Rotating charged AdS black holes solutions in gauged super-gravities

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Talk at 2017-10-12, University of Science and Technology of China

In this talk, I will present a brief summary of my endeavor devoted to finding exact rotating charged AdS solutions with multiple charge parameters in gauged supergravity theories that generalizes the black hole solutions found by Cvetič and Youm for the $D = 4, 5$ case and those by Horowitz and Sen for the $D \geq 4$ case.

I will also mention the related issue for the general $D = 6, 7$ two-charged rotating AdS solutions yet to be found.

I. INTRODUCTION

A. Why $D \geq 4$?

Black holes in higher dimensions have many different and much more fruitful properties than four-dimensional counterparts.

Exact solutions in five dimensions can have very different horizon topology and asymptotic topology structures (rod structure formalism):

- Black holes with horizon S^3-sphere: [R.C. Myers, M.J. Perry, AP **172** (1986) 304]

$$ds^2 \to -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2 + \cos^2\theta\, d\psi^2)$$

  Rotating AdS: [S.W. Hawking, C.J. Hunter, M.M. Taylor-Robinson, PRD **59** (1999) 064005]

  <u>D=5, N=2 STU model</u>

  Static: [G.T. Horowitz, J.M. Maldacena, A. Strominger, PLB **383** (1996) 151]

  Static AdS: [K. Behrndt, M. Cvetič, W.A. Sabra, NPB **553** (1999) 317]

  Rotating charged extension: [M. Cvetič, D. Youm, NPB **476** (1996) 118; PRD **54** (1996) 2612]

  Special rotating charged AdS case: [minimal: D. Klemm, W.A. Sabra, PLB **503** (2001) 147; Z.W. Chong, M. Cvetič, Lü, C.N. Pope, PRL **95** (2005) 161301;
  STU: M. Cvetič, Lü, C.N. Pope, PLB **598** (2004) 273; M. Cvetič, H. Lü, C.N. Pope, PRD **70** (2004) 081502; Z.W. Chong, M. Cvetič, H. Lü, C.N. Pope, PRD **72** (2005) 041901(R); J.W. Mei, C.N. Pope, PLB **658** (2007) 64]

  Most general rotating charged AdS case: [S.Q. Wu, PLB **707** (2012) 286] (Wu black hole)

- Black ring with $S^2 \times S^1$ horizon: [R. Emparan, H.S. Reall, PRL **88** (2002) 101101;
  SUSY: H. Elvang, R. Emparan, D. Mateos, H.S. Reall, PRL **93** (2004) 211302]

$$ds^2 \to -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2 + \cos^2\theta\, d\psi^2)$$

  Double spinning black rings: [A.A. Pomeransky, R.A. Sen'kov, hep-th/0612005; Y. Chen, K. Hong, E. Teo, PRD **84** (2011) 084030]

  Dipole charged spinning black rings: [Y. Chen, K. Hong, E. Teo, JHEP **1206** (2012) 148; A.L. Feldman, A.A. Pomeransky, JHEP **1207** (2012) 141; JHEP **1408** (2014) 059]

- Black string with topology $S^2 \times R^1$ (D=4 black holes trivially added fifth dimension): [G. Compere, S. de Buyl, E. Jamsin, A. Virmani, CQG **26** (2009) 125016; G. Compere, S. de Buyl, S. Stotyn, A. Virmani, JHEP **1011** (2010) 133]

$$ds^2 \to -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) + dz^2$$

- Black lens with $L(n,1)$ topology: [Y. Chen, E. Teo, PRD **78** (2008) 064062; H.K. Kunduri, J. Lucietti, PRL**113** (2014) 211101; PRD **94** (2016) 064007; S. Tomizawa, M. Nozawa, PRD **94** (2016) 044037; S. Tomizawa, T. Okuda, PRD **95** (2017) 064021]

- Kaluza-Klein (KK) black hole:

  Not referring to [V. Frolov, A. Zelnikov, U. Bleyer, AdP **499** (1987) 371; J. Kunz, D. Maison, F. Navarro-Lerida, J. Viebahn, PLB **639** (2006) 95; S.Q. Wu, PRD **83** (2011) 121502R]

  (I) Cohomogeneity-two class of Kaluza-Klein black hole: [D. Rasheed, NPB **454** (1995) 379; F. Larsen, NPB **575** (2000) 211;
  S. Tomizawa, Y. Yasui, Y. Morisawa, CQG **26** (2009) 145006; D.V. Galtsov, N.G. Scherbluk, PRD **79** (2009) 064020; S. Mizoguchi, S. Tomizawa, PRD **84** (2011) 104009; **86** (2012) 024022; S. Tomizawa, S. Mizoguchi, PRD **87** (2013) 024027]

