Quantum error mitigation

Ying Li, 2023

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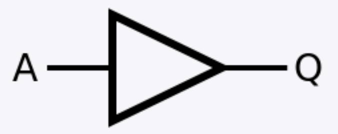
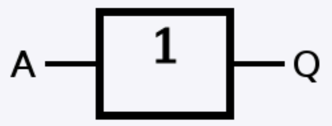
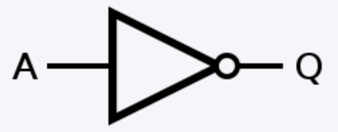
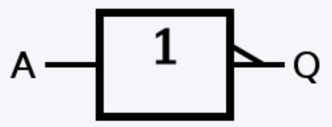
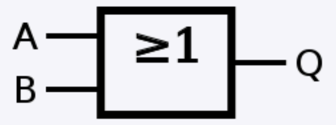
Universal / digital / circuit-based quantum computer

Classical computing with logic gates

<table>
<tr><td>Buffer</td><td></td><td></td><td>A</td><td><table><tr><th>INPUT</th><th>OUTPUT</th></tr><tr><td>A</td><td>Q</td></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr></table></td></tr>
<tr><td>NOT
(inverter)</td><td></td><td></td><td>$\overline{A}$ or $\neg A$</td><td><table><tr><th>INPUT</th><th>OUTPUT</th></tr><tr><td>A</td><td>Q</td></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table></td></tr>
</table>

<table>
<tr><td>AND</td><td></td><td></td><td>$A \cdot B$ or $A \land B$</td><td><table><tr><th colspan="2">INPUT</th><th>OUTPUT</th></tr><tr><td>A</td><td>B</td><td>Q</td></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table></td></tr>
<tr><td>OR</td><td></td><td></td><td>$A + B$ or $A \lor B$</td><td><table><tr><th colspan="2">INPUT</th><th>OUTPUT</th></tr><tr><td>A</td><td>B</td><td>Q</td></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table></td></tr>
</table>

https://en.wikipedia.org/wiki/Logic_gate

- Logic gates are operations on bits.
- {NOT, AND} and {NOT, OR} are universal gate sets.

Qubits and quantum gates

Superposition state of a qubit:

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

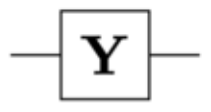
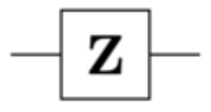
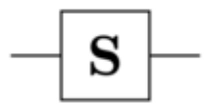
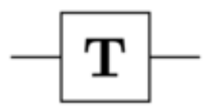
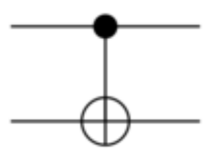
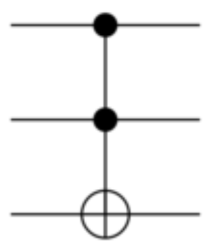
$$= \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} \end{pmatrix}$$

Universal gate sets:

- Clifford gates + one non-Clifford

  $\{\mathrm{H}, \mathrm{T}, \mathrm{CNOT}\}$ or $\{\mathrm{H}, \mathrm{T}, \mathrm{CZ}\}$

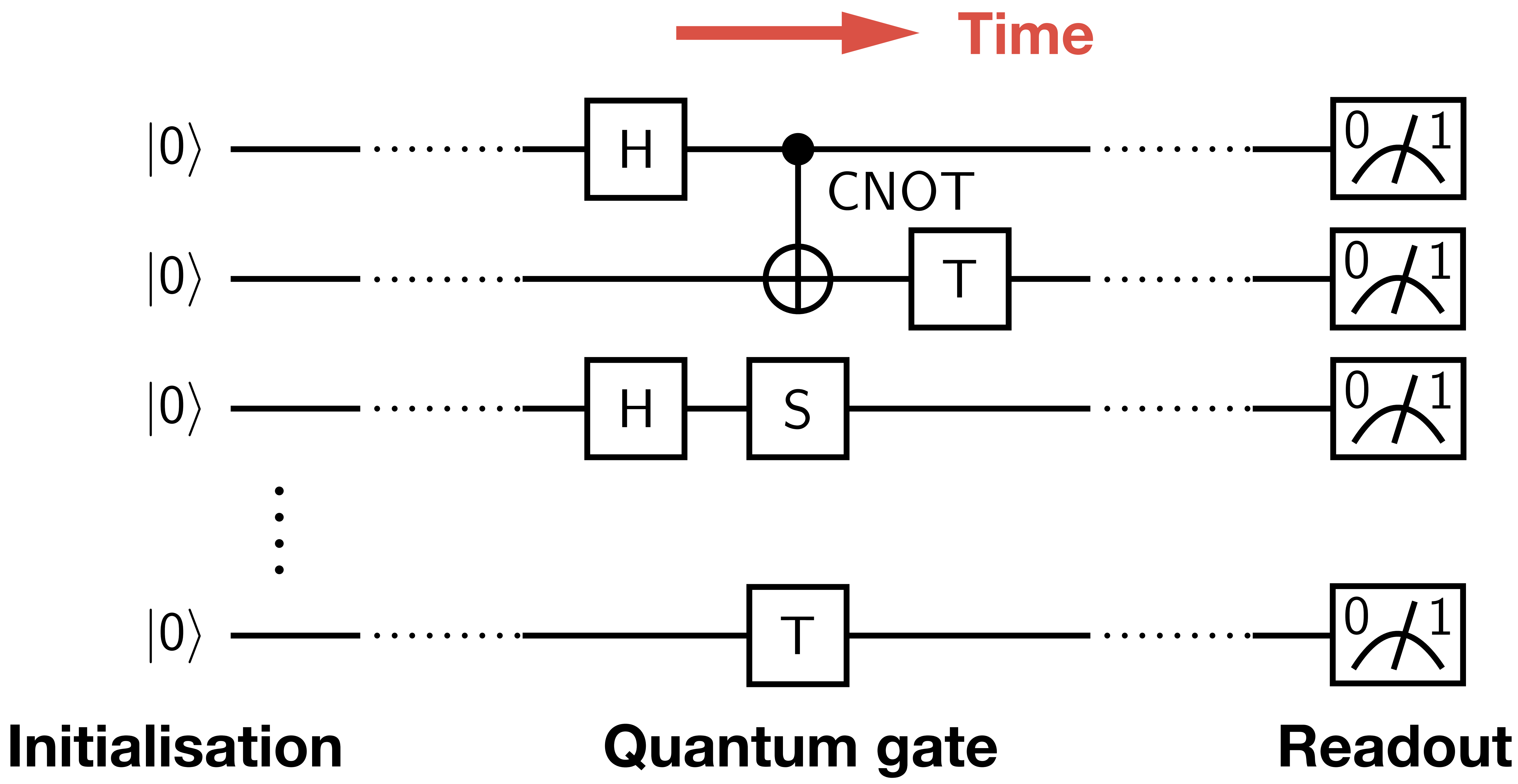
  (Realistic gate sets)
- Toffoli and Hadamard $\{\mathrm{CCNOT}, \mathrm{H}\}$

Operator	Gate(s)	Matrix
Pauli-X (X)		$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)		$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)		$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)		$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$
Controlled Not (CNOT, CX)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z (CZ)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
SWAP		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)		$\begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0 \end{bmatrix}$

https://en.wikipedia.org/wiki/Quantum_logic_gate

Universal quantum computer (Quantum Turing machine)

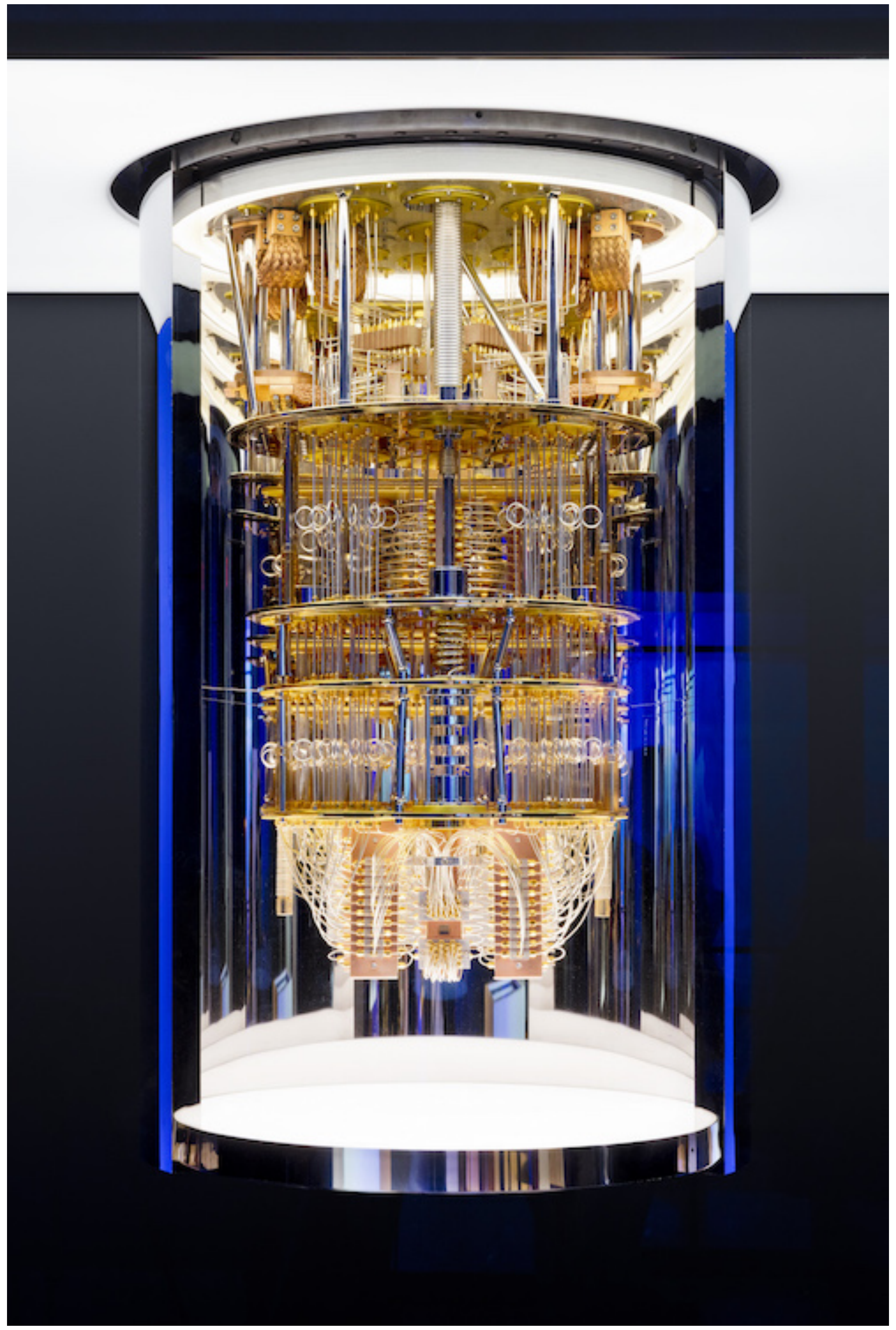
Quantum circuit:



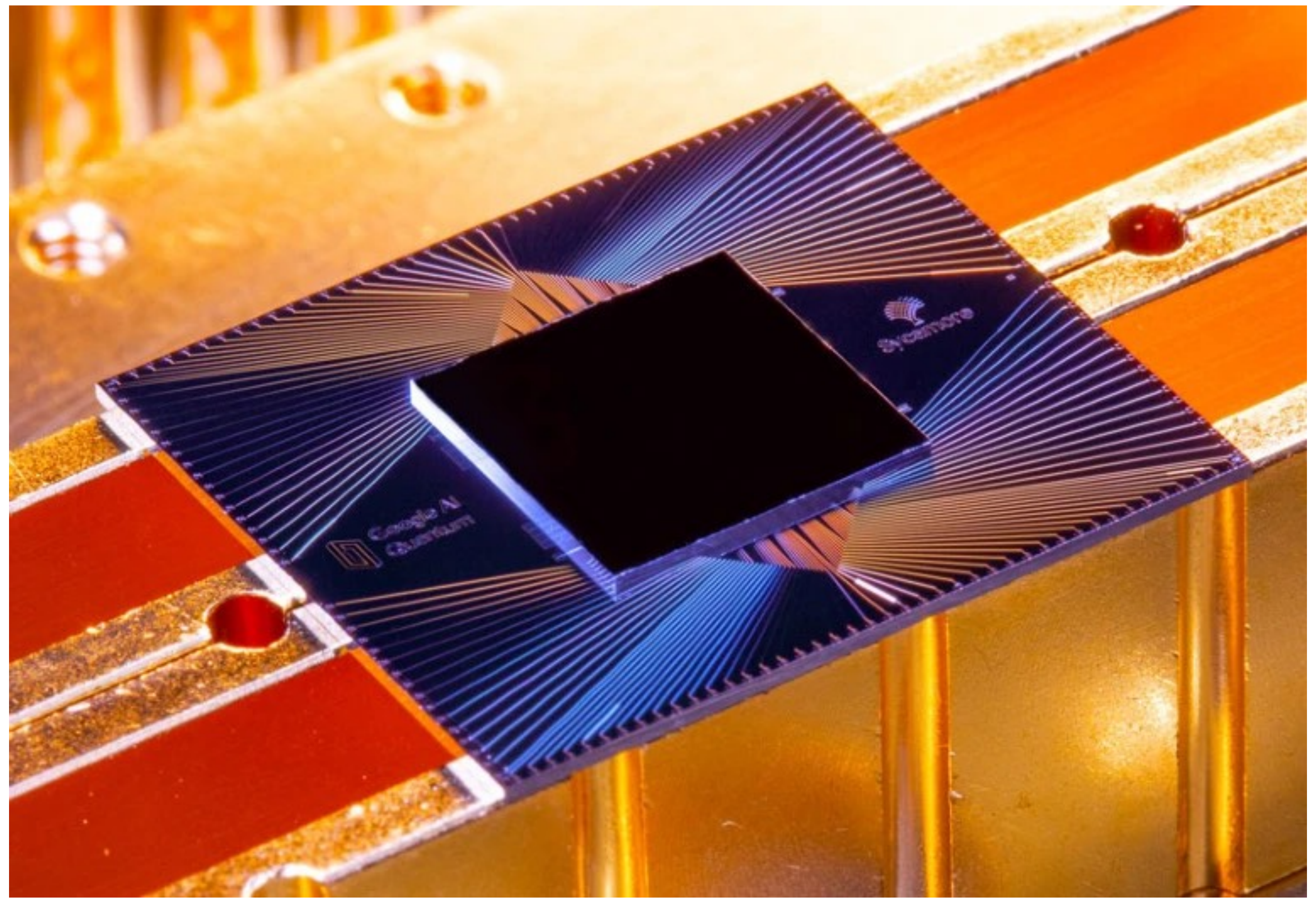
	Classical Computing	Quantum Computing
Information carrier	Bit	Qubit
Operation	Logic gate (Boolean function)	Quantum gate (Unitary transformation)
Universality	General Boolean function	General unitary transformation

David Deutsch, Proceedings of the Royal Society A 400, 97-117 (1985)

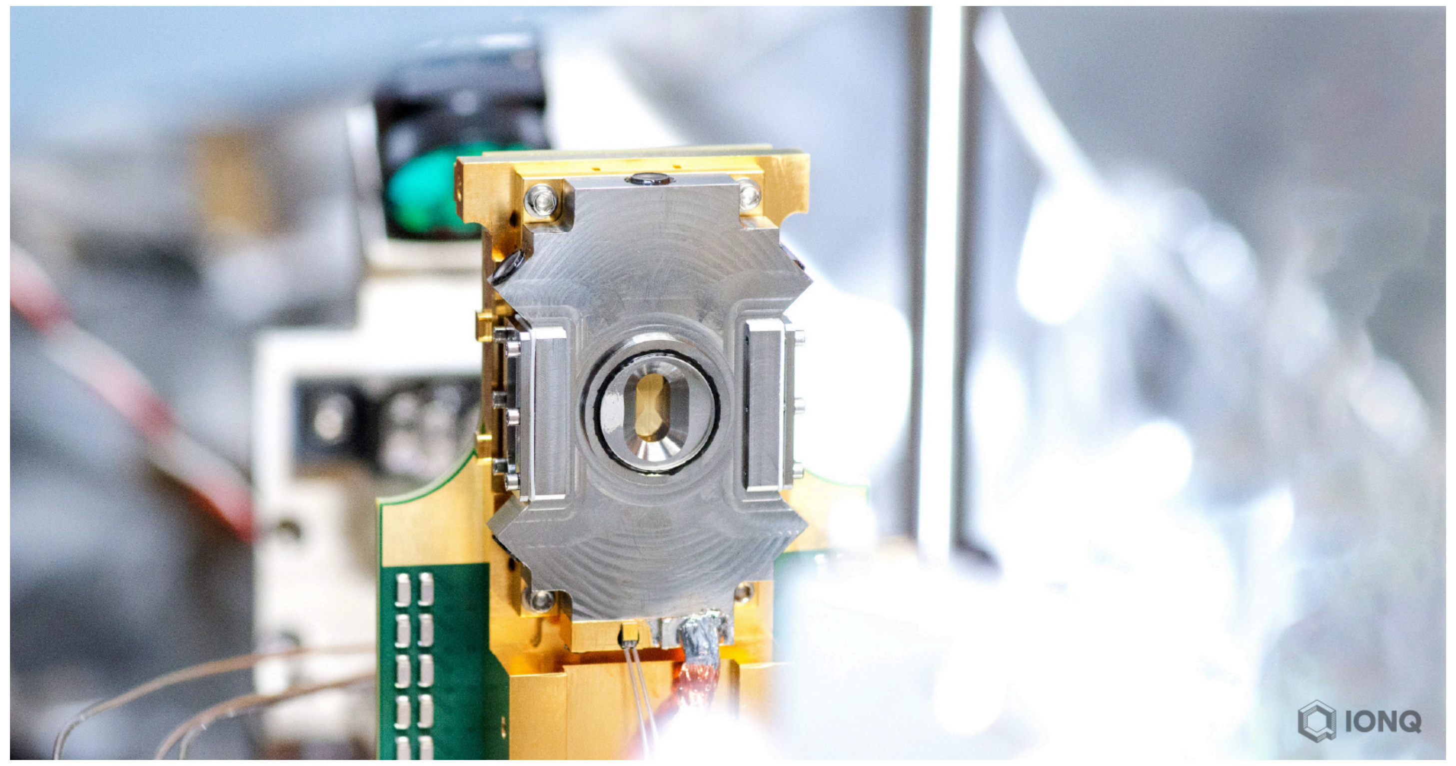
Hardware



IBM Quantum System One
https://qiskit.org/documentation/qc_intro.html



Google's 54-qubit Sycamore chip
https://www.nature.com/articles/d41586-019-03213-z



IonQ's Aria system
https://ionq.com/news/march-21-2022-ionq-aria-coming-to-microsoft-azure-quantum

- Superconducting qubits
  Solid, fast and scalable
- Ion trap
  Accurate, light-matter interface
- Photonics
- Quantum dot
- Neutral atoms
- Majorana fermions
- ……

1. Background
2. What is quantum error mitigation
3. Error-model-based approaches
4. Constraint-based approaches
5. Learning-based approaches

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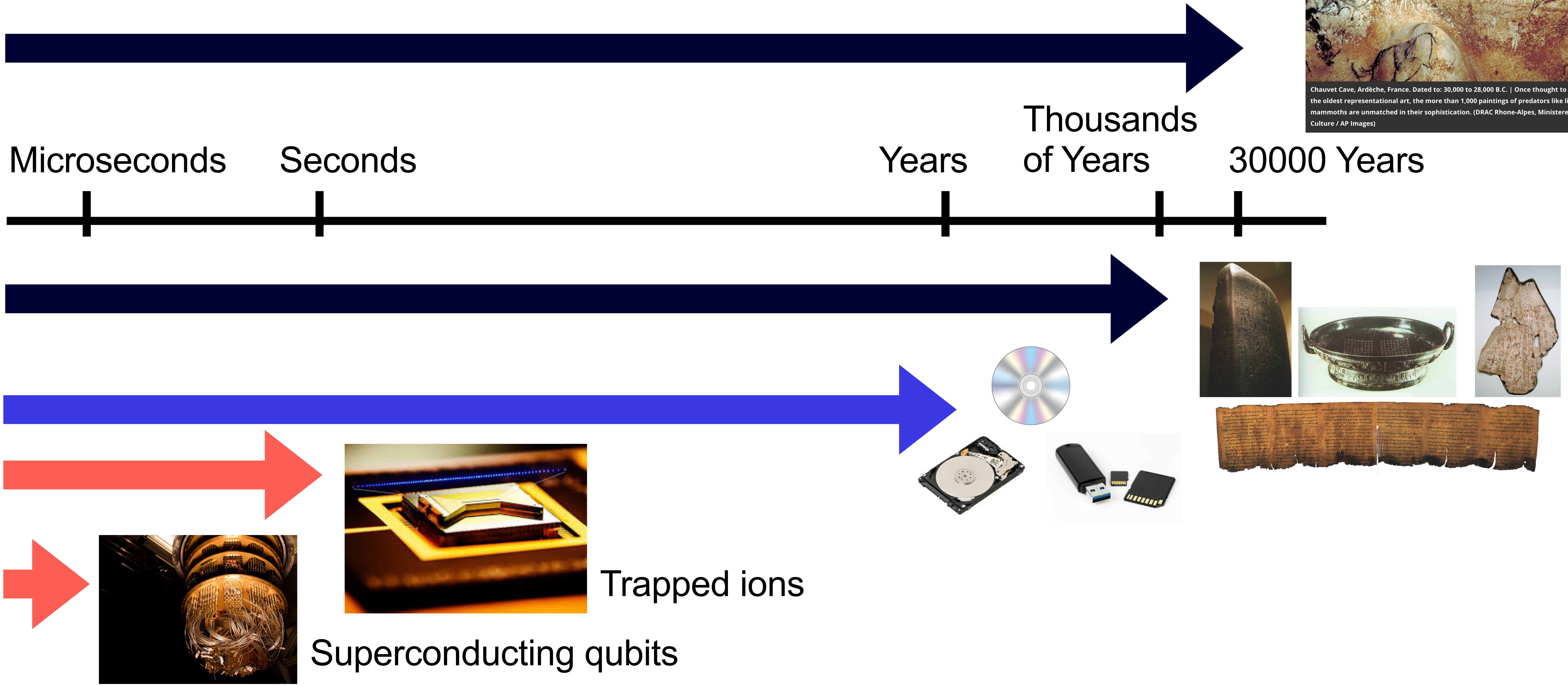
Decoherence and memory time


Chauvet Cave, Ardèche, France. Dated to: 30,000 to 28,000 B.C. | Once thought to house the oldest representational art, the more than 1,000 paintings of predators like lions and mammoths are unmatched in their sophistication. (DRAC Rhone-Alpes, Ministere de la Culture / AP Images)



Resilience of classical information

马未都：最另类的“造假”，把一幅真画一揭为二，变成两幅真品

现在很多艺术作品的造假，目的基本都是为了盈利，按照前辈优秀的真迹，仿照着制作出看起来一模一样的仿制赝品，为了能够以次充好，获得利益。而本文所要说的另类造假，严格地说并不是真正的一种造假，因为做出来的东西其实并不是赝品，东西是真品，因为是真的，就很容易被人粗心大意收入了。这算是一种最另类的造假了，本来只有一个作品，经过作伪者精心仿制之后，竟然分成了两个，甚至三个四个，一下子利润就番了几番。

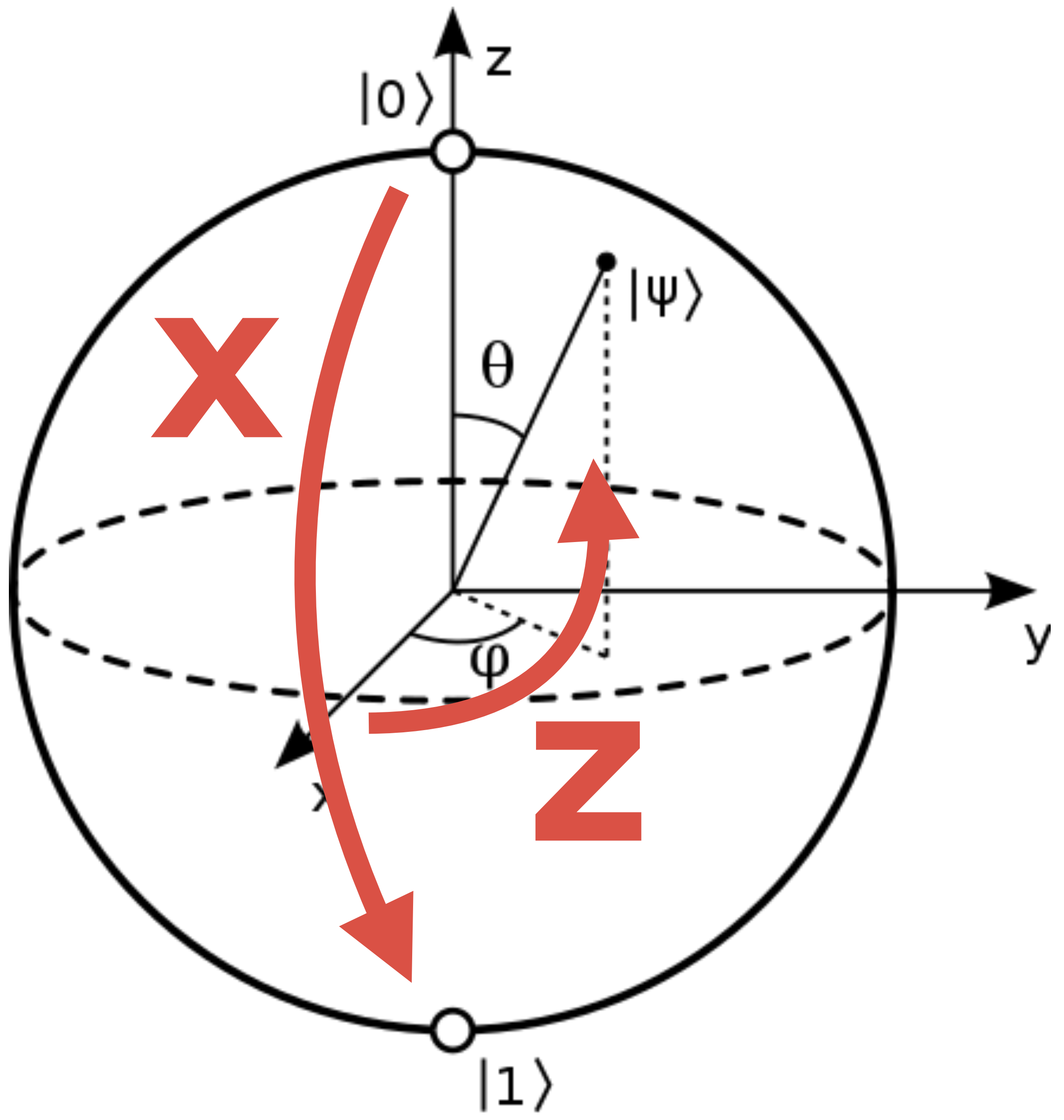
Quantum error correction

Classical error correction:

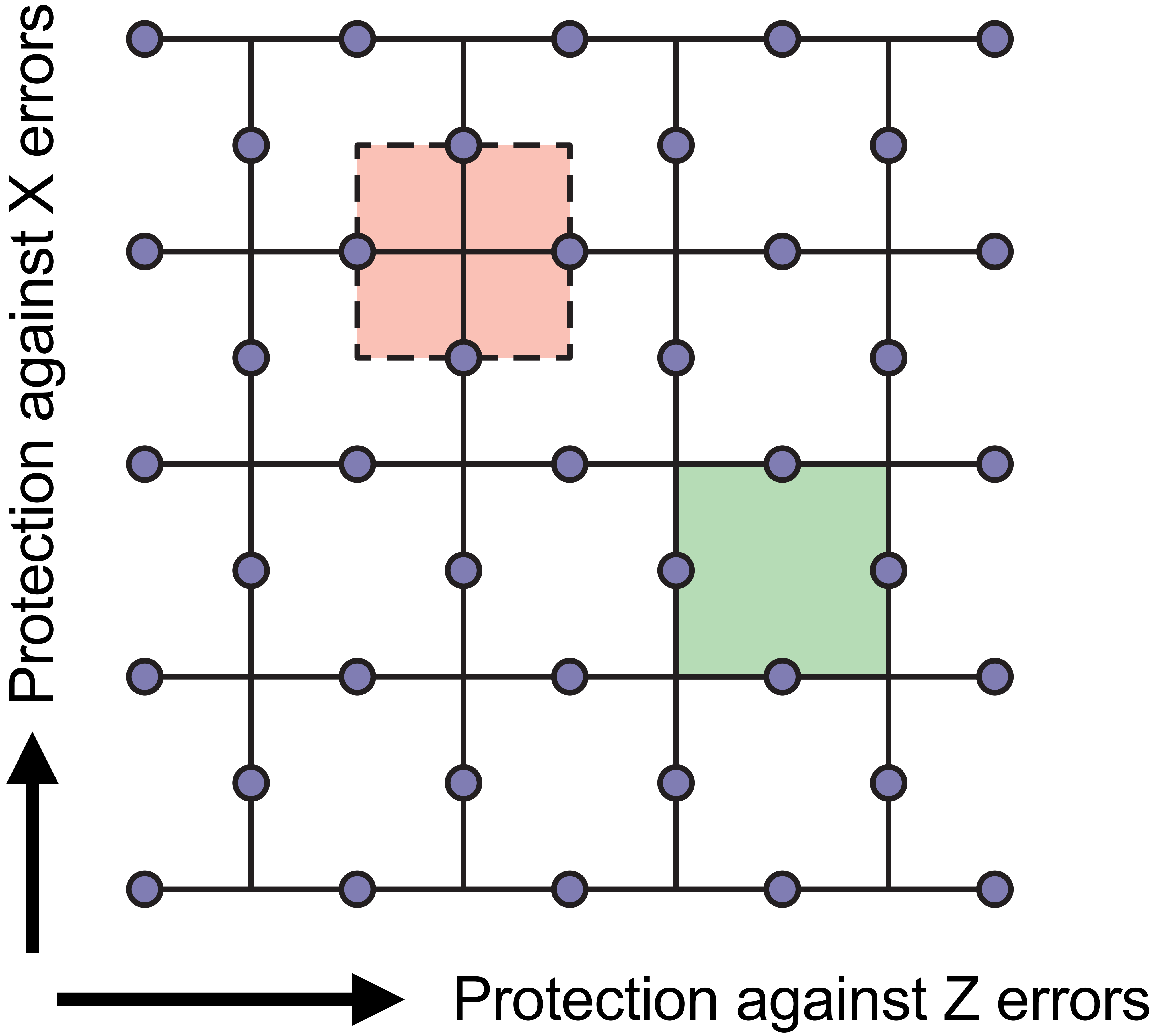
$\mathbf{0} = 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0 \ldots$

$\mathbf{1} = 1\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 1 \ldots$

Two types of errors on qubits:

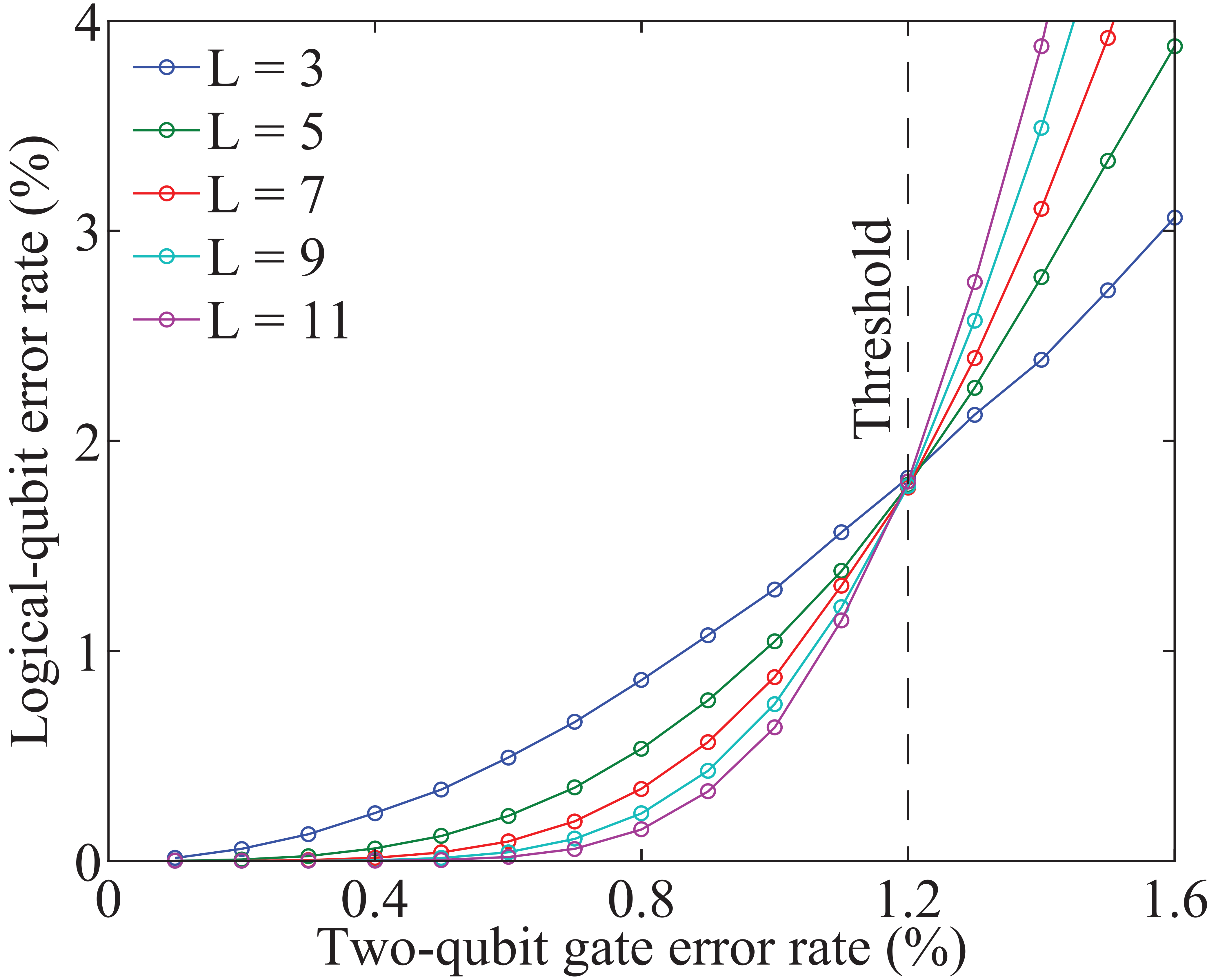


Surface code:



A. Yu. Kitaev, Annals Phys. 303, 2-30 (2003)
Robert Raussendorf and Jim Harrington, Phys. Rev. Lett. 98, 190504 (2007)
Austin G. Fowler, Ashley M. Stephens, and Peter Groszkowski, Phys. Rev. A 80, 052312 (2009)

Surface-code threshold



Thresholds of surface code:

0.75%, Robert Raussendorf and Jim Harrington, 2006

> 1%, David Wang, Austin Fowler and Lloyd Hollenberg, 2011

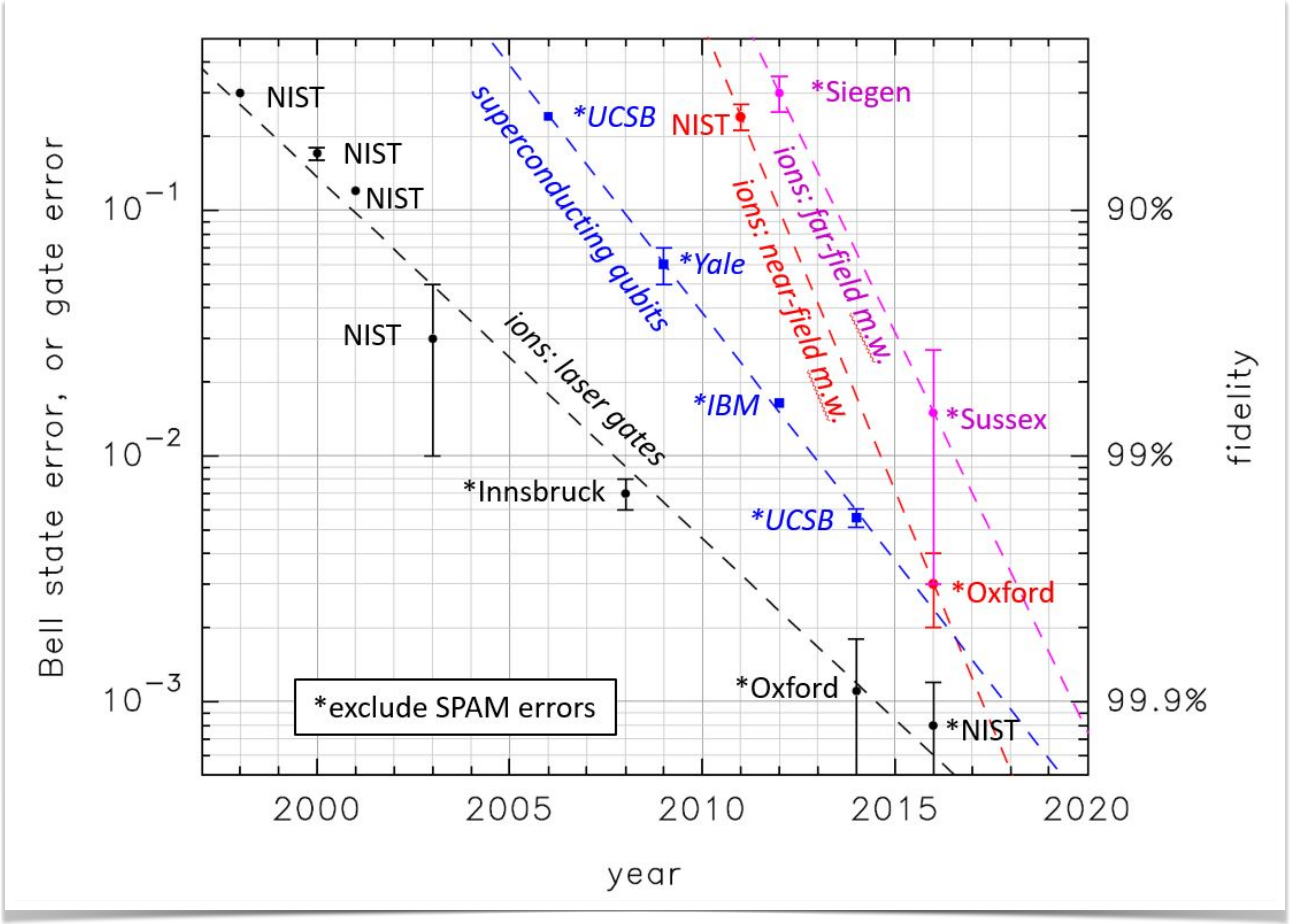
Quantum fault tolerance

Threshold theorem:

- If the physical error rate is lower than the threshold,
- the logical error rate can be suppressed to an arbitrarily low level,
- and the number of physical qubits for encoding is polynomial in one over the logical error rate.

Dorit Aharonov and Michael Ben-Or, arXiv:quant-ph/9611025
Emanuel Knill, Raymond Laflamme, and Wojciech H. Zurek, Science 279, 342-345 (1998)

Error rate over years



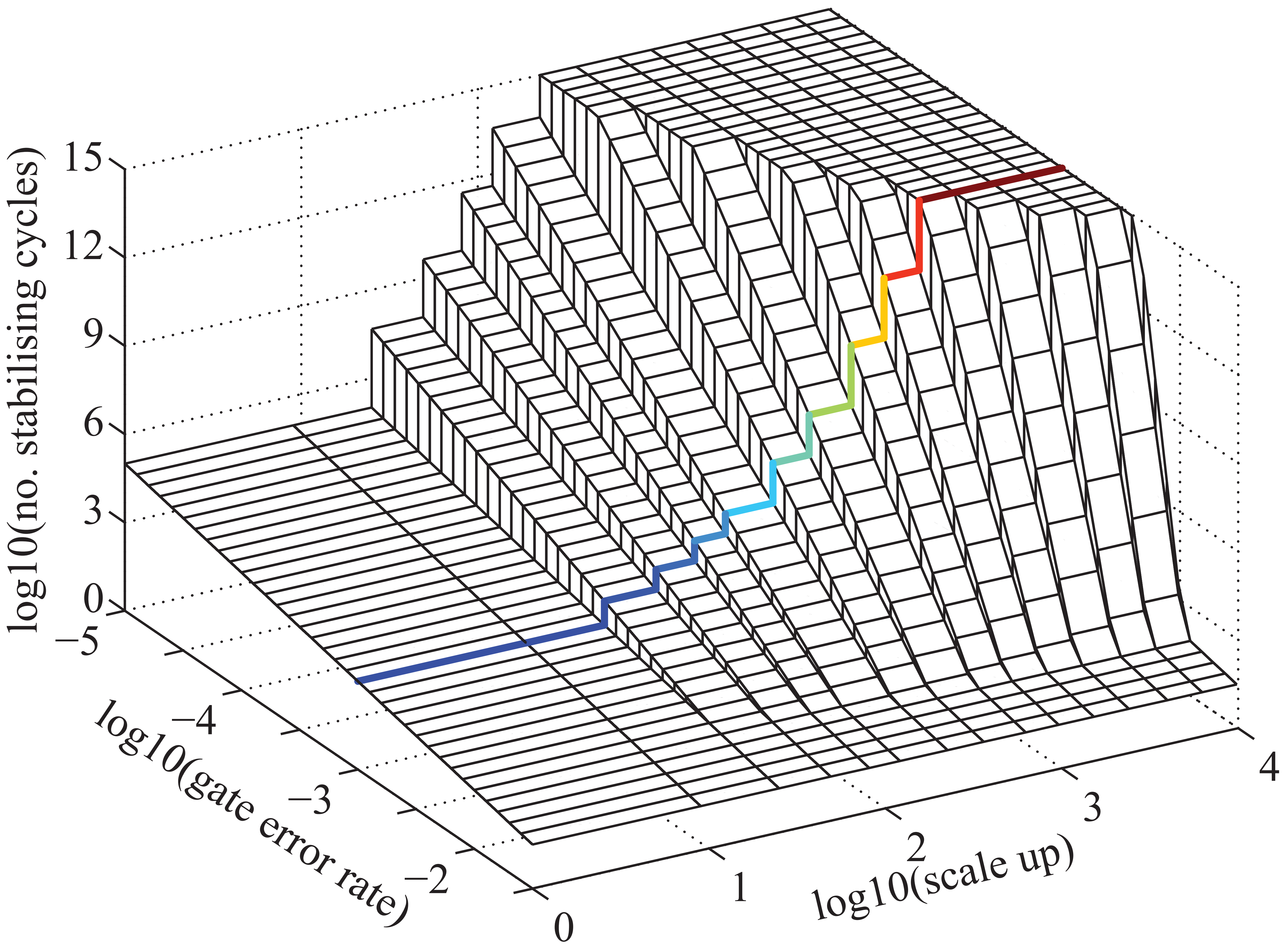
https://nqit.ox.ac.uk/content/ion-traps

Threshold error rate above 1%

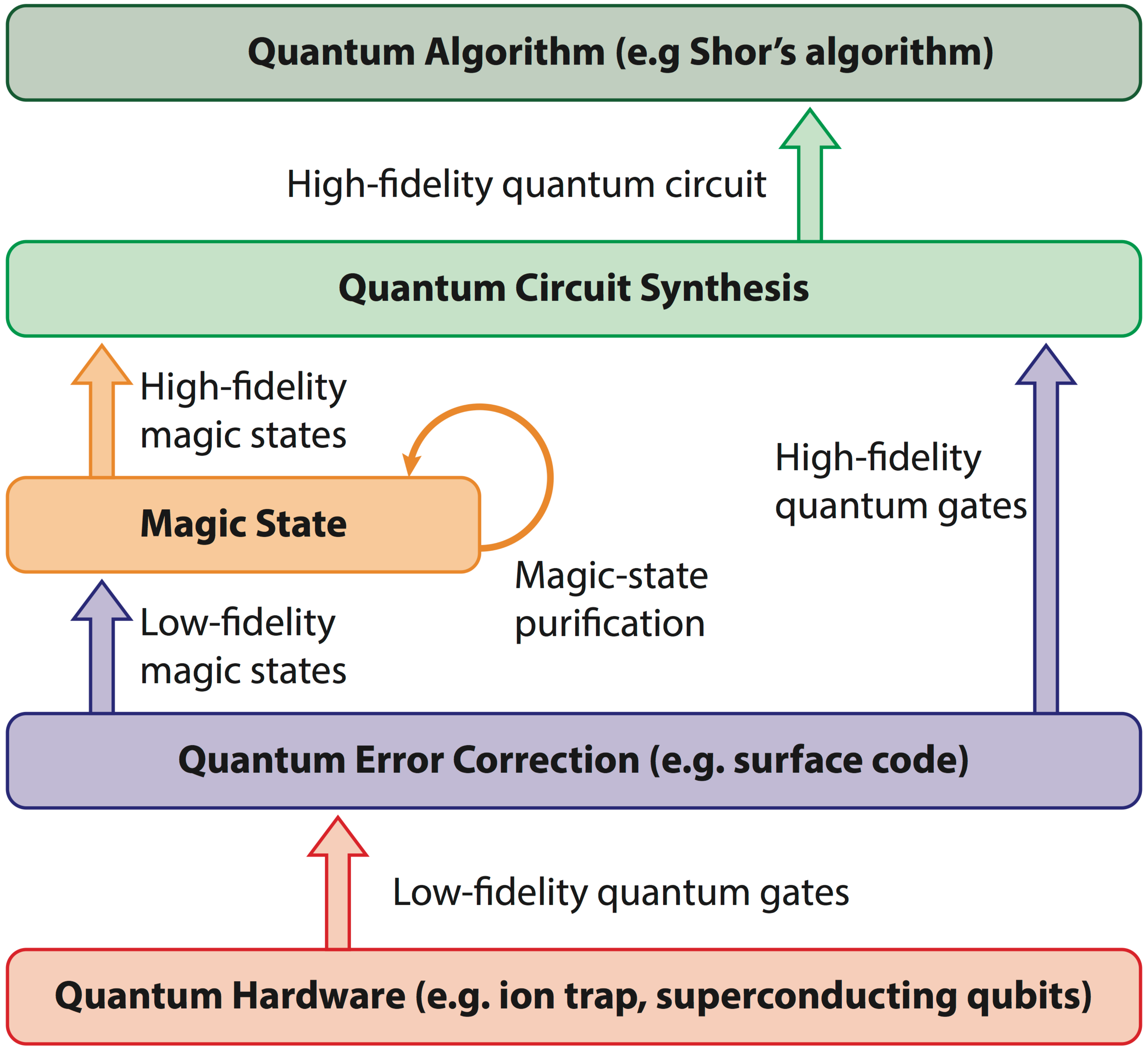
Gap #1 has been closed!!!

Technology today: Error rate below 0.1%

Fault-tolerant quantum computing and the qubit overhead

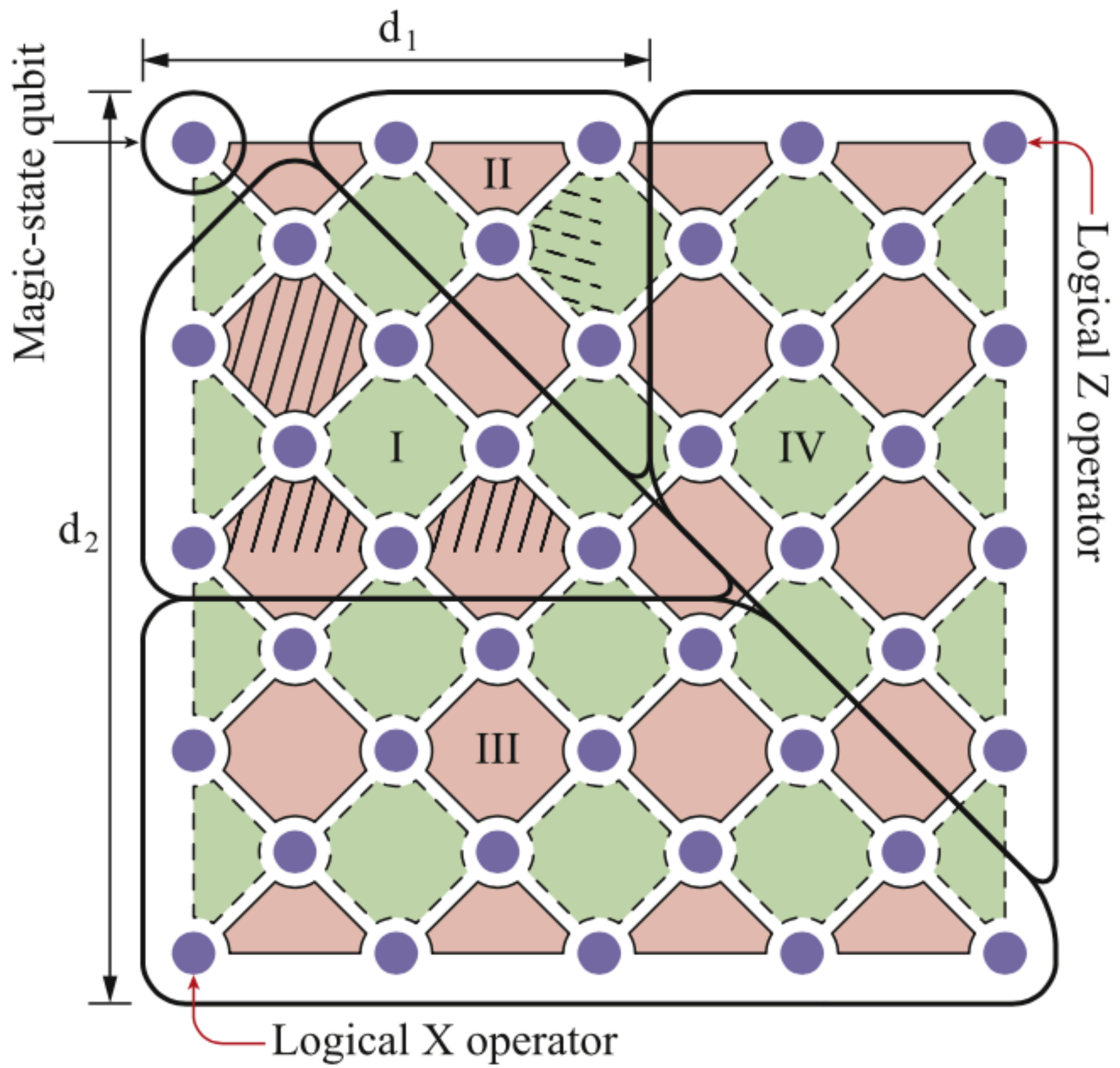


Encoding cost of surface code



Fault-tolerant quantum computing

Magic-state encoding

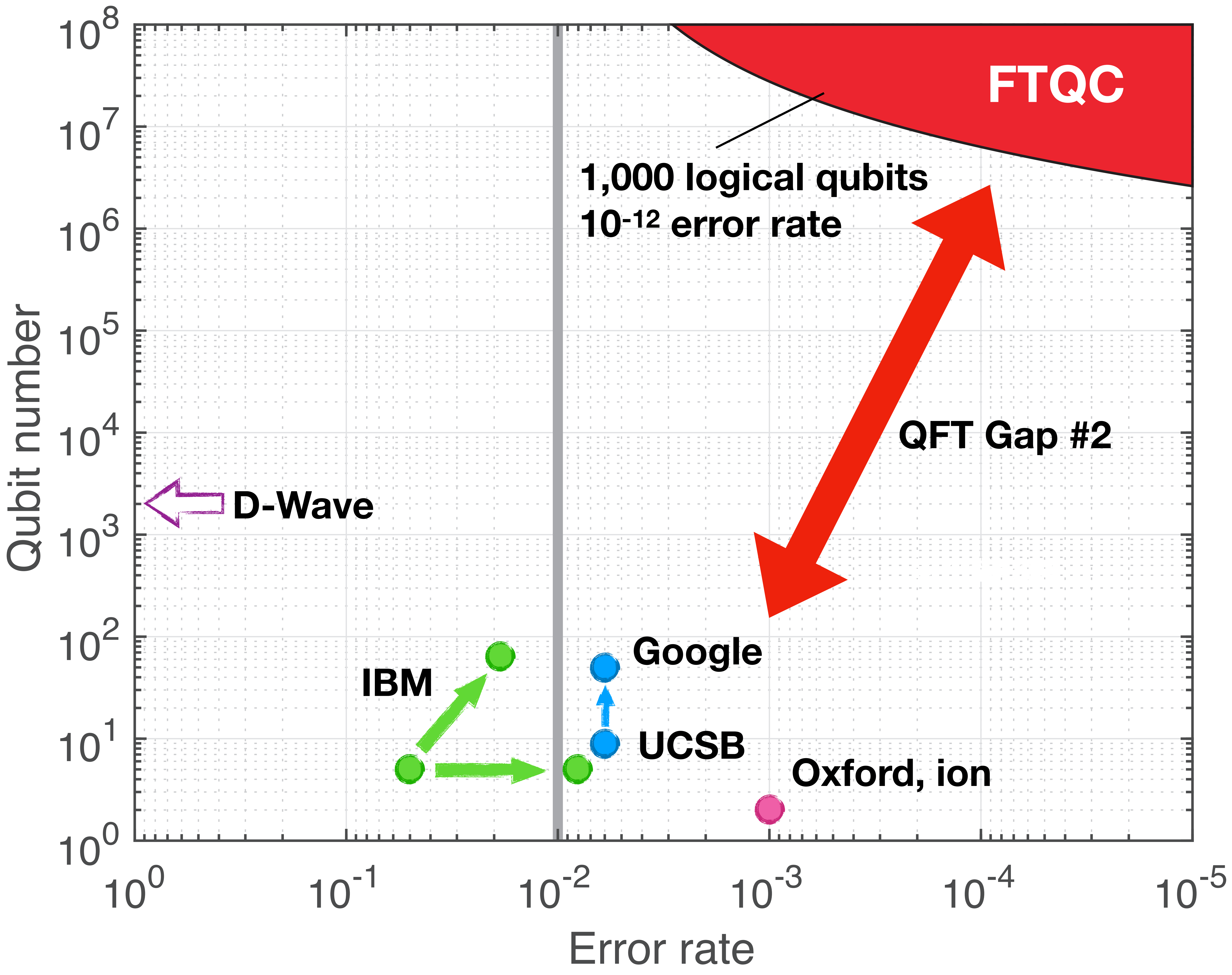


YL, New J. Phys. 17, 023037 (2015)

