optimized perturbation theory –
minimize the higher-order contributions – **PMS**

P.M. Stevenson, Phys. Rev. D23 (1981) 2916

$\frac{\partial \rho_N}{\partial \mu_R} \equiv 0$

Nuclear Physics B 868 (2013) 38–64

How about directly set it to satisfy the RG invariance ?

Question for FAC and PMS
Difficult: how to use them for all orders other than NLO; not easy

Kernel Limitation: These two schemes are only optional/effective and do not answer the question of whether their scales are optimal/physical ?

$$\rho_N = \mathcal{C}_0 \alpha_s^p(\mu) + \sum_{i=1}^{N} \mathcal{C}_i(\mu) \alpha_s^{p+i}(\mu)$$

因此，
通常不将它作为能标方案

FAC: $\sum_{i=1}^{N} \mathcal{C}_i(\mu^{\rm FAC}) \alpha_s^{p+i}(\mu^{\rm FAC}) \equiv 0.$ no higher-order terms

G. Grunberg, Phys.Lett. **B95**, 70 (1980); Phys.Lett. **B110**, 501 (1982); Phys.Rev. **D29**, 2315 (1984).

Depends on $f^{-1}(\rho)$, scheme-independent, but low predictive power

P.M. Stevenson, Phys.Lett. **B100**, 61 (1981); Phys.Rev. **D23**, 2916 (1981); Nucl.Phys. **B203**, 472 (1982);

PMS: $\frac{\partial \rho_N}{\partial \mu} |_{\mu=\mu^{\rm PMS}} \equiv 0$ force it to satisfy RG-invariance at fixed order

理论不完善

Steady over scale but its perturbative convergence is not guaranteed;
Break Symmetry/Reflexivity/Transitivity

In fact, we have found that the present procedures for PMS suggested by Stevenson may have some internal errors when extending to higher order other than one-loop. This part of work is in progress.

At present, we have finished a four-loop comparison with PMS and PMC

但，至少，PMS作为实用性处理，还是不错

On the other hand, we find that PMC satisfies all the following properties

- Satisfies all basic requirements: Existence, Unitary, Symmetry, Transitivity, Reflexivity, which are deduction of RG-invariance

$$\frac{\partial \rho}{\partial \mu_R} \equiv 0$$

- scheme-independent
- a better perturbative convergence due to elimination of renormalon
- consistent with previous QED scale-setting, GM-L
- Almost no scale-dependence even at the fixed order

虽然，仍需进一步论证细节，
我们相信PMC就是我们所想的终极方案

What is PMC ?

采用任意方案及能标按通常方法完成初始化

$Choose\ renormalization\ scheme;\ e.g.\ \alpha_s^R(\mu_R^{\rm init})$

$\downarrow$

$Choose\ \mu_R^{init};\ arbitrary\ initial\ renormalization\ scale$

$\downarrow$

$Identify\ \{\beta_i^R\}-terms\ using\ n_f-terms$

$through\ the\ PMC-BLM\ correspondence\ principle$

order-by-order $\downarrow$

$Shift\ scale\ of\ \alpha_s\ to\ \mu_R^{\rm PMC}\ to\ eliminate\ \{\beta_i^R\}-terms$

$\downarrow$

$Conformal\ Series$

$Result\ is\ independent\ of\ \mu_R^{\rm init}\ and\ scheme\ at\ fixed\ order$

PMC-BLM – one

Phys. Rev. Lett. **109**, 042002 (2012)

R_δ–scheme – two

Phys. Rev. Lett. **110**, 192001 (2013)

Eliminate β–terms

Basic procedures of PMC

核心：强相互作用的强度由β-函数确定

First way
of achieving the goal of PMC

每一阶的耦合常数行为是不同的

Main idea: The renormalization is done via an **order-by-order** manner; The scale setting can also be done following the same way; i.e. the behavior of the running coupling and hence its scale is determined by at least one-higher order terms, thus one can derive the scale by absorbing highest nf-terms into the coupling via a **step-by-step** way.

Consistent with large $\beta 0$-idea, also similar to seBLM (Kataev), but are different

standard procedures for PMC

To set PMC scales up to NNLO, the starting point

$$\rho = r_0 \Big[a_s^n(Q) + (A_1 + A_2 n_f) a_s^{n+1}(Q) + (B_1 + B_2 n_f + B_3 n_f^2) a_s^{n+2}(Q) + (C_1 + C_2 n_f + C_3 n_f^2 + C_4 n_f^3) a_s^{n+3}(Q) + \cdots \Big]$$

free of $a_s = \left(\frac{\alpha_s}{\pi}\right)$

first

to set the effective scale Q^* at LO

$$\rho = r_0 \left[a_s^n(Q^*) + \widetilde{A}_1 a_s^{n+1}(Q^*) + (\widetilde{B}_1 + \widetilde{B}_2 n_f) a_s^{n+2}(Q^*) + (\widetilde{C}_1 + \widetilde{C}_2 n_f + \widetilde{C}_3 n_f^2) a_s^{n+3}(Q^*) + \cdots \right] . \quad (11)$$

second

The second step is to set the effective scale Q^{**} at NLO

$$\rho = r_0 \left[a_s^n(Q^*) + \widetilde{A}_1 a_s^{n+1}(Q^{**}) + \widetilde{\widetilde{B}}_1 a_s^{n+2}(Q^{**}) + (\widetilde{\widetilde{C}}_1 + \widetilde{\widetilde{C}}_2 n_f) a_s^{n+3}(Q^{**}) + \cdots \right] , \quad (12)$$

standard procedures for PMC

LO

NLO

NNLO

final

and the final step is to set the effective scale Q^{***} at NNLO

$$\rho = r_0 \left[a_s^n(Q^*) + \widetilde{A}_1 a_s^{n+1}(Q^{**}) + \widetilde{\widetilde{B}}_1 a_s^{n+2}(Q^{***}) + \widetilde{\widetilde{\widetilde{C}}}_1 a_s^{n+3}(Q^{***}) + \cdots \right] . \quad (13)$$

At least three-effective scales

$$\widetilde{A}_1 = A_1 + \frac{33}{2} A_2 \, , \; \widetilde{\widetilde{B}}_1 = \breve{B}_1 + \frac{33}{2} \breve{B}_2 \, , \; \widetilde{\widetilde{\widetilde{C}}}_1 = \breve{\breve{C}}_1 + \frac{33}{2} \breve{\breve{C}}_2$$

$$\breve{B}_1 = \frac{1}{4n}\left[1089(n+1)A_2^2 + 153nA_2 + 66(n+1)A_1A_2 + (4B_1 - 1089B_3)n\right]$$

$$\breve{B}_2 = \frac{-1}{4n}\left[66(n+1)A_2^2 + 19nA_2 + 4(n+1)A_1A_2 - 4n(B_2 + 33B_3)\right]$$

$$\breve{C}_1 = \frac{1}{64A_2n^2}\Big[-40392C_4n^3 + 143748A_2{}^4(3+5n+2n^2) + 8A_2n^2(8C_1 + 35937C_4 + 5049B_3n) - 13464A_2{}^3n(n^2-3n-7) + 72A_1A_2(1+n)(34A_2n - 242B_3n + 121A_2{}^2(3+2n)) + 3A_2{}^2n(2857n + 352B_1(2+n) - 95832B_3(3+2n))\Big]$$

$$\breve{C}_2 = \frac{1}{192A_2n^2}\Big[22392C_4n^3 - 52272A_2{}^4(3+5n+2n^2)(3+2n) - 24A_2n^2(-8C_2 + 6534C_4 + 933B_3n) - 48A_1A_2(1+n)(19A_2n - 132B_3n + 66A_2{}^2(3+2n)) + A_2{}^2n(-5033n - 192B_1(2+n) + 3168B_2(2+n) + 52272B_3(8+5n)) + 24A_2{}^3n(-1871 + n(-627+311n))\Big]$$

$$\breve{C}_3 = \frac{1}{576A_2n^2}\Big[-2736C_4n^3 + 4752A_2{}^4(3+5n+2n^2) + 144A_2n^2(4C_3 + 198C_4 + 19B_3n) - 912A_2{}^3(n^3-4n) + 288A_1A_2(1+n)(-2B_3n + A_2{}^2(3+2n)) - A_2{}^2n(-325n + 576B_2(2+n) + 9504B_3(5+3n))\Big]$$

$$\breve{\breve{C}}_1 = \frac{1}{4(n+1)\widetilde{A}_1}\left[33(n+2)\widetilde{B}_2(2\widetilde{B}_1 + 33\widetilde{B}_2) + (n+1)(153\widetilde{B}_2 + 4\widetilde{C}_1 - 1089\widetilde{C}_3)\widetilde{A}_1\right]$$

$$\breve{\breve{C}}_2 = \frac{-1}{4(n+1)\widetilde{A}_1}\left[2(n+2)\widetilde{B}_2(2\widetilde{B}_1 + 33\widetilde{B}_2) + (n+1)(19\widetilde{B}_2 - 4(\widetilde{C}_2 + 33\widetilde{C}_3))\widetilde{A}_1\right]$$

Coefficients

standard procedures for PMC

$$\ln\frac{Q^{*2}}{Q^2} = \ln\frac{Q_0^{*2}}{Q^2} + \frac{x\beta_0}{4}\ln\frac{Q_0^{*2}}{Q^2}\, a_s(Q) + \frac{y}{16}\left(\beta_0^2\ln^2\frac{Q_0^{*2}}{Q^2} - \beta_1\ln\frac{Q_0^{*2}}{Q^2}\right) a_s^2(Q) \quad (14)$$

$$\ln\frac{Q^{**2}}{Q^{*2}} = \ln\frac{Q_0^{**2}}{Q^{*2}} + \frac{z\beta_0}{4}\ln\frac{Q_0^{**2}}{Q^{*2}}\, a_s(Q^*) \quad (15)$$

$$\ln\frac{Q^{***2}}{Q^{**2}} = \ln\frac{Q_0^{***2}}{Q^{**2}} \quad (16)$$

where the effective scales $Q_0^{*,**,***}$ are determined so as to eliminate $A_2 n_f$, $\widetilde{B}_2 n_f$ and $\widetilde{\widetilde{C}}_2 n_f$-terms completely, the parameters x and z are used to eliminate the $B_3 n_f^2$ and the $\widetilde{C}_3 n_f^2$ terms respectively, and the parameter y is used to eliminate the $C_4 n_f^3$-term. It is found that

$$\ln\frac{Q_0^{*2}}{Q^2} = \frac{6A_2}{n} \quad (17)$$

$$\ln\frac{Q_0^{**2}}{Q^{*2}} = \frac{6\widetilde{B}_2}{(n+1)\widetilde{A}_1} \quad (18)$$

$$\ln\frac{Q_0^{***2}}{Q^{**2}} = \frac{6\widetilde{\widetilde{C}}_2}{(n+2)\widetilde{\widetilde{B}}_1} \quad (19)$$

and

$$x = \frac{3(n+1)A_2^2 - 6nB_3}{nA_2} \quad (20)$$

$$y = \frac{(n+1)(2n+1)A_2^3 - 6n(n+1)A_2B_3 + 6n^2C_4}{nA_2^2} \quad (21)$$

$$z = \frac{3(n+2)\widetilde{B}_2^2 - 6(n+1)\widetilde{A}_1\widetilde{C}_3}{(n+1)\widetilde{A}_1\widetilde{B}_2} \quad (22)$$

The **effective scales should be a perturbative series of α_s** so as to absorb all n_f-dependent terms properly

The effective scales depends on the scheme.

(Transitivity)
Relations between different scales give **scale displacements** among different schemes

$$\alpha_s^{\overline{MS}}(e^{-5/3}Q^2) = \alpha_s^{GM-L}(Q^2)$$

well-known one-loop relation

Subtle points for setting PMC scales

scales can be set up in a general scheme-independent way

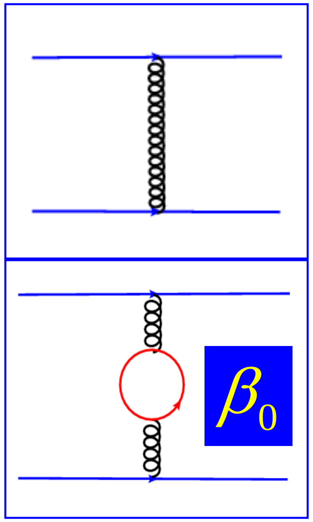
- We shift the initial renormalization scale Q into **effective ones** **until we fully absorb those higher-order terms with n_f-dependence into the running coupling.**

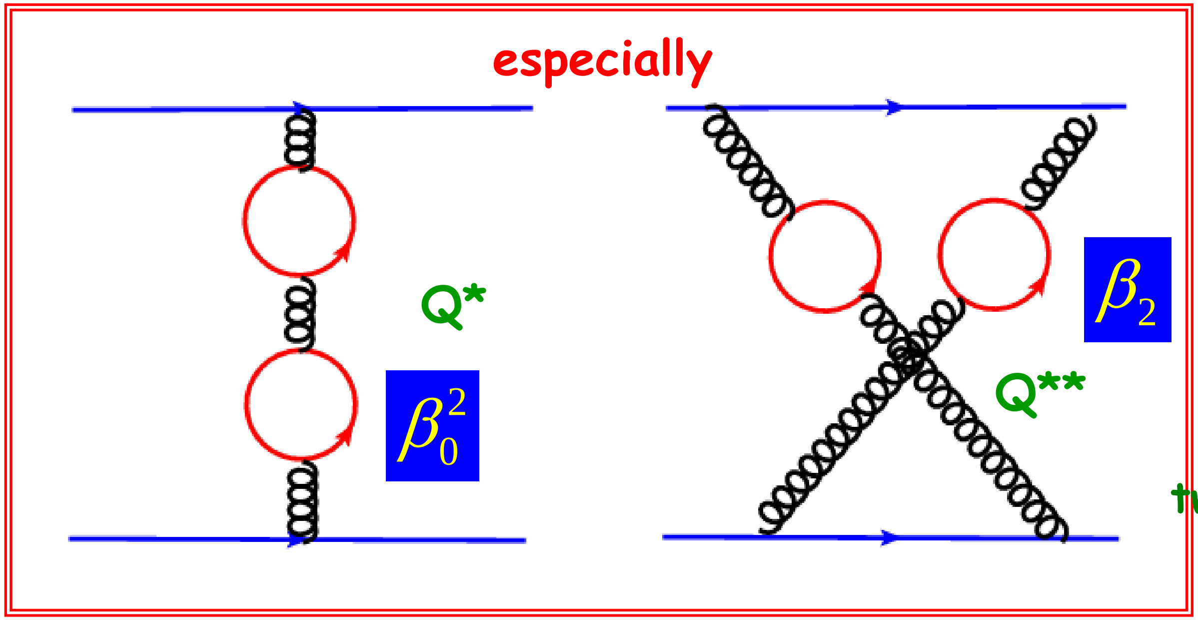
"We need to set at least one effective scale at each order"
Since each order will introduce at least one-new-β-term

"Different terms at the same perturbative order may contribute to different scales Q*, Q, and etc., which depends on how these n_f-terms come from."**

we use nf-terms to identify the β-terms

subtle points





two different PMC scales

Another way: **a unified effective scale Q*** is used for all orders

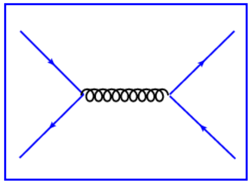
G. Grunberg and A.L. Kataev, Phys.Lett. **B279**, 352(1992).