  (II) Cohomogeneity-one class of Kaluza-Klein black hole with squashed 3-sphere: [G.W. Gibbons, D. L. Wiltshire, AP **167** (1986) 201; **176** (1987) 393E; X.D. Zhu, D. Wu, S.Q. Wu, S.Z. Yang, ScG **46** (2016) 070401; GRG **48** (2016) 154]

$$ds^2 \to -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) + \frac{r_\infty^2}{4}(d\psi + \cos\theta\, d\phi)^2$$

  - (1) $M \neq 0, Q \neq 0, J = 0$: [H. Ishihara, K. Matsuno, PTP **116** (2006) 417]
  - (2) $M \neq 0, Q = 0, J \neq 0$: [T. Wang, NPB **756** (2006) 86]
  - (3) $M \neq 0, Q \neq 0, J \neq 0$: [T. Nakagawa, H. Ishihara, K. Matsuno, S. Tomizawa, PRD **77** (2008) 044040]
  - (4) STU model: [S. Tomizawa, arXiv:1009.3568 [hep-th]; S.Q. Wu, D. Wen, Q.Q. Jiang, S.Z. Yang, PLB **726** (2013) 404]
  - (5) Einstein-Maxwell-Dilaton gravity: [S.S. Yazadjiev, PRD **74** (2006) 024022]
  - (6) Gödel universe: [C. Stelea, K. Schleich, D. Witt, PRD **78** (2008) 124006; S. Tomizawa, H. Ishihara, K. Matsuno, T. Nakagawa, PTP 121 (2009) 823]

- Black hole in Gödel uninverse: [E. G. Gimon, A. Hashimoto, PRL **91** (2003) 021601; S.Q. Wu, PRL **100** (2008) 121301]

Open questions: Is there a non-extremal black ring in Gödel uninverse ?

B. Main contents of this talk

Published work [S.Q. Wu, PRD **83** (2011) 121502R; PLB **705** (2011) 383; **707** (2012) 286] and a lot of still unpublished work done in recent years (2010–2016).

These work generalizes:
(I) M. Cvetič, D. Youm, NPB **476** (1996) 118; PRD **54** (1996) 2612
(II) M. Cvetič, D. Youm, NPB **477** (1996) 449; G.T. Horowitz, A. Sen, PRD **53** (1996) 808
(III) J. Kunz, D. Maison, F. Navarro-Lerida, J. Viebahn, **PLB** 639 (2006) 95
(IV) D. Rasheed, NPB **454** (1995) 379; F. Larsen, NPB **575** (2000) 211
(V) G.W. Gibbons, H. Lü, D.N. Page, C. N. Pope, PRL **93** (2004) 171102; JGP **53** (2005) 49

II. KK GAUGED SUPERGRAVITY [S.Q. WU, PRD 83 (2011) 121502R]

Higher-dimensional generalizations: already-known rotating $D \geq 4$ solutions with arbitrary angular momenta

- (I) Rotating vacuum Myers-Perry solutions ($R_{\mu\nu} = 0 = \Lambda$): [R.C. Myers, M.J. Perry, AP **172** (1986) 304]

- (II) Kerr-de Sitter metrics ($R_{\mu\nu} = (D-1)\Lambda g_{\mu\nu}$): [G.W. Gibbons, H. Lü, D.N. Page, C.N. Pope, PRL **93** (2004) 171102; JGP **53** (2005) 49]

- (III) Rotating charged Kaluza-Klein metrics ($\Lambda = 0 \neq \delta$): [J. Kunz, D. Maison, F. Navarro-Lerida, J. Viebahn, PLB **639** (2006) 95]
- (IV) General rotating charged KK-AdS(-NUT) metrics ($\Lambda \neq 0 \neq \delta$): [S.Q. Wu, PRD **83** (2011) 121502(R)]

Rotating charged KK solutions in (un)gauged supergravity:
(I) Ungauged cases ($\Lambda = 0$)

- $D = 4$: [V. Frolov, A. Zelnikov, U. Bleyer, AdP **44** (1987) 371; A.N. Aliev, H. Cebeci, T. Dereli, PRD **77** (2008) 124022 (generalization with a NUT charge)]
- $D \geq 4$: [J. Kunz, D. Maison, F. Navarro-Lerida, J. Viebahn, PLB **639** (2006) 95]

(II) Gauged cases ($\Lambda \neq 0$)

- $D = 5$, $a \neq 0 = b$: [Z.W. Chong, M. Cvetič, H. Lü, C.N. Pope, PRD **72** (2005) 041901]
- $D = 5$, $a \neq 0 \neq b$: [Z.W. Chong, M. Cvetič, H. Lü, C.N. Pope, PLB **644** (2007) 192]
- $D = 4$: [D.D.K. Chow, CQG **28** (2011) 032001]