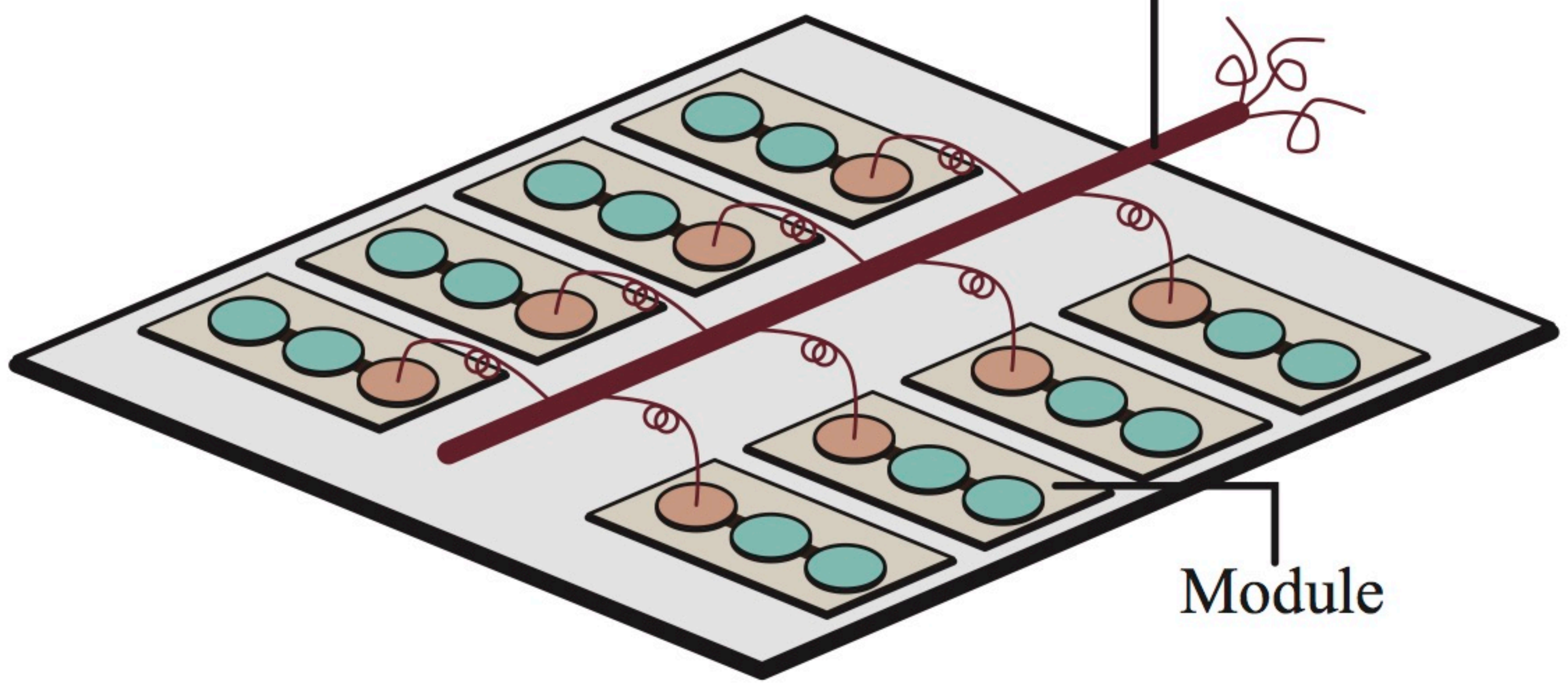
Qubit cost of fault-tolerant quantum computing

	Magic states required		Space-time overhead per magic state in qubit rounds		Physical qubits in factory (and evaluation time) required for time-optimal computation			
					$p_g = 10^{-3}$, $t_{meas/ff} = 0.1 t_{sc}$		$p_g = 10^{-4}$, $t_{meas/ff} = 0.1 t_{sc}$	
Problem	Type	Count	$p_g = 10^{-3}$	$p_g = 10^{-4}$	$t_{sc} = 10^{-3}$ s	$t_{sc} = 10^{-5}$ s	$t_{sc} = 10^{-3}$ s	$t_{sc} = 10^{-5}$ s
1000-bit Shor	Toffoli	$10^{10.60}$	1.41×10^7	5.35×10^5	1.73×10^8 **(6.6 weeks)**	1.73×10^8 **(11 h)**	6.30×10^6 **(6.6 weeks)**	6.30×10^6 **(11 h)**
2000-bit Shor	Toffoli	$10^{11.51}$	1.66×10^7	5.71×10^5	2.18×10^8 **(53 weeks)**	2.18×10^8 **(3.7 days)**	6.97×10^6 **(53 weeks)**	6.97×10^6 **(3.7 days)**
4000-bit Shor	Toffoli	$10^{12.41}$	1.94×10^7	6.12×10^5	2.50×10^8 **(8 years)**	2.50×10^8 **(4.2 weeks)**	7.69×10^6 **(8 years)**	7.69×10^6 **(4.2 weeks)**

Joe O'Gorman and Earl T. Campbell, Phys. Rev. A 95, 032338 (2017)


FTQC
1,000 logical qubits
10-12 error rate
QFT Gap #2
D-Wave
IBM
Google
UCSB
Oxford, ion
Qubit number
Error rate
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^0
10^-1
10^-2
10^-3
10^-4
10^-5

Optical and network quantum computing



Size	Local error rate	Network error rate
0[1]	--	0.001%
1[2]	--	0.01%
2[3,4]	0.1%	1%
3[4]	0.1%	10%
4[4]	0.2%	30%
4[5]	0.775%	10%
5[6]	0.825%	10%
31×31[6]	0.1%	10%
35 (1D)[7]	0.1%	0.1%

[1] **YL**, Peter C. Humphreys, Gabriel J. Mendoza, and Simon C. Benjamin, Phys. Rev. X 5, 041007 (2015)

[2] **YL**, Sean D. Barrett, Thomas M. Stace, and Simon C. Benjamin, Phys. Rev. Lett. 105, 250502 (2010)

[3] **YL**, Daniel Cavalcanti, and Leong Chuan Kwek, Phys. Rev. A 85, 062330 (2012)

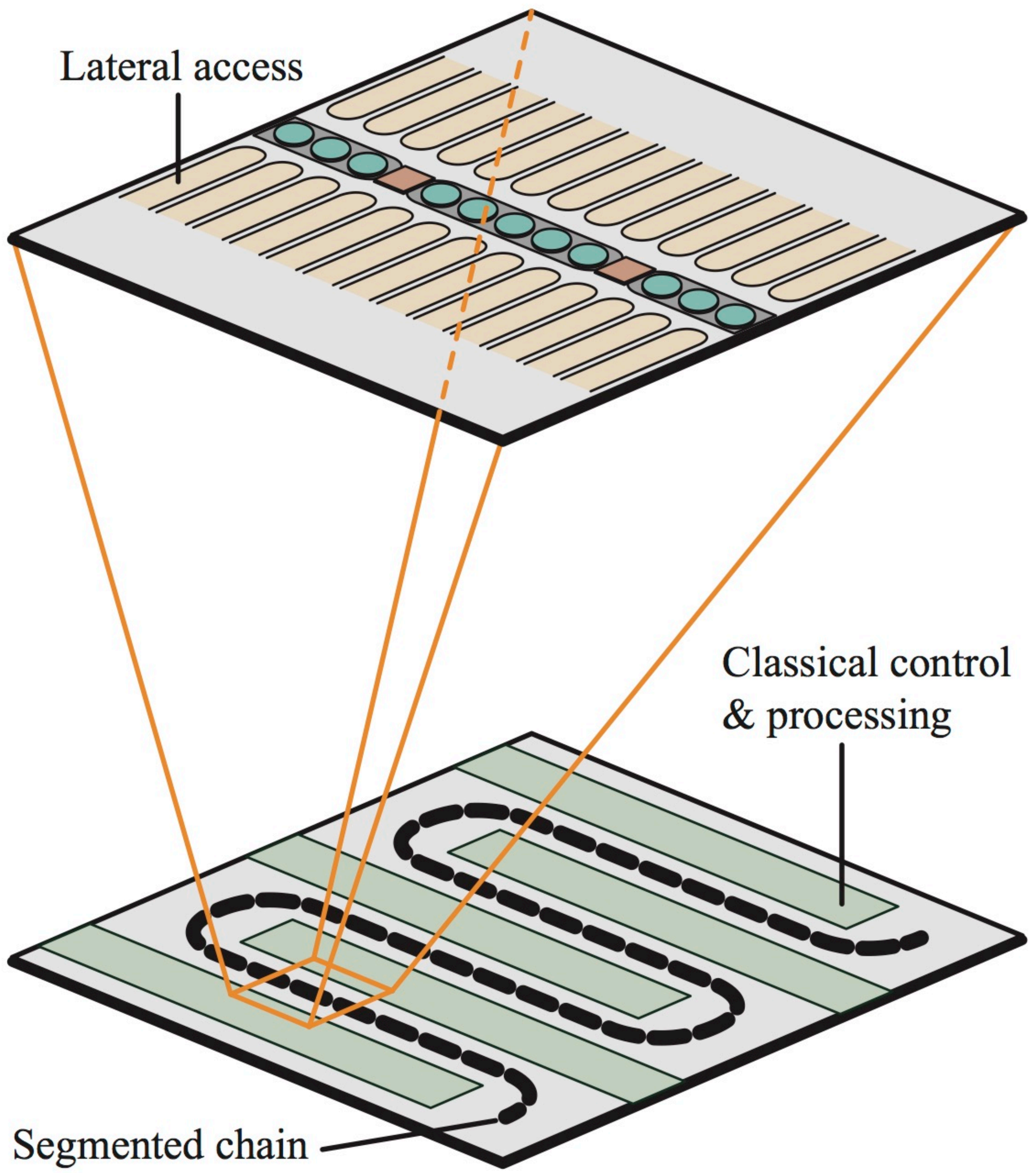
[4] **YL** and Simon C. Benjamin, New J. Phys. 14, 093008 (2012)

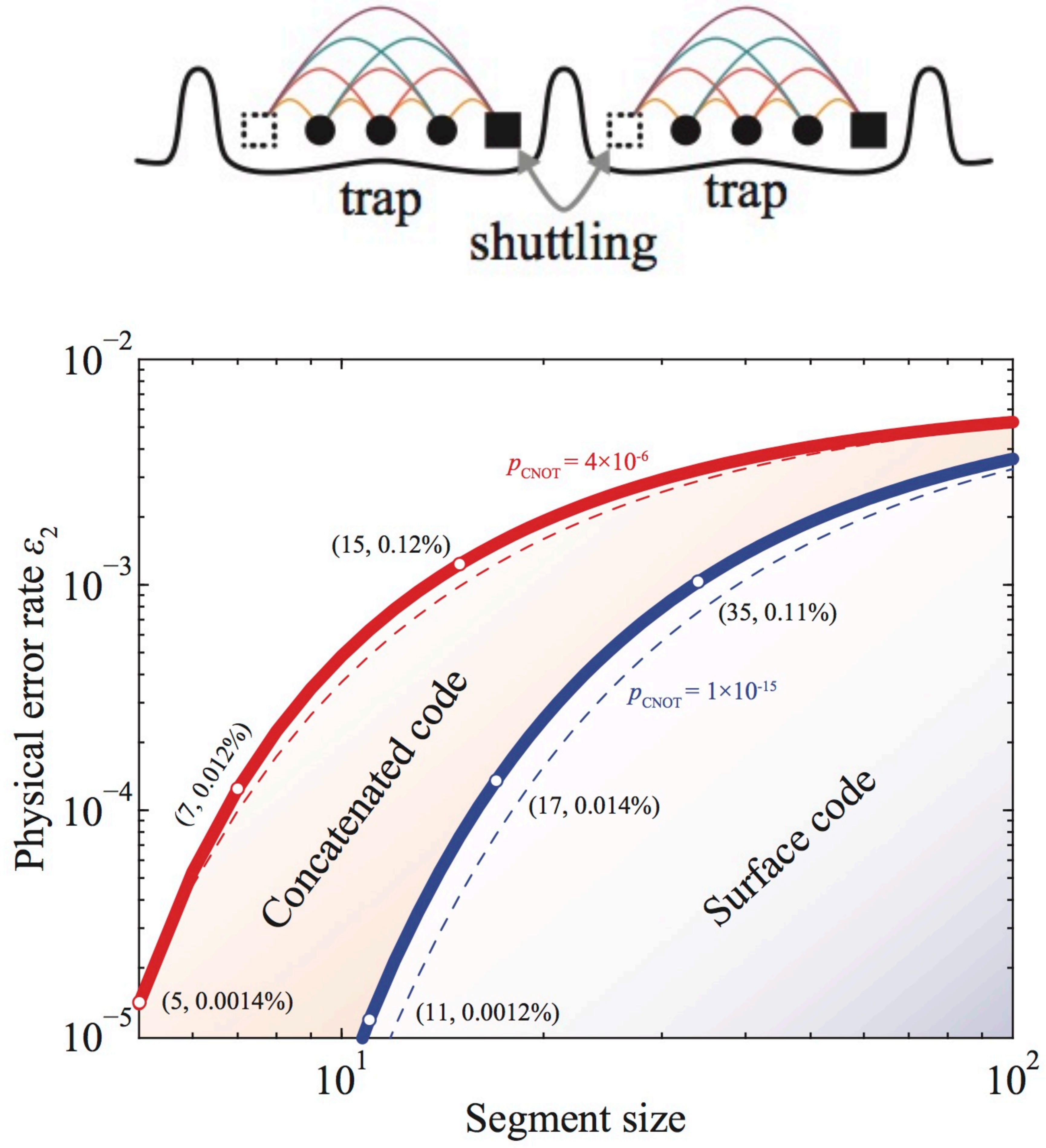
[5] Naomi H. Nickerson, **YL**, and Simon C. Benjamin, Nat. Commun. 4, 1756 (2013)

[6] **YL** and Simon C. Benjamin, Phys. Rev. A 94, 042303 (2016)

[7] **YL** and Simon C. Benjamin, npj Quantum Inf. 4, 25 (2018)

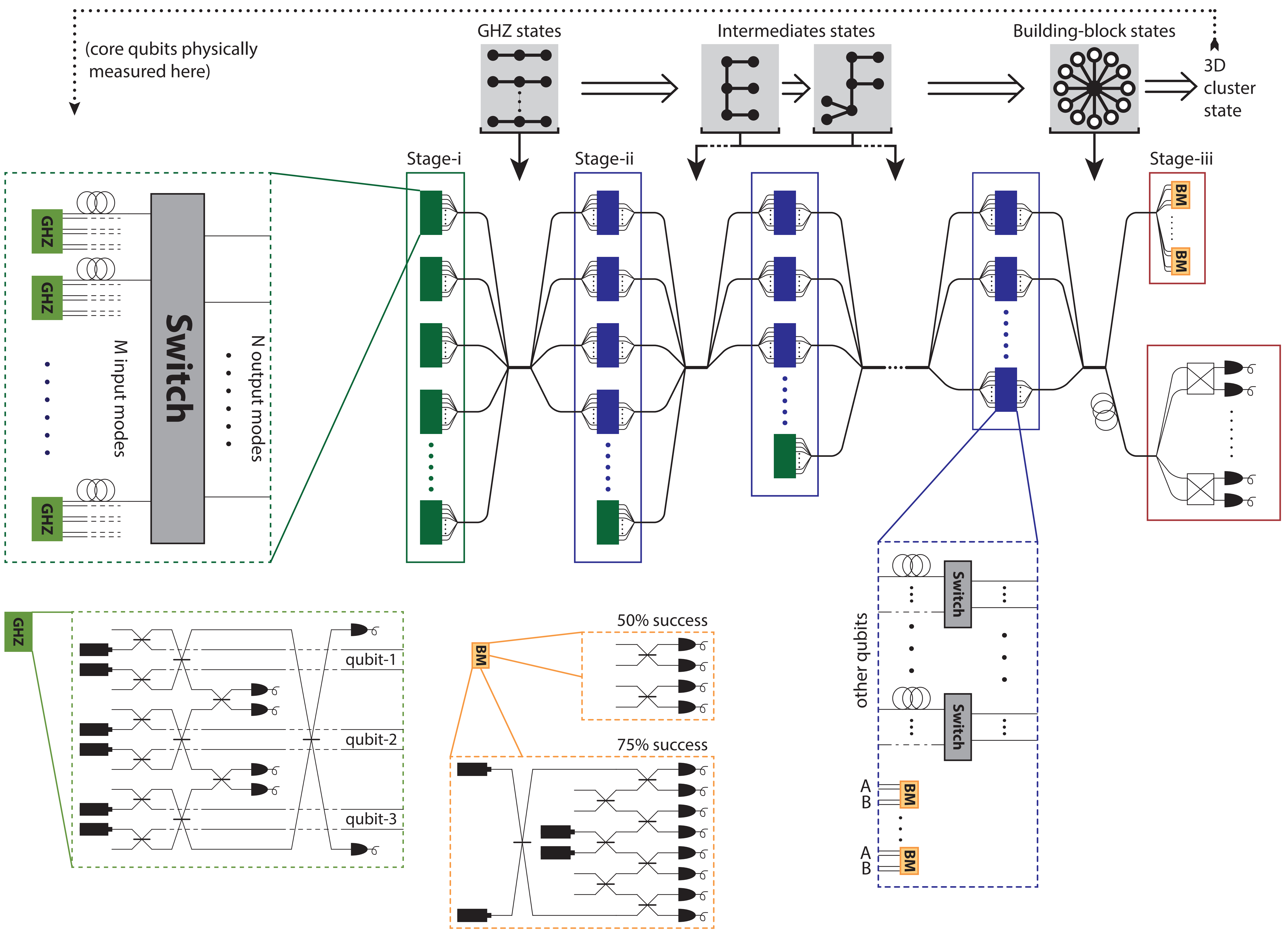
Quantum computing on segmented chain

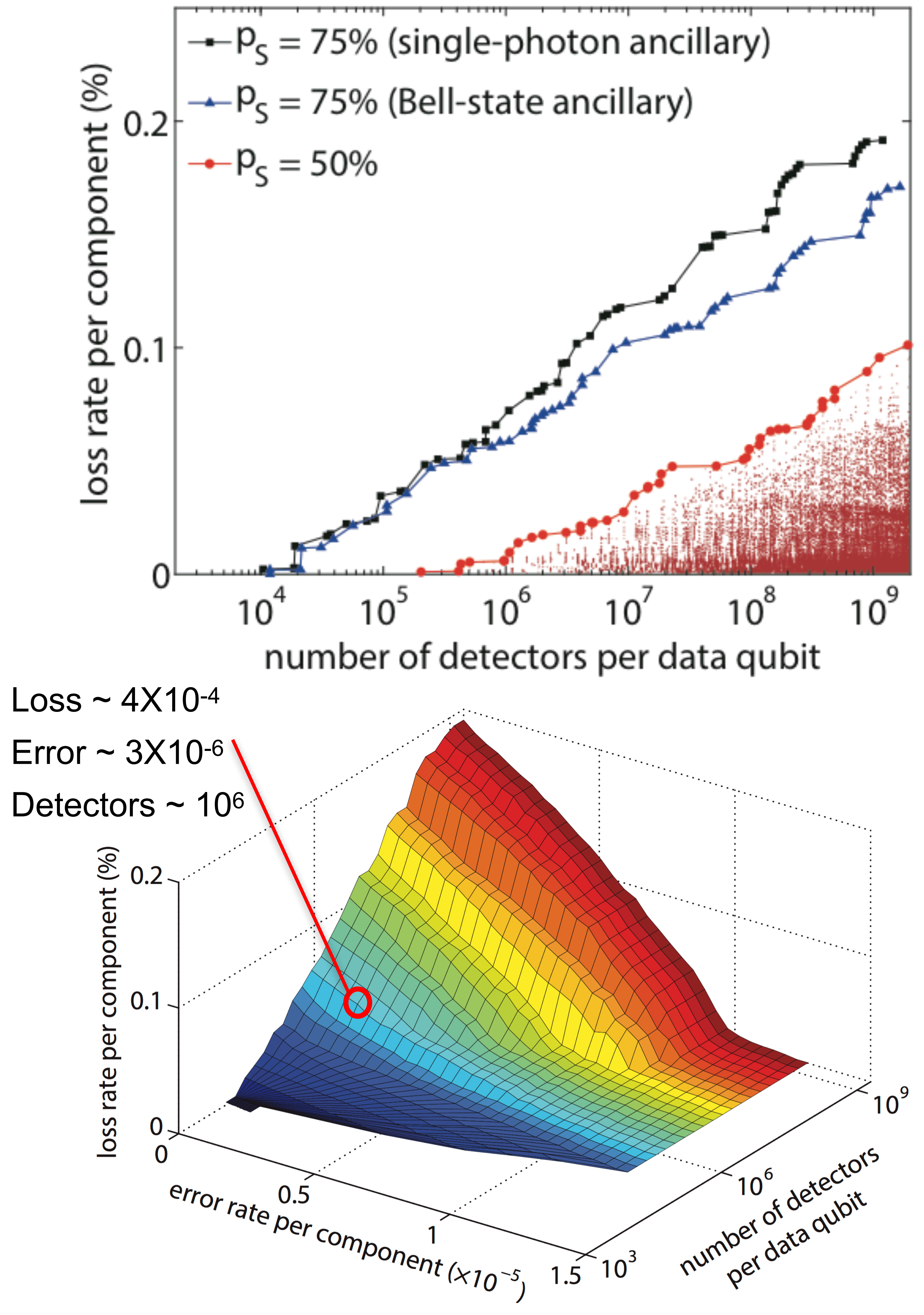




YL and Simon C. Benjamin, npj Quantum Inf. 4, 25 (2018)

