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Is Warm Brane Inflation Possible?

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collaborate with James Dent & Damien Easson

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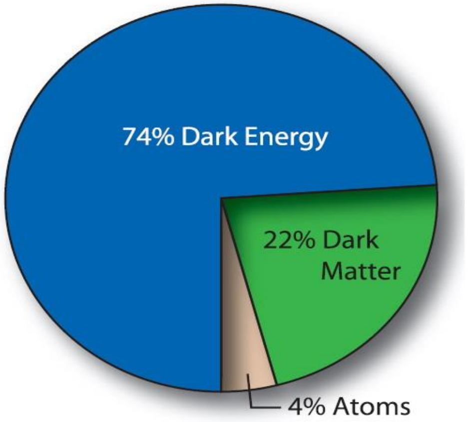
Cosmology: science based on observations



CMB: WMAP, Planck

LSS: SDSS ⇨ hot Big Bang

SuperNovae

Theoretical Problems:

- Horizon
- Flatness
- Unwanted relic
- Structure formation

→ Inflation

Open Questions of inflationary cosmology

1. How to realize inflation at early universe?

2. What is inflaton made of?

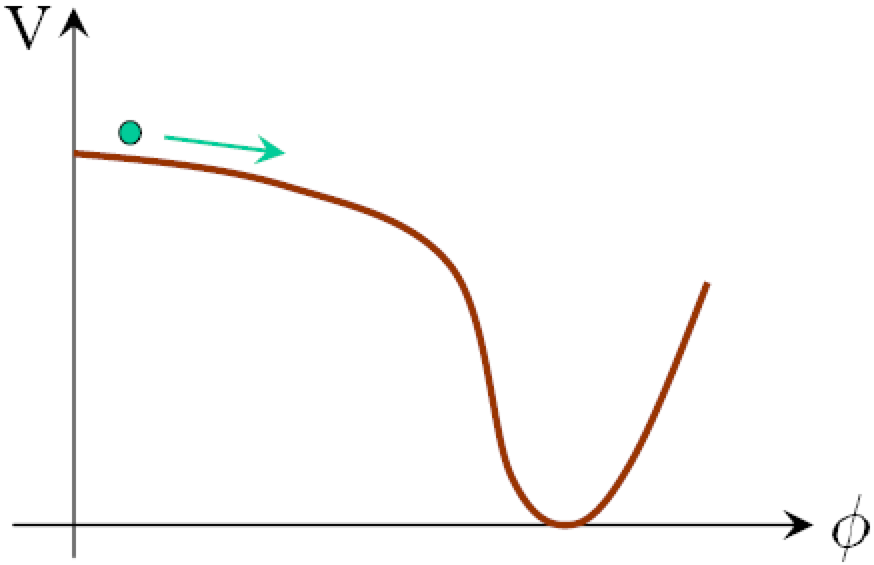
3. Does it fit to observations?

4. How to connect to thermal history?

Adventure is out there!

- Preliminary of Inflationary cosmology

Guth, PRD 23:347,1981;
Linde, PLB 108:389,1982;
Albrecht & Steinhardt, PRL 48:1220,1982



Slow-roll conditions:

$$\epsilon_V = \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \quad << 1$$

$$\eta_V = M_{pl}^2 \frac{V''}{V} \quad << 1$$

Low energy description of string theory

- Preliminary of string cosmology

Type I describes both open and closed, un-oriented strings with SO(32) gauge group with low energy effective theory being N=1 SUGRA.

Type II theories are N = 2 SUGRA effective theories:
IIA contains Dp-branes with p even,
IIB contains Dp-branes with p odd.

A first glance at our universe: 1+3D spacetime
Therefore, it could be embedded into Dp-brane with $p \geq 3$.