The Lagrangian of the Einstein-Maxwell-dilaton system and the solutions can be presented in unified forms for all dimensions:

$$\mathcal{L} \;=\; \sqrt{-g}\Big\{R - \frac{1}{4}e^{-(D-1)\Phi}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}(D-1)(D-2)\partial_\mu\Phi\partial^\mu\Phi + g^2(D-1)\big[(D-3)e^{\Phi} + e^{-(D-3)\Phi}\big]\Big\} \tag{1}$$

General KK-AdS solutions (in a frame <u>nonrotating</u> at infinity):

$$\begin{aligned} ds^2 \;&=\; H^{\frac{1}{D-2}}\Big[-(1+g^2r^2)W\,dt^2 + \frac{U\,dr^2}{V(r)-2mf(r)} + \sum_{i=1}^{N+\epsilon}\frac{r^2+a_i^2}{\chi_i}\,d\mu_i^2 + \sum_{i=1}^{N}\frac{r^2+a_i^2}{\chi_i}\mu_i^2\,d\phi_i^2 \\ &\quad - \frac{g^2}{(1+g^2r^2)W}\Big(\sum_{i=1}^{N+\epsilon}\frac{r^2+a_i^2}{\chi_i}\mu_i\,d\mu_i\Big)^2 + \frac{2m}{UH}\Big(cW\,dt - \sum_{i=1}^{N}\frac{a_i\sqrt{\Xi_i}}{\chi_i}\mu_i^2\,d\phi_i\Big)^2\Big] \end{aligned} \tag{2}$$

$$A \;=\; \frac{2ms}{UH}\Big(cW\,dt - \sum_{i=1}^{N}\frac{a_i\sqrt{\Xi_i}}{\chi_i}\mu_i^2\,d\phi_i\Big)\,, \qquad \Phi = \frac{-1}{D-2}\ln(H) \tag{3}$$

Functions (U, W, H) and $f(r)$ are ($c = \cosh\delta\,, s = \sinh\delta$):

$$U = r^{\epsilon}\sum_{i=1}^{N+\epsilon}\frac{\mu_i^2}{r^2+a_i^2}\prod_{j=1}^{N}(r^2+a_j^2)\,, \quad V(r) = r^{\epsilon-2}(1+g^2r^2)\prod_{i=1}^{N}(r^2+a_i^2)\,, \quad W = \sum_{i=1}^{N+\epsilon}\frac{\mu_i^2}{\chi_i}$$

$$H = 1 + \frac{2ms^2}{U}\,, \quad f(r) = c^2 - s^2(1+g^2r^2)\,, \quad \Xi_i = c^2 - s^2\chi_i\,, \quad \chi_i = 1 - g^2a_i^2$$

Dimension of spacetime: $D = 2N + 1 + \epsilon \geq 4$; Rotation parameters a_i: $N = [(D-1)/2]$; Coordinates: (t, r, μ_i, ϕ_i); N azimuthal angles ϕ_i, each with period 2π; $N + \epsilon = n = [D/2]$ 'Direction cosines' μ_i with constraint: $\sum_{i=1}^{N+\epsilon}\mu_i^2 = 1\,,$ $2\epsilon = 1 + (-1)^D$; $0 \leq \mu_i \leq 1$ $(1 \leq i \leq N)$; (even D) $-1 \leq \mu_{N+1} \leq 1,$ $a_{N+1} = 0$

The solutions have the following metric structure:

$$ds^2 = H^{\frac{1}{D-2}}\Big(d\bar{s}^2 + \frac{2m}{UH}K^2\Big)\,, \qquad A = \frac{2ms}{UH}\,K\,, \qquad \Phi = \frac{-1}{D-2}\ln(H) \tag{4}$$

A prominent discovery of this work is to present (for the first time) the generalization of Kerr-Schild ansatz to KK (super)gravities:

$$g_{\mu\nu} = H^{\frac{1}{D-2}}\Big(\bar{g}_{\mu\nu} + \frac{2m}{UH}k_\mu k_\nu\Big)\,, \quad g^{\mu\nu} = H^{\frac{-1}{D-2}}\Big(\bar{g}^{\mu\nu} - \frac{2m}{U}k^\mu k^\nu\Big)\,, \qquad A_\mu = \frac{2ms}{UH}k_\mu \tag{5}$$

The vector k_μ is a timelike geodesic congruence:

$$\bar{g}^{\mu\nu}k_\mu k_\nu = -s^2\,, \quad k^\mu\bar{\nabla}_\mu k_\nu = k^\mu\bar{\nabla}_\nu k_\mu = 0\,, \qquad k^\mu = \bar{g}^{\mu\nu}k_\nu\,, \quad k_\mu = \bar{g}_{\mu\nu}k^\nu$$

Rewritten in terms of the dilaton scalar, we have

$$g_{\mu\nu} = e^{-\Phi}\bar{g}_{\mu\nu} + \big[e^{-\Phi} - e^{(D-3)\Phi}\big]s^{-2}k_\mu k_\nu\,, \quad g^{\mu\nu} = e^{\Phi}\bar{g}^{\mu\nu} + \big[e^{\Phi} - e^{-(D-3)\Phi}\big]s^{-2}k^\mu k^\nu\,, \quad A_\mu = \big[1 - e^{(D-2)\Phi}\big]s^{-1}k_\mu\,. \tag{6}$$

This ansatz was further studied in our work [S.Q. Wu, H. Wang, PRD **91** (2015) 104031].