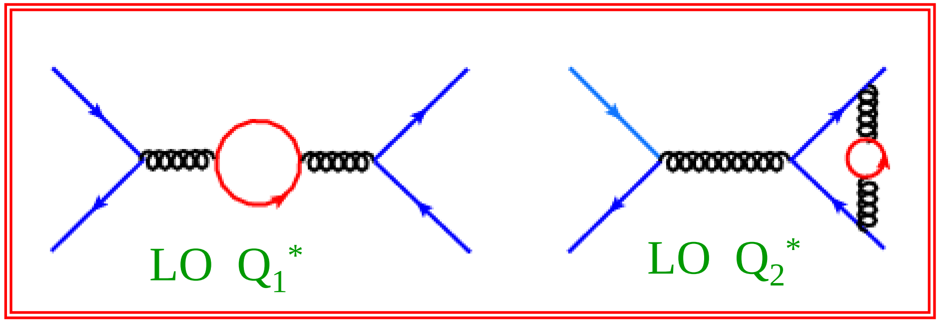
S.V. Mikhailov, JHEP **0706**, 009(2007).

No compelling reason why we should set it in such a naïve way
depression of the initial scale-dependence can not be expected

subtle points

Especially in the threshold region





two different PMC scales

Hard transverse gluon exchange

$$\boxed{\frac{x}{1-e^{-x}}}$$

$$\boxed{\left(1+\frac{\alpha(s\,\beta^2)\,\pi}{4\,\beta}\right)}$$

several % improvement

β-velocity of heavy quark in t-tbar rest frame

$$\boxed{\frac{\pi}{\beta} - Coulomb\ \ terms}$$

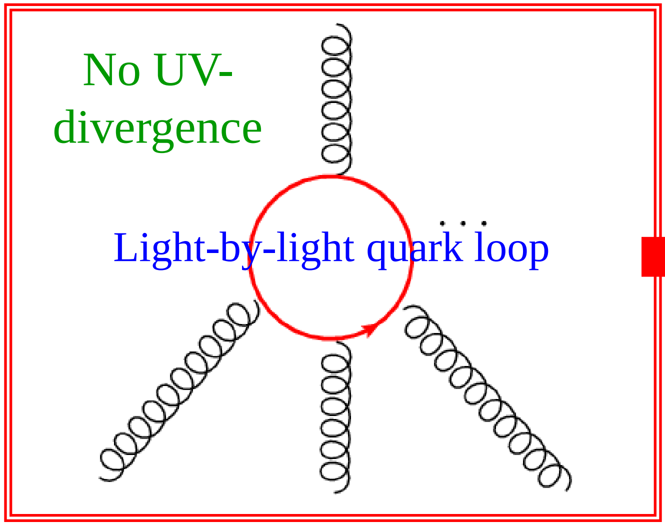
Instantaneous coulomb potential

Sommerfeld's rescattering formula

S.J. Brodsky, A.H. Hoang, J.H. Kuhn and T. Teubner, Phys.Lett. B359, 355(1995)

subtle points

When performing the scale shifts $Q \to Q^*$, $Q^* \to Q^{**}$ and $Q^{**} \to Q^{***}$, we eliminate the n_f-terms associated with the $\{\beta_i\}$-terms completely,



Note

Those n_f-terms, which are irrelevant to the ultra-violet cutoff and have no relation to the β-terms, should be identified and kept separately after the BLM scale setting

subtle points

The BLM – PMC correspondence

PMC, **dealing with the β-series**, provides the principle underlying BLM scale setting.

However to find what's the β-expansion series like ?

1) Since it is more convenient to calculate the n_f-terms (light-quark loops). So usually, we only keep in mind to deal with β-terms, in practice, we directly deal with nf-term.

2) The relation between β and n_f is not in a simple way, i.e. β_2 include the 2-quark -loop, 1-quark-loop and 0-quark-loop contributions. So to get the same n_f-series, the combination of β-term is not unique, which is more adaptable ?

$$\beta_0 = 11 - \frac{2}{3}n_f$$

$$\beta_1 = 102 - \frac{38}{3}n_f$$

$$\beta_2^{\overline{MS}} = \frac{2857}{2} - \frac{5033}{18}n_f + \frac{325}{54}n_f^2$$

$$\beta_3^{\overline{MS}} \simeq 29243.0 - 6964.30n_f + 405.089n_f^2 + 1.49931n_f^3$$

O.V. Tarasov, A.A. Vladimirov and A. Yu Zharkov, Phys.Lett. B**93**, 429(1980); T. van Ritbergen, J.A.M. Vermaseren and S.A. Larin, Phys.Lett. B**400**, 379(1997); M. Czakon, Nucl.Phys. B**710**, 485(2005).

S.V. Mikhailov, JHEP **0706**, 009(2007).

A.L. Kataev and S.V. Mikhailov, Teor.Mat.Fiz. **170**, 174-186 (2012).

$const., \beta_0, \beta_1, \beta_0^2$

$const., \beta_0, \beta_1, (\beta_0\beta_1), \beta_2, \beta_0^2, \beta_0^3$

Up to NNLO / PMC expansion

More explicitly, up to NNLO, the physical observable can be expanded in the $\{\beta_i\}$-series as,

$$\rho = r_0\Big[a_s^n(Q) + (A_1^0 + A_2^0\beta_0)a_s^{n+1}(Q) + (B_1^0 + B_2^0\beta_1 + B_3^0\beta_0^2)a_s^{n+2}(Q) + (C_1^0 + C_2^0\beta_2 + C_3^0\beta_0\beta_1 + C_4^0\beta_0^3)a_s^{n+3}(Q)\Big]. \quad (33)$$

We call it 'The BLM – PMC correspondence'

$$A_1 = A_1^0 + 11A_2^0$$
$$A_2 = -\frac{2}{3}A_2^0$$
$$B_1 = B_1^0 + 102B_2^0 + 121B_3^0$$
$$B_2 = -\frac{2}{3}(19B_2^0 + 22B_3^0)$$
$$B_3 = \frac{4}{9}B_3^0$$
$$C_1 = C_1^0 + \frac{2857}{2}C_2^0 + 1122C_3^0 + 1331C_4^0$$
$$C_2 = -\frac{1}{18}(5033C_2^0 - 3732C_3^0 - 4356C_4^0)$$
$$C_3 = \frac{1}{54}(325C_2^0 + 456C_3^0 + 792C_4^0)$$
$$C_4 = -\frac{8}{27}C_4^0$$

Under such correspondence,

BLM and PMC are related with each other exactly

One-to-One

Second way (R_δ) of achieving the goal of PMC

Main idea: The behavior of the running coupling is determined **by absorbing the whole β-series pertaining to this particular coupling constant into the running coupling at one time**. It provides a natural demonstration for PMC.

Thus, the elimination of δ-terms is equivalent to eliminate β-terms

In the modified minimal subtraction scheme (MS-bar) one subtracts together with the pole a constant [Bardeen, Buras, Duke, Muta (1978) on DIS results]:

$$\ln(4\pi) - \gamma_E$$

This corresponds to a shift in the scale:

$$\mu^2_{\overline{\rm MS}} = \mu^2 \exp(\ln 4\pi - \gamma_E)$$

A finite subtraction from infinity is arbitrary. *Let's make use of this!*

Subtract an arbitrary constant and keep it in your calculation: $\mathcal{R}_\delta$-scheme

Observation

$$\ln(4\pi) - \gamma_E - \delta,$$

$$\mu_\delta^2 = \mu^2_{\overline{\rm MS}} \exp(-\delta) = \mu^2 \exp(\ln 4\pi - \gamma_E - \delta)$$

Observable in the $\mathcal{R}_\delta$-scheme:

$$\rho_\delta(Q^2) = r_0 + r_1 a(\mu) + [r_2 + \beta_0 r_1 \delta] a(\mu)^2 + [r_3 + \beta_1 r_1 \delta + 2\beta_0 r_2 \delta + \beta_0^2 r_1 \delta^2] a(\mu)^3 + \cdots$$

$$\mathcal{R}_0 = \overline{\rm MS}\ , \quad \mathcal{R}_{\ln 4\pi - \gamma_E} = {\rm MS} \qquad \mu^2 = \mu^2_{\overline{\rm MS}} \exp(\ln 4\pi - \gamma_E)\ , \quad \mu^2_{\delta_2} = \mu^2_{\delta_1} \exp(\delta_2 - \delta_1)$$

Renormalization Scheme Equation

$$\frac{d\rho}{d\delta} = -\beta(a)\frac{d\rho}{da} \stackrel{!}{=} 0 \qquad \longrightarrow \ \mathrm{PMC}$$

$$\rho_0(Q^2) = a(\mu_0)^n \sum_{k=0}^{\infty} r_{k+1}(Q^2/\mu_0^2) a(\mu_0)^k ,$$

$$\begin{aligned} a(\mu_0)^k &= a(\mu_\delta)^k + k\beta_0\delta a(\mu_\delta)^{k+1} \qquad (17) \\ &+ k\left[\beta_1\delta + \frac{k+1}{2}\beta_0^2\delta^2\right] a(\mu_\delta)^{k+2} \\ &+ k\left[\beta_2\delta + \frac{2k+3}{2}\beta_0\beta_1\delta^2 + \frac{(k+1)(k+2)}{3!}\beta_0^3\delta^3\right] a(\mu_\delta)^{k+3}. \end{aligned}$$

简单起见： $\mu_0^2 = Q^2$

Shows which term should be absorbed into which coupling

$$\begin{aligned} \rho_\delta(Q^2) =& r_1 a_1(\mu_\delta)^n + [r_2 + n\beta_0 r_1\delta_1] a_2(\mu_\delta)^{n+1} + \left[r_3 + n\beta_1 r_1\delta_1 + (n+1)\beta_0 r_2\delta_2 + \frac{n(n+1)}{2}\beta_0^2 r_1\delta_1^2\right] a_3(\mu_\delta)^{n+2} \\ &+ \left[r_4 + n\beta_2 r_1\delta_1 + (n+1)\beta_1 r_2\delta_2 + (n+2)\beta_0 r_3\delta_3 + \frac{n(3+2n)}{2}\beta_0\beta_1 r_1\delta_1^2 + \frac{(n+1)(n+2)}{2}\beta_0^2 r_2\delta_2^2\right. \\ &\left. + \frac{n(n+1)(n+2)}{3!}\beta_0^3 r_1\delta_1^3\right] a_4(\mu_\delta)^{n+3} + \mathcal{O}(a^5) , \end{aligned} \qquad (18)$$

一阶一阶的展开

$$\begin{aligned} \rho(Q^2) =& r_{1,0} a(Q)^n + [r_{2,0} + n\beta_0 r_{2,1}] a(Q)^{n+1} + \left[r_{3,0} + n\beta_1 r_{2,1} + (n+1)\beta_0 r_{3,1} + \frac{n(n+1)}{2}\beta_0^2 r_{3,2}\right] a(Q)^{n+2} \\ &+ \left[r_{4,0} + n\beta_2 r_{2,1} + (n+1)\beta_1 r_{3,1} + (n+2)\beta_0 r_{4,1} + \frac{n(3+2n)}{2}\beta_0\beta_1 r_{3,2} + \frac{(n+1)(n+2)}{2}\beta_0^2 r_{4,2}\right. \\ &\left. + \frac{n(n+1)(n+2)}{3!}\beta_0^3 r_{4,3}\right] a(Q)^{n+3} + \mathcal{O}(a^{n+4}) , \end{aligned}$$

According to the principal of maximum conformality we must set the scales such to absorb all 'renormalon-terms', i.e. non-conformal terms

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + (\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \beta_2 a(Q)^4 + \cdots)r_{2,1}$$
$$+ (\beta_0^2 a(Q)^3 + \frac{5}{2}\beta_1\beta_0 a(Q)^4 + \cdots)r_{3,2} + (\beta_0^3 + \cdots)r_{4,3}$$
$$+ r_{2,0}a(Q)^2 + 2a(Q)(\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \cdots)r_{3,1}$$
$$+ \cdots$$

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1} + \frac{1}{2}\beta(a)\frac{\partial\beta}{\partial a}r_{3,2} + \cdots + \frac{(-1)^n}{n!}\frac{d^{n-1}\beta}{(d\ln\mu^2)^{n-1}}r_{n+1,n}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1} + \cdots$$

