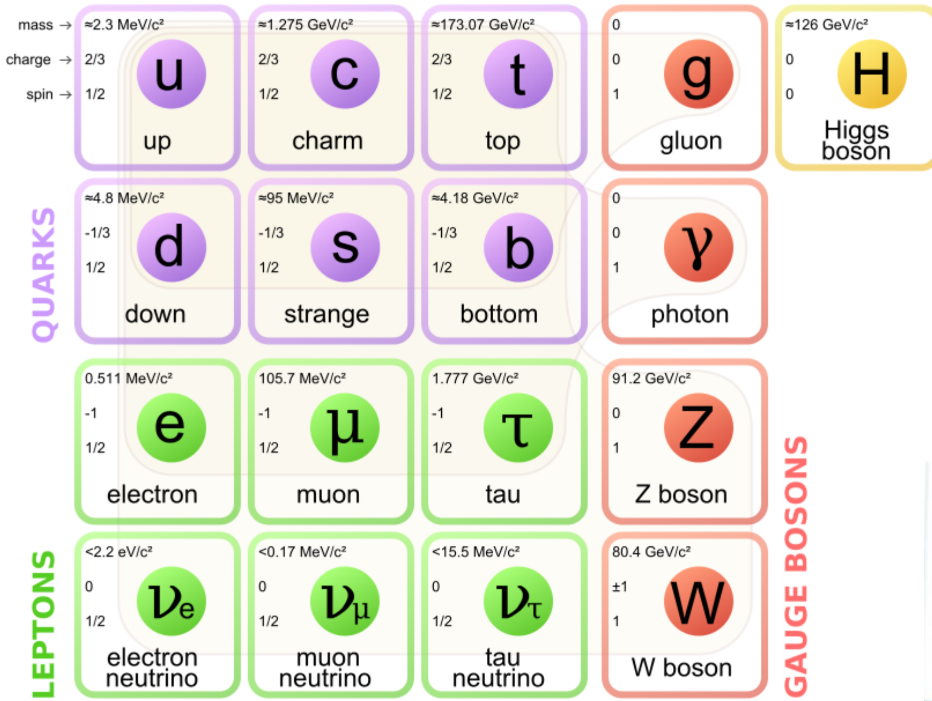
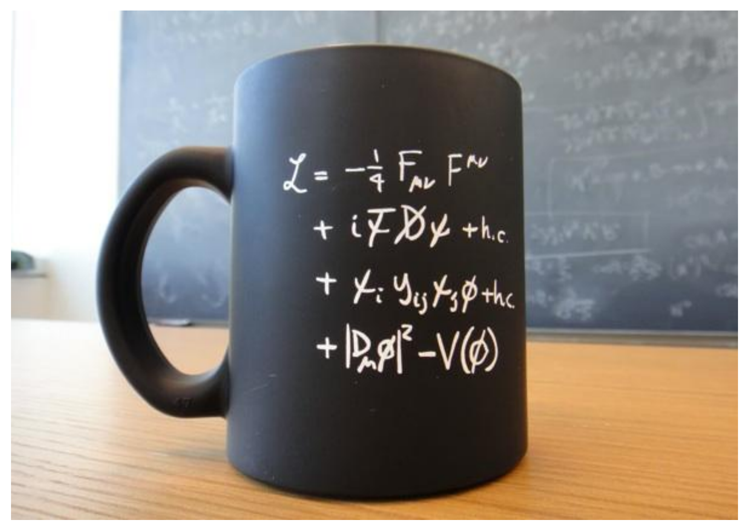
Effective field theory for electroweak and dark matter phenomenology

张宏浩
中山大学

The standard model of particle physics



	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$	3	2	$\frac{1}{6}$
u_R	3	1	$\frac{2}{3}$
d_R	3	1	$-\frac{1}{3}$
$\ell_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$	1	2	$-\frac{1}{2}$
e_R	1	1	-1



Some problems of the standard model

- triviality

$$\lambda(Q) = \frac{\lambda(v)}{1 - \frac{3}{4\pi^3}\log\frac{Q^2}{v^2}\lambda(v)}$$

- hierarchy problem

$$m_h^2 \approx m_{tree}^2 - \frac{3}{8\pi^2}\lambda_t^2\Lambda^2 + \frac{1}{16\pi^2}g^2\Lambda^2 + \frac{1}{16\pi^2}\lambda^2\Lambda^2$$
$$\sim m_{tree}^2 - (200-20-10)(125GeV)^2\left(\frac{\Lambda}{10TeV}\right)^2$$

- too many parameters
- existence of dark matter
- smallness of neutrino mass
- matter/antimatter asymmetry
- vacuum stability

……

The SM may be just an **effective field theory**.

Effective field theory and **the rho parameter**

Gasser-Leutwyler **QCD** chiral Lagrangian for pseudoscalar mesons

$$\mathcal{L}_{eff} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \ldots$$

$$\boxed{\mathcal{L}^{(2)} = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U\chi^\dagger\rangle + h_0 D_\mu\theta D^\mu\theta}$$

$$\chi \equiv 2\,B\,(s + ip)\,, \qquad \langle X\rangle \equiv \mathrm{tr}(X)$$

$$D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + i\,U(v_\mu - a_\mu)$$

$$D_\mu\theta = \partial_\mu\theta + 2\,\mathrm{tr}(a_\mu)$$

Count the external fields as

$$\theta = O(1), \qquad v_\mu, a_\mu = O(p), \qquad s, p = O(p^2)$$

$$\mathcal{L}^{(4)} = \frac{\ell_1}{4}\langle D_\mu U D^\mu U\rangle^2 + \frac{\ell_2}{4}\langle D_\mu U D_\nu U\rangle\langle D^\mu U D^\nu U\rangle + \frac{\ell_3}{4}\langle\chi U^\dagger + U\chi^\dagger\rangle^2 + \frac{\ell_4}{4}\langle D_\mu\chi D^\mu U^\dagger + D_\mu U D^\mu\chi^\dagger\rangle + \ldots$$

Electroweak chiral Lagrangian without the Higgs boson

[Appelquist, Wu. 1993]

$$D_\mu U = \partial_\mu U + ig\frac{\vec{\tau}}{2}\cdot \vec{W}_\mu U - ig' U \frac{\tau_3}{2} B_\mu$$

$$T \equiv U\tau_3 U^\dagger , \qquad V_\mu \equiv (D_\mu U)U^\dagger$$

$$W_{\mu\nu} \equiv \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu]$$

p^2 order:

$$\mathcal{L}_0 \equiv \frac{1}{4} f^2 Tr[(D_\mu U)^\dagger (D^\mu U)] - \frac{1}{4} B_{\mu\nu}B^{\mu\nu} - \frac{1}{2} Tr W_{\mu\nu}W^{\mu\nu}$$

$$\mathcal{L}_1^{'} \equiv \frac{1}{4}\beta_1 g^2 f^2 [Tr(TV_\mu)]^2$$

p^4 order:

$$\mathcal{L}_1 \equiv \frac{1}{2}\alpha_1 g g' B_{\mu\nu} Tr(TW^{\mu\nu}) \qquad \mathcal{L}_2 \equiv \frac{1}{2} i\alpha_2 g' B_{\mu\nu} Tr(T[V^\mu, V^\nu])$$

$$\mathcal{L}_3 \equiv i\alpha_3 g Tr(W_{\mu\nu}[V^\mu, V^\nu]) \qquad \mathcal{L}_4 \equiv \alpha_4 [Tr(V_\mu V_\nu)]^2$$

$$\mathcal{L}_5 \equiv \alpha_5 [Tr(V_\mu V^\mu)]^2 \qquad \mathcal{L}_6 \equiv \alpha_6 Tr(V_\mu V_\nu) Tr(TV^\mu) Tr(TV^\nu)$$

$$\mathcal{L}_7 \equiv \alpha_7 \; Tr(V_\mu V^\mu) Tr(TV_\nu) Tr(TV^\nu) \qquad \mathcal{L}_8 \equiv \frac{1}{4}\alpha_8 \; g^2 \; [Tr(TW_{\mu\nu})]^2$$

$$\mathcal{L}_9 \equiv \frac{1}{2} i\alpha_9 g Tr(TW_{\mu\nu}) Tr(T[V^\mu, V^\nu]) \qquad \mathcal{L}_{10} \equiv \frac{1}{2}\alpha_{10}[Tr(TV_\mu)Tr(TV_\nu)]^2$$

$$\mathcal{L}_{11} \equiv \alpha_{11} g\epsilon^{\mu\nu\rho\lambda} \; Tr(TV_\mu) Tr(V_\nu W_{\rho\lambda})$$

Another **equivalent** formulation of electroweak chiral Lagrangian

[H.H.Zhang, Q.Wang. 06]

Building Blocks

Instead of using the $SU(2)_L$ covariant & $U(1)_Y$ invariant building blocks $T \equiv U\tau^3 U^\dagger$, $V_\mu \equiv (D_\mu U)U^\dagger$, etc., we use the $SU(2)_L$ invariant & $U(1)_Y$ covariant building blocks as follows

$$\tau^3\,, \qquad X_\mu \equiv U^\dagger(D_\mu U)\,, \qquad g_1 B_{\mu\nu}\,,$$
$$\overline{W}_{\mu\nu} \equiv U^\dagger g_2 \frac{\tau^a}{2} W^a_{\mu\nu} U\,.$$

$$S_{\rm eff} = \int d^4x \{\mathcal{L}_0 + \mathcal{L}'_1 + \sum_{i=1}^{19} \mathcal{L}_i - \frac{1}{4} Z_1 (B_{\mu\nu})^2 - \frac{1}{2} Z_2 \mathrm{tr}(\overline{W}_{\mu\nu})^2\}$$

where the explicit terms are given as

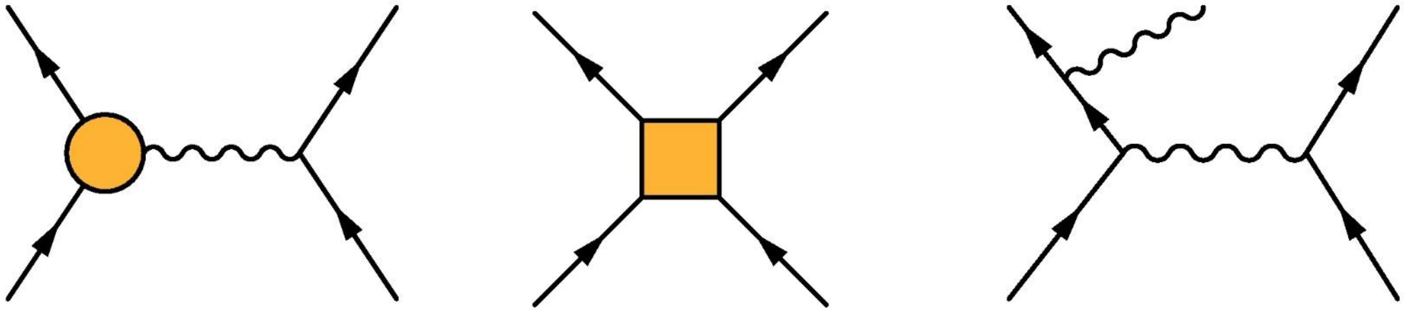
$\mathcal{L}_0$	$-\frac{1}{4} f^2 \mathrm{tr}(X_\mu X^\mu)$
$\mathcal{L}'_1$	$\frac{1}{4}\beta_1 f^2 [\mathrm{tr}(\tau^3 X_\mu)]^2$
$\mathcal{L}_1$	$\frac{1}{2}\alpha_1 g_1 B_{\mu\nu} \mathrm{tr}(\tau^3 \overline{W}^{\mu\nu})$
$\mathcal{L}_2$	$i\alpha_2 g_1 B_{\mu\nu} \mathrm{tr}(\tau^3 X^\mu X^\nu)$
$\mathcal{L}_3$	$2i\alpha_3 \mathrm{tr}(\overline{W}_{\mu\nu} X^\mu X^\nu)$

$\mathcal{L}_4$	$\alpha_4 [\mathrm{tr}(X_\mu X_\nu)]^2$
$\mathcal{L}_5$	$\alpha_5 [\mathrm{tr}(X_\mu X^\mu)]^2$
$\mathcal{L}_6$	$\alpha_6 \mathrm{tr}(X_\mu X_\nu)\mathrm{tr}(\tau^3 X^\mu)\mathrm{tr}(\tau^3 X^\nu)$
$\mathcal{L}_7$	$\alpha_7 \mathrm{tr}(X_\mu X^\mu)\mathrm{tr}(\tau^3 X_\nu)\mathrm{tr}(\tau^3 X^\nu)$
$\mathcal{L}_8$	$\frac{1}{4}\alpha_8 [\mathrm{tr}(\tau^3 \overline{W}_{\mu\nu})]^2$

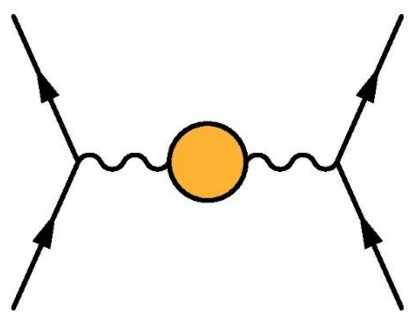
$\mathcal{L}_9$	$i\alpha_9 \mathrm{tr}(\tau^3 \overline{W}_{\mu\nu})\mathrm{tr}(\tau^3 X^\mu X^\nu)$
$\mathcal{L}_{10}$	$\frac{1}{2}\alpha_{10} [\mathrm{tr}(\tau^3 X_\mu)\mathrm{tr}(\tau^3 X_\nu)]^2$
$\mathcal{L}_{11}$	$\alpha_{11} \epsilon^{\mu\nu\rho\lambda} \mathrm{tr}(\tau^3 X_\mu)\mathrm{tr}(X_\nu \overline{W}_{\rho\lambda})$
$\mathcal{L}_{12}$	$\alpha_{12} \mathrm{tr}(\tau^3 X_\mu)\mathrm{tr}(X_\nu \overline{W}^{\mu\nu})$
$\mathcal{L}_{13}$	$\alpha_{13} \epsilon^{\mu\nu\rho\sigma} g_1 B_{\mu\nu} \mathrm{tr}(\tau^3 \overline{W}_{\rho\sigma})$
$\mathcal{L}_{14}$	$2i\alpha_{14} \epsilon^{\mu\nu\rho\sigma} g_1 B_{\mu\nu} \mathrm{tr}(\tau^3 X_\rho X_\sigma)$

Electroweak radiative corrections can be categorized into two classes

Direct corrections (vertex, box and bremsstrahlung corrections)



Oblique corrections (gauge boson propagator corrections)



Oblique corrections can be treated in a self-consistent and model-independent way through an effective lagrangian to incorporate a large class of Feynman diagrams into a few running couplings [Kennedy & Lynn, NPB 322, 1 (1989)]

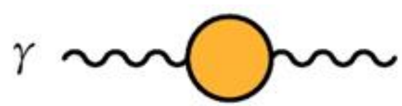
Electroweak **oblique** parameters

[Peskin, Takeuchi. 1991]

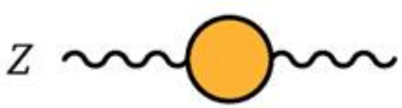
$$\alpha S \equiv 4e^2 \left[\Pi'_{33}(0) - \Pi'_{3Q}(0) \right],$$

$$\alpha T \equiv \frac{e^2}{s^2 c^2 m_Z^2} \left[\Pi_{11}(0) - \Pi_{33}(0) \right],$$

$$\alpha U \equiv 4e^2 \left[\Pi'_{11}(0) - \Pi'_{33}(0) \right],$$

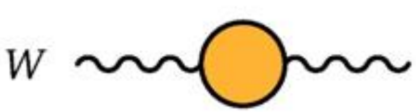


$$= ie^2 \Pi_{QQ}(p^2) g^{\mu\nu} + (p^\mu p^\nu \text{ terms})$$



$$= \frac{ie^2}{s_W c_W} [\Pi_{3Q}(p^2) - s_W^2 \Pi_{QQ}(p^2)] g^{\mu\nu} + (p^\mu p^\nu \text{ terms})$$

$$= \frac{ie^2}{s_W^2 c_W^2} [\Pi_{33}(p^2) - 2s_W^2 \Pi_{3Q}(p^2) + s_W^4 \Pi_{QQ}(p^2)] g^{\mu\nu} + (p^\mu p^\nu \text{ terms})$$



$$= \frac{ie^2}{s_W^2} \Pi_{11}(p^2) g^{\mu\nu} + (p^\mu p^\nu \text{ terms})$$

Peskin

Electroweak Chiral Lagrangian & Oblique Corrections

Three most interesting terms:

$$\mathcal{L}_1' = \frac{1}{4}\beta_1 f^2[\mathrm{tr}(\tau^3 X_\mu)]^2 \,, \qquad \mathcal{L}_1 = \frac{1}{2}\alpha_1 g_1 B_{\mu\nu}\mathrm{tr}(\tau^3 \overline{W}^{\mu\nu}) \,, \qquad \mathcal{L}_8 = \frac{1}{4}\alpha_8[\mathrm{tr}(\tau^3 \overline{W}_{\mu\nu})]^2$$

They are related to the oblique electroweak corrections by

$$S \equiv -16\pi \frac{d}{dq^2}\Pi_{3B}(q^2)\Big|_{q^2=0} = -16\pi\alpha_1$$

$$\alpha T \equiv \frac{e^2}{c^2 s^2 m_Z^2}(\Pi_{11}(0) - \Pi_{33}(0)) = 2\beta_1$$

$$U \equiv 16\pi \frac{d}{dq^2}(\Pi_{11}(q^2) - \Pi_{33}(q^2))\Big|_{q^2=0} = -16\pi\alpha_8$$

where α is the fine structure constant, while c and s are cosine and sine of Weinberg angle respectively.

Electroweak chiral Lagrangian **with the Higgs boson** and the **oblique parameters**

p^2 order terms include: $\mathcal{O}_h = a_h (H^\dagger D_\mu H)(D^\mu H^\dagger H)$

p^4 order terms include:

$$\mathcal{O}_{WB} = a_{WB} (H^\dagger W^a_{\mu\nu} \sigma^a H) B^{\mu\nu}$$

$$\mathcal{O}_{WW} = a_{WW} (H^\dagger W^a_{\mu\nu} \sigma^a H)(H^\dagger W^{b\mu\nu} \sigma^b H)$$

$$\frac{1}{\Lambda^2}(H^\dagger W^a_{\mu\nu}\sigma^a H)B^{\mu\nu} \to S, \quad \frac{1}{\Lambda^2}(H^\dagger D_\mu H)(D^\mu H^\dagger H) \to T, \quad \frac{1}{\Lambda^4}(H^\dagger W^a_{\mu\nu}\sigma^a H)(H^\dagger W^{b\mu\nu}\sigma^b H) \to U$$

$$\boxed{\begin{aligned} S &= \frac{4c_W s_W}{\alpha} a_{WB} v^2 \\ T &= -\frac{1}{2\alpha} a_h v^2 \\ U &= -\frac{4g^2 s_W^2}{\alpha} a_{WW} v^4 \end{aligned}}$$

The **Veltman** ρ **parameter** $\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W}$

measures the relative strength of the neutral and charged currents $\frac{J_Z^\mu J_{Z,\mu}}{J^{\mu+} J_\mu^-}$

An arbitrary number of Higgs multiplets Φ_i with isospin I_i, third component I_i^3, and vacuum expectation values v_i for electroweak symmetry breaking:

$$\rho = \frac{\sum_i \left[I_i \left(I_i + 1 \right) - (I_i^3)^2 \right] v_i^2}{2 \sum_i (I_i^3)^2 v_i^2}$$

[Djouadi, hep-ph/0503172]

↓

$\rho = 1$ for an arbitrary number of **doublet** and **singlet** Higgs fields

(because the model has a custodial $\mathrm{SU}(2)_\mathrm{R}$ globlal symmetry in this case)

↓

Standard model (SM): $\rho = 1$ at tree level

The experimentally measured value for the ρ parameter is extremely close to 1, putting stringent constraints on many new physics models

From the electroweak global fit, $\rho_0 = 1.00037 \pm 0.00023$ [PDG 2016]

Here $\rho_0 \equiv m_W^2/(m_Z^2 \hat{c}_Z^2 \hat{\rho})$ describes new sources of $\mathrm{SU(2)_L}$ breaking that cannot be accounted for by the SM Higgs doublet or m_t effects

It has been argued that the $\rho = 1$ relation is naturally valid up to electroweak radiative corrections in a large class of models in which the Higgs sector has an unbroken $\mathrm{SU(2)_R}$ globlal symmetry, the so-called **custodial symmetry**

Therefore, even for **strong dynamic models**, such as technicolor models, a custodial symmetry can protect the $\rho = 1$ relation to all orders of interactions

[Sikivie, Susskind, Voloshin & Zakharov, NPB 173, 189 (1980)]

Custodial symmetry in the standard model

SM scalar potential $V = -\mu^2 H^\dagger H + \lambda(H^\dagger H)^2$ is a function of $H^\dagger H$

Viewpoint of an $\mathrm{SU(2)_R}$ globlal symmetry:

tensor language

$$(\mathcal{H}^A)_i = \begin{pmatrix} H_i^\dagger \\ H_i \end{pmatrix}, \quad A \text{ is and } \mathrm{SU(2)_R} \text{ indice}$$

$$H^\dagger H = -\frac{1}{2}\varepsilon_{AB}\varepsilon^{ij}(\mathcal{H}^A)_i(\mathcal{H}^B)_j$$

Viewpoint of an SO(4) global symmetry (isomorphic to $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$):

$$H = \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}, \qquad H^\dagger H = \varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2$$

The custodial symmetry is explicitly broken by the Yukawa couplings of fermions at **loop** level.

Custodial symmetry in the SM in **matrix language**

$$H=\begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \sim \left(2, \tfrac{1}{2}\right)$$

$$H^*=\begin{pmatrix} h^- \\ h^{0*} \end{pmatrix} \sim \left(2^*, -\tfrac{1}{2}\right) \quad \rightarrow \quad \tilde{H} \equiv i\sigma_2 H^* = \begin{pmatrix} h^{0*} \\ h^- \end{pmatrix} \sim \left(2, -\tfrac{1}{2}\right)$$

构造块状矩阵 $\Phi \equiv (\underbrace{\tilde{H}}_{\text{超荷}\ -\frac{1}{2}} \ \ \underbrace{H}_{\frac{1}{2}})$

由于 $H^\dagger H = \tilde{H}^\dagger \tilde{H}$

$$\mathrm{tr}(\Phi\Phi^\dagger) = 2H^\dagger H$$

故 $H^\dagger H$ 具有 $SU(2)_L \times SU(2)_R$ global sym.

$$\Phi \rightarrow V_L \Phi V_R^\dagger$$

$$\Phi \equiv \begin{pmatrix} \tilde{H} & H \end{pmatrix}$$

$$\begin{array}{ccc} & \downarrow & \downarrow \\ \text{超荷} & -\frac{1}{2} & \frac{1}{2} \end{array}$$

$$D_\mu \Phi = \partial_\mu \Phi - ig \frac{\sigma^a}{2} W^a_\mu \Phi + \Phi\, ig' \frac{\tau^3}{2} B_\mu$$

$$\leftrightarrow \quad D_\mu H = \left(\partial_\mu - ig \frac{\sigma^a}{2} W^a_\mu - ig' \frac{1}{2} B_\mu\right) H$$

$$\mathrm{tr}\left[(D_\mu \Phi^\dagger)(D^\mu \Phi)\right] = 2 (D_\mu H^\dagger)(D^\mu H)$$

Relations among the rho parameter, the oblique parameter T, and the electroweak chiral Lagrangian

$$\rho - 1 = \alpha\, T = 2\beta_1$$

Usually it is obtained by the 1-loop Feynman diagrammatic calculation.

In this work, we propose a simple nondiagrammatic calculation method.

H.H.Zhang, Eur.Phys.J. C67 (2010) 51-56.

What are used in this method?

- path integral

$$\exp\left(i S_{\mathrm{EW}}[U, W_\mu^a, B_\mu]\right) = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left(i S_{\mathrm{eff}}[U, W_\mu^a, B_\mu, \bar{\psi}, \psi]\right)$$

- electroweak chiral Lagrangian

$$S_{\mathrm{eff}}[V_\mu^a, V_\mu^0, \bar{\psi}_\xi, \psi_\xi] = \int d^4x\, \bar{\psi}_\xi (i\partial\!\!\!/ + v\!\!\!/ + a\!\!\!/\gamma_5 - s - m)\psi_\xi$$

- Schwinger proper time method

$$\mathrm{Re}\,\mathrm{Tr}\ln(D+m) = -\frac{1}{2}\lim_{\Lambda\to\infty}\int d^4x \int_{\frac{1}{\Lambda^2}}^{\infty}\frac{d\tau}{\tau}e^{-\tau m^2}\mathrm{tr}_{c,f,l}\langle x|e^{-\tau(E-\nabla^2)}|x\rangle$$

- Consider the scalar source **s** as **p^0** order rather than **p^2** order.

$$\mathrm{tr}_f\left(s a_\mu s a^\mu\right)$$

$$\mathrm{tr}_f\left[(d_\mu s)(d^\mu s)\right]$$

传统上，人们以为它们是p^6项，此前没有人关注过。

亮点： 只须在**Seely Dewitt展开**中计算出这两项的系数，就可简单地求得rho参数！

Hypothetical heavy fermions:

$$\psi_L \equiv \begin{pmatrix} U \\ D \end{pmatrix}_L \sim (2, 0), \qquad U_R \sim \left(1, \frac{1}{2}\right),$$

$$D_R \sim \left(1, -\frac{1}{2}\right).$$

$$\begin{aligned} S_{\text{eff}}&[U, W_\mu^a, B_\mu, \bar{\psi}, \psi] \\ &= \int d^4x \Big[\bar{\psi}_L \left(i\not{\partial} - g_2 \frac{\tau^a}{2} \not{W}^a \right) \psi_L \\ &\quad + \bar{\psi}_R \left(i\not{\partial} - g_1 \frac{\tau^3}{2} \not{B} \right) \psi_R \\ &\quad - (\bar{\psi}_L U M \psi_R + \bar{\psi}_R M U^\dagger \psi_L) \Big], \end{aligned} \qquad M \equiv \mathrm{diag}(m_U,\ m_D)$$

$$U(x) \to V_L(x) U(x) V_R^\dagger(x),$$

$$\exp\big(i S_{\mathrm{EW}}[U, W_\mu^a, B_\mu]\big)$$
$$= \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\big(i S_{\mathrm{eff}}[U, W_\mu^a, B_\mu, \bar{\psi}, \psi]\big),$$

Chiral decomposition: $U(x) = \xi_L^\dagger(x)\xi_R(x),$

$$\xi_L(x) \to h(x)\xi_L(x)V_L^\dagger(x),$$

$$\xi_R(x) \to h(x)\xi_R(x)V_R^\dagger(x),$$

$$S_{\text{eff}}\left[V_\mu^a, V_\mu^0, \bar{\psi}_\xi, \psi_\xi\right]$$
$$= \int d^4x\, \bar{\psi}_\xi \left(i\partial\!\!\!/ + v\!\!\!/ + a\!\!\!/\gamma_5 - s - m\right)\psi_\xi\,,$$

$$m \equiv (m_U + m_D)/2$$

$$s \equiv \Delta m \cdot \tau^3 \qquad \Delta m \equiv (m_U - m_D)/2$$

$$\psi_\xi(x) = P_L \xi_L(x)\psi_L(x) + P_R \xi_R(x)\psi_R(x),$$

$$v_\mu(x) \equiv -\frac{1}{2}\left[g_2 \frac{\tau^a}{2} V_\mu^a(x) + g_1 \frac{\tau^3}{2} V_\mu^0(x)\right], \qquad g_2 \frac{\tau^a}{2} V_\mu^a(x) \equiv \xi_L \left[g_2 \frac{\tau^a}{2} W_\mu^a(x) - i\partial_\mu\right]\xi_L^\dagger,$$

$$a_\mu(x) \equiv \frac{1}{2}\left[g_2 \frac{\tau^a}{2} V_\mu^a(x) - g_1 \frac{\tau^3}{2} V_\mu^0(x)\right], \qquad g_1 \frac{\tau^3}{2} V_\mu^0(x) \equiv \xi_R \left[g_1 \frac{\tau^3}{2} B_\mu(x) - i\partial_\mu\right]\xi_R^\dagger.$$

$$\psi_\xi(x) \to h(x)\psi_\xi(x),$$

$$g_2\frac{\tau^a}{2}V_\mu^a \to h(x)\left[g_2\frac{\tau^a}{2}V_\mu^a - i\partial_\mu\right]h^\dagger(x),$$

$$g_1\frac{\tau^3}{2}V_\mu^0 \to h(x)\left[g_1\frac{\tau^3}{2}V_\mu^0 - i\partial_\mu\right]h^\dagger(x).$$

$$g_2\frac{\tau^a}{2}V_\mu^a(x) \equiv \xi_L\left[g_2\frac{\tau^a}{2}W_\mu^a(x) - i\partial_\mu\right]\xi_L^\dagger,$$

$$g_1\frac{\tau^3}{2}V_\mu^0(x) \equiv \xi_R\left[g_1\frac{\tau^3}{2}B_\mu(x) - i\partial_\mu\right]\xi_R^\dagger.$$

$$\Rightarrow$$

$$g_2\frac{\tau^a}{2}V_\mu^a(x) - g_1\frac{\tau^3}{2}V_\mu^0(x) = -i\xi_R X_\mu \xi_R^\dagger,$$

$$X_\mu \equiv U^\dagger(D_\mu U)$$

$$a_\mu = -\tfrac{i}{2}\xi_R X_\mu \xi_R^\dagger \qquad s = \xi_R \Delta m \cdot \tau^3 \xi_R^\dagger$$

$$v_\mu = \frac{i}{2}\xi_R X_\mu \xi_R^\dagger - \xi_R g_1 \frac{\tau^3}{2} B_\mu \xi_R^\dagger + i\xi_R\left(\partial_\mu \xi_R^\dagger\right)$$

$$\begin{aligned}\mathrm{tr}\left(sa_\mu sa^\mu\right) = &-\frac{1}{4}\Delta m^2 \mathrm{tr}\left(\tau^3 X_\mu\right)\mathrm{tr}\left(\tau^3 X^\mu\right) \\ &+\frac{1}{4}\Delta m^2 \mathrm{tr}\left(X_\mu X^\mu\right), \\ \mathrm{tr}\left[(d_\mu s)(d^\mu s)\right] &= -\mathrm{tr}\left[s d_\mu\left(d^\mu s\right)\right] \\ &= \frac{1}{2}\Delta m^2 \mathrm{tr}\left(\tau^3 X_\mu\right)\mathrm{tr}\left(\tau^3 X^\mu\right) \\ &\quad - \Delta m^2 \mathrm{tr}\left(X_\mu X^\mu\right)\end{aligned}$$

where

$$\begin{aligned} d_\mu f &\equiv \partial_\mu f - i[v_\mu, f] \\ &= \xi_R(\partial_\mu F)\xi_R^\dagger + \xi_R\left(\frac{1}{2}X_\mu + i g_1 \frac{\tau^3}{2} B_\mu\right)F\xi_R^\dagger \\ &\quad + \xi_R F\left(-\frac{1}{2}X_\mu - i g_1 \frac{\tau^3}{2}B_\mu\right)\xi_R^\dagger \\ &= \xi_R\left\{(D_\mu F) + \frac{1}{2}[X_\mu, F]\right\}\xi_R^\dagger, \end{aligned}$$

for any chiral rotated field $f \equiv \xi_R F \xi_R^\dagger$

$$
\begin{aligned}
i S_{\mathrm{EW}}[U, W_\mu^a, B_\mu] &= \ln \mathrm{Det}(D+m) \\
&= \mathrm{Tr} \ln (D+m)
\end{aligned}
$$

where $\quad D \equiv \not\partial - i \not v - i \not a \gamma_5 - s$

$$
\begin{aligned}
&\mathrm{Re}\,\mathrm{Tr} \ln (D+m) \\
&= \frac{1}{2} \mathrm{Tr} \ln [(D^\dagger + m)(D+m)] \\
&= -\frac{1}{2} \lim_{\Lambda \to \infty} \int d^4 x \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{d\tau}{\tau} \mathrm{tr}_{c,f,l} \langle x | e^{-\tau (D^\dagger + m)(D+m)} | x \rangle ,
\end{aligned}
$$