III. GENERAL NONEXTREMAL ROTATING CHARGED ADS BLACK HOLES IN 5-DIMENSIONAL $N = 2, U(1)^3$ GAUGED SUPERGRAVITY

In 1996, Cvetič and Youm [M. Cvetič, D. Youm, PRD **54** (1996) 2612; NPB **476** (1996) 118] generated a three-charged rotating black hole solution from the $D = 5$ neutral rotating Myers-Perry black hole solution [R.C. Myers, M.J. Perry, AP **172** (1986) 304]. The AdS generalization of the latter solution was found by Hawking, *et al* [S.W. Hawking, C.J. Hunter, M.M. Taylor-Robinson, PRD **59** (1999) 064005].

There were a lot of great attempts to combine the above solutions into just one solution, but only achieved successfully in some special cases [M. Cvetič, H. Lü, C.N. Pope, PLB **598** (2004) 273; PRD **70** (2004) 081502; Z.W. Chong, M. Cvetič, H. Lü, C.N. Pope, PRD **72** (2005) 041901(R); PLB **644** (2007) 192; J.W. Mei, C.N. Pope, PLB **658** (2007) 64].

The $D = 5$ minimal gauged supergravity solution was found in [Z.W. Chong, M. Cvetič, H. Lü, C.N. Pope, PRL **95** (2005) 161301]. It includes as a special case the supersymmetric version given in [D. Klemm, W.A. Sabra, PLB **503** (2001) 147].

It should be noted that a general supersymmetric rotating and charged AdS_5 solution with two independent rotation and three unequal charge parameters in $D = 5, U(1)^3$ gauged supergravity theory had been found in [H.K. Kunduri, J. Lucietti, H.S. Reall, JHEP **0604** (2006) 036].

A. $D = 5, N = 2$ STU gauged supergravity

The five-dimensional Lagrangian for the bosonic sector of the $D = 5, N = 2, U(1)^3$ gauged supergravity coupled to two vector multiplets can be written as:

$$\mathcal{L} = \sqrt{-g}\Big[R + 4g^2(X_1 + X_2 + X_3) - 3(\partial\varphi_1)^2 - (\partial\varphi_2)^2 - \frac{1}{4}\sum_{I=1}^{3} X_I^2 F_I^2\Big] + \frac{1}{4}\varepsilon^{\mu\nu\rho\sigma\lambda} F_{1\mu\nu} F_{2\rho\sigma} A_{3\lambda} \,. \tag{7}$$

In terms of two scalar fields φ_1 and φ_2, $X_I = H_I/(H_1H_2H_3)^{1/3}$ can be parameterized as:

$$X_1 = e^{\varphi_1+\varphi_2} \,, X_2 = e^{\varphi_1-\varphi_2} \,, X_3 = e^{-2\varphi_1} \,.$$

B. Previous solution [S.Q. Wu, PLB 707 (2012) 286]

Here I will review the AdS extension of the $D = 5$ rotating charged Cvetič-Youm black holes. The most general solution with three unequal charges successfully found in [S.Q. Wu, PLB **707** (2012) 286] reads

$$\begin{aligned} ds^2 = (H_1H_2H_3)^{1/3}\Big[&- \frac{(1+g^2r^2)\Delta_\theta}{\chi_a\chi_b}\,dt^2 + \Sigma\Big(\frac{r^2\,dr^2}{\Delta_r} + \frac{d\theta^2}{\Delta_\theta}\Big) + \frac{(r^2+a^2)\sin^2\theta}{\chi_a}\,d\phi^2 + \frac{(r^2+b^2)\cos^2\theta}{\chi_b}\,d\psi^2 \\ &+ \frac{2ms_1^2}{\Sigma H_1(s_2^2-s_1^2)(s_3^2-s_1^2)}K_1^2 + \frac{2ms_2^2}{\Sigma H_2(s_1^2-s_2^2)(s_3^2-s_2^2)}K_2^2 + \frac{2ms_3^2}{\Sigma H_3(s_1^2-s_3^2)(s_2^2-s_3^2)}K_3^2\Big] \,, \end{aligned} \tag{8}$$

three $U(1)$ Abelian gauge fields and two scalar fields are

$$A_i = \frac{2m}{\Sigma H_i}K_i \,, \quad \varphi_1 = \frac{1}{6}\ln\Big(\frac{H_1H_2}{H_3}\Big) \,, \quad \varphi_2 = \frac{1}{2}\ln\Big(\frac{H_1}{H_2}\Big) \,, \qquad H_I = 1 + \frac{2ms_I^2}{\Sigma} \,, \tag{9}$$

