Holographic vortex pair annihilation in superfluid turbulence

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Based mainly on arXiv:1412.8417 with:
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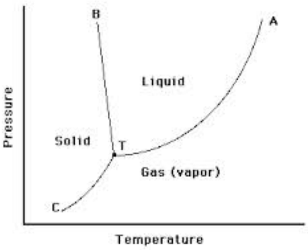
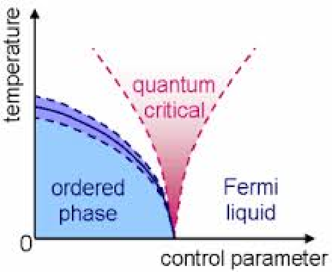
1. Motivation and introduction
2. Holographic model of superfluids
3. Quantized vortex and quantum turbulence in holographic superfluids
4. Vortex pair annihilation in holographic superfluid turbulence
5. Conclusion

- The physical world is partially unified by remarkable RG flow in QFT
  - High Energy Physics: IR→UV(Reductionism)
  - Condensed Matter Physics: UV→IR(Emergence)
    - Thermal Phase Transition
    - Quantum Phase Transition





- Another seemingly distinct part is gravitation, which is understood as geometry by general relativity

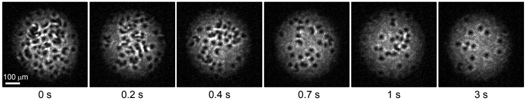
Remarkably, with AdS/CFT correspondence, general relativity can also geometrize renormalization flow in particular when the quantum field theory is strongly coupled, namely

$$GR = RG.$$

In this sense, the world is further unified by AdS/CFT duality. This talk will focus on its particular application to condensed matter physics by general relativity.

Vortex pair annihilation in holographic superfluid turbulence

[Shin *et.al.* arXiv:1403.4658]



Gross-Pitaevskii equation

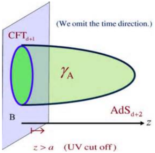
$$(i-\eta)\hbar\partial_t\varphi = \left(-\frac{\nabla^2}{2m} + V(x,y,t) + g|\varphi|^2 - \mu\right)\varphi$$

What AdS/CFT is I: Dictionary

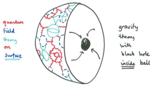
$$Z_{CFT}[J] = S_{AdS}[\phi](J=\phi)$$

What AdS/CFT is II: Implications

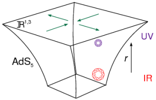
- Entanglement entropy for boundary QFT is equal to the extremal surface area in the bulk gravity



- Finite temperature field theory with finite chemical potential is dual to charged black hole



- AdS boundary corresponds to QFT at UV fixed point and the bulk horizon corresponds to IR fixed point



Why AdS/CFT III: String theory

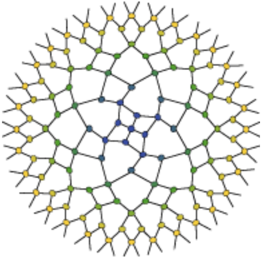
- Maldacena duality
- ABJM duality
- High spin gravity

Why AdS/CFT I: General relativity

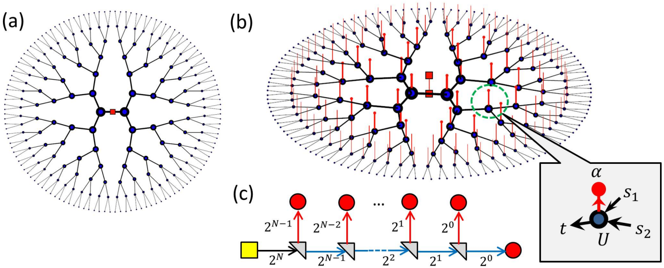
- Bousso's covariant entropy bound
- Bekenstein-Hawking's black hole thermodynamics
- Brown-York's surface tensor formulation of quasilocal energy and conserved charges
- Brown-Henneaux's asymptotic symmetry analysis for three dimensional gravity
- Minwalla's Gravity/Fluid correspondence

Why AdS/CFT II: Quantum field theory

- MERA



- EHM



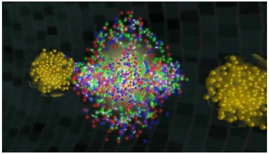
Why AdS/CFT is useful

It is a machine, mapping a hard quantum many-body problem to an easy classical few-body one.

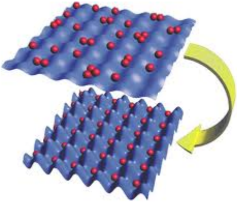
- Strongly coupled systems
- Non-equilibrium behaviors

Towards applied AdS/CFT I

- AdS/QCD



- AdS/CMT
  Non-Fermi liquids, superfluids and superconductors, charge density waves, thermalization and many-body localization...



