2017年9月7日　中国科学技术大学

Investigation on the holographic superconducting system and its negative refraction

Qiyuan Pan

Hunan Normal University

(with *J. Jing, B. Wang, H. Yu* and *S. Chen*)

中科大交叉学科理论研究中心

Outline:

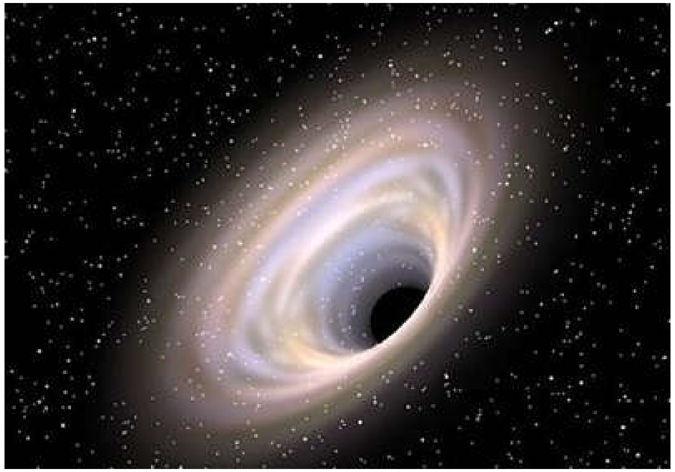
1. **Introduction**

2. **Holographic superconductors**

3. **Negative refraction in the holographic superconductors**

4. **Conclusions and discussions**

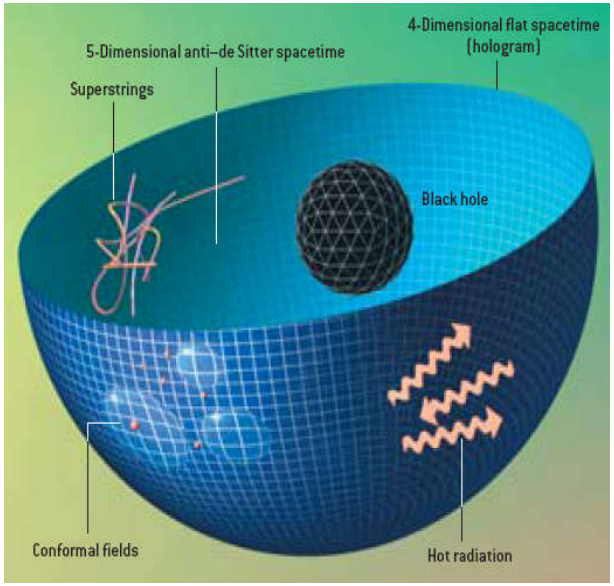
1、 Introduction



Black hole

High Tc superconductor

a gravity theory in a *n*-dimensional anti-de Sitter (AdS) spacetime	⟷ **AdS/CFT correspondence** ⟷	a conformal field theory on the (*n-1*)-dimensional boundary of AdS



AdS/CFT dictionary

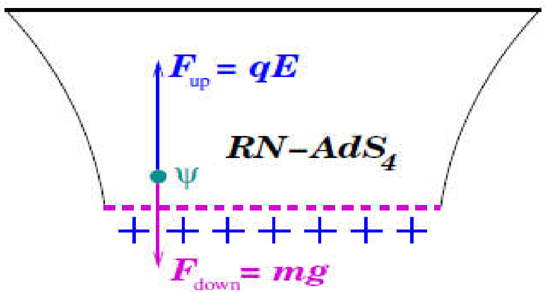
Gravity	Superconductor
Black hole	Temperature
Charged scalar field	Condensate

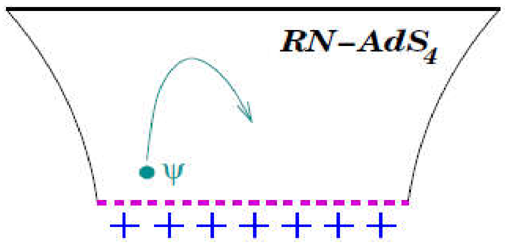
the emergence of the scalar hair in the bulk AdS black hole ⟷ the formation of a charged condensation in the boundary dual CFTs