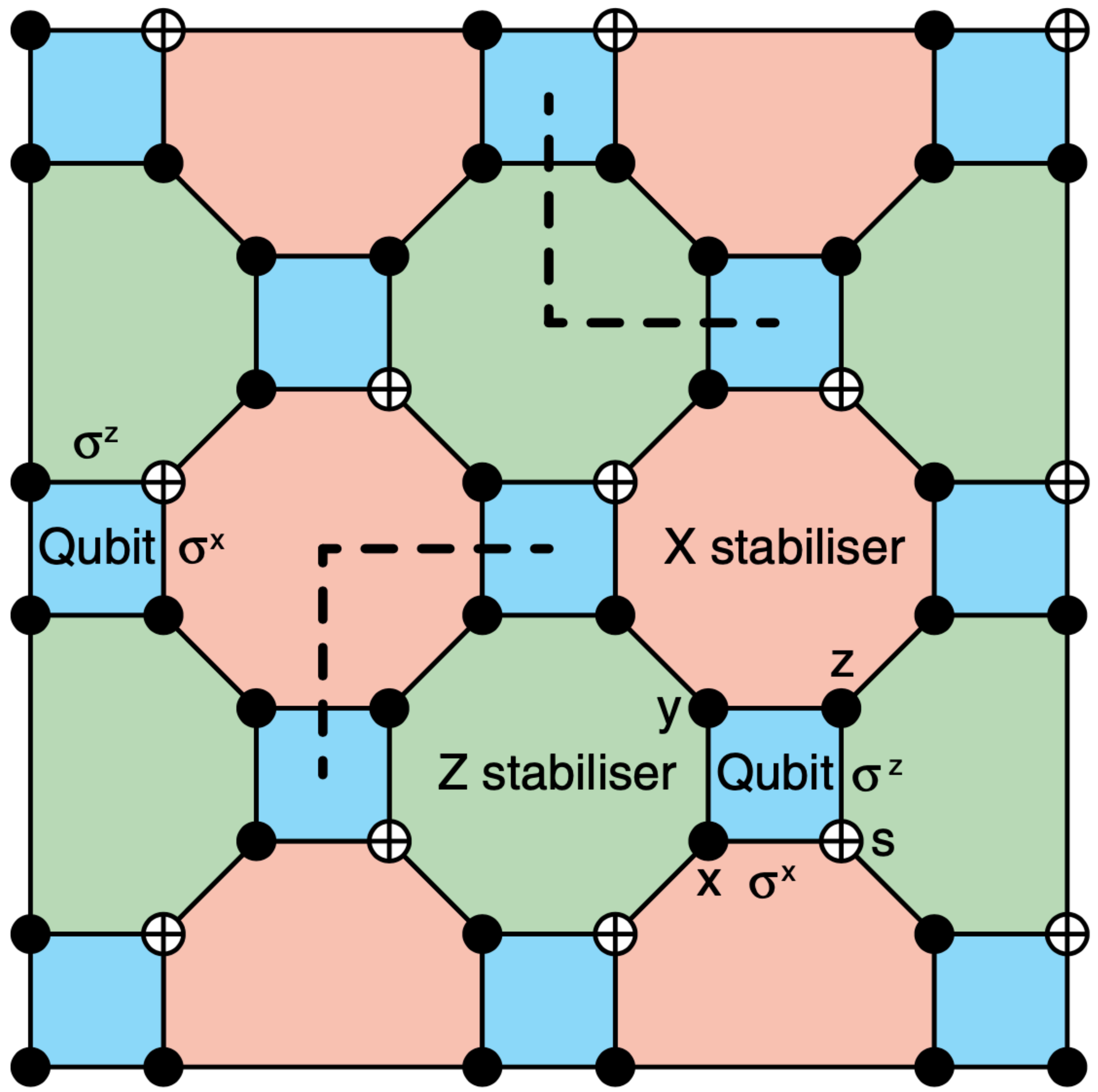
Linear optical quantum computing

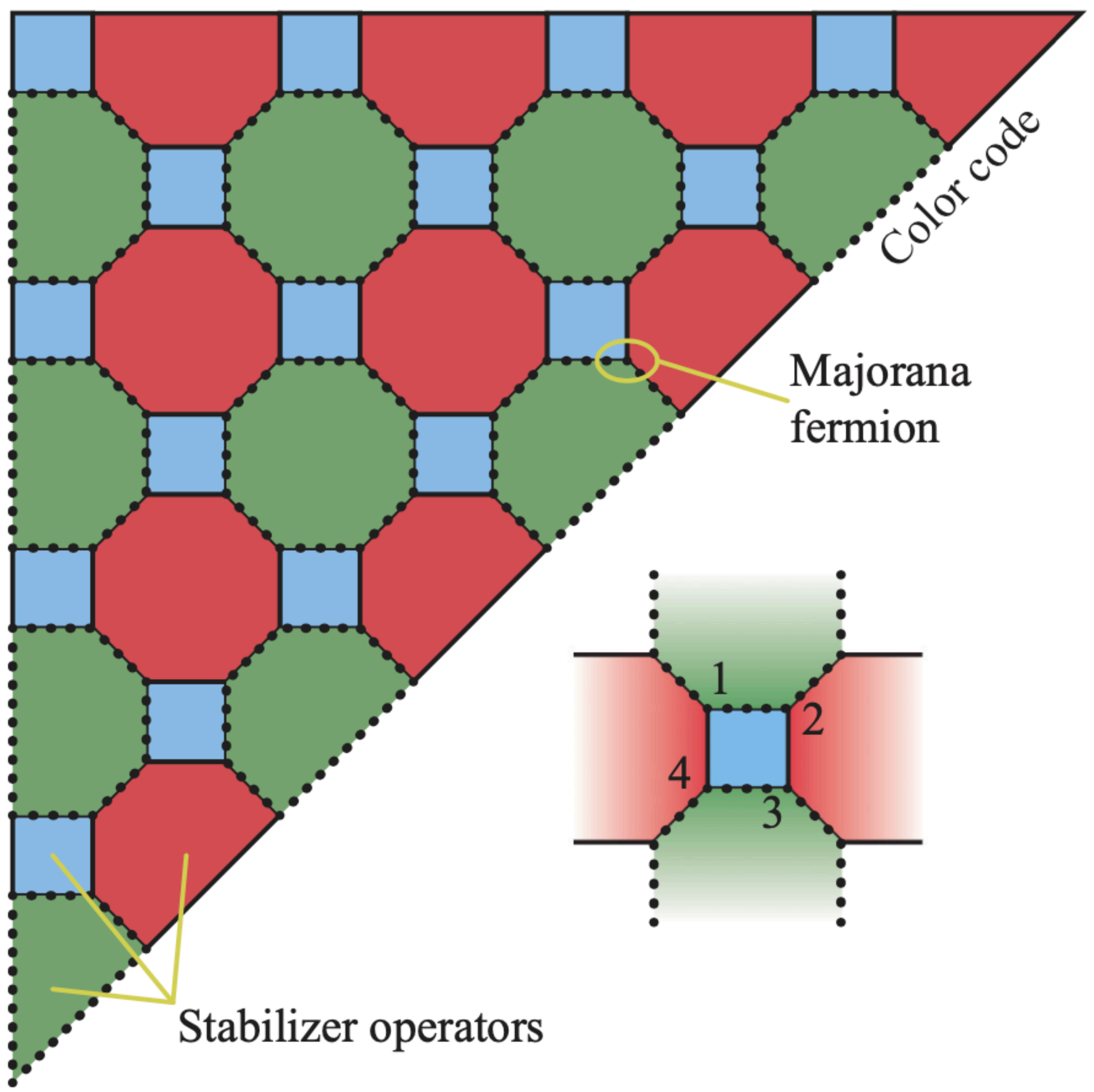


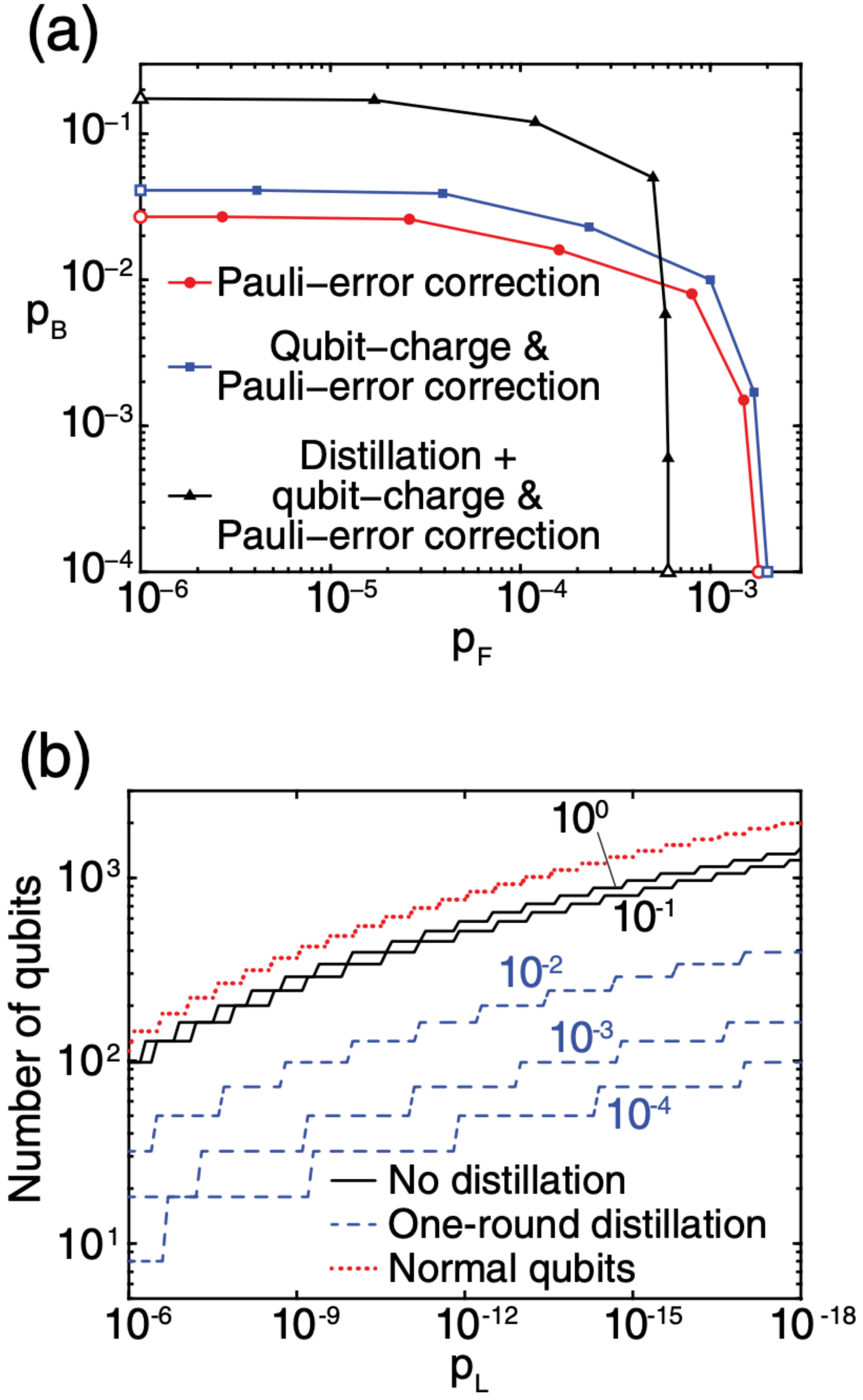


YL, Peter C. Humphreys, Gabriel J. Mendoza, and Simon C. Benjamin, Phys. Rev. X 5, 041007 (2015)

Majorana fermion quantum computing







YL, Phys. Rev. Lett. 117, 120403 (2016)
YL, Phys. Rev. A 98, 012336 (2018)

1. Background
2. What is quantum error mitigation
3. Error-model-based approaches
4. Constraint-based approaches
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What is quantum error mitigation

Quantum error mitigation refers to methods for

- attaining the correct computing result using data from quantum circuits already affected by errors,
- instead of preventing errors from happening.

What is quantum error mitigation

- ***Hardware and control optimisation***: Minimising physical errors
- ***Quantum error correction***: Minimising logical errors
- ***Quantum error mitigation***: Minimising the impact of errors

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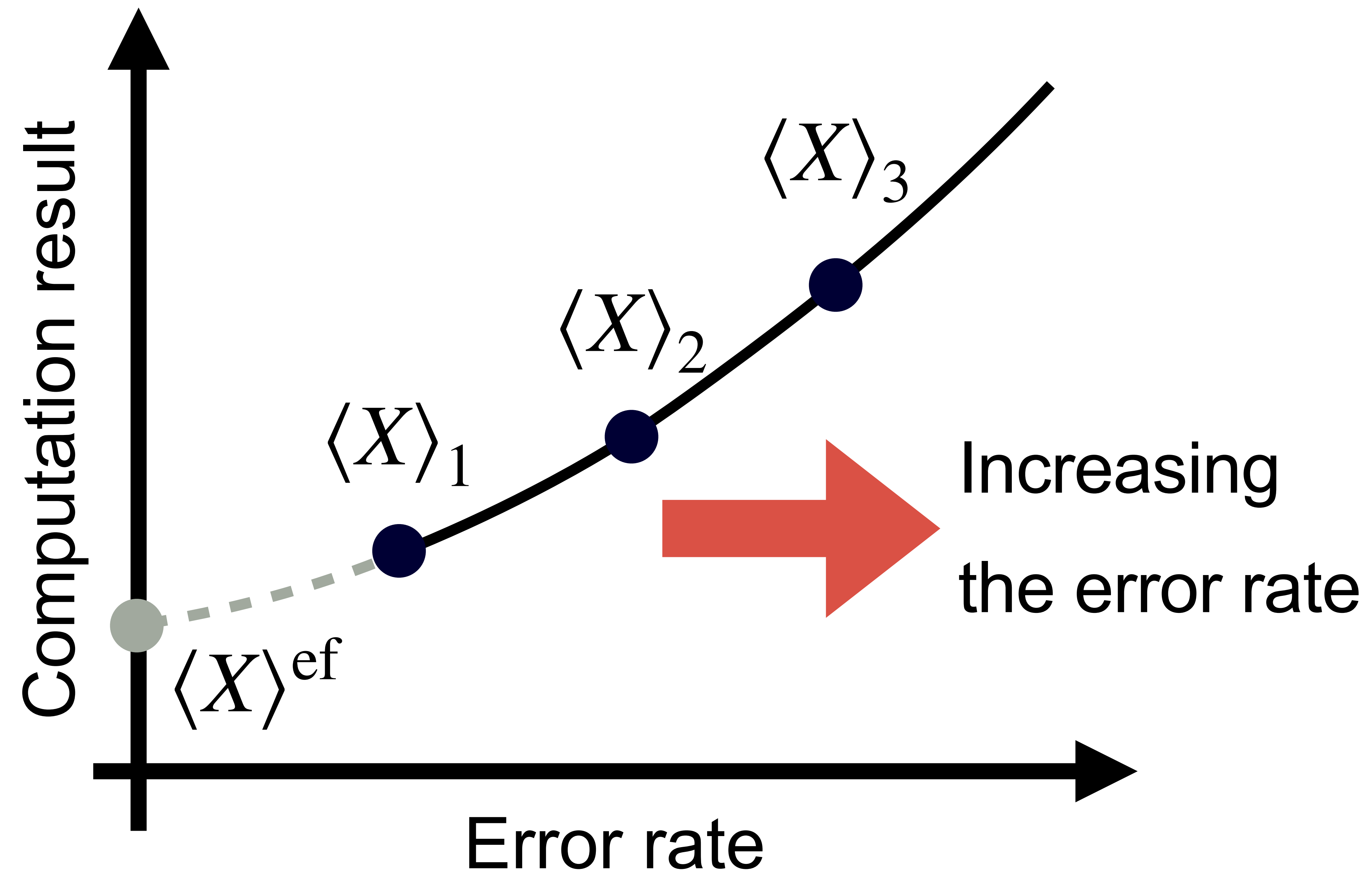
Error extrapolation (Zero-noise extrapolation)

Deliberately make the errors ***worse!***

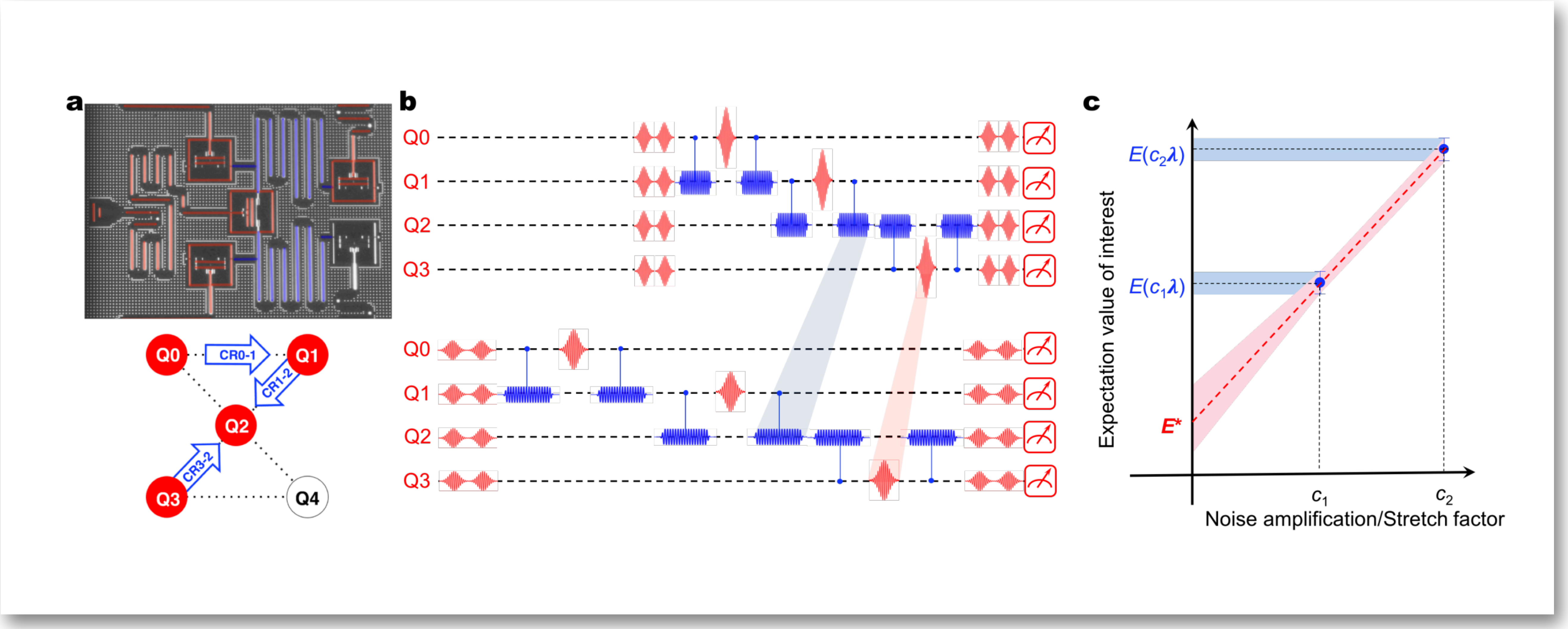


Quantum Error mitigation formula:

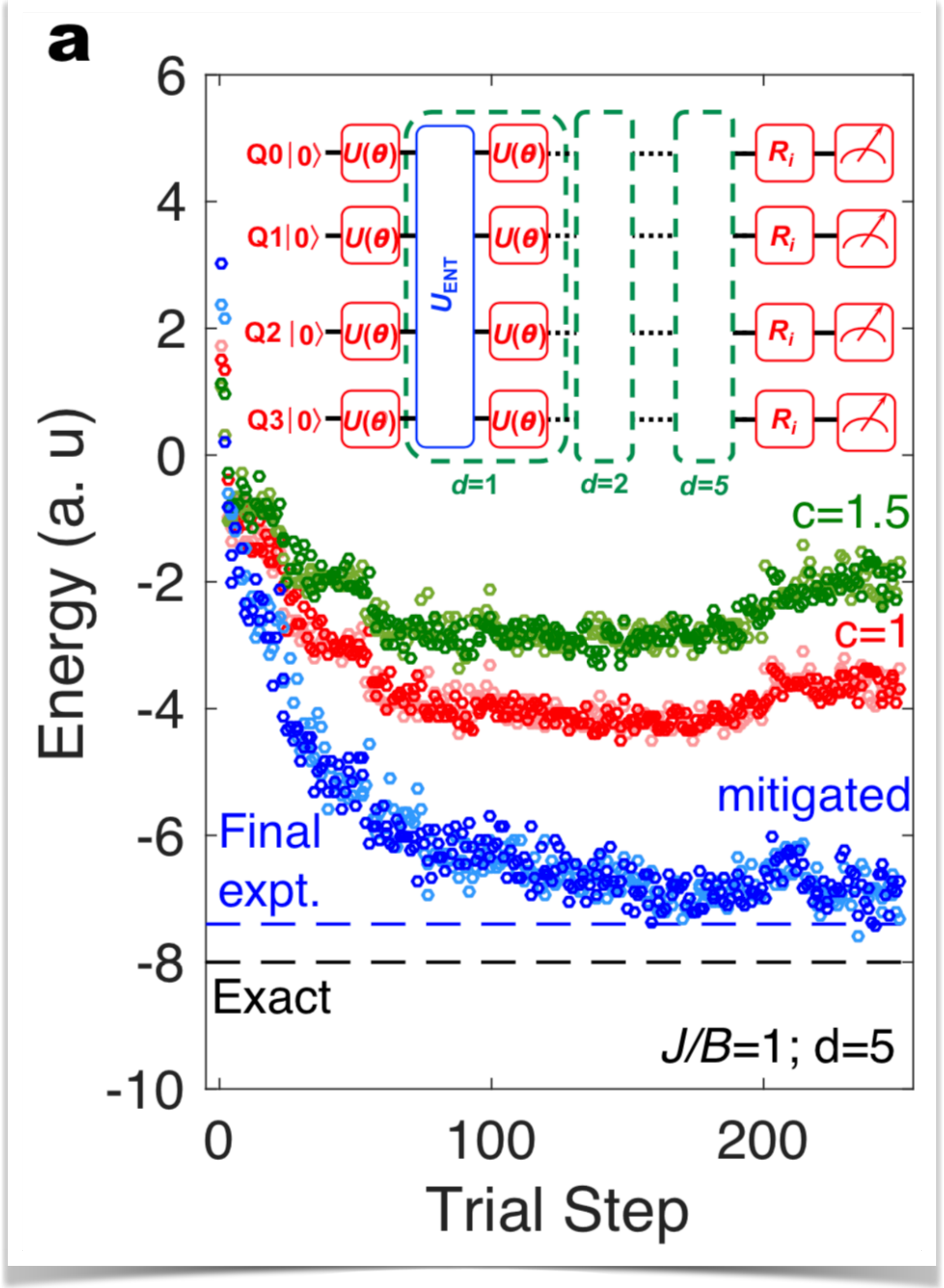
$$\langle X \rangle^{\text{em}} = F(\langle X \rangle_1, \langle X \rangle_2, \langle X \rangle_3, \ldots)$$

YL and Simon C. Benjamin, Phys. Rev. X 7, 021050 (2017)
Kristan Temme, Sergey Bravyi, and Jay M. Gambetta, Phys. Rev. Lett. 119, 180509 (2017)

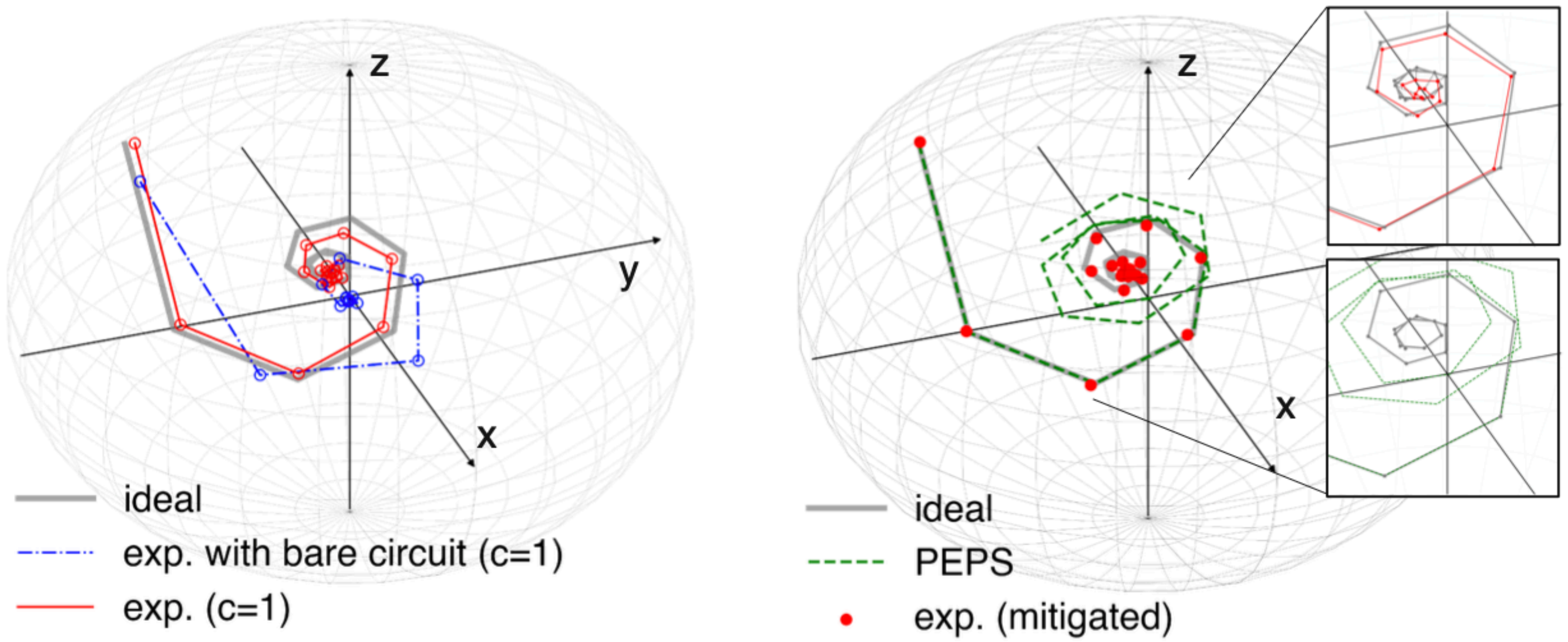
Error extrapolation (Zero-noise extrapolation)



A. Kandala, K. Temme, A. D. Corcoles, A. Mezzacapo, J. M. Chow, and J. M. Gambetta, Nature 567, 491 (2019)



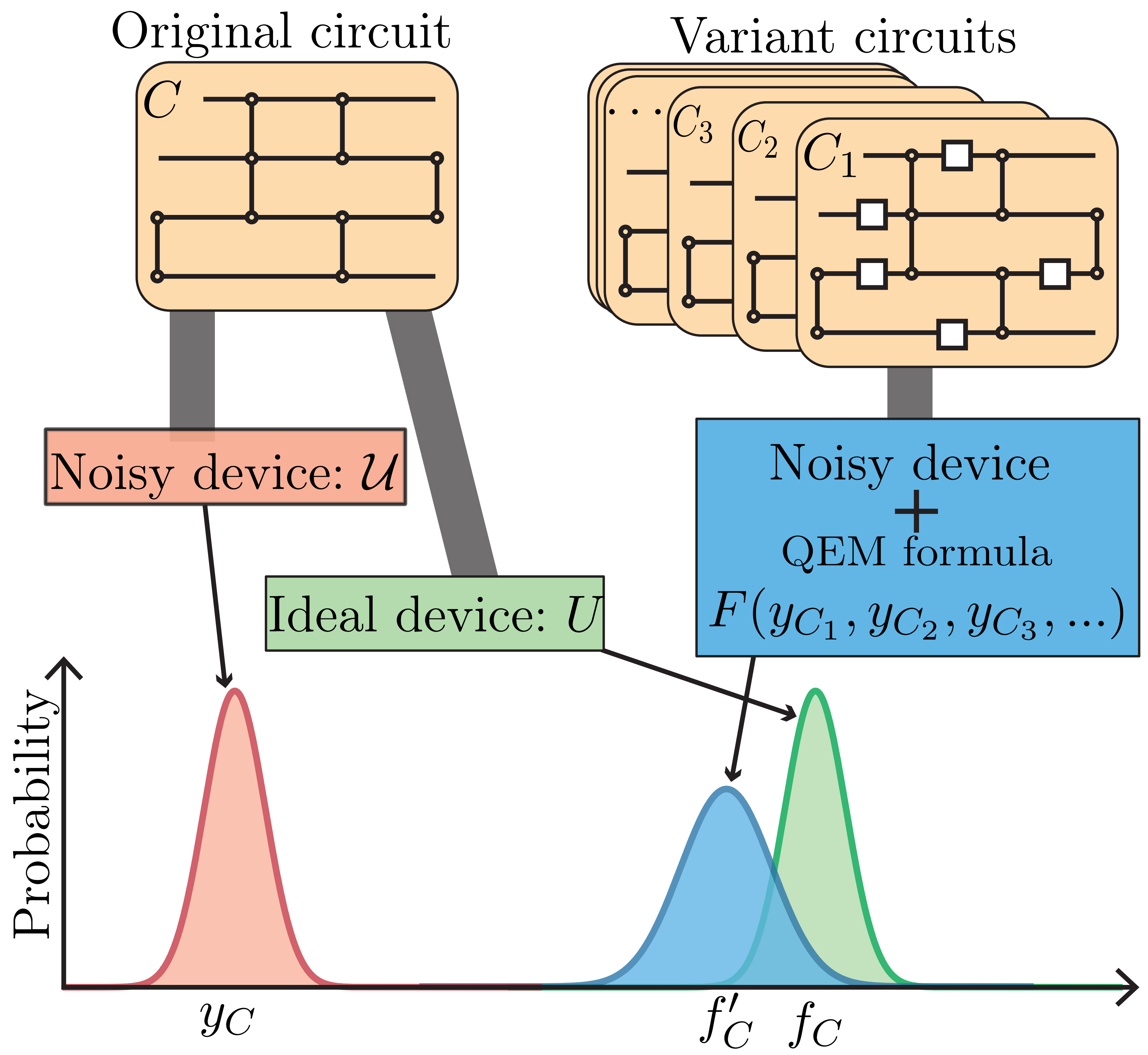
Error extrapolation (Zero-noise extrapolation)



- 26 spin 2D Ising spin lattice
- Digital quantum simulation with Trotterisation
- Circuit depth of 60 and 1080 CNOT gates

Youngseok Kim *et al.*, arXiv:2108.09197

General formalism of quantum error mitigation



Error-free original circuit $\mathbf{C}$:

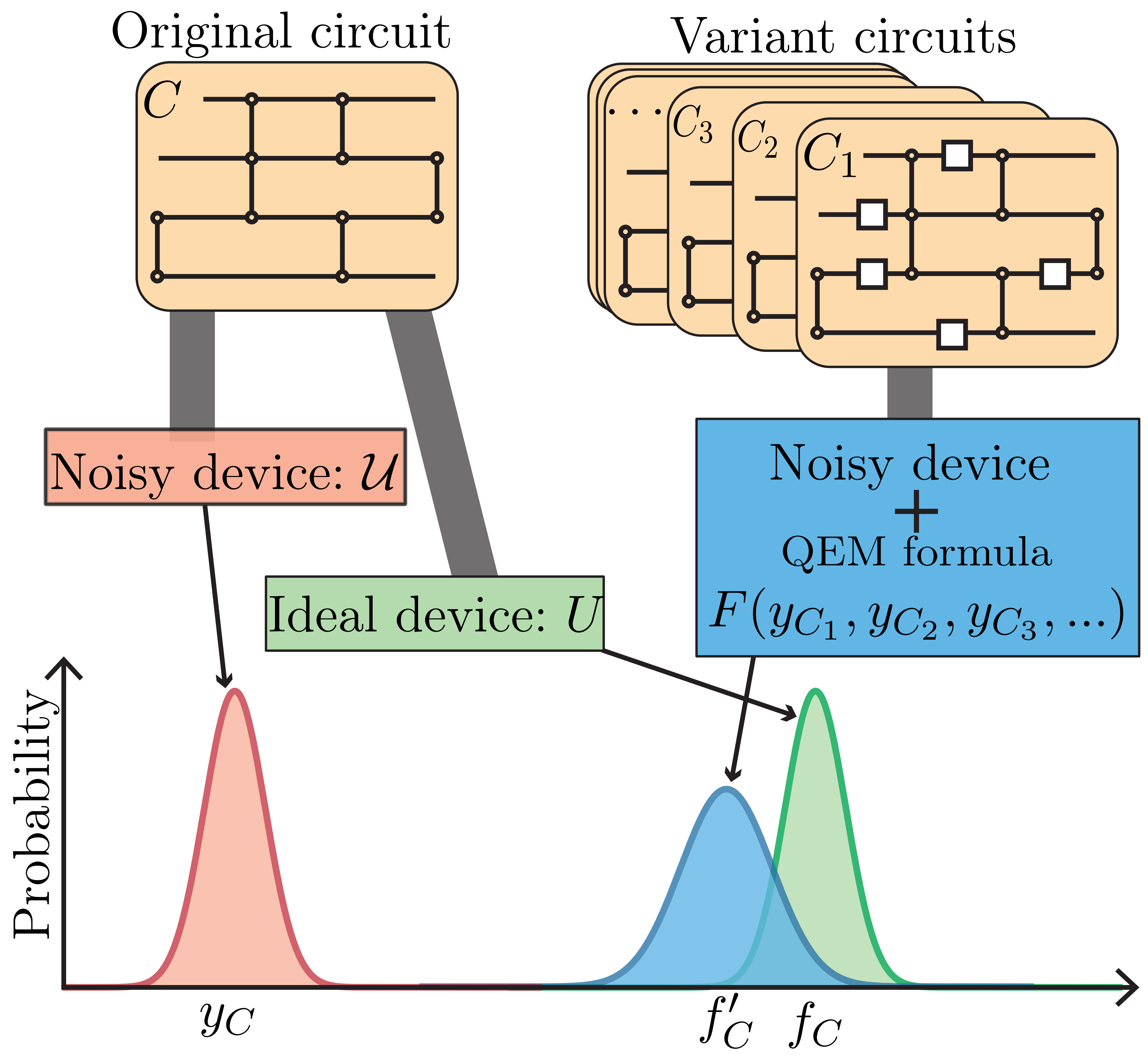
- Initial state: $\rho^{\text{ef}} = |0\rangle\langle 0|^{\otimes n}$
- TPCP map of the circuit: $\mathscr{M}^{\text{ef}}$
- Measured observable: O^{ef}
- Error-free computation result: $f_{\mathbf{C}} = \text{Tr}\left[O^{\text{ef}}\mathscr{M}^{\text{ef}}\left(\rho^{\text{ef}}\right)\right]$

Noisy original circuit $\mathbf{C}$:

- Initial state: ρ
- TPCP map of the circuit: $\mathscr{M}$
- Measured observable: O
- Error-free computation result: $y_{\mathbf{C}} = \text{Tr}\left[O\mathscr{M}\left(\rho\right)\right]$

Dayue Qin, Yanzhu Chen, and **YL**, arXiv:2112.06255
Dayue Qin, Xiaosi Xu, and **YL**, An overview of quantum error mitigation formulas, Chinese Phys. B 31 090306 (2022)

General formalism of quantum error mitigation



Noisy variant circuit $\mathbf{C}_k$:

- Initial state: ρ
- TPCP map of the circuit: $\mathscr{M}_k$
- Measured observable: O_k
- Error-free computation result: $y_{\mathbf{C}_k} = \mathrm{Tr}\left[O_k \mathscr{M}_k(\rho)\right]$

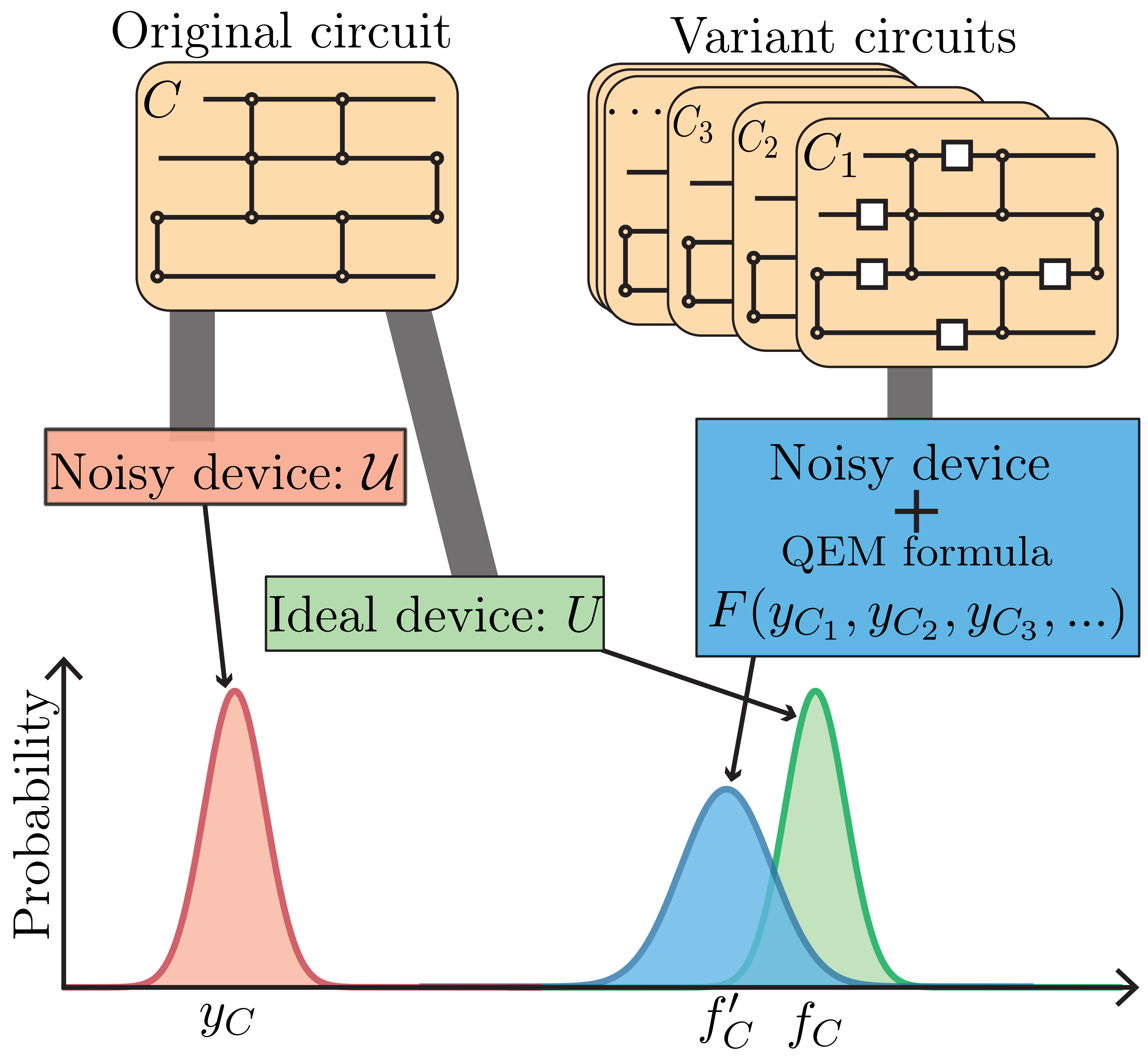
Error mitigation formula:

$$f'_{\mathbf{C}} = F(y_{\mathbf{C}_1}, y_{\mathbf{C}_2}, y_{\mathbf{C}_3}, \ldots)$$

Dayue Qin, Yanzhu Chen, and **YL**, arXiv:2112.06255

Dayue Qin, Xiaosi Xu, and **YL**, An overview of quantum error mitigation formulas, Chinese Phys. B 31 090306 (2022)

General formalism of quantum error mitigation



- Error before error mitigation: $y_{\mathbf{C}} - f_{\mathbf{C}}$
- Error after error mitigation: $f'_{\mathbf{C}} - f_{\mathbf{C}}$
- *Variances*

Dayue Qin, Yanzhu Chen, and **YL**, arXiv:2112.06255

Dayue Qin, Xiaosi Xu, and **YL**, An overview of quantum error mitigation formulas, Chinese Phys. B 31 090306 (2022)

Quasi-probability decomposition and probabilistic error cancellation

Quasi-probability decomposition: $O^{\mathrm{ef}}\mathscr{M}^{\mathrm{ef}}\left(\rho^{\mathrm{ef}}\right) = \sum_k q_k O_k \mathscr{M}_k\left(\rho\right)$

Error mitigation formula: $f'_C = \sum_k q_k y_{C_k}$

Probabilistic error cancellation (Monte Carlo): $f'_C = \mathrm{Cost} \times \sum_k \mathrm{sgn}\left(q_k\right) p_k y_{C_k}$

$$p_k = |q_k|/\mathrm{Cost}, \quad \mathrm{Cost} = \sum_k |q_k|, \quad \mathrm{Variance} \propto \mathrm{Cost}^2$$

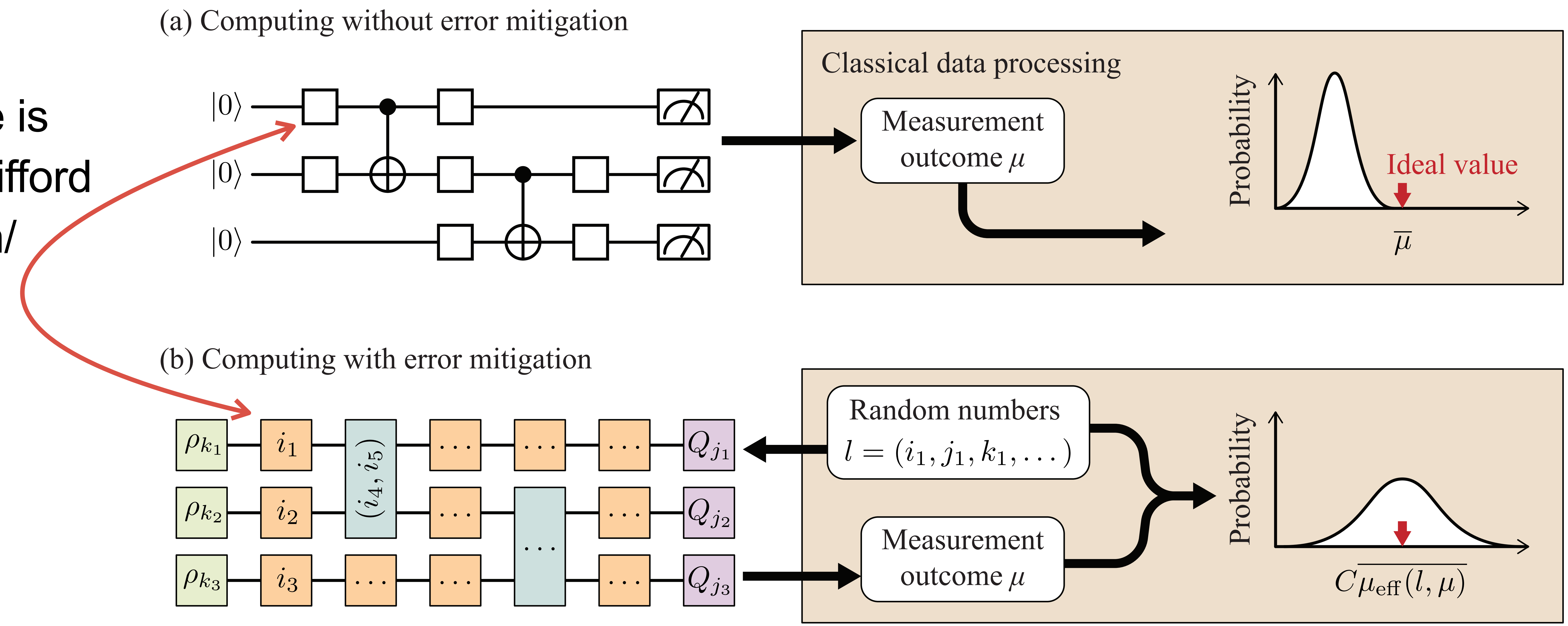
Kristan Temme, Sergey Bravyi, and Jay M. Gambetta, Phys. Rev. Lett. 119, 180509 (2017)

Suguru Endo, Simon C. Benjamin, and **YL**, Phys. Rev. X 7 8, 031027 (2018)

Probabilistic error cancellation

$$f'_{\mathbf{C}} = q_1 y_{\mathbf{C}_1} + q_2 y_{\mathbf{C}_2} + q_3 y_{\mathbf{C}_3} + \cdots$$

The original gate is replaced by a Clifford gate/Initialisation/ Measurement



Provable effectiveness:
The bias can be completely eliminated.

Kristan Temme, Sergey Bravyi, and Jay M. Gambetta, Phys. Rev. Lett. 119, 180509 (2017)
Suguru Endo, Simon C. Benjamin, and **YL**, Phys. Rev. X 7 8, 031027 (2018)

Probabilistic error cancellation - Universal operation set

1	$[\mathbb{1}]$ (no operation)
2	$[\sigma^{\mathrm{x}}] = [R_{\mathrm{x}}]^2$
3	$[\sigma^{\mathrm{y}}] = [R_{\mathrm{x}}]^2[R_{\mathrm{z}}]^2$
4	$[\sigma^{\mathrm{z}}] = [R_{\mathrm{z}}]^2$
5	$[R_{\mathrm{x}}] = [\frac{1}{\sqrt{2}}(\mathbb{1} + i\sigma^{\mathrm{x}})] = [H][S]^3[H]$
6	$[R_{\mathrm{y}}] = [\frac{1}{\sqrt{2}}(\mathbb{1} + i\sigma^{\mathrm{y}})] = [R_{\mathrm{z}}]^3[R_{\mathrm{x}}][R_{\mathrm{z}}]$
7	$[R_{\mathrm{z}}] = [\frac{1}{\sqrt{2}}(\mathbb{1} + i\sigma^{\mathrm{z}})] = [S]^3$
8	$[R_{\mathrm{yz}}] = [\frac{1}{\sqrt{2}}(\sigma^{\mathrm{y}} + \sigma^{\mathrm{z}})] = [R_{\mathrm{x}}][R_{\mathrm{z}}]^2$
9	$[R_{\mathrm{zx}}] = [\frac{1}{\sqrt{2}}(\sigma^{\mathrm{z}} + \sigma^{\mathrm{x}})] = [R_{\mathrm{z}}][R_{\mathrm{x}}][R_{\mathrm{z}}]$
10	$[R_{\mathrm{xy}}] = [\frac{1}{\sqrt{2}}(\sigma^{\mathrm{x}} + \sigma^{\mathrm{y}})] = [R_{\mathrm{x}}]^2[R_{\mathrm{z}}]$
11	$[\pi_{\mathrm{x}}] = [\frac{1}{2}(\mathbb{1} + \sigma^{\mathrm{x}})] = [R_{\mathrm{z}}]^3[R_{\mathrm{x}}]^3[\pi][R_{\mathrm{x}}][R_{\mathrm{z}}]$
12	$[\pi_{\mathrm{y}}] = [\frac{1}{2}(\mathbb{1} + \sigma^{\mathrm{y}})] = [R_{\mathrm{x}}][\pi][R_{\mathrm{x}}]^3$
13	$[\pi_{\mathrm{z}}] = [\frac{1}{2}(\mathbb{1} + \sigma^{\mathrm{z}})] = [\pi]$
14	$[\pi_{\mathrm{yz}}] = [\frac{1}{2}(\sigma^{\mathrm{y}} + i\sigma^{\mathrm{z}})] = [R_{\mathrm{z}}]^3[R_{\mathrm{x}}]^3[\pi][R_{\mathrm{x}}]^3[R_{\mathrm{z}}]$
15	$[\pi_{\mathrm{zx}}] = [\frac{1}{2}(\sigma^{\mathrm{z}} + i\sigma^{\mathrm{x}})] = [R_{\mathrm{x}}][\pi][R_{\mathrm{x}}]^3[R_{\mathrm{z}}]^2$
16	$[\pi_{\mathrm{xy}}] = [\frac{1}{2}(\sigma^{\mathrm{x}} + i\sigma^{\mathrm{y}})] = [\pi][R_{\mathrm{x}}]^2$

TABLE I. Sixteen basis operations. Gates $[R_{\mathrm{x}}]$ and $[R_{\mathrm{y}}]$ can be derived from $[H]$ and $[S]$, and other operations can be derived from $[\pi]$, $[R_{\mathrm{x}}]$ and $[R_{\mathrm{y}}]$.

Suguru Endo, Simon C. Benjamin, and **YL**, Phys. Rev. X 7 8, 031027 (2018)

Probabilistic error cancellation - Consistency

Standard Quantum Theory Representation

(*Unknown*)

Initial state $|\bar{\rho}_k\rangle\rangle$

Operation $\bar{\mathcal{O}}_i$

Measured quantity $\langle\langle \bar{Q}_j|$

Gate Set Tomography

$\langle\langle \bar{Q}_j|\bar{\mathcal{O}}_i|\bar{\rho}_k\rangle\rangle = \langle\langle \hat{Q}_j|\hat{\mathcal{O}}_i|\hat{\rho}_k\rangle\rangle$

Gate Set Tomography Representation

(*Known*)

Initial state $|\hat{\rho}_k\rangle\rangle = S|\bar{\rho}_k\rangle\rangle$

Operation $\hat{\mathcal{O}}_i = S\bar{\mathcal{O}}_i S^{-1}$

Measured quantity $\langle\langle \hat{Q}_j| = \langle\langle \bar{Q}_j|S^{-1}$

Quasi-probability decomposition

Quantum circuit

Initial state $|\hat{\rho}^{(0)}\rangle\rangle = \sum_k q_k|\hat{\rho}_k\rangle\rangle$

Operation $\hat{\mathcal{O}}^{(0)} = \sum_i q_i\hat{\mathcal{O}}_i$

Measured quantity $\langle\langle \hat{Q}^{(0)}| = \sum_j q_j\langle\langle \hat{Q}_j|$

Quantum computing without error

$\langle\langle \bar{Q}^{(0)}|\bar{\mathcal{O}}^{(0)}|\bar{\rho}^{(0)}\rangle\rangle$
$= \langle\langle \hat{Q}^{(0)}|\hat{\mathcal{O}}^{(0)}|\hat{\rho}^{(0)}\rangle\rangle$

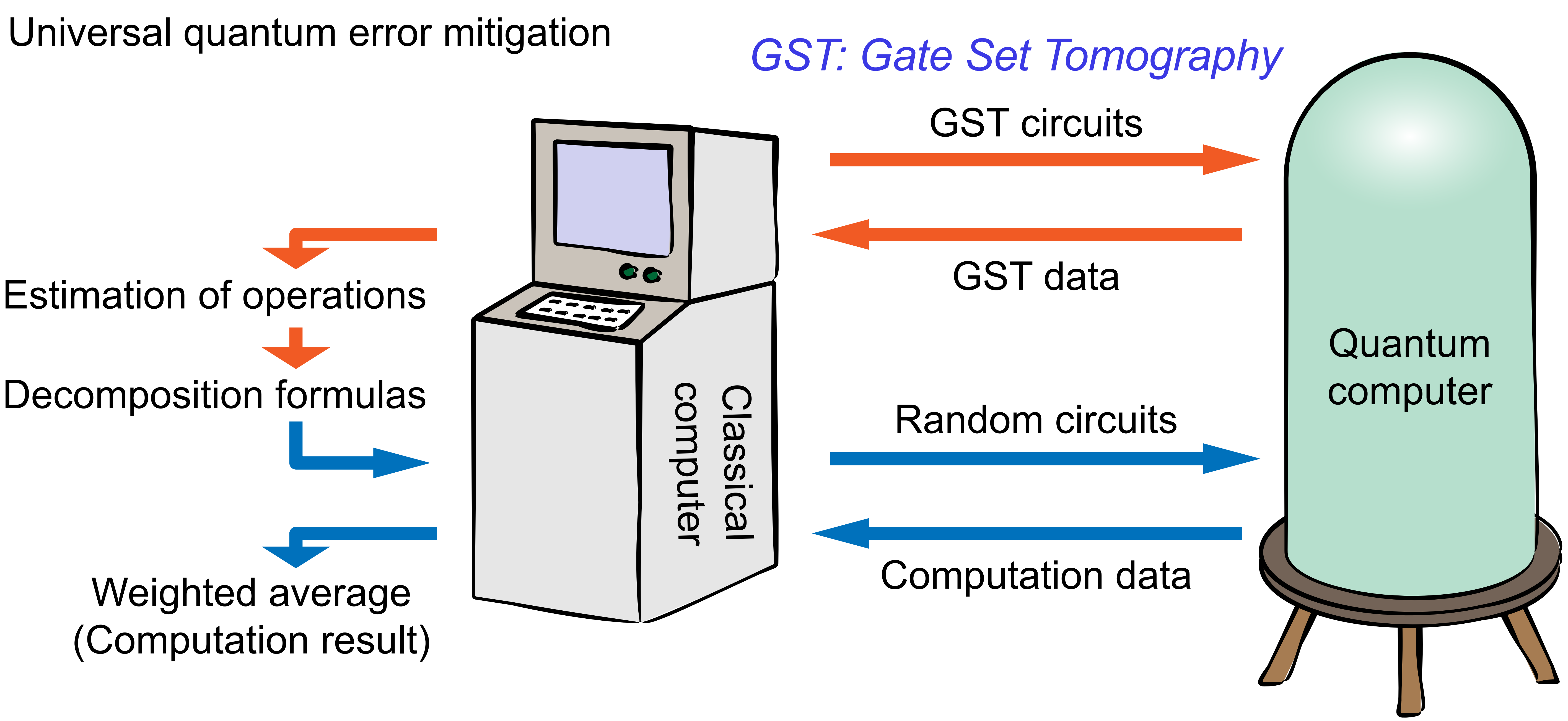
Initial state $|\bar{\rho}^{(0)}\rangle\rangle = S^{-1}|\hat{\rho}^{(0)}\rangle\rangle$

Operation $\bar{\mathcal{O}}^{(0)} = S^{-1}\hat{\mathcal{O}}^{(0)}S$

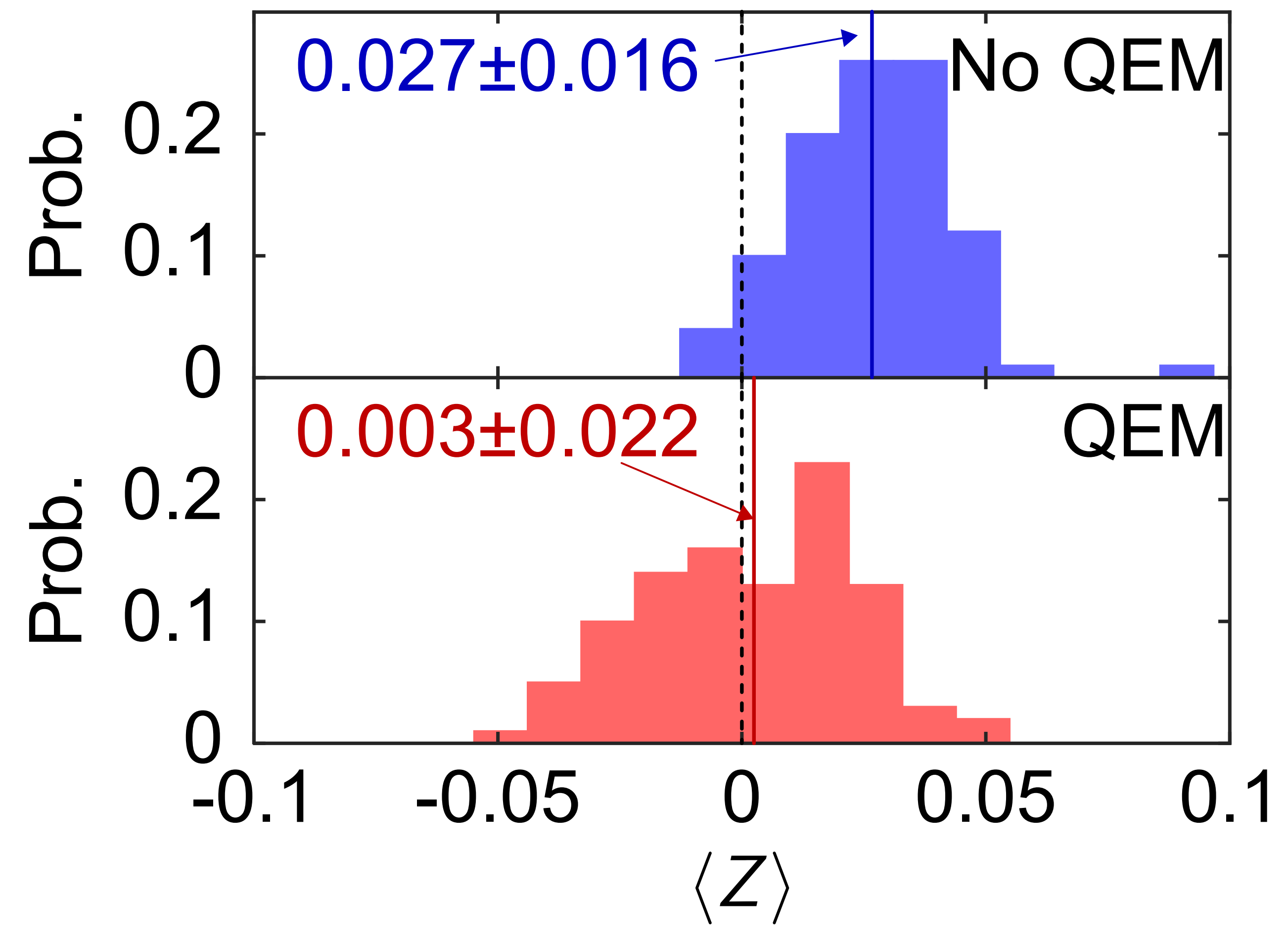
Measured quantity $\langle\langle \bar{Q}^{(0)}| = \langle\langle \hat{Q}^{(0)}|S$

Suguru Endo, Simon C. Benjamin, and **YL**, Phys. Rev. X 7 8, 031027 (2018)