- AdS/???

Towards applied AdS/CFT II

- Towards less symmetric configurations
- Towards fully numerical relativity regimes

Numerical Holography!

- Action of model

  [Hartnoll, Herzog, and Horowitz,arXiv:0803.3295,0810.6513]

$$S=\frac{1}{16\pi G}\int_{\mathcal{M}}d^4x\sqrt{-g}[R+\frac{6}{L^2}+\frac{1}{q^2}(-\frac{1}{4}F_{ab}F^{ab}-|D\Psi|^2-m^2|\Psi|^2)]. \quad (1)$$

- Background metric

$$ds^2=\frac{L^2}{z^2}[-f(z)dt^2-2dtdz+dx^2+dy^2],f(z)=1-(\frac{z}{z_h})^3. \quad (2)$$

- Heat bath temperature

$$T=\frac{3}{4\pi z_h}. \quad (3)$$

- Equations of motion

$$D_aD_a\Psi-m^2\Psi=0,\nabla_aF^{ab}=i(\bar{\Psi}D^b\Psi-\Psi\overline{D^b\Psi}). \quad (4)$$

- Asymptotical behavior at AdS boundary

$$A_\nu = a_\nu + b_\nu z + o(z), \tag{5}$$

$$\Psi = \frac{1}{L}[\phi z + z^2\psi + o(z^2)]. \tag{6}$$

- AdS/CFT dictionary

$$\langle J^\nu \rangle = \frac{\delta S_{ren}}{\delta a_\nu} = \lim_{z\to 0}\frac{\sqrt{-g}}{q^2}F^{z\nu}, \tag{7}$$

$$\langle O \rangle = \frac{\delta S_{ren}}{\delta \phi} = \lim_{z\to 0}[\frac{z\sqrt{-g}}{Lq^2}\overline{D^z\Psi} - \frac{z\sqrt{-\gamma}}{L^2q^2}\bar{\Psi}]$$

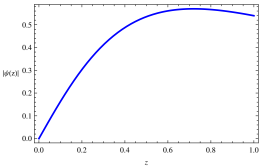
$$= \frac{1}{q^2}(\bar{\psi} - \dot{\bar{\phi}} - ia_t\bar{\phi}), \tag{8}$$

where

$$S_{ren} = S - \frac{1}{Lq^2}\int_{\mathcal{B}}\sqrt{-\gamma}|\Psi|^2 \tag{9}$$

is the renormalized action by holography.

Phase transition to a superfluid



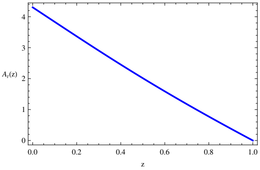
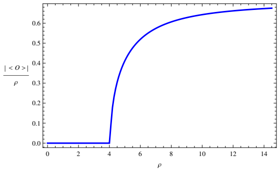


Figure: The profile of amplitude of scalar field and electromagnetic potential for the superconducting phase at the charge density $\rho = 4.7$.



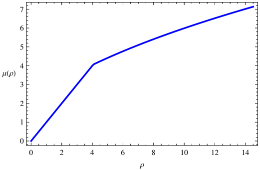


Figure: The condensate and chemical potential as a function of charge density with the critical charge density $\rho_c = 4.06(\mu_c = 4.07)$.

Quantized vortex in superfluids

With the superfluid velocity defined as

$$\mathbf{u} = \frac{\mathbf{j}}{|\psi|^2}, \mathbf{j} = \frac{i}{2}(\bar{\psi}\partial\psi - \psi\partial\bar{\psi}), \tag{10}$$

the winding number w of a vortex is determined by

$$w = \frac{1}{2\pi}\oint_\gamma d\mathbf{x}\cdot\mathbf{u}, \tag{11}$$

In particular, close to the core of a single vortex with winding number w, the condensate

$$\bar{\psi} \propto (\mathbf{z}-\mathbf{z_0})^w, w > 0 \tag{12}$$

$$\psi \propto (\mathbf{z}-\mathbf{z_0})^{-w}, w < 0 \tag{13}$$

with $\mathbf{z}$ the complex coordinate and $\mathbf{z_0}$ the location of the core.