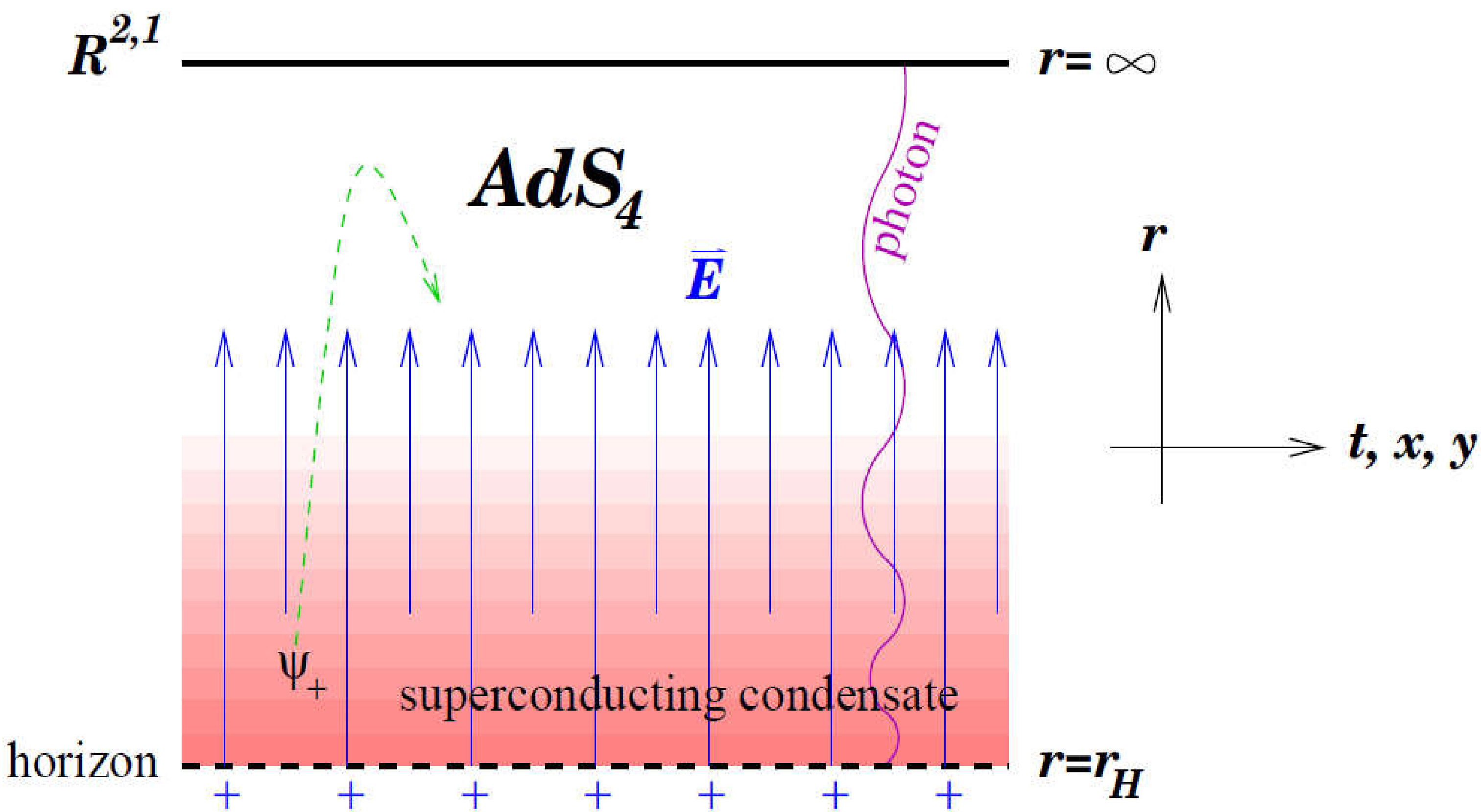
Need to find a black hole which has scalar hair at low temperatures, but no hair at high temperatures! ------How to find…?

