5d rank 2 SCFTs
and
5-brane webs

电子科技大学
University of Electronic Science and Technology of China

Sung-Soo Kim 金晟洙 (UESTC)

2018-12-14

USTC

中国科学技术大学
University of Science and Technology of China

Hirotaka Hayashi (Tokai univ.), Kimyeong Lee (KIAS), Futoshi Yagi (SWJTU)
arXiv: 1801.03916, 1806.10569, 1812.xxxxx

- In this talk, we focus on 5d N=1 supersymmetric gauge theories (with 8 supercharges).
  - superconformal at UV fixed point

Seiberg 97

- 5-brane web diagram in Type IIB string theory has provided a powerful tool to study these 5d N=1 gauge theories.

Aharony, Hanany 97
Aharony, Hanany, Kol 97

- E.g. Non-perturbative aspects and various dualities could be explicitly seen from their 5-brane constructions.
  - also quantitatively: partition function

- A 5-brane web for **$SU(N)_k$** gauge theories is obtained from N parallel D5 branes between 2 NS5 branes.
  - for small Chern-Simons level k

- With the introduction of an orientifold 5-plane (or O7), one may realize an **Sp(N)** or an **SO(N)** gauge group.

Brunner, Karch 97,
Brandhuber, Itzhaki, Sonnenschein, Theisen, Yankielowizc 97
Hanany, Zaffaroni 99
Zafrir 15

- Classification of 6d SCFTs
  - (2,0) ADE classification Witten'95,Strominger'95
  - (1,0) classification via F-theory Heckman,Morrison,Rudelius,Vafa:1502

- Possibly, all 5d SCFTs could be obtained from a circle compactification with some twist from 6d theories, full classification is still not complete
  - dualities
  - symmetries

- Recently there has been progress on classification of 5d SCFTs:

[17 Jefferson- HC Kim-Vafa-Zafrir]

- From the field theory:
  milder constraint on the theory in the Coulomb branch
  with **the positivity of the monopole string tension**
  —> a classification of 5d SCFTs of a single gauge group.

[18 Jefferson-Katz-HC Kim-Vafa]

- geometric construction
  M-theory on local shrinkable Calabi-Yau threefold
  Rank 2 theories are classified (with duality).

- In this talk, we propose **all the 5-brane webs for rank 2 SCFTs**

  e.g.,

  G_2 + 6F

  $SU(3)_9$

  $SU(3)_{3/2}$ + 1 Sym

- Employ them to **test duality** between different gauge descriptions

  e.g.,

  Pure G_2 $\longleftrightarrow$ $SU(3)_7$

  G_2 + 2F $\longleftrightarrow$ $SU(3)_6$ + 2F $\longleftrightarrow$ $Sp(2)_\pi$ + 2AS

  Sp(2) + 1AS + 8F $\longleftrightarrow$ $SU(3)_{3/2}$ + 9F

Plan

- Review of 5d N =1 gauge theories
- Type IIB (p,q) 5-brane webs: e.g. rank 1 theories.
- Explicit construction of 5-brane web
  - G_2 - $SU(3)_7$
  - G_2-SU(3)-Sp(2) sequences
  - $SU(3)_{3/2}$ + 9F = Sp(2) + 1AS + 8F
  - SU(3) + 1 Sym
- Conclusion

5d N = 1 gauge theory

5d N=1 supersymmetric gauge theories with gauge group G:

8 Supercharges and SU(2) R-symmetry

Matter content

Vector multiplet: $A_\mu, \phi;\ \lambda, \bar{\lambda}$

Hypermultiplet: $q^A, \bar{q}_A;\ \psi, \bar{\psi}$ (A=1,2 $SU(2)_R$ index)

Coulomb branch is parametrized by vev of vec. mult. scalar ϕ

Higgs branch is parametrized by vev of hypermultiplet scalars

Global symmetry

G_{Hypers} x $U(1)_{Instatons}$

Instanton solitons with conserved current j= *(F∧F)

5d Superconformal theories (SCFTs)

A certain class of 5d N=1 theories have **non-trivial UV fixed point**

[96 Seiberg]
[96 Morrison-Seiberg]
[97 Intriligator-Morrison-Seiberg]

with enhanced global symmetry

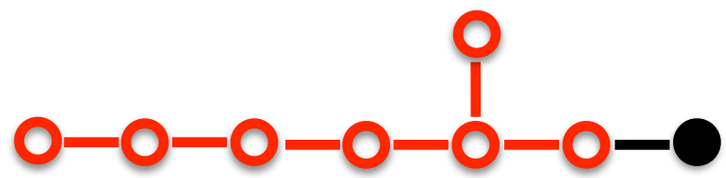
$$G_{\text{Hypers}} \times U(1)_{\text{Instantons}} \subset G_{\text{Global}}$$

massless hypers + massless instanton

An example: $SU(2)$ theory with $N_f \leq 7$ hypermultiplets in fund. representation

(flavors)

$$SO(2N_f) \times U(1)_I \subset E_{N_f+1}$$



flavor symmetry Instanton (charged under spinor representation)

Prepotential

- Along the Coulomb branch (G -> $U(1)^{\text{rank}(G)}$), the theory is described by abelian low energy effective theory which is characterized by prepotential.

- Prepotential is at most cubic and 1-loop exact:

$$\mathcal{F} = \frac{1}{2g_0^2} h_{ij}\phi^i\phi^j + \frac{\kappa}{6} d_{ijk}\phi^i\phi^j\phi^k + \frac{1}{12}\Big(\sum_{\text{Roots}} |R\cdot\phi|^3 - \sum_f \sum_{w\in W_f} |w\cdot\phi + m_f|^3 \Big)$$

$h_{ij} = \text{Tr}(T_i T_j),\ d_{abc} = \frac{1}{2}\text{Tr}T_a(T_bT_c + T_cT_b), \quad W_f = \text{Weight of } G \text{ in the rep. } r_f$

[96 Morrison-Seiberg]

[97 Intriligator-Morrison-Seiberg]

Magnetic monopole string tension: $\phi_{D_i} = \partial_i \mathcal{F}$

Effective coupling: $\tau_{ij} = \partial_i\partial_j \mathcal{F}$

Coulomb branch metric: $d^2 s = \tau_{ij}\, d\phi^i d\phi^j$

An example: SU(2) Prepotential

- With massless hypers

$$\mathcal{F}_{SU(2)} = \frac{1}{g_0^2}a^2 + \frac{1}{6}(8 - N_f)a^3$$

Magnetic monopole string tension: $\phi_{D_i} = \partial_i \mathcal{F}$ $\qquad \frac{a}{2}\left(\frac{4}{g_0^2} + (8 - N_f)a\right)$

Effective coupling: $\tau_{ij} = \partial_i \partial_j \mathcal{F}$ $\qquad \frac{2}{g_0^2} + (8 - N_f)a$

Notice that $N_f > 8$, Coulomb branch is not well defined
—> theory is now well defined.

M-theory/String constructions

Large class of 5d SCFTs can be engineered by M-theory or string theory.

M- theory on non-compact Calabi-Yau 3 fold (CY3) with some compact cycles shrunk to a point

[Witten'96],[Morrison,Seiberg'96],
[Douglas,Katz,Vafa'96]
[Katz,Klemm,Vafa'96],

M2 wrapping compact 2-cycles <==> BPS particle mass
= vol(2-cycles)

M5 wrapping compact 4-cycles <==> monopole string tension
= vol(4-cycles)

Type IIB brane configuration

[Aharony,Hanany'97]
[Aharony,Hanany,Kol'97],
[DeWolfe,Hanany,Iqbal,Katz'99]

BPS configuration with D5 and NS5 branes with their bound states [next slide]

Two descriptions are equivalent.

[Leung,Vafa'97]

Type IIB (p,q) 5-brane

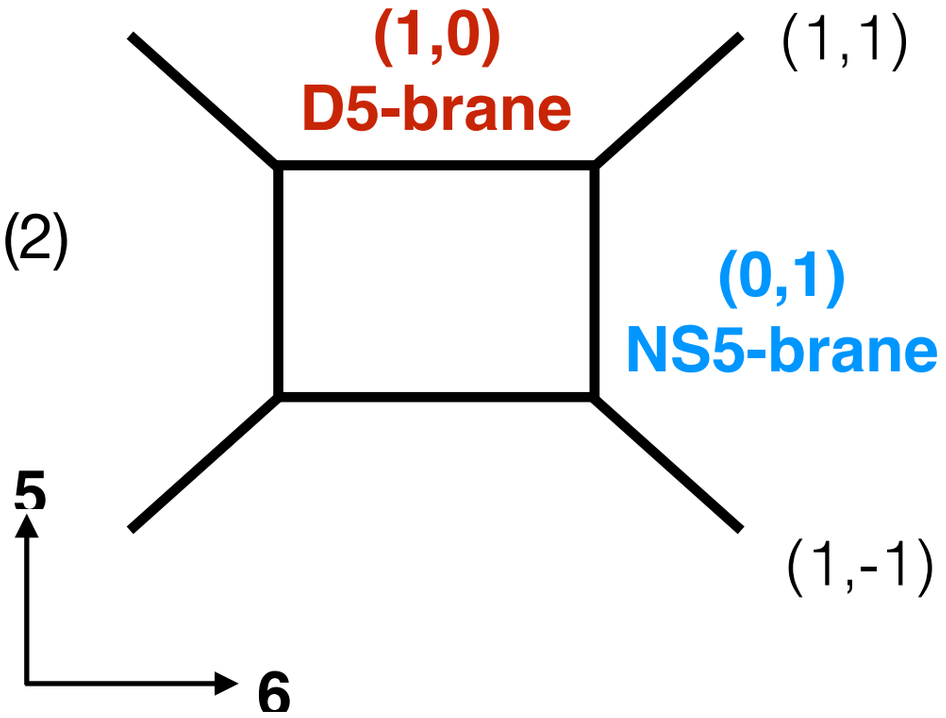
In 5d, D5 and NS5 make a configuration looks like a web: **(p,q) 5-brane web**

- charge conserving, tension is balanced

	0	1	2	3	4	5	6	7	8	9
NS5	–	–	–	–	–	–				
D5	–	–	–	–	–		–			
(p,q)	–	–	–	–	–	-	-			

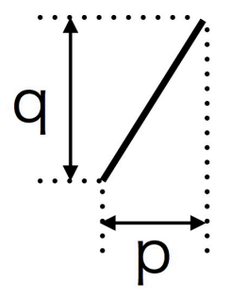
5d world vol.

e.g., SU(2)



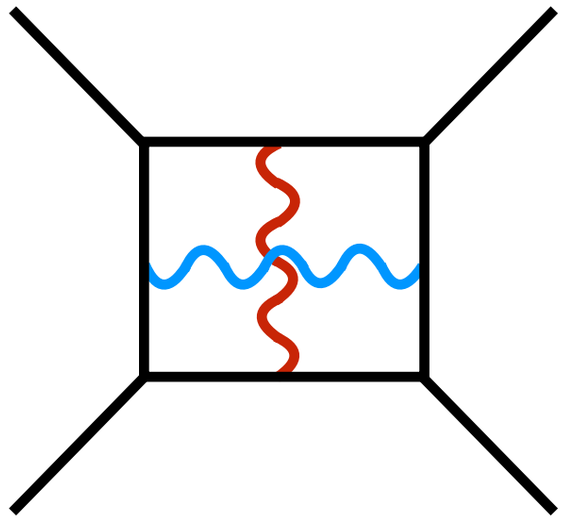
(p,q) plane

- (p,q) charges: p D5 charge + q NS5 charge,
  (p,q) 5-brane has a slope q/p



5d pure SU(2) gauge theory

- Coulomb branch : a = < ϕ >/2



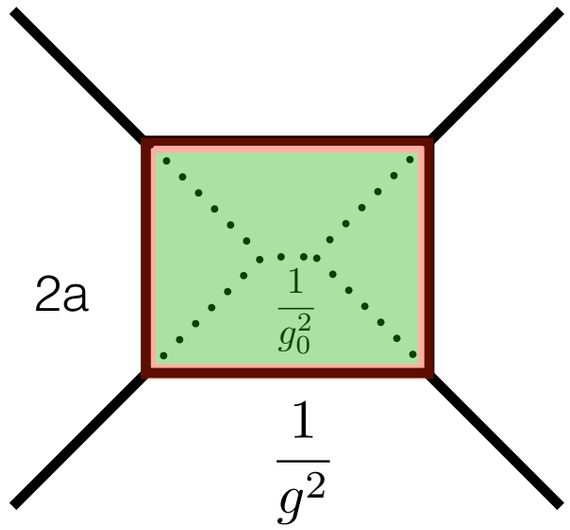
F1 SU(2) **W-bosons**

D1 SU(2) **Instantons**

- **Monopole string tension**

Area

$$2a\left(\frac{1}{g_0^2}+2a\right)$$



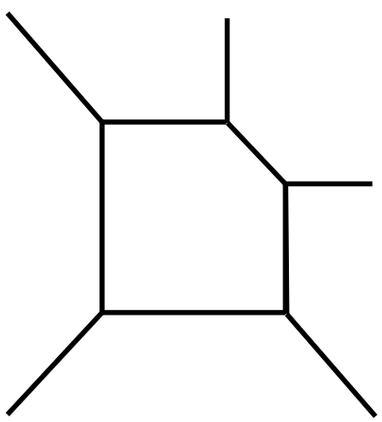
- **Effective couplings**

Circumference

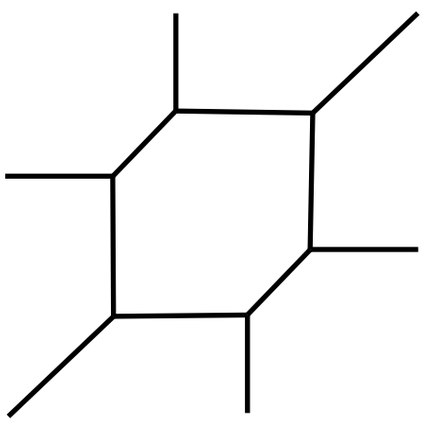
$$\frac{2}{g_0^2}+8a$$

Rank 1 theories

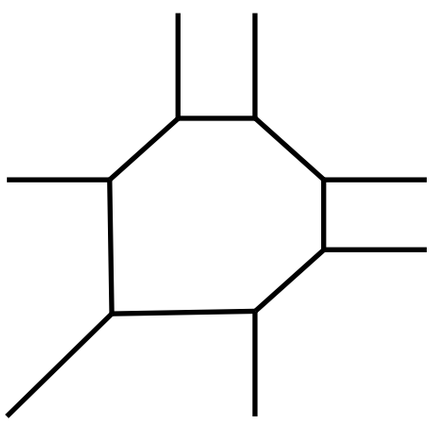
SU(2) theory with $N_f \leq 7$ flavors



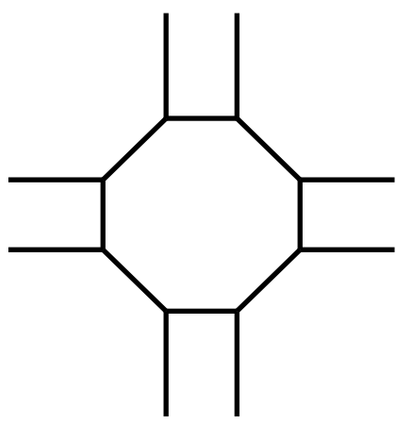
$N_f = 1$



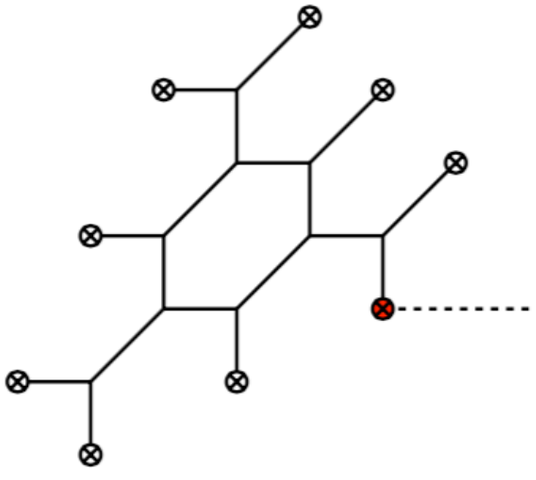
$N_f = 2$



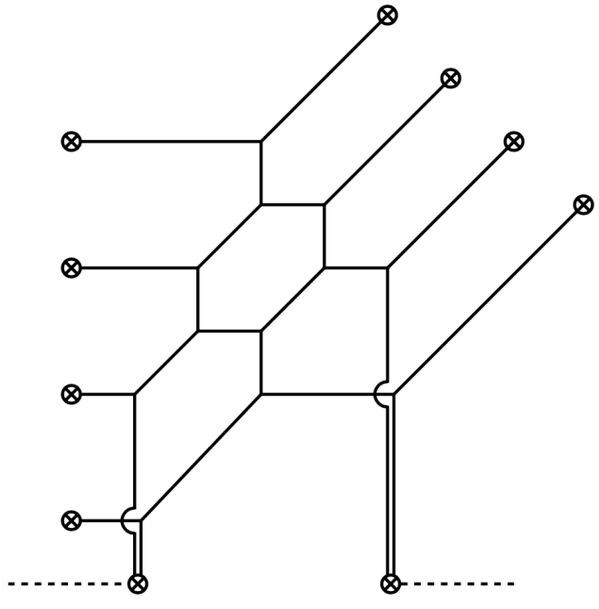
$N_f = 3$



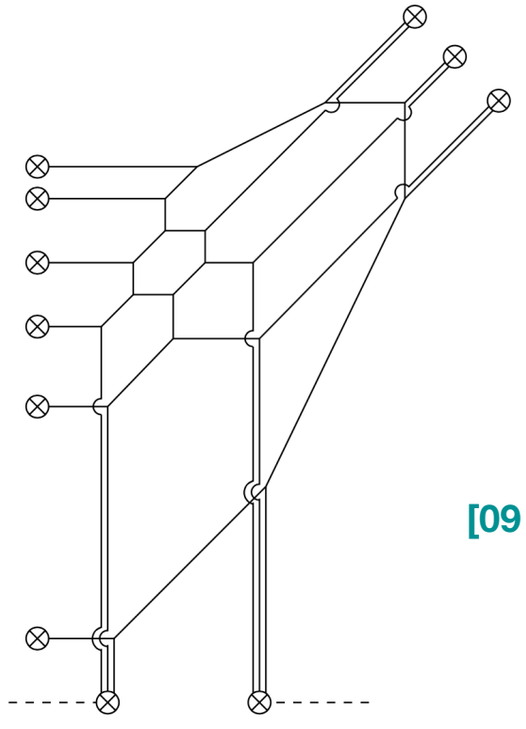
$N_f = 4$



$N_f = 5$



$N_f = 6$



$N_f = 7$

[09 Benini-Benvenuti -Tachikawa]

Q: What happens if we add one more flavor?