Schwinger proper time method

$$(D^\dagger + m)(D + m) = E - \nabla^2 + m^2$$

where
$$\begin{aligned}
E &\equiv -2ms - 2im\phi\!\!\!/\,\gamma_5 + \frac{i}{4}[\gamma^\mu, \gamma^\nu]R_{\mu\nu} \\
&\quad + \gamma^\mu d_\mu s + i\gamma^\mu \{s, a_\mu\}\gamma_5 + s^2, \\
\nabla_\mu &\equiv \partial_\mu - i v_\mu - i a_\mu \gamma_5, \\
R_{\mu\nu} &\equiv i[\nabla_\mu, \nabla_\nu] \\
&= V_{\mu\nu} + (d_\mu a_\nu - d_\nu a_\mu)\gamma_5 - i[a_\mu, a_\nu], \\
V_{\mu\nu} &\equiv i[\partial_\mu - i v_\mu, \partial_\nu - i v_\nu] \\
&= \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu], \\
d_\mu s &\equiv \partial_\mu s - i[v_\mu, s].
\end{aligned}$$

Seely–DeWitt expansion

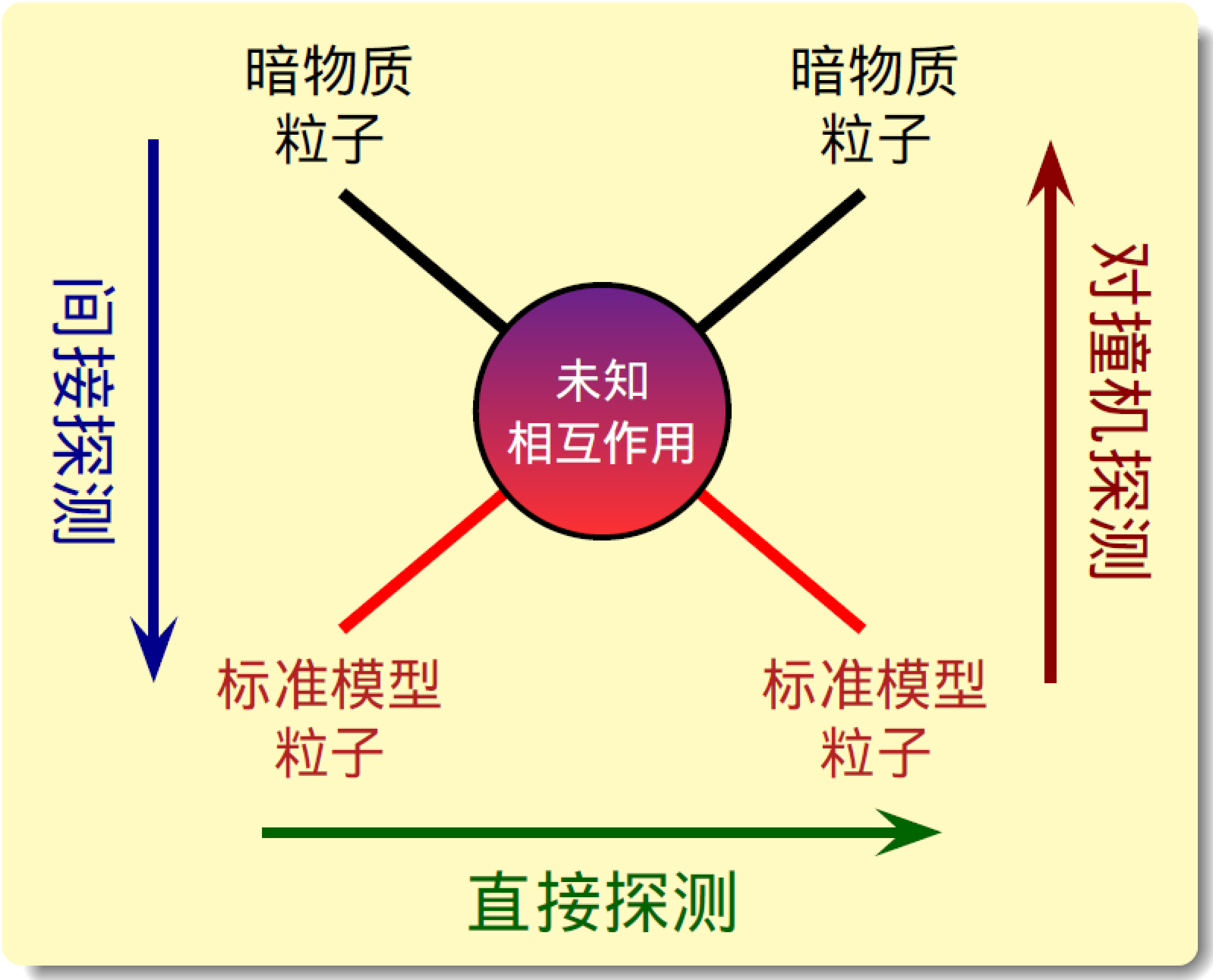
$$
\begin{aligned}
&\langle x|e^{-\tau(E-\nabla^2)}|x\rangle \\
&\quad = \frac{1}{16\pi^2}\left\{\frac{1}{\tau^2} - \frac{E}{\tau}\right. \\
&\qquad + \left(\frac{1}{2}E^2 - \frac{1}{6}[\nabla_\mu,[\nabla^\mu,E]] - \frac{1}{12}R_{\mu\nu}R^{\mu\nu}\right) \\
&\qquad + \tau\left(-\frac{1}{6}E^3 + \frac{1}{12}([\nabla_\mu,[\nabla^\mu,E]]E\right. \\
&\qquad \left.\left. + E[\nabla_\mu,[\nabla^\mu,E]] + [\nabla_\mu,E][\nabla^\mu,E])\right) + \frac{\tau^2}{24}E^4\right\} \\
&\qquad + \mathcal{O}(\tau^3). \qquad (37)
\end{aligned}
$$

$$
\begin{aligned}
&\mathrm{Re}\,\mathrm{Tr}\ln(D+m) \\
&\ni \frac{i}{2}\frac{N_c}{16\pi^2}\int d^4x_M \lim_{\Lambda\to\infty}\left\{\left[4\left(-\gamma-\ln\frac{m^2}{\Lambda^2}\right)-8\right]\right. \\
&\quad\times\left[-\frac{1}{4}\Delta m^2 \mathrm{tr}(\tau^3 X_\mu)\mathrm{tr}(\tau^3 X^\mu)\right] \\
&\quad\left.+\left[2\left(-\gamma-\ln\frac{m^2}{\Lambda^2}\right)-\frac{4}{3}\right]\frac{1}{2}\Delta m^2 \mathrm{tr}(\tau^3 X_\mu)\mathrm{tr}(\tau^3 X^\mu)\right\} \\
&= i\frac{N_c}{24\pi^2}\Delta m^2\int d^4x_M \mathrm{tr}(\tau^3 X_\mu)\mathrm{tr}(\tau^3 X^\mu), \qquad (50)
\end{aligned}
$$

➡ $$\mathcal{L}_0' = \frac{1}{4}\beta_1 f^2[\mathrm{tr}(\tau^3 X_\mu)]^2 = \frac{N_c}{24\pi^2}\Delta m^2[\mathrm{tr}(\tau^3 X_\mu)]^2$$

➡ $$\beta_1 = \frac{N_c}{96\pi^2}\frac{e^2}{s^2c^2}\frac{(m_U-m_D)^2}{M_Z^2},$$ ➡ $$\rho = 1+\frac{N_c}{48\pi^2}\frac{e^2}{s^2c^2}\frac{(m_U-m_D)^2}{M_Z^2}$$

Effective field theory and **dark matter**



暗物质
粒子
暗物质
粒子
间接探测
对撞机探测
未知
相互作用
标准模型
粒子
标准模型
粒子
直接探测

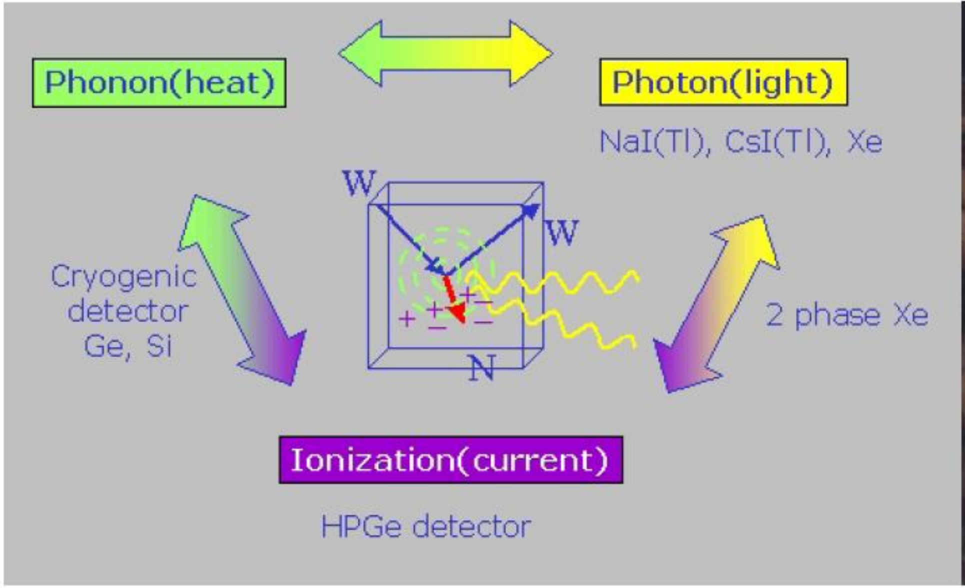
暗物质的直接探测

暗物质粒子＋原子核 → 暗物质粒子＋原子核

测量原子核被暗物质粒子散射后导致的反冲信号 (光、热、电)

实验：DAMA, CoGeNT, XENON, CDMS, LUX, CDEX, PANDAX, ……

为屏蔽宇宙线背景，一般在深层地下实验室进行实验

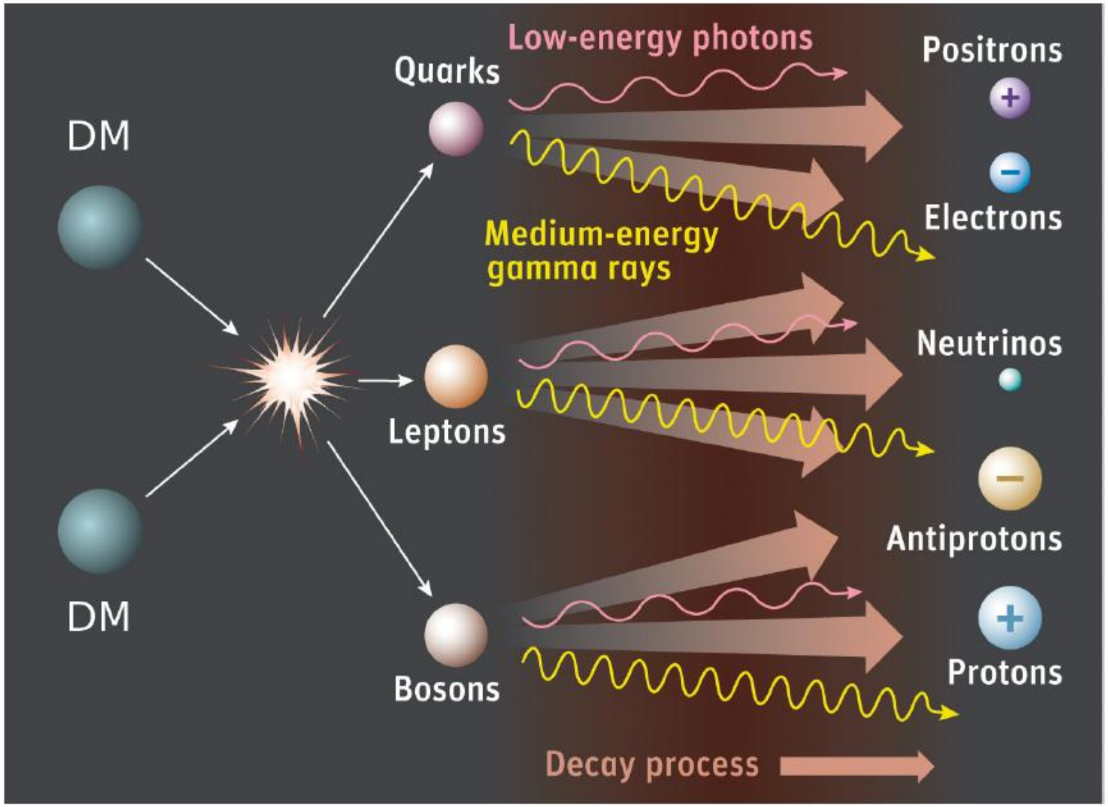


暗物质的间接探测

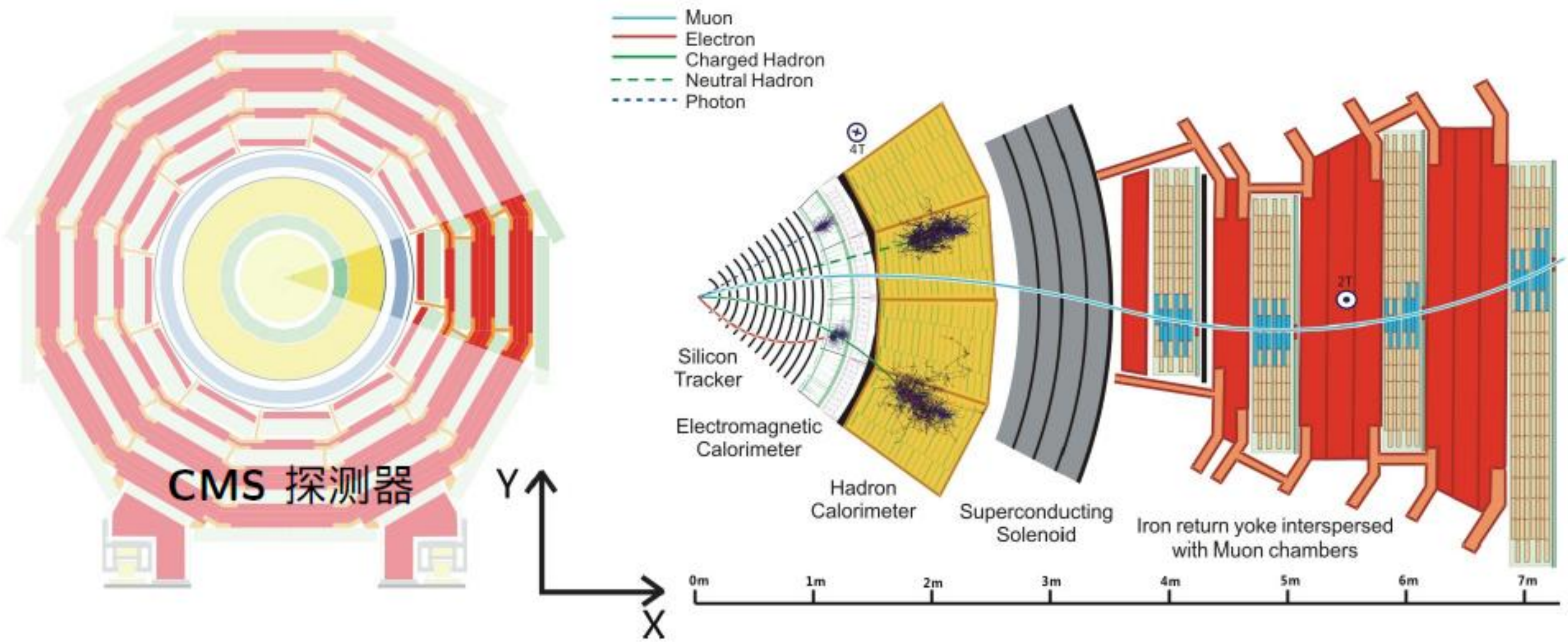
暗物质粒子 (+暗物质粒子) → 荷电宇宙线, γ射线, 中微子

暗物质粒子的湮灭或衰变过程额外贡献到这些粒子当中

相关实验：**PAMELA, ATIC, Fermi, IceCube, AMS-02, DAMPE, ……**



高能对撞机的探测器与粒子重建



	γ	$e^{\pm}$	$\mu^{\pm}$	带电强子	中性强子	中微子和暗物质粒子
径迹探测器	×	√	√	√	×	×
电磁量能器	√	√	×	×	×	×
强子量能器	×	×	×	√	√	×
μ 子探测器	×	×	√	×	×	×

$\Rightarrow$ 丢失能量 $\not{E}$ 或 $\not{E}_{\mathrm{T}}$

暗物质的对撞机探测

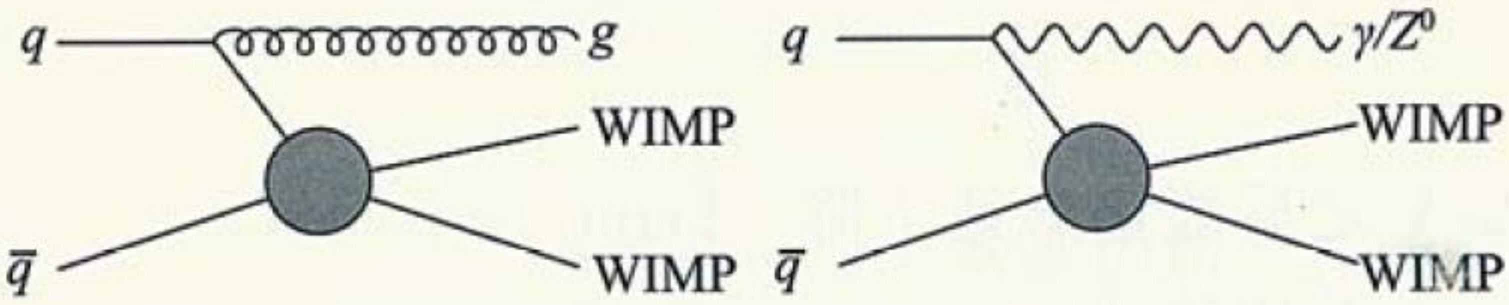


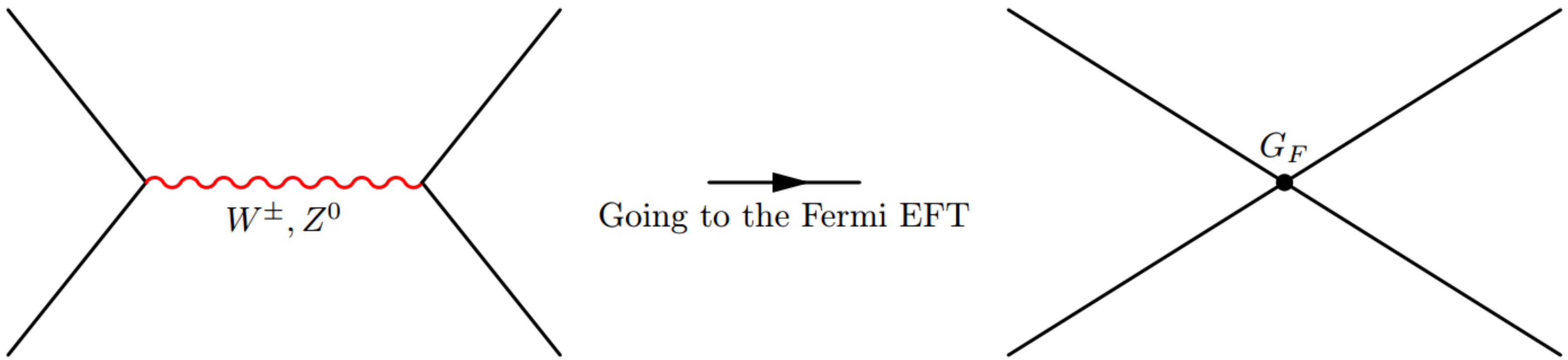
图 5　强子对撞机上产生 WIMP 对的同时伴随产生单喷注（左）或单光子/Z^0（右）示意图

Fig. 5　Schematic diagram of WIMP pair production associating with monojet (left diagram) or mono-gamma/Z^0 (right diagram) production at hadron collider

有效场论的一个简单例子

- 当相互作用过程的**转移动量远小于媒介粒子质量**时，相互作用可以用**有效场论**描述。

例：从标准模型→费米理论



- 使用有效场论，通常能够把握研究问题的基本特征，**与模型细节相对无关**。

$$S_{\text{eff}}[\phi] = \sum_i c_i \int d^4x\, \mathcal{O}_i(x),$$

Wilson系数 $$c_i(\mu = \Lambda) = \frac{\alpha_i}{\Lambda^{\Delta_i - 4}},$$

$$\text{with } \alpha_i \sim \mathcal{O}(1)$$

不同的完全理论对应于不同的Wilson系数，但在低能标的差别并不明显。

暗物质与标准模型费米子的有效相互作用

- 当相互作用过程的转移动量远小于媒介粒子质量时，相互作用可以用有效理论描述。
- 使用有效理论，通常能够把握研究问题的基本特征，与模型细节相对无关。
- 我们分别假定暗物质由自旋为 0, 1/2, 1 或 3/2 的粒子组成，用有效算符描述暗物质粒子与标准模型费米子的相互作用，把直接探测、间接探测和宇宙学丰度的实验结果综合在一起进行比较
- 我们发现，在不同情况下，各类暗物质实验的灵敏度有些差别，具有一定的互补性

Nucl.Phys. B854 (2012) 350-374
Nucl.Phys. B860 (2012) 115-151

自旋为 0 的暗物质粒子

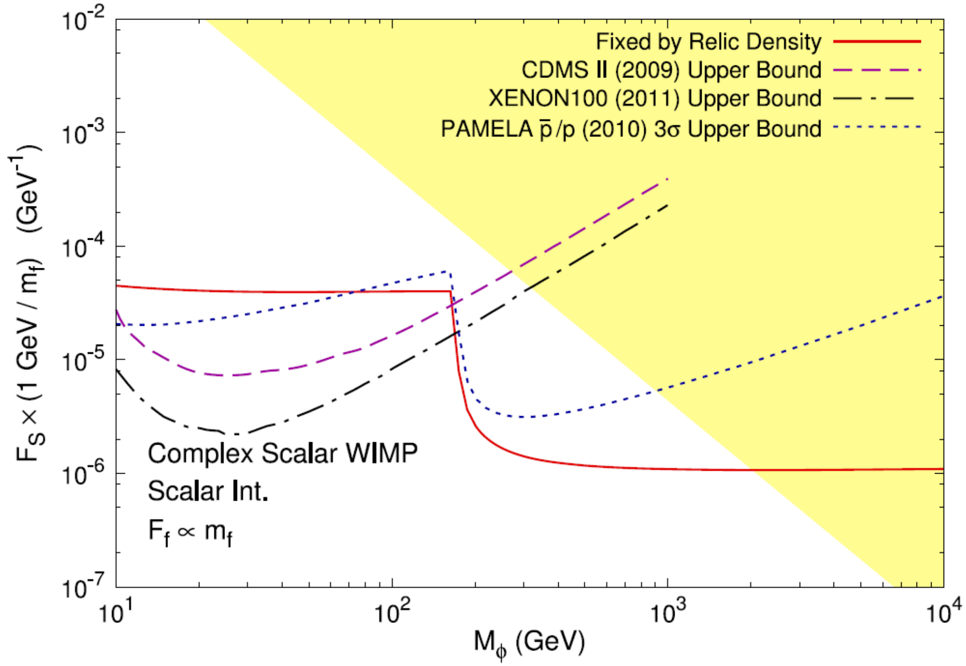
$$\mathcal{L}_{\mathrm{S}} = \sum_f \frac{F_{\mathrm{S},f}}{\sqrt{2}} \phi^\dagger \phi \bar{f} f$$

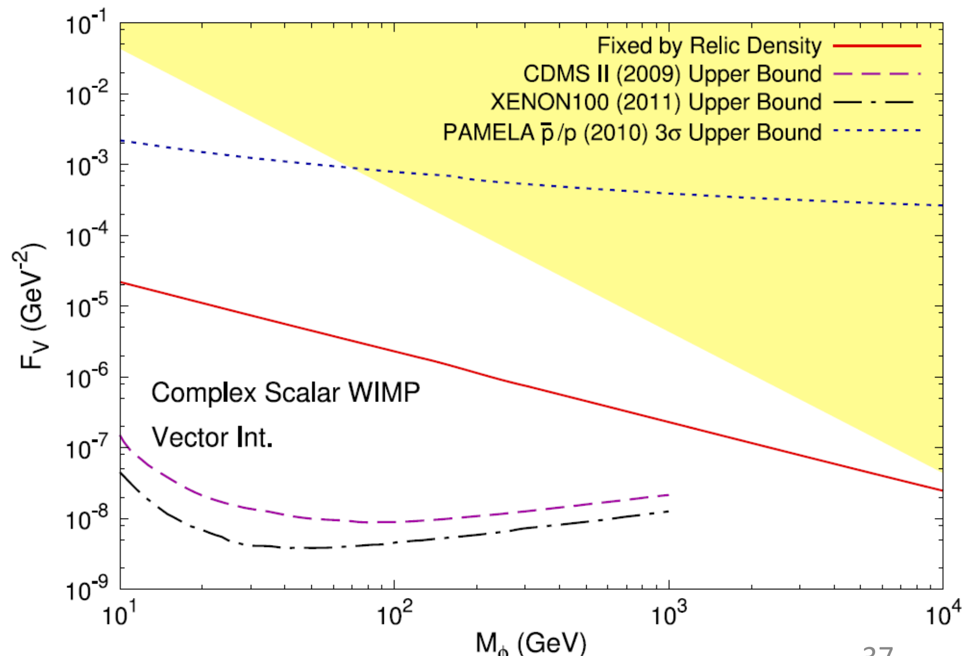
$$\mathcal{L}_{\mathrm{V}} = \sum_f \frac{F_{\mathrm{V},f}}{\sqrt{2}} (\phi^\dagger i \overleftrightarrow{\partial_\mu} \phi) \bar{f} \gamma^\mu f$$

$$\mathcal{L}_{\mathrm{SP}} = \sum_f \frac{F_{\mathrm{SP},f}}{\sqrt{2}} \phi^\dagger \phi \bar{f} i \gamma_5 f$$

$$\mathcal{L}_{\mathrm{VA}} = \sum_f \frac{F_{\mathrm{VA},f}}{\sqrt{2}} (\phi^\dagger i \overleftrightarrow{\partial_\mu} \phi) \bar{f} \gamma^\mu \gamma_5 f$$

黄色区域为考虑到可微扰条件时有效理论不成立的区域





自旋为 1/2 的暗物质粒子

$$\mathcal{L}_{\mathrm{S}} = \sum_f \frac{G_{\mathrm{S},f}}{\sqrt{2}} \bar{\chi}\chi \bar{f} f$$

$$\mathcal{L}_{\mathrm{P}} = \sum_f \frac{G_{\mathrm{P},f}}{\sqrt{2}} \bar{\chi}\gamma_5\chi \bar{f}\gamma_5 f$$

$$\mathcal{L}_{\mathrm{V}} = \sum_f \frac{G_{\mathrm{V},f}}{\sqrt{2}} \bar{\chi}\gamma^\mu\chi \bar{f}\gamma_\mu f$$

$$\mathcal{L}_{\mathrm{A}} = \sum_f \frac{G_{\mathrm{A},f}}{\sqrt{2}} \bar{\chi}\gamma^\mu\gamma_5\chi \bar{f}\gamma_\mu\gamma_5 f$$

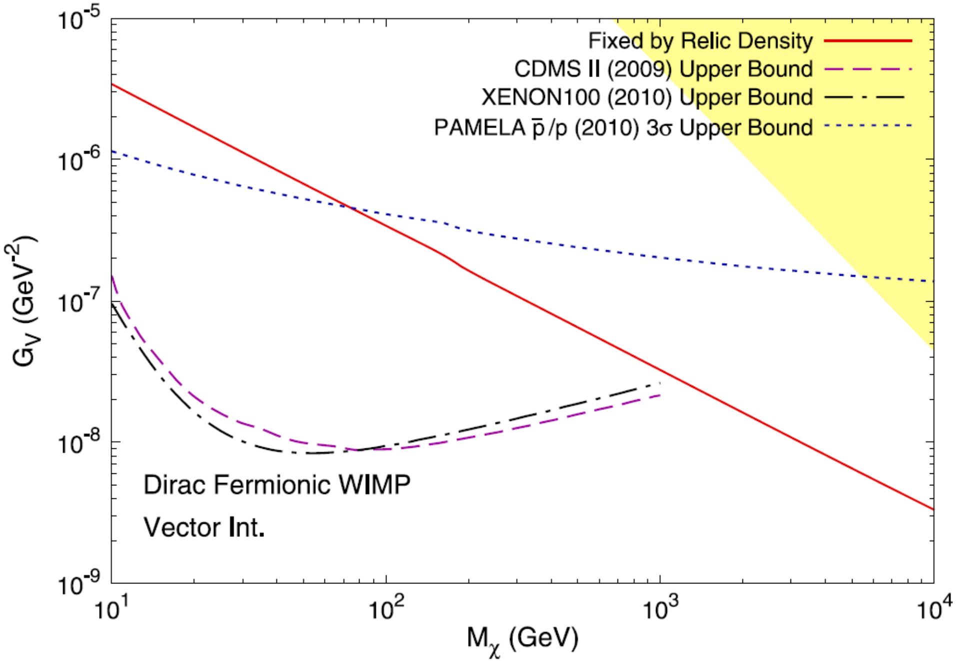
$$\mathcal{L}_{\mathrm{T}} = \sum_f \frac{G_{\mathrm{T},f}}{\sqrt{2}} \bar{\chi}\sigma^{\mu\nu}\chi \bar{f}\sigma_{\mu\nu} f$$