Gubser (08) consider the action with a Maxwell field and a charged complex scalar field : S.S. Gubser, *PRD* 78, 065034 (2008)

$$\mathcal{L} = \frac{1}{2\kappa^2}\left[R - \frac{1}{4}F_{\mu\nu}^2 - |(\partial_\mu - iqA_\mu)\psi|^2 + \frac{6}{L^2} - m^2|\psi|^2\right]$$







For lower temperatures T<Tc (critical temperature), the gravitational dual is a black hole with a nonvanishing scalar hair!

2、 Holographic superconductors

2.1 Holographic superconductor in the probe limit (s-wave)

Hartnoll et al. *PRL* 101, 031601 (2008)

A Maxwell field and a charged complex scalar field with Lagrangian density：

$$\mathcal{L} = -\frac{1}{4}F^{ab}F_{ab} - V(|\Psi|) - |\partial\Psi - iA\Psi|^2$$

The potential: $V(|\Psi|) = -\frac{2|\Psi|^2}{L^2}$

The planar Schwarzschild–anti-de Sitter black hole：

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2)$$

The metric function: $f = \frac{r^2}{l^2} - \frac{M}{r}$

The equation of motion:

$$\Psi'' + \left(\frac{f'}{f} + \frac{2}{r}\right)\Psi' + \frac{\Phi^2}{f^2}\Psi + \frac{2}{L^2 f}\Psi = 0$$

$$\Phi'' + \frac{2}{r}\Phi' - \frac{2\Psi^2}{f}\Phi = 0$$

Properties of the dual field theory can be read off from the asymptotic behavior of the solution

Integrating out to infinity, these solutions behave as

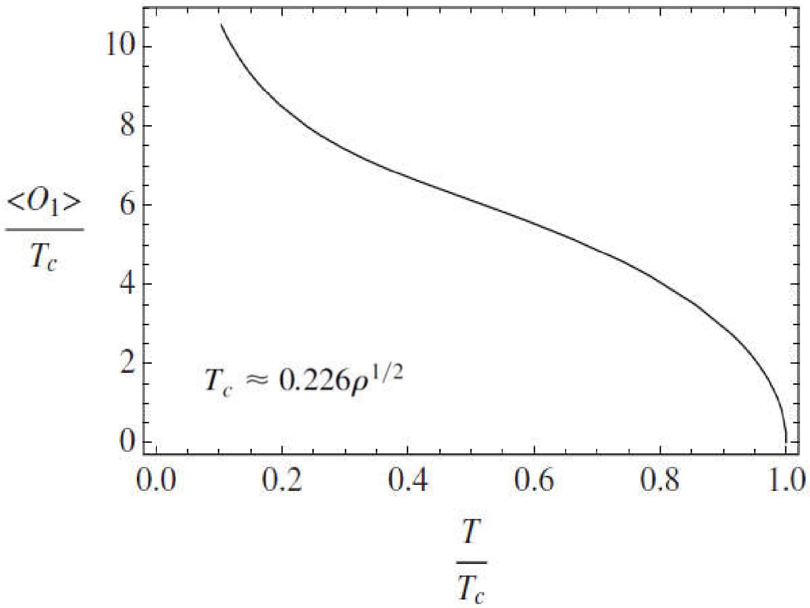
$$\Psi = \frac{\Psi^{(1)}}{r} + \frac{\Psi^{(2)}}{r^2} + \cdots$$