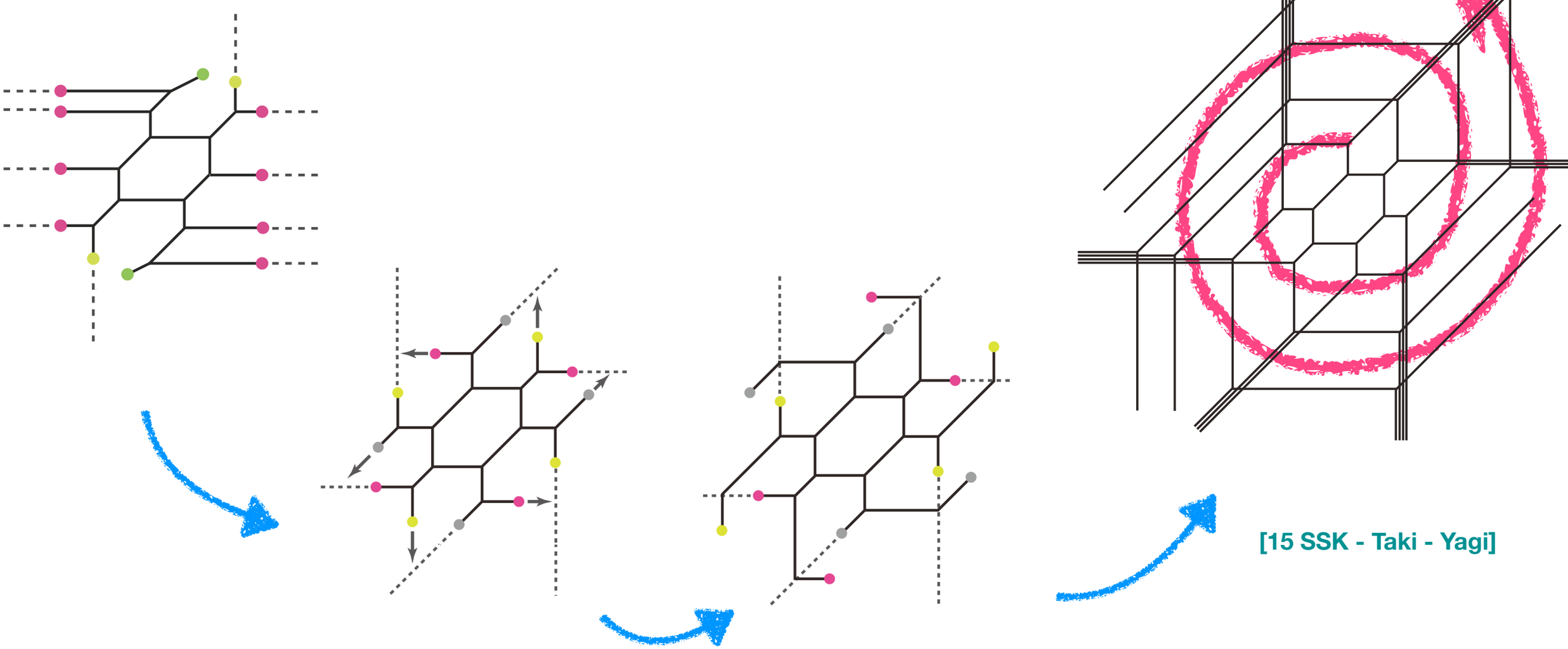
New type of 5-brane configuration

When the number of hypermultiplets reach the marginal value,

- It is known that the theory has the UV fixed point **in 6d**.

  e.g., SU(2) theory with $N_f = 8$ flavors <==> 6d E-string theory on a circle

- A new type of (p,q) 5-brane configuration emerges: Tao☯ diagram

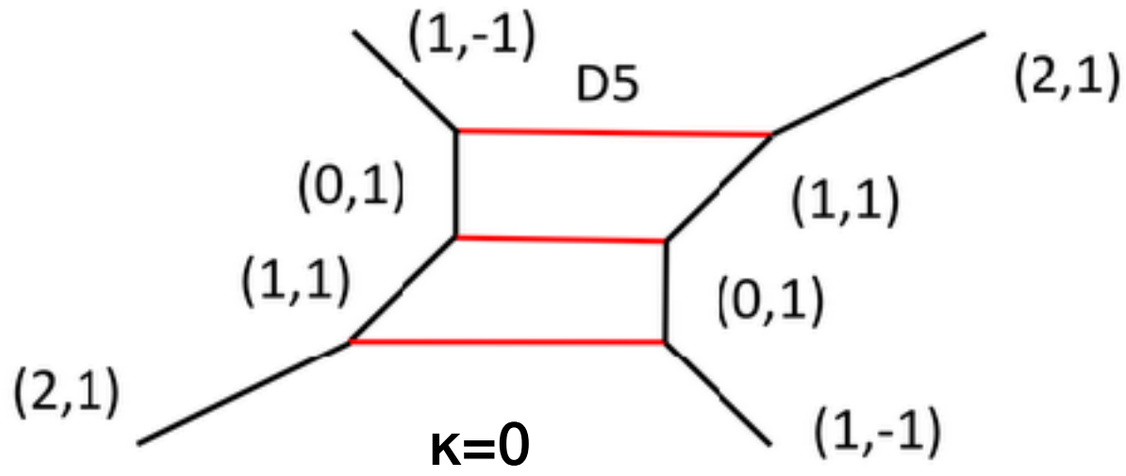


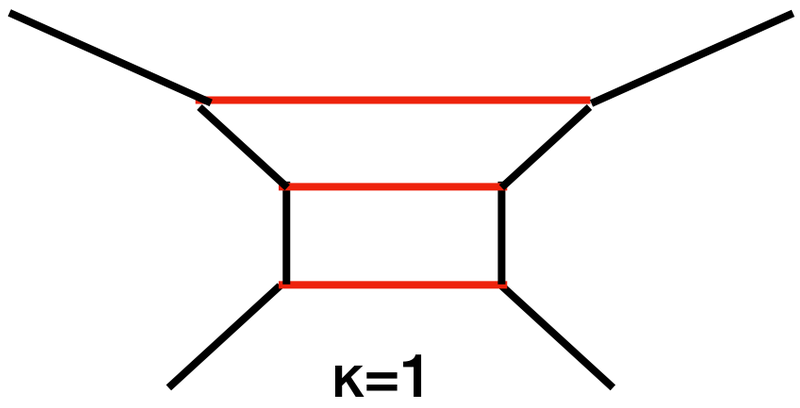
[15 SSK - Taki - Yagi]

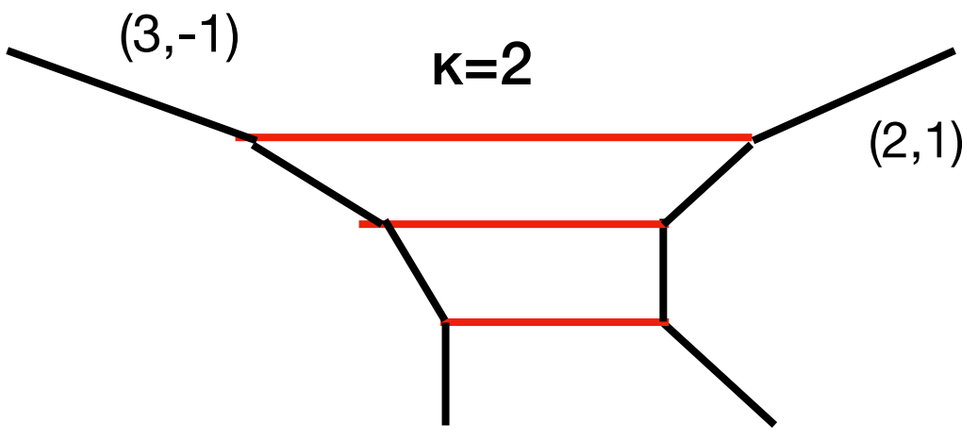
Explicit construction of 5-brane webs for 5d rank 2 theories:

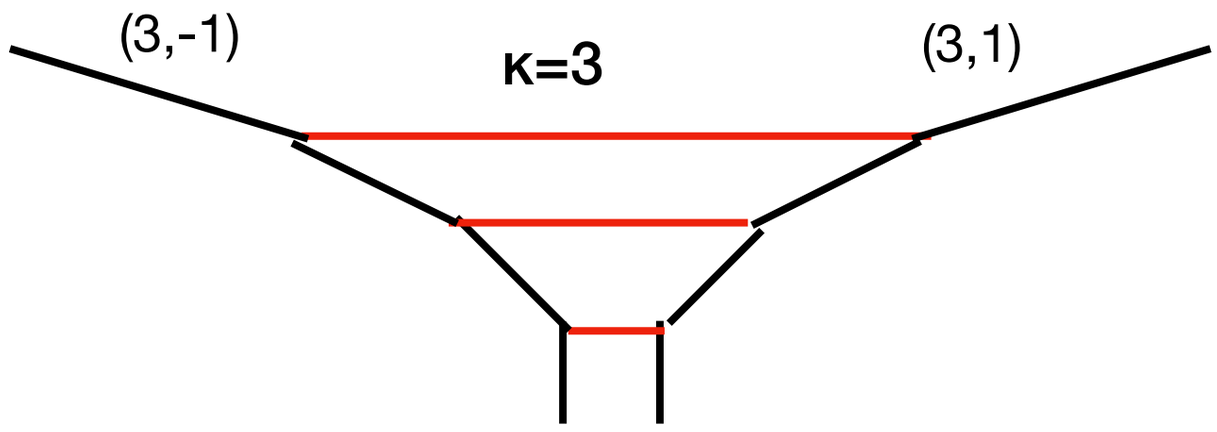
SU(3), Sp(2)=SO(5), G_2

- 5d pure SU(3) gauge theory with κ=0,1,2,3







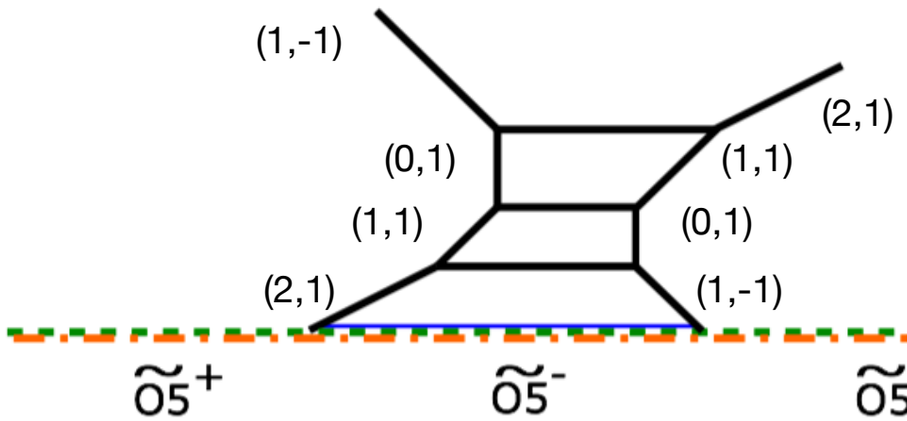


5d SO(7) gauge theories

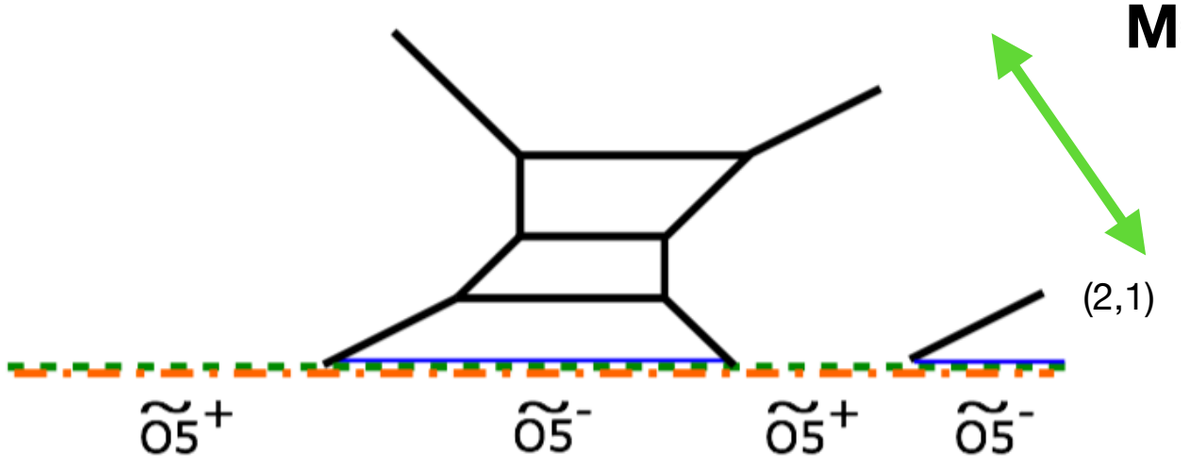
$\widetilde{O5}^-$ = **O5$^-$ + 1/2 D5 + 1/2 D7 cut** [Zafrir'15]

$\widetilde{O5}^+$ = **O5$^+$ + 1/2 D7 cut**

Pure SO(7)