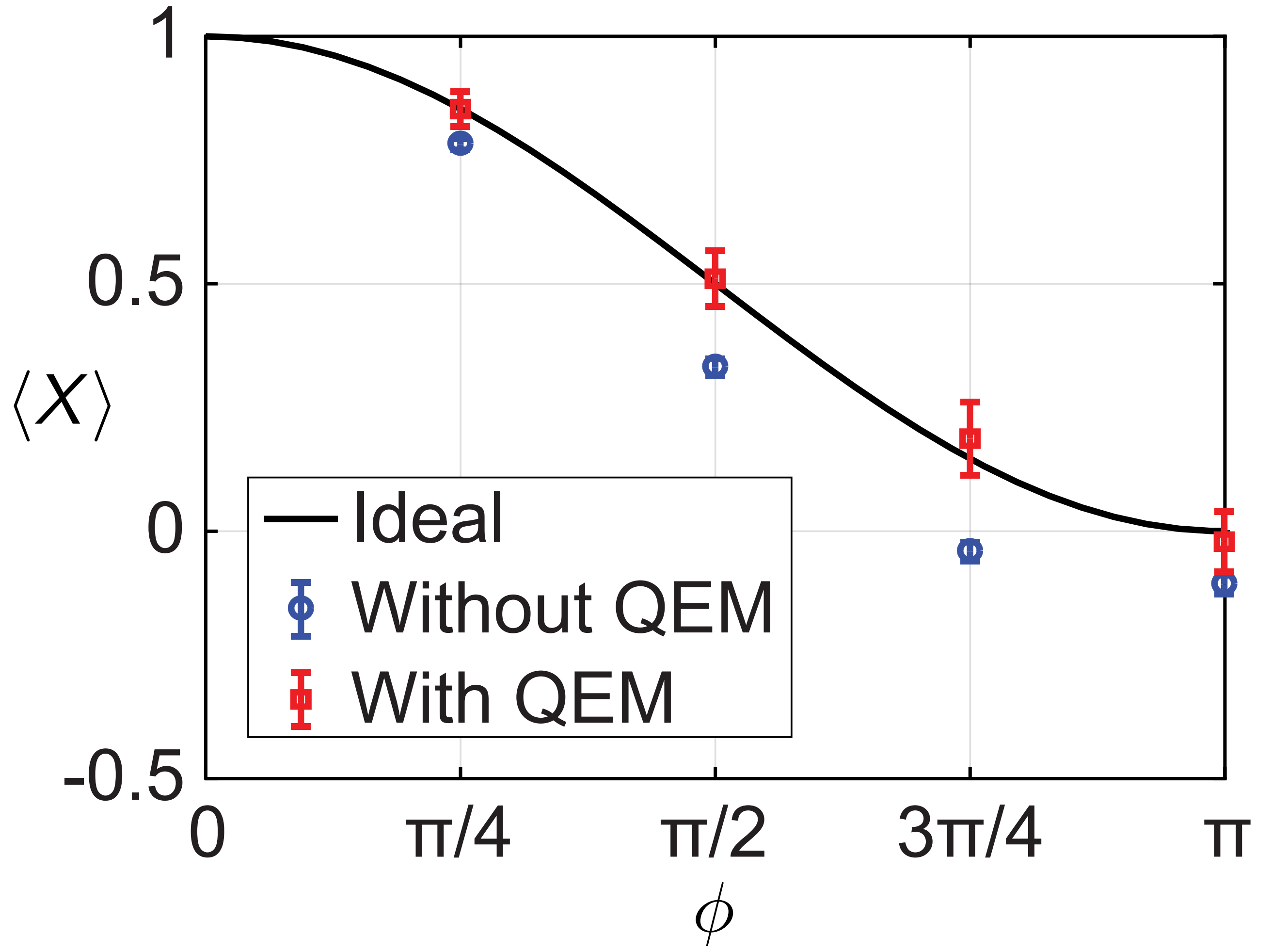
Probabilistic error cancellation



(f) One-qubit computation results

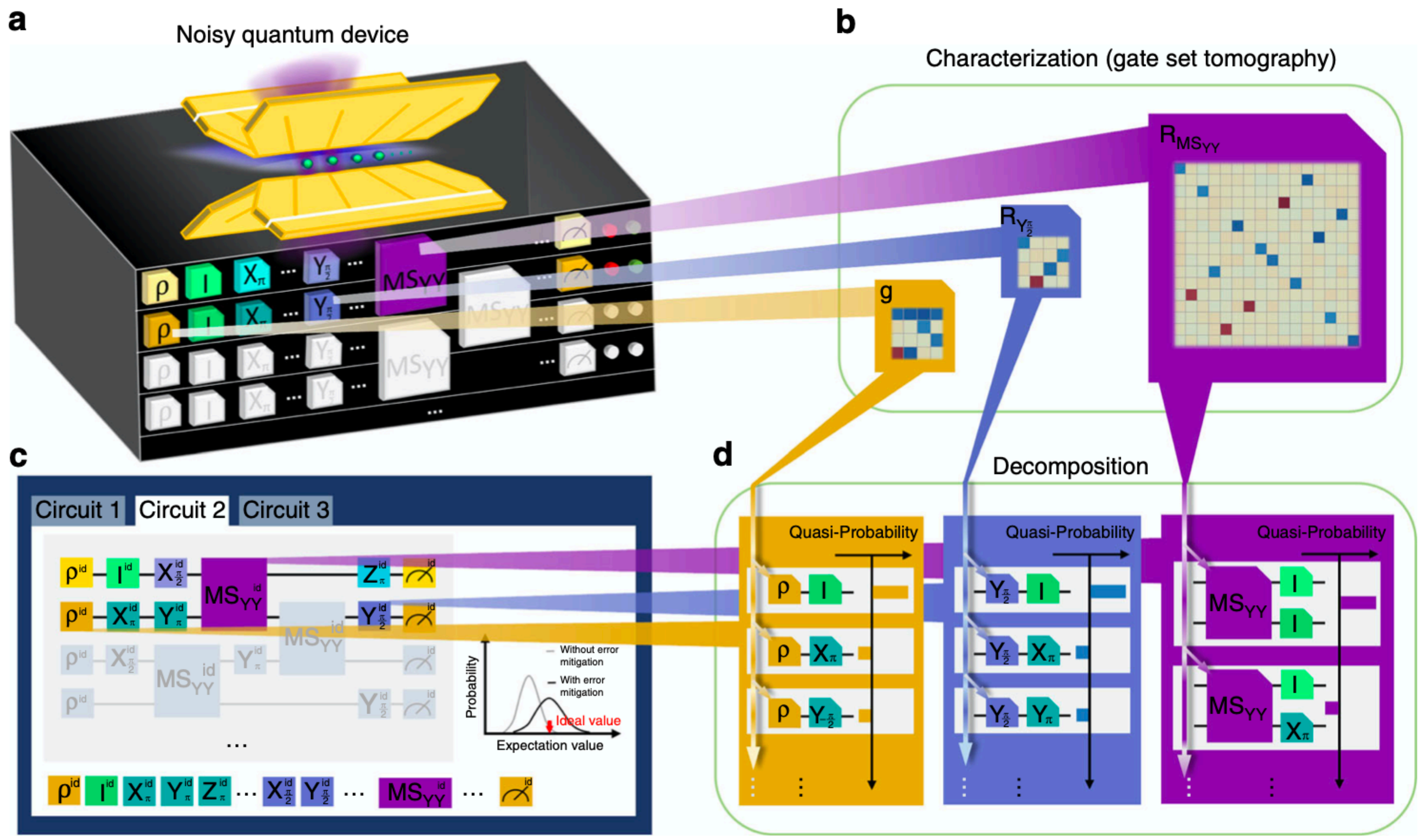


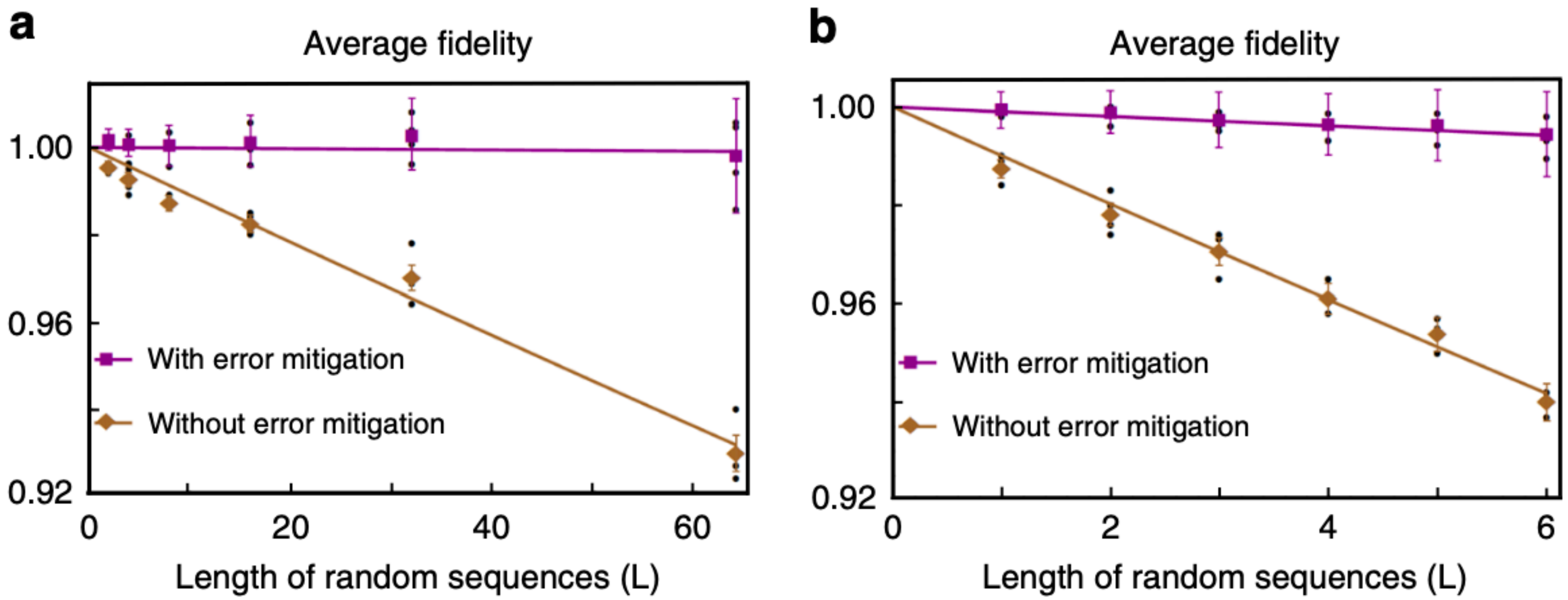
(g) Two-qubit computation results



Chao Song, Jing Cui, H. Wang, J. Hao, H. Feng, and **YL**, Sci. Adv. 5, eaaw5686 (2019)

Probabilistic error cancellation



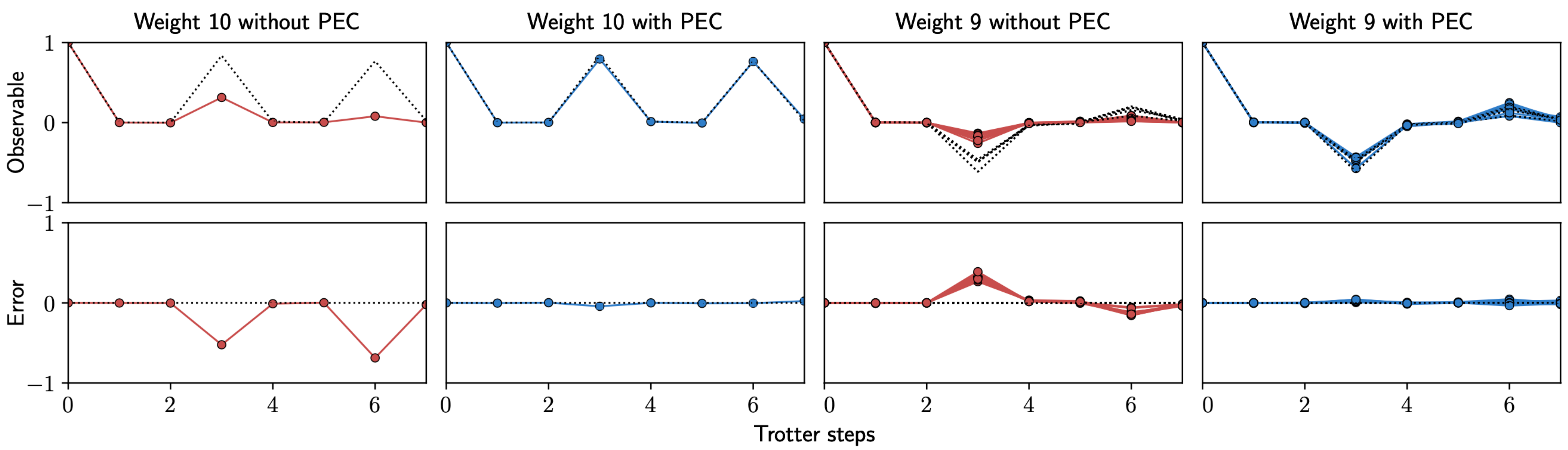


Shuaining Zhang, Yao Lu, Kuan Zhang, Wentao Chen, **YL**, Jing-Ning Zhang, and Kihwan Kim, Nature Communications 11, 587 (2020)

Probabilistic error cancellation

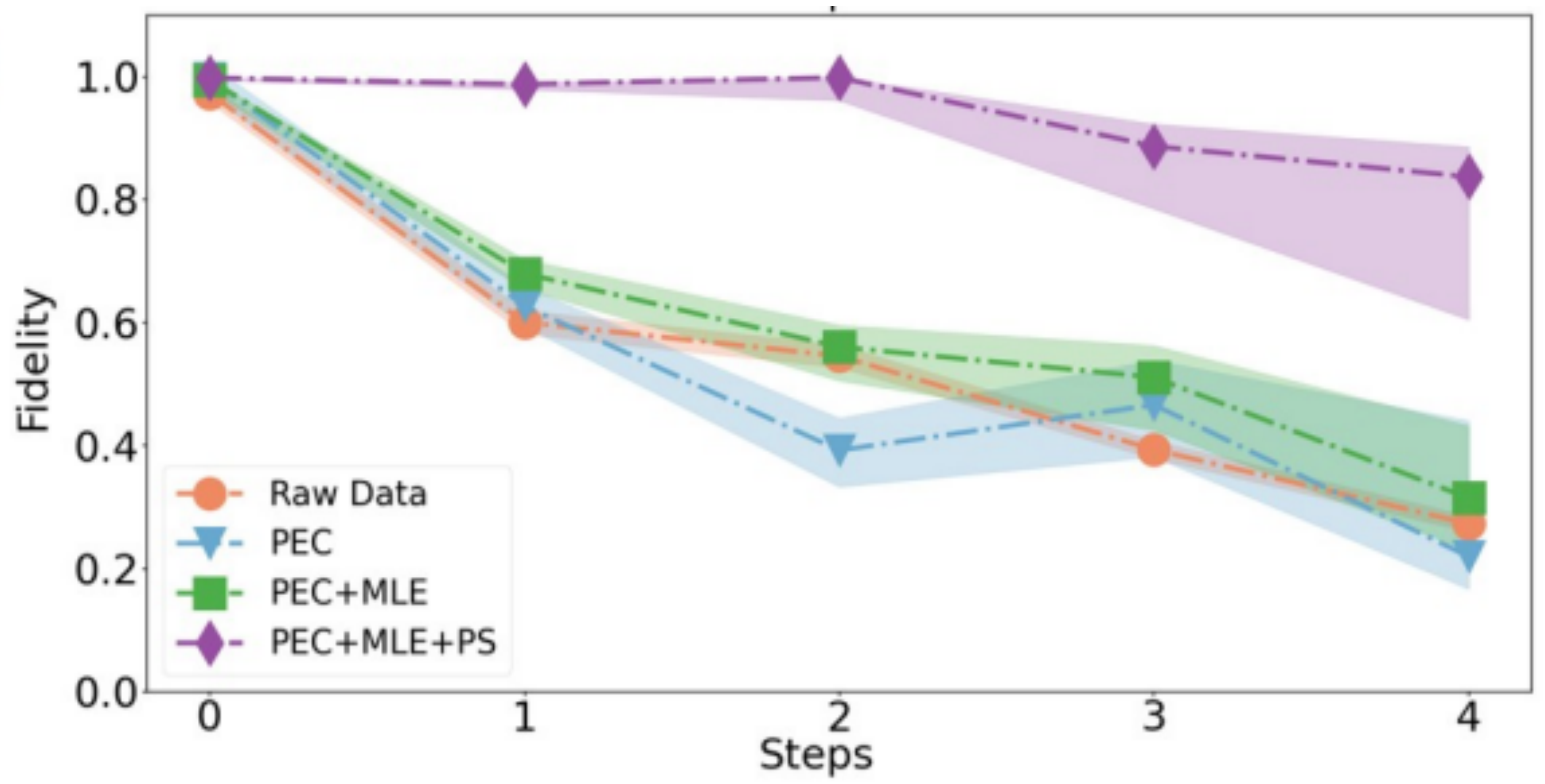
Ten superconducting qubits:

Ewout van den Berg, Zlatko K. Minev, Abhinav Kandala, Kristan Temme, arXiv:2201.09866



Four ion-trap qubits:

Wentao Chen, Shuaining Zhang, Jialiang Zhang, Xiaolu Su, Yao Lu, Kuan Zhang, Mu Qiao, Ying Li, Jing-Ning Zhang, Kihwan Kim, arXiv:2302.10436



Cost of quantum error mitigation

$$f'_{\mathbf{C}} = q_1 y_{\mathbf{C}_1} + q_2 y_{\mathbf{C}_2} + q_3 y_{\mathbf{C}_3} + \cdots$$

Monte Carlo summation:

$$\mathrm{Var} \propto (|q_1| + |q_2| + |q_3| + \cdots)^2 \simeq \exp(4Np)$$

$Np =$ Gate number $\times$ Error rate $\lesssim 1$

1. Background
2. What is quantum error mitigation
3. Error-model-based approaches
4. Constraint-based approaches
5. Learning-based approaches

Constraint-based approaches

- Quantum error correction — Stabiliser group
- Symmetry verification
- Purification of fermion correlations
- Purification of quantum states
- ……

Symmetry-based quantum error mitigation

The final state $|\psi\rangle$ has the symmetry P_S (projection operator), i.e. $P_S|\psi\rangle = |\psi\rangle$.

State with noise ρ_{n}

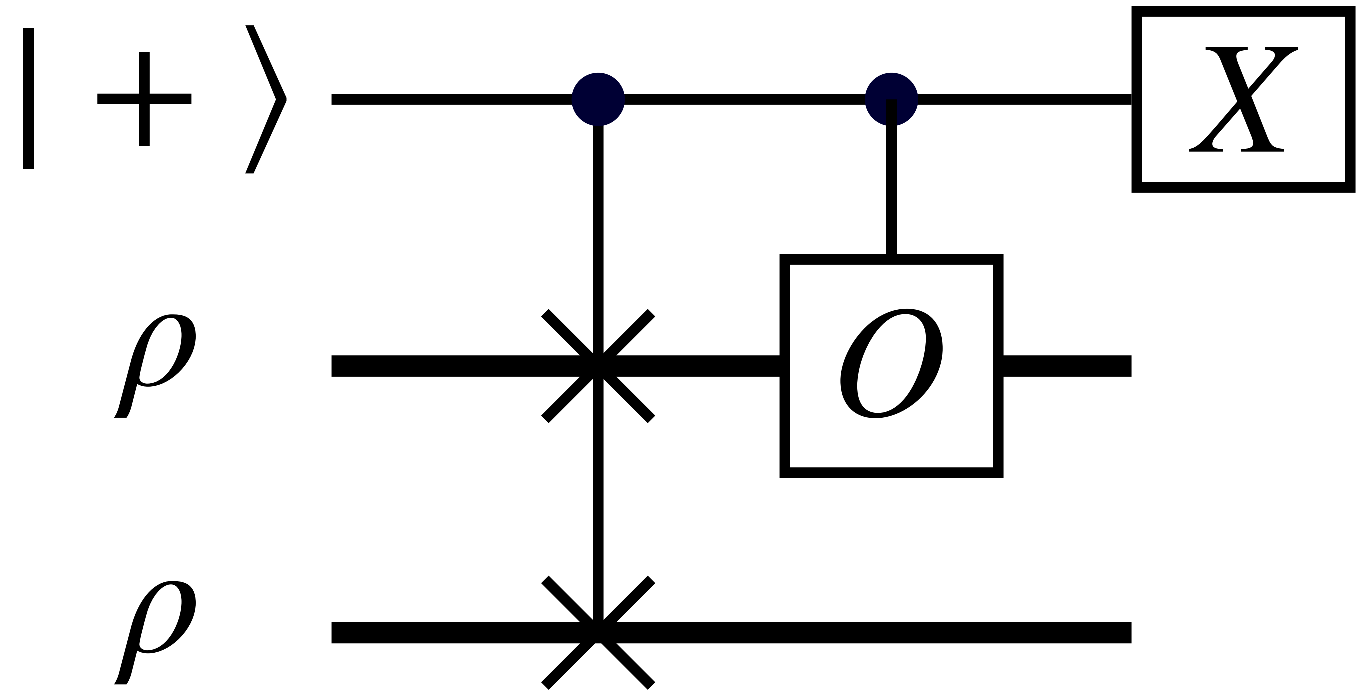
Error mitigated state $\rho_{\mathrm{em}} = \dfrac{P_S \rho_{\mathrm{n}} P_S}{\mathrm{Tr}\left(P_S \rho_{\mathrm{n}} P_S\right)}$

Sam McArdle, Xiao Yuan, and Simon Benjamin, Phys. Rev. Lett. 122, 180501 (2019)
X. Bonet-Monroig, R. Sagastizabal, M. Singh, and T. E. O'Brien, Phys. Rev. A 98, 062339 (2018)

Purification and virtual distillation

Purified state:

$$\rho \to \frac{\rho^2}{\mathrm{Tr}\left(\rho^2\right)}$$



- Two copies of the state, qubit overhead
- Controlled-swap operation must be error-free

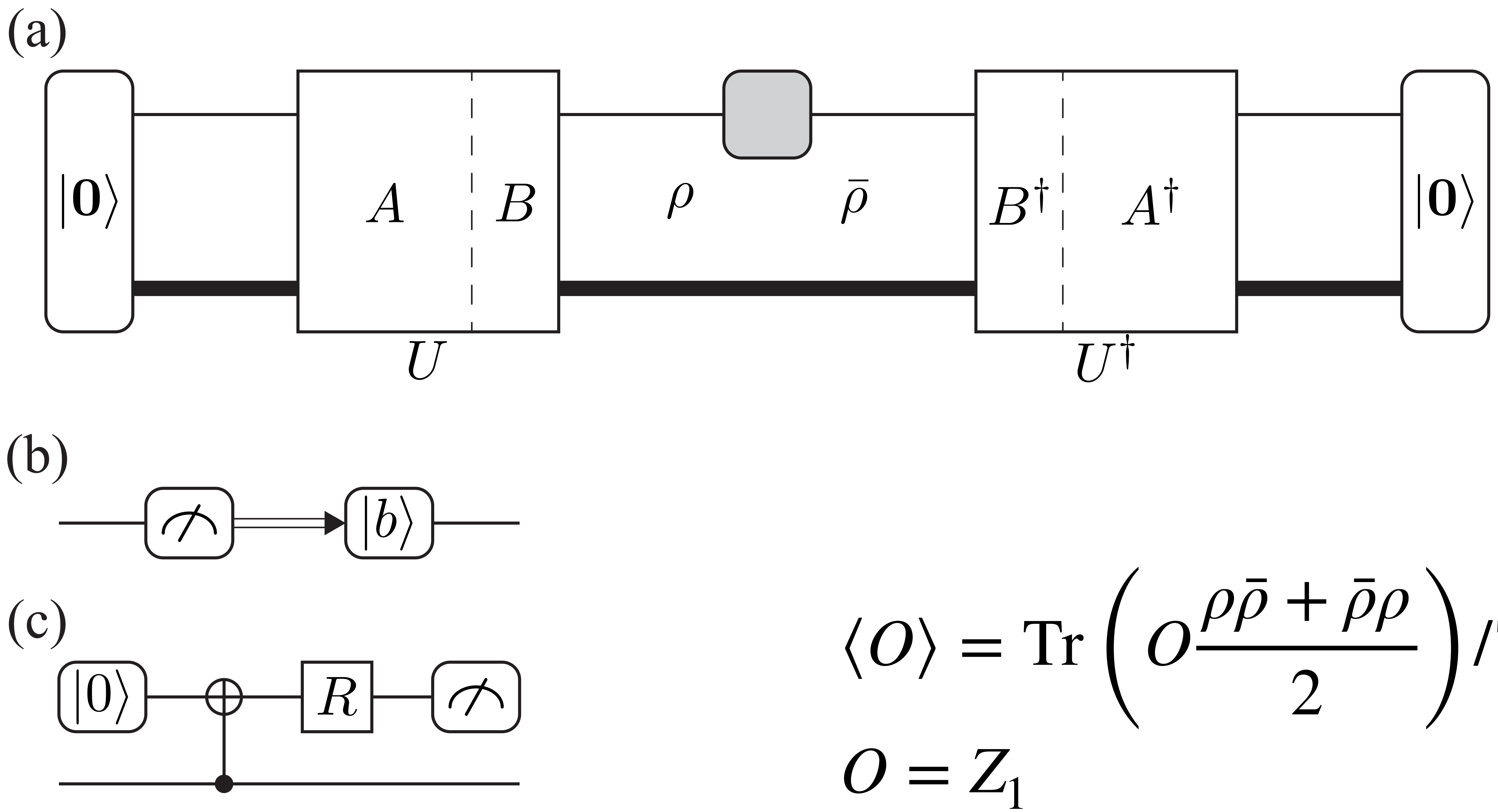
Bálint Koczor, Phys. Rev. X 11, 031057 (2021)
William J. Huggins *et al.*, Phys. Rev. X 11, 041036 (2021)
Piotr Czarnik, Andrew Arrasmith, Lukasz Cincio, Patrick J. Coles, arXiv:2102.06056

Purification and virtual distillation

$$\rho = (1-p)\,|\psi\rangle\langle\psi| + p\,|\psi_\perp\rangle\langle\psi_\perp|$$

$$\rho \rightarrow \frac{\rho^2}{\mathrm{Tr}\left(\rho^2\right)} = \frac{(1-p)^2\,|\psi\rangle\langle\psi| + p^2\,|\psi_\perp\rangle\langle\psi_\perp|}{(1-p)^2 + p^2}$$