$$\Phi = \mu - \frac{\rho}{r} + \cdots$$

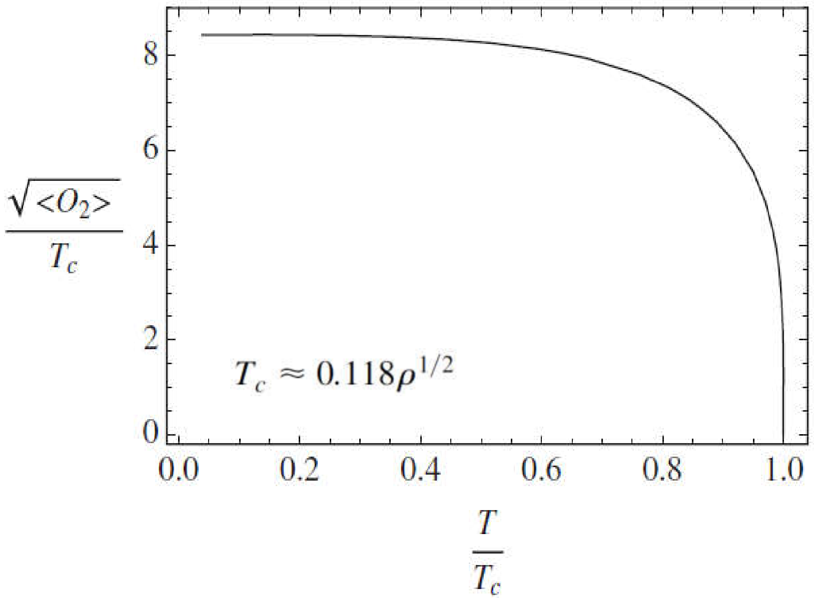
μ: chemical potential ; ρ: charge density

The condensate of the scalar operator $\mathcal{O}$ in the field theory dual to the field Ψ is given by

$$\langle \mathcal{O}_i \rangle = \sqrt{2}\Psi^{(i)}, \quad i = 1, 2$$



$$\langle \mathcal{O}_1 \rangle \approx 9.3\, T_c\, (1 - T/T_c)^{1/2}, \qquad T \to T_c$$