SO(7) + spinor matter

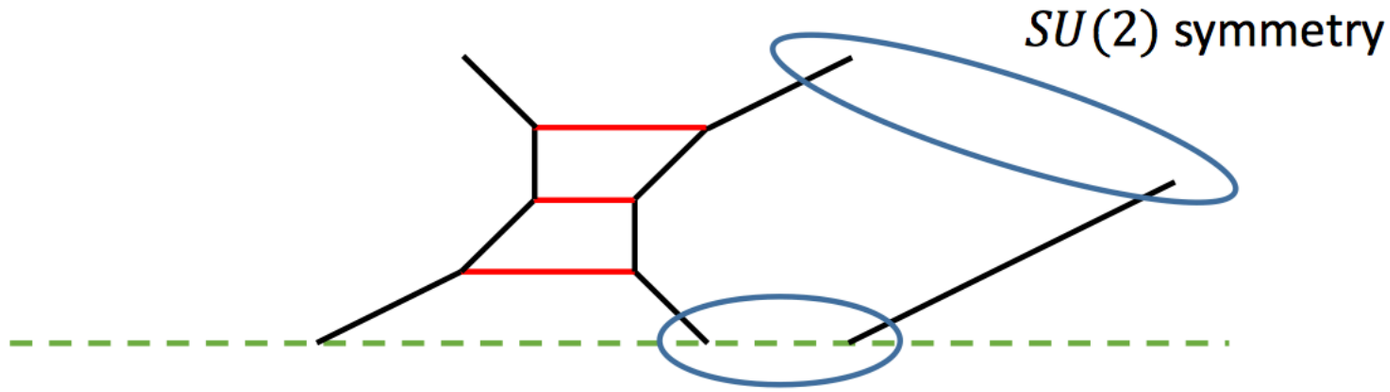


Ms: mass of Spinor

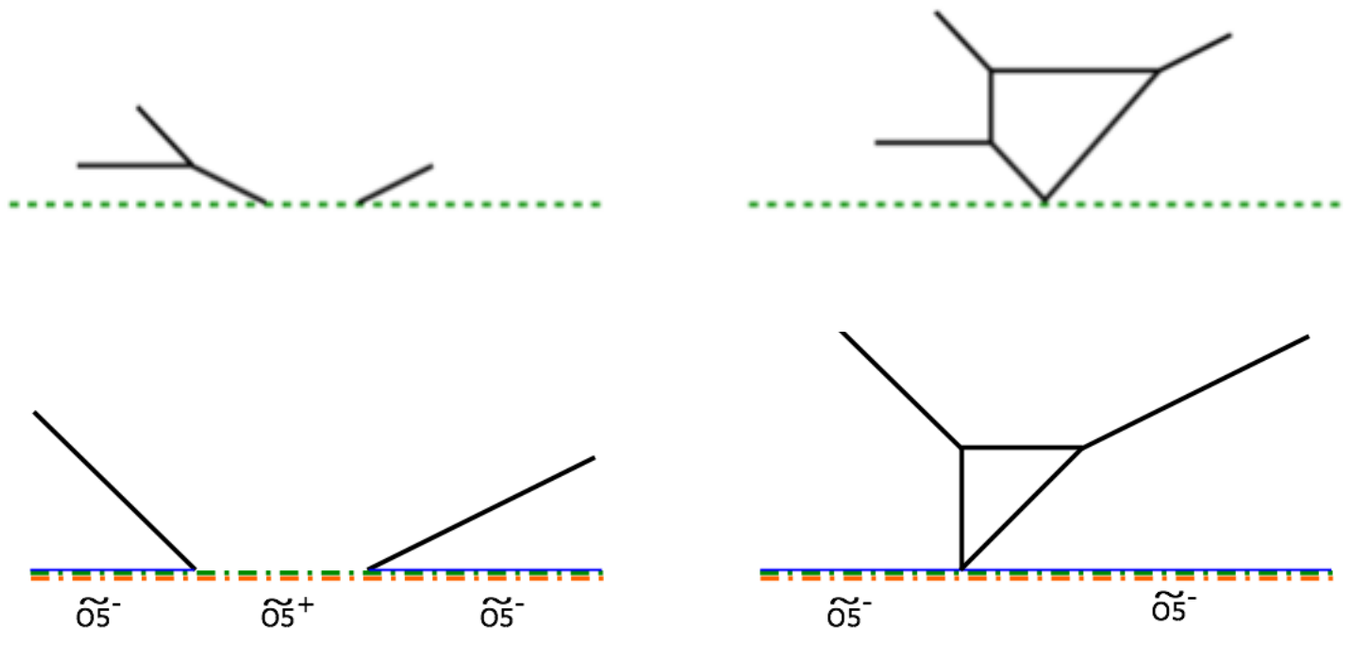
Sp(0) instantons=spinors

G_2 = Higgsing of SO(7) with spinor

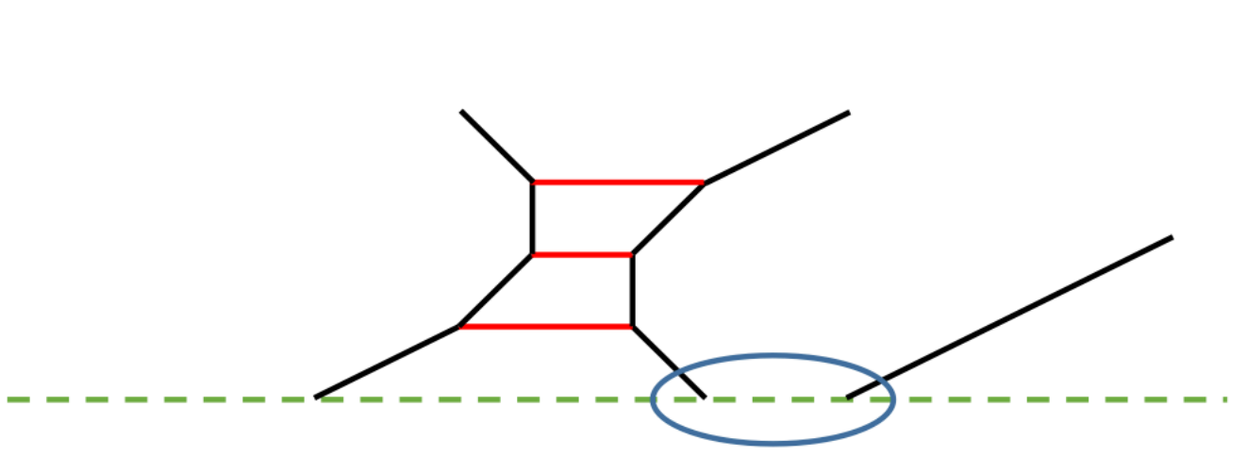
Ms -> 0 : Higgs branch

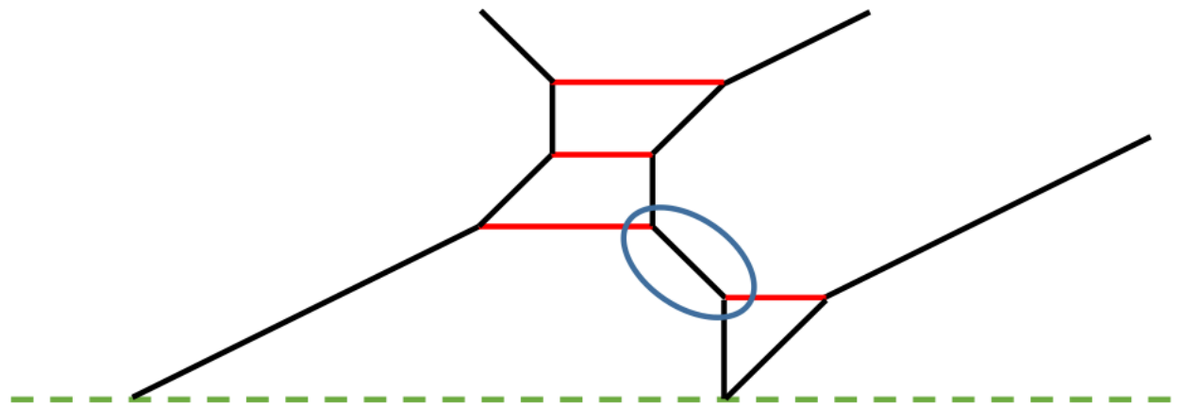


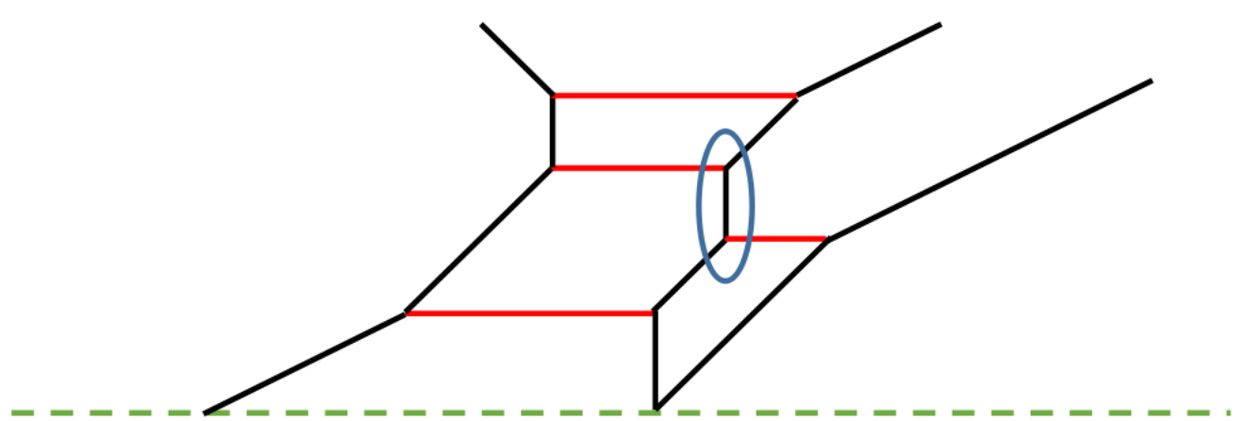
Generalized flop transitions [Hayashi-SSK-Lee-Yagi'17]