Low energy description of string theory

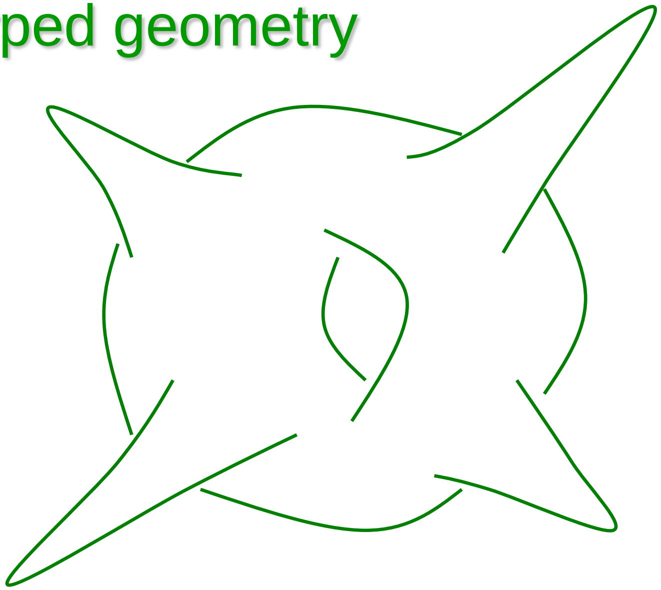
- Preliminary of string cosmology

Difficulty in string theories: Hard to obtain 4D de-Sitter solution with stabilized moduli. No inflation!

In 2003 KKLT mechanism was proposed to illustrate the possibility of 4D dS solution.

Kachru, Kallosh, Linde, Trivedi, 2003

- Flux compactification leads to warped geometry



Giddings, Kachru, Polchinski, 2001;
Klebanov, Strassler, 2000;
Randall, Sundrum, 1999;…

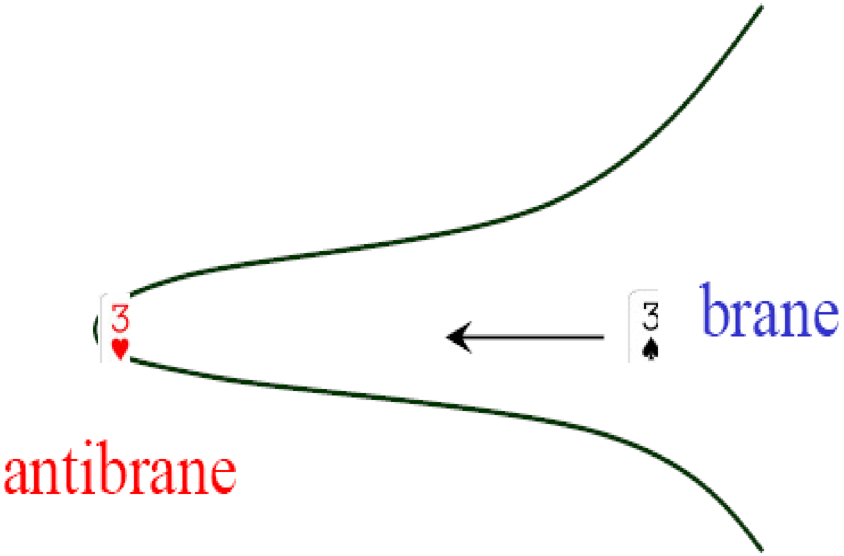
6D CY manifold

$$ds^2 = \frac{dx^\mu dx_\mu}{\sqrt{h(r)}} + \sqrt{h(r)}\left(dr^2 + r^2 ds^2_{T_{1,1}}\right)$$

$$h(r) \simeq \frac{R^4}{r^4}, \qquad R^4 = \frac{27}{4}\pi g_s N \alpha'^2, \quad N = MK$$

A first glance of KKLMMT brane inflation

- Brane motion in a warped throat



Anti-D3 is fixed at the tip of throat
and uplift AdS into dS;
D3 moves as a free object.

KKLMMT, 2003

$T_3 \to h^4 T_3$, h is the warp factor.

The inflaton has a mass

$$m_\phi^2 = 2H^2$$

Observations: $m_\phi^2 \leq 10^{-2} H^2$

- The **eta**-problem

$$\left.\begin{array}{l}\text{Moduli stabilization;}\\ \text{Volume-scalar coupling;}\end{array}\right\} \quad V \sim \pm H^2 \phi^2 \longrightarrow \eta \sim 1$$

Copeland, Liddle, Lyth, Stewart, Wands, 1994.