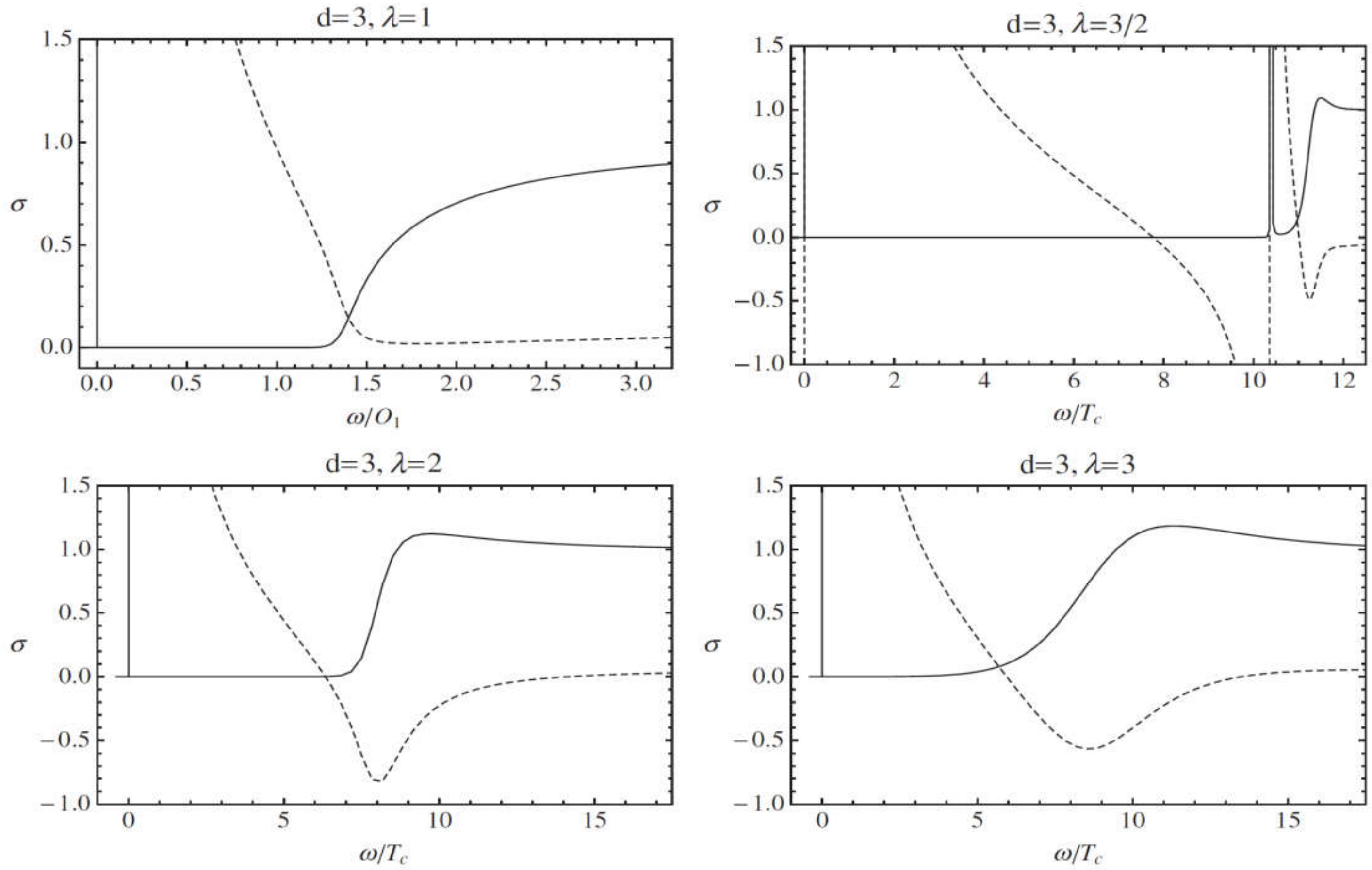


$$\langle \mathcal{O}_2 \rangle \approx 144\, T_c^2\, (1 - T/T_c)^{1/2}, \quad T \to T_c$$

It is expected that this condensate will lead to superconductivity

Conductivity of holographic superconductors with various condensates

Horowitz et al. *PRD* 78, 126008 (2008)



For the case of d=3: the solid line is the real part and dashed is imaginary

For all cases with $\lambda > \lambda_{\rm BF}$, we find

$$\boxed{\frac{\omega_g}{T_c} \approx 8}$$

ω_g: the gap frequency;

$\lambda_{\rm BF}$: corresponds to the case of lower bound called the Breitenlohner-Freedman (BF) bound for the scalar mass

A robust feature: Since the corresponding BCS value is 3.5, this shows that the energy to break apart the condensate is more than twice the weakly coupled value

2.2 Holographic superconductor away from the probe limit (s-wave)

Hartnoll et al. *JHEP* 12, 015 (2008)
Brihaye et al. *PRD* 81, 126008 (2010)

The Lagrangian density for a Maxwell field and a charged complex scalar field coupled to gravity:

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F^{ab}F_{ab} - V(|\psi|) - |\nabla\psi - iqA\psi|^2$$

The metric ansatz:

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2\left(dx^2 + dy^2\right)$$

together with

$$A = \phi(r)dt\,, \qquad \psi = \psi(r)$$

The equations of motion:

$$\psi'' + \left(\frac{g'}{g} - \frac{\chi'}{2} + \frac{2}{r}\right)\psi' + \frac{q^2\phi^2 e^\chi}{g^2}\psi - \frac{1}{2g}V'(\psi) = 0$$

$$\phi'' + \left(\frac{\chi'}{2} + \frac{2}{r}\right)\phi' - \frac{2q^2\psi^2}{g}\phi = 0$$