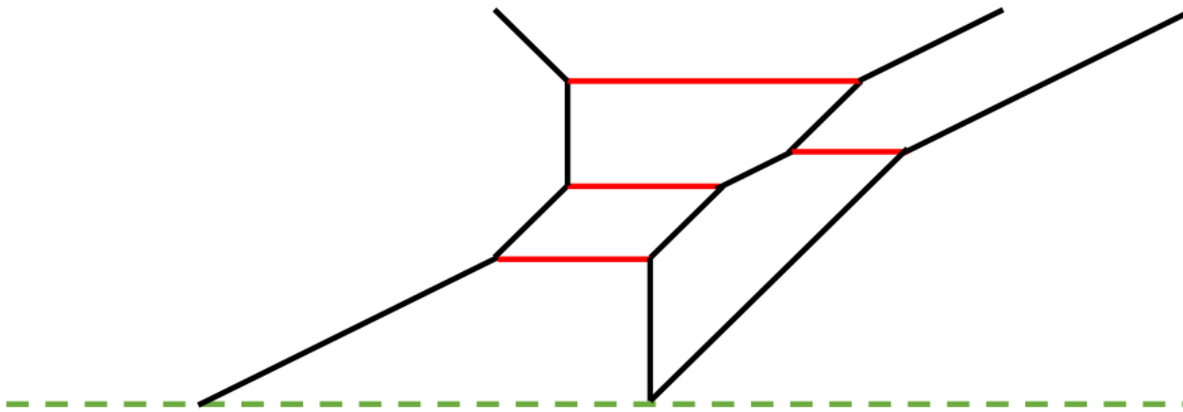


G2 = Higgsing of SO(7) with spinor

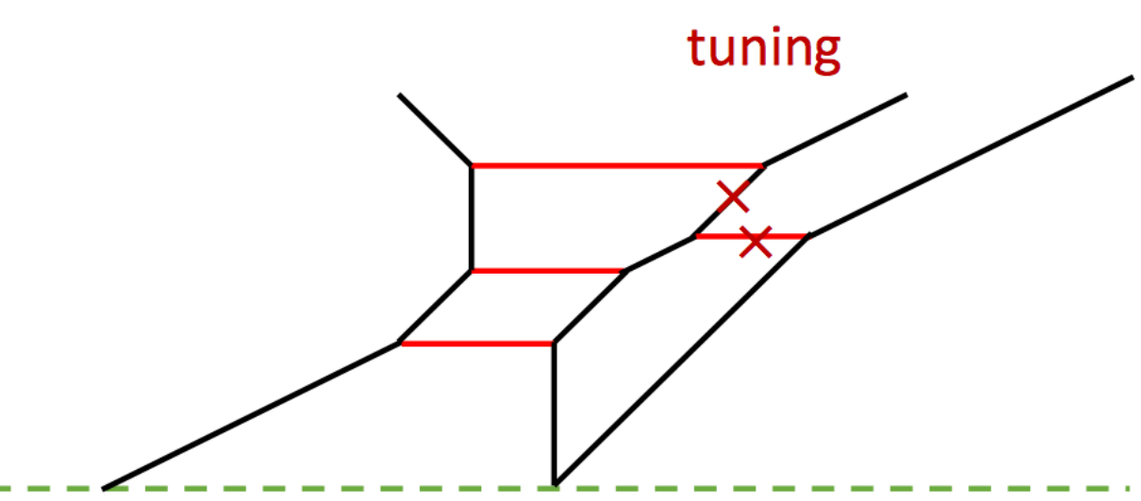




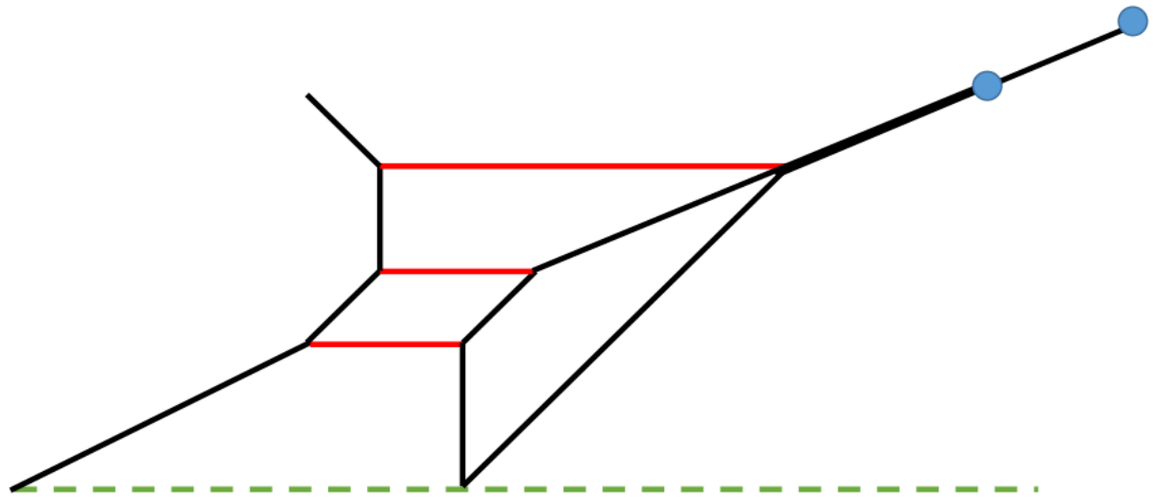


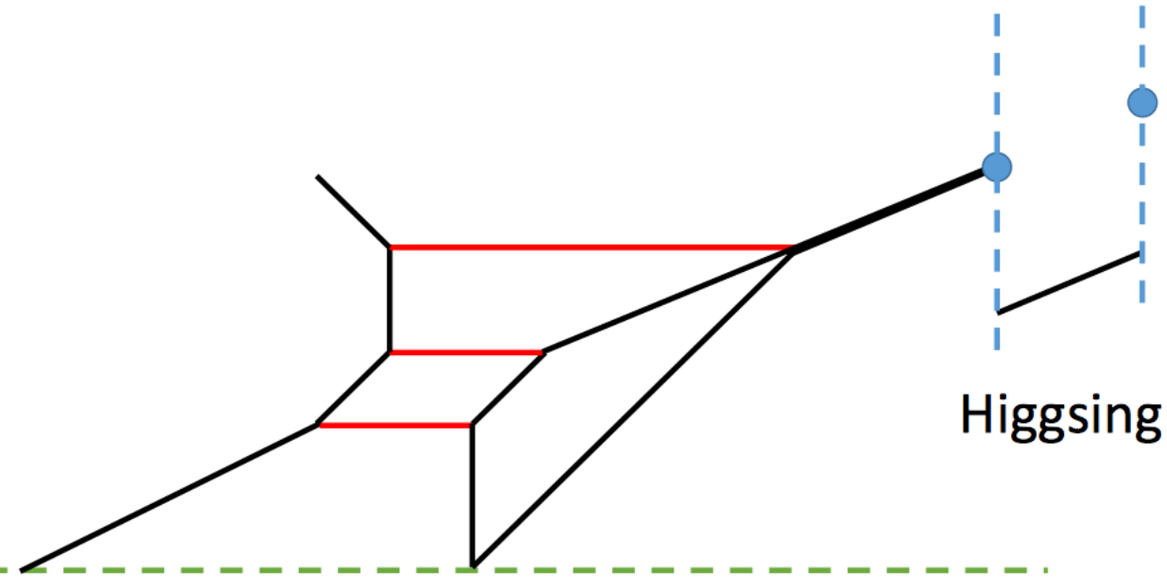


G2 = Higgsing of SO(7) with spinor

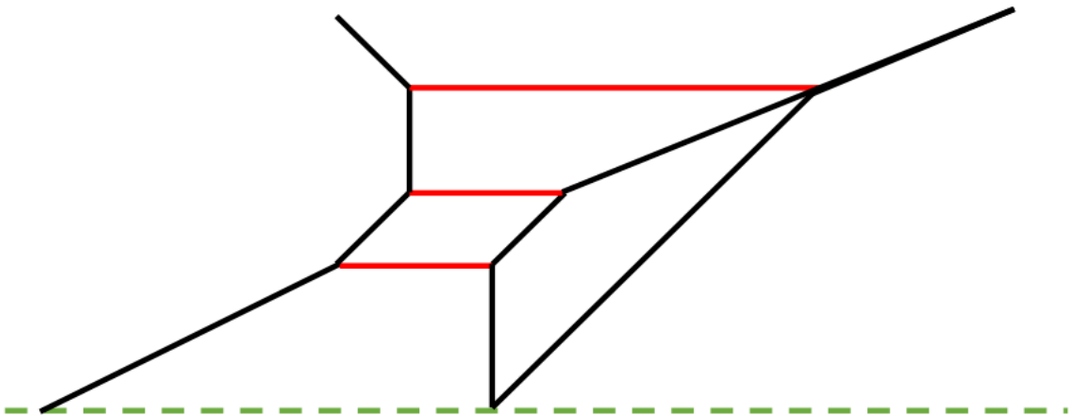


5d SO(7) with a massless spinor





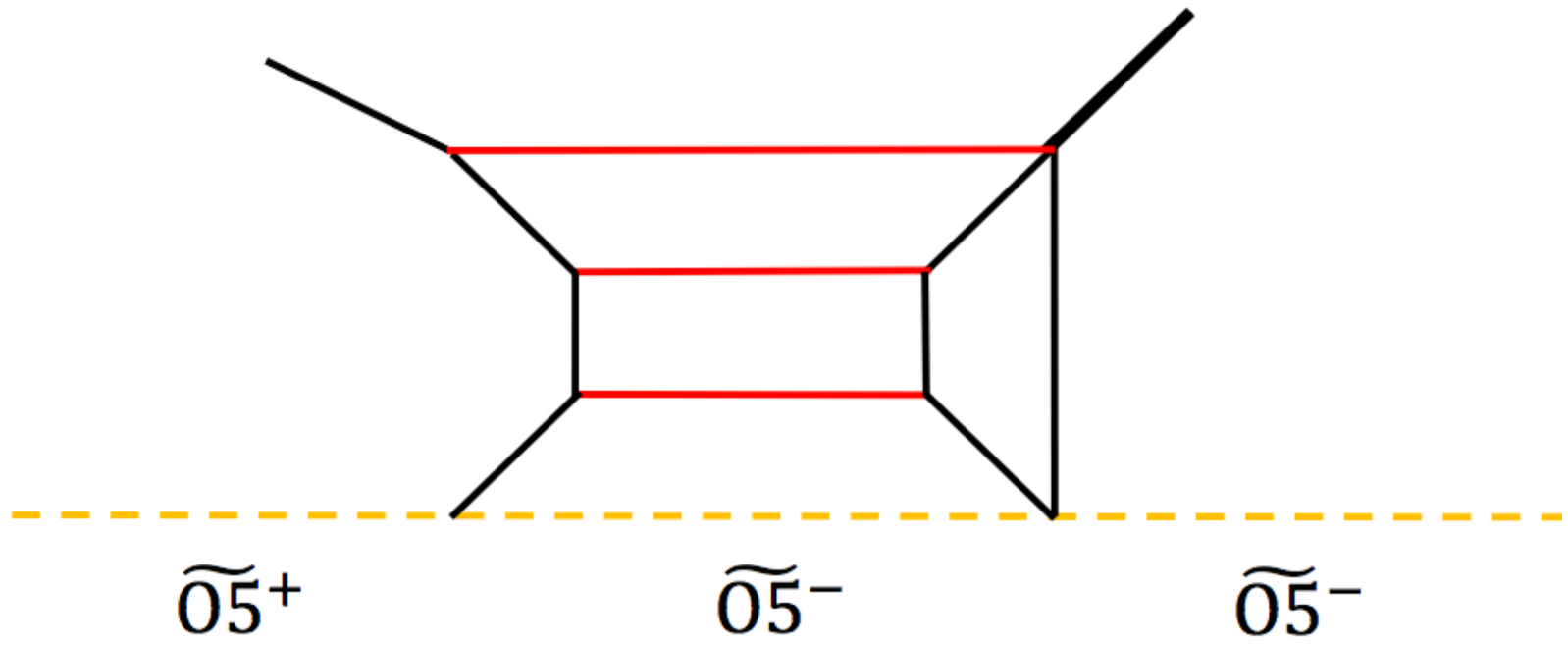
5d G_2 without matter



$\widetilde{O5}^+$ $\widetilde{O5}^-$ $\widetilde{O5}^-$

Pure G_2 gauge theory

After an SL(2,Z)



This 5-brane web reproduces the correct monopole string tension,
consistent with the prepotential

• A 5-brane web configuration for the 5d pure G_2

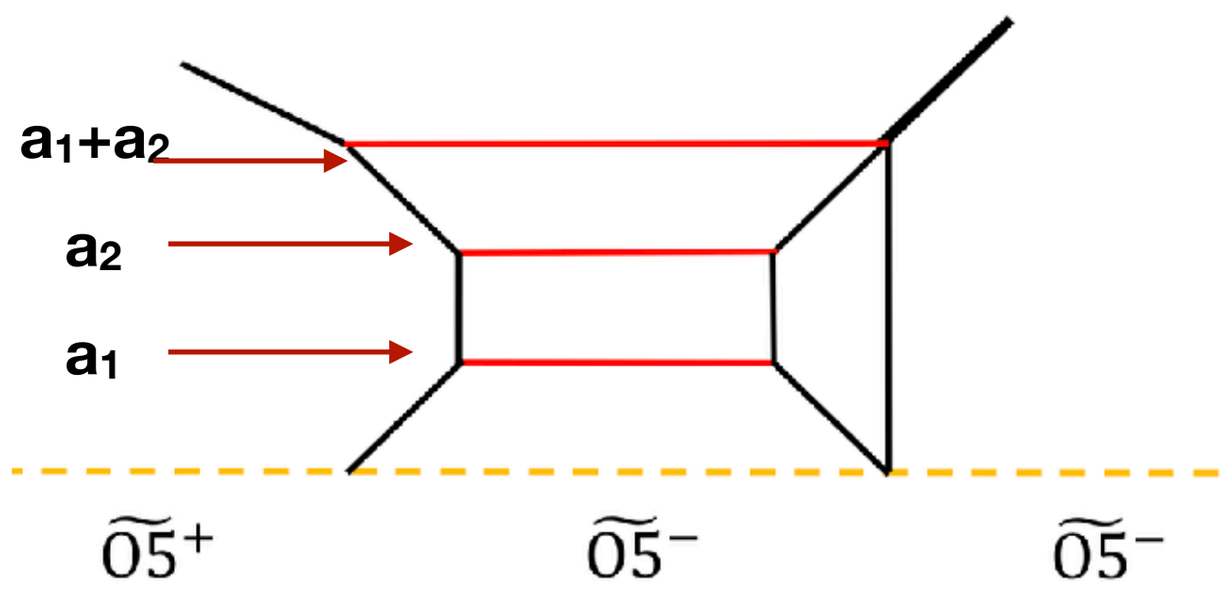
- G_2 +N_F Fundamental hyper
- Prepotential $\mathcal{F} = \frac{m}{2} h_{ij}\phi_i\phi_j + \frac{1}{12}\Big(\sum_{\mathbf{R}} |\mathbf{R}\cdot\phi|^3 - \sum_f \sum_{\mathbf{w}\in\mathbf{W}_f} |\mathbf{w}\cdot\phi|^3\Big)$

$$2\phi_1 - 3\phi_2 = a_2 - a_1,\ -\phi_1 + 2\phi_2 = a_1$$

$$\mathcal{F}_{G_2} = m_0(\phi_1^2 - 3\phi_1\phi_2 + 3\phi_2^2) + \frac{4}{3}\phi_1^3 - 4\phi_1^2\phi_2 + 3\phi_1\phi_2^2 + \frac{4}{3}\phi_2^3$$

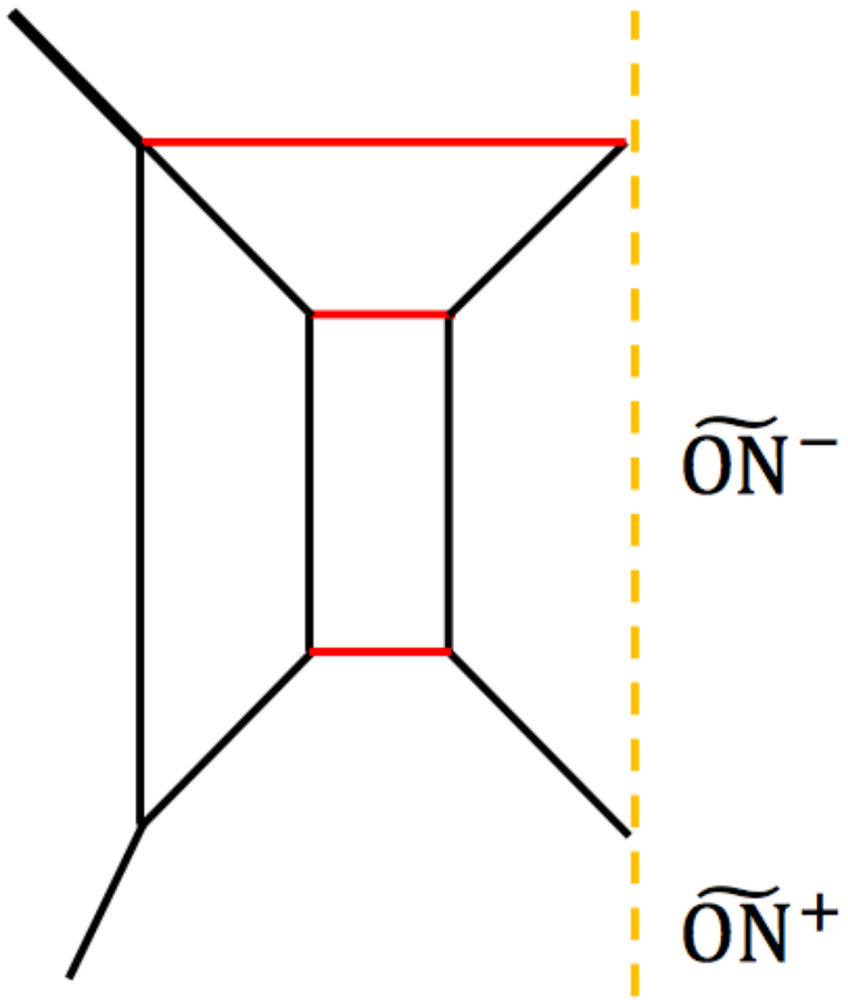
- Monopole tension

$$T_1 = \partial\mathcal{F}/\partial\phi_1,\ T_2 = \partial\mathcal{F}/\partial\phi_2$$



Duality

S-dual to the 5-brane web for the pure G_2 gauge theory:



3 color D5 branes

SU(3) gauge theory

One may confirm the Chern-Simons level by computing the effective prepotential of the 5d theory

Or, we can compare the tension of a monopole string tension by a derivative of the effective prepotential

In 5-brane web, the monopole string tension corresponds to the area of a face.

We confirmed that this SU(3) gauge theory has $\kappa = 7$

$SU(3)_7$ gauge theory

- SU(N)κ +N_F Fundamental hyper

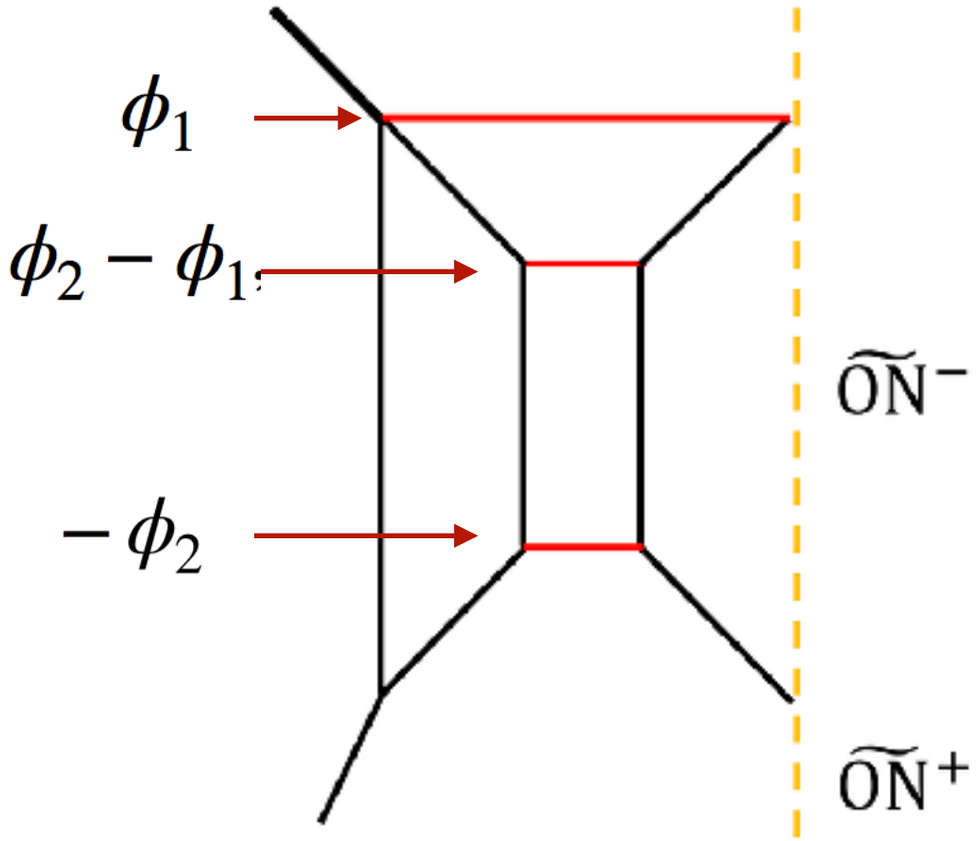
- Prepotential $$a_1 \geq a_2 \geq \cdots \geq a_N, \ \sum_{i=1}^{N} a_i = 0$$

$$\mathcal{F}_{SU(N)} = \sum_{i=1}^{N} \left(\frac{m}{2} a_i^2 + \frac{\kappa}{6} a_i^3 \right) + \frac{1}{6} \sum_{i<j}^{N} |a_i - a_j|^3 - \frac{1}{12} \sum_{F}^{N_F} \sum_{i}^{N} |a_i - m_F|^3$$

$$\mathcal{F}_{SU(3)_7} = m_0(\phi_1^2 - \phi_1\phi_2 + \phi_2^2) + \frac{4}{3}\phi_1^3 + 3\phi_1^2\phi_2 - 4\phi_1\phi_2^2 + \frac{4}{3}\phi_2^3$$