- Dangerous KK relics

Barnaby, Burgess, Cline, 2005;
Kofman, Yi, 2005

- Unwanted entropy production

Brandenberger, Frey, Lorenz, 2007;
Brandenberger, Dasgupta, Davis, 2008

Inflation models in string cosmology

D3/D7 involving a shift symmetry
Dasgupta, Herdeiro, Hirano & Kallosh, 2002

DBI inflation
Silverstein & Tong, 2003

Racetrack inflation with unfixed moduli,
Blanco-Pillado, et.al., 2004

Giant inflation via brane/flux anni.
DeWolfe, Kachru & Verlinde, 2004

Spinflation with angular motion
Easson, Gregory, Mota, Tasinato & Zavala, 2007

Inflation models in **non-IIB** string theories

… …

DBI Inflation

- Basic idea:

Inflation is realized by a mobile D3-brane with a relativistic speed in a warped throat;

Slow-roll can be relaxed by a small sound speed.

- The effective action:

$$S = \int d^4x \sqrt{-g} \left[-T(\phi)\sqrt{1-\partial^\mu \phi \partial_\mu \phi / T(\phi)} + T(\phi) - V(\phi) \right]$$

ϕ: radial position of the probe brane

Reduced brane tension $T(\phi) = T_3^{1/2} h^4$

Silverstein & Tong, PRD70:103505,2004;
Alishahiha, Silverstein & Tong,PRD70:123505,2004

DBI Inflation: Formalism

- Background:

$$c_s = \sqrt{1 - \dot{\phi}^2 / T(\phi)}$$

$$\rho = T(\phi)(c_s^{-1} - 1) + V(\phi)$$

$$p = T(\phi)(1 - c_s) - V(\phi)$$

- Slow-roll conditions:

$$\varepsilon \equiv 2c_s M_p^2 \left(\frac{H'}{H} \right)^2$$

$$\eta \equiv 2c_s M_p^2 \frac{H''}{H}$$

$$s \equiv \frac{\dot{c}_s}{Hc_s}$$

- Perturbations:

$$P_S = \frac{H^2}{8\pi^2 M_p^2 c_s \varepsilon} \qquad n_S - 1 = 2\eta - 4\varepsilon - 2s$$

$$P_T = \frac{2H^2}{\pi^2 M_p^2} \qquad n_T = -2\varepsilon$$

$$f_{NL} = \frac{35}{108}\left(\frac{1}{c_s^2} - 1 \right) \qquad r = 16 c_s \varepsilon$$

- Useful formulae:

$$\left(\frac{\Delta\phi}{M_p} \right)^2 \approx \frac{r}{8} (\Delta N)^2$$

$$1 - n_s = \frac{r}{4}\sqrt{1 + 3f_{NL}} - \frac{2s}{3f_{NL}} + \frac{\dot{T}}{HT}$$

DBI Inflation: Results

Feature:

- A relativistic mobile probe brane yields $c_s^2 << 1$

Benefits:

- A realization of K-inflation from string theory;
- A potential solution to the eta-problem;
- Plentiful phenomena in the early universe

Main predictions:

- Large nonlinearity parameter of equilateral shape;

$$f_{NL} = \frac{35}{108}\left(\frac{1}{c_s^2} - 1\right)$$

- Tensor perturbation is doubly suppressed

$$r = 16 c_s \varepsilon$$

DBI Inflation: Constraints

For $V = \frac{1}{2}m^2\phi^2$

Baumann & McIlister, 2006;
Chen, Sarangi, Tye & Xu,2006
Lidsey & Huston, 2007

- Upper bound on r due to a theoretical constraint

$$\left(\frac{\Delta\phi}{M_p}\right)^2 \approx \frac{r}{8}\left(\Delta N\right)^2$$

$$\left(\frac{\Delta\phi}{M_{\rm P}}\right)^2_* < \frac{T_3\kappa_{10}^2(\Delta\rho_*)^2}{|\Delta V_{6,*}|} \quad \Rightarrow \quad r < 10^{-7}$$

- Lower bound on r from observational constraint

$$1 - n_s = \frac{r}{4}\sqrt{1+3f_{NL}} - \frac{2s}{3f_{NL}} + \frac{\dot{T}}{HT}$$

$$r_* > \frac{4(1-n_s)}{\sqrt{3f_{NL}}} > \frac{1-n_s}{8} \quad \Rightarrow \quad r > 0.002$$

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Inconsistence!