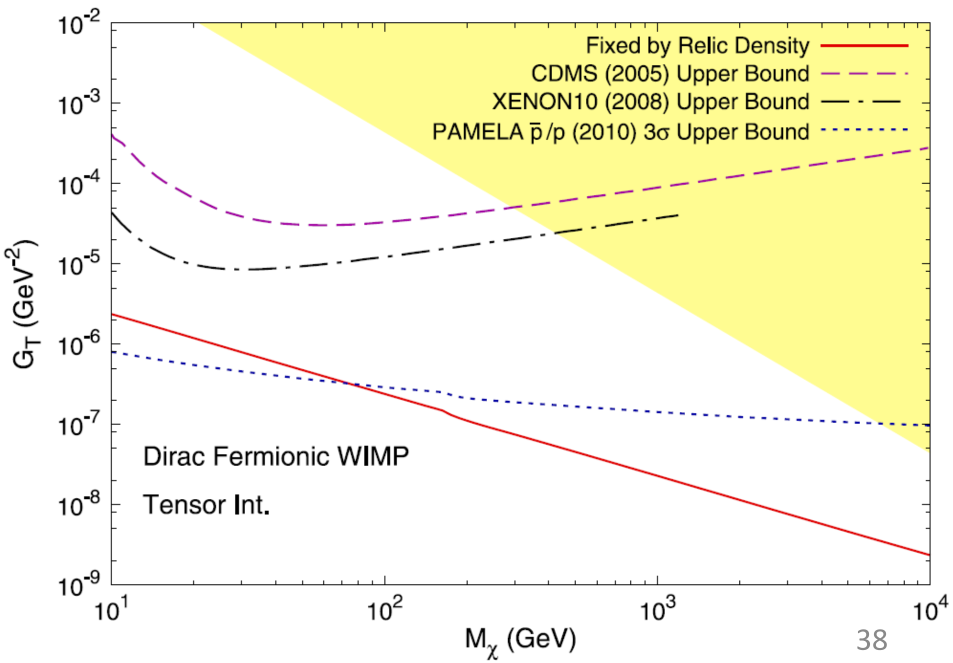
$$\mathcal{L}_{\mathrm{SP}} = \sum_f \frac{G_{\mathrm{SP},f}}{\sqrt{2}} \bar{\chi}\chi \bar{f} i\gamma_5 f$$

$$\mathcal{L}_{\mathrm{PS}} = \sum_f \frac{G_{\mathrm{PS},f}}{\sqrt{2}} \bar{\chi} i\gamma_5\chi \bar{f} f$$

$$\mathcal{L}_{\mathrm{VA}} = \sum_f \frac{G_{\mathrm{VA},f}}{\sqrt{2}} \bar{\chi}\gamma^\mu\chi \bar{f}\gamma_\mu\gamma_5 f$$

$$\mathcal{L}_{\mathrm{AV}} = \sum_f \frac{G_{\mathrm{AV},f}}{\sqrt{2}} \bar{\chi}\gamma^\mu\gamma_5\chi \bar{f}\gamma_\mu f$$





自旋为 1 的暗物质粒子

$$\mathcal{L}_S = \sum_f \frac{K_{S,f}}{\sqrt{2}} X_\mu^* X^\mu \bar{f} f$$

$$\mathcal{L}_V = \sum_f \frac{K_{V,f}}{\sqrt{2}} (X_\nu^* i \overleftrightarrow{\partial_\mu} X^\nu) \bar{f} \gamma^\mu f$$

$$\mathcal{L}_T = \sum_f \frac{K_{T,f}}{\sqrt{2}} i (X_\mu^* X_\nu - X_\nu^* X_\mu) \bar{f} \sigma^{\mu\nu} f$$

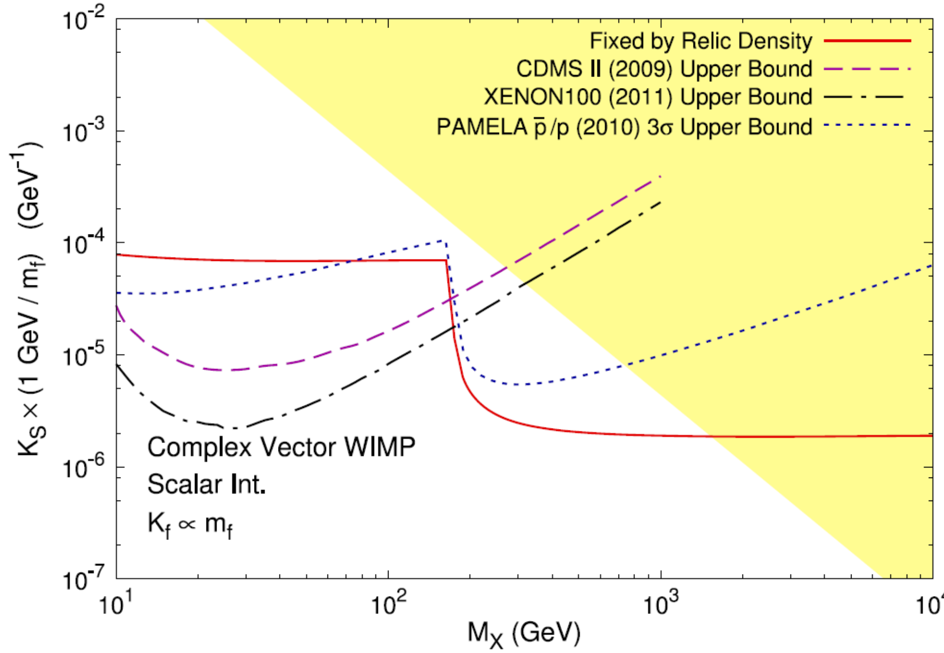
$$\mathcal{L}_{SP} = \sum_f \frac{K_{SP,f}}{\sqrt{2}} X_\mu^* X^\mu \bar{f} i \gamma_5 f$$

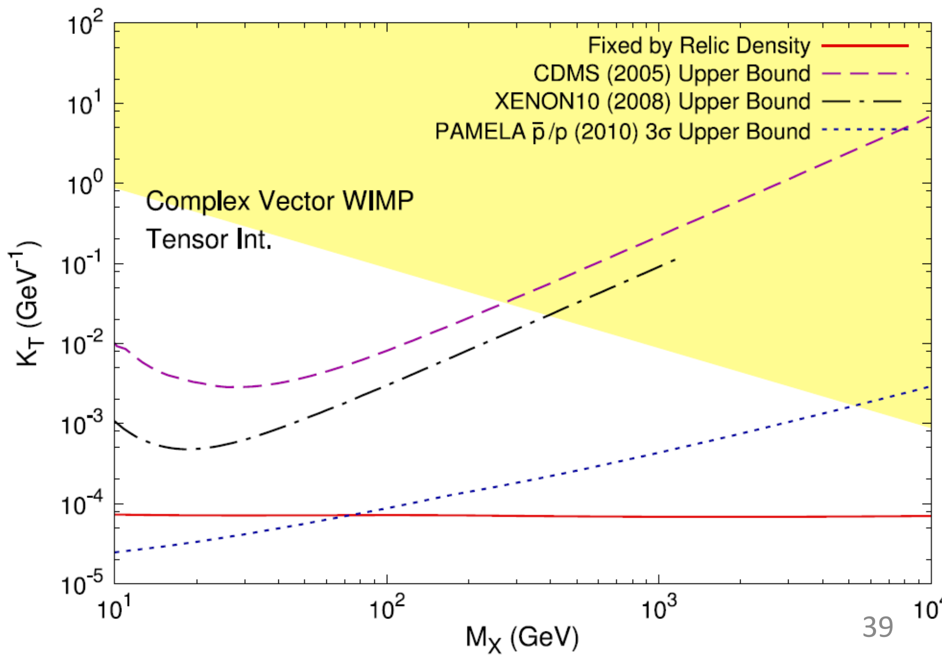
$$\mathcal{L}_{VA} = \sum_f \frac{K_{VA,f}}{\sqrt{2}} (X_\nu^* i \overleftrightarrow{\partial_\mu} X^\nu) \bar{f} \gamma^\mu \gamma_5 f$$

$$\mathcal{L}_{\tilde{V}} = \sum_f \frac{\tilde{K}_{V,f}}{\sqrt{2}} \varepsilon^{\mu\nu\rho\sigma} (X_\mu^* \overleftrightarrow{\partial_\nu} X_\rho) \bar{f} \gamma_\sigma f$$

$$\mathcal{L}_{\widetilde{VA}} = \sum_f \frac{\tilde{K}_{VA,f}}{\sqrt{2}} \varepsilon^{\mu\nu\rho\sigma} (X_\mu^* \overleftrightarrow{\partial_\nu} X_\rho) \bar{f} \gamma_\sigma \gamma_5 f$$

$$\mathcal{L}_{\tilde{T}} = \sum_f \frac{\tilde{K}_{T,f}}{\sqrt{2}} \varepsilon^{\mu\nu\rho\sigma} i (X_\mu^* X_\nu - X_\nu^* X_\mu) \bar{f} \sigma_{\rho\sigma} f$$





自旋为 3/2 的暗物质粒子

$$\mathcal{L}_{\mathrm{S}} = \sum_f \frac{G_{\mathrm{S},f}}{\sqrt{2}} \bar{\chi}^\mu \chi_\mu \bar{f} f$$

$$\mathcal{L}_{\mathrm{P}} = \sum_f \frac{G_{\mathrm{P},f}}{\sqrt{2}} \bar{\chi}^\mu \gamma_5 \chi_\mu \bar{f} \gamma_5 f$$

$$\mathcal{L}_{\mathrm{V}} = \sum_f \frac{G_{\mathrm{V},f}}{\sqrt{2}} \bar{\chi}^\rho \gamma^\mu \chi_\rho \bar{f} \gamma_\mu f$$

$$\mathcal{L}_{\mathrm{A}} = \sum_f \frac{G_{\mathrm{A},f}}{\sqrt{2}} \bar{\chi}^\rho \gamma^\mu \gamma_5 \chi_\rho \bar{f} \gamma_\mu \gamma_5 f$$

$$\mathcal{L}_{\mathrm{T1}} = \sum_f \frac{G_{\mathrm{T1},f}}{\sqrt{2}} \bar{\chi}^\mu \sigma^{\rho\sigma} \chi_\mu \bar{f} \sigma_{\rho\sigma} f$$

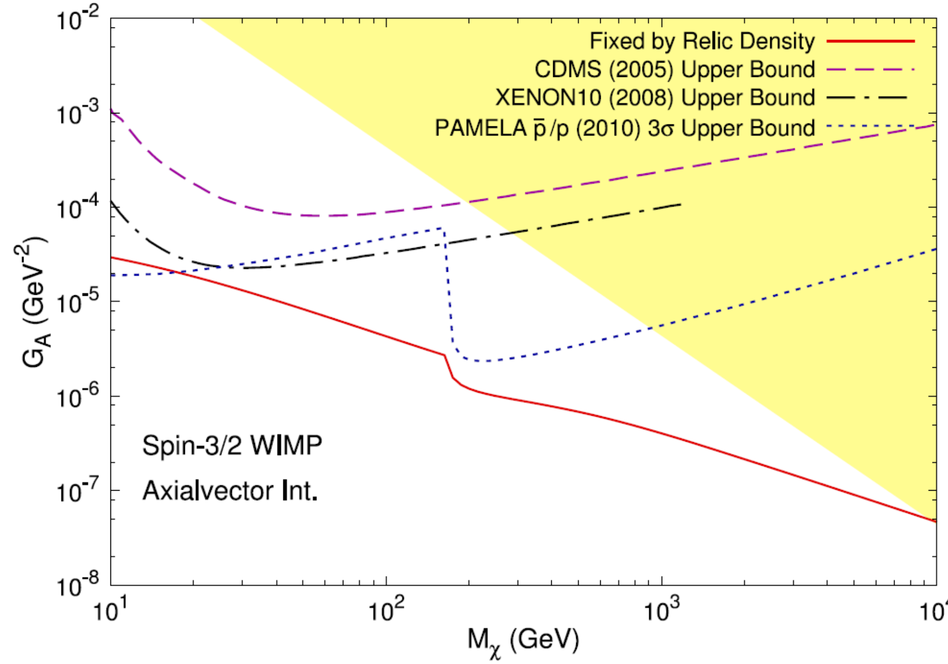
$$\mathcal{L}_{\mathrm{T2}} = \sum_f \frac{G_{\mathrm{T2},f}}{\sqrt{2}} i\left(\bar{\chi}^\mu \chi^\nu - \bar{\chi}^\nu \chi^\mu\right) \bar{f} \sigma_{\mu\nu} f$$

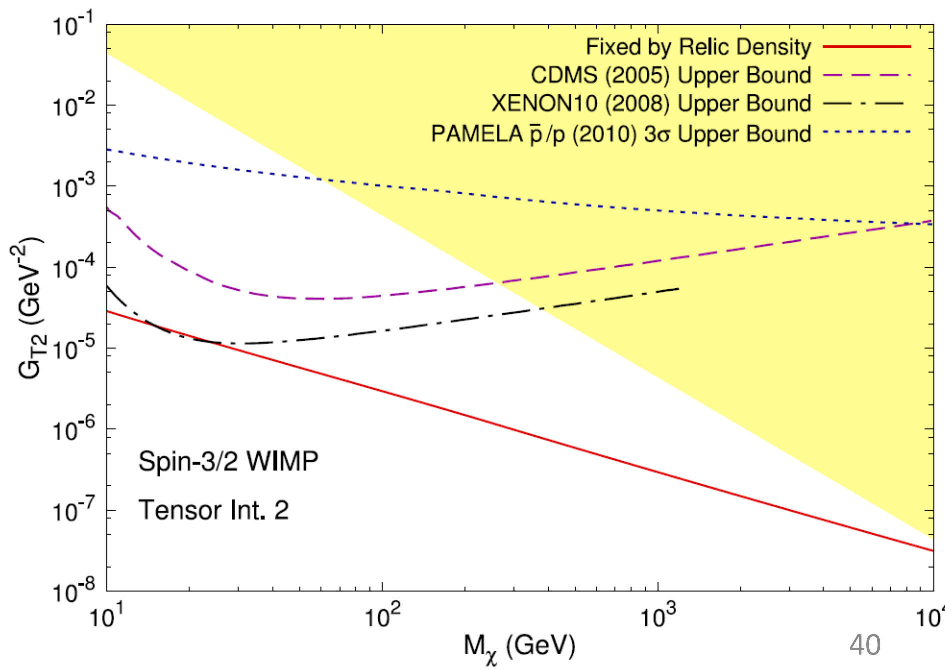
$$\mathcal{L}_{\mathrm{T3}} = \sum_f \frac{G_{\mathrm{T3},f}}{\sqrt{2}} \varepsilon^{\mu\nu\rho\sigma} i\left(\bar{\chi}_\mu \chi_\nu - \bar{\chi}_\nu \chi_\mu\right) \bar{f} \sigma_{\rho\sigma} f$$

$$\mathcal{L}_{\mathrm{T4}} = \sum_f \frac{G_{\mathrm{T4},f}}{\sqrt{2}} \varepsilon^{\mu\nu\rho\sigma} \bar{\chi}^\tau \sigma_{\mu\nu} \chi_\tau \bar{f} \sigma_{\rho\sigma} f$$

$$\mathcal{L}_{\mathrm{T5}} = \sum_f \frac{G_{\mathrm{T5},f}}{\sqrt{2}} \left(\bar{\chi}^\mu \gamma_5 \chi^\nu - \bar{\chi}^\nu \gamma_5 \chi^\mu\right) \bar{f} \sigma_{\mu\nu} f$$

$$\mathcal{L}_{\mathrm{T6}} = \sum_f \frac{G_{\mathrm{T6},f}}{\sqrt{2}} \varepsilon^{\mu\nu\rho\sigma} \left(\bar{\chi}_\mu \gamma_5 \chi_\nu - \bar{\chi}_\nu \gamma_5 \chi_\mu\right) \bar{f} \sigma_{\rho\sigma} f$$





spin-3/2 WIMP can be described by a vector–spinor χ_α^μ

rank-1 Rarita–Schwinger equations

$$(i\partial\!\!\!/ - M_\chi)\chi^\mu = 0, \qquad \gamma_\mu \chi^\mu = 0$$

where the spinor indices have been suppressed

$$\sum_{s=1}^{4} u_s^\mu(p)\bar{u}_s^\nu(p) = (p\!\!\!/ + M_\chi)\left(P^{\mu\nu} - \frac{1}{3}P^{\mu\rho}P^{\nu\sigma}\gamma_\rho\gamma_\sigma\right)$$

$$\sum_{s=1}^{4} v_s^\mu(p)\bar{v}_s^\nu(p) = (p\!\!\!/ - M_\chi)\left(P^{\mu\nu} - \frac{1}{3}P^{\mu\rho}P^{\nu\sigma}\gamma_\rho\gamma_\sigma\right)$$

$$P^{\mu\nu} \equiv g^{\mu\nu} - p^\mu p^\nu / p^2$$

Minimal dark matter

已经有这么多暗物质模型了

在高能物理数据库SPIRES，“暗物质”论文数量多达四千多篇

Neutrino (excluded), sterile neutrino, right-handed neutrino, neutralino, higgsino, bino, photino, wino, gravitino, sneutrino, possibly split or anthropic, right-handed sneutrino, scalar singlet, singlino, Kaluza Klein LKP: graviton_1, photon_1, neutrino_1, Z_1, Z', axion, axino, B-balls, Q-balls, odd-balls, inflatino, quintissencino, scalar condensate, Pseudo-Goldstone, ultra light PG, radion, radino, modulus, modulinos, Planck relicts, quark nugget, encapsulated atoms, top bound state, shadow matter, mirror matter, branon, branino, normal matter on folded brane or on another brane or membrane or D-brane or p-brane, cosmic string, cosmic necklace, mini black hole, soliton, monopole, techni-baryon, techni-meson, Chaplygin gas, fuzzy DM, WIMPzilla, familion, familin CP pseudoscalar, preon, dilaton, doubly-charged lepton, degenerate fermion, kination, H dibaryon, crypton, hiddenon, heterotic, d-quark from Wilson lines, 4th generation, ...

为什么还要考虑这一个暗物质候选模型：最小暗物质模型？

最小暗物质模型只有一个参数：暗物质质量M，具有可预言性！

最小暗物质 (minimal dark matter) 模型

[Cirelli, *et al.*, hep-ph/0512090]

- 在标准模型基础上引入一个 $SU(2)_L \times U(1)_Y$ **多重态**
- 多重态的**电中性分量**是暗物质候选粒子

Add to the SM extra particles $\mathcal{X}$ + h.c. Search for assignement of quantum numbers (gauge charges, spin) that give a as-perfect-as-possible DM candidate:

1. Cosmologically stable
2. Only one parameter: M
3. Lightest component is neutral.
4. Allowed by DM searches

$$\mathscr{L} = \mathscr{L}_{\rm SM} + c \begin{cases} \bar{\mathcal{X}}(i\not{D} + M)\mathcal{X} & \text{when } \mathcal{X} \text{ is a spin 1/2 fermionic multiplet} \\ |D_\mu \mathcal{X}|^2 - M^2|\mathcal{X}|^2 & \text{when } \mathcal{X} \text{ is a spin 0 bosonic multiplet} \end{cases}$$

Simple because no other term is compatible with gauge/Lorentz invariance

EWSB induces a well-defined and non-trivial phenomenology. M fixed by $\Omega_{\rm DM}$.

最小暗物质 (minimal dark matter) 模型

[Cirelli, *et al.*, hep-ph/0512090]

- 在标准模型基础上引入一个 $SU(2)_L \times U(1)_Y$ 多重态
- 多重态的电中性分量是暗物质候选粒子
- 电弱圈图修正使带电分量的质量比中性分量大
- 若多重态所在表示维数 n 足够高，电弱规范对称性会禁戒多重态与标准模型粒子的耦合，使暗物质稳定

费米子：$n \geq 5$　　　标量：$n \geq 7$

这是一种偶然对称性 (accidental symmetry)，比人为地引入 Z_2 对称性的一般方法显得更加自然

- 多重态的引入会影响弱耦合常数 g_2 的跑动。如果要求 g_2 能够**一直到普朗克能标都不遇到朗道极点**，可以给出 n 的上限。

  Majorana费米子：$n \leqslant 5$　　实标量： $n \leqslant 8$

- 不过，标量多重态的情况没这么简单。标量多重态存在多种**自相互作用**，以及与 Higgs 场的相互作用。
- 最近有研究[Hamada, *et al*., arXiv:1505.01721]表明，七重态标量的**四次自相互作用耦合常数在 10^8 GeV 能标处就会遇到朗道极点**。
- 我们进一步研究了在什么情况下可以**将朗道极点提升到更高能标**上。　　**Phys.Rev. D92 (2015) 115004**

七重态实标量最小暗物质模型

七重态实标量 $\Phi = \frac{1}{\sqrt{2}}(\Delta^{(3)},\ \Delta^{(2)},\ \Delta^{(1)},\ \Delta^{(0)},\ \Delta^{(-1)},\ \Delta^{(-2)},\ \Delta^{(-3)})^{\mathrm{T}}$

动能项和规范耦合项

$$
\begin{aligned}
\mathcal{L}_1 = &\ (D_\mu\Phi)^\dagger D^\mu\Phi \\
= &\ \frac{1}{2}(\partial_\mu\Delta^{(0)})^2 + \sum_{Q=1}^{3}(\partial_\mu\Delta^{(Q)})(\partial^\mu\Delta^{(-Q)}) + \sum_{Q=1}^{3}(QeA^\mu + Qg_2c_W Z^\mu)\Delta^{(-Q)} i\overleftrightarrow{\partial_\mu}\Delta^{(Q)} \\
&-g_2W^{+,\mu}(\sqrt{3}\Delta^{(-3)}i\overleftrightarrow{\partial_\mu}\Delta^{(2)} + \sqrt{5}\Delta^{(-2)}i\overleftrightarrow{\partial_\mu}\Delta^{(1)} + \sqrt{6}\Delta^{(-1)}i\overleftrightarrow{\partial_\mu}\Delta^{(0)}) \\
&-g_2W^{-,\mu}(\sqrt{3}\Delta^{(-2)}i\overleftrightarrow{\partial_\mu}\Delta^{(3)} + \sqrt{5}\Delta^{(-1)}i\overleftrightarrow{\partial_\mu}\Delta^{(2)} + \sqrt{6}\Delta^{(0)}i\overleftrightarrow{\partial_\mu}\Delta^{(1)}) \\
&+(e^2A_\mu A^\mu + g_2^2c_W^2Z_\mu Z^\mu + 2eg_2c_W A_\mu Z^\mu)\sum_{Q=1}^{3}Q^2\Delta^{(Q)}\Delta^{(-Q)} \\
&+g_2^2W_\mu^+W^{-,\mu}[6(\Delta^{(0)})^2 + 11\Delta^{(1)}\Delta^{(-1)} + 8\Delta^{(2)}\Delta^{(-2)} + 3\Delta^{(3)}\Delta^{(-3)}] \\
&-g_2^2\{W_\mu^+(s_WA^\mu + c_WZ^\mu)(\sqrt{6}\Delta^{(0)}\Delta^{(-1)} + 3\sqrt{5}\Delta^{(1)}\Delta^{(-2)} + 5\sqrt{3}\Delta^{(2)}\Delta^{(-3)}) \\
&+W_\mu^+W^{+,\mu}[3(\Delta^{(-1)})^2 - \sqrt{30}\Delta^{(0)}\Delta^{(-2)} - \sqrt{15}\Delta^{(1)}\Delta^{(-3)}] + \mathrm{h.c.}\}
\end{aligned}
$$

势能项 $V = \mu^2 H^\dagger H + m^2\Phi^\dagger\Phi + \lambda(H^\dagger H)^2 + \lambda_2(\Phi^\dagger\Phi)^2$

$+\lambda_3(H^\dagger H)(\Phi^\dagger\Phi) + \frac{\lambda_4}{48}(\Phi^\dagger T^a T^b \Phi)^2$

这一项被大多数文献忽略

Higgs 场破缺后，第五项会贡献到七重态树图质量上：

$$m_0^2 = m^2 + \frac{\lambda_3 v^2}{2}$$

当 $m_0 >> m_Z$ 时，一圈图引起的质量劈裂为

$$m_Q - m_0 = Q^2\Delta m, \quad \Delta m = \alpha_2 m_W \sin^2(\theta_W/2) \simeq 167\ \mathrm{MeV}$$

λ_3 为零时，暗物质遗迹密度(relic density)观测值对应于

m_0 = 8.8 TeV (不考虑索末菲效应)

m_0 = 25 TeV (考虑索末菲效应)

标准模型耦合常数的 beta 函数

$$\beta_{g_1}^{\rm SM} = \frac{1}{16\pi^2}\frac{41}{10}g_1^3, \quad \beta_{g_2}^{\rm SM} = \frac{1}{16\pi^2}\left(-\frac{19}{6}\right)g_2^3, \quad \beta_{g_3}^{\rm SM} = \frac{1}{16\pi^2}(-7)g_3^3$$

$$\beta_{y_t}^{\rm SM} = \frac{1}{16\pi^2}y_t\left(\frac{9}{2}y_t^2 - \frac{9}{4}g_2^2 - \frac{17}{20}g_1^2 - 8g_3^2\right)$$

$$\beta_{\lambda}^{\rm SM} = \frac{1}{16\pi^2}\left\{24\lambda^2 - 6y_t^4 + \frac{3}{8}\left[2g_2^4 + \left(g_2^2 + \frac{3}{5}g_1^2\right)^2\right] + \lambda\left(-9g_2^2 - \frac{9}{5}g_1^2 + 12y_t^2\right)\right\}$$

在 $\Lambda = 25\ \mathrm{TeV}$ 处加入七重态实标量之后的 beta 函数

$$\beta_{g_1} = \beta_{g_1}^{\rm SM}, \quad \beta_{g_2} = \beta_{g_2}^{\rm SM} + \frac{1}{16\pi^2}\frac{14}{3}g_2^3, \quad \beta_{g_3} = \beta_{g_3}^{\rm SM}, \quad \beta_{y_t} = \beta_{y_t}^{\rm SM}$$

$$\beta_{\lambda} = \beta_{\lambda}^{\rm SM} + \frac{1}{16\pi^2}\frac{7}{2}\lambda_3^2, \quad \beta_{\lambda_2} = \frac{1}{16\pi^2}[30\lambda_2^2 + 2\lambda_3^2 + \frac{45}{2}\lambda_4^2 + 51\lambda_2\lambda_4 - 144g_2^2\lambda_2]$$

$$\beta_{\lambda_3} = \frac{1}{16\pi^2}\left[12\lambda\lambda_3 + 18\lambda_2\lambda_3 + 4\lambda_3^2 + \frac{51}{2}\lambda_3\lambda_4 + 36g_2^4 - \lambda_3\left(\frac{153}{2}g_2^2 + \frac{9}{10}g_1^2 - 6y_t^2\right)\right]$$

$$\beta_{\lambda_4} = \frac{1}{16\pi^2}\left[288g_2^4 + \frac{255}{8}\lambda_4^2 + 24\lambda_2\lambda_4 - 144g_2^2\lambda_4\right]$$ （系数非常大！）

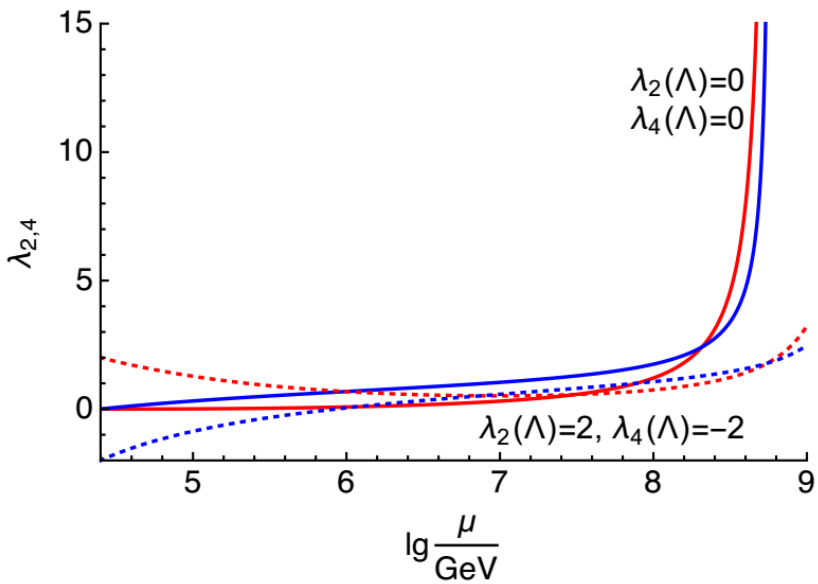
真空稳定性条件：

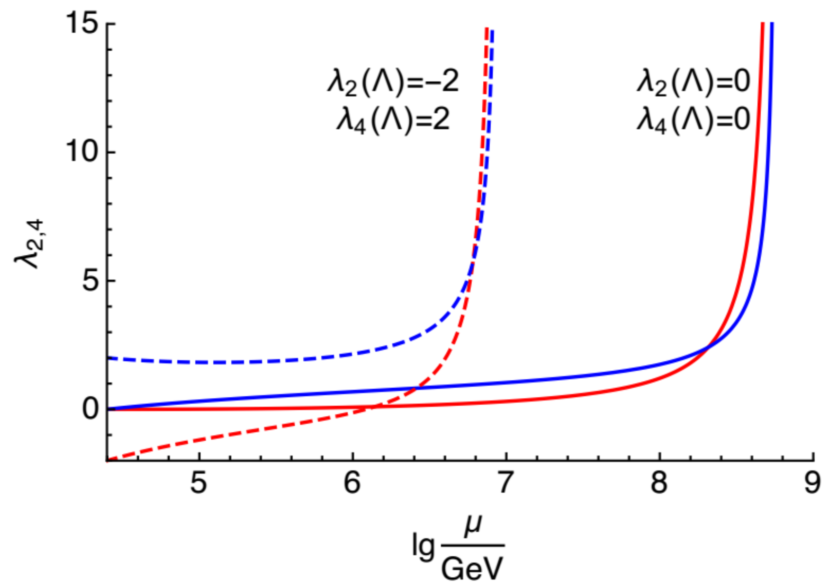
对于 $\lambda_4 \geq 0$

$$\begin{cases} \lambda \geq 0, \\ \lambda_2 + \lambda_4 \geq 0, \\ \lambda_3 + 2\sqrt{\lambda(\lambda_2 + \lambda_4)} \geq 0. \end{cases}$$

对于 $\lambda_4 < 0$

$$\begin{cases} \lambda \geq 0, \\ \lambda_2 + \frac{57}{32}\lambda_4 \geq 0, \\ \lambda_3 + 2\sqrt{\lambda\left(\lambda_2 + \frac{57}{32}\lambda_4\right)} \geq 0. \end{cases}$$





若 $\lambda_2(\Lambda) = \lambda_4(\Lambda) = 0$ 朗道极点能标 $\Lambda_{\text{LP}} \sim 10^8 - 10^9$ GeV

若 $\lambda_2(\Lambda) = 2$ $\lambda_4(\Lambda) = -2$ (不满足真空稳定性)，$\Lambda_{\text{LP}} \sim 10^{10}$ GeV

7-3-5 模型

为了提升朗道极点能标，引入1个三重态费米子场和1个五重态费米子场，与七重态标量场发生 Yukawa 耦合

$$
\begin{aligned}
\mathcal{L}_{\text{yuk}} = & -\sqrt{15} y \Phi_{ijklmn} \overline{\Psi_L^{ijkl}} \Sigma_{R,m'n'} \varepsilon^{mm'} \varepsilon^{nn'} \\
& -(y_\Sigma)_{ab} \overline{l^i_{a,L}} (\Sigma_{b,R})_{ij} H_k \varepsilon^{jk} + \text{h.c.}
\end{aligned}
$$