- Monopole tension $$a_1 = \phi_1, a_2 = \phi_2 - \phi_1, a_3 = \phi_3 - \phi_2, \cdots, a_N = -\phi_{N-1}$$

$$T_i = \partial\mathcal{F}/\partial\phi_i, \ i = 1,2,\cdots,N-1$$



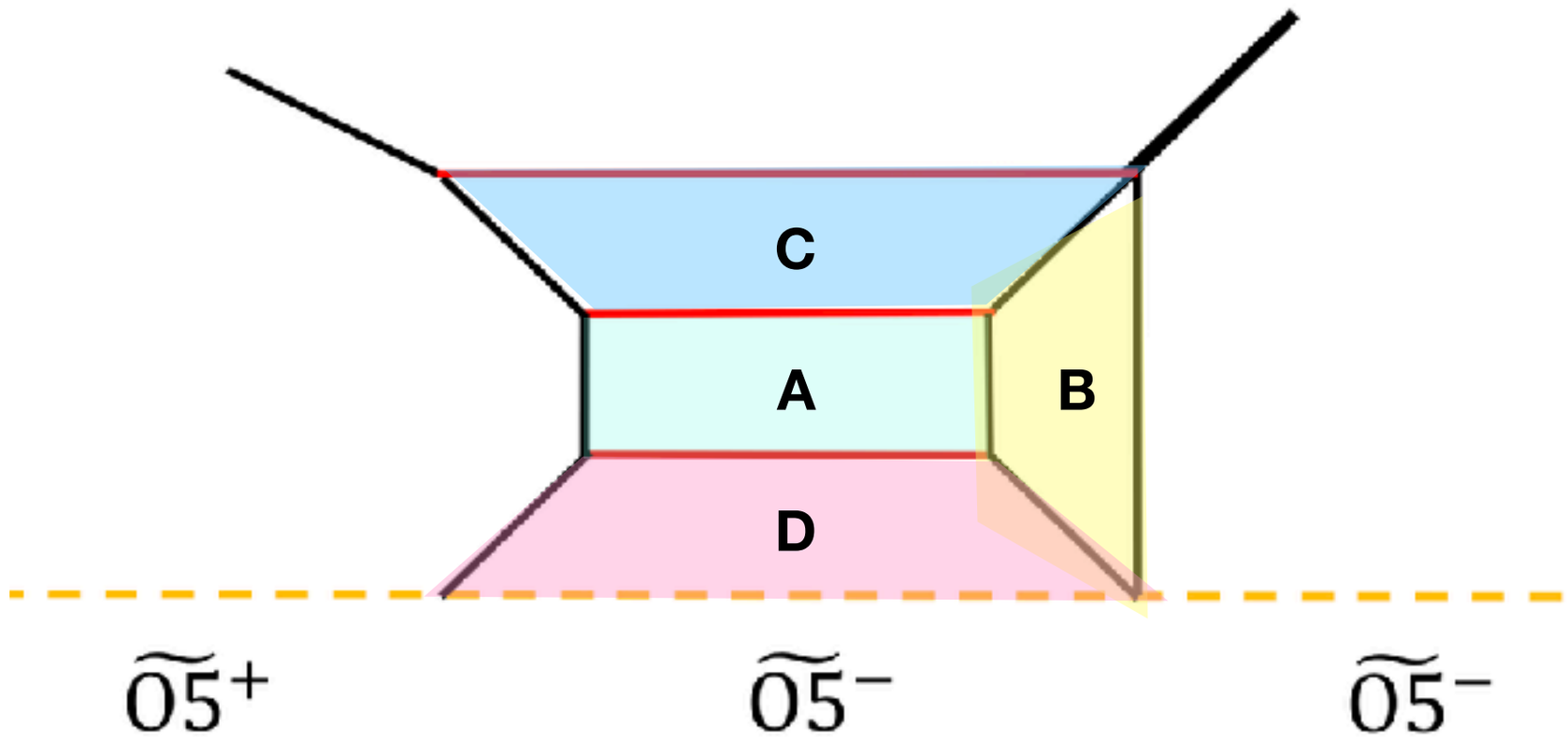
- Duality between G_2 & $SU(3)_7$

Parameter Map

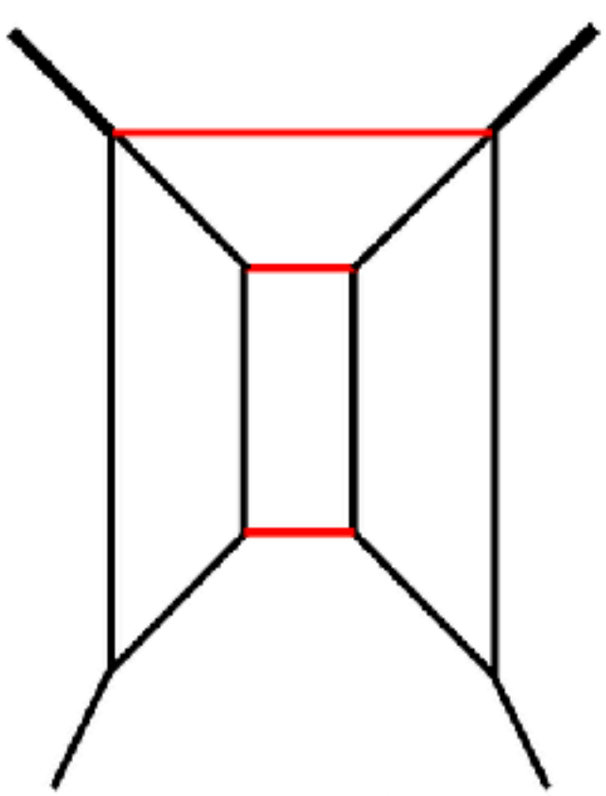
$$m_0^{SU(3)} = -\frac{m_0^{G_2}}{3},$$
$$\phi_1^{SU(3)} = \phi_2^{G_2} + \frac{1}{3} m_0^{G_2},$$
$$\phi_2^{SU(3)} = \phi_1^{G_2} + \frac{2}{3} m_0^{G_2},$$

Monopole tension

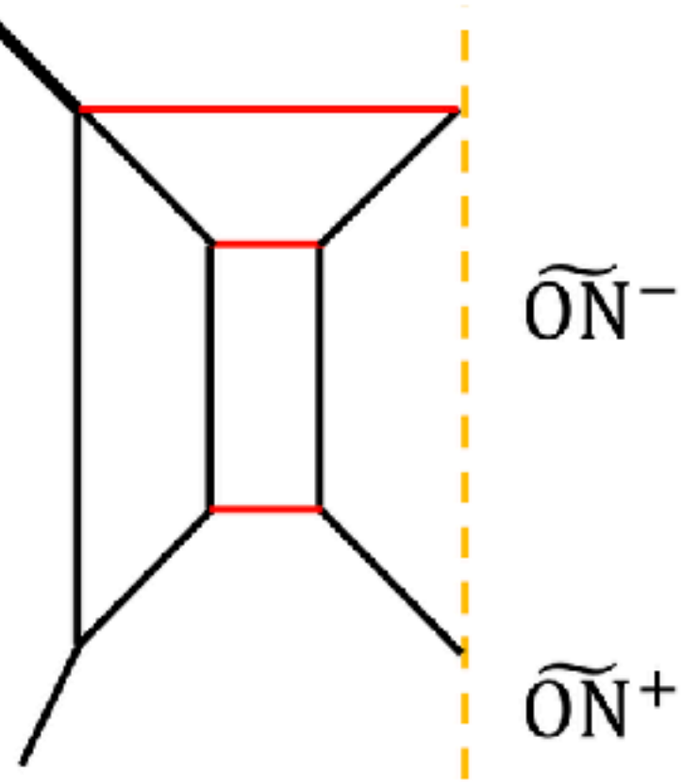
$$T_1^{SU(3)_7} = T_2^{G_2} = \mathbf{B} + \mathbf{C} + \mathbf{2D}$$
$$T_2^{SU(3)_7} = T_1^{G_2} = \mathbf{A}$$



- In fact, we can further increase the Chern-Simons levels.

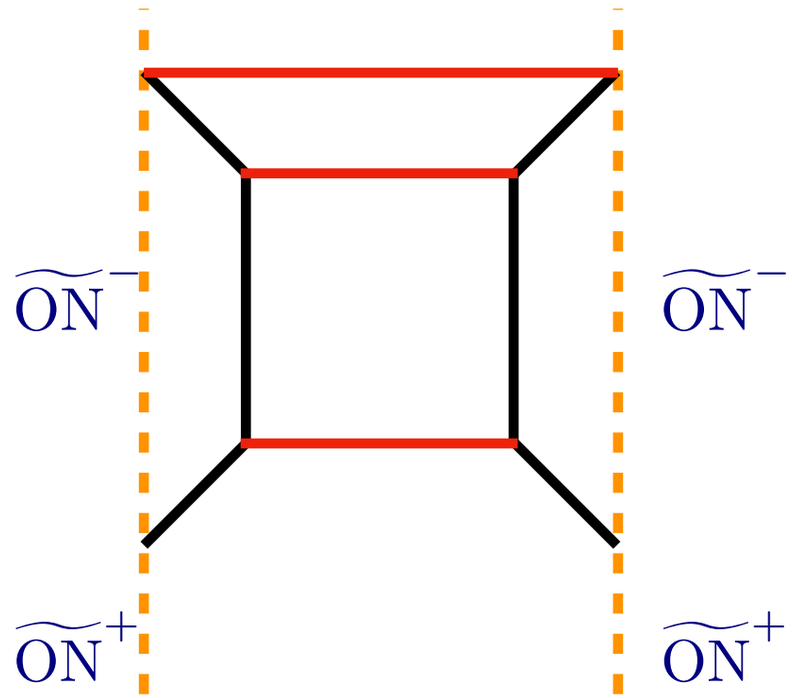


$\kappa = 5$



$\kappa = 7$

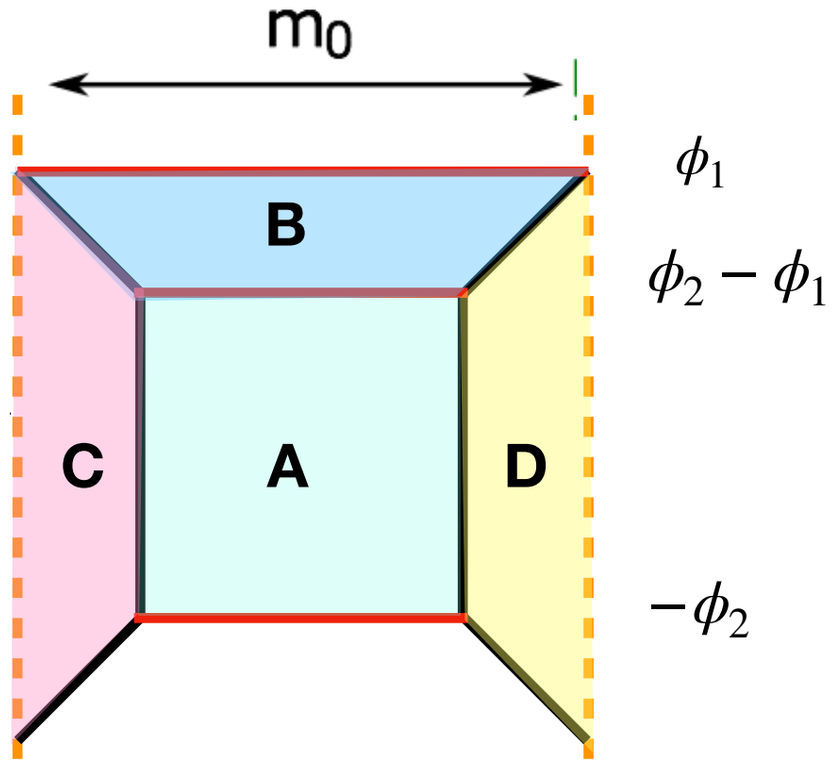
- A 5-brane web diagram for SU(3) with Chern-Simons level κ=9



- The $SU(3)_9$ theory has been found and constructed from geometry.

Jefferson, Kim, Vafa, Zafrir 17
Jefferson, Katz, Kim, Vafa 18

- Check: compare monopole tension from the prepotential and the 5-brane webs.
  - Two independent D3 brane faces
  - A, B + 2C+ 2D



$$\mathcal{F}_{SU(3)_9} = m_0\left(\phi_1^2 - \phi_1\phi_2 + \phi_2^2\right) + \frac{4}{3}\phi_1^3 + 4\phi_1^2\phi_2 - 5\phi_1\phi_2^2 + \frac{4}{3}\phi_2^3,$$

$$\frac{\partial \mathcal{F}_{SU(3)_9}}{\partial \phi_1} = (2\phi_1 - \phi_2)(m_0 + 2\phi_1 + 5\phi_2) \qquad = \mathbf{A}$$

$$\frac{\partial \mathcal{F}_{SU(3)_9}}{\partial \phi_2} = (-\phi_1 + 2\phi_2)(m_0 - 4\phi_1 + 2\phi_2) \qquad = \mathbf{B + 2C + 2D}$$

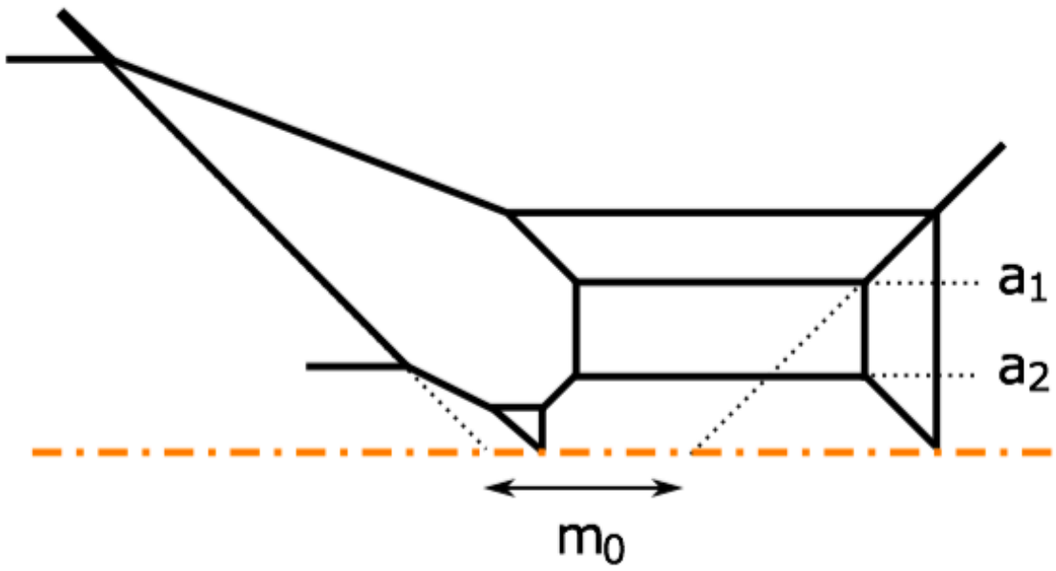
G_2 -SU(3) -Sp(2) sequences

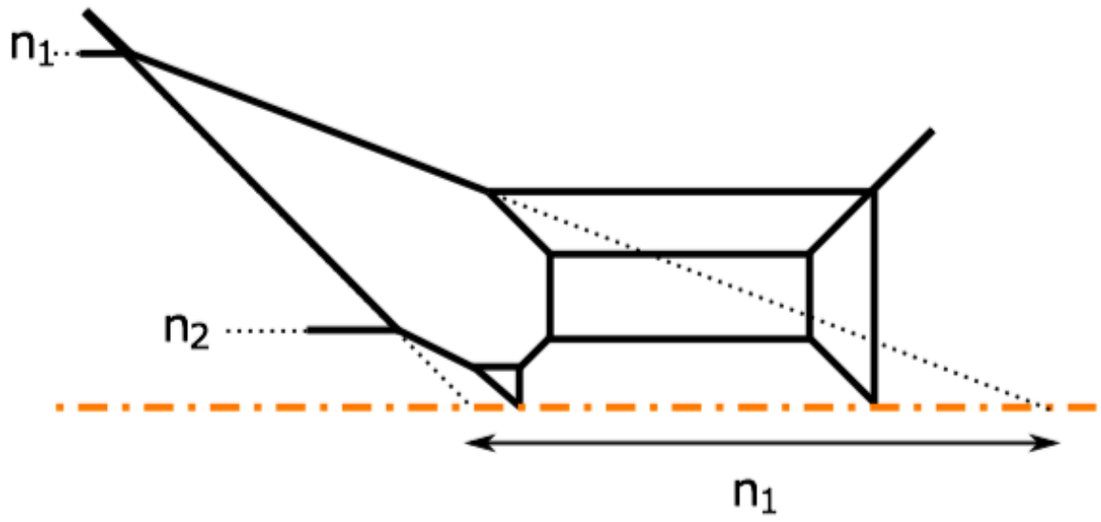
G_2 + 2F, SU(3)+ 2F, Sp(2)+2AS

⋮

G_2 + 6F, SU(3)+ 6F, Sp(2)+2AS+4F

- G_2 + 2F
  - SO(7)+ 3S → G_2+ 2F+ 2 singlets (after Higgsing with a spinor)

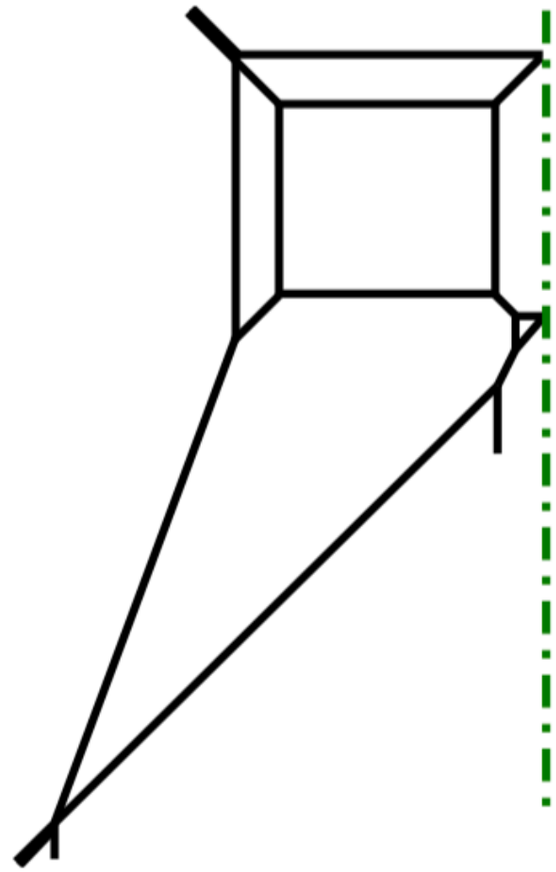




Area and the monopole string tension agree.