Multi-field models in brane inflation

- Extra moduli modes of background geometry
- Non-radial dof of a single brane
- Multiple branes

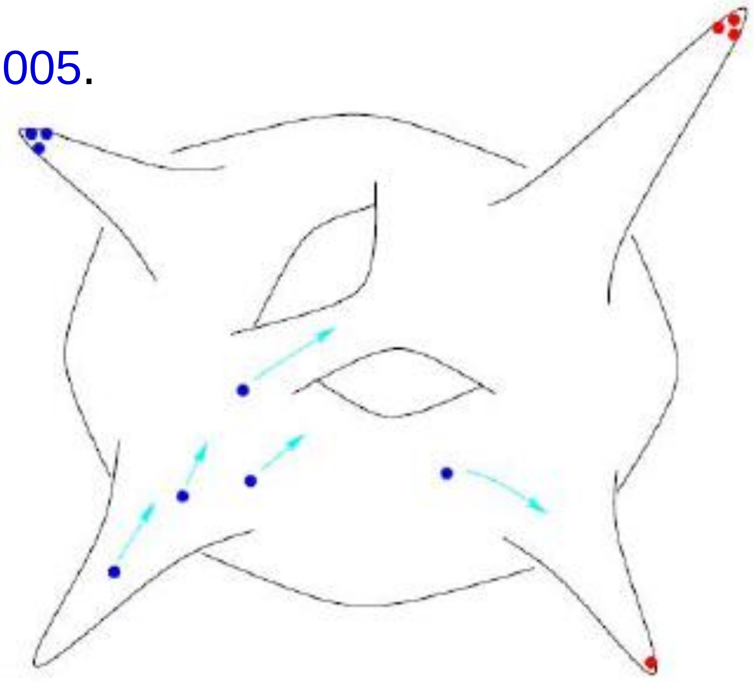
Inflation with multiple branes

- Basic idea:

  Slow-roll can be relaxed by introducing multiple degrees of freedom;
  Inflation is realized by a collection of branes in warped throats

KKLT version: Cline & Stoica,PRD72:126004,2005.

DBI version: CYF & Xia, PLB 677:226,2009;
CYF & Xue, PLB 680:395,2009



- Features:

  Brane positions as inflatons;
  Brane annihilation or collision as ending;
  Warped compactifications

- Motivations:

  To circumvent fine-tuning problem in usual brane inflation;
  To develop cosmological perturbation theory into a generic case;
  To find more testable predictions of string cosmology

- Dynamics of the branes:

$$S = \int d^4x \sqrt{-g} \left[\sum_I P_I(X_I, \phi_I) \right]$$

which involves N scalar fields, with

$$P_I(X_I, \phi_I) = \frac{1}{f(\phi_I)} \left[1 - \sqrt{1 - 2f(\phi_I)X_I} \right] - V_I(\phi_I)$$

- Consider the case of IR type potential: $V_I = V_{0I} - \frac{1}{2} m_I^2 \phi_I^2$

We have the solution:

$$\phi_I = -\frac{\sqrt{\lambda}}{t} \left(1 - \frac{9H^2}{2m_I^4 t^2} + \cdots \right)$$

and obtain a series of small sound speeds for these branes

$$c_{sI} \equiv \sqrt{1 - 2f(\phi_I)X_I} \sim 0$$

- Primordial perturbations:

(we consider a double-brane model)

Curvature perturbations:

$$\mathcal{R} \simeq \frac{\mathcal{N}^2}{2\pi\sqrt{\lambda_I}}\left(1 + \frac{27H^4}{2m_1^2 m_2^2 \mathcal{N}^2}\right)$$

Entropy perturbations:

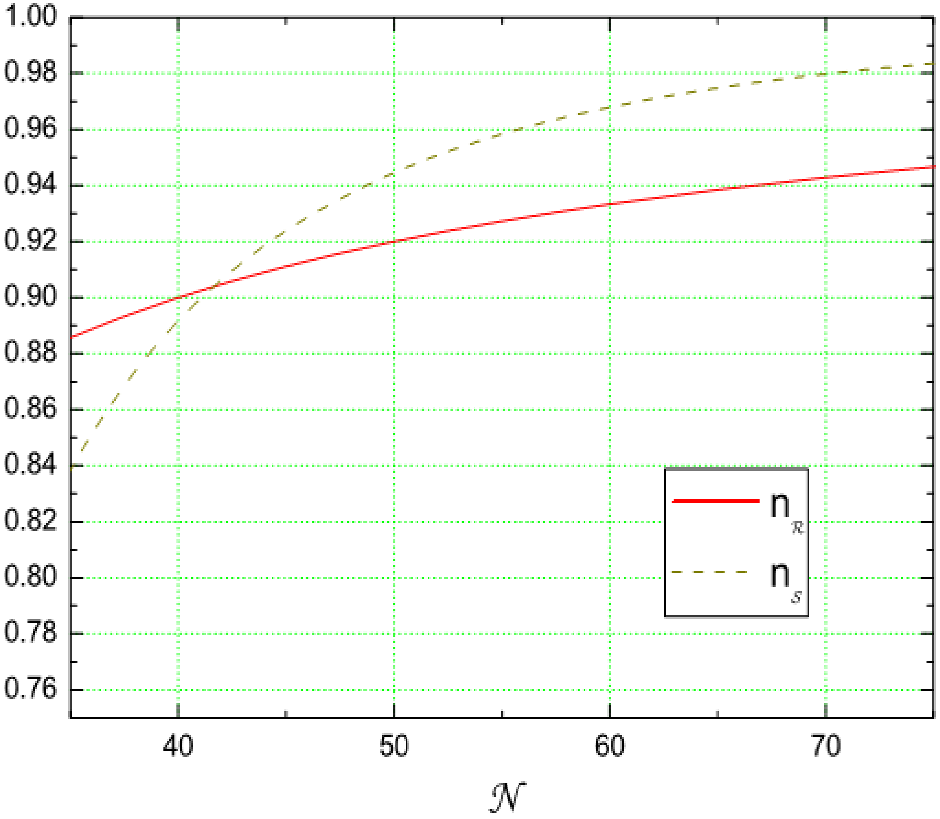
$$\mathcal{S} \simeq \frac{27H^4(m_1^2 - m_2^2)}{4\pi\sqrt{\lambda_I} m_1^3 m_2^3} + O(\mathcal{N}^{-2})$$

Non-Gaussianity of local type:

$$f_{\rm NL}^{\rm local} \sim \frac{q}{\mathcal{N}_1}$$

Non-Gaussianity of equilateral type:

$$f_{\rm NL} \sim 1/c_{s2}^2$$



Benefits of multiple brane inflation:

- Explaining the current observations well and relaxing the eta-problem as usual DBI inflation;
- Cooperating with BM and LH bounds

(BM bound can be enhanced due to multiple branes);

- Plentiful phenomena in the early universe

Main predictions:

- A homogeneous, isotropic and flat universe;
- Nearly scale-invariant CMB spectrum;
- Sizable local non-gaussianity & large equilateral one;
- Possible scale-invariant entropy perturbations → curvaton brane

Unclear issues:

- Back-reaction
- Reheating process is still unclear

Li, CYF & Piao, 2008

Zhang, CYF & Piao, 2009

CYF & Wang, 2010

Warm Brane Inflation

CYF, Dent & Easson, 2010

Bastero-Gil, Berera, Rosa, 2011

Preliminaries

- Basic picture of Warm Inflation:

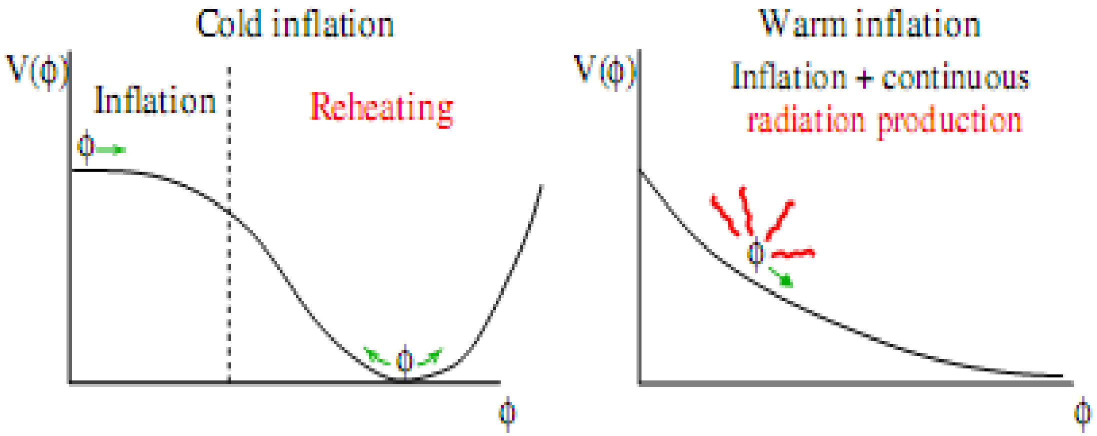
The paradigm of warm inflation suggests that, radiation can be preserved during inflation through a dissipative term, and the inflationary phase could smoothly end into a radiation dominated era without a separate reheating phase, by the process of the vacuum energy falling faster than the radiation energy.

Berera & Fang, 1995
Berera, 1995

- The key equation of motion:

$$\ddot{\phi} + [3H + \Upsilon]\dot{\phi} - \frac{1}{a^2(t)}\nabla^2\phi + V'(\phi) = \xi$$

where Υ is a dissipative term and ξ is a thermal noise.



Fang, 1980
Moss, 1985
Yokoyama & Maeda, 1988
…
Berera, Moss & Ramos, 2009

Warm Brane Inflation

CYF, Dent & Easson, 2010

- Basic equation $\frac{d}{dt}\left(\frac{\dot{\phi}}{c_s}\right) + (3H+\Gamma)\frac{\dot{\phi}}{c_s} - \frac{c_s}{a^2}\vec{\nabla}^2\phi + \frac{f'}{f}(1-c_s) + V' = 0$

- Slow ro $\epsilon = 2c_s m_p^2\left(\frac{H'}{H}\right)^2$: $\eta = 2c_s m_p^2\frac{H''}{H}$ $s = -2m_p^2\frac{c_s'}{c_s}\frac{H'}{H}$ $\alpha = 2c_s m_p^2\frac{\Gamma'}{\Gamma}\frac{H'}{H}$

These parameters are required to be less than $1+\gamma$, where $\gamma \equiv \Gamma/3H$

- Conservation of Energy and slow roll approximation yield

$$\dot{\rho}_r + 4H\rho_r = \Gamma\frac{\dot{\phi}}{c_s}$$

$$(3H+\Gamma)\dot{\phi} + c_s V' \simeq 0$$

- The exit mechanism: $\epsilon = 1 + \gamma$

It is unnecessary to require the probe brane annihilates with anti-brane at the infrared cutoff. Thus no dangerous tachyons and KK relics.

Warm Brane Inflation

- Perturbation analysis

$$\ddot{u}_k + (3H + \Gamma)\dot{u}_k + \frac{c_s^2 k^2}{a^2} u_k \simeq \theta_k$$

where u_k is canonical pert variable, rhs is the thermal noise satisfying

$$a^3 \langle \theta_{k_1}(t_1)\theta_{-k_2}(t_2) \rangle = 2\Gamma T (2\pi)^3 \delta(\vec{k}_1 - \vec{k}_2)\delta(t_1 - t_2)$$