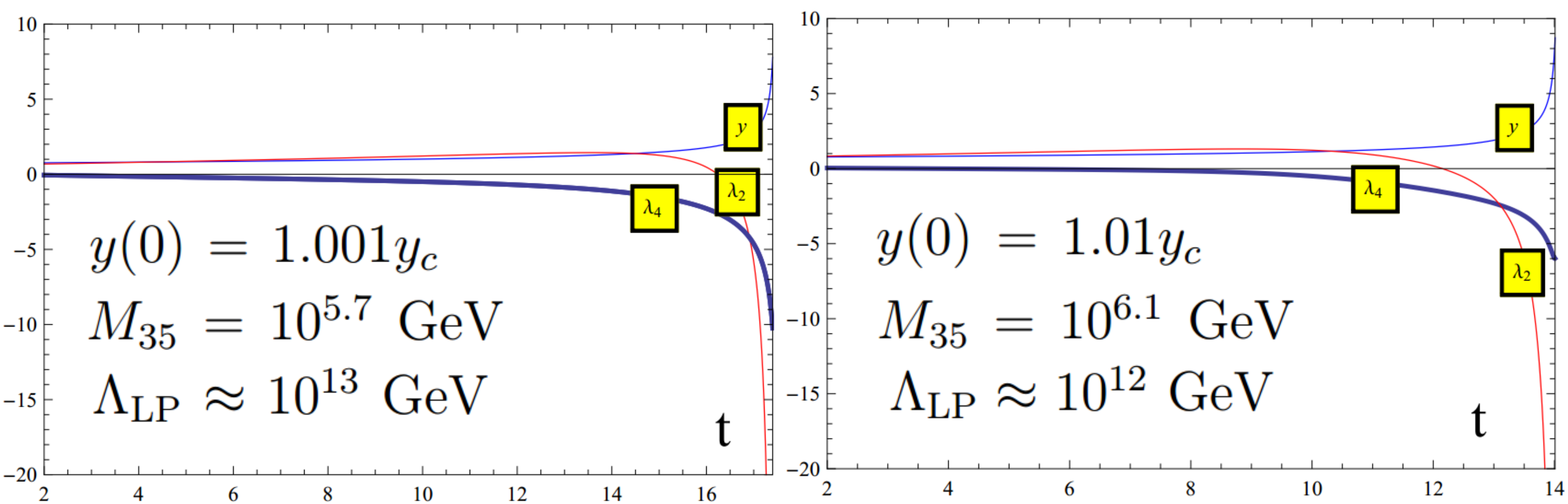
三重态费米子场可通过第二项构建 type-III seesaw 机制

对 beta 函数的影响：

$$
\begin{aligned}
\delta\beta_{g_2} &= \frac{8 g_2^3}{16\pi^2} \\
\delta\beta_{\lambda_2} &= \frac{1}{16\pi^2}(-54 y^4 + 40 y^2 \lambda_2) \\
\delta\beta_{\lambda_3} &= \frac{20 y^2 \lambda_3}{16\pi^2}
\end{aligned}
$$

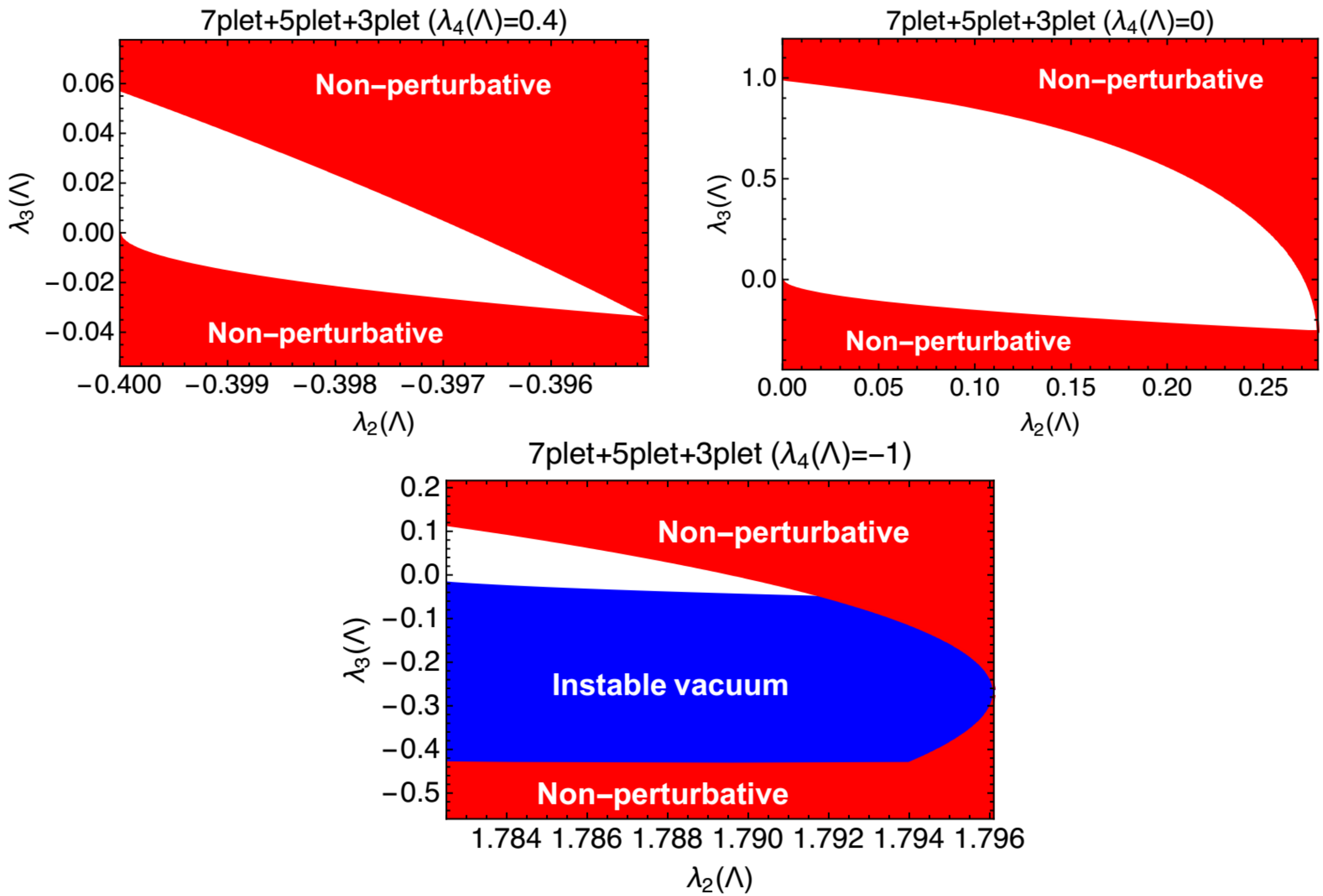
$$
\begin{aligned}
\delta\beta_{\lambda_4} &= \frac{1}{16\pi^2}(-96 y^4 + 40 y^2 \lambda_4) \\
\beta_y &= \frac{y}{16\pi^2}(25 y^2 - 24 g_2^2)
\end{aligned}
$$

我们发现，**y 的跑动与 g_2 的跑动强烈相关**。当 y 的取值为 $y_c \equiv \left(\frac{24+b_2}{25}\right)^{1/2} g_2(0)$ 可得 $\Lambda_{\rm LP}^{(y)} = \Lambda_{\rm LP}^{(g_2)}$。否则 y 的朗道极点将早于 g_2 的朗道极点出现。若允许精细调节初值，至多可将朗道极点能标推迟到 $\sim 10^{14}$ GeV

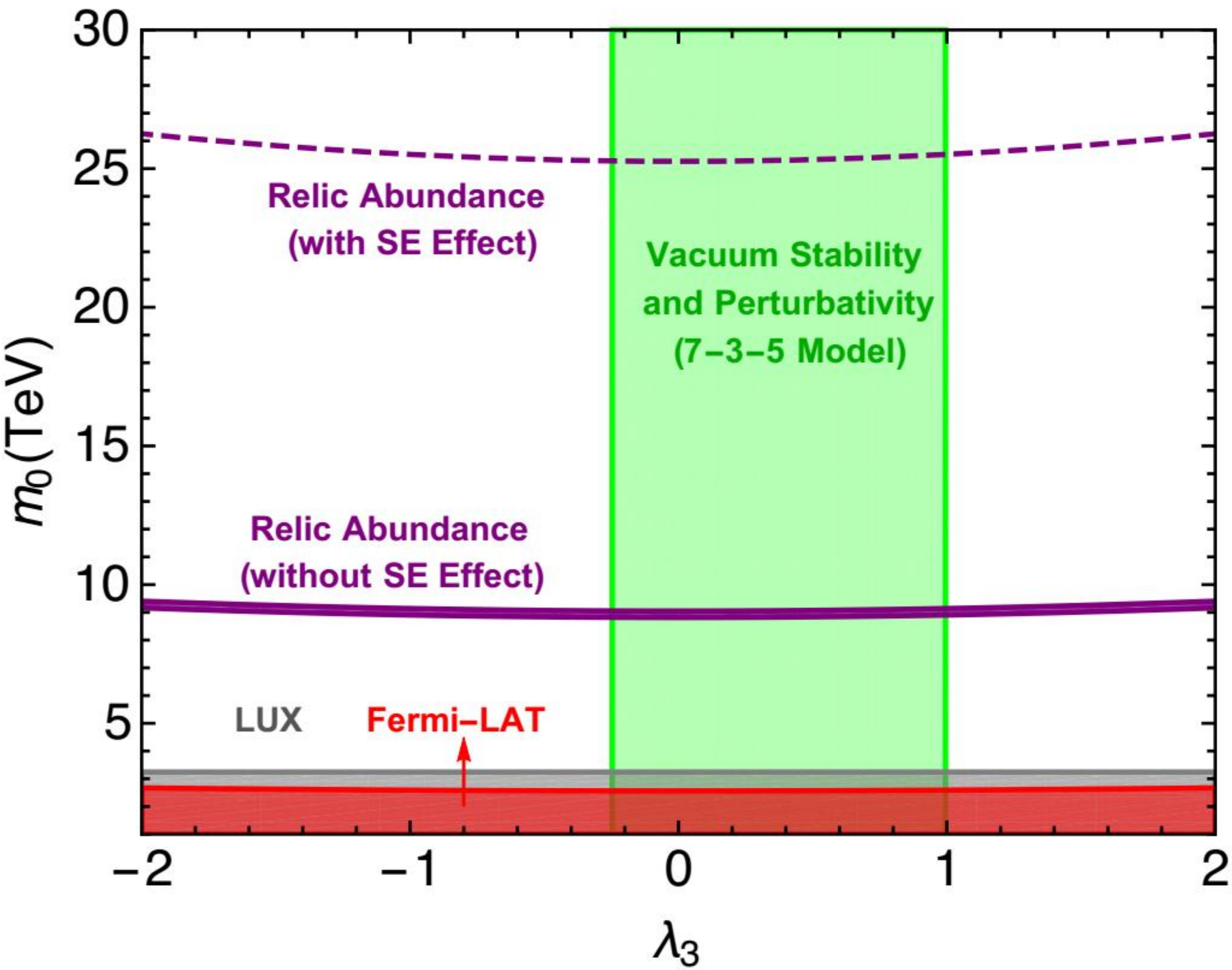


$t \equiv \ln(\mu/M_{35})$

精细调节 y 的初值，将耦合常数演化至 10^{14} GeV（略低于朗道极点能标），则可用真空稳定性和微扰性条件限制七重态的耦合常数



直接探测实验 LUX、间接探测实验 Fermi-LAT、真空稳定性和微扰性条件对模型参数的限制



Composite Higgs and dark matter

QCD and spontaneous breaking of chiral symmetry

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{\psi}(i\not{D} - m)\psi, \quad \text{with} \quad \psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

Since $m \ll 1$, the Lagrangian has an approximate $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$ global symmetry.

Strong dynamics leads to $\langle \bar{\psi}\psi \rangle \neq 0 \quad \Rightarrow \quad SU(2)_L \times SU(2)_R \,/\, SU(2)_{L+R}$

$$\mathcal{L}_{\mathrm{eff}} = \frac{f^2}{4} \mathrm{tr}\left[(\partial_\mu U)(\partial^\mu U) \right] + \frac{f^2}{4} \mathrm{tr}\left(\chi U^+ + U \chi^+ \right) + \cdots$$

Technicolor

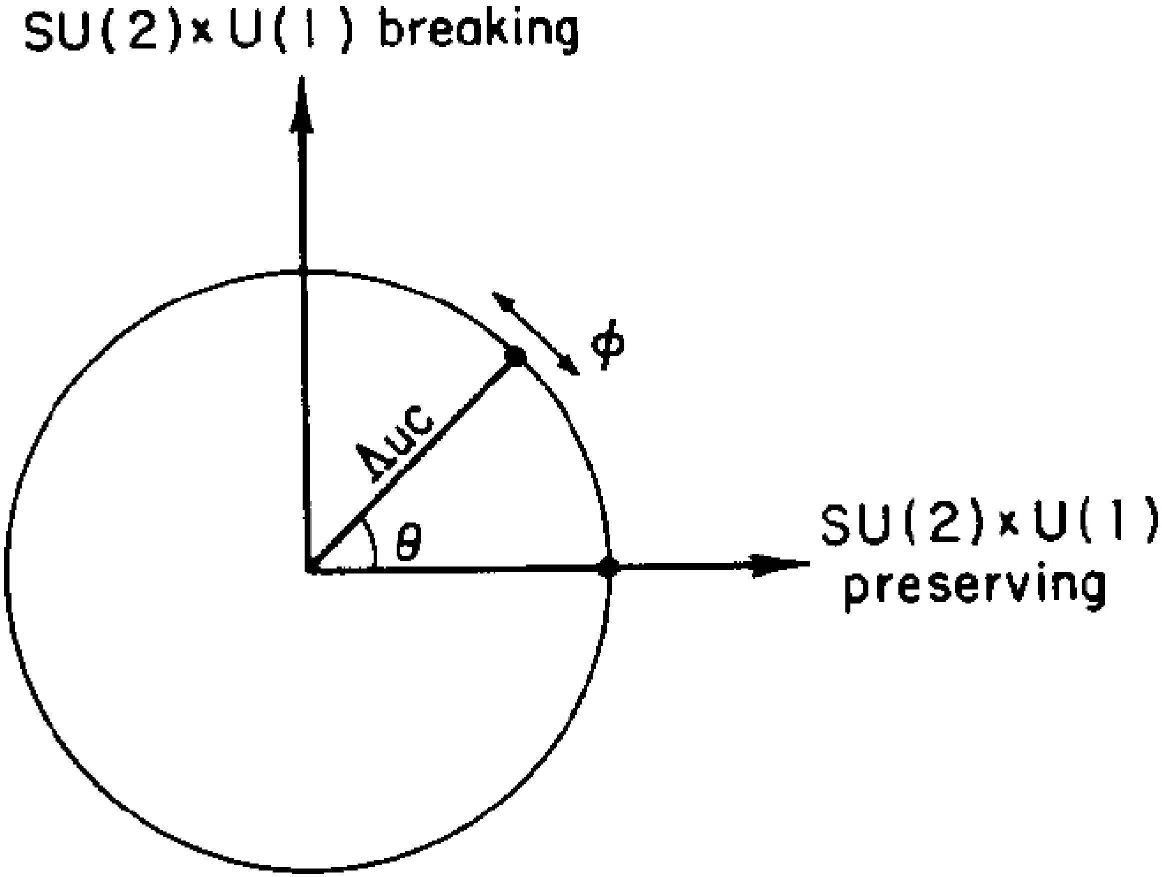
Technicolor is a scaled-up version of QCD (Weinberg 79, Susskind 79)

Techiquarks:	$SU(N)_{TC}$	$SU(2)_L$	$SU(2)_R$
$\psi_L = \begin{pmatrix} U \\ D \end{pmatrix}_L \sim$	N	2	1
$\psi_R = \begin{pmatrix} U \\ D \end{pmatrix}_R \sim$	N	1	2

No Higgs! $\langle \bar{\psi}\psi \rangle \neq 0 \Rightarrow SU(2)_L \times SU(2)_R / SU(2)_{L+R}$

$$\Lambda_{\mathrm{TC}} \sim 10^3 \Lambda_{\mathrm{QCD}}, \quad F_\pi \sim 246\,\mathrm{GeV}, \quad M_W^2 = \frac{g^2 F_\pi^2}{4}, \quad M_Z^2 = \frac{(g^2 + g'^2) F_\pi^2}{4}$$

Techinfermion condensate could be a mixture of its Goldstone limit and technicolor limit



Dugan, Georgi, Kaplan 1984

Various symmetry breaking patterns

$\mathcal{G}$	$\mathcal{H}$	C	N_G	$\mathbf{r}_{\mathcal{H}} = \mathbf{r}_{\mathrm{SU(2)\times SU(2)}}$ $(\mathbf{r}_{\mathrm{SU(2)\times U(1)}})$	Ref.
SO(5)	SO(4)	✓	4	$\mathbf{4} = (\mathbf{2},\mathbf{2})$	[11]
SU(3) × U(1)	SU(2) × U(1)		5	$\mathbf{2}_{\pm 1/2} + \mathbf{1}_0$	[10, 35]
SU(4)	Sp(4)	✓	5	$\mathbf{5} = (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	[29, 47, 64]
SU(4)	$[\mathrm{SU}(2)]^2 \times \mathrm{U}(1)$	✓*	8	$(\mathbf{2},\mathbf{2})_{\pm 2} = 2 \cdot (\mathbf{2},\mathbf{2})$	[65]
SO(7)	SO(6)	✓	6	$\mathbf{6} = 2 \cdot (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	–
SO(7)	G_2	✓*	7	$\mathbf{7} = (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$	[66]
SO(7)	SO(5) × U(1)	✓*	10	$\mathbf{10}_0 = (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$	–
SO(7)	$[\mathrm{SU}(2)]^3$	✓*	12	$(\mathbf{2},\mathbf{2},\mathbf{3}) = 3 \cdot (\mathbf{2},\mathbf{2})$	–
Sp(6)	Sp(4) × SU(2)	✓	8	$(\mathbf{4},\mathbf{2}) = 2 \cdot (\mathbf{2},\mathbf{2})$	[65]
SU(5)	SU(4) × U(1)	✓*	8	$\mathbf{4}_{-5} + \bar{\mathbf{4}}_{+5} = 2 \cdot (\mathbf{2},\mathbf{2})$	[67]
SU(5)	SO(5)	✓*	14	$\mathbf{14} = (\mathbf{3},\mathbf{3}) + (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1})$	[9, 47, 49]
SO(8)	SO(7)	✓	7	$\mathbf{7} = 3 \cdot (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	–
SO(9)	SO(8)	✓	8	$\mathbf{8} = 2 \cdot (\mathbf{2},\mathbf{2})$	[67]
SO(9)	SO(5) × SO(4)	✓*	20	$(\mathbf{5},\mathbf{4}) = (\mathbf{2},\mathbf{2}) + (\mathbf{1}+\mathbf{3},\mathbf{1}+\mathbf{3})$	[34]
$[\mathrm{SU}(3)]^2$	SU(3)		8	$\mathbf{8} = \mathbf{1}_0 + \mathbf{2}_{\pm 1/2} + \mathbf{3}_0$	[8]
$[\mathrm{SO}(5)]^2$	SO(5)	✓*	10	$\mathbf{10} = (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	[32]
SU(4) × U(1)	SU(3) × U(1)		7	$\mathbf{3}_{-1/3} + \bar{\mathbf{3}}_{+1/3} + \mathbf{1}_0 = 3 \cdot \mathbf{1}_0 + \mathbf{2}_{\pm 1/2}$	[35, 41]
SU(6)	Sp(6)	✓*	14	$\mathbf{14} = 2 \cdot (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{3}) + 3 \cdot (\mathbf{1},\mathbf{1})$	[30, 47]
$[\mathrm{SO}(6)]^2$	SO(6)	✓*	15	$\mathbf{15} = (\mathbf{1},\mathbf{1}) + 2 \cdot (\mathbf{2},\mathbf{2}) + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3})$	[36]

1401.2457

General model setup

- $2N_f$ fermions ψ^i charged under some gauge Group G_{TC}.
- Global flavor symmetry $G_F = \mathrm{SU}(2N_f)$ or $\mathrm{SU}(N_f) \times \mathrm{SU}(N_f)$,
- Non-abelian G_{TC}, asymptotic freedom $\longrightarrow$ ψ^i condense in the IR,

$$\langle \psi^i \psi^j \rangle \sim \Sigma^{ij} \neq 0 \quad \Rightarrow \quad G_F \to H \tag{1}$$

  where H is a subgroup of G_F.
  - ψ^i: real reps. of $G_{TC} \longrightarrow \mathrm{SU}(2N_f) \to \mathrm{SO}(2N_f)$,
  - ψ^i: pseudo-real reps. of $G_{TC} \longrightarrow \mathrm{SU}(2N_f) \to \mathrm{Sp}(2N_f)$.
  - ψ^i: complex reps. of $G_{TC} \longrightarrow \mathrm{SU}(N_f) \times \mathrm{SU}(N_f) \to \mathrm{SU}(N_f)$.
- pNGBs, coset space G_F/H,

$$U = \mathrm{e}^{i\Pi(\phi)}, \qquad \Pi(\phi) = \sum_i \phi_i X^i \tag{2}$$

- EW gauge group $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subset H$, Higgs doublet $\subset$ pNGBs.

Sp(2N) group: brief introduction

- $\mathrm{Sp}(2N) = \mathrm{Sp}(2N, C) \cap \mathrm{SU}(2N)$, $2N \times 2N$ matrices U satisfy

$$UEU^T = E, \qquad E = \begin{pmatrix} & \mathbb{1}_{N\times N} \\ -\mathbb{1}_{N\times N} & \end{pmatrix}, \tag{3}$$

$$or \quad U = \mathrm{e}^{i\theta^a \tilde{S}^a}, \qquad \tilde{S}^a E + E(\tilde{S}^a)^T = 0 \tag{4}$$

- Choice of E is not unique,

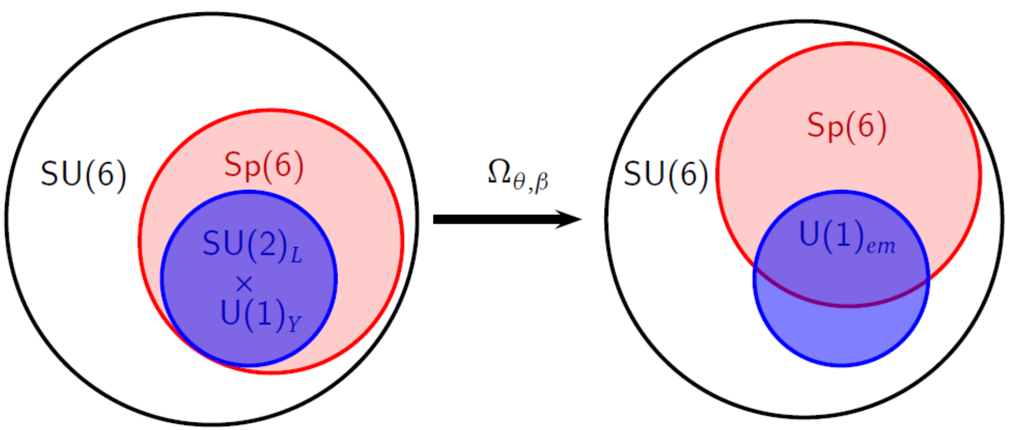
$$\Sigma_B = \begin{pmatrix} \mathcal{J} & & \\ & \pm\mathcal{J} & \\ & & \ddots \end{pmatrix}, \quad \mathcal{J} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \begin{matrix} \Sigma_B = OEO^T, \quad S^a = O\tilde{S}^a O^{-1}, \\ S^a \Sigma_B + \Sigma_B (S^a)^T = 0 \end{matrix} \tag{5}$$

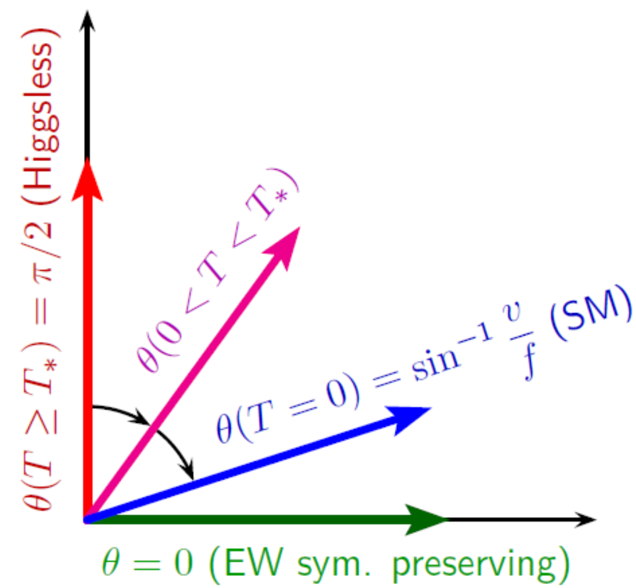
- Degree of freedom:
  $\mathrm{SU}(2N) \sim 4N^2 - 1$, $\mathrm{Sp}(2N) \sim 2N^2 + N$, $\mathrm{SU}(2N)/\mathrm{Sp}(2N) \sim (N-1)(2N+1)$.
- SU(4)/Sp(4): minimal model [E. Katz (2005), B. Gripaio (2009), M. Frigerio (2012), G. Cacciapaglia (2014)],
- SU(6)/Sp(6): 2HDM or Singlet-Doublet-Triplet Model.

一种新的暗物质产生机制

最近，我们提出了一种新的产生暗物质与希格斯的理论机制，并用一个SU(6)/Sp(6)简化模型阐述了该机制的具体实现。

➢ C.F. Cai, **H.H. Zhang***, et al, *Higgs Boson Emerging from the Dark*, **Phys. Rev. Lett.** 125 (2020) no.2, 021801.

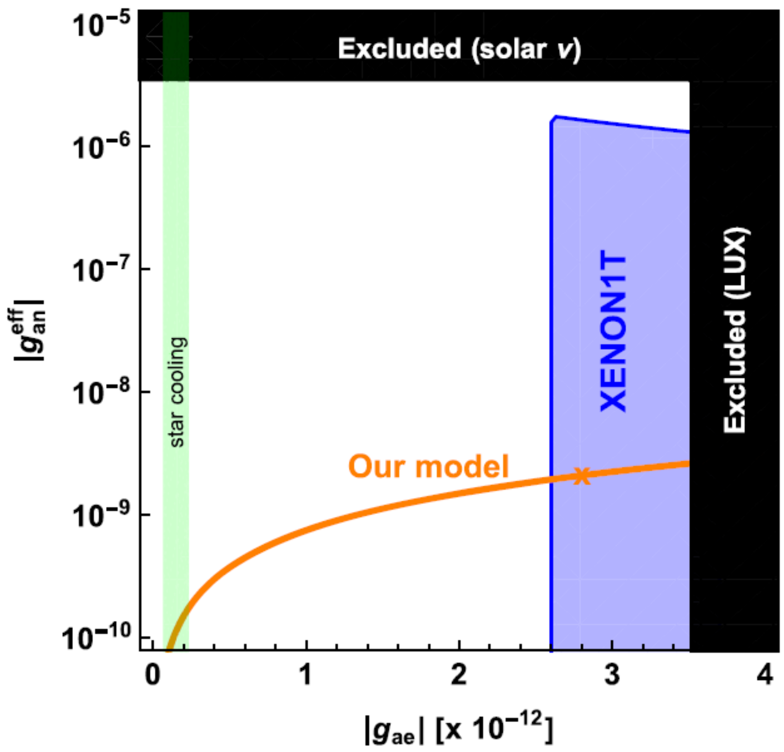




可以解释XENON1T实验信号

我们模型预言的η标量粒子，很好地解释了XENON1T实验合作组发现的疑似超出信号。

➢ C.F. Cai, **H.H. Zhang***, et al, *XENON1T solar axion and the Higgs boson emerging from the dark,* **Phys.Rev.D** 102 (2020) no.7, 075018.



pseudo-Nambu-Goldstone dark matter

Thermal Dark Matter

✨ Conventionally, **dark matter (DM)** is assumed to be a **thermal relic** remaining from the early Universe

🌙 DM relic abundance observation

👉 Particle mass $m_\chi \sim \mathcal{O}(\text{GeV}) - \mathcal{O}(\text{TeV})$

Interaction strength $\sim$ **weak strength**

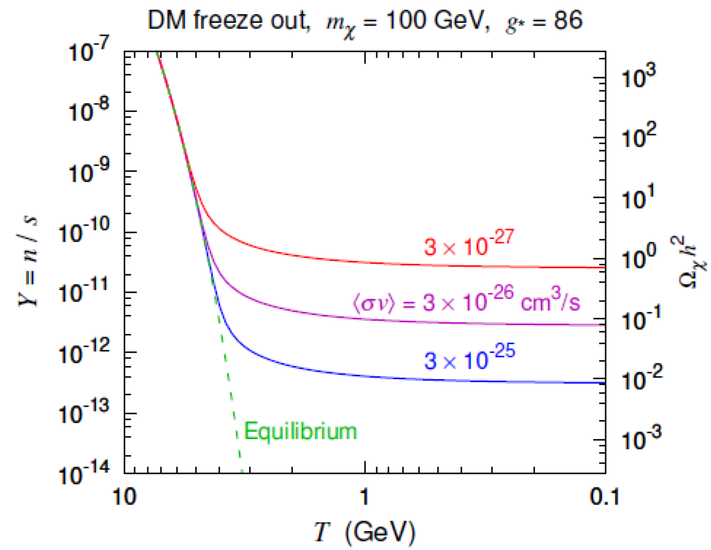
"Weakly interacting massive particles"

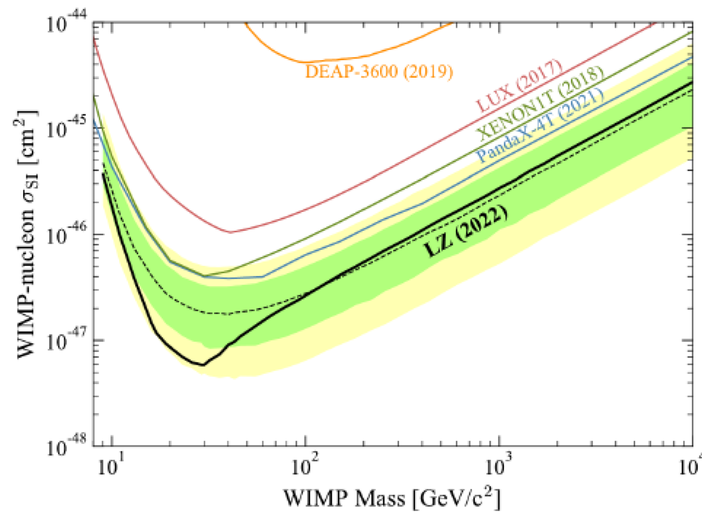
"WIMPs"

🔍 **Direct detection** for WIMPs

👉 **No robust signal found so far**

☁️ **Great challenge** to the thermal dark matter paradigm





[LZ Coll., 2207.03764]

pseudo-Nambu-Goldstone Dark Matter

[Gross, Lebedev, Toma, 1708.02253, PRL]

✏️ **Standard model (SM) Higgs doublet** H, **complex scalar** S (SM singlet)

💡 Scalar potential respects a **softly broken global** $\mathrm{U}(1)$ **symmetry** $S \to e^{i\alpha} S$

🌓 $\mathrm{U}(1)$ **symmetric:** $V_0 = -\frac{\mu_H^2}{2}|H|^2 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_H}{2}|H|^4 + \frac{\lambda_S}{2}|S|^4 + \lambda_{HS}|H|^2|S|^2$

🌗 **Soft breaking:** $V_{\mathrm{soft}} = -\frac{\mu_S'^2}{4} S^2 + \mathrm{H.c.}$

⭐ V_{soft} is **special**, and it can be justified by a **proper ultraviolet (UV) completion**

✏️ H and S develop vacuum expectation values (VEVs) v and v_s

$$H \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad S = \frac{1}{\sqrt{2}}(v_s + s + i\chi)$$

🔥 The **soft breaking term** V_{soft} give a mass to χ: $m_\chi = \mu_S'$

💥 The **DM candidate** χ is a **stable pseudo-Nambu-Goldstone boson (pNGB)**

🌙 Rotate **CP-even Higgs bosons** h and s to **mass eigenstates** h_1 and h_2

$$\begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \quad m_{h_1,h_2}^2 = \frac{1}{2}\left(\lambda_H v^2 + \lambda_S v_s^2 \mp \frac{\lambda_S v_s^2 - \lambda_H v^2}{\cos 2\theta}\right)$$

DM-nucleon Scattering

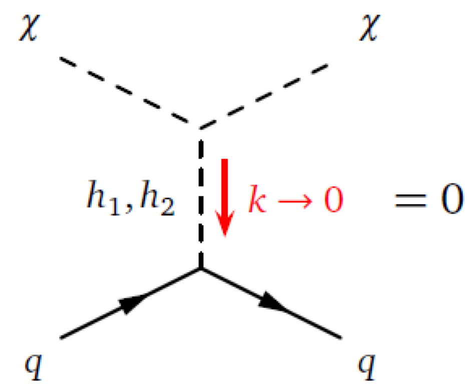
[Gross, Lebedev, Toma, 1708.02253, PRL]

✨ **DM-quark** interactions induce **DM-nucleon** scattering in direct detection

✏️ **DM-quark scattering amplitude** from Higgs portal interactions

$$\mathcal{M}(\chi q \to \chi q) \propto \frac{m_q s_\theta c_\theta}{v v_s}\left(\frac{m_{h_1}^2}{t - m_{h_1}^2} - \frac{m_{h_2}^2}{t - m_{h_2}^2}\right)$$

$$= \frac{m_q s_\theta c_\theta}{v v_s}\frac{t(m_{h_1}^2 - m_{h_2}^2)}{(t - m_{h_1}^2)(t - m_{h_2}^2)}$$



🔥 **Zero momentum transfer limit** $t = k^2 \to 0, \quad \mathcal{M}(\chi q \to \chi q) \to 0$

👉 DM-nucleon scattering cross section **vanishes** at tree level

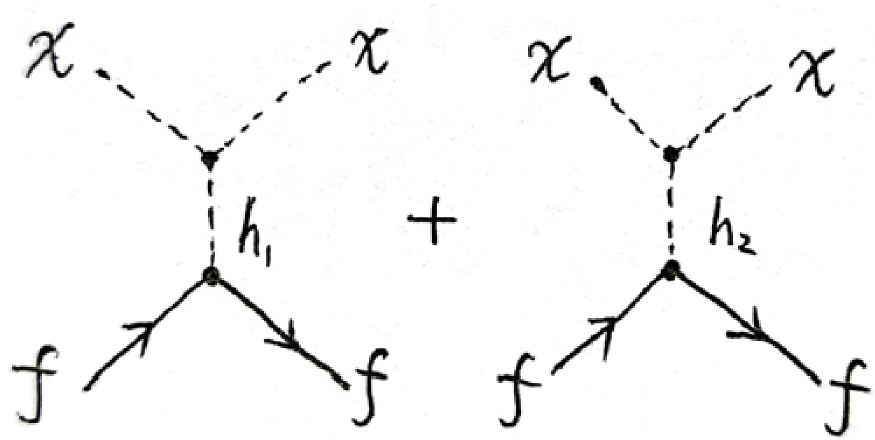
💡 Tree-level interactions of a **pNGB** are generally **momentum-suppressed**

🌥️ **One-loop corrections** typically lead to $\sigma_{\chi N}^{\rm SI} \lesssim \mathcal{O}(10^{-50})\ \mathrm{cm}^2$

[Azevedo *et al.*, 1810.06105, JHEP; Ishiwata & Toma, 1810.08139, JHEP]

👉 **Beyond capability** of current and near future direct detection experiments

The tree-level Feynman diagrams for scattering of the dark matter χ on the SM fermions f involve the t-channel exchange a single h_1 or h_2:



The interaction operators relevant to this scattering are:

$$h_1\chi^2, \quad h_2\chi^2, \quad h_1\bar{f}f, \quad h_2\bar{f}f$$

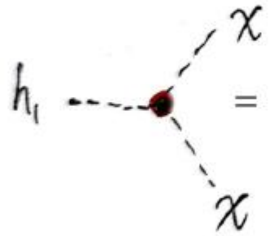
Let us figure out their coupling constants in this model.