where

$$\begin{aligned} K_i = \frac{s_ic_1c_2c_3}{c_i}\frac{\sqrt{\Xi_{1a}\Xi_{2a}\Xi_{3a}\Xi_{1b}\Xi_{2b}\Xi_{3b}}}{\sqrt{\Xi_{ia}\Xi_{ib}}}\Big(&\frac{c_i^2}{c_1c_2c_3}\frac{\Delta_\theta}{\chi_a\chi_b}\,dt - \frac{\Xi_{ia}}{\sqrt{\Xi_{1a}\Xi_{2a}\Xi_{3a}}}\frac{a\sin^2\theta}{\chi_a}\,d\phi - \frac{\Xi_{ib}}{\sqrt{\Xi_{1b}\Xi_{2b}\Xi_{3b}}}\frac{b\cos^2\theta}{\chi_b}\,d\psi\Big) \\ + \frac{c_is_1s_2s_3}{s_i}\sqrt{\Xi_{ia}\Xi_{ib}}\Big(&-\frac{c_1c_2c_3}{c_i^2}\frac{g^2ab\Delta_\theta}{\chi_a\chi_b}\,dt + \frac{\sqrt{\Xi_{1a}\Xi_{2a}\Xi_{3a}}}{\Xi_{ia}}\frac{b\sin^2\theta}{\chi_a}\,d\phi + \frac{\sqrt{\Xi_{1b}\Xi_{2b}\Xi_{3b}}}{\Xi_{ib}}\frac{a\cos^2\theta}{\chi_b}\,d\psi\Big) \,, \end{aligned}$$

in which, $\Xi_{ia} = c_i^2 - s_i^2\chi_a$, $\Xi_{ib} = c_i^2 - s_i^2\chi_b$, and

$$\begin{aligned}
\Delta_r &= (r^2+a^2)(r^2+b^2)(1+g^2r^2) - 2mr^2 + 2mg^2\Big\{(s_1^2+s_2^2+s_3^2)r^4 - (s_1^2s_2^2+s_1^2s_3^2+s_2^2s_3^2)\big[(a^2+b^2-2m)r^2 \\
&\quad + a^2b^2(2+g^2r^2)\big] + s_1^2s_2^2s_3^2\Big([(a+b)^2-2m][(a-b)^2-2m] - 2g^2a^2b^2(2r^2+2m+a^2+b^2) + g^4a^4b^4\Big) \\
&\quad + 2mg^2a^2b^2\big[s_2^2(s_1+s_3)^2 - s_1^2s_3^2\big]\big[s_2^2(s_1-s_3)^2 - s_1^2s_3^2\big]\Big\}\,, \\
\Sigma &= r^2 + a^2\cos^2\theta + b^2\sin^2\theta\,, \quad \Delta_\theta = 1 - g^2(a^2\cos^2\theta + b^2\sin^2\theta)\,, \qquad \chi_a = 1-g^2a^2\,, \quad \chi_b = 1-g^2b^2\,.
\end{aligned}$$

C. New simple solution

A new simple form for the solution with an arbitrary parameter w was found during 2015.6.4-7.5

Three gauge potentials and two scalar fields are still given by (9). It is interesting to note that the above metric can also be cast into the form (8).

This solution is much more simple than the previous one especially in the $w = 1, 0$ cases.

D. A special case: $w = 1$ and $b = a$

In the case $w = 1$ and $b = a$, if we set

$$\rho^2 = \frac{r^2+a^2}{\chi}\,, \quad \mu = \frac{2m}{\chi}\,, \qquad \gamma = 0, \quad \ell = a\,, \quad \chi = 1-g^2a^2\,, \tag{10}$$

then the above solution recovers the equal-rotation solution given in [M. Cvetič, H. Lü, C.N. Pope, PRD **70** (2004) 081502].

However, I don't find out the relation of the three equal-charge case to the minimal gauged supergravity solution found in [Z.W. Chong, M. Cvetič, H. Lü, C.N. Pope, PRL **95** (2005) 161301]. One could anticipate to find such a three-charged rotating AdS solution that reproduces the minimal gauged supergravity solution.

IV. BEYOND STU MODEL — NEW STU-W^{2}U

The five-dimensional Lagrangian for the bosonic sector of the $D = 5, N = 2$ ungauged supergravity coupled to three vector multiplets is omitted here.

Several static solutions with or without squashed horizons, static AdS case, and equal rotating black hole solutions have been found.

Our recent interest is to construct singly rotating and general rotating charged black hole with two-unequal rotation parameters, but this task is very hard and little essential progress has been made.

V. GENERAL NON-EXTREMAL ROTATING DYONIC ADS BLACK HOLE SOLUTION IN FOUR-DIMENSIONAL KALUZA-KLEIN GAUGED SUPERGRAVITY

A. Dyonic KK black holes (without NUT charge) (These solutions appears unable to be cast into Kerr-Schild form or any kind of its generalization!