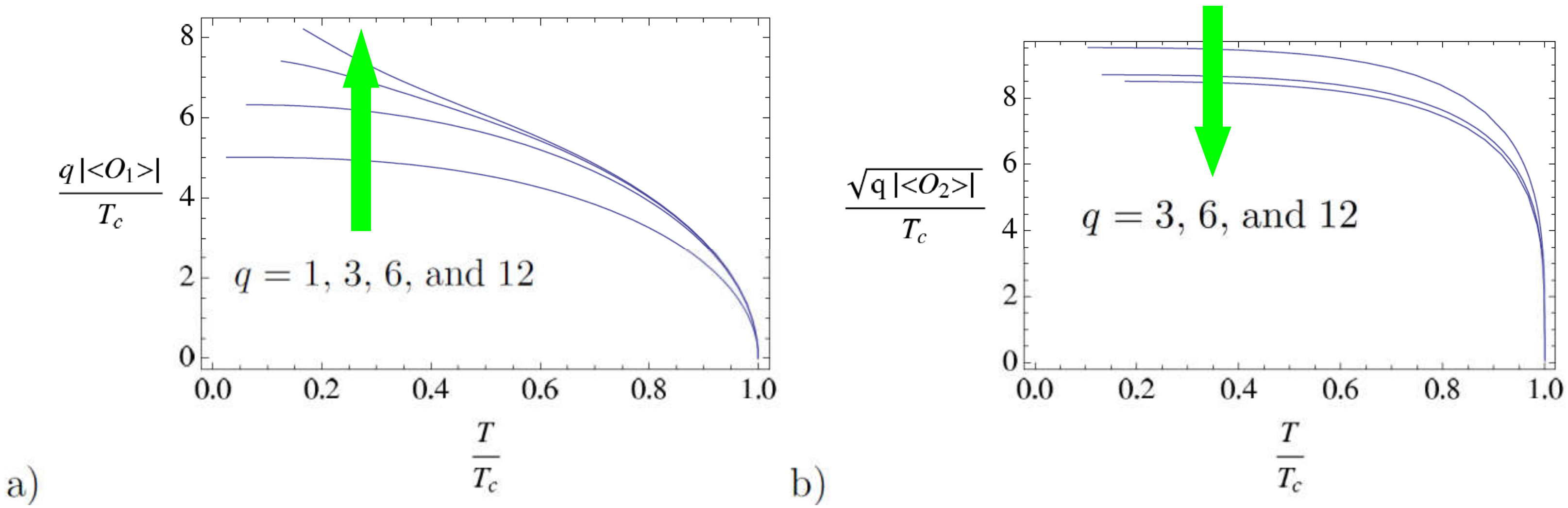
$$\chi' + r\psi'^2 + \frac{rq^2\phi^2\psi^2 e^\chi}{g^2} = 0$$

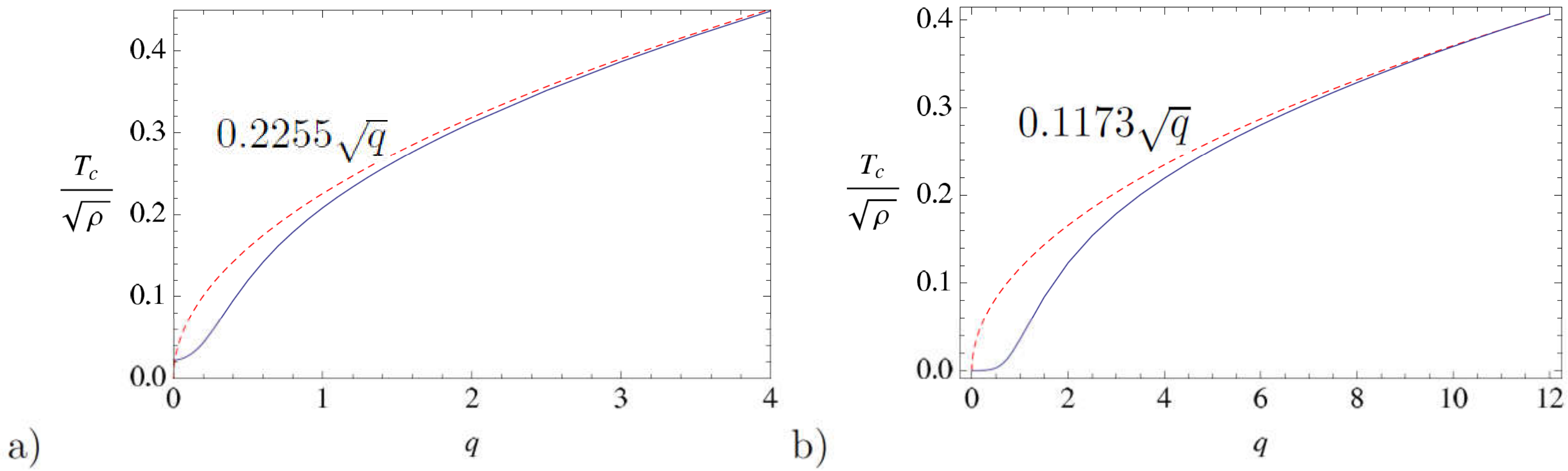
$$\frac{1}{2}\psi'^2 + \frac{\phi'^2 e^\chi}{4g} + \frac{g'}{gr} + \frac{1}{r^2} - \frac{3}{gL^2} + \frac{V(\psi)}{2g} + \frac{q^2\psi^2\phi^2 e^\chi}{2g^2} = 0$$