There are only two terms in the potential V contributing to the effective operators $h_i\chi^2$ $(i = 1, 2)$:

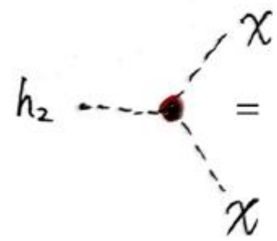
$$V \supset \frac{\lambda_S}{2}|S|^4 \supset \frac{\lambda_S v_s}{2} s\chi^2, \qquad V \supset \lambda_{HS}|H|^2|S|^2 \supset \frac{\lambda_{HS} v}{2} h\chi^2$$

$$\begin{aligned}
\Rightarrow \quad \mathcal{L} \supset -V \supset & -\frac{\lambda_S v_s}{2} s\chi^2 - \frac{\lambda_{HS} v}{2} h\chi^2 \\
&= -\frac{1}{2}\chi^2 \begin{pmatrix} \lambda_{HS} v & \lambda_S v_s \end{pmatrix} \begin{pmatrix} h \\ s \end{pmatrix} \\
&= -\frac{1}{2}\chi^2 \begin{pmatrix} \lambda_{HS} v & \lambda_S v_s \end{pmatrix} \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \\
&= -\frac{1}{2}(\lambda_{HS} v c_\theta - \lambda_S v_s s_\theta) h_1\chi^2 \\
&\quad - \frac{1}{2}(\lambda_{HS} v s_\theta + \lambda_S v_s c_\theta) h_2\chi^2 \\
&= \frac{1}{2}\frac{m_1^2 s_\theta}{v_s} h_1\chi^2 - \frac{1}{2}\frac{m_2^2 c_\theta}{v_s} h_2\chi^2
\end{aligned}$$

From the interactions we can read off the Feynman rules:



$$= i\frac{m_1^2 s_\theta}{v_s}$$



$$= -i\frac{m_2^2 c_\theta}{v_s}$$

We have used the relations:

$$\lambda_{HS} v c_\theta - \lambda_S v_S s_\theta = -\frac{m_1^2 s_\theta}{v_S}, \quad \lambda_{HS} v s_\theta + \lambda_S v_S c_\theta = \frac{m_2^2 c_\theta}{v_S}$$

$$\begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix}\begin{pmatrix} \lambda_H v^2 & \lambda_{HS} v v_s \\ \lambda_{HS} v v_s & \lambda_s v_s^2 \end{pmatrix}\begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} = \begin{pmatrix} m_1^2 & \\ & m_2^2 \end{pmatrix}$$

$$\rightarrow \underbrace{\begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix}\begin{pmatrix} \lambda_H v^2 & \lambda_{HS} v v_s \\ \lambda_{HS} v v_s & \lambda_s v_s^2 \end{pmatrix}}_{\begin{pmatrix} * & \lambda_{HS} v v_s c_\theta - \lambda_s v_s^2 s_\theta \\ * & \lambda_{HS} v v_s s_\theta + \lambda_s v_s^2 c_\theta \end{pmatrix}} = \underbrace{\begin{pmatrix} m_1^2 & \\ & m_2^2 \end{pmatrix}\begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix}}_{\begin{pmatrix} * & -m_1^2 s_\theta \\ * & m_2^2 c_\theta \end{pmatrix}}$$

$$\rightarrow \begin{cases} \lambda_{HS} v v_s c_\theta - \lambda_s v_s^2 s_\theta = -m_1^2 s_\theta \\ \lambda_{HS} v v_s s_\theta + \lambda_s v_s^2 c_\theta = m_2^2 c_\theta \end{cases}$$

Considering the Higgs-fermion interactions in the SM

$$\mathcal{L} \supset -\sum_f \frac{m_f}{v} h \bar{f} f$$

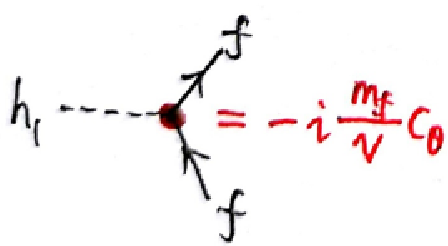
together with

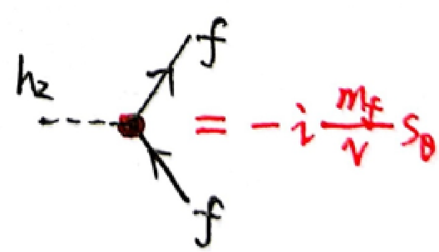
$$\begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \quad \Rightarrow \quad h = h_1 c_\theta + h_2 s_\theta$$

we have

$$\mathcal{L} \supset -\sum_f \frac{m_f}{v} (h_1 c_\theta + h_2 s_\theta) \bar{f} f$$

from which we can read off the Feynman rules:

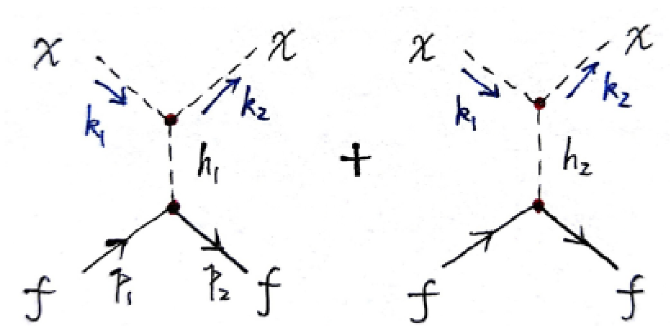




Thus, the tree-level $\chi + f \to \chi + f$ scattering amplitude is

$$
\begin{aligned}
& i\mathcal{M}\left[\chi(k_1) + f(p_1) \to \chi(k_2) + f(p_2)\right] \\
&= \bar{u}(p_2)\left(-i\frac{m_f}{v}c_\theta\right)u(p_1)\frac{i}{q^2 - m_1^2}\, i\frac{m_1^2 s_\theta}{v_s} \\
&\quad + \bar{u}(p_2)\left(-i\frac{m_f}{v}s_\theta\right)u(p_1)\frac{i}{q^2 - m_2^2}\left(-i\frac{m_2^2 c_\theta}{v_s}\right) \\
&= i\bar{u}(p_2)u(p_1)\frac{m_f c_\theta s_\theta}{v v_s}\left(\frac{m_1^2}{q^2 - m_1^2} - \frac{m_2^2}{q^2 - m_2^2}\right) \\
&= i\bar{u}(p_2)u(p_1)\frac{m_f c_\theta s_\theta}{v v_s}\frac{t(m_1^2 - m_2^2)}{(t - m_1^2)(t - m_2^2)} \xrightarrow{t \to 0} 0
\end{aligned}
$$

where $q \equiv k_1 - k_2 = p_2 - p_1$ and $t \equiv q^2$.



The previous calculation can be written in terms of matrices.
The Feynman rules in matrix language are

- **Propagator**

$$\left\langle \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \begin{pmatrix} h_1 & h_2 \end{pmatrix} \right\rangle = \begin{pmatrix} \frac{i}{q^2-m_1^2} & \\ & \frac{i}{q^2-m_2^2} \end{pmatrix} = \begin{pmatrix} \frac{i}{t-m_1^2} & \\ & \frac{i}{t-m_2^2} \end{pmatrix}$$

- **Vertices**

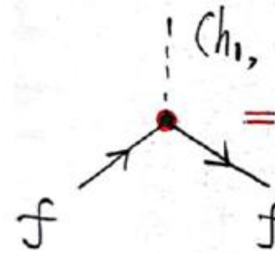
$$\mathcal{L} \supset \frac{1}{2}\frac{m_1^2 s_\theta}{v_s} h_1 \chi^2 - \frac{1}{2}\frac{m_2^2 c_\theta}{v_s} h_2 \chi^2$$

$$= \frac{1}{2}\chi^2 \left(\frac{m_1^2 s_\theta}{v_s}, \ -\frac{m_2^2 c_\theta}{v_s} \right) \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

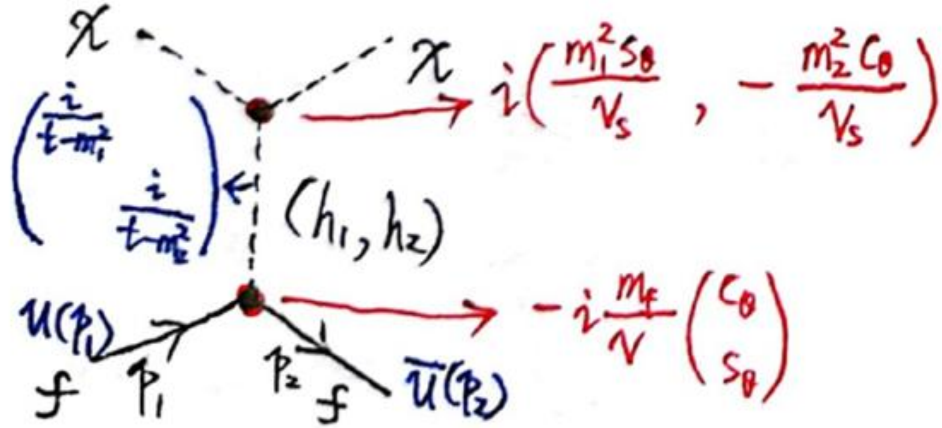
$$= i \left(\frac{m_1^2 s_\theta}{v_s}, \ -\frac{m_2^2 c_\theta}{v_s} \right)$$

$$\mathcal{L} \supset -\sum_f \frac{m_f}{v} (h_1 c_\theta + h_2 s_\theta) \bar{f} f$$

$$= -\sum_f \frac{m_f}{v} \bar{f} f \, (h_1, \ h_2) \begin{pmatrix} c_\theta \\ s_\theta \end{pmatrix}$$



$$= -i \frac{m_f}{v} \begin{pmatrix} c_\theta \\ s_\theta \end{pmatrix}$$



Thus, the tree-level $\chi + f \to \chi + f$ scattering amplitude is

$$i\mathcal{M} = i\begin{pmatrix} \frac{m_1^2 s_\theta}{v_s} & -\frac{m_2^2 c_\theta}{v_s} \end{pmatrix} \begin{pmatrix} \frac{i}{t-m_1^2} & \\ & \frac{i}{t-m_2^2} \end{pmatrix} \left(-i\frac{m_f}{v}\right) \begin{pmatrix} c_\theta \\ s_\theta \end{pmatrix} \bar{u}(p_2)u(p_1)$$

$$= i\bar{u}(p_2)u(p_1)\frac{m_f c_\theta s_\theta}{v v_s}\left(\frac{m_1^2}{t-m_1^2} - \frac{m_2^2}{t-m_2^2}\right) \xrightarrow{t\to 0} 0$$

We can also calculate in the interaction basis (*i.e.*, in terms of the states *h* and *s*). The Feynman rules in this basis are

$$\begin{pmatrix} h \\ s \end{pmatrix} = \underbrace{\begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}}_{\equiv\, O} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = O \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \quad \Longleftrightarrow \quad (h \;\; s) = (h_1 \;\; h_2)\, O^T$$

➢ **Propagator**

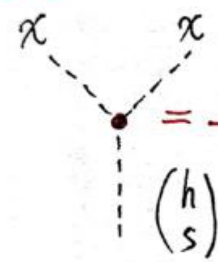
$$\left\langle \begin{pmatrix} h \\ s \end{pmatrix} (h \;\; s) \right\rangle = O \left\langle \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} (h_1 \;\; h_2) \right\rangle O^T$$

$$= O \begin{pmatrix} \frac{i}{q^2 - m_1^2} & \\ & \frac{i}{q^2 - m_2^2} \end{pmatrix} O^T$$

➢ **Vertices**

$$\mathcal{L} \supset -\frac{1}{2}\lambda_{HS} v\, h\chi^2 - \frac{1}{2}\lambda_s v_s\, s\chi^2$$

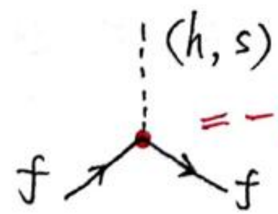
$$= -\frac{1}{2}\chi^2 \left(\lambda_{HS} v,\ \lambda_s v_s\right) \begin{pmatrix} h \\ s \end{pmatrix}$$



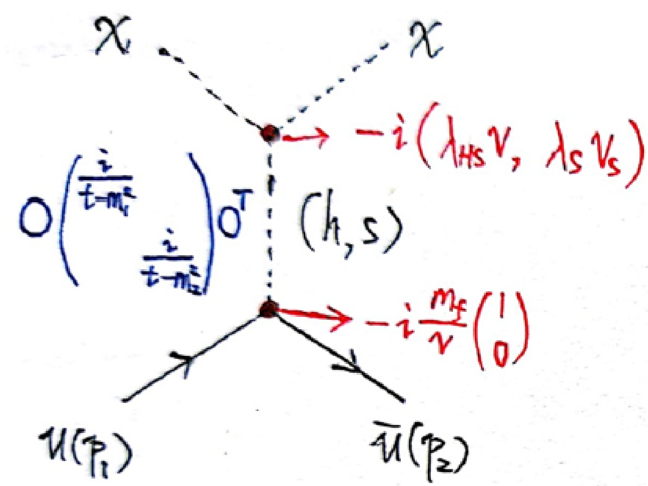
$$= -i\left(\lambda_{HS} v,\ \lambda_s v_s\right)$$

$$\mathcal{L} \supset -\sum_f \frac{m_f}{v} h\bar{f}f$$

$$= -\sum_f \frac{m_f}{v} \bar{f}f\, (h,\ s) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



$$= -i\frac{m_f}{v} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



$$i\mathcal{M} = -i\begin{pmatrix}\lambda_{HS}v & \lambda_S v_s\end{pmatrix} O \begin{pmatrix}\frac{i}{t-m_1^2} & \\ & \frac{i}{t-m_2^2}\end{pmatrix} O^T \left(-i\frac{m_f}{v}\right)\begin{pmatrix}1\\0\end{pmatrix}\bar{u}(p_2)u(p_1)$$

$$\xrightarrow{t\to 0} i\frac{m_f}{v}\bar{u}(p_2)u(p_1)\begin{pmatrix}\lambda_{HS}v & \lambda_S v_s\end{pmatrix} O \begin{pmatrix}\frac{1}{m_1^2} & \\ & \frac{1}{m_2^2}\end{pmatrix} O^T \begin{pmatrix}1\\0\end{pmatrix}$$

$$= i\frac{m_f}{v}\bar{u}(p_2)u(p_1)\begin{pmatrix}\lambda_{HS}v & \lambda_S v_s\end{pmatrix}\left(M^2_{\text{even}}\right)^{-1}\begin{pmatrix}1\\0\end{pmatrix}$$

$$= i\frac{m_f}{v}\bar{u}(p_2)u(p_1)\begin{pmatrix}\lambda_{HS}v & \lambda_S v_s\end{pmatrix}\frac{1}{\det\left(M^2_{\text{even}}\right)}\begin{pmatrix}\lambda_S v_s^2 & -\lambda_{HS}vv_s \\ -\lambda_{HS}vv_s & \lambda_H v^2\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$$

$$
\begin{aligned}
i\mathcal{M} &\xrightarrow{t\to 0} i\frac{m_f}{v}\frac{\bar{u}(p_2)u(p_1)}{\det\left(M^2_{\text{even}}\right)}\begin{pmatrix}\lambda_{HS}v & \lambda_S v_s\end{pmatrix}\begin{pmatrix}\lambda_S v_s^2 & -\lambda_{HS}vv_s \\ -\lambda_{HS}vv_s & \lambda_H v^2\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \\
&= i\frac{m_f}{v}\frac{\bar{u}(p_2)u(p_1)}{\det\left(M^2_{\text{even}}\right)}\begin{pmatrix}0 & *\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \\
&= 0
\end{aligned}
$$

Once again, we see that the tree-level $\chi + f \to \chi + f$ scattering amplitude vanishes in the limit of zero momentum transfer.

pNBG DM and Two Higgs Doublets

[XM Jiang, CF Cai, ZH Yu, YP Zeng, **HH Zhang**, 1907.09684, PRD]

- **Two Higgs doublets** Φ_1 and Φ_2 with $Y = 1/2$, **complex scalar singlet** S
- Scalar potential respects a **softly broken global** $\mathrm{U}(1)$ **symmetry** $S \to e^{i\alpha} S$
- Scalar potential constructed with Φ_1 and Φ_2

$$V_1 = m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 - m_{12}^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{\lambda_5}{2}[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2]$$

- $\mathrm{U}(1)$ **symmetric** potential terms involving S

$$V_2 = -m_S^2|S|^2 + \frac{\lambda_S}{2}|S|^4 + \kappa_1|\Phi_1|^2|S|^2 + \kappa_2|\Phi_2|^2|S|^2$$

- Quadratic term **softly breaking** the global $\mathrm{U}(1)$: $V_{\text{soft}} = -\frac{m_S'^2}{4}S^2 + \mathrm{H.c.}$
- Φ_1, Φ_2, and S develop VEVs v_1, v_2 and v_s

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ (v_1 + \rho_1 + i\eta_1)/\sqrt{2} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ (v_2 + \rho_2 + i\eta_2)/\sqrt{2} \end{pmatrix}, \quad S = \frac{v_s + s + i\chi}{\sqrt{2}}$$

- χ is a **stable pNGB** with $m_\chi = m_S'$, acting as a **DM candidate**

Physical Scalars

Rotations of **charged scalars** and CP-**odd scalars**:

$$\begin{pmatrix}\phi_1^+\\ \phi_2^+\end{pmatrix} = R(\beta)\begin{pmatrix}G^+\\ H^+\end{pmatrix},\quad \begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix} = R(\beta)\begin{pmatrix}G^0\\ a\end{pmatrix},\quad R(\beta) = \begin{pmatrix}\cos\beta & -\sin\beta\\ \sin\beta & \cos\beta\end{pmatrix},\quad \tan\beta = \frac{v_2}{v_1}$$

$G^\pm$ and G^0 are **massless Nambu-Goldstone bosons** eaten by $W^\pm$ and Z

$H^\pm$ and a are **physical states**

Mass terms for CP-**even scalars** $\mathcal{L}_{\text{mass}} \supset -\frac{1}{2}\begin{pmatrix}\rho_1, & \rho_2, & s\end{pmatrix}\mathcal{M}^2_{\rho s}\begin{pmatrix}\rho_1\\ \rho_2\\ s\end{pmatrix}$

$$\mathcal{M}^2_{\rho s} = \begin{pmatrix}\lambda_1 v_1^2 + m_{12}^2\tan\beta & \lambda_{345}v_1v_2 - m_{12}^2 & \kappa_1 v_1 v_s\\ \lambda_{345}v_1v_2 - m_{12}^2 & \lambda_2 v_2^2 + m_{12}^2\cot\beta & \kappa_2 v_2 v_s\\ \kappa_1 v_1 v_s & \kappa_2 v_2 v_s & \lambda_S v_s^2\end{pmatrix},\quad \lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$$

$$\begin{pmatrix}\rho_1\\ \rho_2\\ s\end{pmatrix} = O\begin{pmatrix}h_1\\ h_2\\ h_3\end{pmatrix},\quad O^{\mathrm{T}}\mathcal{M}^2_{\rho s}O = \mathrm{diag}(m_{h_1}^2, m_{h_2}^2, m_{h_3}^2),\quad m_{h_1} \le m_{h_2} \le m_{h_3}$$

One of h_i should behave like the **125 GeV SM Higgs boson**

Four Types of Yukawa Couplings

If all fermions with the same quantum numbers just couple to the one **same** Higgs doublet, **flavor-changing neutral currents (FCNCs)** will be **absent** at tree level

[Glashow & Weinberg, PRD 15, 1958 (1977); Paschos, PRD 15, 1966 (1977)]

Yukawa interactions for the **fermion mass eigenstates**

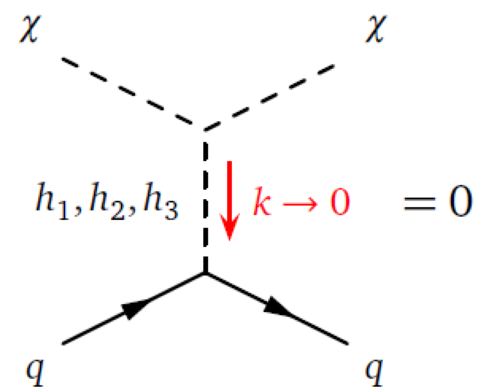
$$\mathcal{L}_{\mathrm{Y}} = \sum_{f=\ell_j,d_j,u_j} \left[-m_f \bar{f} f - \frac{m_f}{v} \left(\sum_{i=1}^{3} \xi^f_{h_i} h_i \bar{f} f + \xi^f_a a \bar{f} i \gamma_5 f \right) \right]$$

$$- \frac{\sqrt{2}}{v} [H^+ (\xi^{\ell_i}_a m_{\ell_i} \bar{\nu}_i P_{\mathrm{R}} \ell_i + \xi^{d_j}_a m_{d_j} V_{ij} \bar{u}_i P_{\mathrm{R}} d_j + \xi^{u_i}_a m_{u_i} V_{ij} \bar{u}_i P_{\mathrm{L}} d_j) + \mathrm{H.c.}]$$

	Type I	Type II	Lepton specific	Flipped
$\xi^{\ell_j}_{h_i}$	$O_{2i}/\sin\beta$	$O_{1i}/\cos\beta$	$O_{1i}/\cos\beta$	$O_{2i}/\sin\beta$
$\xi^{d_j}_{h_i}$	$O_{2i}/\sin\beta$	$O_{1i}/\cos\beta$	$O_{2i}/\sin\beta$	$O_{1i}/\cos\beta$
$\xi^{u_j}_{h_i}$	$O_{2i}/\sin\beta$	$O_{2i}/\sin\beta$	$O_{2i}/\sin\beta$	$O_{2i}/\sin\beta$
$\xi^{\ell_j}_a$	$\cot\beta$	$-\tan\beta$	$-\tan\beta$	$\cot\beta$
$\xi^{d_j}_a$	$\cot\beta$	$-\tan\beta$	$\cot\beta$	$-\tan\beta$
$\xi^{u_j}_a$	$-\cot\beta$	$-\cot\beta$	$-\cot\beta$	$-\cot\beta$

Vanishing of DM-nucleon Scattering

💡 Take the **type-I Yukawa couplings** as an example



⭐ **Higgs portal** interactions $\mathcal{L}_{h_i\chi^2} = \dfrac{1}{2}\sum_{i=1}^{3} g_{h_i\chi^2}\, h_i\chi^2$

$$g_{h_i\chi^2} = -\kappa_1 v_1 O_{1i} - \kappa_2 v_2 O_{2i} - \lambda_S v_s O_{3i}$$

✨ **DM-quark scattering** amplitude

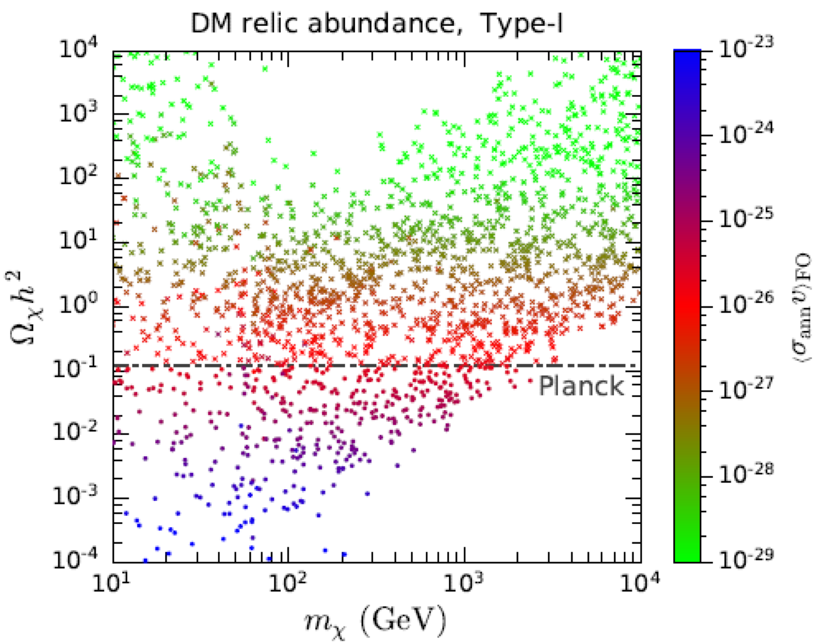
$$\mathcal{M}(\chi q \to \chi q) \propto \frac{m_q}{v\sin\beta}\left(\frac{g_{h_1\chi^2}O_{21}}{t-m_{h_1}^2} + \frac{g_{h_2\chi^2}O_{22}}{t-m_{h_2}^2} + \frac{g_{h_3\chi^2}O_{23}}{t-m_{h_3}^2}\right)$$

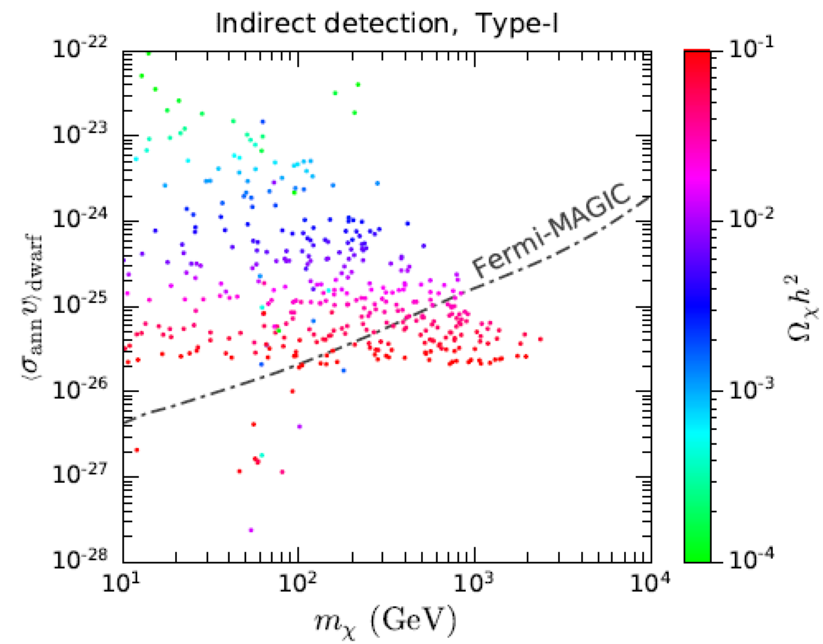
$$\xrightarrow{t\to 0} \frac{m_q}{v\sin\beta}\begin{pmatrix}\kappa_1 v_1 & \kappa_2 v_2 & \lambda_S v_s\end{pmatrix} O \begin{pmatrix} m_{h_1}^{-2} & & \\ & m_{h_2}^{-2} & \\ & & m_{h_3}^{-2}\end{pmatrix} O^{\mathrm{T}} \begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$= \frac{m_q}{v\sin\beta}\begin{pmatrix}\kappa_1 v_1 & \kappa_2 v_2 & \lambda_S v_s\end{pmatrix} (\mathcal{M}_{\rho s}^2)^{-1} \begin{pmatrix}0\\1\\0\end{pmatrix} = 0$$

Interaction basis expression

DM Relic Abundance and Indirect Detection





- **Planck** observed DM relic abundance $\Omega_{\rm DM}h^2 = 0.1186 \pm 0.0020$

  [Planck coll., 1502.01589, Astron. Astrophys.]