(I) Ungauged cases ($g = 0$)

- static dyonic KK solution: [G.W. Gibbons, D.L. Wiltshire, AP **167** (1986) 201]
- rotating dyonic KK solutions: [D. Rasheed, NPB **454** (1995) 379; F. Larsen, NPB **575** (2000) 211]

(II) Gauged cases ($g \neq 0$)

- static dyonic KK-AdS solution: [H. Lü, Y. Pang, C.N. Pope, JHEP **11** (2013) 033]
  thermodynamics: adding (X, Y) pair

- general static AdS solution (STU gauged supergravity): [D.D.K. Chow, G. Compère, PRD **89** (2014) 065003] mass is not integrable!

B. Electric KK black holes:

Rotating charged KK solutions in 4D (un)gauged supergravity:
(I) Ungauged cases ($g = 0 = N$): [V. Frolov, A. Zelnikov, U. Bleyer, AdP **44** (1987) 371]
(II) Gauged cases ($g \neq 0$): [D.D.K. Chow, CQG **28** (2011) 032001; S.Q. Wu, PRD **83** (2011) 121502(R)]

Here I present exact solutions for general nonextremal rotating, charged AdS black holes with a cosmological constant (no NUT charge) in 4D gauged KK supergravity. Our rotating dyonic Kaluza-Klein AdS_4 black holes have two generalizations of: $g = 0$: Rasheed-Larsen solution; and $a = 0$: JHEP **11** (2013) 033.

The Lagrangian of the Einstein-Maxwell-dilaton system is:

$$\mathcal{L} = \sqrt{-g}\Big[R - \frac{3}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{4}e^{-3\phi}F_{\mu\nu}F^{\mu\nu} + 3g^2(e^{\phi} + e^{-\phi})\Big] \tag{11}$$

$$F = dA\,, \quad \widetilde{F} = d\widetilde{A} = e^{-3\phi}\,{}^\star F\,. \tag{12}$$

Special cases

1) $p = q = 2m$: (neutral) Kerr-AdS
2) $a = 0, g \neq 0$: (non-rotating) [JHEP **11** (2013) 033]
3) $g = 0, a \neq 0$: (ungauged) Rasheed-Larsen
4) $p = 2m, q = 2m\cosh^2\delta$: electric-charged [CQG **28** (2011) 032001]

VI. GENERAL NON-EXTREMAL ROTATING CHARGED ADS BLACK HOLE SOLUTIONS WITH 4 INDEPENDENT CHARGE PARAMETERS IN $D = 4, N = 2, U(1)^4$ GAUGED SUPERGRAVITY

In 1996, Cvetič and Youm [M. Cvetič, D. Youm, PRD **54** (1996) 2612] only presented the metric of a four-charged rotating black hole solution with each two independent electric and magnetic parameters. Later, Chong, *et al* [Z.W. Chong, M. Cvetič, H. Lü, C. N. Pope, NPB **717** (2005) 246] obtained the complete solution especially for four gauge potentials generated from the Kerr black hole solution [R.P. Kerr, PRL **11** (1963) 237]. They also generated its NUT-charge generalization.

The pairwise equal charge case contains two special cases:
1) Eintein-Maxwell-Dilaton-Axion (EMDA), namely, Kerr-Sen black hole [A. Sen, PRL **69** (1992) 1006; A. Garcia, D. Galtsov, O. Kechkin, PRL **74** (1995) 1276];
2) Kerr-Newman black hole with equal electric and magnetic charges. They also constructed its AdS extension.

Recently, Chow and Compère [D.D.K. Chow, G. Compère, CQG **31** (2014) 022001; PRD **90** (2014) 025029] generalized the solution to the case with with each four independent electric and magnetic parameters.

A. $D = 4, N = 2$ STU gauged supergravity

The maximal four-dimensional $N = 8, SO(8)$ gauged supergravity can be consistently truncated to $U(1)^4$ gauged supergravity, which is $N = 2$ gauged supergravity coupled to three Abelian vector multiplets. The bosonic fields of this truncation are the graviton $g_{\mu\nu}$, four U(1) gauge fields A_I, three dilatons φ_i and three axions χ_i, with $I = i, 4$ and $i = 1, 2, 3$.
The four-dimensional Lagrangian for the bosonic sector of the $N = 2$ STU supergravity coupled to three vector multiplets can be written as