Dual-state purification



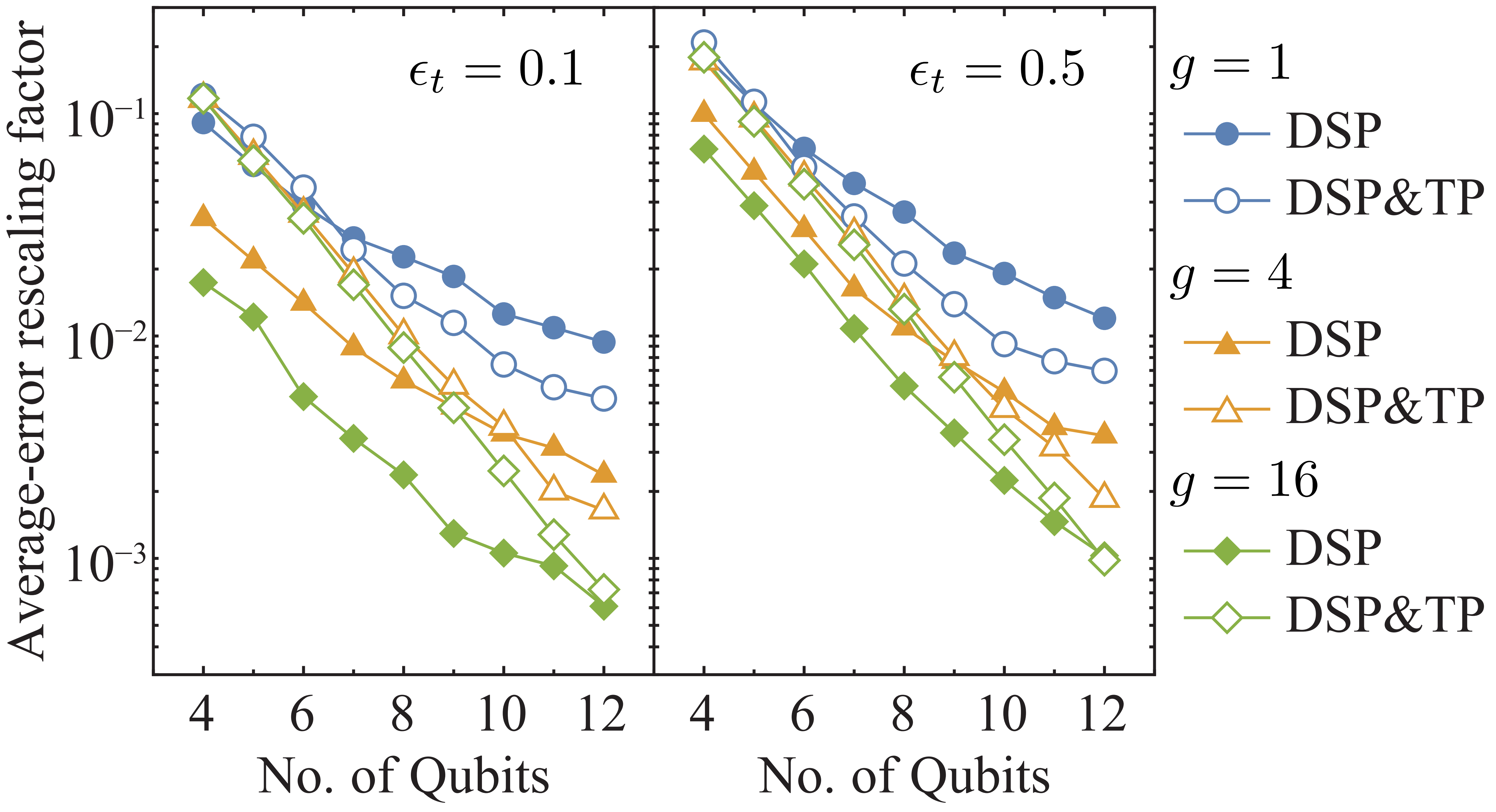
$$\langle O\rangle = \mathrm{Tr}\left(O\frac{\rho\bar{\rho}+\bar{\rho}\rho}{2}\right)/\mathrm{Tr}\left(\frac{\rho\bar{\rho}+\bar{\rho}\rho}{2}\right) = \frac{\langle Z_a\rangle_0}{1+\langle X_a\rangle_0}$$

$$O = Z_1$$

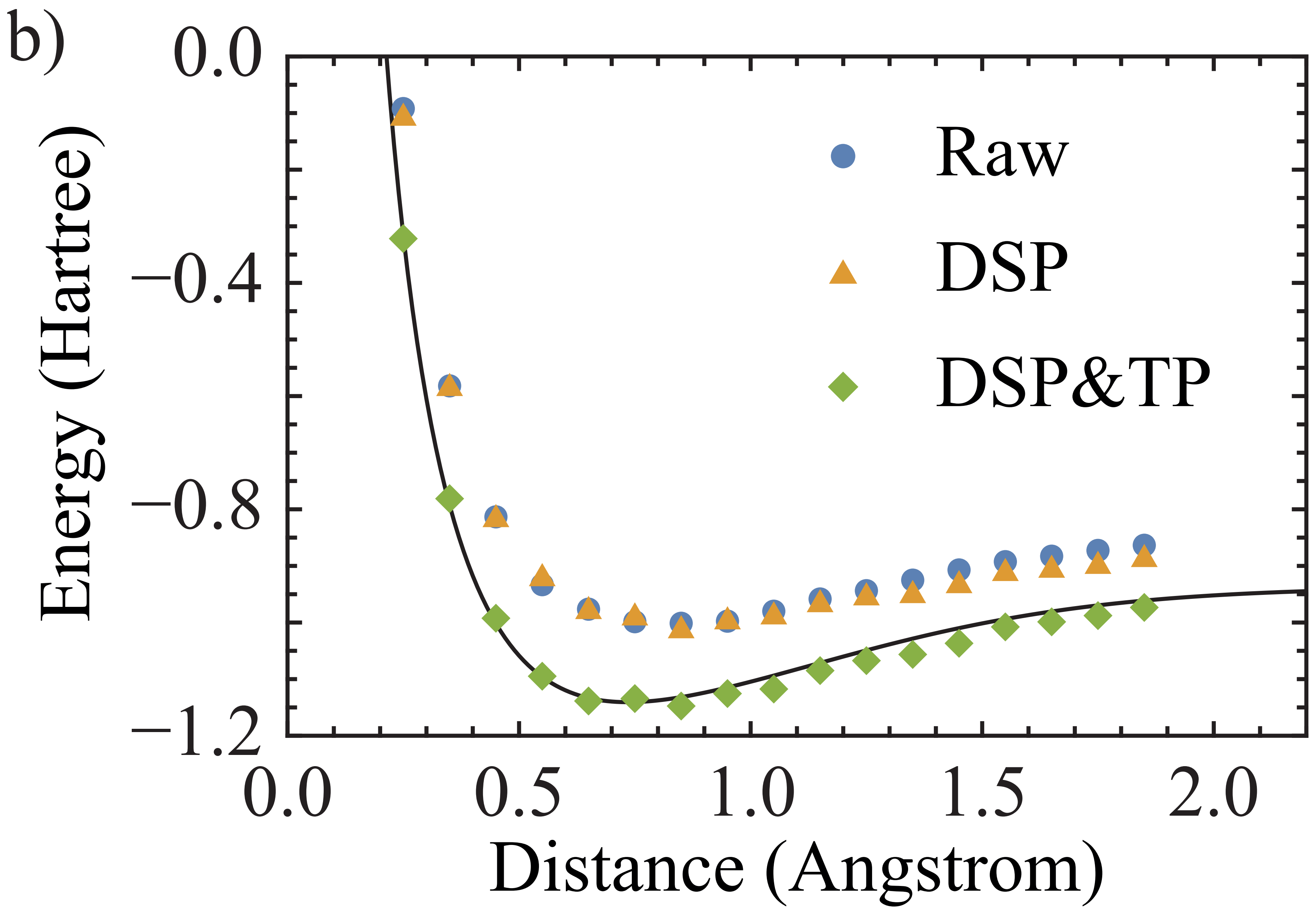
Mingxia Huo and **YL**, Phys. Rev. A 105, 022427 (2022)

Dual-state purification

Random circuit test



Experiment on *ibmq_athens*



Mingxia Huo and **YL**, Phys. Rev. A 105, 022427 (2022)

1. Background
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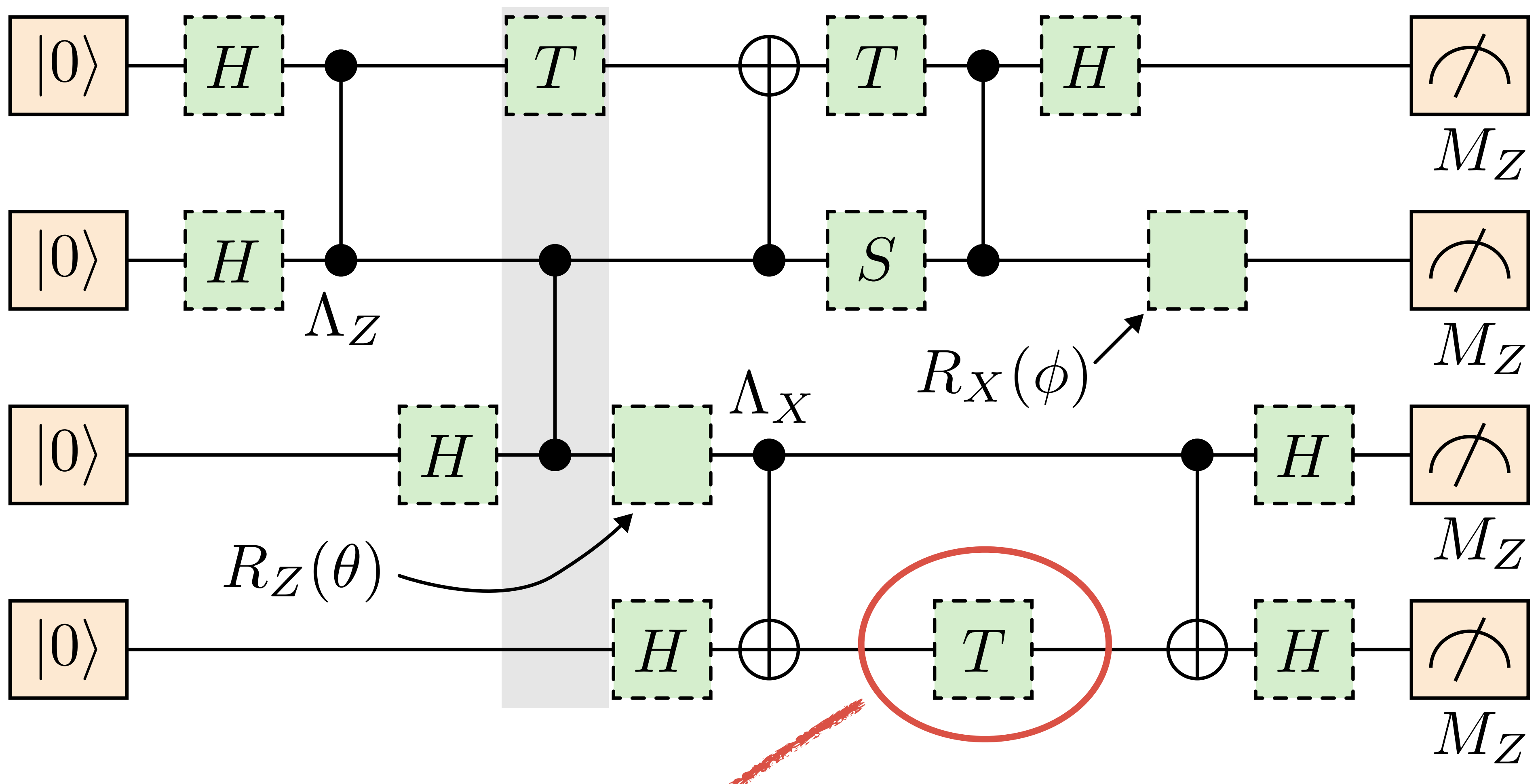
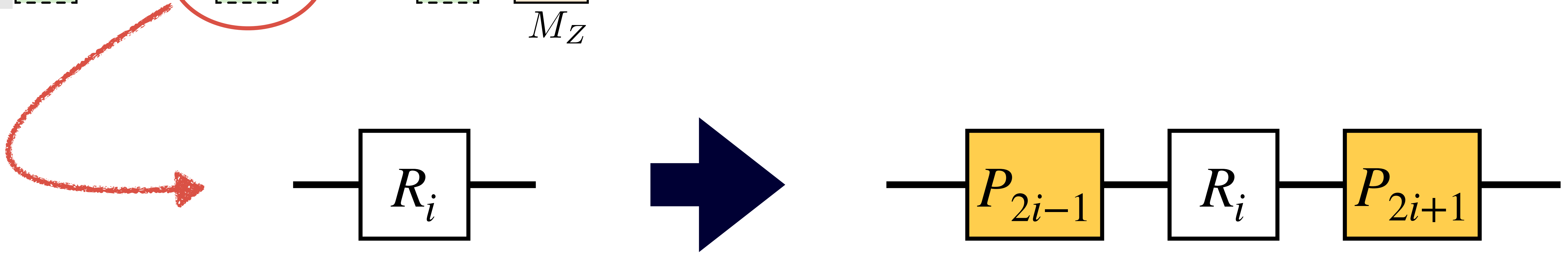
5. Learning-based approaches

Learning-based approach



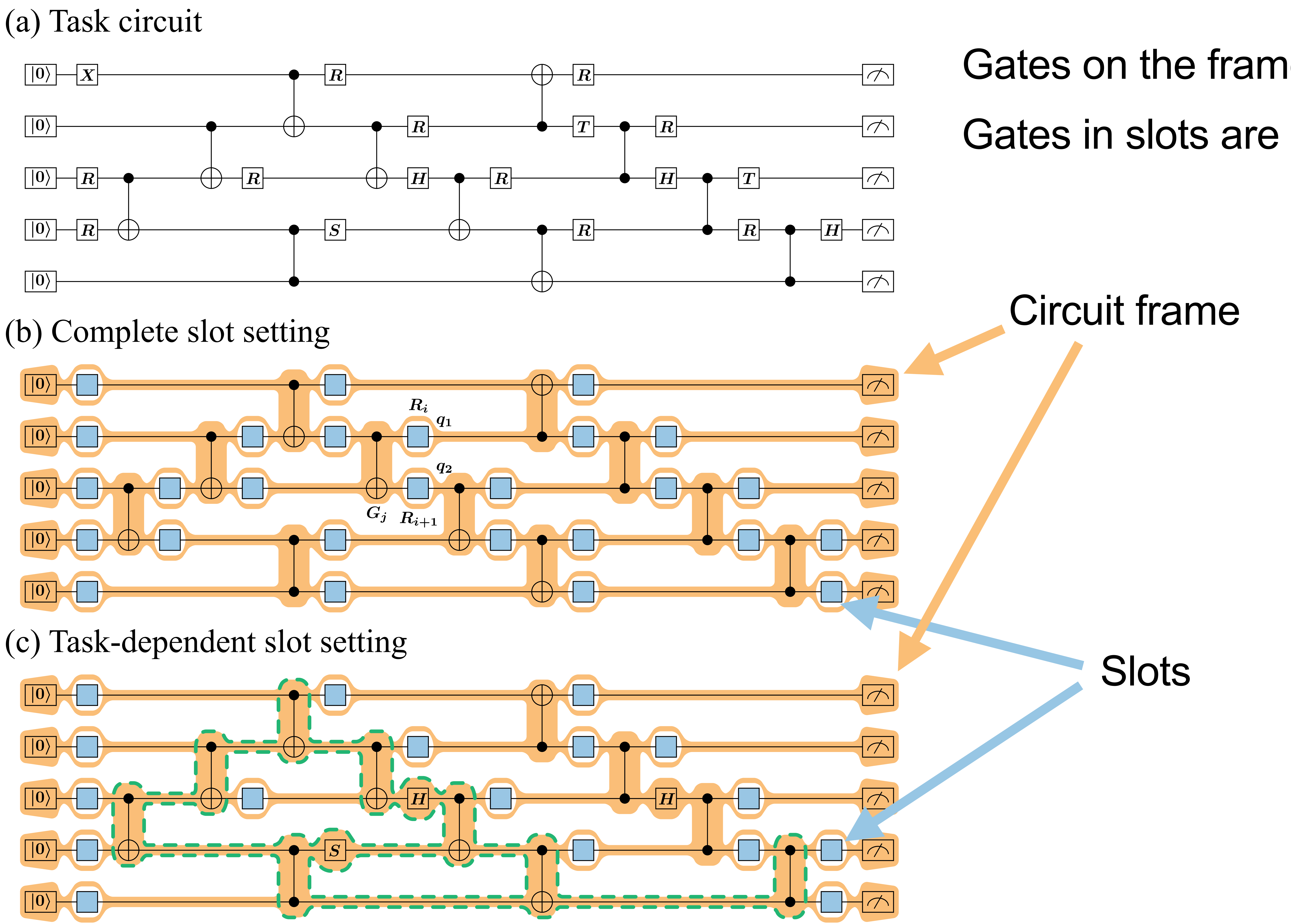
$$f'_C = q_1 y_{C_1} + q_2 y_{C_2} + q_3 y_{C_3} + \cdots$$

- $\mathrm{Loss}(q)$ = difference between f'_C and f_C
- Minimise $\mathrm{Loss}(q)$



Armands Strikis, Dayue Qin, Yanzhu Chen, Simon C. Benjamin, and **YL**, PRX Quantum 2, 040330 (2021)

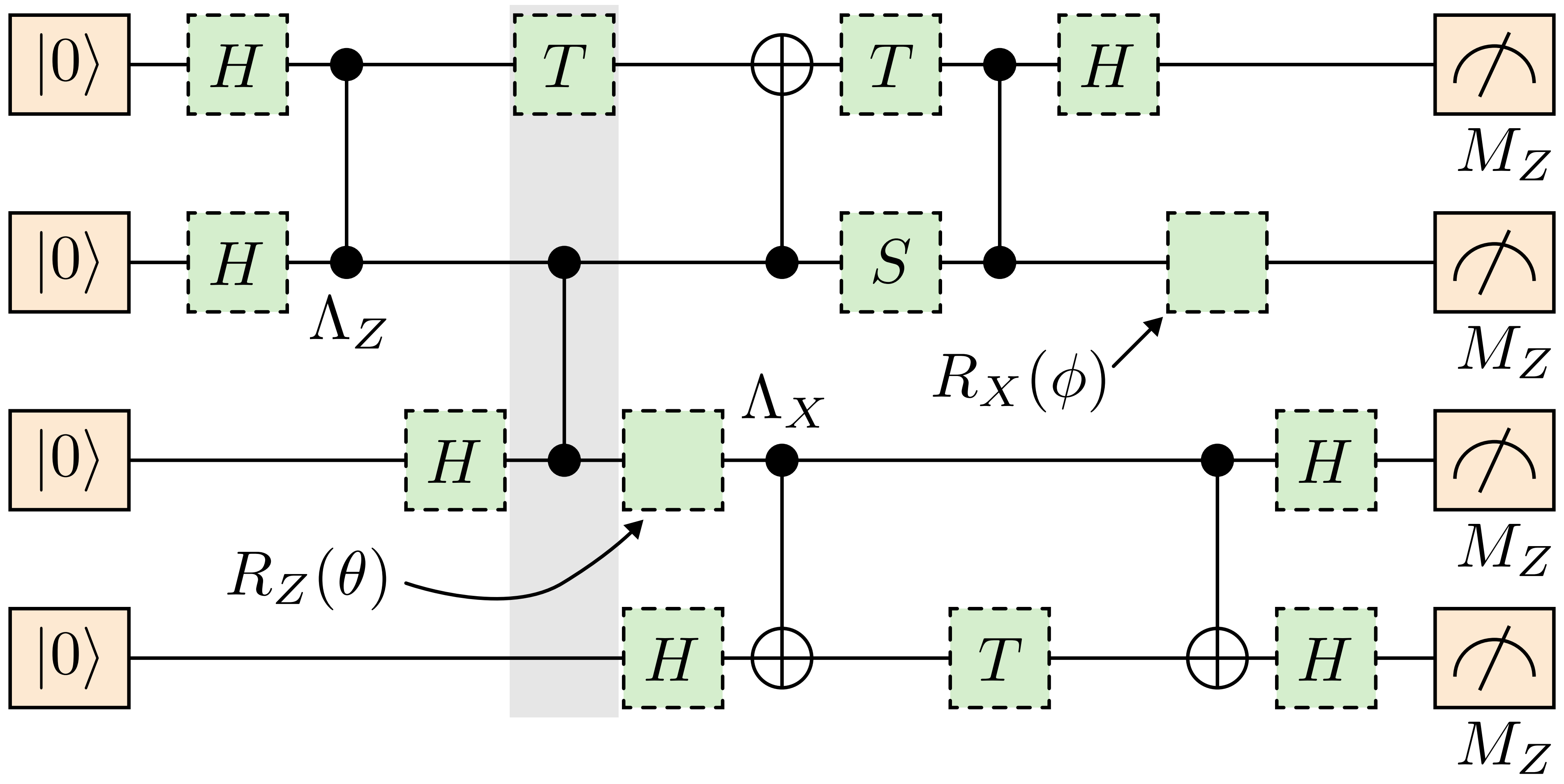
Circuit frame



Gates on the frame are fixed.

Gates in slots are variable.

Quadratic error loss



$$L_{\mathbb{R}} = \mathrm{E}\left[(f'_{\mathbf{C}} - f_{\mathbf{C}})^2\right]_{\mathbf{C}\in\mathbb{R}}$$

- $\mathbb{R}$ is a subset of circuits; all two-qubit gates are Clifford
- $\mathbb{R} = \mathbb{C}$, single-qubit gates are Clifford (Clifford sampling)
- $\mathbb{R} = \mathbb{U}$, single-qubit gates are general unitaries (Unitary sampling)

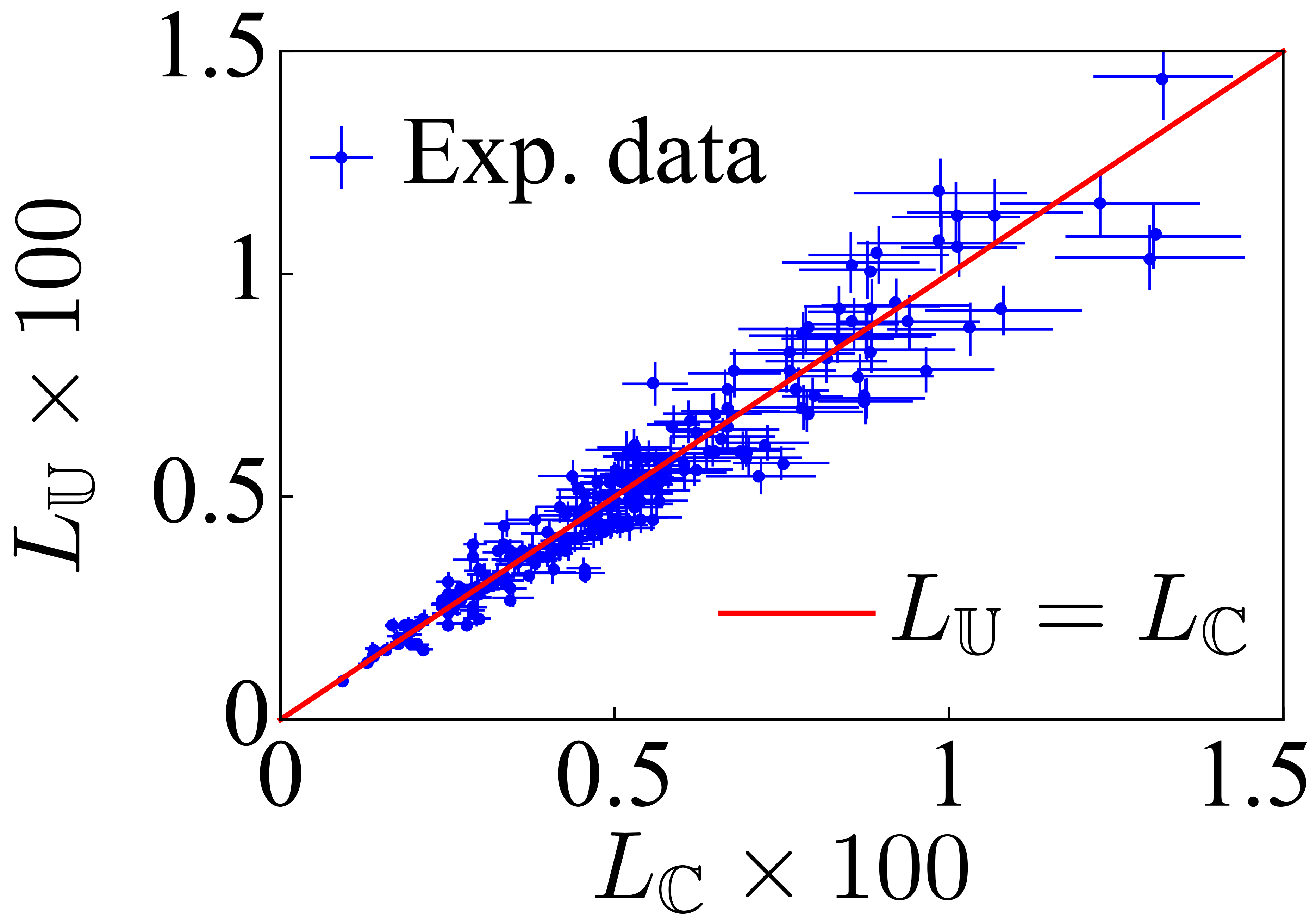
Armands Strikis, Dayue Qin, Yanzhu Chen, Simon C. Benjamin, and **YL**, PRX Quantum 2, 040330 (2021)

Unitary sampling and Clifford sampling

- f_{C} and f'_{C} are $\mathrm{Hom}(1,1)$

  (Assumption: Single-qubit-gate errors are gate-independent)

- $(f'_{\mathrm{C}} - f_{\mathrm{C}})^2$ is $\mathrm{Hom}(2,2)$
- $L_{\mathbb{U}} = L_{\mathbb{C}}$



Demonstrated with four superconducting qubits;

Randomly generated circuit with up to 10 layers of two-qubit gates;

Observable is a single-qubit Pauli operator.