Application of PMC and its interesting features

I) Top pair production

$$\boxed{p+p(\bar{p}) \;\rightarrow\; Q+\bar{Q}+X}$$

$$q+\bar{q} \;\rightarrow\; Q+\bar{Q}+X$$

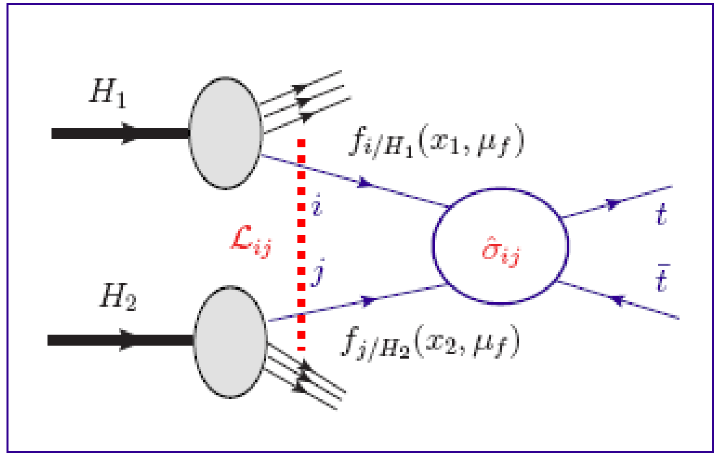
$$g+g \;\rightarrow\; Q+\bar{Q}+X$$

$$g+q \;\rightarrow\; Q+\bar{Q}+X$$

$$\sigma(S,m^2) = \sum_{ij} \int dx_1\, dx_2\, \hat{\sigma}_{ij}(s,m^2,\mu^2)\, f_i(x_1,\mu^2)\, f_j(x_2,\mu^2)$$

A) Total cross-section

Hadronic Cross-Section



如何确定
因子化能标
还在考虑当中

$$\sigma_{H_1 H_2 \to t\bar{t}X} = \sum_{i,j} \int_{4m_t^2}^{S} ds \, \mathcal{L}_{ij}(s, S, \mu_f) \hat{\sigma}_{ij}(s, \alpha_s(\mu_r), \mu_r, \mu_f),$$

Parton luminosity

$$\mathcal{L}_{ij} = \frac{1}{S} \int_{s}^{S} \frac{d\hat{s}}{\hat{s}} f_{i/H_1}(x_1, \mu_f) f_{j/H_2}(x_2, \mu_f)$$

$$a_s = \frac{\alpha_s}{\pi}$$

Subprocess Cross-Section NNLO

$$\hat{\sigma}_{ij} = \frac{1}{m_t^2} \Big\{ \underbrace{f_{ij}^0(\rho, Q) a_s^2(Q)}_{\text{LO}} + \underbrace{f_{ij}^1(\rho, Q) a_s^3(Q)}_{\text{NLO}} + \underbrace{f_{ij}^2(\rho, Q) a_s^4(Q)}_{\text{NNLO}} \Big\},$$

PMC scale-setting

NLO

$$f^1_{ij}(\rho,Q) = [A_{1ij} + B_{1ij} n_f] + D_{1ij}\left(\frac{\pi}{v}\right)$$

$$\boxed{A_{0ij} = f^0_{ij}(\rho,Q)}$$

NNLO

$$f^2_{ij}(\rho,Q) = [A_{2ij} + B_{2ij} n_f + C_{2ij} n_f^2] + [D_{2ij} + E_{2ij} n_f]\left(\frac{\pi}{v}\right) + F_{2ij}\left(\frac{\pi}{v}\right)^2$$

$$m_t^2 \hat{\sigma}_{ij} = A_{0ij} a_s^2(Q_1^*) + \left[\tilde{A}_{1ij}\right] a_s^3(Q_1^*) + \left[\tilde{A}_{2ij} + \tilde{B}_{2ij} n_f\right] a_s^4(Q_1^*) + D_{1ij}\left(\frac{\pi}{v}\right) a_s^3(Q_2^*) + \left[\tilde{D}_{2ij}\right]\left(\frac{\pi}{v}\right) a_s^4(Q_2^*) + F_{2ij}\left(\frac{\pi}{v}\right)^2 a_s^4(Q_2^*).$$

first step

$$\begin{aligned} m_t^2 \hat{\sigma}_{ij} &= A_{0ij} a_s^2(Q_1^*) + \left[\tilde{A}_{1ij}\right] a_s^3(Q_1^{**}) + \left[\tilde{\tilde{A}}_{2ij}\right] a_s^4(Q_1^{**}) + D_{1ij}\left(\frac{\pi}{v}\right) a_s^3(Q_2^*) + \left[\tilde{D}_{2ij}\left(\frac{\pi}{v}\right) + F_{2ij}\left(\frac{\pi}{v}\right)^2\right] a_s^4(Q_2^*) \\ &= A_{0ij} a_s^2(Q_1^*) + \left[\tilde{A}_{1ij}\right] a_s^3(Q_1^{**}) + \left[\tilde{\tilde{A}}_{2ij}\right] a_s^4(Q_1^{**}) \\ &\quad + \left(\frac{\pi}{v}\right) D_{1ij}\left[\frac{2\kappa}{1-\exp(-2\kappa)}\right] a_s^3(Q_2^*), \end{aligned}$$

second step

Sommerfeld rescattering

general formula

Q-initial scale

LO PMC scale

$$\ln\frac{Q_1^{*2}}{Q^2} = \ln\frac{Q_{10}^{*2}}{Q^2} + \frac{\chi}{4}\beta_0 \ln\frac{Q_{10}^{*2}}{Q^2} a_s(Q)$$

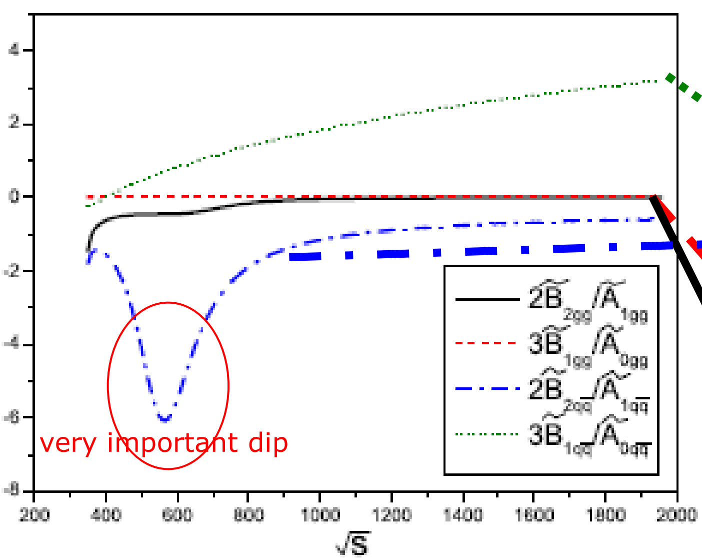
$$\ln\frac{Q_{10}^{*2}}{Q^2} = \frac{3B_{1ij}}{A_{0ij}}, \quad \chi = \frac{9B_{1ij}^2 - 12A_{0ij}C_{2ij}}{2A_{0ij}B_{1ij}}$$

perturbative series

NLO PMC scale

$$\boxed{\ln\frac{Q_1^{**2}}{Q_1^{*2}} = \frac{2\tilde{B}_{2ij}}{\tilde{A}_{1ij}}}$$

most important greatly improve the NLO estimation



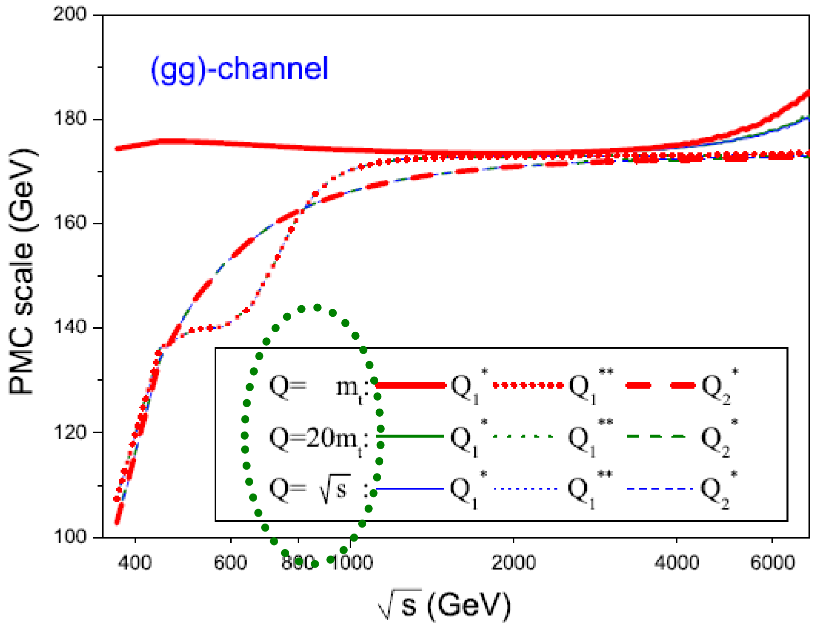
determines ⇒ $q\bar{q}$ – *channel* LO *scale*

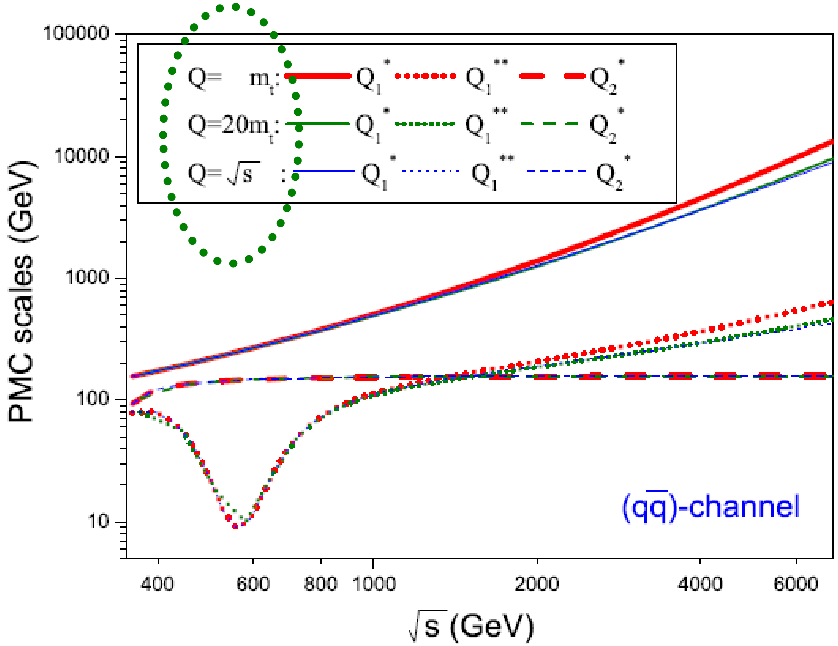
determines ⇒ $q\bar{q}$ – *channel* NLO *scale*

determines ⇒ gg – *channel* LO *scale*

determines ⇒ gg – *channel* NLO *scale*

slight change for gg-channel





initial-scale dependence for t-tbar total cross-section at NNLO

Why is there quite small initial-scale dependence ?

There is remaining scale dependence at any fixed-order.
But the effects from unknown-terms are highly suppressed,
because all unknown β-terms are absorbed into the higher-order of PMC-scales themselves. Exponentially suppressed !

	Conventional scale-setting				PMC scale-setting			
	LO	NLO	NNLO	*total*	LO	NLO	NNLO	*total*
$(q\bar{q})$-channel	4.989	0.975	0.489	6.453	4.841	1.756	-0.063	6.489
(gg)-channel	0.522	0.425	0.155	1.102	0.520	0.506	0.148	1.200
(gq)-channel	0.000	-0.0366	0.0050	-0.0316	0.000	-0.0367	0.0050	-0.0315
$(g\bar{q})$-channel	0.000	-0.0367	0.0050	-0.0315	0.000	-0.0366	0.0050	-0.0316
sum	5.511	1.326	0.654	7.489	5.3613	2.188	0.095	7.626

TABLE I. Total cross-sections (in unit: pb) for the top-quark pair production at the Tevatron with $\sqrt{S} = 1.96$ TeV. For the conventional scale-setting, we set the renormalization scale $\mu_r \equiv Q$. For the PMC scale-setting, we set the initial renormalization scale $\mu_r^{\rm init} = Q$. Here $Q = m_t = 172.9$ GeV and the central CT10 as the PDF [51].

	Conventional scale-setting				PMC scale-setting			
	LO	NLO	NNLO	*total*	LO	NLO	NNLO	*total*
$(q\bar{q})$-channel	23.283	3.374	1.842	28.527	22.244	7.127	-0.765	28.429
(gg)-channel	78.692	45.918	10.637	135.113	78.399	53.570	8.539	142.548
(gq)-channel	0.000	-0.401	1.404	1.025	0.000	-0.408	1.403	1.006
$(g\bar{q})$-channel	0.000	-0.420	0.235	-0.186	0.000	-0.424	0.235	-0.188
sum	101.975	48.471	14.118	164.594	100.643	59.865	9.414	171.796

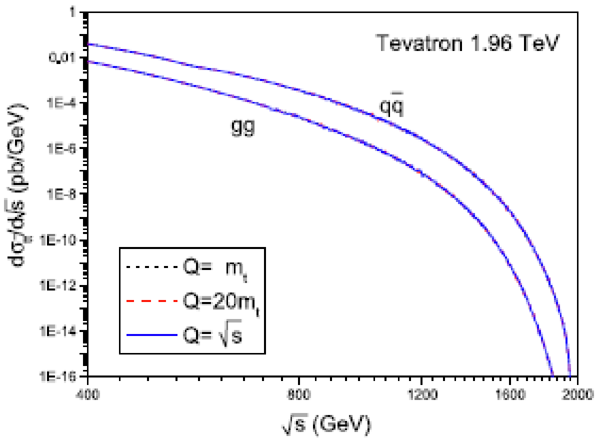
TABLE II. Total cross-sections (in unit: pb) for the top-quark pair production at the LHC with $\sqrt{S} = 7$ TeV. For the conventional scale-setting, we set the renormalization scale $\mu_r \equiv Q$. For the PMC scale-setting, we set the initial renormalization scale $\mu_r^{\rm init} = Q$. Here $Q = m_t = 172.9$ GeV and the central CT10 as the PDF [51].