$$\begin{aligned}\mathcal{L} \;=\; & R \star 1\!\!1 + g^2\sum_{i=1}^{3}\big[e^{-\varphi_i} + e^{\varphi_i}\big(1 + \chi_i^2\big)\big] \star 1\!\!1 - \frac{1}{2}\sum_{i=1}^{3}\big({}^\star d\varphi_i \wedge d\varphi_i + e^{2\varphi_i}\,{}^\star d\chi_i \wedge d\chi_i\big) \\ & -\frac{1}{2}\big(X_1{}^\star F_1 \wedge F_1 + X_2{}^\star F_2 \wedge F_2 + X_3{}^\star\mathcal{F}_1 \wedge \mathcal{F}_1 + X_4{}^\star\mathcal{F}_2 \wedge \mathcal{F}_2\big) - \chi_2(F_1 \wedge \mathcal{F}_1 + F_2 \wedge \mathcal{F}_2)\,,\end{aligned} \tag{13}$$

where

$$X_1 = e^{\varphi_1-\varphi_2-\varphi_3}\,, \quad X_2 = e^{\varphi_1-\varphi_2+\varphi_3}\,, \quad X_3 = e^{-\varphi_1-\varphi_2+\varphi_3}\,, \quad X_4 = e^{-\varphi_1-\varphi_2-\varphi_3}\,.$$

[Note that in the above, I have made the following changes relative to the convention of Ref. [NPB **717** (2005) 246] in order to cast the scalar fields in a unified expression: $(\varphi_1, \varphi_2) \to (\varphi_2, \varphi_1)$, $(\chi_1, \chi_2) \to (\chi_2, -\chi_1)$].

B. Including a nonzero cosmological constant to the $D = 4$ rotating charged Cvetič-Youm black holes

The AdS extension of the $D = 4$ Cvetič-Youm solution was reported in 2015 but still unpublished.

C. Further generalizations

During 2015-2016, I had also obtained the AdS extension of the rotating charged Chow-Compère black holes with 8 charges. Due to the complexity of the expressions, this work has been neither published nor reported yet, and I will not present it.

Extensions of the AdS versions of the Cvetič-Youm and Chow-Compère black holes to include nonzero NUT charge were also finished in 2015.

Further to add an acceleration parameter to the above solutions is my main task in the next years. These will generalize PD metric [J.F. Plebański, M. Demiański, AP **98** (1976) 98] to N=2 supergravity.
In the case of C-metric, partial work had been done in [H. Lü, Justin F. Vazquez-Poritz, JHEP **1412** (2014) 057; PRD **90** (2015) 064004].

The C-metric is a special three-soliton limit of the double-Schwarzschild black hole; The most general Plebański-Demiański solution without a cosmological constant or the Spinning C-metric is a three-soliton limit of the double-Kerr(-Newman)-NUT black hole.

Open questions: A double black hole solution on the flat Minkowski background can be expressed in the Weyl form.

What is the double black hole solution on the (A)dS background? Whether it could be expressed in a Weyl-like coordinates?

VII. TWO-CHARGE ADS BLACK HOLES WITH A SINGLE ROTATION IN GAUGED SUPERGRAVITY IN ALL HIGHER DIMENSIONS

In 1996, Cvetič and Youm [M. Cvetič, D. Youm, NPB 477 (1996) 449] also generated the two-charged rotating black hole solutions from the neutral rotating Myers-Perry black hole solution in all higher dimensions [R.C. Myers, M.J. Perry, AP **172** (1986) 304]. The (A)dS and NUT-charge generalizations of the latter solution was found in [G.W. Gibbons, H. Lü, D.N. Page, C. N. Pope, PRL **93** (2004) 171102; JGP **53** (2005) 49; W. Chen, H. Lü, C.N. Pope, CQG **23** (2006) 5323; NPB **762** (2007) 38].

Previous partial success has been achieved only in the singly rotation case and in the equal-charge case. In particular, Chow [D.D.K. Chow, arXiv:1108.5139] constructed two-charge singly rotating AdS solutions in $4 \leq D \leq 7$ dimensions. This includes as special cases for $D = 4$ [D.D.K. Chow, CQG **28** (2011) 175004], $D = 5$ [S.Q. Wu, PLB **707** (2012) 286] and $D = 7$ [S.Q. Wu, PLB **705** (2011) 383].

I will consider two-charge singly rotating AdS solutions in all dimensions. This will extend the Horowitz-Sen [G.T. Horowitz, A. Sen, PRD **53** (1996) 808] solution to include a nonzero cosmological constant.