- **Colored dots:** $\Omega_\chi h^2$ is **equal** or **lower** than observation

- **Colored crosses:** χ is **overproduced**, contradicting standard cosmology

- The parameter points with $m_\chi \gtrsim 100$ GeV and $\Omega_\chi h^2 \sim 0.1$ are **not excluded** by Fermi-LAT and MAGIC γ-ray observations of dwarf spheroidal galaxies

  [MAGIC & Fermi-LAT, 1601.06590, JCAP]

Gravitational Waves from First-order Phase Transition

[Z Zhang, CF Cai, XM Jiang, YL Tang, ZH Yu, **HH Zhang**, 2102.01588, JHEP]

🔭 The discovery of **gravitational waves (GWs)** by LIGO in 2015 opens a new window to new physics models

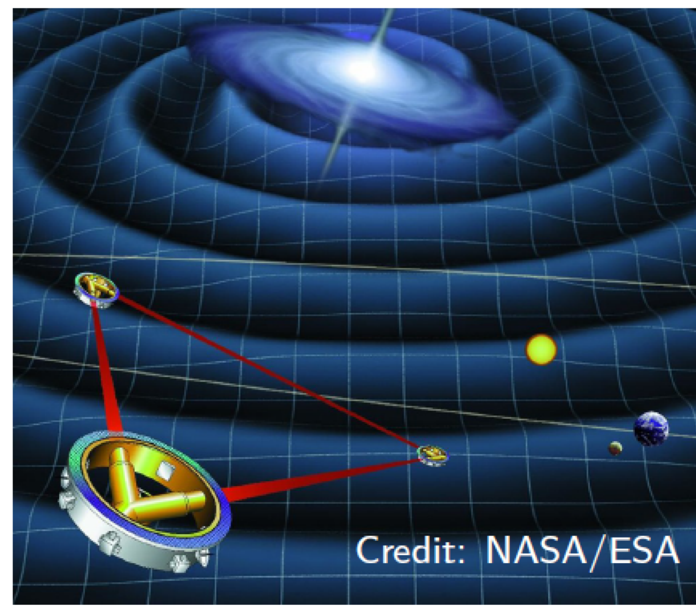
🔥 Introducing new scalar fields may change the **electroweak phase transition** to be a **first-order phase transition (FOPT)**

💥 Cosmological FOPT may induce a **stochastic GW background** with $f \sim$ mHz

👉🏻 Potential signals in **future space-borne GW interferometers** like **TianQin**, **Taiji**, **LISA**, **BBO**, and **DECIGO**

😟 However, the **original pNGB DM model** can only result in **second-order** phase transitions

[Kannike & Raidal, 1901.03333, PRD]



🤔 The situation may be different in the **2HDM extension of pNGB DM**

Effective Potential

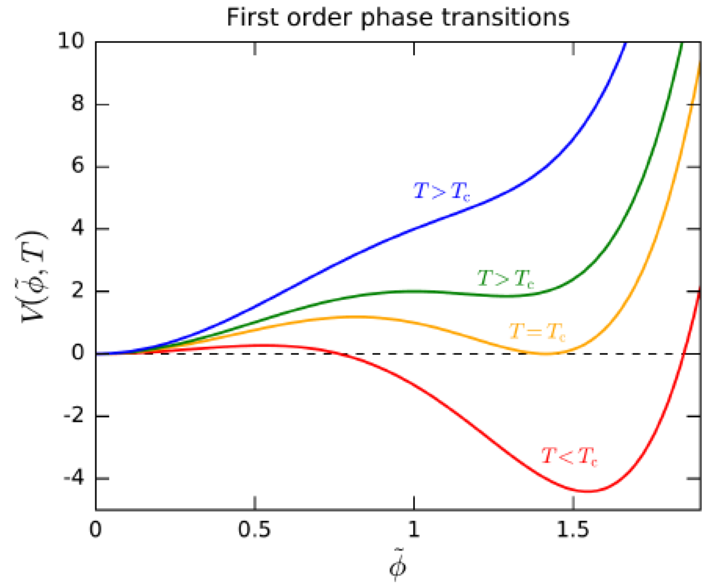
🌶️ Different **local minima** in the **effective potential** V_{eff} of the scalar fields

👉 Different **phases** 👉 **Phase transitions**

🌸 We assume that only the CP-even neutral scalar fields (ρ_1, ρ_2, s) develop VEVs in the cosmological history

🐉 As a function of the **classical background fields** $(\tilde{\rho}_1, \tilde{\rho}_2, \tilde{s})$ and the **temperature** T,

$$V_{\text{eff}}(\tilde{\rho}_1, \tilde{\rho}_2, \tilde{s}, T) = V_0 + V_1 + V_{\text{CT}} + V_{1\text{T}} + V_{\text{D}}$$



🍅 Tree-level potential V_0

🍒 1-loop zero-temperature corrections V_1

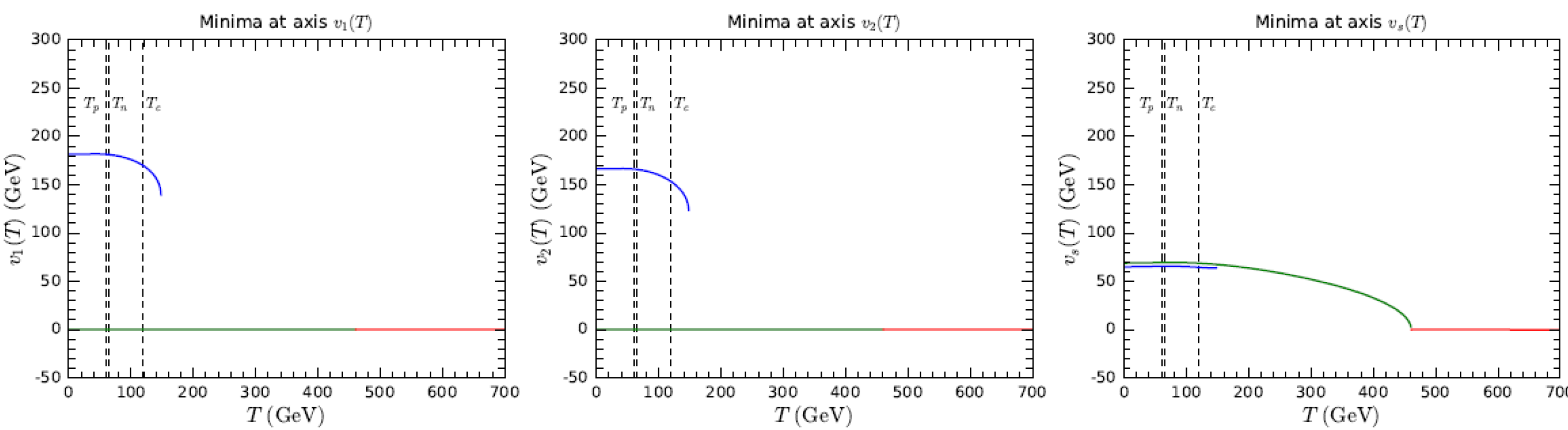
🍑 Counter terms V_{CT} for keeping the VEV positions and the renormalized mass-squared matrix of the CP-even neutral scalars

🍓 1-loop finite-temperature corrections $V_{1\text{T}}(T)$

🍉 Daisy diagram contributions $V_{\text{D}}(T)$ beyond 1-loop at finite temperatures

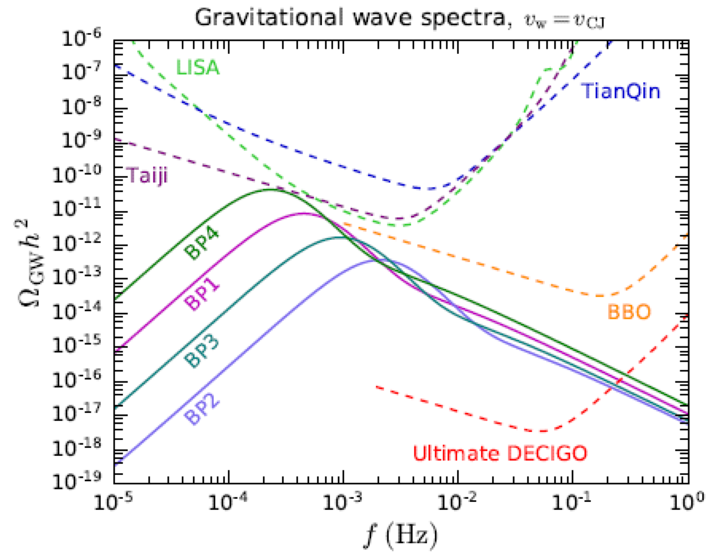
Temperature Evolution of Local Minima

- We utilize `CosmoTransitions` to analyze the phase transitions
- At sufficiently high temperatures, the only minimum in the effective potential is the **gauge symmetric minimum** $(\tilde{\rho}_1, \tilde{\rho}_2, \tilde{s}) = (0, 0, 0)$
- As the Universe cools down, **extra minima** may appear
- **Multi-step** cosmological phase transitions typically occur in this model
- If there are two coexisted minima separated by a **high barrier**, a **strong FOPT** could take place, resulting in **stochastic gravitational waves**
- At last, the system is trapped at the **true vacuum** $(\tilde{\rho}_1, \tilde{\rho}_2, \tilde{s}) = (v_1, v_2, v_s)$



Benchmark Points (BPs)

	BP1	BP2	BP3	BP4
Type	I	I	II	II
v_s (GeV)	542.40	384.26	64.987	138.82
m_χ (GeV)	117.88	78.191	134.03	76.678
m_{12}^2 (10^4 GeV^2)	2.0210	0.015876	17.696	15.042
$\tan\beta$	2.8616	3.2654	0.91655	1.1732
λ_1	2.1496	2.1882	1.5297	0.87839
λ_2	0.80887	0.85479	1.2074	0.80222
λ_3	2.3925	2.2628	1.5741	2.8002
λ_4	3.0027	1.4715	5.3967	4.4643
λ_5	-6.2187	-4.0567	-7.8556	-7.5755
λ_S	3.4048	2.5502	6.0689	4.8644
κ_1	-1.4852	1.0295	0.80378	-0.38075
κ_2	1.1727	-1.2142	-0.83745	-0.14591
m_{h_1} (GeV)	125.11	91.459	125.38	124.87
m_{h_2} (GeV)	282.02	124.77	158.83	307.56
m_{h_3} (GeV)	1014.5	641.83	650.98	582.08
m_a (GeV)	664.75	496.49	911.87	874.04
$m_{H^\pm}$ (GeV)	402.96	280.94	655.60	631.66
$\langle\sigma_{\text{ann}} v\rangle_{\text{dwarf}}$ (10^{-26} cm^3/s)	1.30	0.368	1.72	0.682
α	**0.240**	**0.160**	**0.181**	**0.346**
$\tilde{\beta}^{-1}$ (10^{-2})	**1.33**	**0.402**	**0.771**	**2.15**
T_{p} (GeV)	55.3	74.9	60.2	47.2
SNR_{LISA}	**96.6**	**37.7**	**60.1**	**120**
$\text{SNR}_{\text{Taiji}}$	**83.3**	**23.9**	**42.3**	**155**
$\text{SNR}_{\text{TianQin}}$	**5.50**	**2.39**	**3.07**	**9.20**



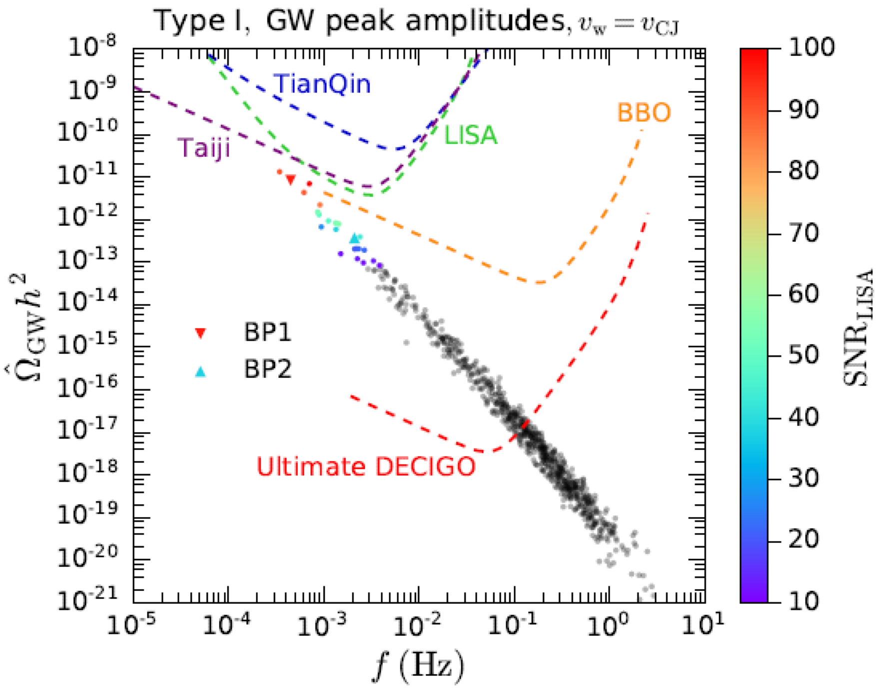
For a practical observation time $\mathcal{T}$, the **signal-to-noise ratio** is

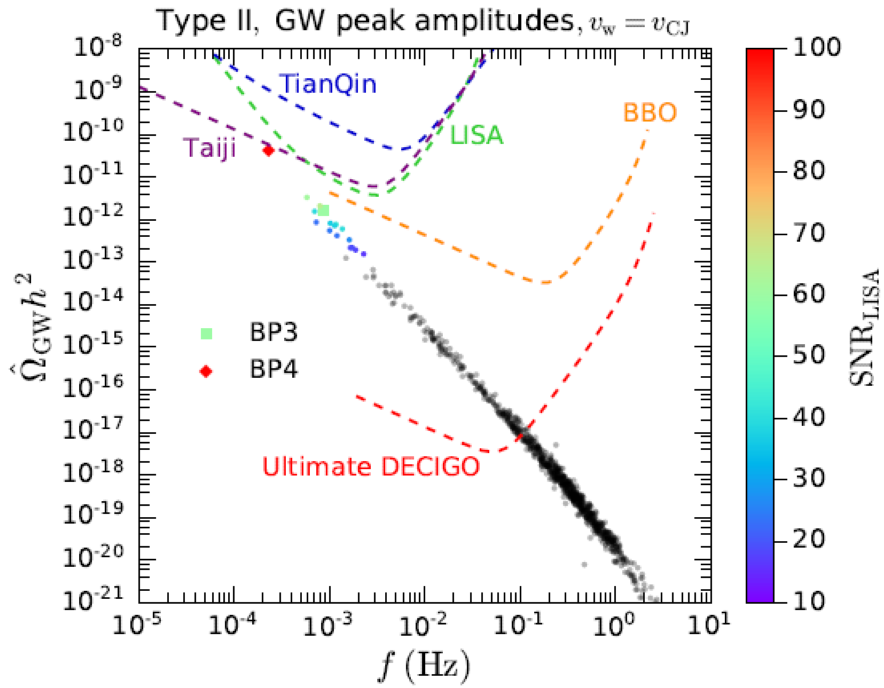
$$\text{SNR} \equiv \sqrt{\mathcal{T} \int_{f_{\min}}^{f_{\max}} \frac{\Omega_{\text{GW}}^2(f)}{\Omega_{\text{sens}}^2(f)}\, df}$$

Take $\mathcal{T} = 3$ yr for LISA, Taiji, TianQin

The detection threshold can be chosen as $\text{SNR}_{\text{thr}} = 10$

Peak Amplitudes and Signal-to-noise Ratios





🎨 The **colored points** leads to $\mathrm{SNR}_{\mathrm{LISA}} > 10$, promising to be probed by **LISA**

🦄 Based on current information, the sensitivity of **Taiji** could be similar to **LISA**, while **TianQin** would be less sensitive due to its **shorter arm length**

🎯 Far future plans aiming at $f \sim \mathcal{O}(0.1)$ Hz, like **BBO** and **DECIGO**, may explore much more parameter points

总结

- 用手征有效场论计算电弱精细参数时，要重新分析 数幂，不能照搬QCD的手征拉氏量的数幂。

- 暗物质有效模型可用于模型无关的研究，便于与 各种实验比照。在讨论各种暗物质模型的直接探测唯象时，可以很方便地将模型参数与有效算符顶点建立联系，并讨论其直接探测限制。

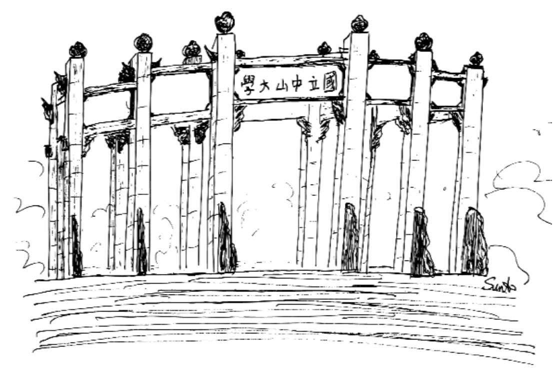
- 七重态最小暗物质模型存在朗道极点等一些理论问题，加入费米子可得到一定程度的缓解。

- 复合希格斯模型在解决规范等级问题的同时，还可以自然地提供暗物质候选粒子，并通过对称性破缺的残余对称性保护其稳定；暗物质和希格斯可能都是pseudo-Goldstone玻色子，它们在宇宙早期的产生机制可能是非平庸的。

- pNGB暗物质可以自然地解释暗物质直接探测的零结果，扩充这类模型的标量部分（例如增加二重态标量）仍然可以保留这种相消机制，而且有更丰富的唯象学，还可望产生一级相变引力波信号。

2023

中山大学物理学院
诚聘英才



目
录
CATALOG

01
学院简介

发展规划
02

03
人才政策

01

第一部分

学院简介

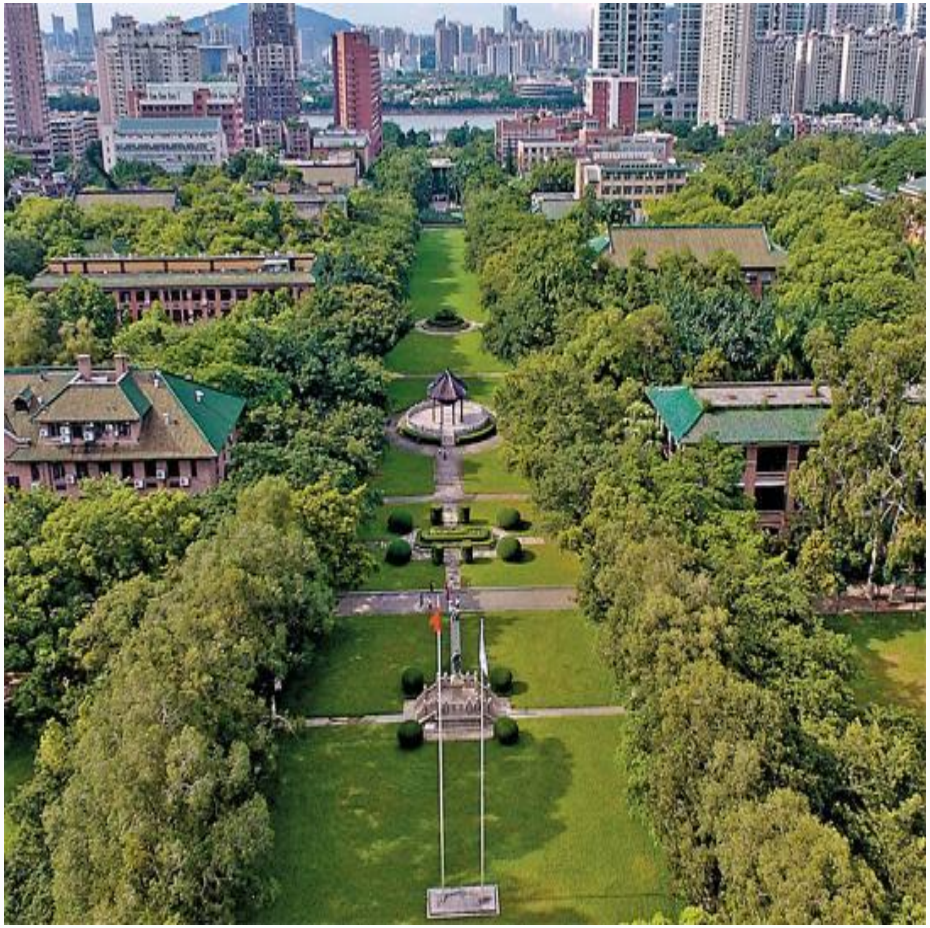
PART
01

中山大学物理学院坐落于中山大学广州校区

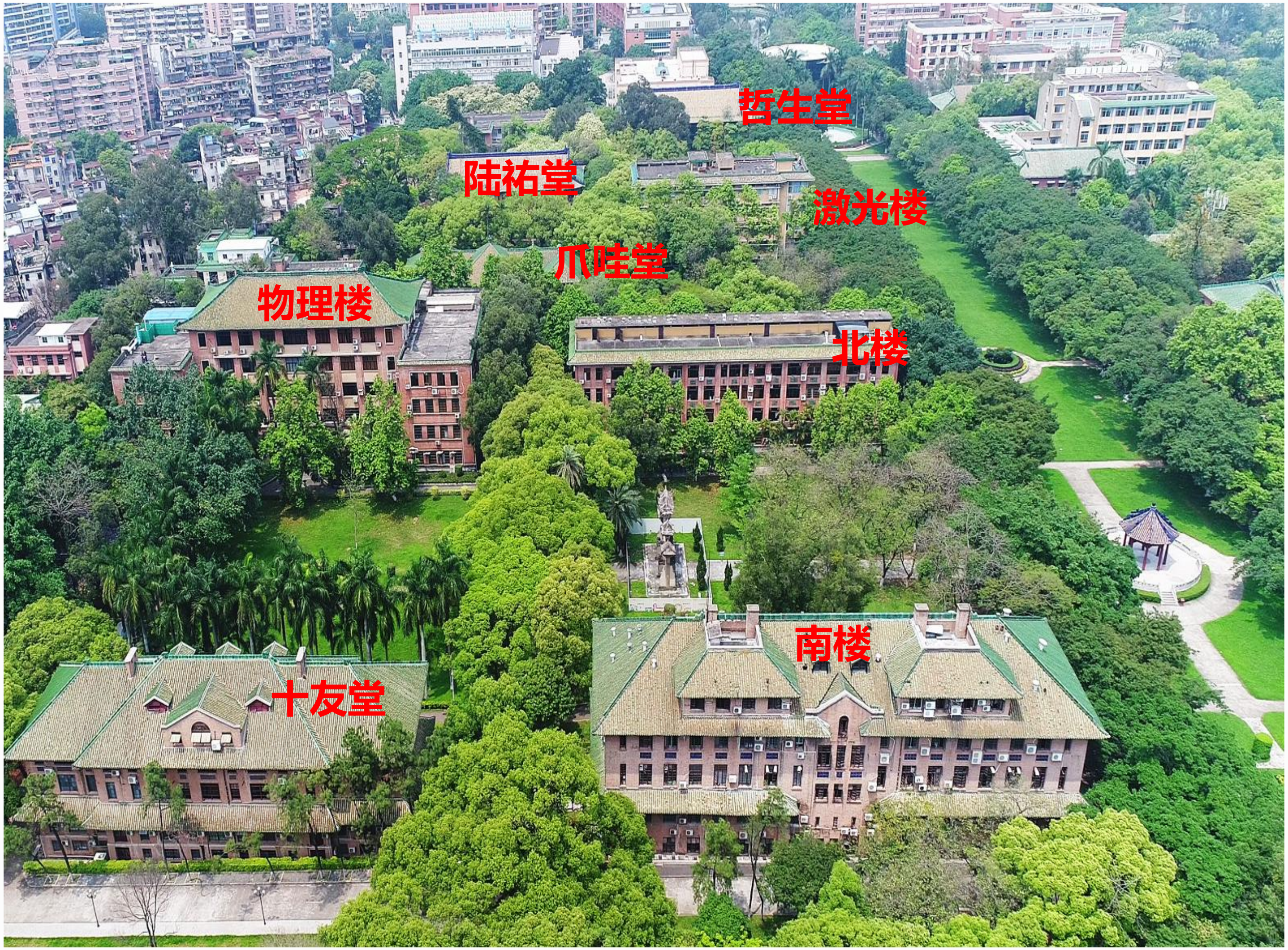
集云山珠水之秀，沐岭南开明之风。

物理学科始建于1924年创校之初，1952年并岭南大学物理系，1981年恢复理论物理和光学博士点，1984年筹建首批国家重点实验室之一“超快速激光光谱学国家重点实验室”，1985年成立国家首批博士后流动站之一“物理学博士后流动站”，1993年获批建设国家物理学人才培养基地，1998年物理学科成为首批博士、硕士学位授权一级学科。在第三轮学科评估中，物理学科高被引论文总量全国排名第三。

广州校区南校园



学院概貌



339栋

冼为坚堂

历史沿革

1924
建校时首批院系

1952
同岭南大学物理系合并

1978
培植出电子学系

1996
成立物理科学与工程技术学院

1998
物理学成为我校首批一级学科

2005
以力学学科为基础成立工学院

2006
成立光学与光学工程系、微电子学系

2015
物理学院成立

现有学科

主要学科方向

凝聚态物理

曾为我国电介质物理、固体缺陷与内耗、固体发光等领域的发展做出了重要贡献。2002年入选国家重点学科。拥有一支由国家杰青、国家青年拔尖、青年千人等人才组成的科研、教学队伍。当前瞄准国际科学研究前沿和国家重大需求，开展低维与表面物理、多铁物理、强关联物理、拓扑量子材料、低维量子物性、中子散射物理等方向的前沿科学研究。

理论物理

传统优势学科，广东省攀峰计划重点学科。1981 年首批恢复博士学位授予点。建国初期李华钟、郭硕鸿等人在理论物理方向做出有影响的工作。70年代本学科在规范场和引力理论方面的研究在国内领先并具有国际影响。80代初杨振宁先生创办了中山大学高等学术中心，大力发展理论物理。近年来，本学科在关联电子体系、统计物理、量子信息、纳米电子学、粒子物理与场论等前沿方向取得了有国际影响力的成果，开发有自主知识产权软件，受到了同行的广泛关注。本学科目前有教师 20余 人，拥有国家杰青、973 首席、青年千人、广东特支计划领军人才等杰出人才与广州天河二号超算中心、东莞散裂中子源、大亚湾中微子实验等大科学装置有长期的合作。

光　学

由著名光谱物理学家高兆兰教授创建，是国内首批重点学科，首批博士点和首批博士后建站单位。1984年建立国内首批国家重点实验室-超快速激光光谱学国家重点实验室。上世纪90年代，成为了当时国际上为数不多的知名超快研究中心之一。如今围绕量子调控和光场调控中的重大科学问题和关键核心技术开展研究。现有专任教师20余人，包括长江学者计划、国家杰青、青年千人、国家优青等国家级人才。承担多项国家重大科学研究计划、国家重点研究计划、

粒子物理与原子核物理

实验和理论粒子物理方向现有教师10余人，其中青年千人3人，研究覆盖粒子物理、核物理、天体物理等方面的前沿领域，尤其是在中微子物理、天基宇宙射线探测和高能重离子物理等方向已经初具规模。凭借广东省的核电优势，该团队在国家的两个基于核反应堆的中微子物理大科学工程项目江门和大亚湾中微子实验中承担探测器研发、模拟以及物理分析任务。同时，深度参与欧洲核子中心对撞机实验、中国锦屏实验室的深地实验、北京正负电子对撞机实验等国际重要实验。

学科发展

2019年学科排名

- 软科排名：10
- QS排名：8
- ARWU排名：10
- USNEWS: 9

中山大学物理学第四轮学科评估A-

评估结果	学校代码及名称
A+	10001 北京大学
	10358 中国科学技术大学
A	10003 清华大学
	10246 复旦大学
	10248 上海交通大学
	10284 南京大学
A-	10055 南开大学
	10183 吉林大学
	10335 浙江大学
	10486 武汉大学
	10487 华中科技大学
	10558 中山大学

人才队伍

现有教职员工情况

类别	人数
教授	45
副教授	63
讲师	2
专职科研人员	8
博士后	28
工程技术人员	21
行政人员	12

人才工程

长江学者特聘教授：4人

国家杰出青年科学基金获得者：3人

国家重大科学研究计划首席科学家：2人

青年长江特聘教授：1人

优秀青年科学基金获得者： 4人

国家重点研发计划青年科学家：

支撑平台

类别	名称
国家重点实验室	光电材料与技术国家重点实验室（共建）
国家基金委	物理学人才培养基地
国家级实验教学示范中心	中山大学物理实验教学中心
广东省重点实验室	广东省光伏技术重点实验室
广东省重点实验室	广东省磁电物性分析与器件重点实验室
广东省工程中心	广东省绿色电力变换及智能控制工程技术研究中心
广州市重点实验室	集成光子系统与应用重点实验室
校级研究所	中山大学激光与光谱学研究所
校级研究所	中山大学凝聚态物理研究所
校级研究所	中山大学太阳能系统研究所