Quantum turbulence in superfluids I

$$\partial_z(\partial_z A_t - \partial \cdot \mathbf{A}) = i(\bar{\Phi}\partial_z\Phi - \Phi\partial_z\bar{\Phi}), \quad \Phi = \frac{\Psi}{z} \tag{14}$$

once $\mathbf{A}$ is given at $t = 0$. For convenience but without loss of generality, we shall set the initial value $\mathbf{A} = 0$. With the above initial data and boundary conditions, the later time behavior of bulk fields can be obtained by the following evolution equations

$$\begin{aligned}
\partial_t\partial_z\Phi &= iA_t\partial_z\Phi + \frac{1}{2}[i\partial_z A_t\Phi + f\partial_z^2\Phi + f'\partial_z\Phi \\
&\quad +(\partial - i\mathbf{A})^2\Phi - z\Phi], \qquad (15)\\
\partial_t\partial_z\mathbf{A} &= \frac{1}{2}[\partial_z(\partial A_t + f\partial_z\mathbf{A}) + (\partial^2\mathbf{A} - \partial\partial\cdot\mathbf{A}) \\
&\quad -i(\bar{\Phi}\partial\Phi - \Phi\partial\bar{\Phi})] - \mathbf{A}\bar{\Phi}\Phi, \qquad (16)\\
\partial_t\partial_z A_t &= \partial^2 A_t + f\partial_z\partial\cdot\mathbf{A} - \partial_t\partial\cdot\mathbf{A} - 2A_t\bar{\Phi}\Phi \\
&\quad +if(\bar{\Phi}\partial_z\Phi - \Phi\partial_z\bar{\Phi}) - i(\bar{\Phi}\partial_t\Phi - \Phi\partial_t\bar{\Phi}). \qquad (17)
\end{aligned}$$

Quantum turbulence in superfluids II

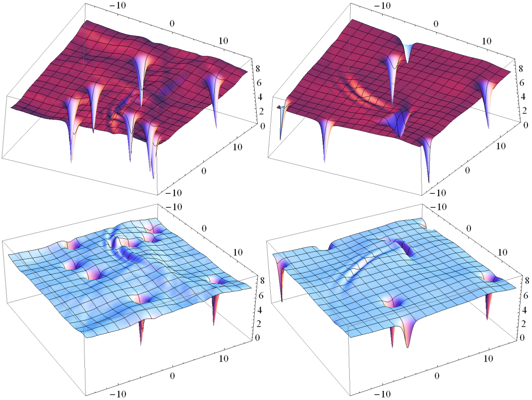


Figure: The bottom and top view of absolute value of turbulent condensate $|\langle O\rangle|$ for the superfluid at the chemical potential $\mu = 6.25$, where the left plot is for $t = 100$ and the right plot is for $t = 200$. The vortex cores are located at the position where the condensate vanishes, the shock waves are seen as the ripples, and the grey soliton is identified in the form of the bending structure with the condensate depleted.

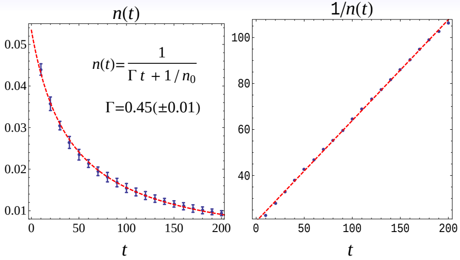


Figure: The temporal evolution of averaged vortex number density in the turbulent superfluid over 12 groups of data with randomly prepared initial conditions at the chemical potential $\mu = 6.25$

$$\frac{dn(t)}{dt} = -\Gamma n(t)^2, \tag{18}$$

where $\Gamma = \frac{vd}{2}$ with v the velocity of vortices and d cross section if the vortices can be regarded as a gas of particles.

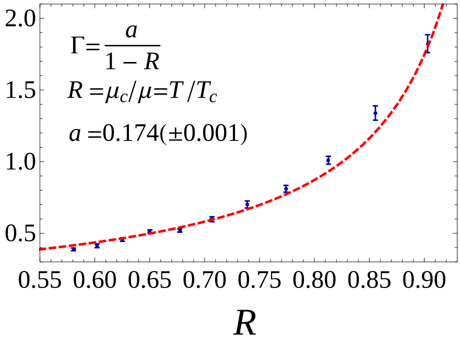


Figure: The variation of decay rate with respect to the chemical potential (equally spaced from $\mu = 4.5$ to $\mu = 7.0$).

Conclusion

- The decrease of vortex number can be well described by two-body decay due to vortex pair annihilation from a very early time on
- The decay rate is decreased (increased) with the chemical potential (temperature)
- The decay rate near the critical temperature is in good agreement with the mean field theory calculation
- Power law fit indicates that our holographic superfluid turbulence exhibits an obvious different decay pattern from that demonstrated in the real experiment
- Holography offers a first principles method for one to understand vortex dynamics by its gravity dual

Thanks for your attention!

Reminder

International Workshop
on Condensed Matter Physics and AdS/CFT
at Kavli IPMU, Japan(May 24-May 30, 2015)

http://indico.ipmu.jp/indico/conferenceDisplay.py?ovw=TrueconfId=49

Rene Meyer (Kavli IPMU), Shin Nakamura (Chuo U./ISSP), Hirosi Ooguri (Caltech/ Kavli IPMU), Masaki Oshikawa (ISSP), Masahito Yamazaki (Kavli IPMU)