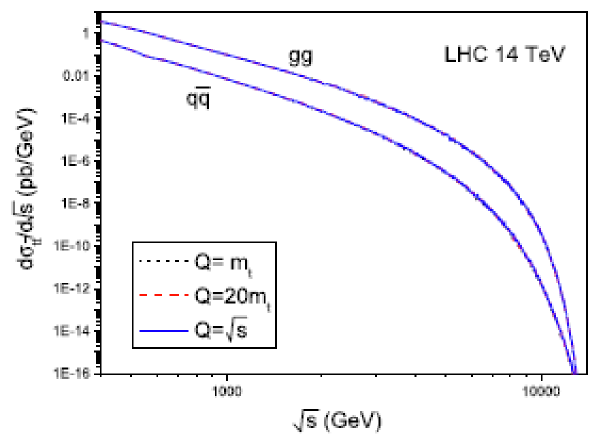
A proper NLO scale is clearly very important !

especially to understand the ttbar-asymmetry

total differential cross-section versus √s (very small initial scale-dependence)

$$\boxed{d\sigma_{t\bar{t}}/d\sqrt{s} \propto \left(2\sqrt{s}\mathcal{L}_{ij}\hat{\sigma}_{ij}\right)}$$





total cross-section almost unchanged !

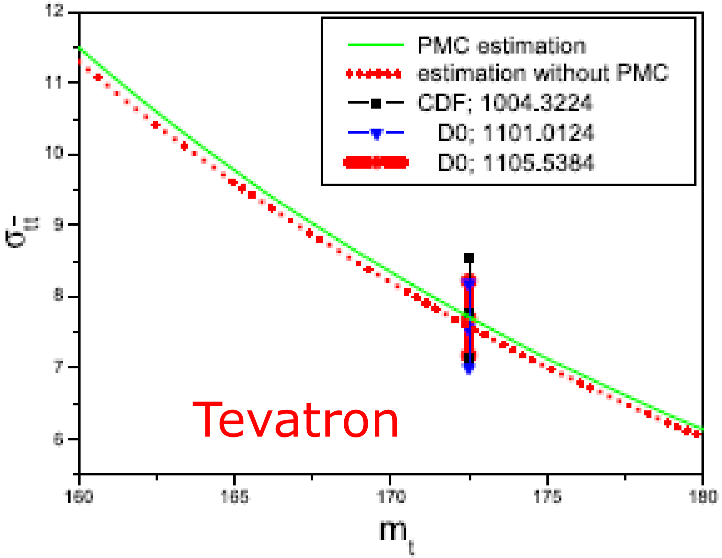
	PMC scale-setting					Conventional scale-setting		
	$Q=m_t/4$	$Q=m_t/2$	$Q=m_t$	$Q=2m_t$	$Q=4m_t$	$\mu_r\equiv m_t/2$	$\mu_r\equiv m_t$	$\mu_r\equiv 2m_t$
Tevatron (1.96 TeV)	7.620(5)	7.622(5)	7.626(3)	7.622(6)	7.623(6)	7.742(5)	7.489(3)	7.199(5)
LHC (7 TeV)	171.6(1)	171.7(1)	171.8(1)	171.7(1)	171.7(1)	168.8(1)	164.6(1)	157.5(1)
LHC (14 TeV)	941.8(8)	941.9(8)	941.3(5)	941.4(8)	941.4(8)	923.8(7)	907.4(4)	870.9(6)

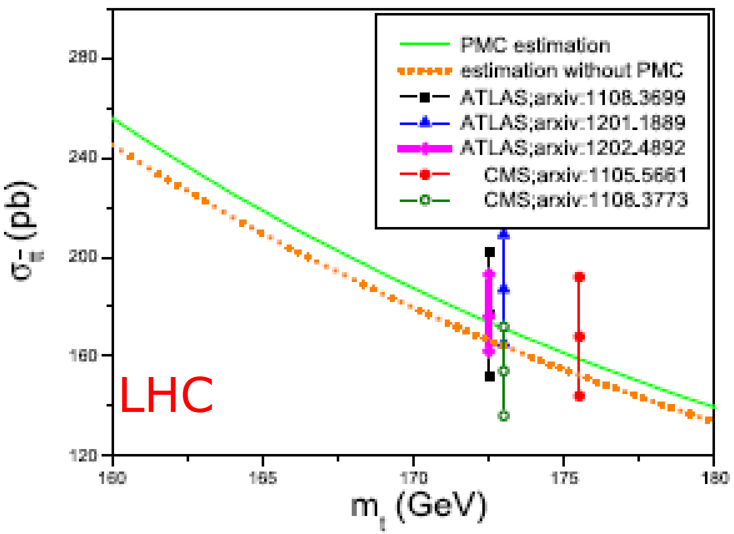
unchanged, within error 10^{-3}

3%-4%

$10m_t$ ($20m_t$) =>15%(19%)

NNLO





This shows the conventional scale m_t is a lucky guess for the total cross-section.

However it well underestimates the ttbar-asymmetry !

B) Forward-Backward asymmetry

$$A_{FB}^{t\bar{t}} = \frac{\sigma(y_t^{t\bar{t}} > 0) - \sigma(y_t^{t\bar{t}} < 0)}{\sigma(y_t^{t\bar{t}} > 0) + \sigma(y_t^{t\bar{t}} < 0)} \qquad A_{FB}^{p\bar{p}} = \frac{\sigma(y_t^{p\bar{p}} > 0) - \sigma(y_t^{p\bar{p}} < 0)}{\sigma(y_t^{p\bar{p}} > 0) + \sigma(y_t^{p\bar{p}} < 0)}$$

previous SM estimation **under conventional scale-setting**

1108.3341

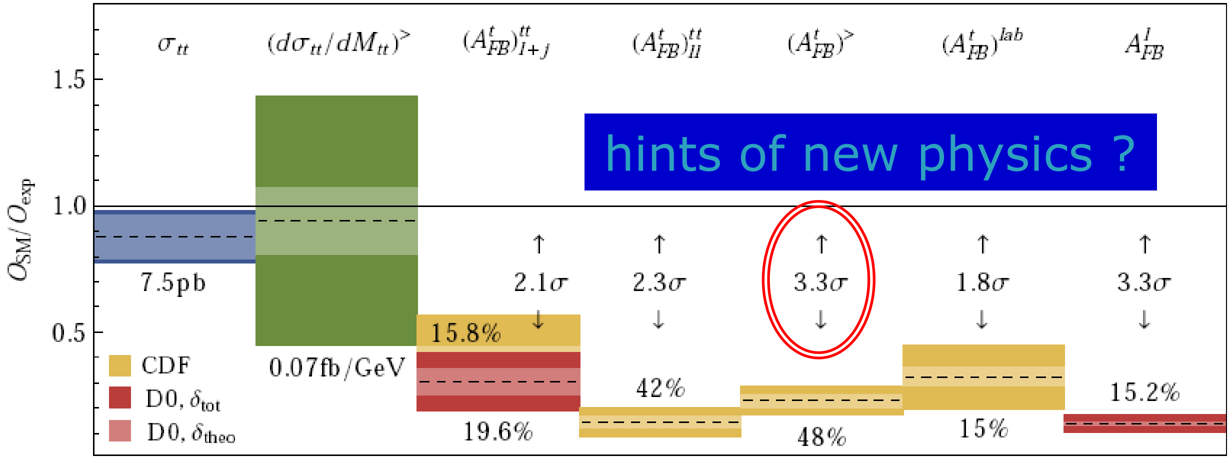


Figure 1: Top-antitop production at the Tevatron. The ratio O_{SM}/O_{exp} is displayed for the total cross section $\sigma_{t\bar{t}}$ and its invariant mass distribution $(d\sigma/dM_{t\bar{t}})^{>}$ for $M_{t\bar{t}} \in [0.8, 1.4]$ TeV. The inclusive asymmetry in the parton frame is shown for the lepton + jets channel, $(A_{FB})_{l+j}^{t\bar{t}}$, besides its bin $(A_{FB}^t)^{>}$ for high invariant mass $M_{t\bar{t}} > 0.45$ TeV, as well as for the dilepton channel, $(A_{FB}^t)_{ll}^{t\bar{t}}$. The asymmetry in the laboratory frame is denoted by $(A_{FB}^t)^{\rm lab}$, and A_{FB}^l is the charged lepton asymmetry. Numbers correspond to the central measured values [1].

A more detailed comparison of SM and exp.

CDF

$$A_{FB}^{t\bar{t},\mathrm{CDF}} = (15.8 \pm 7.5)\% \quad A_{FB}^{p\bar{p},\mathrm{CDF}} = (15.0 \pm 5.5)\%$$

$$A_{FB}^{t\bar{t}}(M_{t\bar{t}} > 450 \text{ GeV}) = (47.5 \pm 11.4)\%$$

T. Aaltonen *et al.*, CDF Collaboration, Phys.Rev. D**83**, 112003 (2011).

W. Bernreuther and Z.G. Si, Nucl.Phys. B**837**, 90 (2010).

QCD NLO

$$A_{FB}^{t\bar{t}} \simeq 7\% \text{ and } A_{FB}^{p\bar{p}} \simeq 5\% \quad A_{FB}^{t\bar{t}}(M_{t\bar{t}} > 450 \text{ GeV}) \sim 8.8\%$$

QCD NLO + EW ~20%

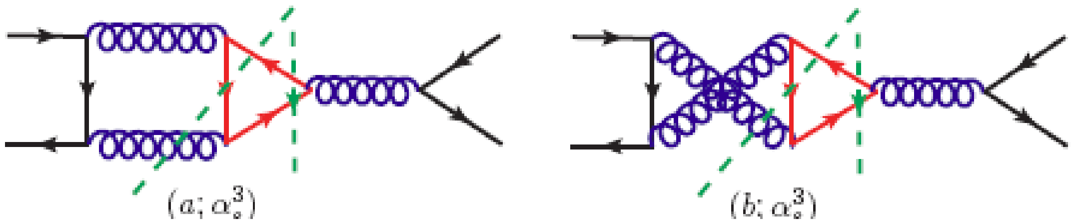
$A_{FB}^{t\bar{t}}(A_{FB}^{p\bar{p}}) \sim 9\%$ (7%) [31, 32] and $A_{FB}^{t\bar{t}}(M_{t\bar{t}} > 450\ \text{GeV}) \sim 12.8\%$ [32].

J.H. Kuhn and G. Rodrigo, JHEP **1201**, 063(2012).
W. Hollik and D. Pagani, Phys.Rev. D**84**, 093003(2011).

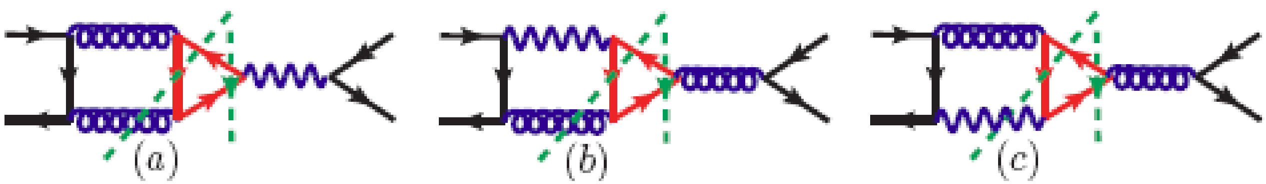
Maybe new physics, but any new source of asymmetry should not break the good-agreement with total CS ~5%

why the conventional scale-setting gives small asymmetry ?

dominant asymmetric $q\bar{q}$ – *channel*



NLO



~20%

	Conventional scale-setting				PMC scale-setting			
	LO	NLO	NNLO	*total*	LO	NLO	NNLO	*total*
$(q\bar{q})$-channel	4.890	0.963	0.483	6.336	4.748	1.727	-0.058	6.417
(gg)-channel	0.526	0.440	0.166	1.132	0.524	0.525	0.160	1.208
(gq)-channel	0.000	-0.0381	0.0049	-0.0332	0.000	-0.0381	0.0049	-0.0332
$(g\bar{q})$-channel	0.000	-0.0381	0.0049	-0.0332	0.000	-0.0381	0.0049	-0.0332
sum	5.416	0.985	0.659	7.402	5.272	2.176	0.112	7.559

TABLE I. Total cross-sections (in unit: pb) for the top-quark pair production at the Tevatron with $p\bar{p}$-collision energy $\sqrt{S} = 1.96$ TeV. For conventional scale-setting, we set the renormalization scale $\mu_R \equiv Q$. For PMC scale-setting, we set the initial renormalization scale $\mu_R^{\rm init} = Q$. Here we take $Q = m_t = 172.9$ GeV and use the MSRT 2004-QED parton distributions [54] as the PDF.