- $G_2 + 2F$

Again, S-dual diagram has 3 color D5-diagram, suggesting an SU(3)



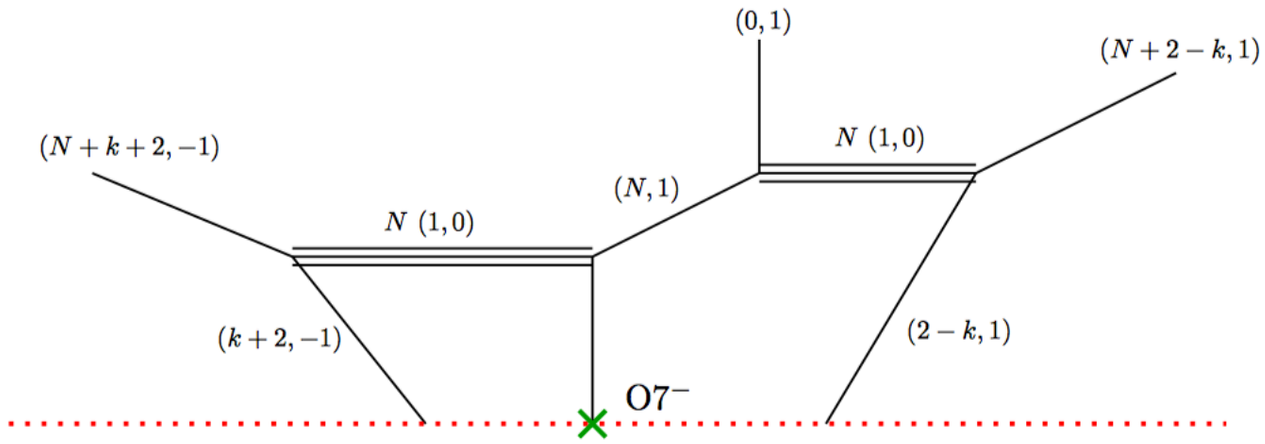
Flavors: external D5 (or D7)

- $SU(3)_6$+ 2F

SU(3)+ 2F = SU(3) + 2AS

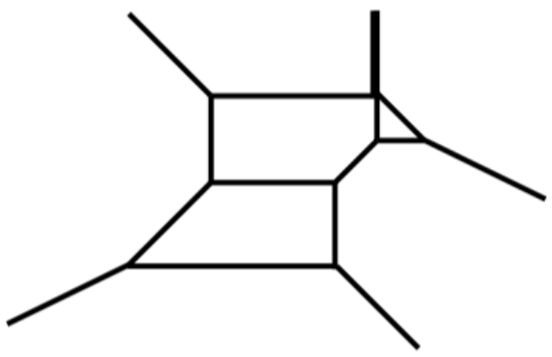
$$\underset{3}{\square} = \underset{\bar{3}}{\begin{array}{|c|}\hline\\\hline\\\hline\end{array}}$$

Antisymmetric matter of SU(3) = $O7^-$ with NS5

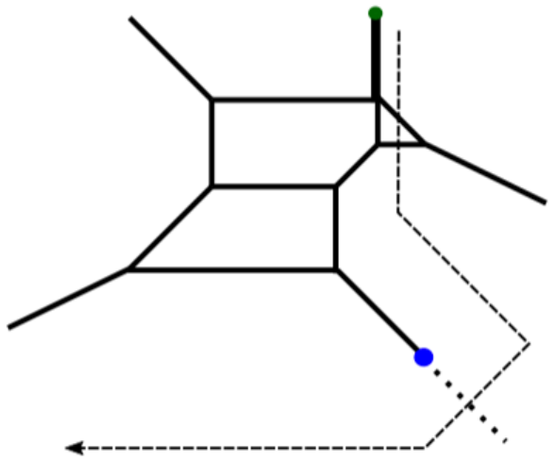


A new way of obtaining AS of SU(3)

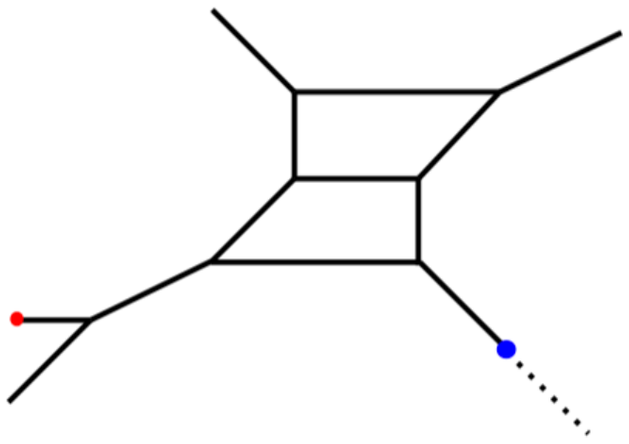
Antisymmetric matter of SU(3) = Higgsing of SU(3) - SU(2)



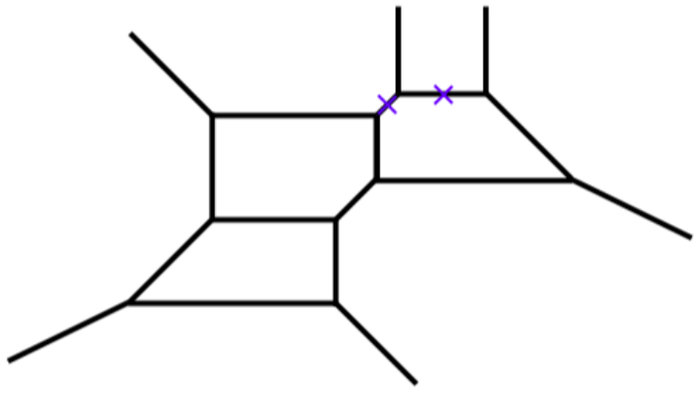
(a)



(b)



(c)



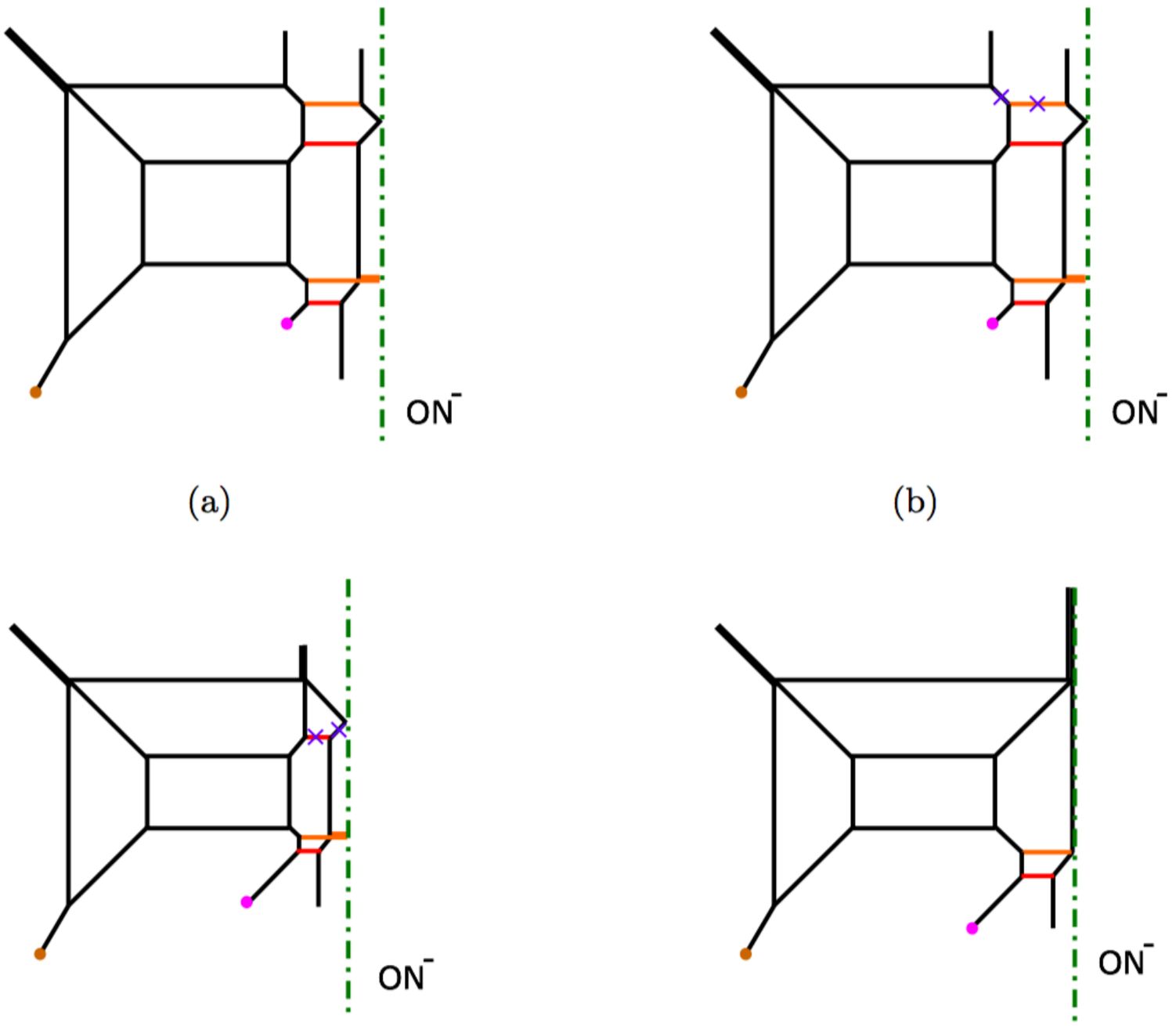
(d)

$SU(3)_{-1} \times SU(2)$ Higgsed to $SU(3)_{1/2} + 1AS = SU(3)_{1/2} + 1F$

- $SU(3)_6$+ 2F

- SU(2)x$SU(3)_3$xSU(2) Higgsed to $SU(3)_6$+ 2AS

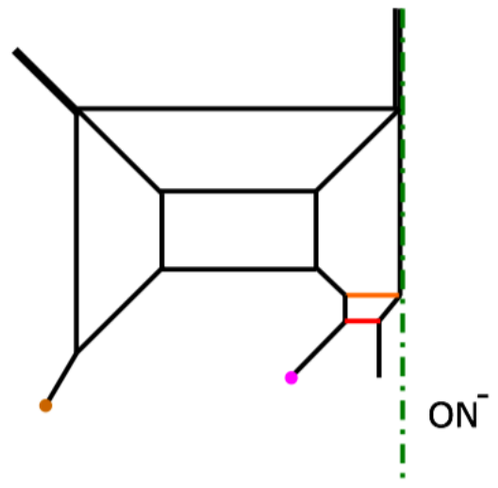
[99 Hanany-Zaffaroni]



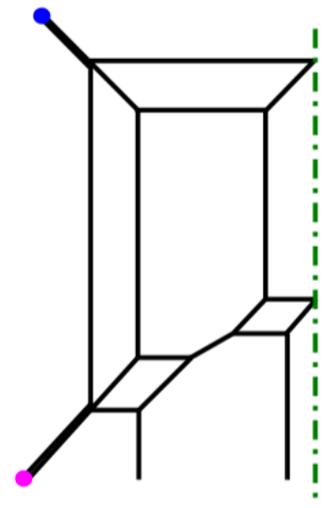
(a) (b)

(c) (d)

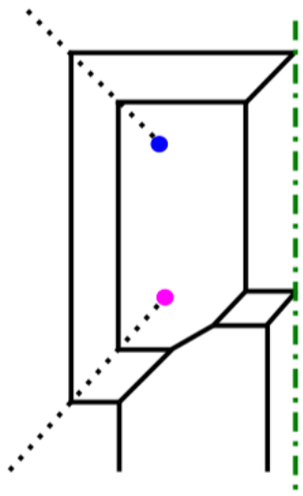
- $Sp(2)_\pi + 2AS$



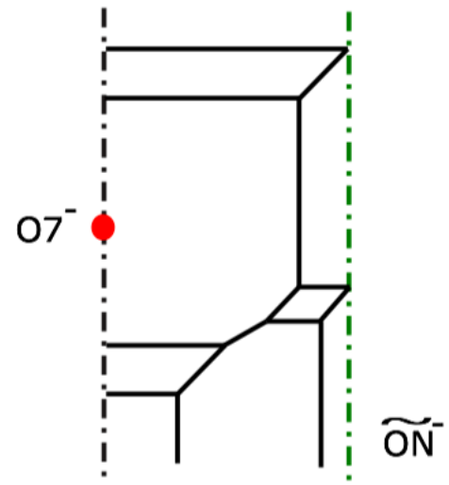
$SU(3)_6 + 2F$



Deform with flop transitions



Pulling in two 7-branes



[1,-1] and [1,1] 7-branes to O7⁻-plane

[96 Sen]

Hence, the 5-brane webs shows the non-trivial duality between G_2-SU(3)-Sp(2) theories

$$\mathbf{G_2 + 2F} \quad \longleftrightarrow \quad \mathbf{SU(3)_6 + 2F} \quad \longleftrightarrow \quad \mathbf{Sp(2)_\pi + 2AS}$$

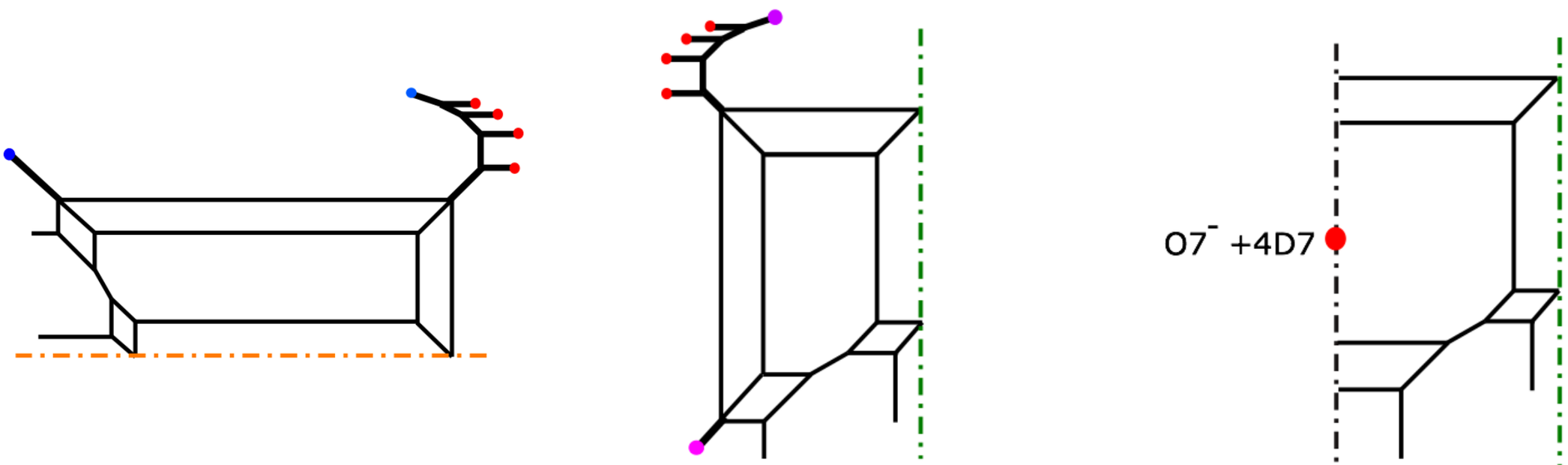
The **duality maps** can be read off from the 5-brane webs:

$$
\begin{aligned}
m_0^{SU(3)} &= \frac{m_{\mathbf{F},1}^{G_2} + m_{\mathbf{F},2}^{G_2}}{2}, \\
m_{\mathbf{AS},1}^{SU(3)} &= \frac{1}{3}\left(-m_0^{G_2} + m_{\mathbf{F},1}^{G_2} - 2m_{\mathbf{F},2}^{G_2}\right), \\
m_{\mathbf{AS},2}^{SU(3)} &= \frac{1}{3}\left(-m_0^{G_2} - 2m_{\mathbf{F},1}^{G_2} + m_{\mathbf{F},2}^{G_2}\right), \\
\phi_1^{SU(3)} &= \phi_2^{G_2} + \frac{1}{3}\left(m_0^{G_2} - m_{\mathbf{F},1}^{G_2} - m_{\mathbf{F},2}^{G_2}\right), \\
\phi_2^{SU(3)} &= \phi_1^{G_2} + \frac{1}{3}\left(2m_0^{G_2} - 2m_{\mathbf{F},1}^{G_2} - 2m_{\mathbf{F},2}^{G_2}\right),
\end{aligned}
$$

$$
\begin{aligned}
m_0^{Sp(2)} &= -\frac{m_0^{G_2}}{2}, \\
m_{\mathbf{AS},1}^{Sp(2)} &= m_{\mathbf{F},1}^{G_2}, \\
m_{\mathbf{AS},2}^{Sp(2)} &= m_{\mathbf{F},2}^{G_2}, \\
\phi_1^{Sp(2)} &= \phi_2^{G_2} + \frac{1}{2}m_0^{G_2}, \\
\phi_2^{Sp(2)} &= \phi_1^{G_2} + m_0^{G_2}.
\end{aligned}
$$