02

第二部分

发展规划

PART
02

学科规划

3

2

3

国内第一方阵
世界一流学科

◆巩固传统三大优势学科：
光学、凝聚态物理、理论物理

◆ 推进两大特色新兴方向：
软物质物理、能源物理

◆ 结合区域优势大力发展三个重点研究领域：
粒子物理、中子科学、超快强激光物理

◆ 专人教师规模150人
◆ 力争达到50%以上教师获得国家级人才工程

物理学公共科研大平台建设

凝聚态物理

光学

粒子物理

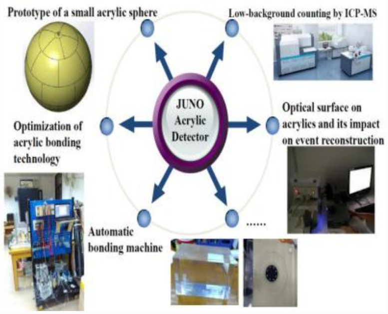
软物质物理

学术交流中心

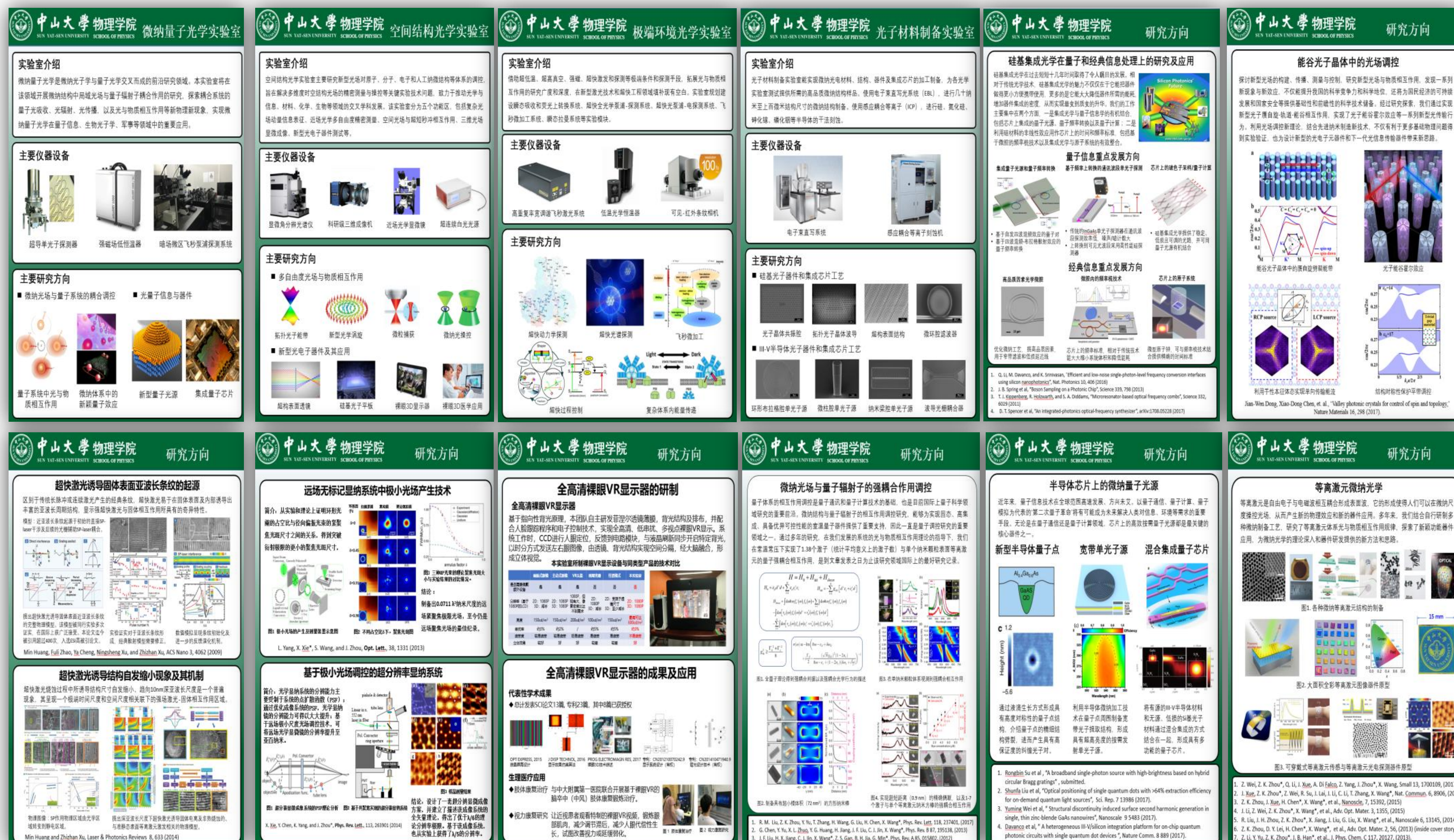
建设世界顶尖的物理学公共科研大平台

实现物理学公共科研平台从无到有、科研人员“拎包入住”。

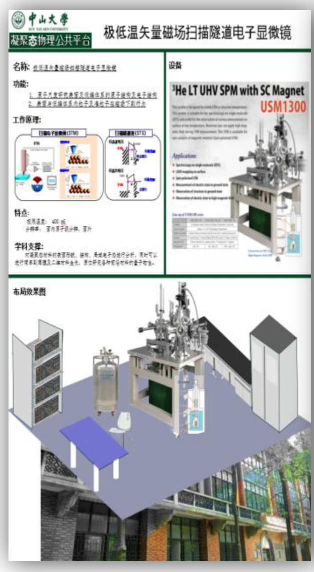


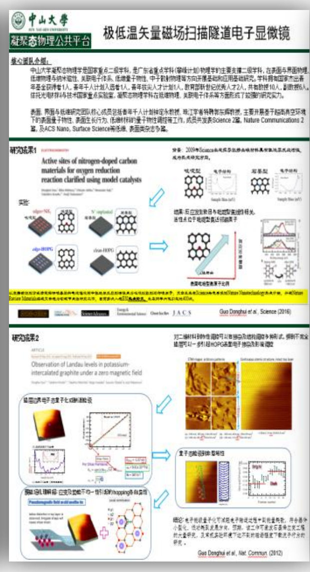



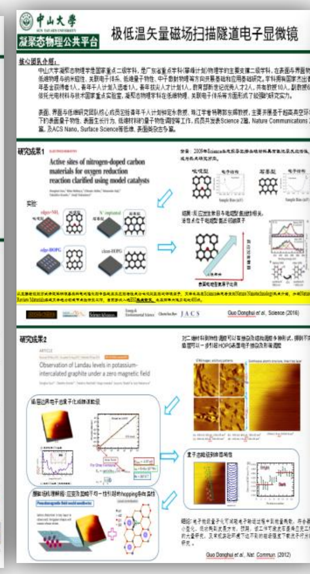
物理学公共科研大平台建设

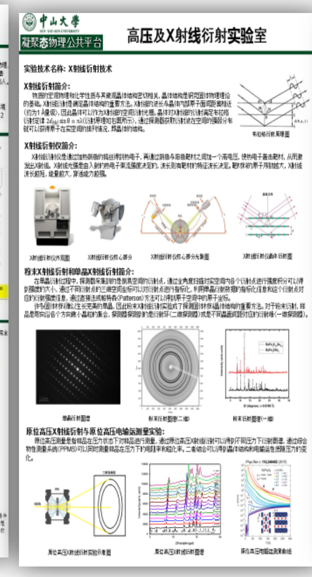


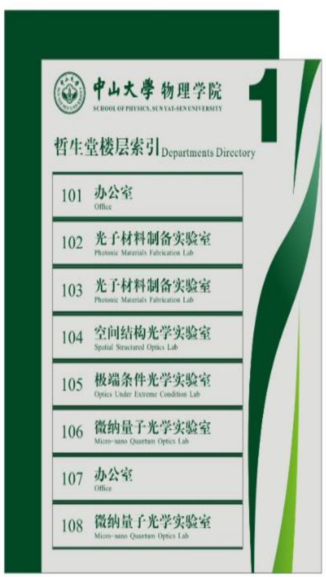
物理学公共科研大平台建设







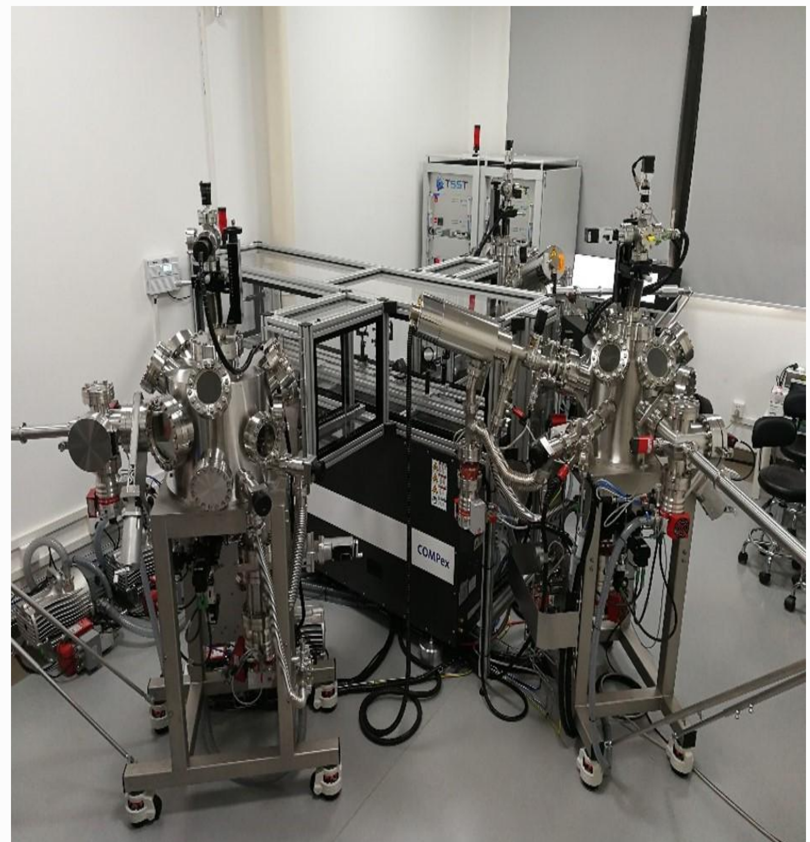




物理学公共科研大平台建设

1.综合物性测量系统 PPMS

2.激光分子束外延



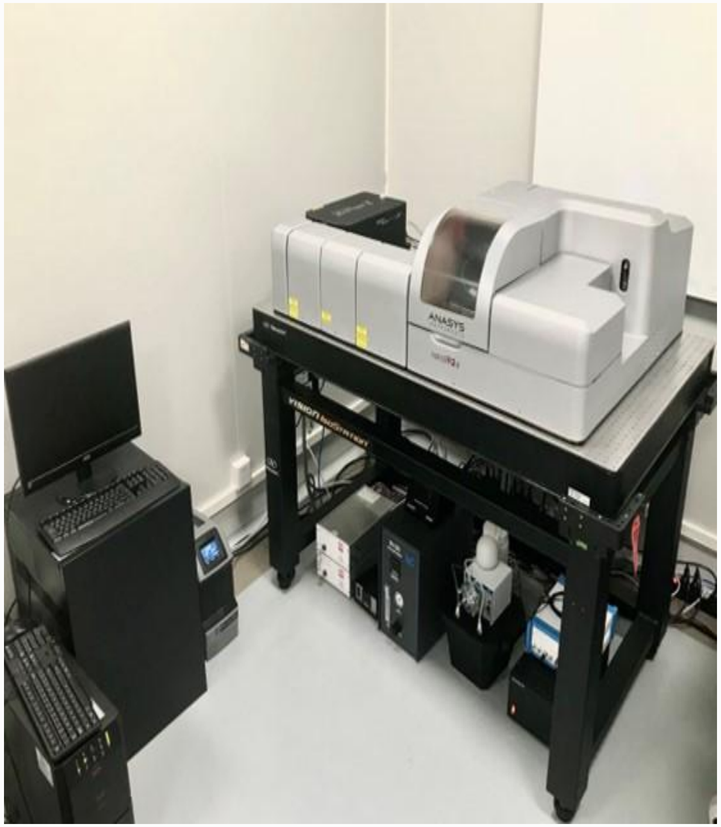
物理学公共科研大平台建设

3. 极低温矢量磁场扫描隧道电子显微镜

4. 高温高压光学浮区炉

物理学公共科研大平台建设

5.纳米探针红外光诱导共振分析 NanoIR

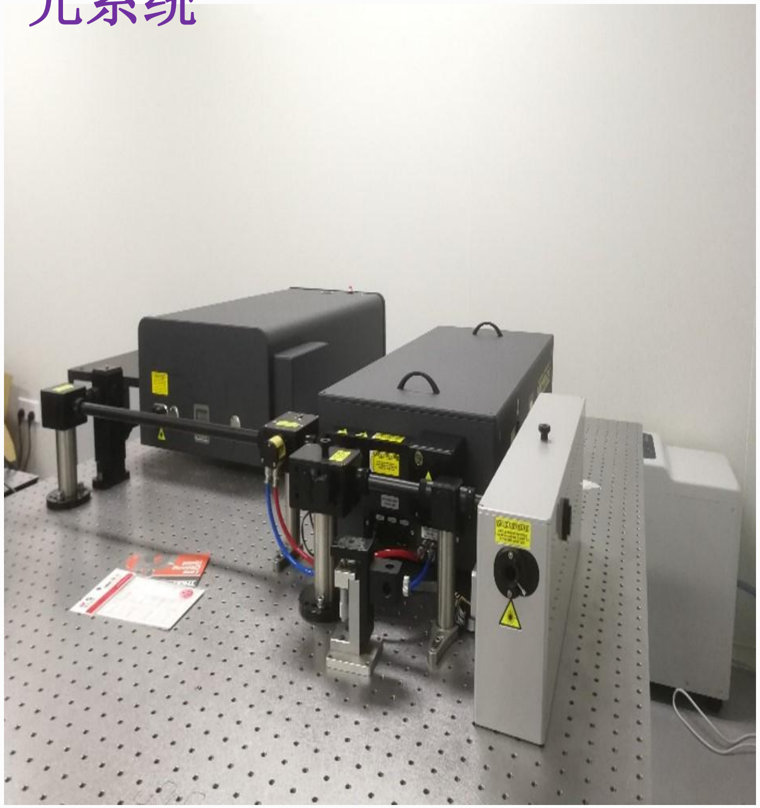


6.集成光镊系统

物理学公共科研大平台建设

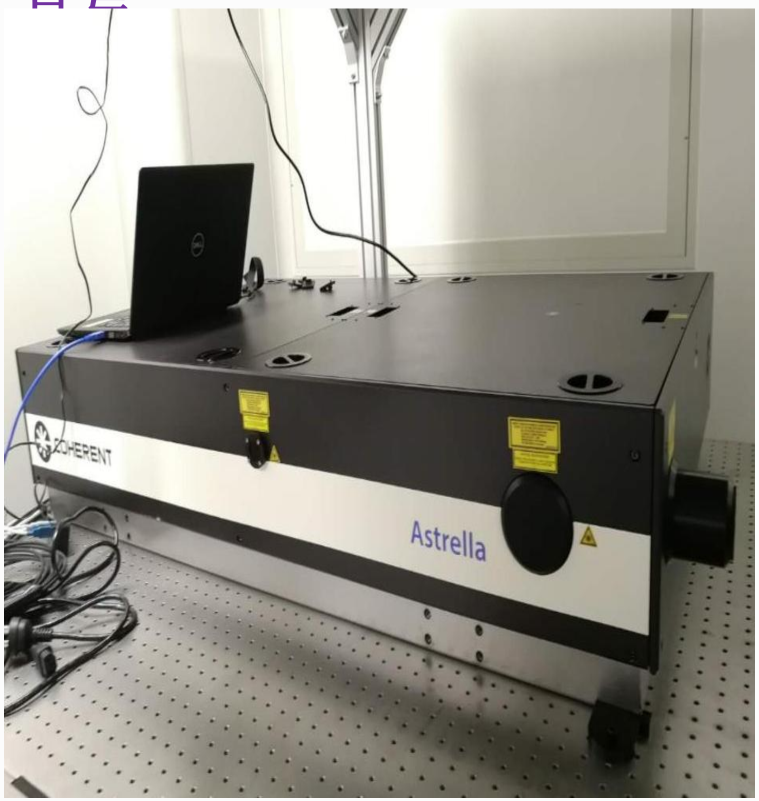
7.人工智能平台

8.高重复率宽调谐掺镱飞秒激光系统

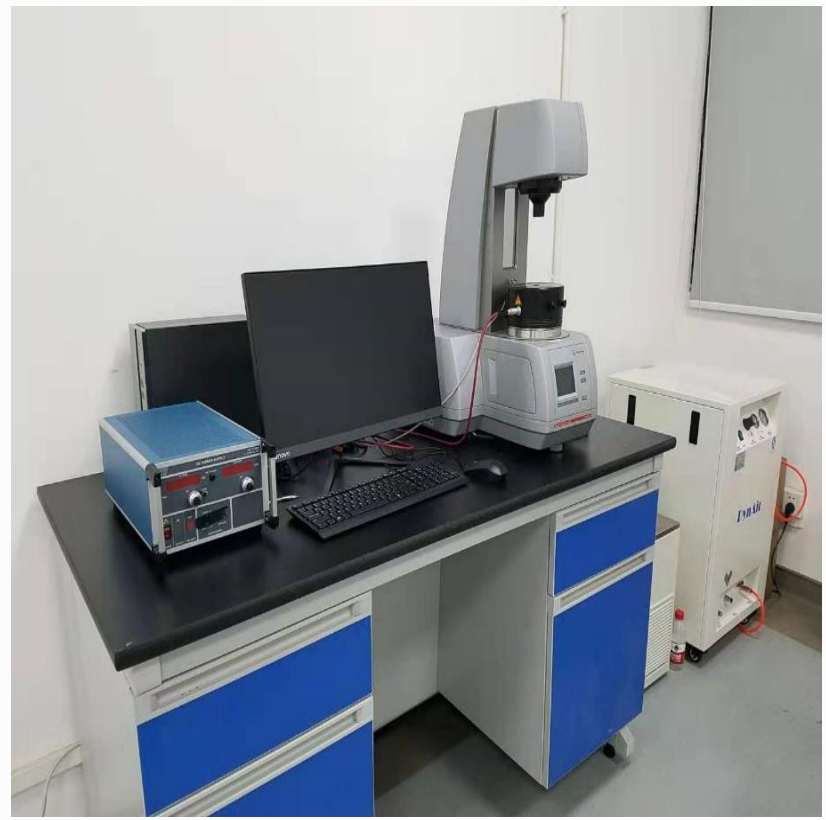


物理学公共科研大平台建设

9.一体式钛宝石飞秒激光放大器及一体式光学参量放大器电子束直写



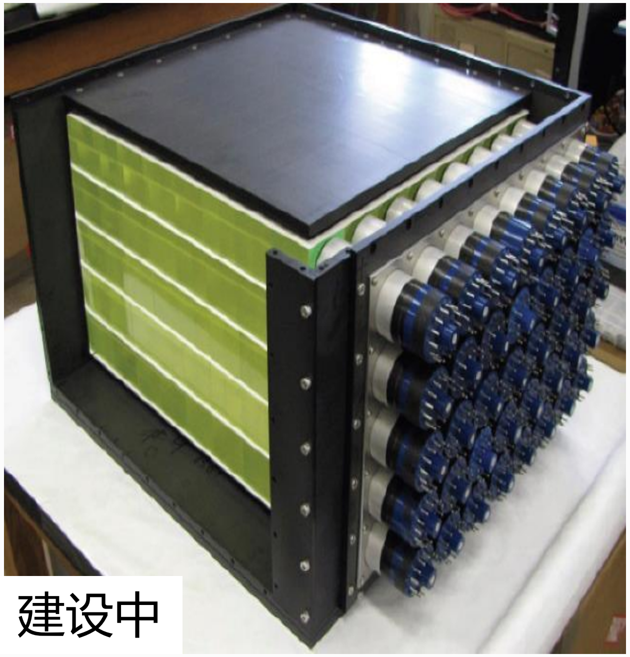
10.流变仪



物理学公共科研大平台建设

11.电子束直写

12.小型中微子探测平台



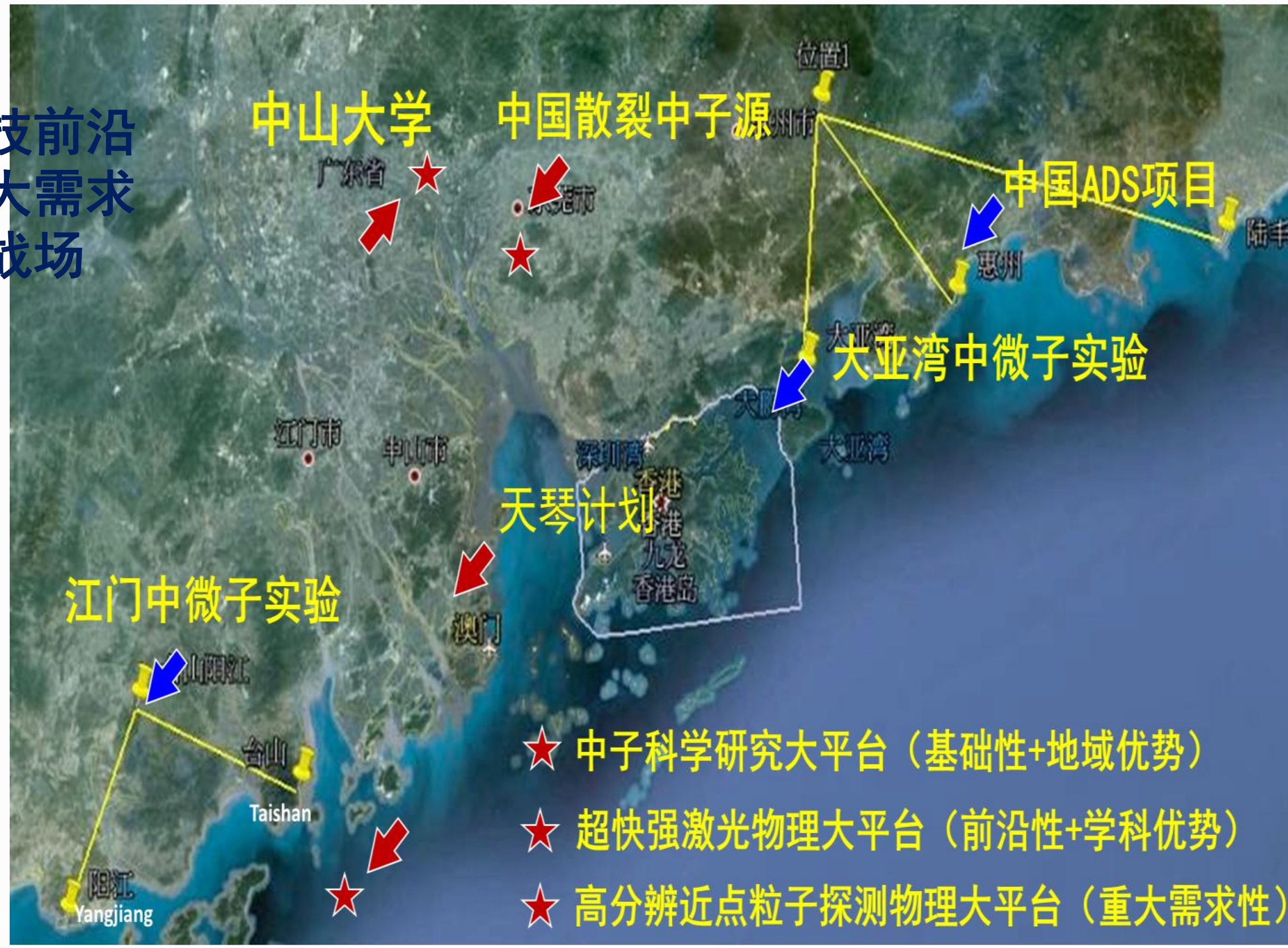
建设中

“三大”建设

三个面向：
- 面向世界科技前沿
- 面向国家重大需求
- 面向经济主战场

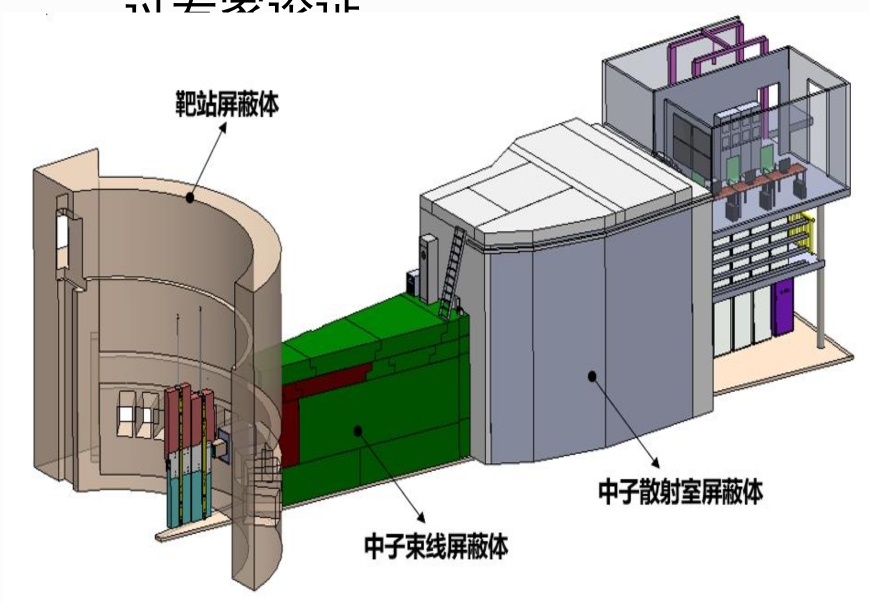
“三大”建设：
- 大团队
- 大平台
- 大项目

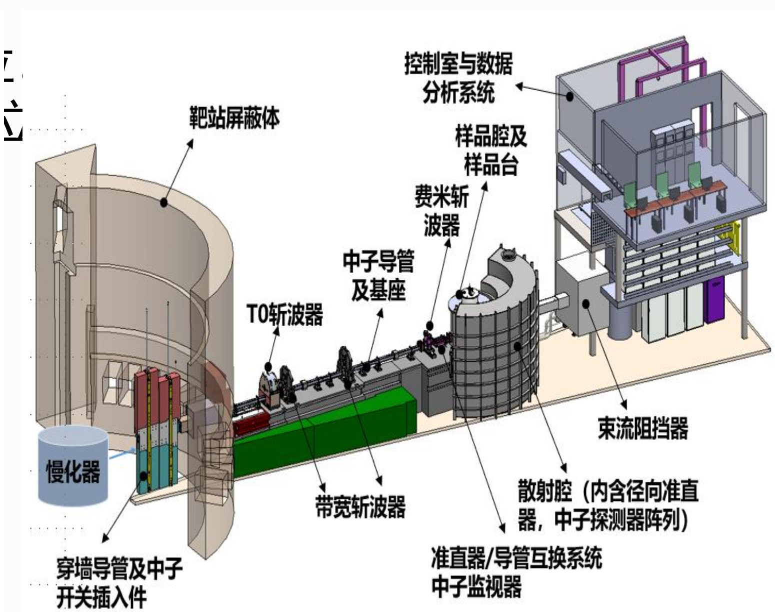
大平台建设：
- 中子科学
- 超快强激光
- 粒子物理
- ……



中子科学与技术中心建设

- 2017年12月13日，中山大学罗俊校长与高能所王贻芳所长签署合作协议，正式确定建设“高能直接几何非弹性中子散射谱仪”，将成为中国首个高能非弹性中子散射谱仪。
- 2018年1月20-23日，在中山大学召开谱仪建设讨论会（澳大利亚谱仪建设专家于德洪来访）。
- 2019年9月3日，高能直接几何非弹性中子散射飞行时间谱仪项目通过专家论证