A consistent perturbative-order-analysis of the asymmetry

$$A_{FB} = \frac{\alpha_s^3 \overset{\uparrow\, NLO-q\bar{q}}{N_1} + \alpha_s^4 \overset{\uparrow\, NNLO-q\bar{q}}{N_2} + \mathcal{O}(\alpha_s^5)}{\alpha_s^2 D_0 + \alpha_s^3 D_1 + \alpha_s^4 D_2 + \mathcal{O}(\alpha_s^5)}$$

$$= \frac{\alpha_s}{D_0}\left[N_1 + \alpha_s\left(N_2 - \frac{D_1 N_1}{D_0}\right) + \alpha_s^2\left(\frac{D_1^2 N_1}{D_0^2} - \frac{D_1 N_2}{D_0} - \frac{D_2 N_1}{D_0}\right) + \cdots\right]$$

total LO *total NLO* *total NNLO*

Using conventional scale-setting

$$[\alpha_s^2 D_0 : \alpha_s^3 D_1 : \alpha_s^4 D_2 \simeq 1 : 18\% : 12\%]$$

$$[\alpha_s^3 N_1 : \alpha_s^4 N_2 \sim 1 : 50\%]$$

same importance

$N_1 D_1 / D_0$ term and the N_2 term

present SM estimation is estimated by $\boxed{A_{FB} = \frac{N_1}{D_0}\alpha_s.}$

we just call it LO asymmetry

Using PMC scale-setting

$$[\alpha_s^2 D_0 : \alpha_s^3 D_1 : \alpha_s^4 D_2 \simeq 1 : 41\% : 2\%]$$

$$[\alpha_s^3 N_1 : \alpha_s^4 N_2 \sim 1 : 3\%]$$

NNLO-terms N_2, D_2 are highly suppressed and negligible

$$A_{FB} = \frac{\alpha_s}{D_0}\left[N_1 - \alpha_s\left(\frac{D_1 N_1}{D_0}\right) + \alpha_s^2\left(\frac{D_1^2 N_1}{D_0^2}\right)\right]$$

we just call it NNLO asymmetry

It is natural to assume all the higher orders are also negligible

resummed

$$A_{FB} = \frac{\alpha_s^3 N_1}{\alpha_s^2 D_0 + \alpha_s^3 D_1}$$

include the electro-weak

$$A_{FB} = \frac{\alpha_s^3 N_1 + \alpha_s^2 \alpha \tilde{N}_1 + \alpha^2 \tilde{N}_0}{\alpha_s^2 D_0 + \alpha_s^3 D_1}$$

final formula

short notation as HP

W. Hollik and D. Pagani, Phys.Rev. D84, 093003(2011).

The results obtained by using **conventional scale-setting** can be greatly improved by using PMC :

$$A_{FB}^{t\bar{t},\mathrm{PMC}} = \left\{\frac{\sigma^{\mathrm{tot,HP}}_{H_1H_2\to t\bar{t}X}}{\sigma^{\mathrm{tot,PMC}}_{H_1H_2\to t\bar{t}X}}\right\}\left\{\frac{\overline{\alpha}_s{}^3\left(\overline{\mu}_R^{\mathrm{PMC,NLO}}\right)}{\alpha_s^{HP\,3}\left(\mu_R^{\mathrm{conv}}\right)}A_{FB}^{t\bar{t},\mathrm{HP}}|_{\mathcal{O}(\alpha_s^3)} + \frac{\overline{\alpha}_s{}^2\left(\overline{\mu}_R^{\mathrm{PMC,NLO}}\right)}{\alpha_s^{HP\,2}\left(\mu_R^{\mathrm{conv}}\right)}A_{FB}^{t\bar{t},\mathrm{HP}}|_{\mathcal{O}(\alpha_s^2\alpha)} + A_{FB}^{t\bar{t},\mathrm{HP}}|_{\mathcal{O}(\alpha^2)}\right\} \quad (11)$$

$$A_{FB}^{p\bar{p},\mathrm{PMC}} = \left\{\frac{\sigma^{\mathrm{tot,HP}}_{H_1H_2\to t\bar{t}X}}{\sigma^{\mathrm{tot,PMC}}_{H_1H_2\to t\bar{t}X}}\right\}\left\{\frac{\overline{\alpha}_s{}^3\left(\overline{\mu}_R^{\mathrm{PMC,NLO}}\right)}{\alpha_s^{HP\,3}\left(\mu_R^{\mathrm{conv}}\right)}A_{FB}^{p\bar{p},\mathrm{HP}}|_{\mathcal{O}(\alpha_s^3)} + \frac{\overline{\alpha}_s{}^2\left(\overline{\mu}_R^{\mathrm{PMC,NLO}}\right)}{\alpha_s^{HP\,2}\left(\mu_R^{\mathrm{conv}}\right)}A_{FB}^{p\bar{p},\mathrm{HP}}|_{\mathcal{O}(\alpha_s^2\alpha)} + A_{FB}^{p\bar{p},\mathrm{HP}}|_{\mathcal{O}(\alpha^2)}\right\} \quad (12)$$

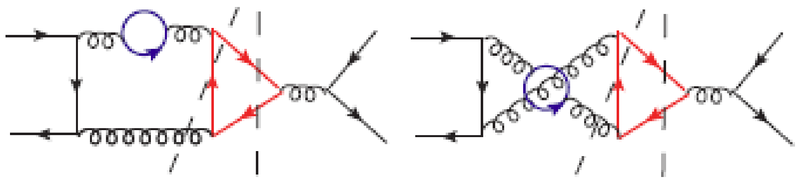


FIG. 6. Dominant cut diagrams for the n_f-terms at the α^4-order of the $(q\bar{q})$-channel, which are responsible for the smaller effective NLO PMC scale $\overline{\mu}_R^{\mathrm{PMC,NLO}}$, where the solid circles stand for the light-quark loops.

a global PMC scale for NLO

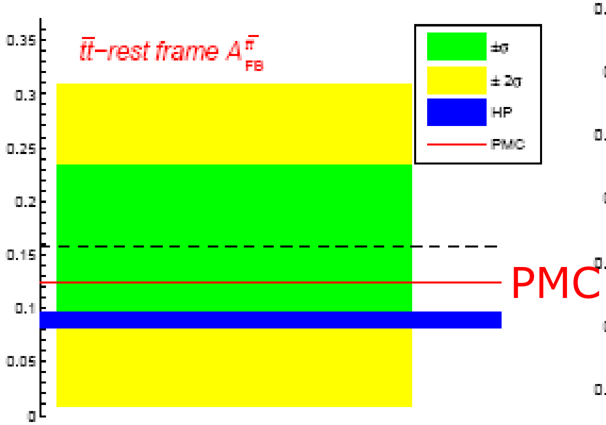
$$\overline{\mu}_R^{\mathrm{PMC,effective}} \simeq \exp(-9/10)m_t \sim 70\ \mathrm{GeV}\ ,$$

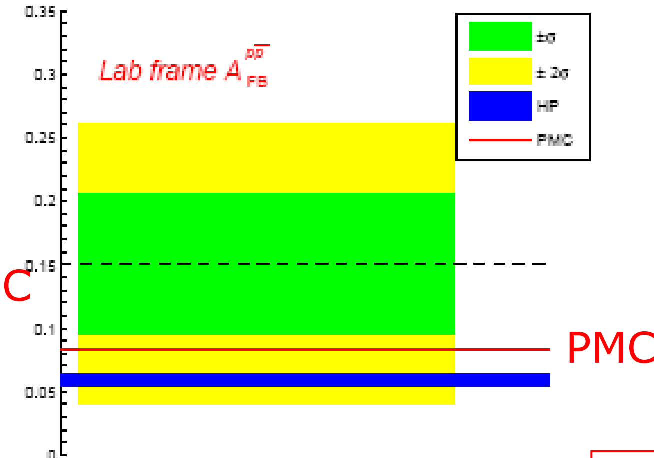
$$\boxed{\overline{\alpha}_s\left(\overline{\mu}_R^{\mathrm{PMC,NLO}}\right) = 0.1228.}$$

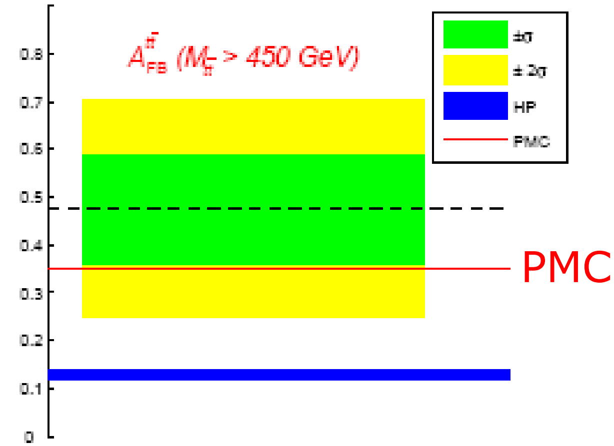
$$\alpha_s^{HP}(m_t) \simeq 0.098\ [31, 32].$$

[31] J.H. Kuhn and G. Rodrigo, JHEP **1201**, 063(2012).
[32] W. Hollik and D. Pagani, Phys.Rev. D**84**, 093003(2011).

around 1σ-error is obtained







$$\boxed{A_{FB}^{t\bar{t},\mathrm{PMC}} \simeq 12.7\% \; ; \quad A_{FB}^{p\bar{p},\mathrm{PMC}} \simeq 8.39\%}$$

$$\overline{\alpha}_s\left(\overline{\mu}_R^{\mathrm{PMC,NLO}}\right) = 0.1460$$

$$\overline{\mu}_R^{\mathrm{PMC,NLO}} \sim \exp(-19/10)m_t \simeq 26 \text{ GeV}.$$

$$\boxed{A_{FB}^{t\bar{t},PMC}(M_{t\bar{t}} > 450 \text{ GeV}) \simeq 35.0\%}$$

HP

$$A_{FB}^{t\bar{t}} = (9.7, 8.9, 8.3)\%, \qquad A_{FB}^{p\bar{p}} = (6.4, 5.9, 5.4)\%.$$

$$A_{FB}^{t\bar{t}}(M_{t\bar{t}} > 450 \text{ GeV}) = (13.9, 12.8, 11.9)\%,$$

$$A_{FB}^{t\bar{t},\mathrm{CDF}} = (15.8 \pm 7.5)\% \quad A_{FB}^{p\bar{p},\mathrm{CDF}} = (15.0 \pm 5.5)\%$$

$$A_{FB}^{t\bar{t}}(M_{t\bar{t}} > 450 \text{ GeV}) = (47.5 \pm 11.4)\%$$

CDF

T. Aaltonen *et al.*, CDF Collaboration, Phys.Rev. D**83**, 112003 (2011).

II) PMC Scale setting for 3-jets events at LO

R. K. Ellis *et Al*, Nucl. Phys. B178, 421-456 (1981)

S.J. Brodsky and L.D. Giustino, arXiv: 1107.0338.

$$\boxed{e^+e^- \to q + \bar{q} + g \text{ up to NLO}}$$

$$e^+e^- \to q + \bar{q} + g \text{ up to NLO}$$

$$\frac{1}{\overline{\sigma}_0}\frac{d\sigma^{(s)}+d\sigma^3}{dy} = \int_y^{1-2y} dz \int_y^{1-y-z} dx\, T[1-x-z,x,z]\alpha_s(s)$$

y: the maximum virtuality of the jet

$$\left[1-\frac{\alpha_s(s)}{\pi}\left(\frac{\beta_0}{4}\left(\ln[x]+\ln[z]-\frac{5}{3}\right)+\cdots\right)\right]$$

$$= \alpha_s(s)\left[T(y)-\frac{\alpha_s(s)}{\pi}\left(\left(C(y)-\frac{5}{3}T(y)\right)\frac{\beta_0}{4}+\cdots\right)\right]$$

$$= T(y)\alpha_s(s)\left[1-\frac{\alpha_s(s)}{\pi}\left(\frac{1}{4}\left(\frac{C(y)}{T(y)}-\frac{5}{3}\right)\beta_0+\cdots\right)\right]$$

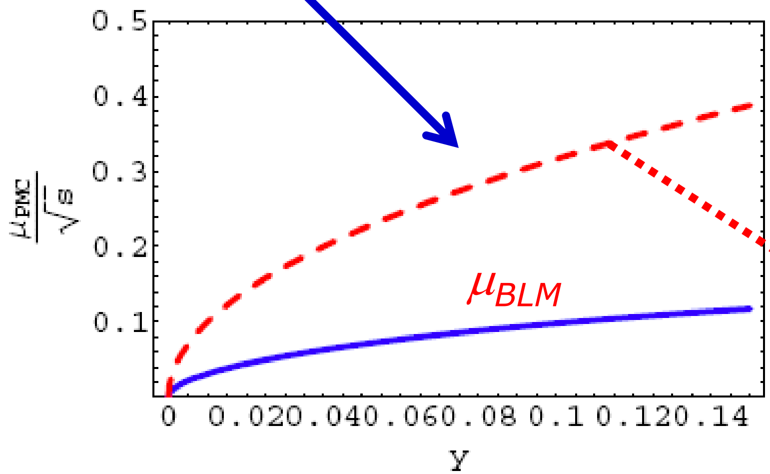
$$= T(y)\alpha_s(\mu^2_{BLM})+\cdots$$

$$\boxed{\mu^2_{BLM} = s\times\exp\left(-\frac{5}{3}+\frac{C(y)}{T(y)}\right)}$$

LO-BLM/PMC scale

Physical scale should decrease with the decrement of jet energy

Smaller PMC-scale improve perturbative expansion

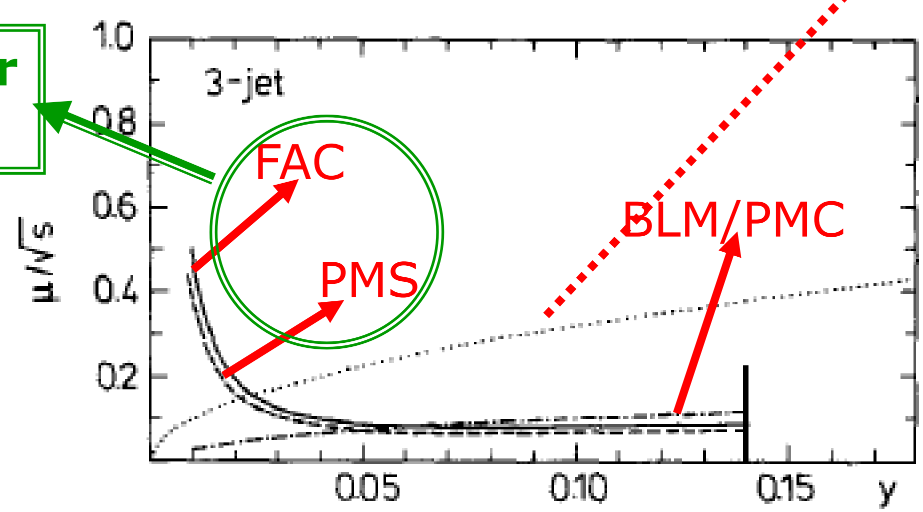


GM-L scale

$\mu/\sqrt{s} = \sqrt{y}$

gluon jet

Wrong behavior At small *y*



$r_1(\mu) = 0.$ **FAC**

$\frac{dR_N}{d\mu} = 0.$ **PMS**

G. Kramer and B. Lampe, Z. Phys. C 39, 101 (1988).

III) Scale setting for R(Q) at NNLO and a comparison of different scale-setting schemes