Adding 4 more Fundamental hypers yields **marginal** theories of G_2-SU(3)-Sp(2) sequences

G_2 + 6F ⟷ **$SU(3)_4$ + 6F** ⟷ **Sp(2) + 2AS+ 4F**



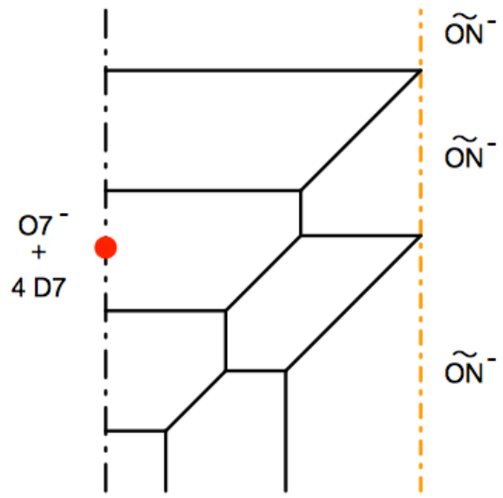
Sp(2)+ 1AS + 8 F

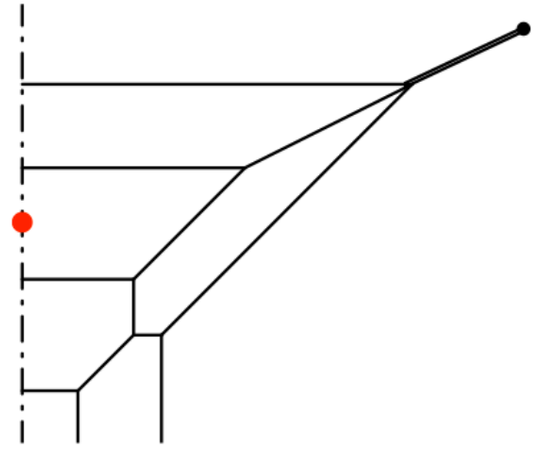
and

$SU(3)_{3/2}$ + 9 F

From Sp(2)+ 2AS + 4F, we decouple 1 AS to get

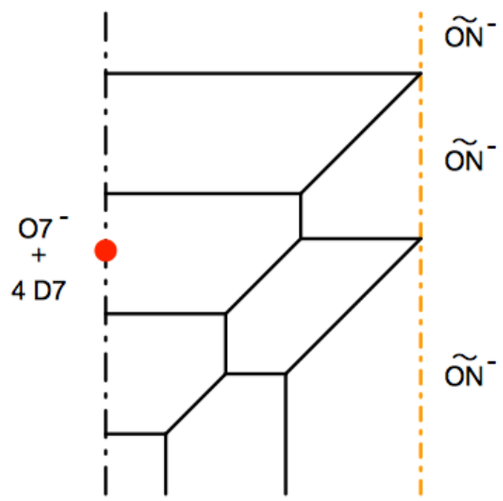
Sp(2)+ 1AS + 4F :

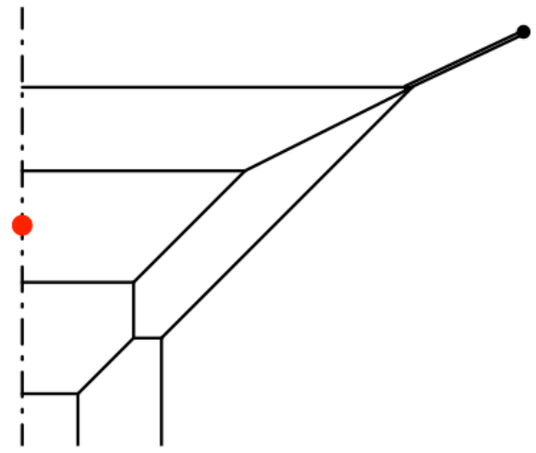




From Sp(2)+ 2AS + 4F, we decouple 1 AS to get

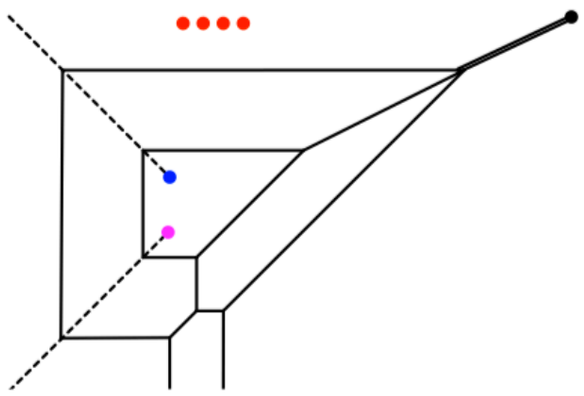
Sp(2)+ 1AS + 4F :



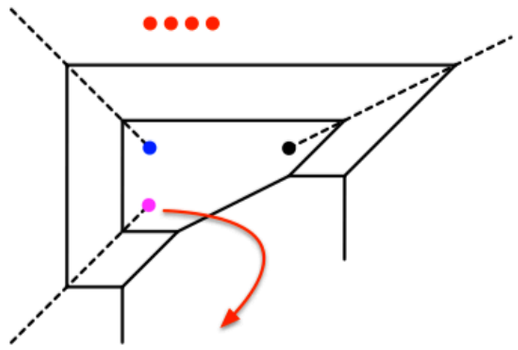


Then resolving O7-plane and moving 7-branes gives

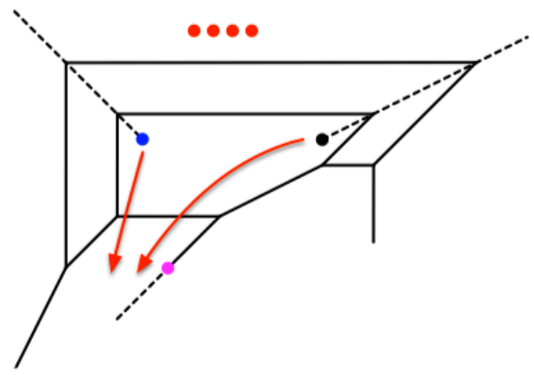
$SU(3)_{7/2}$+5F :



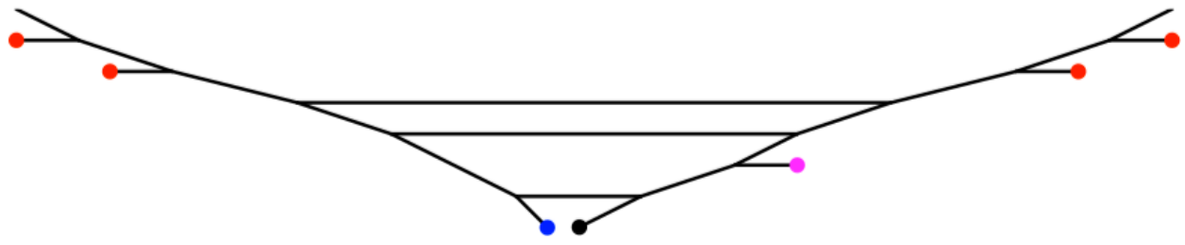
(a)



(b)

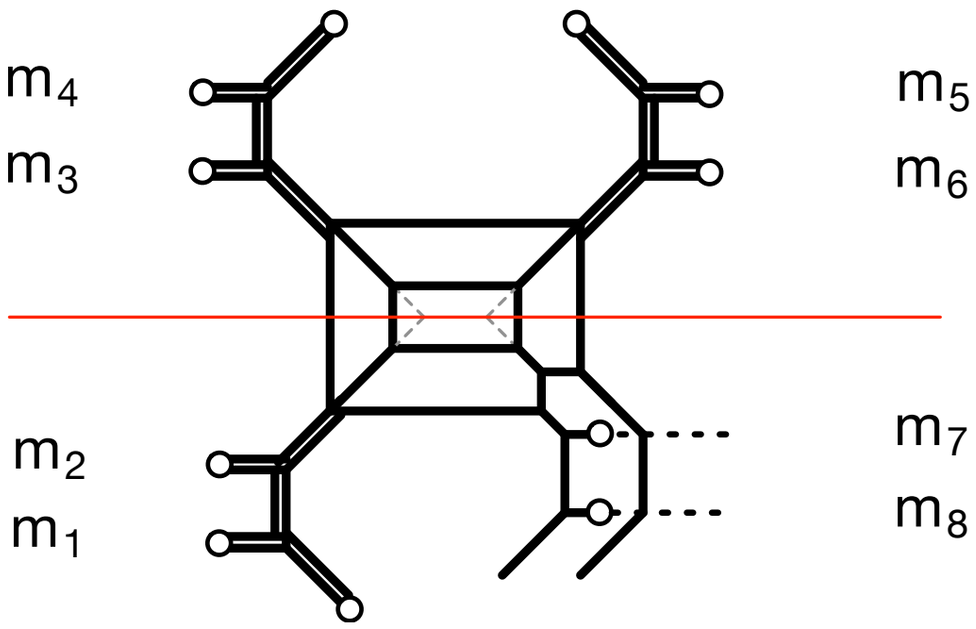


(c)

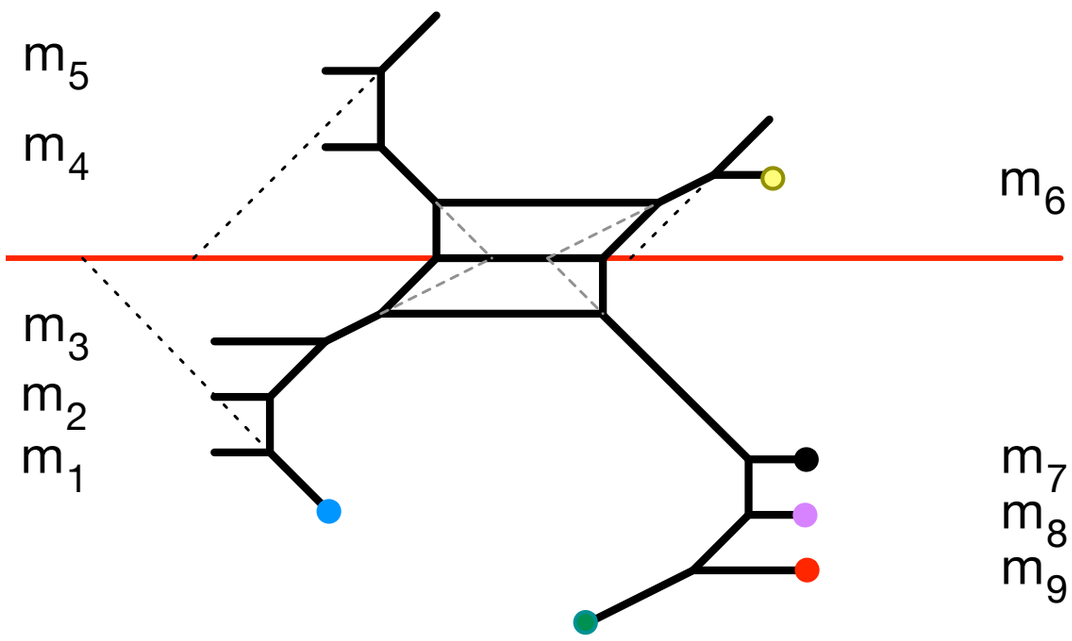


Adding 4 more Fundamental hypers gives

Sp(2)+ 1AS + 8F

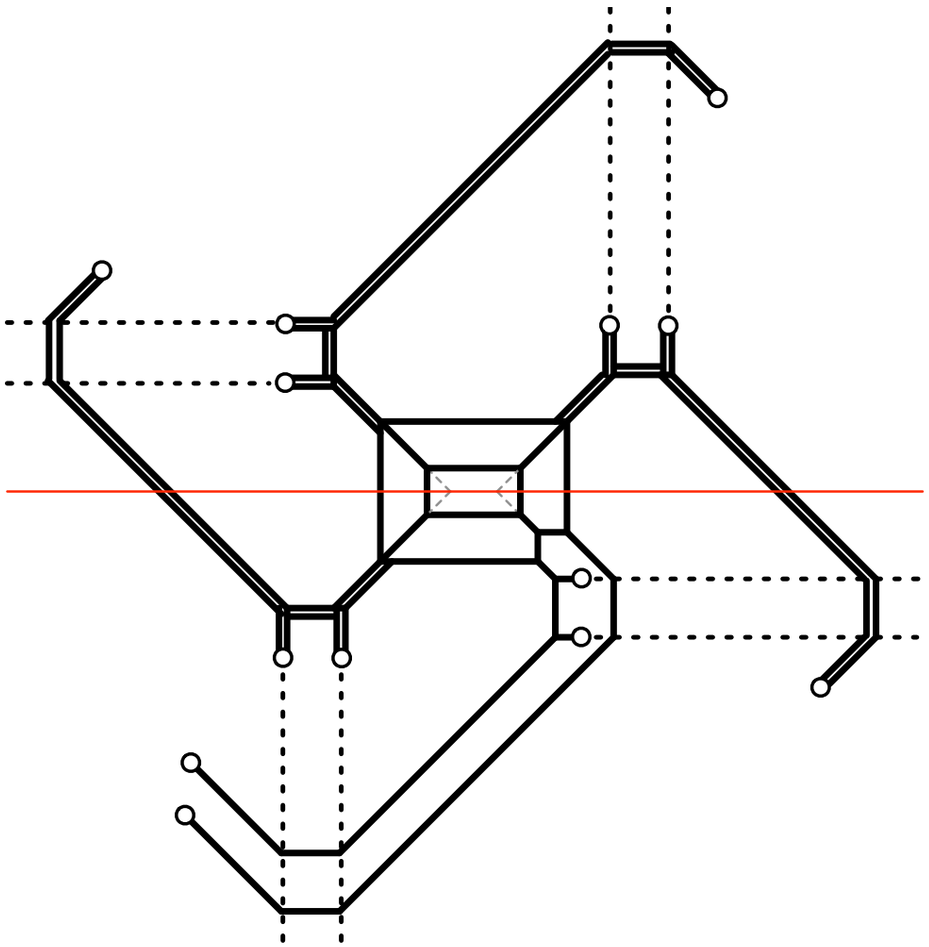


$SU(3)_{3/2}$ + 9F

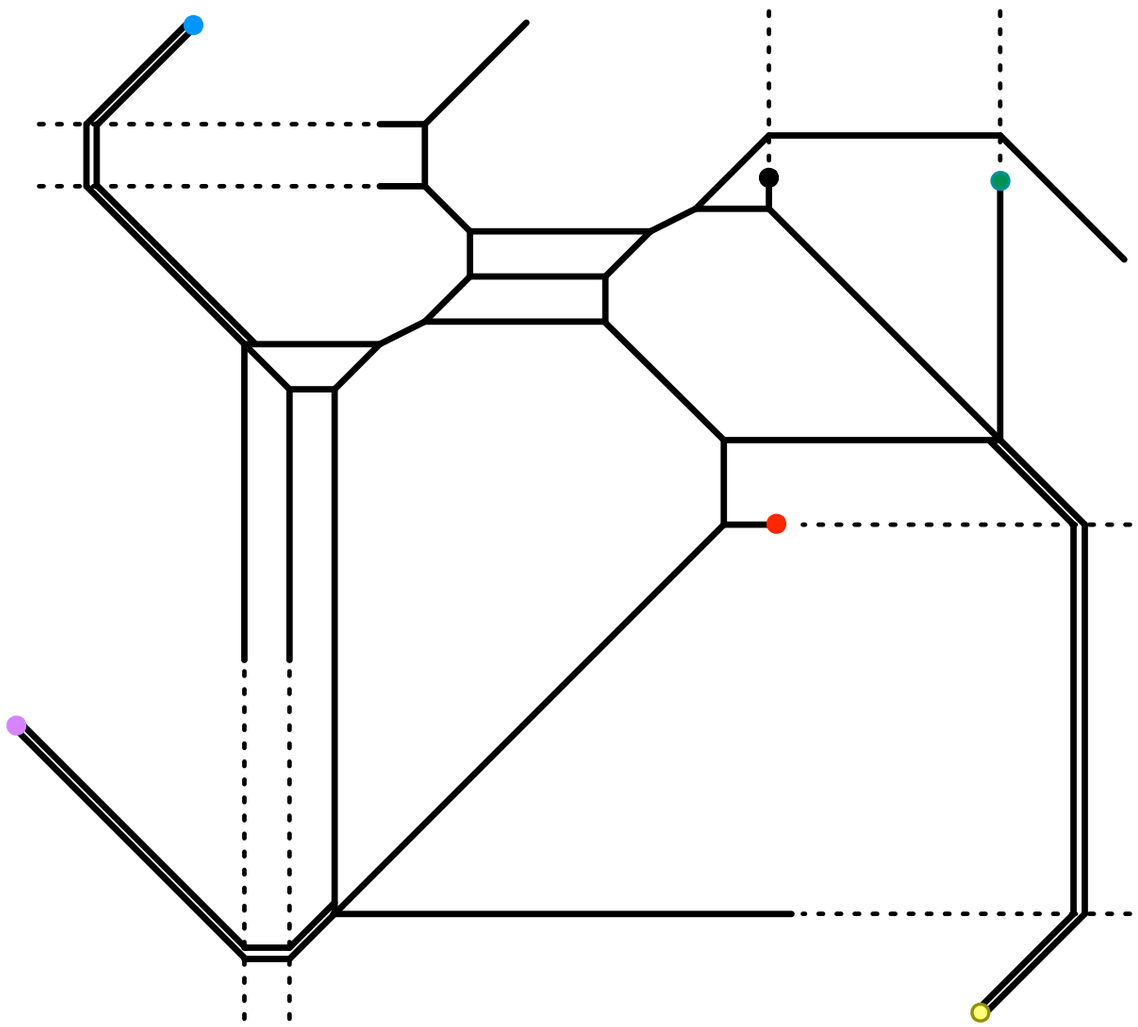


These marginal theories have a periodic spiral structure.
In fact, their UV completion is 6d **rank 2 E-string theory**