A. The Lagrangian

The gauged Lagrangian suitable for all $D \geq 4$ dimensions is

$$\begin{aligned}\mathcal{L}_D \;=\;& R\star\mathbb{1} + g^2\big[(D-3)^2X_1X_2 + 2(D-3)(X_1X_2)^{-(D-3)/2}(X_1+X_2) + (5-D)(X_1X_2)^{-(D-3)}\big]\star\mathbb{1}\\ &-(D-2)\,{}^\star d\phi_1\wedge d\phi_1 - {}^\star d\phi_2\wedge d\phi_2 - \frac{1}{2}(X_1^{-2}\,{}^\star F_1\wedge F_1 + X_2^{-2}\,{}^\star F_2\wedge F_2) - \frac{1}{2}(X_1X_2)^2\,{}^\star\mathcal{F}_{(D-3)}\wedge\mathcal{F}_{(D-3)}\\ &-F_1\wedge F_2\wedge\mathcal{C}_{(D-4)} \qquad +\cdots\,. \end{aligned} \tag{14}$$

where the field strengths are given in terms of 1-form potentials A_i and a (D-4)-form potential $\mathcal{C}_{(D-4)}$ by $F_i = dA_i$ and $\mathcal{F}_{(D-3)} = d\mathcal{C}_{(D-4)}$. The scalars are the combinations:

$$X_1 = e^{-\phi_1-\phi_2}\,, \quad X_2 = e^{-\phi_1+\phi_2}\,.$$

B. Previous black hole solutions

The D-dimensional singly rotating black hole solutions with two independent charges in arbitrary higher dimensions are omitted here also.

The solutions and action are valid for all $D \geq 4$ ($3 < D < 12$ verified!)

C. Two-charged singly rotating black holes in $D = 7$ gauged supergravity [S.Q. Wu, PLB 705 (2011) 383]

The $D = 7$ gauged Lagrangian is

$$\begin{aligned}\mathcal{L}_7 \;=\;& R\star\mathbb{1} + 2g^2\big[8X_1X_2 + 4(X_1X_2)^{-2}(X_1+X_2) - (X_1X_2)^{-4}\big]\star\mathbb{1} - 5\,{}^\star d\phi_1\wedge d\phi_1 - {}^\star d\phi_2\wedge d\phi_2\\ &-\frac{1}{2}(X_1^{-2}\,{}^\star F_1\wedge F_1 + X_2^{-2}\,{}^\star F_2\wedge F_2) - \frac{1}{2}(X_1X_2)^2\,{}^\star\mathcal{F}_{(4)}\wedge\mathcal{F}_{(4)} + (F_1\wedge F_2 - g\mathcal{F}_{(4)})\wedge\mathcal{C}_{(3)}\,,\end{aligned} \tag{15}$$

supplemented with the first-order self-duality equation

$$(X_1X_2)^2\star\mathcal{F}_{(4)} = -2g\mathcal{C}_{(3)} - \mathcal{H}_{(3)}\,, \tag{16}$$

in which $\mathcal{F}_{(4)} = d\mathcal{C}_{(3)}$ and $\mathcal{H}_{(3)} = d\mathcal{B}_{(2)} - (A_1\wedge F_2 + A_2\wedge F_1)/2$.

The metric, two $U(1)$ Abelian gauge potentials and two scalars are given in the last subsection with

$$d\Omega_3^2 = d\psi^2 + \cos^2\psi d\zeta^2 + \sin^2\psi d\xi^2\,, \tag{17}$$

while the non-vanishing components of the 2-form potential and 3-form potential are

$$\mathcal{B}_{t\phi} = \frac{ms_1s_2\Delta_\theta a\sin^2\theta}{r^2\Sigma\chi}\Big(\frac{1}{H_1} + \frac{1}{H_2}\Big)\,, \qquad \mathcal{C}_{t\theta\phi} = g\frac{2ms_1s_2a\sin\theta\cos\theta}{r^2\chi}\,, \quad \mathcal{C}_{\psi\zeta\xi} = \frac{2ms_1s_2a\cos^4\theta}{\Sigma}\sin\psi\cos\psi\,. \tag{18}$$

D. New simple black hole solutions in all $D \geq 4$

A new simple solution is omitted.

E. General $D = 6, 7$ two-charge rotating AdS black hole solution?

So far, the two-charged rotating AdS black hole solutions only have been partially solved in the $D = 6, 7$ cases: 1) The above two-charged singly-rotating AdS cases; 2) Two-charged AdS solution with equal rotation parameters in $D = 7$ dimensions [Z.W. Chong, M. Cvetič, H. Lü, C.N. Pope, PLB **626** (2005) 215]; and 3) Two-equal-charge rotating AdS solutions in $D = 6, 7$ dimensions [D.D.K. Chow, CQG **27** (2010) 065004; **25** (2008) 175010].

The most general two-charged rotating solutions in $D = 6, 7$ gauged supergravity are still unknown up to date. A simple way that modifies the functions H_i can retain our two-charge metric ansatz.

Thanks your patience very much!

谨此沉痛追悼刚刚不幸病逝的
闫沐霖教授！

唁 电

惊悉闫沐霖教授仙逝，深感悲痛。

闫兄有深厚的数学物理功底，做了很多高水平的研究工作，对推动我国的相对论和粒子物理研究作出了贡献。多年来我一直把他看作挚友和师长。

去年闫兄来京，曾送我他的英文著作，当时觉得他非常健康。这次听到他去世的消息，深感意外和震惊。

愿他的灵魂安乐。

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2017.10.10