$$\frac{\sigma(e^+e^- \to \text{hadrons}, Q)}{\sigma(e^+e^- \to \mu^+\mu^-, Q)} \equiv R(Q)$$

$$R_{e^+e^-}(Q) = 3\sum_q e_q^2 \Big[1 + \left(a^{\overline{MS}}(Q)\right) + (1.9857 - 0.1152 n_f) \left(a^{\overline{MS}}(Q)\right)^2$$
$$+ \left(-6.63694 - 1.20013 n_f - 0.00518 n_f^2 - 1.240 \frac{(\sum_q e_q)^2}{3\sum_q e_q^2} \right) \left(a^{\overline{MS}}(Q)\right)^3$$
$$+ \left(-156.61 + 18.77 n_f - 0.7974 n_f^2 + 0.0215 n_f^3 + C \frac{(\sum_q e_q)^2}{3\sum_q e_q^2} \right) \left(a^{\overline{MS}}(Q)\right)^4 \Big]$$

C is for singlet contribution and is small

As usual, we set *C*=0

P.A. Baikov, K.G. Chetyrkin and J.H. Kuhn, Phys.Rev.Lett.**101**, 012002(2008); arXiv:0906.2987[hep-ph]; K. Nakamura et al. (Particle Data Group), J.Phys. G**37**, 075021 (2010).

$$R_{e^+e^-}(Q) = 3\sum_q e_q^2 \Big[1 + \left(a_s^{\overline{MS}}(Q^*)\right) + \widetilde{A} \left(a_s^{\overline{MS}}(Q^{**})\right)^2$$
$$+ \widetilde{\widetilde{B}} \left(a_s^{\overline{MS}}(Q^{***})\right)^3 + \widetilde{\widetilde{C}} \left(a_s^{\overline{MS}}(Q^{***})\right)^4 \Big], (44)$$

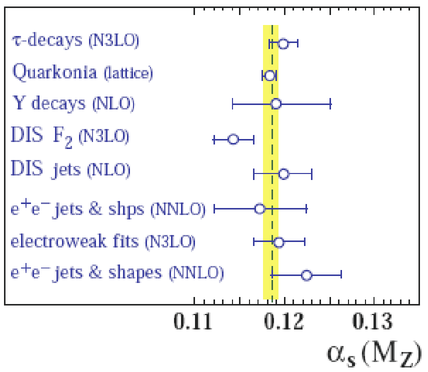
• If taking the experimental results for R(Q)

From the experimental value, $r_{e^+e^-}(31.6GeV) = \frac{3}{11}R_{e^+e^-}(31.6GeV) = 1.0527 \pm 0.0050$ [26], we obtain

$$\Lambda'^{tH}_{\overline{MS}} = 412^{+206}_{-161}\mathrm{MeV}$$
$$\Lambda_{\overline{MS}} = 359^{+181}_{-140}\mathrm{MeV}$$

PMC

$$\alpha_s^{\overline{MS}}(M_Z) = 0.129^{+0.009}_{-0.010}.$$



is consistent with those obtained from e^+e^- colliders, i.e. $\alpha_s^{\overline{MS}}(M_Z) = 0.13 \pm 0.005 \pm 0.03$ by the CLEO Collaboration [28] and $\alpha_s^{\overline{MS}}(M_Z) = 0.1224 \pm 0.0039$ from the jet shape analysis

S. Bethke, Eur.Phys.J. C64, 689 (2009)

Eur.Phys.J. C**64**, 689 (2009)
$215 \pm 9MeV$
wrong 4-loop coefficient

- **Inversely, if taking the value of $\alpha_s(M_z)$**

$$\alpha_s^{\overline{MS}}(M_Z) = 0.1184 \pm 0.0007$$

Four-loop

$$\Lambda_{\overline{MS}}^{\prime tH}|_{n_f=5} = 245^{+9}_{-10}\ \mathrm{MeV\ and}\ \Lambda_{\overline{MS}}|_{n_f=5} = 213^{+19}_{-8}\ \mathrm{MeV}$$

- **Discuss the four-loop uncertainty caused by *C***

(C-> c_3) => using scheme-equation

Even take *C* to have the value comparable with other terms at the same order

$$\boxed{|c_3^R| = 16.1239.}$$

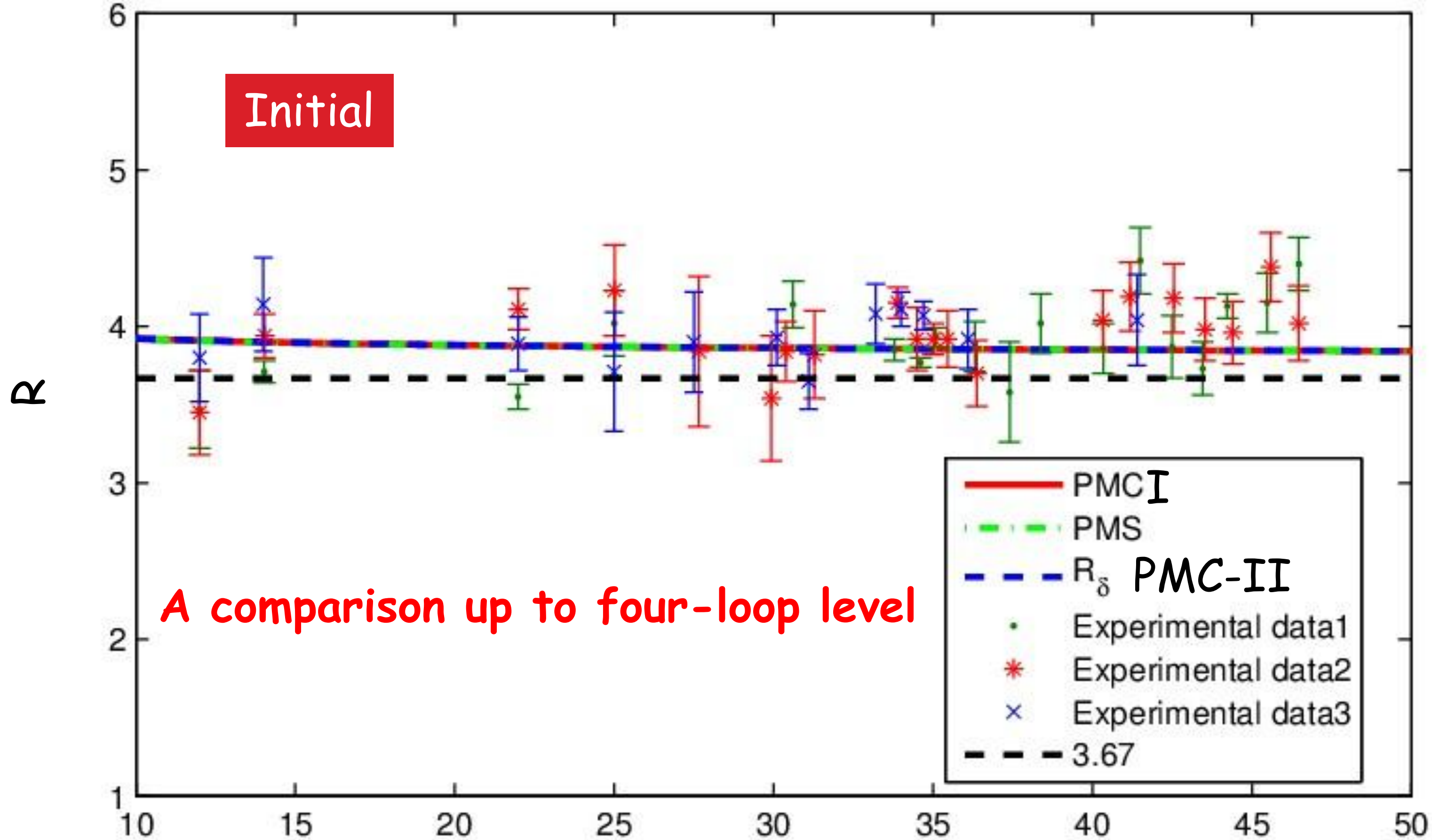
Several percent around **2%** of experimental error

$$a_s^R[31.6GeV] = 0.0665 \pm 0.0063$$

R. Marshall, Z.Phys. C**43**, 595 (1989).

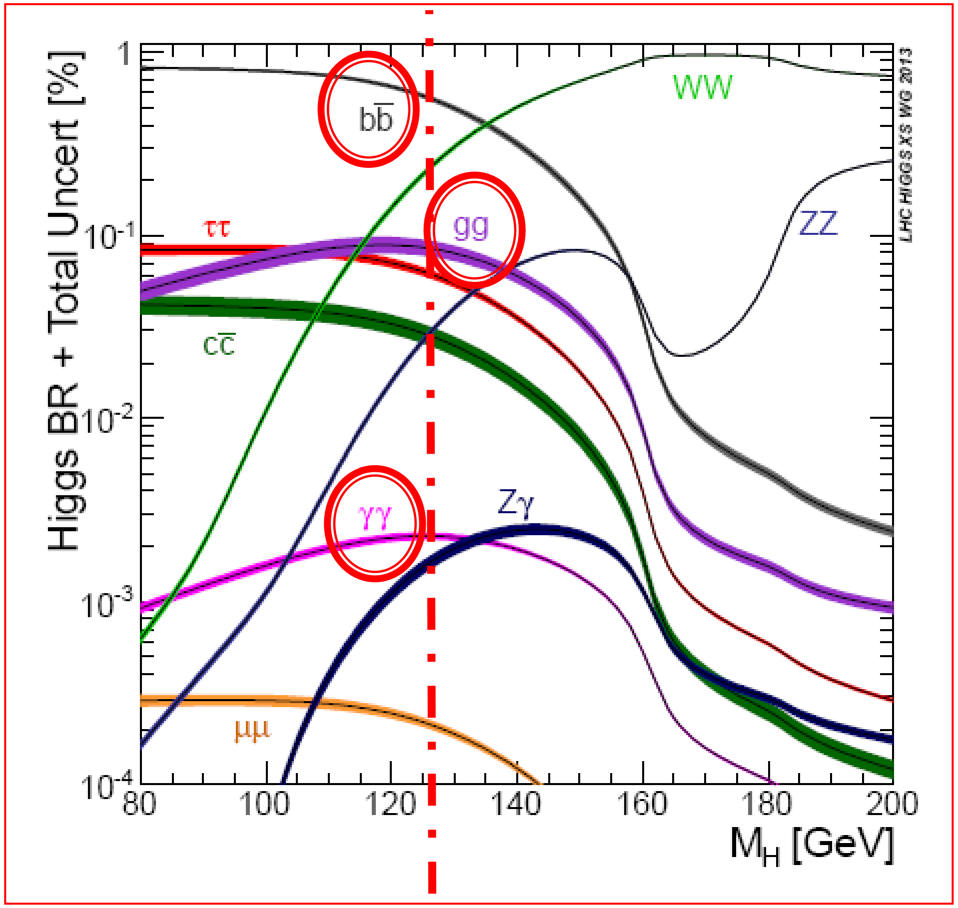
	$a^R = 0.07276$	$a^R = 0.06645$	$a^R = 0.06016$
$c_3^R = -16.1239$	10.3512 (726)	11.6325 (477)	13.1768 (287)
$c_3^R = 0$	10.3932 (717)	11.6673 (471)	13.2054 (284)
$c_3^R = +16.1239$	10.4348 (707)	11.7020 (466)	13.2338 (282)

A comparison of PMC and PMS



This shows that at the four loop-level, two methods of PMC can obtain the same results; while, PMC and PMS also consistent with each other in this sense.