Sp(2)+ 1AS + 8F

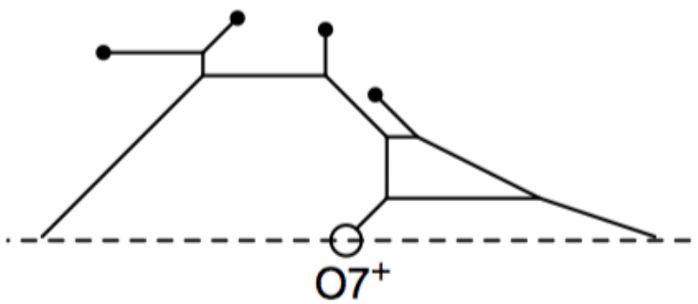


$SU(3)_{3/2}$ + 9F

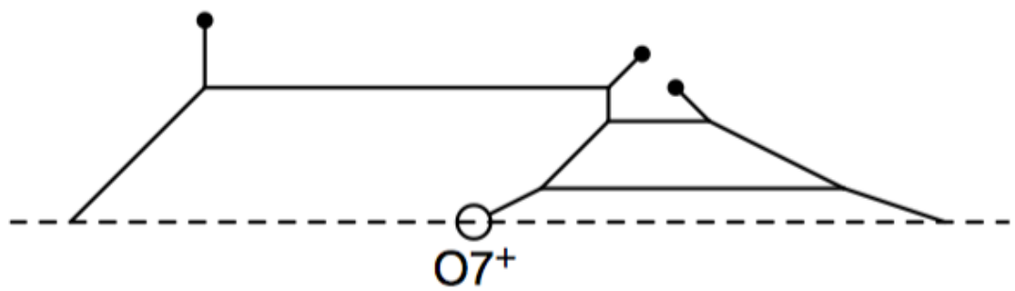


SU(3) + 1 Sym

Two marginal theories of SU(3)+1Sym : $O7^+$

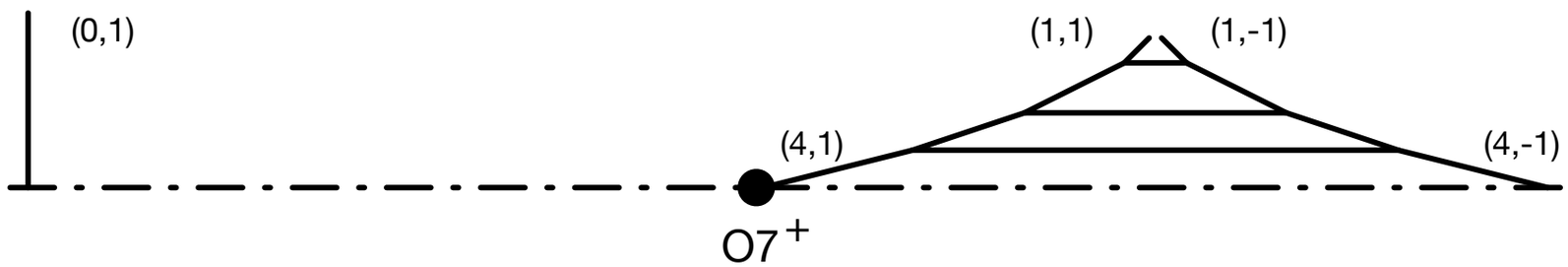


$$SU(3)_0 + 1\mathbf{Sym} + 1\mathbf{F}$$

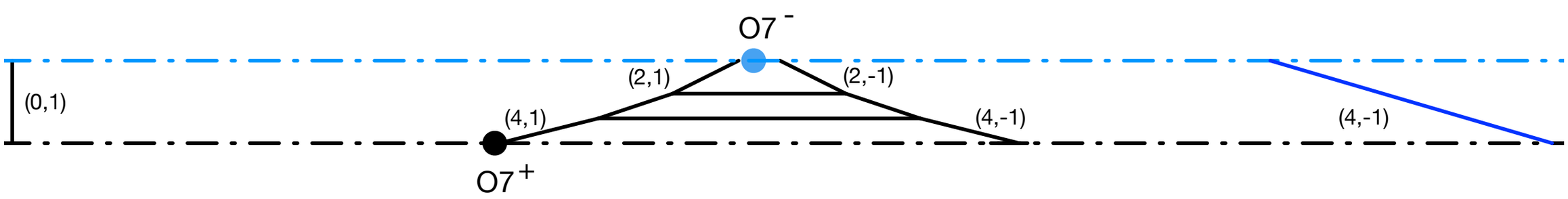


$$SU(3)_{\frac{3}{2}} + 1\mathbf{Sym}$$

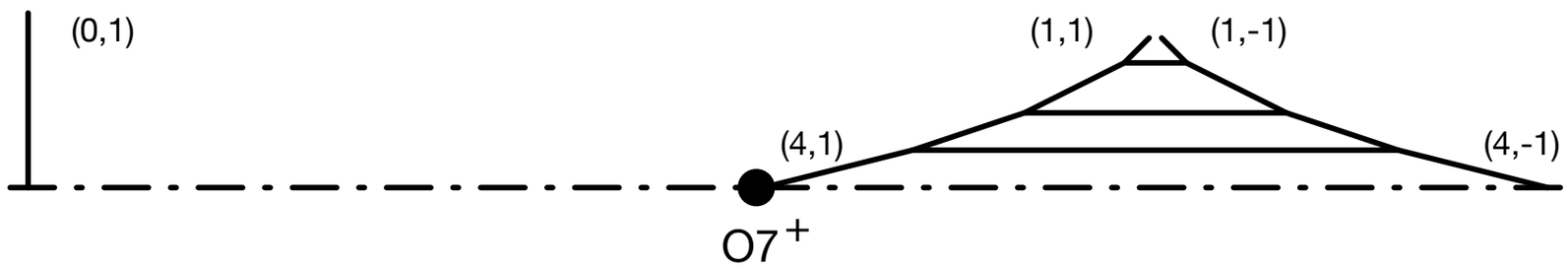
In particular, $SU(3)_{\frac{3}{2}} + 1\mathbf{Sym}$



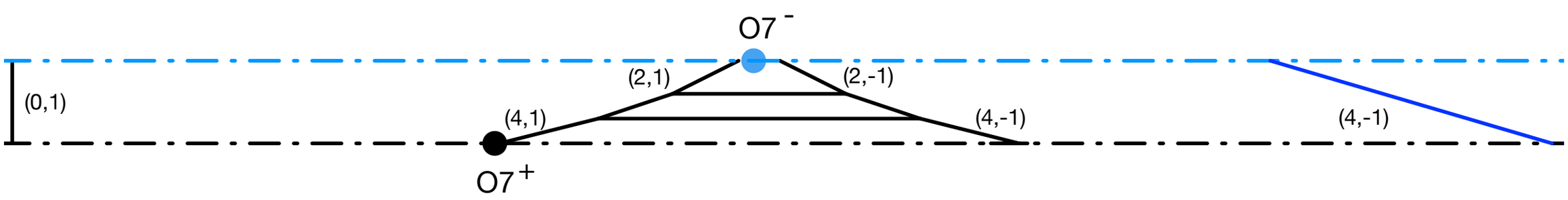
Making O7- -plane from two 7-branes:



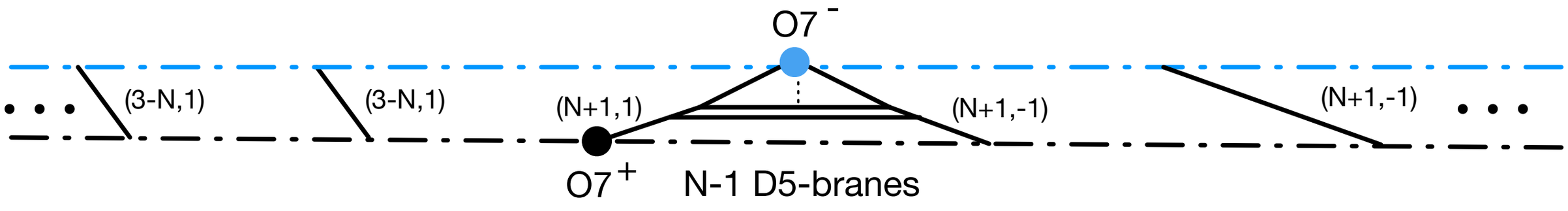
In particular, $SU(3)_{\frac{3}{2}} + 1\mathbf{Sym}$



Making O7- -plane from two 7-branes:



A **new periodic structure** appears, and con be generalized to rank N



Conclusion

- Obtained 5-brane webs for rank 2 SCFTs (including G2)
- Checked various dualities from 5-brane web
- Read off Duality map between different theories in the Coulomb phase.

— —

- Partition functions and duality check?
- Higher rank SCFTs
- Higgs branch?
- Down to 4d theories

Fig. 137
$SU(3)_9$

Fig. 88
$SU(3)_0$+10**F**
$Sp(2)$+10**F**
[4]-$SU(2)$-$SU(2)$-[4]

Fig. 89
$SU(3)_{\frac{1}{2}}$+9**F**
$Sp(2)$+9**F**
[3]-$SU(2)$-$SU(2)$-[4]

Fig. 78
$SU(3)_{\frac{3}{2}}$+9**F**
$Sp(2)$+1**AS**+8**F**
[2]-$SU(2)$-$SU(2)$-[5]

Fig. 99
$SU(3)_0$+8**F**
[3]-$SU(2)$-$SU(2)$-[3]

Fig. 90
$SU(3)_1$+8**F**
$Sp(2)$+8**F**
[2]-$SU(2)$-$SU(2)$-[4]

Fig. 79
$SU(3)_2$+8**F**
$Sp(2)$+1**AS**+7**F**
[1]-$SU(2)$-$SU(2)$-[5]

Fig. 125
$SU(2)_0$-$SU(2)$-[5]

Fig. 100
$SU(3)_{\frac{1}{2}}$+7**F**
[2]-$SU(2)$-$SU(2)$-[3]

Fig. 91
$SU(3)_{\frac{3}{2}}$+7**F**
$Sp(2)$+7**F**
[1]-$SU(2)$-$SU(2)$-[4]

Fig. 80
$SU(3)_{\frac{5}{2}}$+7**F**
$Sp(2)$+1**AS**+6**F**
$SU(2)_\pi$-$SU(2)$-[5]

Fig. 126
[4]-$SU(2)$-$SU(2)_0$

Fig. 108
$SU(3)_0$+6**F**
[2]-$SU(2)$-$SU(2)$-[2]

Fig. 101
$SU(3)_1$+6**F**
[1]-$SU(2)$-$SU(2)$-[3]

Fig. 92
$SU(3)_2$+6**F**
$Sp(2)$+6**F**
$SU(2)_\pi$-$SU(2)$-[4]

Fig. 81
$SU(3)_3$+6**F**
$Sp(2)$+1**AS**+5**F**

Fig. 66
$SU(3)_4$+6**F**
$Sp(2)$+2**AS**+4**F**
G_2+6**F**

Fig. 127
[3]-$SU(2)$-$SU(2)_0$

Fig. 109
$SU(3)_{\frac{1}{2}}$+5**F**
[1]-$SU(2)$-$SU(2)$-[2]

Fig. 102
$SU(3)_{\frac{3}{2}}$+5**F**
$SU(2)_\pi$-$SU(2)$-[3]

Fig. 93
$SU(3)_{\frac{5}{2}}$+5**F**
$Sp(2)$+5**F**

Fig. 82
$SU(3)_{\frac{7}{2}}$+5**F**
$Sp(2)$+1**AS**+4**F**

Fig. 67
$SU(3)_{\frac{9}{2}}$+5**F**
$Sp(2)$+2**AS**+3**F**
G_2+5**F**

Fig. 128
[2]-$SU(2)$-$SU(2)_0$

Fig. 115
$SU(3)_0$+4**F**
[1]-$SU(2)$-$SU(2)$-[1]

Fig. 110
$SU(3)_1$+4**F**
$SU(2)_\pi$-$SU(2)$-[2]

Fig. 103
$SU(3)_2$+4**F**

Fig. 94
$SU(3)_3$+4**F**
$Sp(2)$+4**F**

Fig. 83
$SU(3)_4$+4**F**
$Sp(2)$+1**AS**+3**F**

Fig. 68
$SU(3)_5$+4**F**
$Sp(2)$+2**AS**+2**F**
G_2+4**F**

Fig. 129
[1]-$SU(2)$-$SU(2)_0$

Fig. 116
$SU(3)_{\frac{1}{2}}$+3**F**
$SU(2)_\pi$-$SU(2)$-[1]

Fig. 111
$SU(3)_{\frac{3}{2}}$+3**F**

Fig. 104
$SU(3)_{\frac{5}{2}}$+3**F**

Fig. 95
$SU(3)_{\frac{7}{2}}$+3**F**
$Sp(2)$+3**F**

Fig. 84
$SU(3)_{\frac{9}{2}}$+3**F**
$Sp(2)$+1**AS**+2**F**

Fig. 69
$SU(3)_{\frac{11}{2}}$+3**F**
$Sp(2)$+2**AS**+1**F**
G_2+3**F**

Fig. 73
$Sp(2)_0$+3**AS**

Fig. 130
$SU(2)_0$-$SU(2)_0$

Fig. 131
$SU(2)_\pi$-$SU(2)_0$

Fig. 121
$SU(3)_0$ + 2**F**
$SU(2)_\pi$-$SU(2)_\pi$

Fig. 117
$SU(3)_1$+2**F**

Fig. 112
$SU(3)_2$+2**F**

Fig. 105
$SU(3)_3$+2**F**

Fig. 96
$SU(3)_4$+2**F**
$Sp(2)$+2**F**

Fig. 85
$SU(3)_5$+2**F**
$Sp(2)$+1**AS**+1**F**

Fig. 70
$SU(3)_6$+2**F**
$Sp(2)_\pi$+2**AS**
G_2+2**F**

Fig. 74
$Sp(2)_0$+2**AS**

Fig. 134
$SU(3)_0$+1**Sym**+1**F**

Fig. 136
$SU(3)_{\frac{3}{2}}$+1**Sym**

Fig. 135
$SU(3)_{\frac{1}{2}}$+1**Sym**

Fig. 132
"$\mathbb{F}_1 \cup$ dP$_2$"

Fig. 122
$SU(3)_{\frac{1}{2}}$+1**F**

Fig. 118
$SU(3)_{\frac{3}{2}}$+1**F**

Fig. 113
$SU(3)_{\frac{5}{2}}$+1**F**

Fig. 106
$SU(3)_{\frac{7}{2}}$+1**F**

Fig. 97
$SU(3)_{\frac{9}{2}}$+1**F**
$Sp(2)$+1**F**

Fig. 86
$SU(3)_{\frac{11}{2}}$+1**F**
$Sp(2)_\pi$+1**AS**

Fig. 71
$SU(3)_{\frac{13}{2}}$+1**F**
G_2+1**F**

Fig. 75
$Sp(2)_0$+1**AS**

Fig. 133
"$\mathbb{F}_2 \cup$ dP$_1$"

Fig. 124
$SU(3)_0$

Fig. 123
$SU(3)_1$

Fig. 119
$SU(3)_2$

Fig. 114
$SU(3)_3$

Fig. 107
$SU(3)_4$

Fig. 98
$SU(3)_5$
$Sp(2)_\pi$

Fig. 87
$SU(3)_6$

Fig. 72
$SU(3)_7$
G_2

Fig. 76
$Sp(2)_0$

Fig. 120
"$\mathbb{F}_3 \cup \mathbb{P}^2$"

Fig. 77
"$\mathbb{F}_6 \cup \mathbb{P}^2$"