- Sovling the above equation through Green function method, we finally get the primordial power spectrum and spectral index

$$P_\zeta \simeq \frac{\sqrt{3\pi}\gamma^{\frac{5}{2}} H T}{4 m_p^2 c_s \epsilon}$$

$$n_s - 1 \equiv \frac{d \ln P_\zeta}{d \ln k} \simeq \frac{1}{\gamma}\left(-\frac{3}{4}\epsilon + \frac{3}{2}\eta - \frac{9}{4}\alpha - \frac{7}{4}s \right)$$

Warm Brane Inflation

- Constraints from theory and observation:

$$\left.\begin{array}{ll}\text{throat volume constraint} & r < \frac{8P_\zeta}{\sqrt{3}\pi^{\frac{5}{2}}\,\mathrm{Vol}_5\,\gamma\lambda^2\Delta\mathcal{N}^6} \\ & \\ \text{lower bound on r} & r > \frac{8(1-n_s-\frac{\alpha}{2\gamma})}{75\gamma^{\frac{3}{2}}\lambda\sqrt{3f_{NL}}}\end{array}\right\} \quad \alpha > 2\gamma(1-n_s)$$

- Modified Lyth bound:

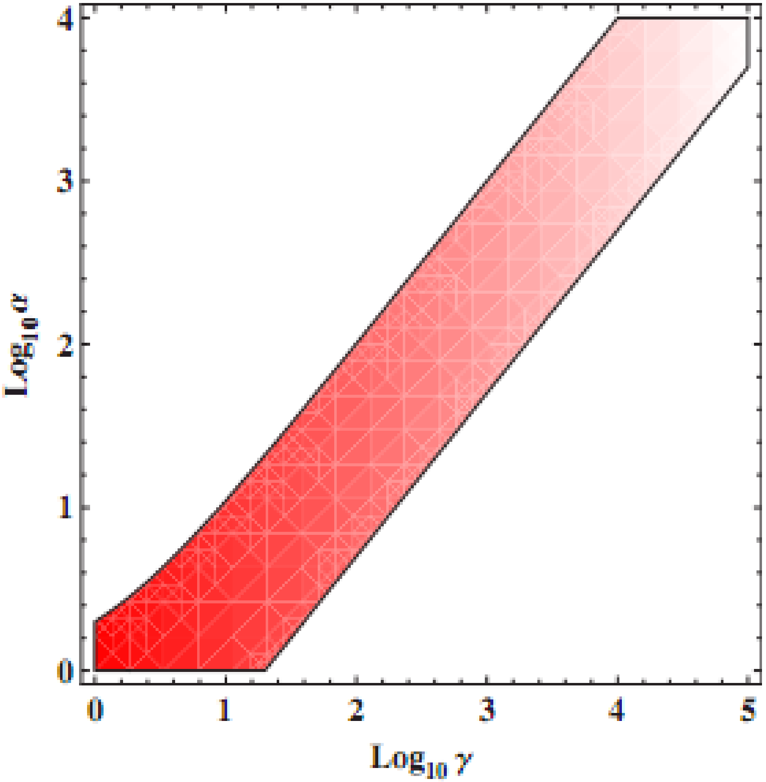
$$\Delta\phi = \frac{\dot{\phi}}{H}\Delta\mathcal{N} \simeq 2.75 r^{\frac{1}{2}}\bar{\Gamma}^{\frac{5}{4}}\bar{T}^{\frac{1}{2}}m_p\Delta\mathcal{N}$$

For sufficiently large dissipation, it is possible to have a large variation of the inflaton field even if the tensor-to-scalar ratio is unobservable. This interesting characteristic behavior of warm inflation is present even in the non-DBI limit.

Warm Brane Inflation

- Numerical result:

Phase space of γ and α from observational and theoretical constraints



Warm Brane Inflation

Features:

- It provides another approach to entering radiation domination, and might alleviate the problem of unwanted relic production in usual brane inflation;

- The origin of primordial curvature perturbation is a thermal noise, and thus different predictions for observations are obtained;

- The slow roll conditions are relaxed by a large value of parameter gamma, and thus a potential phase space could be found.

Summary

- A brief review of brane inflation models

- Current difficulties in brane inflation
    - Eta problem
    - Inconsistency

- Multi-brane Inflation

- Warm brane inflation

Happiness only real when shared
from “Into the wild”

Thank You !