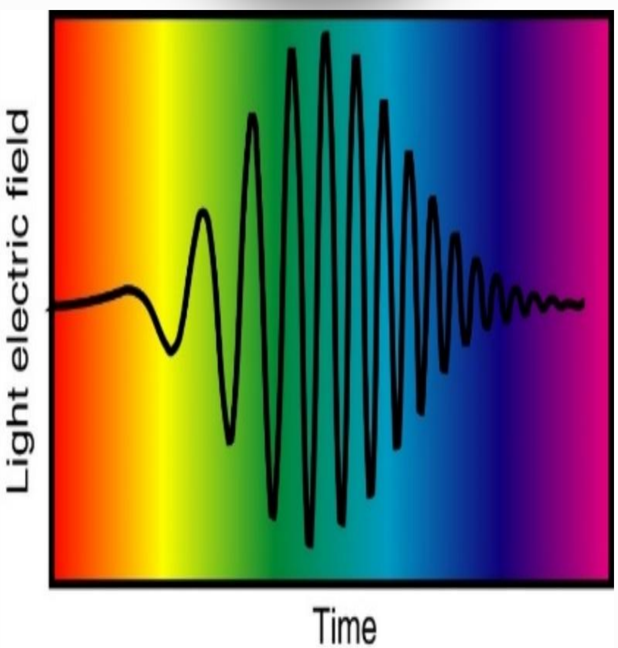


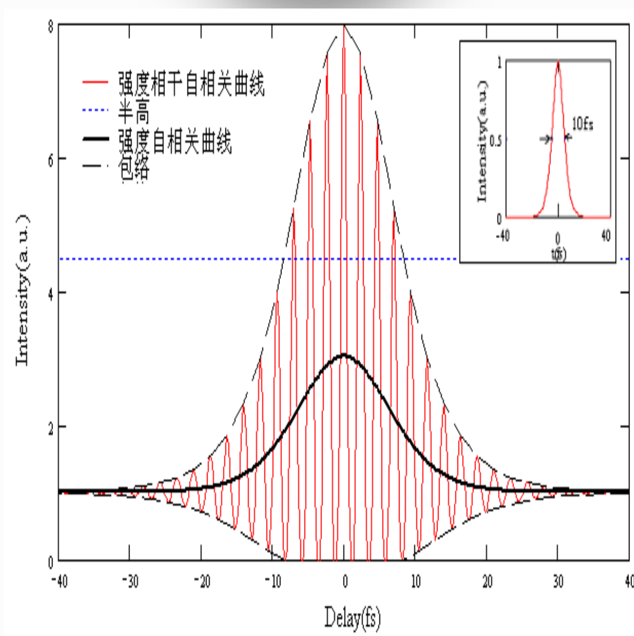
“超快强激光物理大平台”建设

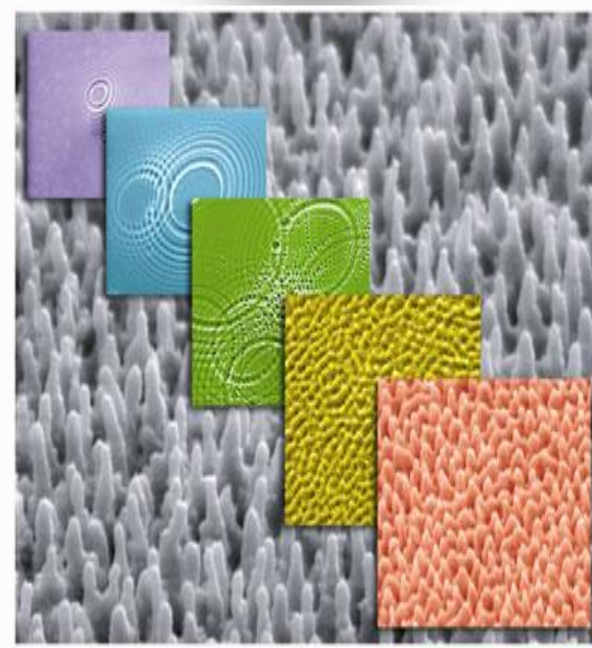
超快激光科学

超快激光技术

超快激光应用



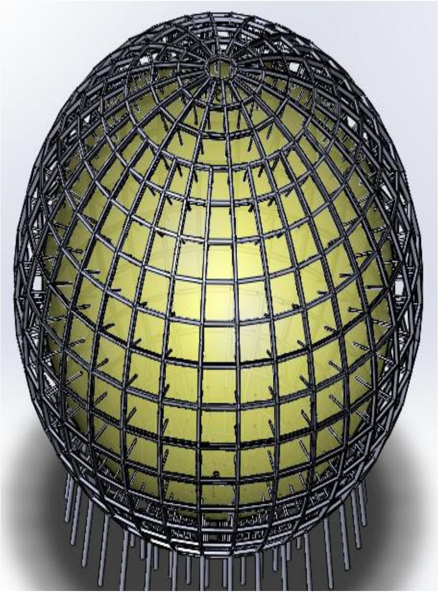




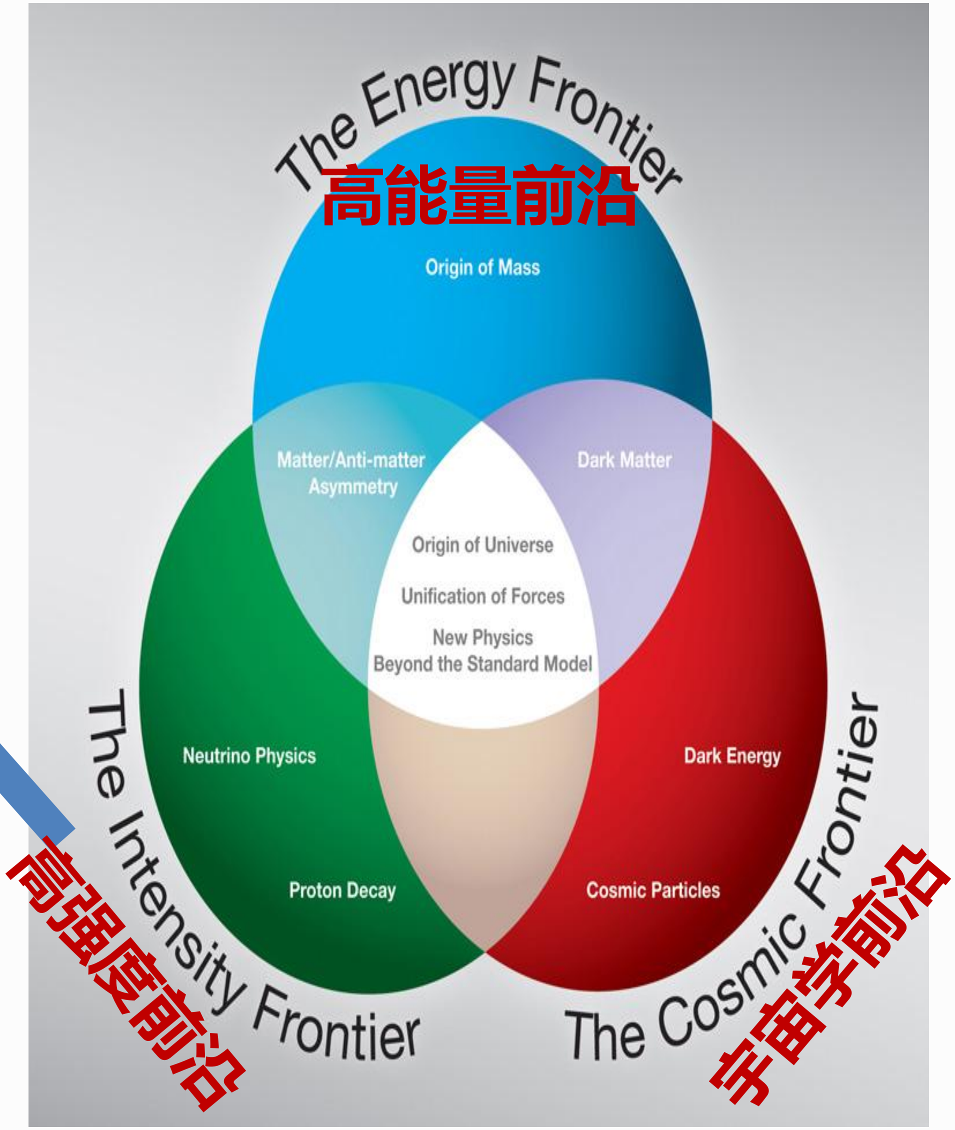
“高分辨近点粒子探测物理大平台”建设

江门近点探测器

- 依托国家大科学项目：前沿
- 新型中微子探测技术：



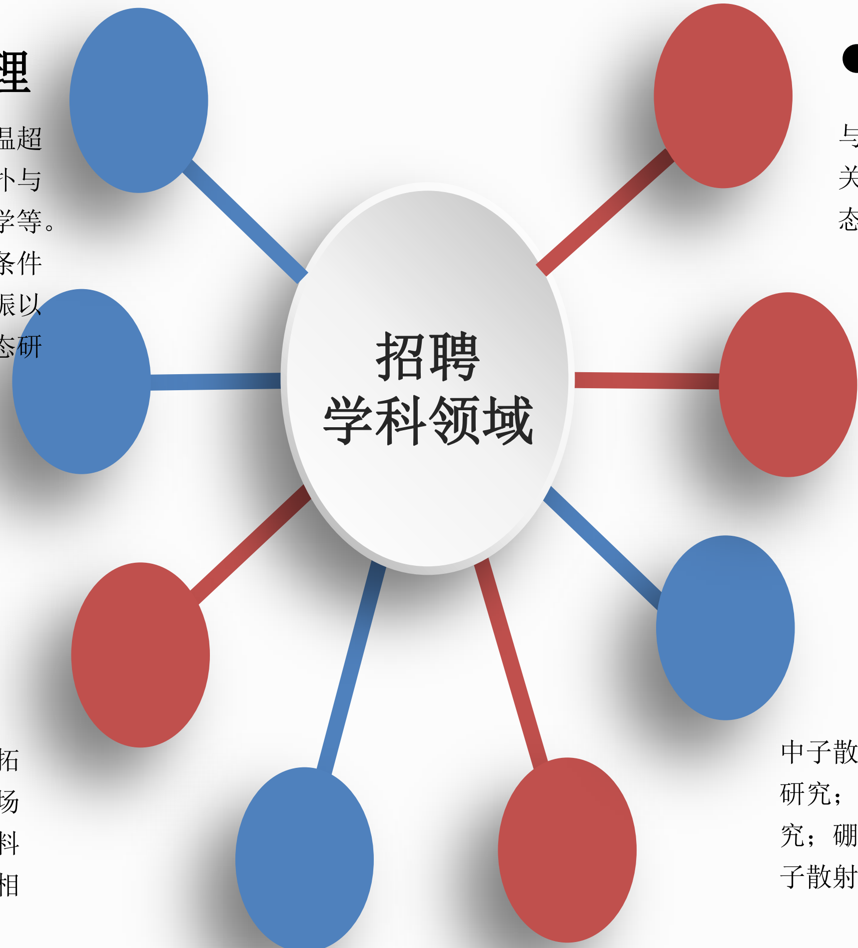
江门中微子实验是高强度前沿的典型代表、
回答最基础的中微子质量问题



03

第三部分
人才政策

PART
02



凝聚态物理

包括（但不限于）中子散射物理、高温超导、低维与表面物理、多铁物理、拓扑与量子材料、关联电子体系、自旋电子学等。优先考虑有角分辨光电子能谱、极端条件下输运测量、中子散射、缪子自旋共振以及基于先进光源大科学装置开展凝聚态研究经验的申请

理论物理

量子多体物理（含量子多体纠缠、关联电子体系、冷原子物理、量子磁性等），统计物理与计算物理，粒子物理与场论，量子信息与量子力学基础，数学物理及其他新兴学科方向等。

光学

重点招聘（但不限于）量子光学、拓扑光学、成像光学、超快光学、强场物理与阿秒激光、薄膜光学、超材料光学、与材料、信息、生物、能源相关学科交叉方向。

粒子物理与原子核物理

高能物理理论、唯象和实验（包括探测器硬件、电子学、计算软件和数据分析等）的各类人才。

软物质与生物物理

与生物大分子、细胞、神经与脑科学等相关的生物物理研究方向人才以及与软凝聚态理论与实验相关研究方向人才。

能源物理

新型太阳能利用及光电转换材料物理、太阳电池的陷光结构与钝化机制及其数值模拟、太阳电池新理论和新结构设计、高效晶体硅太阳电池、新型薄膜太阳电池材料与器件物理、光谱转换材料与理论、纳米技术与太阳电池相关交叉学科等。

中子科学与技术

中子散射基础科学研究；中子散射工程应用研究；中子散射实验方法和数据处理方法研究；硼中子俘获疗法（BNCT）相关研究；中子散射谱仪建设。

超快强激光技术

强场激光物理，飞秒激光加工，THz产生及应用，深紫外阿秒激光技术及超快动力学研究。

招聘岗位

“百人计划”教授、副教授

取得国内外知名大学博士学位；具有重要国际学术影响的领军人才或具有较高学术造诣的中青年杰出人才或具有较好发展潜力的青年学术骨干。

专职科研系列人员

海内外知名高校或研究机构的优秀博士后或博士，或具有海内外知名高校或研究机构工作经历的人员。年龄不超过38周岁。

博士后研究人员

年龄不超过35周岁。获得博士学位不超过3年，具备较高的学术水平和较强的科研能力的海内外优秀博士。

发展支持

“百人计划”科研基本启动费：　　办公业务费

“百人计划”科研启动费：

按需申请，[illegible]

经费支持

空间支持

薪酬待遇

绩效奖励

相应岗位及薪酬、安家补贴

住房补贴及周转房

子女入园入学支持

独立办公空间

公共实验平台

设科研奖励和人才引进奖励，根据年度工作表现给予一次性年终绩效奖励

新建教师公寓

人才项目

各类人才可依托学校申请国家和广东省各类人才项目

1. 国家“长江学者”项目
2. 国家“海外优青”项目
3. 教育部“青年长江学者”项目
4. 国家特支计划青年拔尖人才
5. **博士后创新人才支持计划**
   国家提供63万元/人/两年。
6. **博士后“香江学者计划”、“澳门青年学者计划”**
   国家提供（36万元人民币和36万元港币/澳门元）/人/两年。
7. **博士后“中德博士后交流项目”**
   国家提供（ 30万元人民币和1500欧/月） /人 / 两年。

人才项目

各类人才可依托学校申请国家和广东省各类人才项目

8 **博士后国际交流计划**
入选派出项目，国家提供资助经费30万元、国（境）外接收单位提供资助经费30万元（或接近）；入选学术交流项目，国家提供资助经费3万元/次。

9 **中国博士后科学基金项目**
特别资助项目15万元；面上一等资助8万元，面上二等资助5万元。

10 **广东省青年优秀科研人才国际培养计划**
广东省资助40万元。

11 **广东特支计划青年拔尖人才**
广东省提供一次性补助10万元。

12 **广东省珠江学者青年项目**
广东省提供津贴8万元/年，共提供3年。

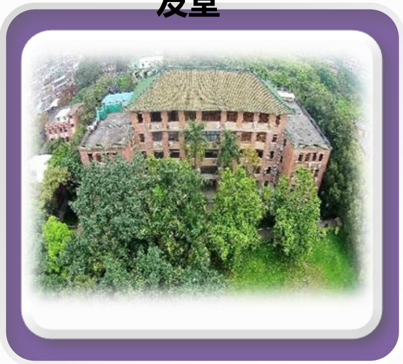
13 **广东省珠江人才计划**
（1）领军人才：每名资助600万元，包括500万元专项工作经费和100万元（税后）住房补贴。
（2）青年拔尖人才：分A、B两类给予资助，按实际年薪酬收入的1倍给予生活补贴，A类每年最高不超过50万元，B类每年最高不超过30万元。对从事应用研究和技术开发的可连续资助5年，对从事基础研究的可连续资助10年。
（3）海外青年人才引进计划（博士后资助项目）：广东省提供生活补贴30万元/年，共两年。出站后承诺留粤工作3年以上的，广东省提供住房补贴40万元。

物理学院大家庭

行政办公：十友堂

物理学公共科研平台：哲生堂

大学物理实验教学中心：陆祐堂



学院实验室：物理楼

教师办公室:爪哇堂

教师办公室：高等学术中心（冼为坚堂）

物理学院欢迎您

中山大学国际青年学者论坛——物理学院分论坛

- 珠海论坛：每年上半年6月中旬
- 深圳论坛：每年下半年12月中旬

联系我们

中山大学物理学院诚聘英才！

申请人可通过电子邮件或电话联系我院负责人，并提供个人学术简历、发表论文清单、研究方向、研究计划以及3封或以上同行专家推荐信。
专职科研人员和博士后也可以直接与我院现有相关科研团队联系。

联系地址： 广州市新港西路135号中山大学物理学院（十友堂）,510275
联系人： 沈杏艳　shxingy@mail.sysu.edu.cn　020-84113293
郭丽丽　guolli5@mail.sysu.edu.cn　020-84113305
郭东辉　guodonghui@mail.sysu.edu.cn
学院主页： http://spe.sysu.edu.cn

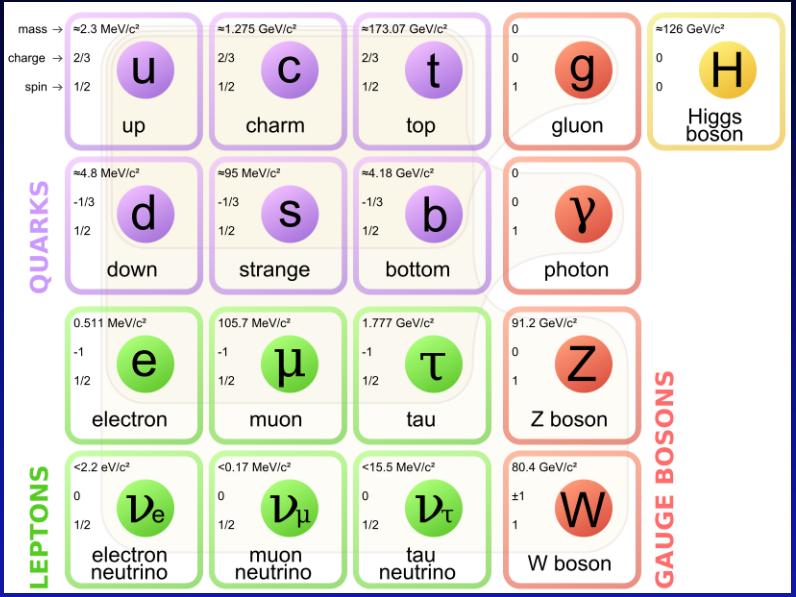
中山大学物理学院

理论粒子物理研究团队介绍

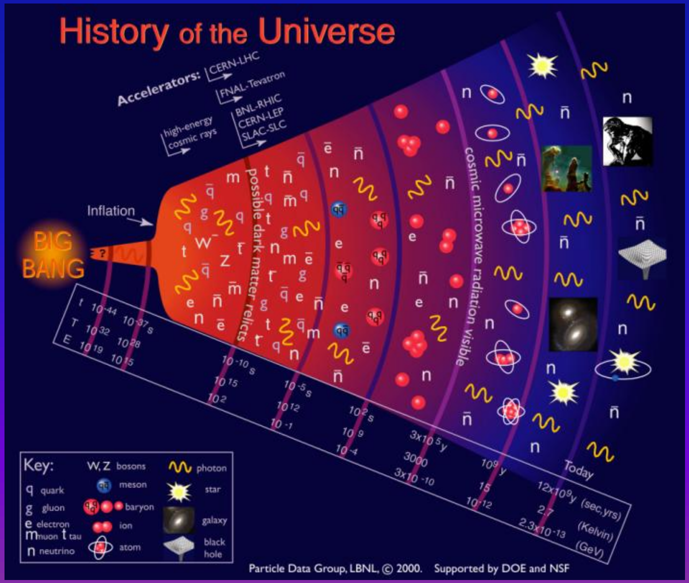
团队负责人：张宏浩

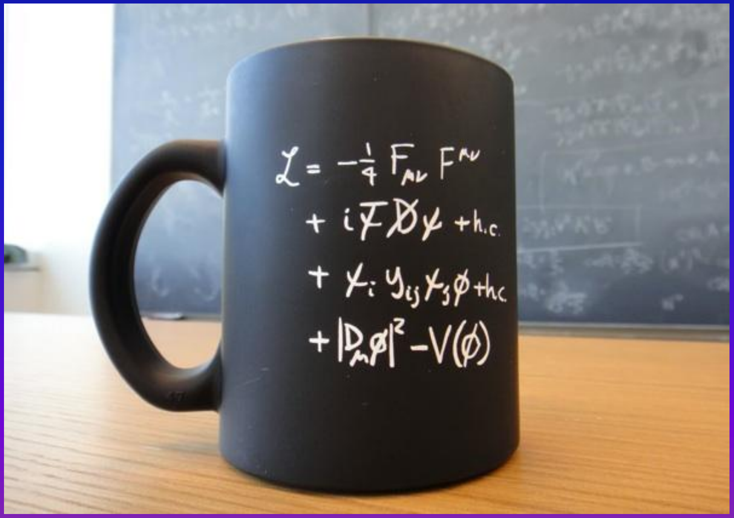
团队骨干：
潘逸文，余钊焕，汤亦蕾，
蔡成丰，江学敏，龙光波，陈丰之，赵亚，Seishi Enomoto，……

The standard model of particle physics



	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$	3	2	$\frac{1}{6}$
u_R	3	1	$\frac{2}{3}$
d_R	3	1	$-\frac{1}{3}$
$\ell_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$	1	2	$-\frac{1}{2}$
e_R	1	1	-1





Some **problems** of the standard model

- triviality

$$\lambda(Q) = \frac{\lambda(v)}{1 - \frac{3}{4\pi^3}\log\frac{Q^2}{v^2}\lambda(v)}$$

- hierarchy problem

$$m_h^2 \approx m_{tree}^2 - \frac{3}{8\pi^2}\lambda_t^2\Lambda^2 + \frac{1}{16\pi^2}g^2\Lambda^2 + \frac{1}{16\pi^2}\lambda^2\Lambda^2$$
$$\sim m_{tree}^2 - (200 - 20 - 10)(125GeV)^2\left(\frac{\Lambda}{10TeV}\right)^2$$

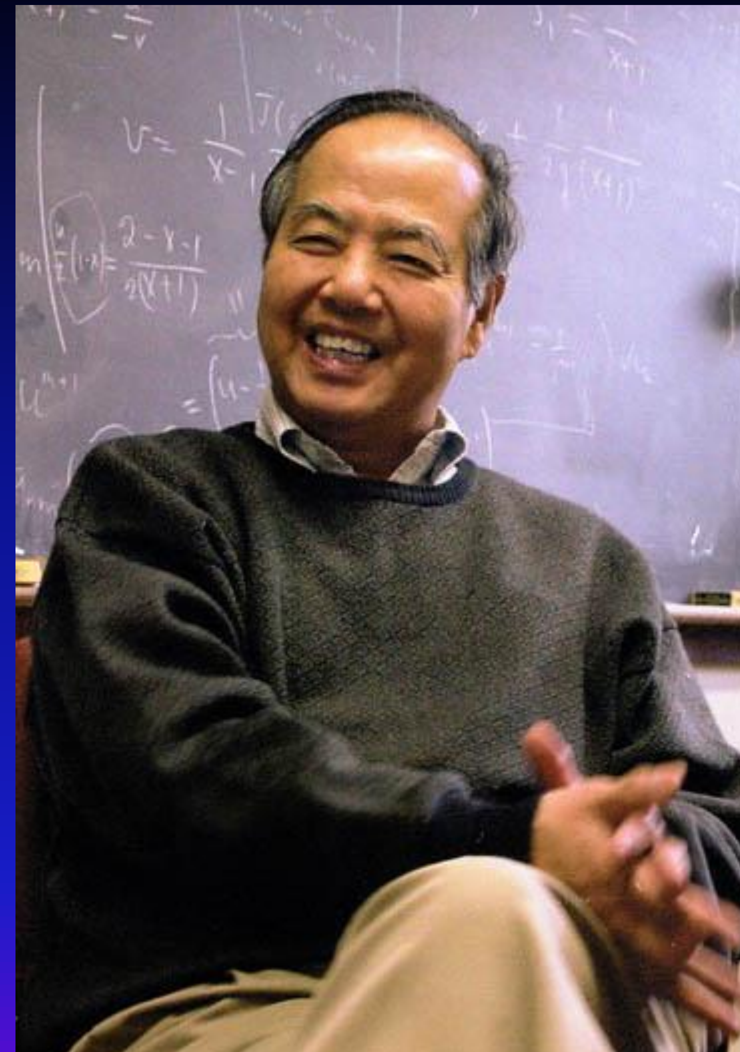
- too many parameters
- existence of **dark matter**
- smallness of neutrino mass
- matter/antimatter asymmetry
- vacuum stability

……

The SM may NOT be a final theory.

暗物质是笼罩20世纪末和21世纪初现代物理学的最大乌云，它将预示着物理学的又一次革命。

——李政道



关于李政道先生引言的参考文献：
秦波，精确宇宙学时代的暗物质问题，《现代物理知识》2009(5):17-24

所谓**暗物质**、暗能量就是非常稀奇的事物，这里面我想是可能引出基本物理学中革命性的发展来的……

假如一个年轻人，他觉得自己一生的目的就是要做革命性的发展的话，他应该去学习天文物理学。

——杨振宁

关于杨振宁先生引言的参考文献：
秦波，精确宇宙学时代的暗物质问题，《现代物理知识》2009(5):17-24

团队负责人：张宏浩 教授

- 1998.9-2007.7　清华大学本科，直博
- 2007.7-2010.12　中山大学讲师
- 2011.1-2016.12　中山大学副教授
- 2017.1-现在　中山大学教授

■ 研究方向：粒子物理理论与唯象学，量子场论

■ 研究兴趣：暗物质，复合希格斯，超出标准模型的新物理，宇宙起源与演化，量子场论的新方法与新技术

■ 代表性研究成果：

- 是国际上最早研究暗物质有效模型的学者之一，成果被国际同行大量引用
- 发现了一种产生暗物质和复合希格斯的新机制，成果在顶尖期刊PRL发表

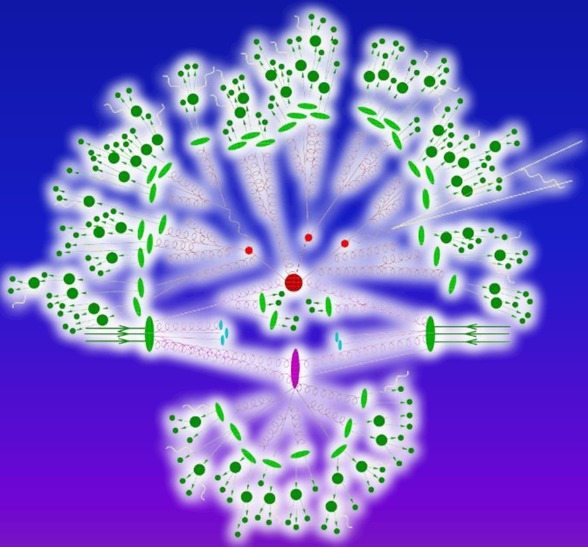
是物理学一级学科博士生导师，每年拟招2名硕士新生

欢迎对我们团队的研究方向感兴趣的优秀学生联系：zhh98@mail.sysu.edu.cn

团队骨干：余钊焕 副教授

- 中山大学本科，2006-2010
- 中国科学院高能物理研究所博士， 2010-2015
- 澳大利亚墨尔本大学博士后，2015-2017
- 中山大学物理学院副教授， 2018-现在

■ 研究兴趣：粒子物理唯象学，涉及暗物质、对撞机、超出标准模型的新物理



■ 研究简介：
提出性质良好的新物理模型，探索新物理理论在实验上可能存在的信号，讨论来自当前实验的限制和未来实验的灵敏度。已在 PRL、JHEP、PRD、NPB、CPC 上发表学术论文 20 多篇，总引用次数超过 900 次。

■ 招生计划：每年 1 至 2 名硕士生

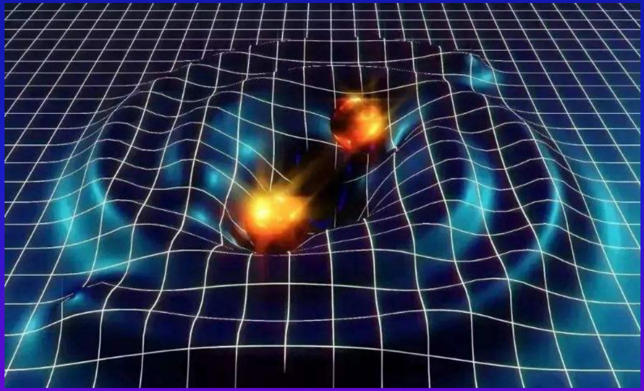
团队骨干：汤亦蕾 副教授

- 中国科学院理论物理研究所 博士 2010-2015
- 北京大学 博士后 2015-2017
- 韩国KRF项目资助 博士后 2017-2019
- 中山大学物理学院 副教授 2019-现在

- 研究兴趣：粒子物理唯象学，涉及暗物质、对撞机、右手中微子、超出标准模型的新物理

- 已在PRD、JHEP等期刊发表文章18篇

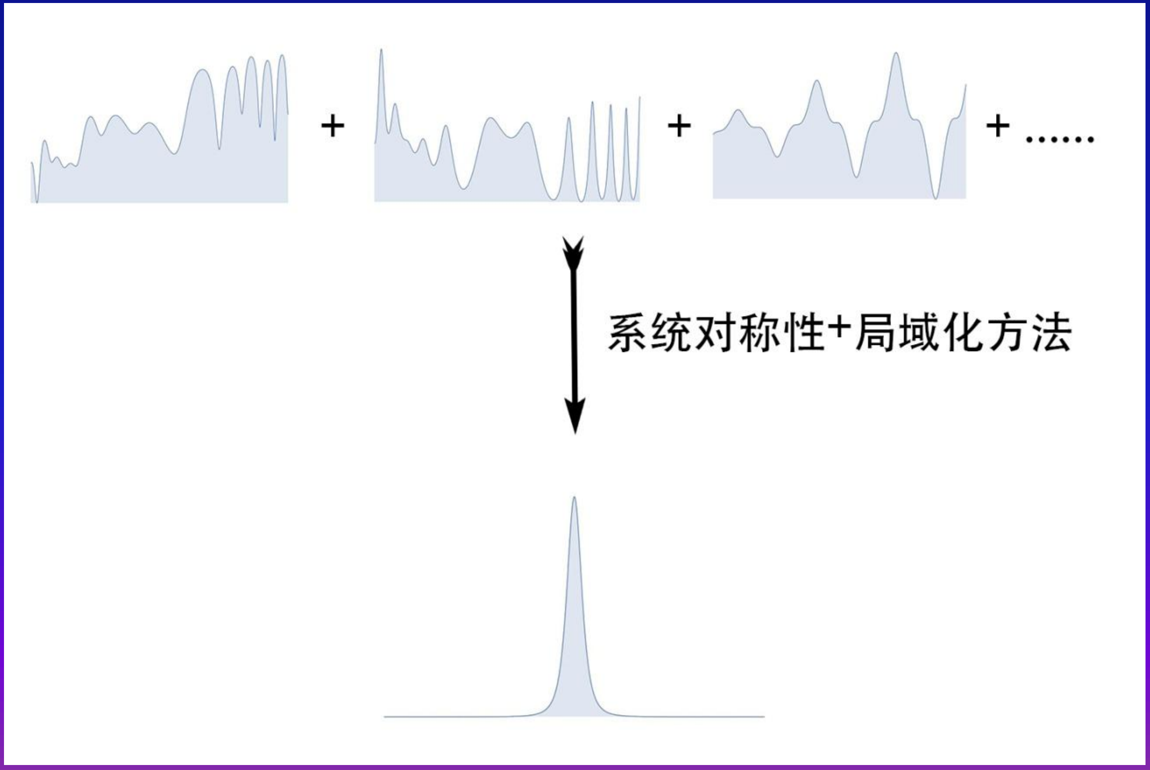
- 招生计划：每年 1 至 2 名硕士生



超对称局域化

超对称场论：**大量可以精确求解的模型**

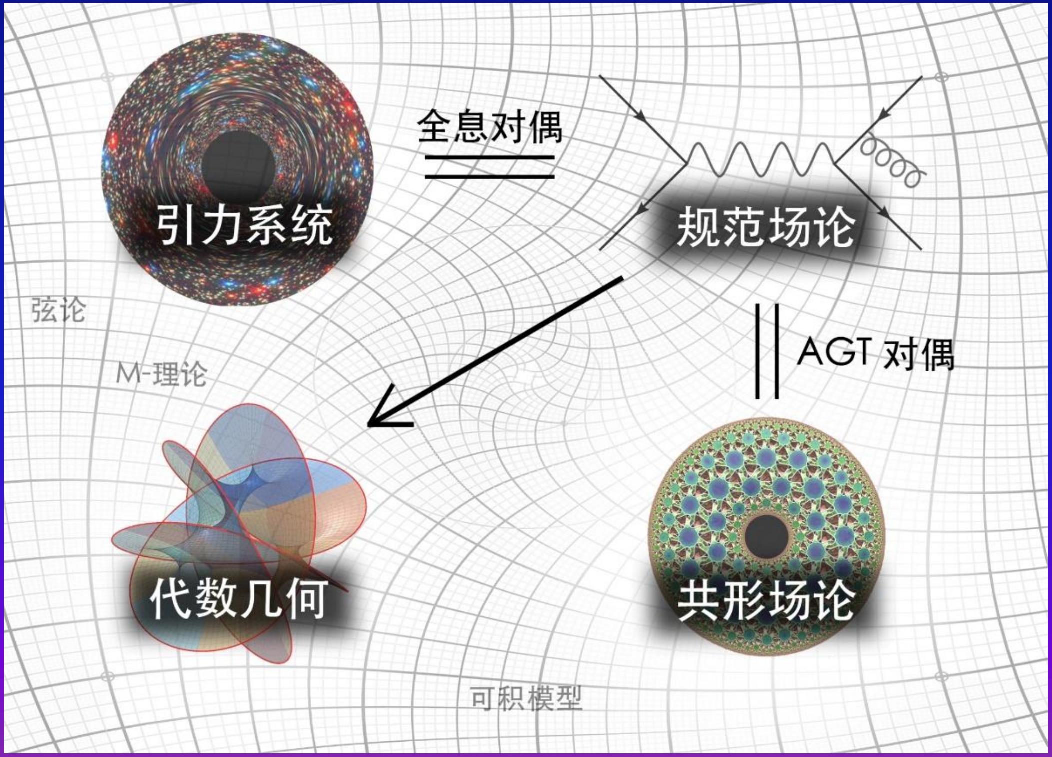
场论中极端复杂的**无穷维、无穷阶微绕、非微扰问题**，可以利用对称性把问题**局域化**为简单的关于孤立子、瞬子的**有限维**问题，并**严格求解**。



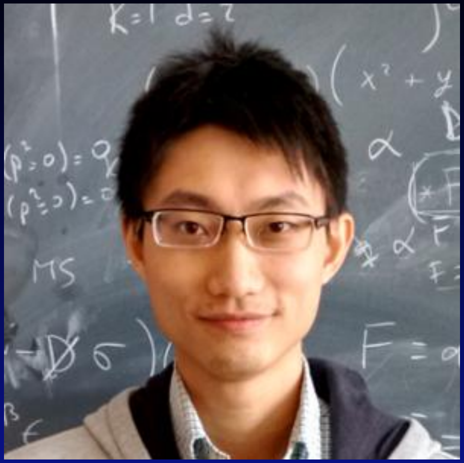
对偶

数之不尽的物理系统，连成一张名为“对偶”的巨网。

相互对偶的物理系统，描述性质迥异的物质，运行在不同的时空结构上，但它们的物理量却有完全**严格的一一对应**，暗示着实际上它们是**同一个物理规律**的不同表象。

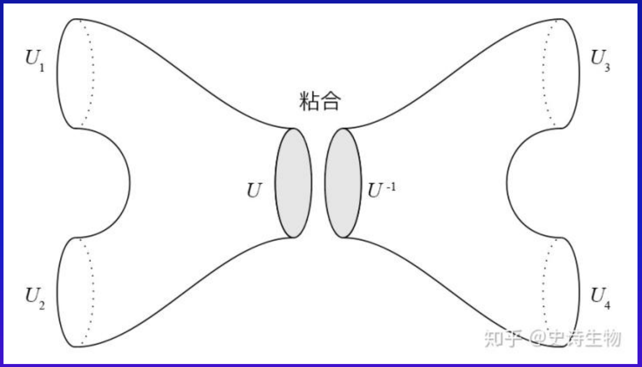


团队骨干：潘逸文 副教授



- 中山大学本科，2006-2010
- 纽约州立大学-石溪分校博士， 2010-2015
- 瑞典乌普萨拉大学博士后，2015-2017
- 中山大学物理学院副教授， 2018-现在

- 研究兴趣：粒子物理理论，涉及超对称局域化、4d/5d AGT 对偶、4d N=2 超对称共形理论与手征代数对偶，2维共形场论



- 招生计划：每年 1 至 2 名硕士生

团队骨干：

- 蔡成丰 博士后：复合希格斯模型
- 江学敏 博士后：暗物质
- 龙光波 博士后：轴子与类轴子的天文学探测
- 陈丰之 博士后：粒子物理唯象学
- 赵　亚 博士后：中微子理论
- Seishi Enomoto 研究员：宇宙起源与演化
- …… …… ……

- 已毕业博士：蔡成丰，冯劼
- 已毕业硕士：骆柱，劳珏斌
- 在读研究生：博士生3人，硕士生每年级2人或以上

欢迎你加入！

欢迎联系：zhh98@mail.sysu.edu.cn