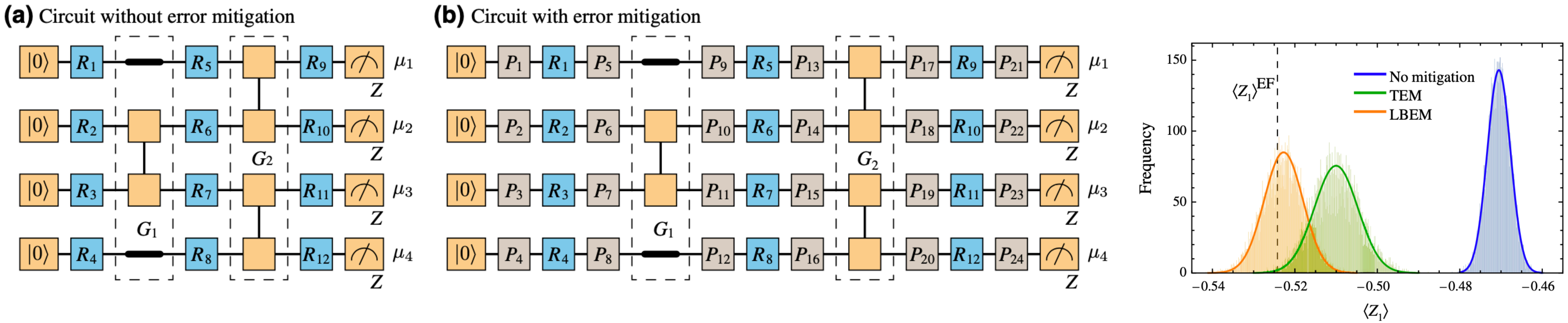
Zhen Wang, Yanzhu Chen, Zixuan Song, Dayue Qin, Hekang Li, Qiujiang Guo, H. Wang, Chao Song, and **YL**, Phys. Rev. Lett. 126, 080501 (2021)

Learning-based quantum error mitigation

A parametrised error mitigation formula: $f'_{\mathbf{C}}(\lambda) = F(y_{\mathbf{C}_1}, y_{\mathbf{C}_2}, \ldots, \lambda)$

Find optimal λ by minimising the error loss $L_{\mathbb{C}}(\lambda) = \mathrm{E}\left[\left(f'_{\mathbf{C}} - f_{\mathbf{C}}\right)^2\right]_{\mathbf{C}\in\mathbb{R}}$

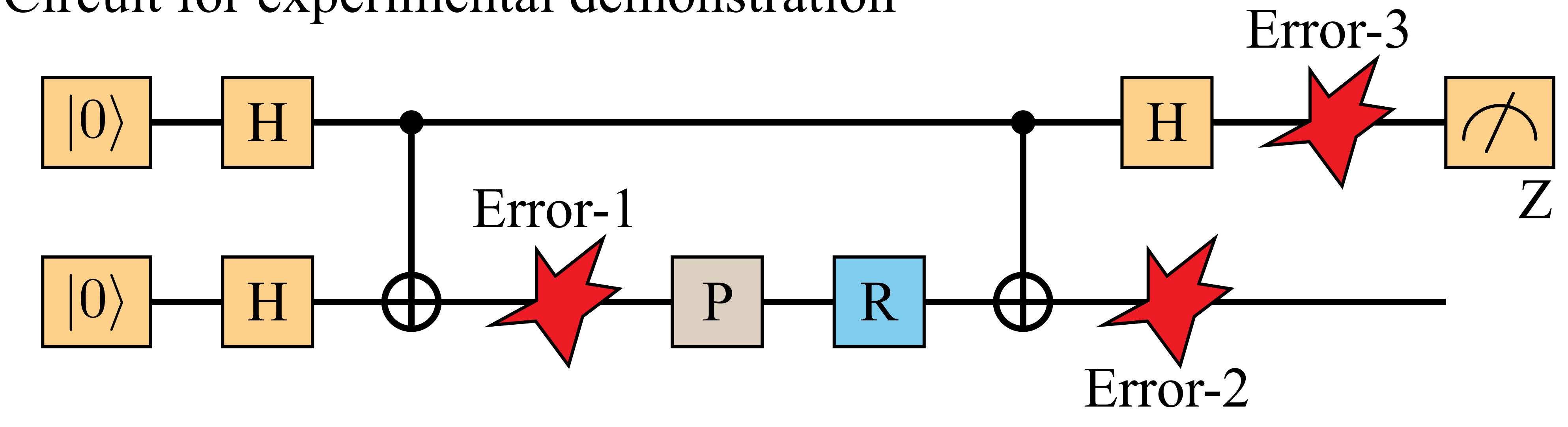
$f_{\mathbf{C}}$ can be evaluated on a classical computer



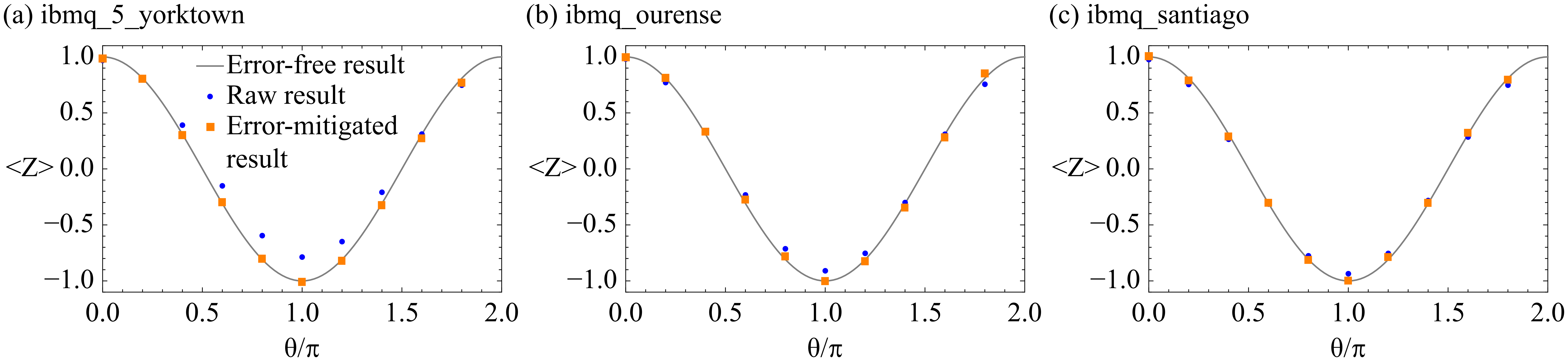
Armands Strikis, Dayue Qin, Yanzhu Chen, Simon C. Benjamin, and **YL**, PRX Quantum 2, 040330 (2021)

Demonstration on IBMQ

Circuit for experimental demonstration

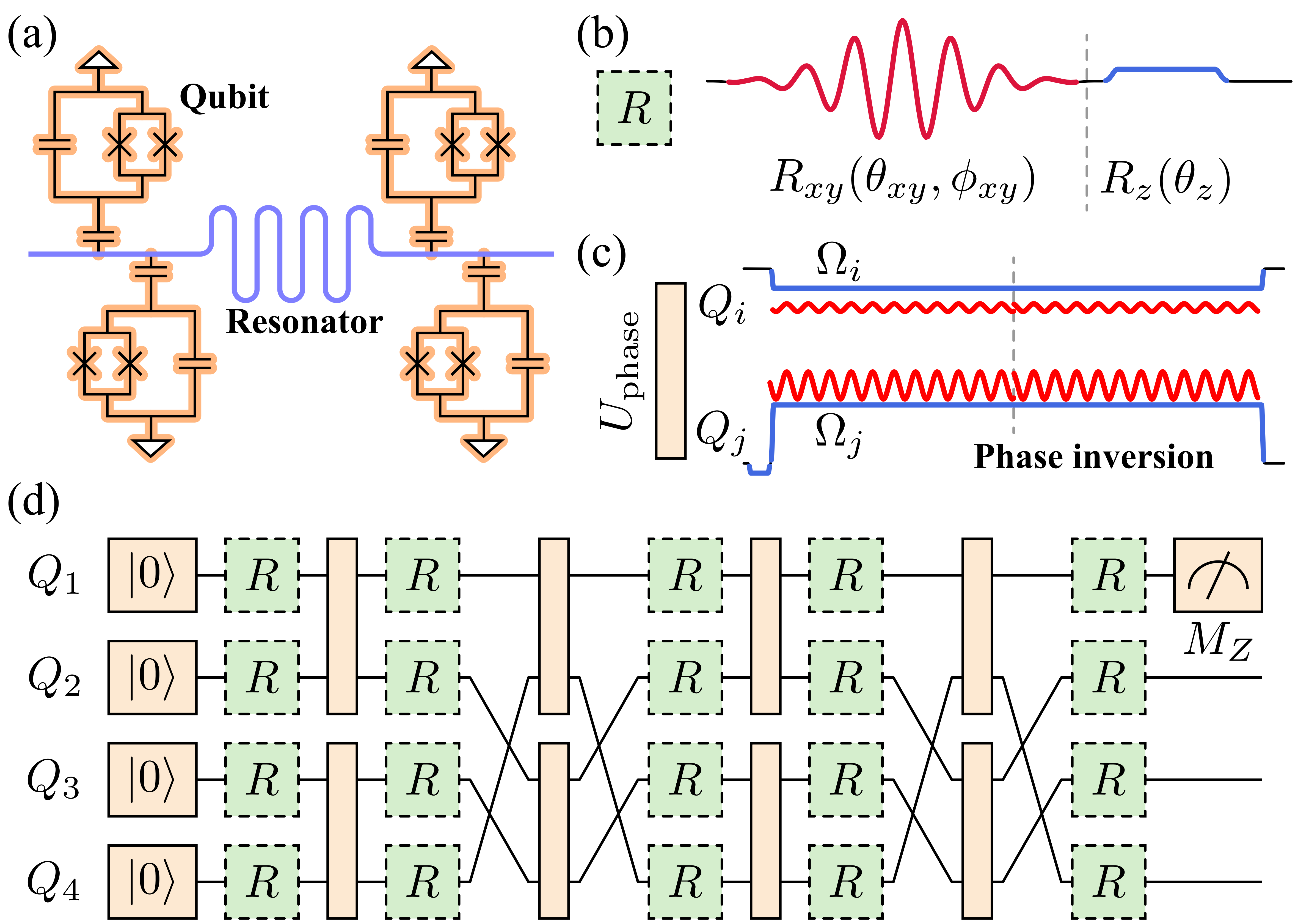


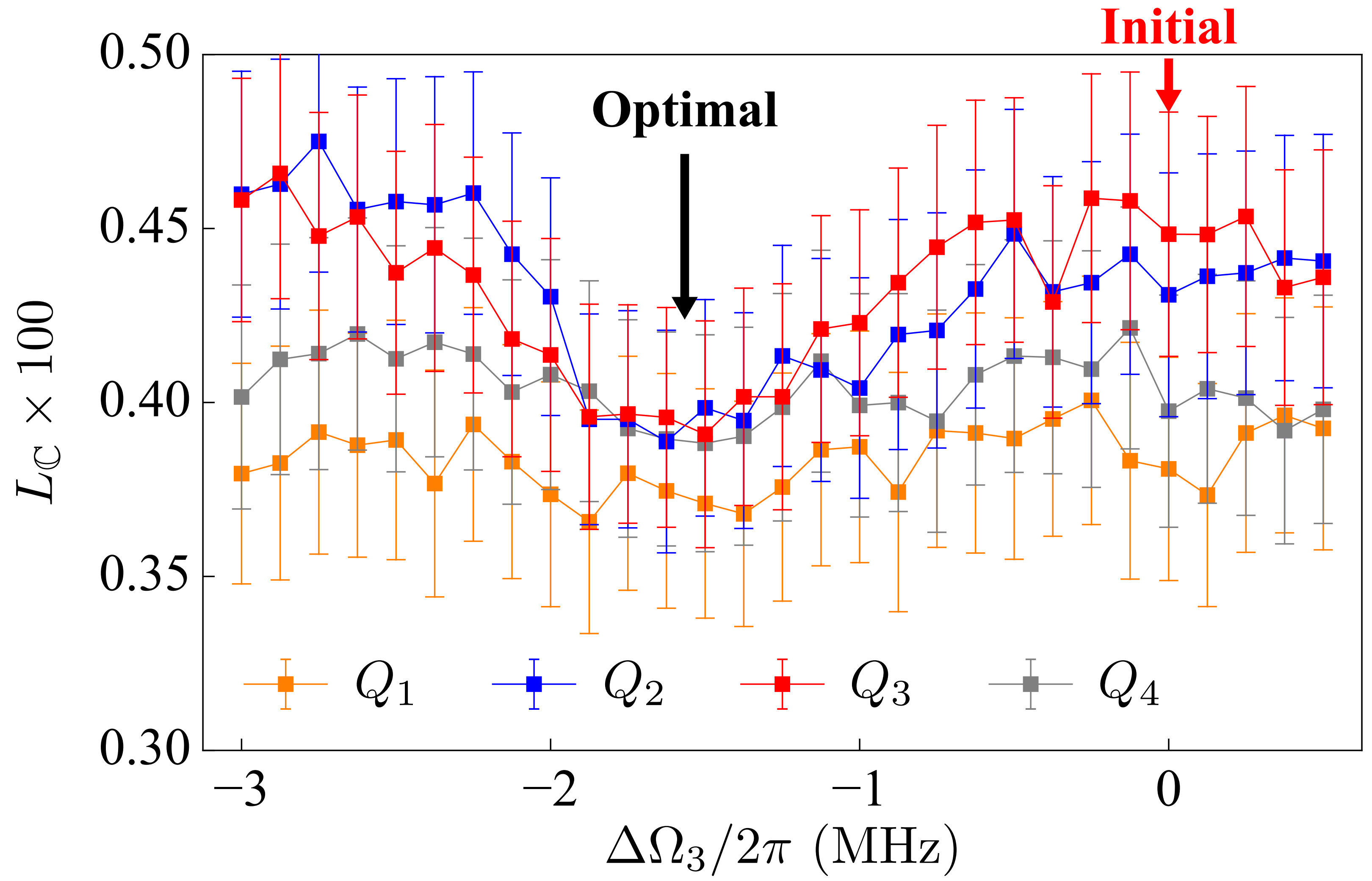
	Training	Test
No. of circuits	24	40

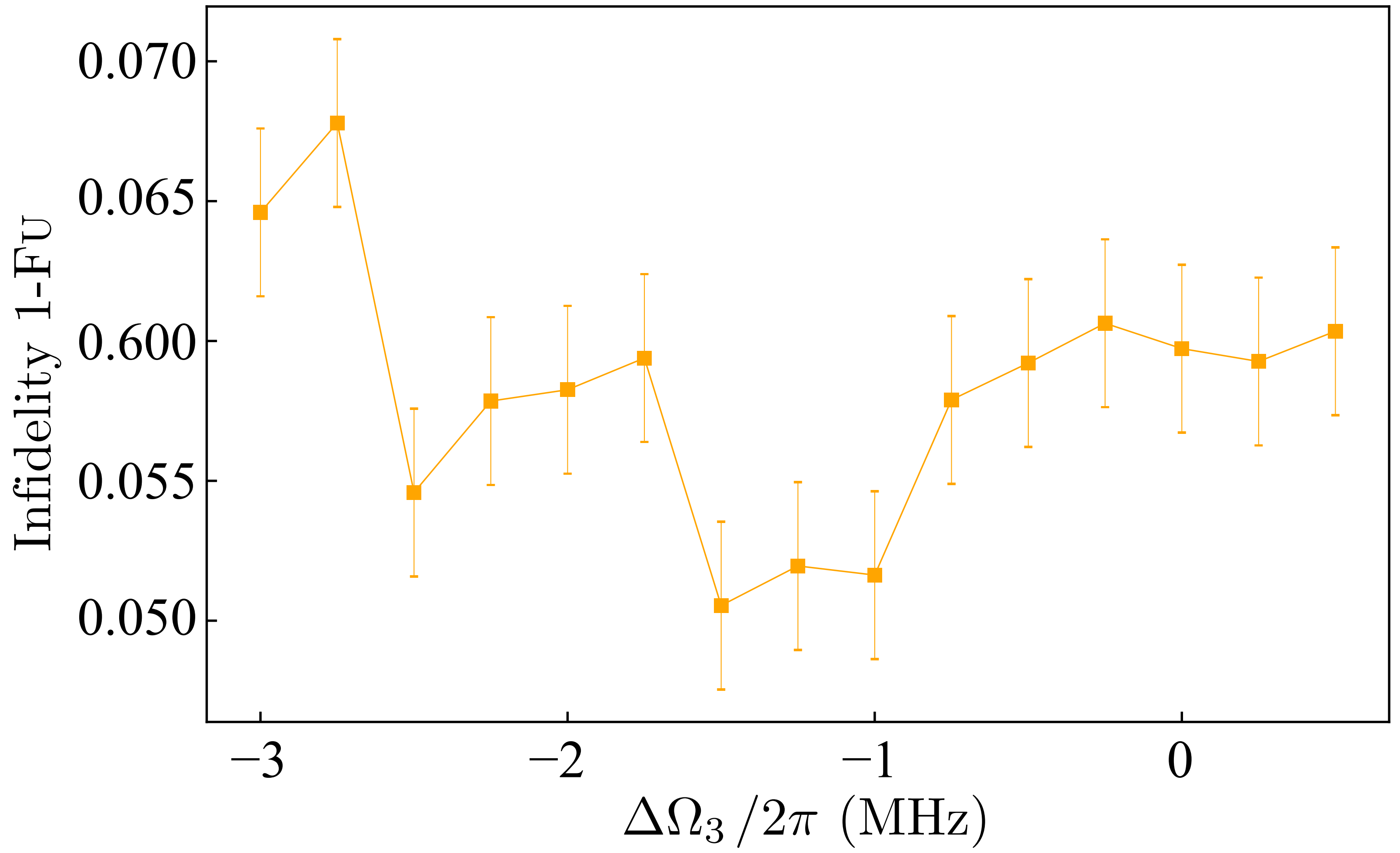


Armands Strikis, Dayue Qin, Yanzhu Chen, Simon C. Benjamin, and **YL**, PRX Quantum 2, 040330 (2021)

Application to Rabi frequency optimisation







Zhen Wang, Yanzhu Chen, Zixuan Song, Dayue Qin, Hekang Li, Qiujiang Guo, H. Wang, Chao Song, and **YL**, Phys. Rev. Lett. 126, 080501 (2021)

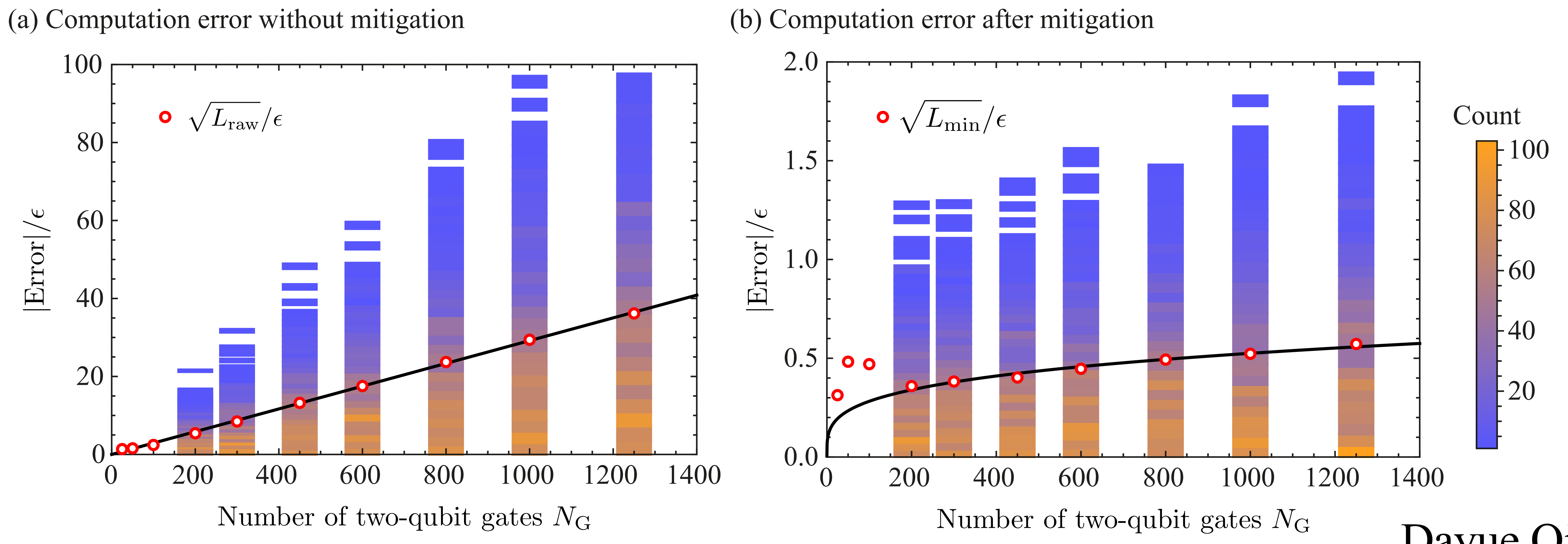
Phenomenological global depolarising error model

Global depolarising error model: $\mathscr{M}\left(\rho\right) = (1-\epsilon)[U]\left(\rho\right) + \epsilon 2^{-n} I^{\otimes n}$

Simplest error mitigation formula: $f'_C = a y_C$

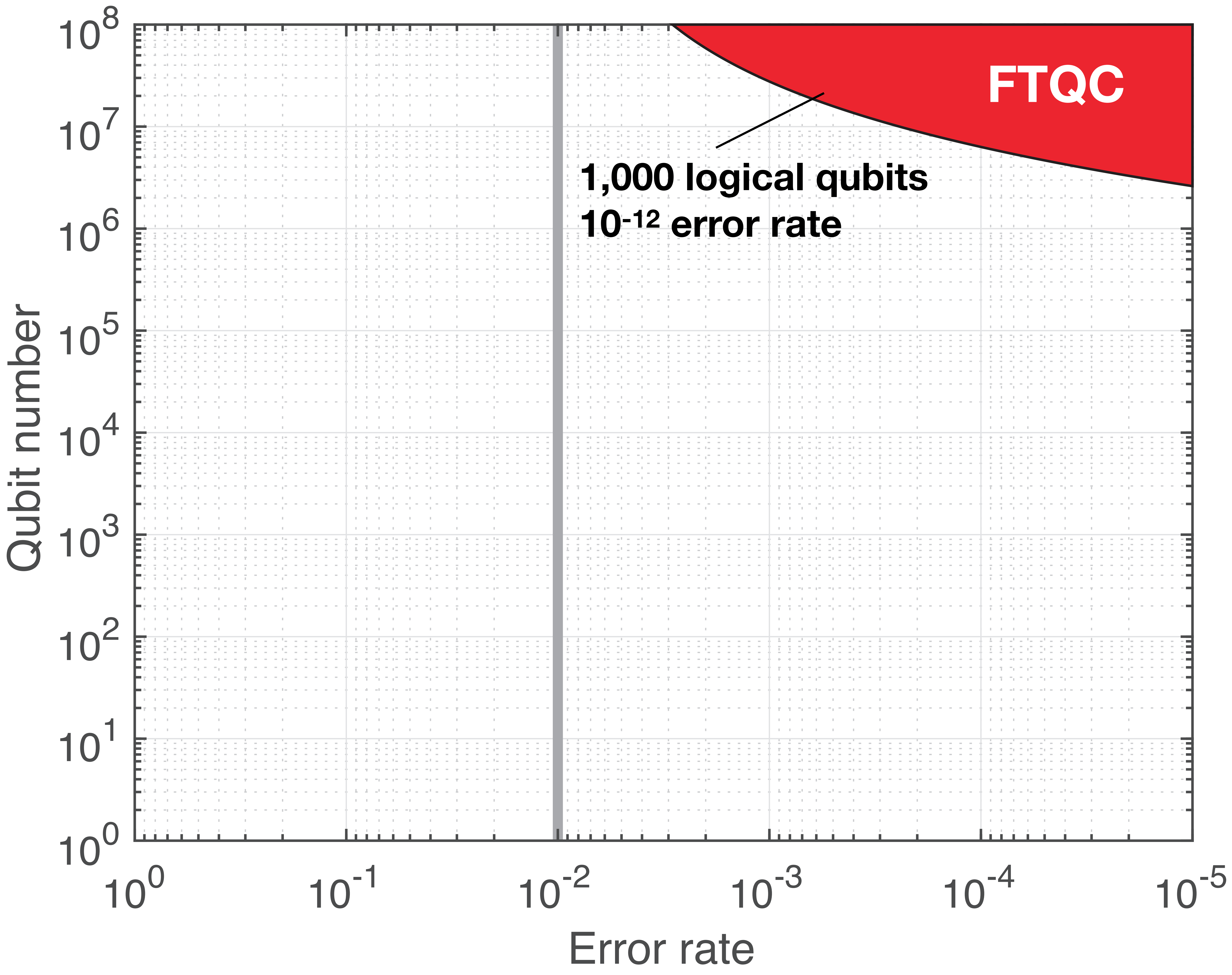
Piotr Czarnik, Andrew Arrasmith, Patrick J. Coles, Lukasz Cincio, Quantum 5, 592 (2021)

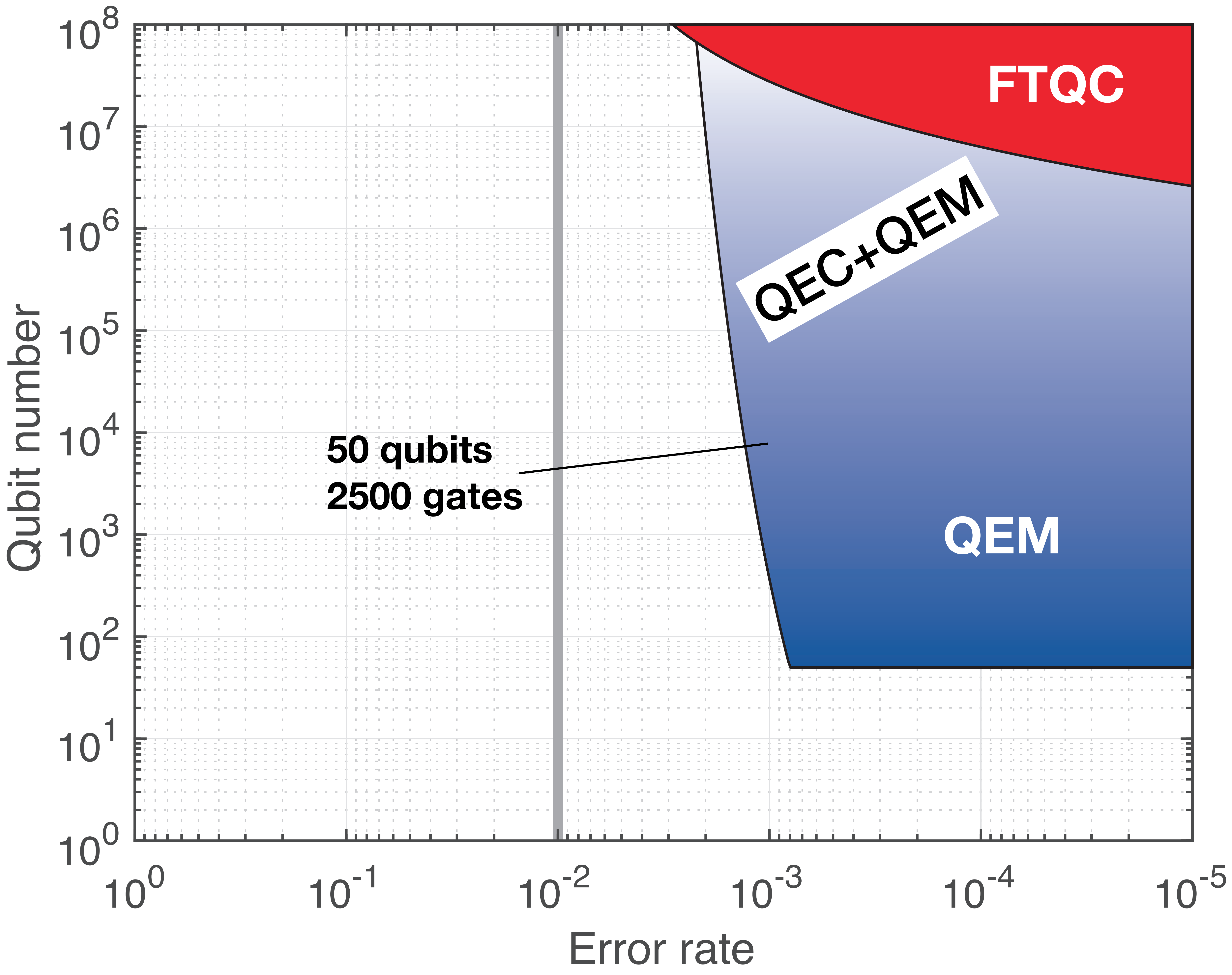
The global depolarising error model is a better approximation when the gate number is larger.



N scaling to $\sqrt{N}$ scaling

Dayue Qin, Yanzhu Chen, and **Ying Li**, arXiv:2112.06255


FTQC
1,000 logical qubits
10-12 error rate
Qubit number
Error rate
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^-1
10^-2
10^-3
10^-4
10^-5



FTQC
QEC+QEM
QEM
50 qubits
2500 gates
Qubit number
Error rate
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^0
10^-1
10^-2
10^-3
10^-4
10^-5

Thank you!