IV) Scale setting for Higgs decays



$$\Gamma(H \to b\bar{b}) = \frac{3G_F M_H m_b^2(M_H)}{4\sqrt{2}\pi}\left[1 + c_{1,0}\, a_s(M_H) + (c_{2,0} + c_{2,1} n_f)\, a_s^2(M_H) + (c_{3,0} + c_{3,1} n_f + c_{3,2} n_f^2)\, a_s^3(M_H) + (c_{4,0} + c_{4,1} n_f + c_{4,2} n_f^2 + c_{4,3} n_f^3)\, a_s^4(M_H) + \mathcal{O}(a_s^5)\right],$$

Note: The quark masses are neglected for two loop calculation, so all nf-terms should be absorbed into the coupling constant.

$$\begin{aligned}\Gamma(H \to b\bar{b}) = \frac{3G_F M_H m_b^2(M_H)}{4\sqrt{2}\pi}\Big\{ & 1 + r_{1,0}(\mu_r^{\rm init})\, a_s(\mu_r^{\rm init}) + \left[r_{2,0}(\mu_r^{\rm init}) + \beta_0 r_{2,1}(\mu_r^{\rm init})\right]\, a_s^2(\mu_r^{\rm init}) \\ & + \left[r_{3,0}(\mu_r^{\rm init}) + \beta_1 r_{2,1}(\mu_r^{\rm init}) + 2\beta_0 r_{3,1}(\mu_r^{\rm init}) + \beta_0^2 r_{3,2}(\mu_r^{\rm init})\right]\, a_s^3(\mu_r^{\rm init}) + \left[r_{4,0}(\mu_r^{\rm init})\right. \\ & + \beta_2 r_{2,1}(\mu_r^{\rm init}) + 2\beta_1 r_{3,1}(\mu_r^{\rm init}) + \frac{5}{2}\beta_1\beta_0 r_{3,2}(\mu_r^{\rm init}) + 3\beta_0 r_{4,1}(\mu_r^{\rm init}) \\ & \left. + 3\beta_0^2 r_{4,2}(\mu_r^{\rm init}) + \beta_0^3 r_{4,3}(\mu_r^{\rm init})\right]\, a_s^4(\mu_r^{\rm init}) + \mathcal{O}(a_s^5)\Big\}.\end{aligned}$$

$$\Gamma(H \to b\bar{b}) = \frac{3G_F M_H m_b^2(M_H)}{4\sqrt{2}\pi}\left[1 + r_{1,0}(\mu_r^{\rm init}) a_s(Q_1) + r_{2,0}(\mu_r^{\rm init}) a_s^2(Q_2) + r_{3,0}(\mu_r^{\rm init}) a_s^3(Q_3) + r_{4,0}(\mu_r^{\rm init}) a_s^4(Q_4)\right] \quad (15)$$

$$Q_1 = \mu_r^{\rm init} \exp\left\{\frac{1}{2}\frac{-r_{2,1}(\mu_r^{\rm init}) + \frac{r_{3,2}(\mu_r^{\rm init})}{2}\frac{\partial\beta}{\partial a_s} - \frac{r_{4,3}(\mu_r^{\rm init})}{3!}\left[\beta\frac{\partial^2\beta}{\partial a_s^2} + \left(\frac{\partial\beta}{\partial a_s}\right)^2\right]}{r_{1,0}(\mu_r^{\rm init}) - \frac{r_{2,1}(\mu_r^{\rm init})}{2}\left(\frac{\partial\beta}{\partial a_s}\right) + \frac{r_{3,2}(\mu_r^{\rm init})}{4}\left(\frac{\partial\beta}{\partial a_s}\right)^2 + \frac{1}{3!}\left[\beta\frac{\partial^2\beta}{\partial a_s^2} - \frac{1}{2}\left(\frac{\partial\beta}{\partial a_s}\right)^2\right]\frac{r_{2,1}^2(\mu_r^{\rm init})}{r_{1,0}(\mu_r^{\rm init})}}\right\}$$

$$Q_2 = \mu_r^{\rm init} \exp\left\{\frac{1}{2}\frac{-r_{3,1}(\mu_r^{\rm init}) + \frac{r_{4,2}(\mu_r^{\rm init})}{2}\left[\frac{\partial\beta}{\partial a_s} + \frac{\beta}{a_s}\right]}{r_{2,0}(\mu_r^{\rm init}) - \frac{r_{3,1}(\mu_r^{\rm init})}{2}\left[\frac{\partial\beta}{\partial a_s} + \frac{\beta}{a_s}\right]}\right\},$$

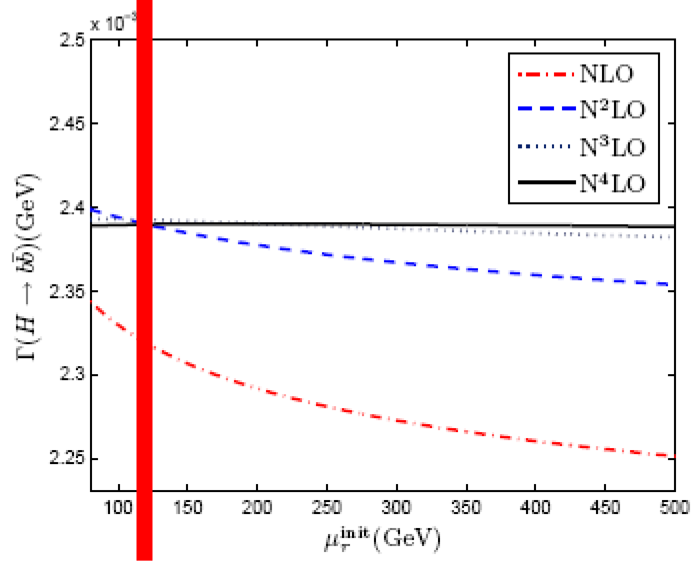
$$Q_3 = \mu_r^{\rm init} \exp\left\{\frac{1}{2}\frac{-r_{4,1}(\mu_r^{\rm init})}{r_{3,0}(\mu_r^{\rm init})}\right\},$$

Total cross sections almost unchanged

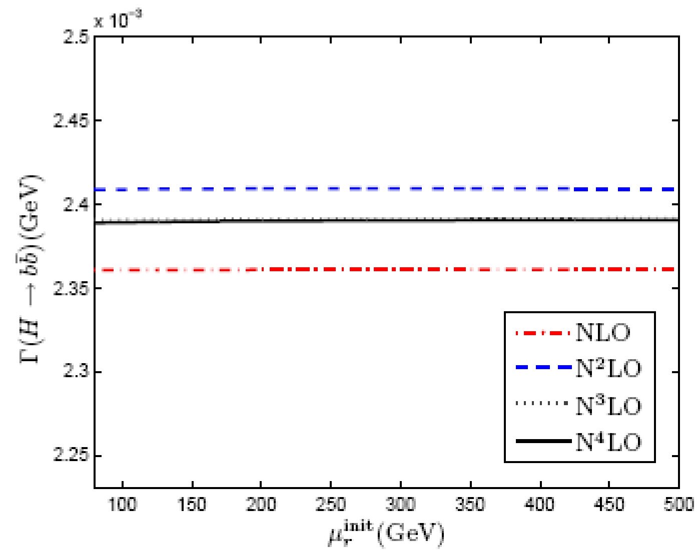
	Conventional scale setting						PMC scale setting					
	LO	NLO	N^2LO	N^3LO	N^4LO	Total	LO	NLO	N^2LO	N^3LO	N^4LO	Total
Γ_i (KeV)	1924.28	391.74	72.38	3.73	-2.65	2389.48	1924.28	436.23	48.12	-18.12	-1.38	2389.13
$\Gamma_i/\Gamma_{\rm tot}$	80.53%	16.39%	3.03%	0.16%	-0.11%		80.54%	18.26%	2.01%	-0.76%	-0.06%	

TABLE I. Decay width for the process $H \to b\bar{b}$ up to four-loop level. For conventional scale setting, we set the renormalization scale $\mu_R \equiv M_H$. For PMC scale setting, we set the initial renormalization scale $\mu_R^{\rm init} = M_H$. Here Γ_i stands for the decay width at each perturbative order with $i =$ LO, NLO and etc., $\Gamma_{\rm tot}$ stands for the total decay width. $M_H = 126$ GeV.

Guess work: agree at NLO



Agree with conventional wisdom



PMC estimation

Shows exactly LO, NLO contributions

A comparison with several BLM-extension methods

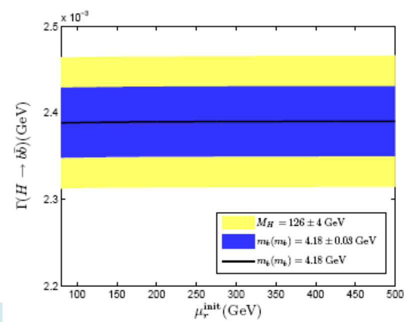
	$\Gamma_{\rm tot}$	$\Gamma_{\rm LO}/\Gamma_{\rm tot}$	$\Gamma_{\rm NLO}/\Gamma_{\rm tot}$	$\Gamma_{\rm N^2LO}/\Gamma_{\rm tot}$	$\Gamma_{\rm N^3LO}/\Gamma_{\rm tot}$	$\Gamma_{\rm N^4LO}/\Gamma_{\rm tot}$
conventional scale setting	2.389 MeV	80.53%	16.39%	3.03%	0.16%	−0.11%
seBLM	2.389 MeV	80.56%	18.25%	1.99%	-0.72%	-0.08%
PMC-I	2.388 MeV	80.58%	18.26%	2.03%	-0.76%	-0.10%
R_δ-scheme	2.389 MeV	80.54%	18.26%	2.01%	-0.76%	-0.06%
BKM [66]	2.75 MeV	74.5%	17.7%	5.3%	1.8%	0.7%
FAPT with $l=2$ [67]	2.38 MeV	79.5%	16.2%	4.3%	-	-
FAPT with $l=3$ [67]	2.44 MeV	78.5%	16.1%	4.2%	1.2%	-

TABLE III. A comparison of several approaches for calculating the perturbative coefficients of $H\to b\bar{b}$, where the predictions of the PMC-I scheme, the R_δ-scheme and the seBLM scheme, together with the ones derived under conventional scale setting, are presented. Here Γ_i stands for the decay width at each perturbative order with $i=$ LO, NLO and etc., $\Gamma_{\rm tot}$ stands for the total decay width. The initial renormalization scale is taken as $M_H=126$ GeV. To be useful reference, the results of Refs.[66, 67] for the FAPT scheme and the BKM scheme are also presented.

	$\Gamma_{\rm NLO}$ (KeV)		
$\mu_r^{\rm init}$	$M_H/2$	M_H	$2M_H$
Conventional scale setting	435.42	391.73	356.18
seBLM [64]	435.95	435.95	435.95
PMC-I [49]	435.03	435.99	436.06
R_δ-scheme [54]	436.12	436.23	436.32

TABLE IV. Initial scale dependence for $\Gamma_{\rm NLO}$ of $H\to b\bar{b}$. Here Γ_i stands for the decay width at each perturbative order with $i=$ LO, NLO and etc. The predictions of the PMC-I, R_δ and seBLM schemes are almost independent of $\mu_r^{\rm init}$. The cases for higher order decay widths Γ_{N^2LO}, Γ_{N^3LO} and Γ_{N^4LO} are the similar. $M_H=126$ GeV.

These three ways are consistent with each other



$$\Gamma(H\to b\bar{b}) = 2.389 \pm 0.073 \pm 0.041 \text{ MeV},$$
$$\Gamma(H\to gg) = 0.373 \pm 0.030 \text{ MeV}.$$

The conditions for (H->gg) up to three loop level are similar to the case of (H->b+bbar)

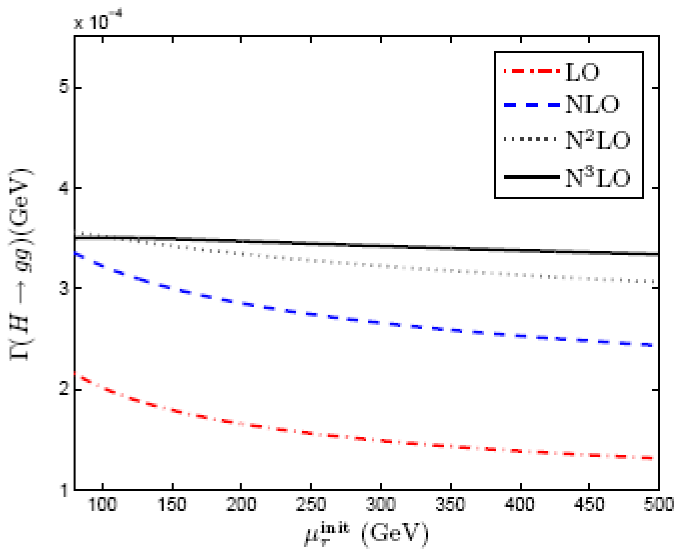


FIG. 5. Total decay width $\Gamma(H \to gg)$ up to three-loop level under conventional scale setting versus the renormalization scale $\mu_r \equiv \mu_r^{\rm init}$. The dash-dot, dashed, dotted and solid lines are for LO, NLO, N^{2}LO and N^{3}LO estimations, respectively.

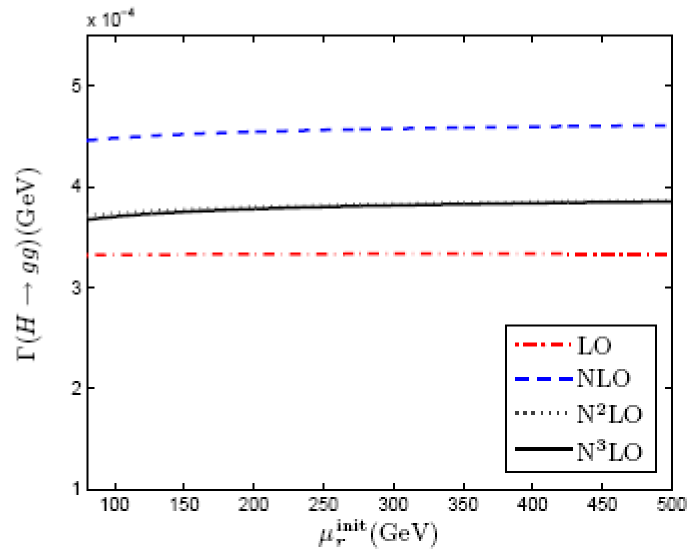


FIG. 6. Total decay width $\Gamma(H \to gg)$ up to three-loop level after PMC scale setting versus the initial renormalization scale $\mu_r^{\rm init}$. The dash-dot, dashed, dotted and solid lines are for LO, NLO, N^{2}LO and N^{3}LO estimations, respectively.

$H \to \gamma\gamma$

Dominant process for finding Higgs

$$\Gamma(H \to \gamma\gamma) = \frac{M_H^3}{64\pi}\left[A_{\rm LO} + A_{\rm NLO}(\mu_r^{\rm init})\frac{\alpha_s(\mu_r^{\rm init})}{\pi} + A_{\rm NNLO}(\mu_r^{\rm init})\left(\frac{\alpha_s(\mu_r^{\rm init})}{\pi}\right)^2 + A_{\rm EW}\frac{\alpha}{\pi}\right], \quad (2.2)$$

where $\mu_r^{\rm init}$ stands for an arbitrary initial choice of renormalization scale [1], and under the conventional scale setting, it is usually fixed to be the typical momentum of the process, e.g. $\mu_r \equiv \mu_r^{\rm init} = m_H$. The coefficients are defined as,

$$A_{\rm LO} = \left(A_W^{(0)}(\tau_W) + \hat{A}_t A_t^{(0)}(\tau_t) + A_f^{(0)}(\tau_f)\right)^2, \quad (2.3)$$

$$A_{\rm NLO}(\mu_r^{\rm init}) = 2\sqrt{A_{\rm LO}}\hat{A}_t A_t^{(1)}(\tau_t), \quad (2.4)$$

$$A_{\rm NNLO}(\mu_r^{\rm init}) = 2\sqrt{A_{\rm LO}}\; Re\left[\hat{A}_t A_t^{(2)}(\tau_t)\right] + \left(\hat{A}_t A_t^{(1)}(\tau_t)\right)^2, \quad (2.5)$$

$$A_{\rm EW} = 2\sqrt{A_{\rm LO}} A_{\rm EW}^{(1)}. \quad (2.6)$$

All coefficients can be changed into the forms with pole mass, which provides **a better platform for testing the idea that own β-terms involving coupling constant can be absorbed into the coupling**

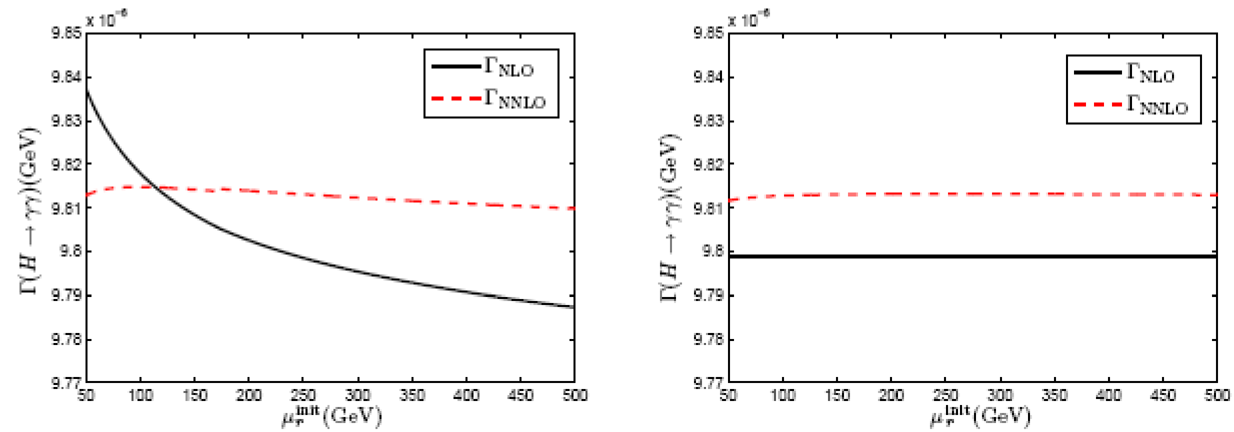


Figure 1. Decay width versus the initial renormalization scale $\mu_R^{\rm init}$ for Higgs decay $H \to \gamma\gamma$ under the conventional scale setting (left) and the PMC scale setting (right). The solid and dashed lines stand for the total decay widths up to NLO level and NNLO level, respectively.

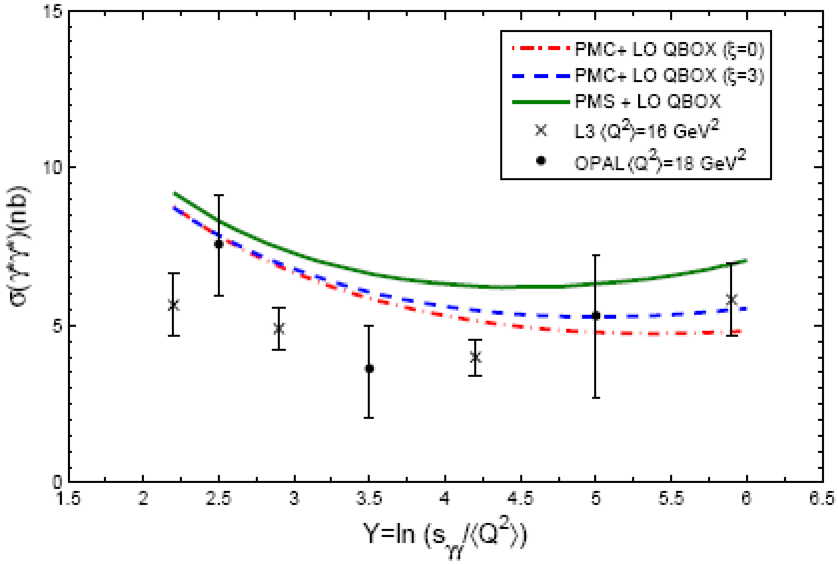
	Decay width of NLO terms (10^{-3} KeV)		
$\mu_r^{\rm init}$	$M_H/2$	M_H	$2M_H$
Conventional scale setting	180.1	162.0	148.0
PMC scale setting	148.7	148.7	148.7

Scale Independence

$$\Gamma(H \to \gamma\gamma)|_{\rm pole} = \left(9.650 \times 10^{-6} + \left(1.620^{+0.180}_{-0.140}\right) \times 10^{-7} + \left(2.200^{-18.585}_{+12.486}\right) \times 10^{-9}\right){\rm GeV}, \quad (3.2)$$

$$\Gamma(H \to \gamma\gamma)|_{\rm PMC} = \left(9.650 \times 10^{-6} + 1.487 \times 10^{-7} + 1.415 \times 10^{-8}\right){\rm GeV}, \quad (3.3)$$

V) Scale setting for QCD pomeron at the next-to-leading order level



New point: Slight gauge dependent

$$\omega(Q^2,0) = 0.15 \sim 0.18.$$

	PMC		
$\mu_R^{\rm init}$	$Q/2$	Q	$2Q$
$Q^2 = 1\ GeV^2$	0.149	0.149	0.149
$Q^2 = 15\ GeV^2$	0.176	0.176	0.176
$Q^2 = 100\ GeV^2$	0.175	0.175	0.175

$$\sigma(\gamma^*\gamma^*) = 8.51 \text{ nb} \quad \text{for } \xi = 0,$$
$$\sigma(\gamma^*\gamma^*) = 10.06 \text{ nb} \quad \text{for } \xi = 1,$$
$$\sigma(\gamma^*\gamma^*) = 11.12 \text{ nb} \quad \text{for } \xi = 3.$$

$\sqrt{s_{\gamma\gamma}} = 1$ TeV,

$\sqrt{s_{\gamma\gamma}} = 2$ TeV, we obtain

$$\sigma(\gamma^*\gamma^*) = 10.50 \text{ nb} \quad \text{for } \xi = 0,$$
$$\sigma(\gamma^*\gamma^*) = 12.67 \text{ nb} \quad \text{for } \xi = 1,$$
$$\sigma(\gamma^*\gamma^*) = 14.19 \text{ nb} \quad \text{for } \xi = 3.$$

Varying Q^2 within the region of $[1, 100]$ GeV^2, we obtain $\omega^{\rm PMC}_{\rm MOM}(Q^2,0) \in [0.082, 0.158]$ for the Landau gauge of $\xi = 0$, $\omega^{\rm PMC}_{\rm MOM}(Q^2,0) \in [0.124, 0.168]$ for the Feynman gauge of $\xi = 1$, and $\omega^{\rm PMC}_{\rm MOM}(Q^2,0) \in [0.149, 0.176]$ for the Fried-Yennie gauge of $\xi = 3$, respectively.

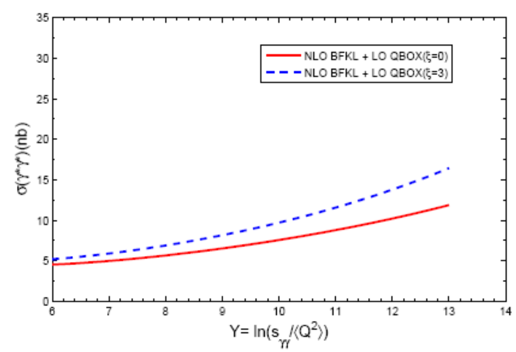
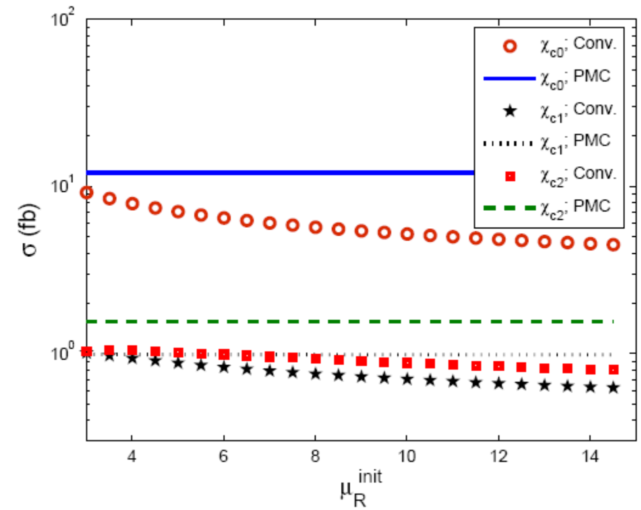


FIG. 5. The energy dependence of the total cross section of highly virtual photon-photon collisions with $Q^2 = 20GeV^2$ predicted by NLO BFKL under PMC scale setting for future linear colliders. Where the uncertainty comes from the choice of gauge in the MOM scheme.

VI) Scale setting for J/ψ +χ{cJ} Production at the B Factories

	Conventional scale setting			PMC scale setting		
$\mu_R^{\rm init}$	2 m_c	$\sqrt{s}/2$	$\sqrt{s}$	2 m_c	$\sqrt{s}/2$	$\sqrt{s}$
σ_t^0 (fb)	9.23	6.81	5.21	12.14	12.14	12.14
σ_t^1 (fb)	1.01	0.85	0.70	0.99	0.99	0.99
σ_t^2 (fb)	1.53	1.26	1.03	1.56	1.56	1.56

	$J/\psi + \chi_{c0}$	$\psi' + \chi_{c0}$
Belle $\sigma \times B^{\chi_{c0}}[> 2]$ [22]	$16 \pm 5 \pm 4$	$17 \pm 8 \pm 7$
Belle $\sigma \times B^{\chi_{c0}}[> 2(0)]$ [20]	$6.4 \pm 1.7 \pm 1.0$	$12.5 \pm 3.8 \pm 3.1$
BaBar $\sigma \times B^{\chi_{c0}}[> 2]$ [21]	$10.3 \pm 2.5^{+1.4}_{-1.8}$	$\sim$
Wang, Ma and Chao [5]	9.5	4.1
Dong, Feng and Jia [6]	8.62	4.98
Our result	$12.14^{+2.67}_{-2.43}$	$5.14^{+1.13}_{-1.02}$



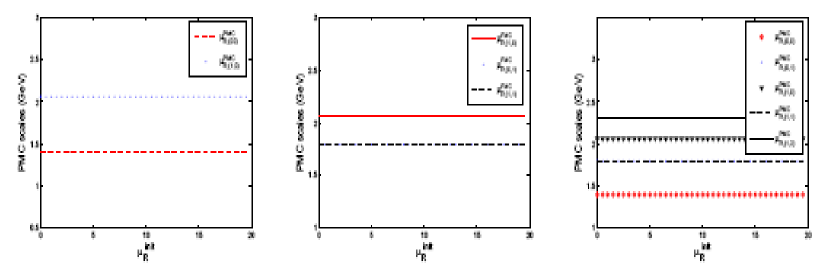


FIG. 2: PMC scales versus the initial renormalization scale $\mu_R^{\rm init}$ for the polarized cross sections of the $e^+ + e^- \to J/\psi + \chi_{cJ}$ processes with $J = 0, 1, 2$.

Summary and Outlook

I. **PMC** provides self-consistent way to set the effective scales, which leads to **scheme-independent result**. QCD is not confromal, however one can use the PMC to convert a PQCD series with the corresponding conformal QCD series.

II. A combination of PMC to Extended-RGE can be used to derive a **precise QCD estimation**.

III. Top-pair production total cross-section agree with exp data.

IV. Top-pair asymmetries are within 1σ-error. **SM is OK ?**

V. A new approach to achieve the PMC goal is suggested.

V. By applying PMC to **Higgs decay, Pomeron, J/ψ+χ_{cJ} production (polarized or unpolarized),** and etc., we show PMC works well.

Since it suppress an important systematic error, PMC shall have too many applications for high energy processes …….

PMC后，我们没有理由再将理论与实验差别归于理论误差（虽然还有其它的误差源），从而判断是否真的应当引入新物理以及新物理能占到多大的份额

Still, we have many points to be clarified

I) The inner connection conformal symmetry to β-terms ?
II) In low energy, what's the preferred behavior of α_s ?
III) Automation ?
IV) Factorization scale ?
V) A convenient way to deal with NNLO nf-terms only ?
VI) A detailed discussion of PMS and PMC ?
VII) Are all the PMS steps OK ?
VIII) Possible questions of PMC, such as pQCD convergence
IX) A detailed discussion of scheme dependence under PMC ?
Why a physical scheme, such as MOM, is better than other schemes in certain cases.

……………………………

欢迎大家使用并深化PMC；知无不言，言无不尽，共同发展

Thanks