The simple potential: $V(\psi) = -\frac{2}{L^2}\psi^2$

The analytic solutions (AdS RN black hole) for $T > T_c$:

$$\chi = \psi = 0\,, \qquad g = r^2 - \frac{1}{r}\left(r_+^3 + \frac{\rho^2}{4r_+}\right) + \frac{\rho^2}{4r^2}\,, \qquad \phi = \rho\left(\frac{1}{r_+} - \frac{1}{r}\right)$$





The dashed red line is the probe limit (naively extrapolated to all q)

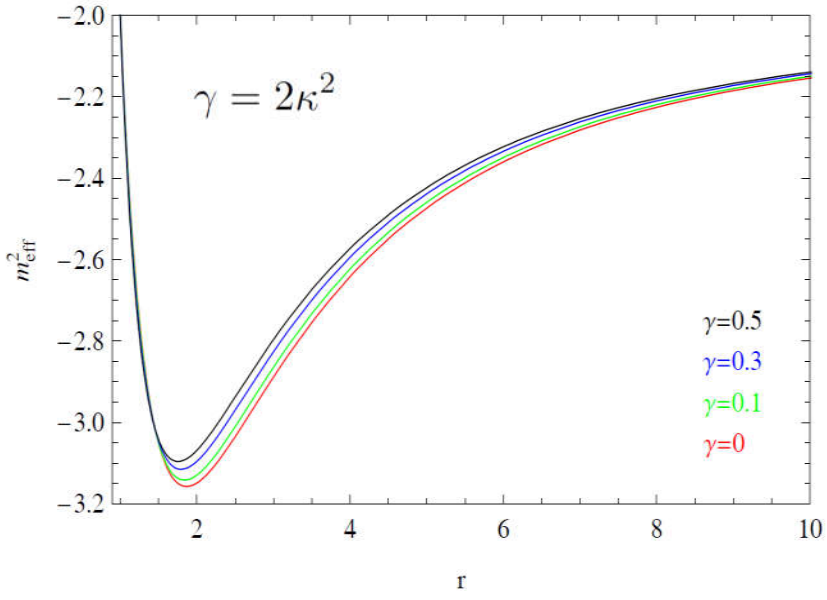
Analytical study on holographic superconductors with backreactions by using Sturm-Liouville eigenvalue method

Pan, Jing, Wang, and Chen, *JHEP* 06, 087 (2012)

	$<\mathcal{O}_1>$		$<\mathcal{O}_2>$	
$\kappa=0$	$0.2250\rho^{1/2}$	$0.2255\rho^{1/2}$	$0.1170\rho^{1/2}$	$0.1184\rho^{1/2}$
$\kappa=0.05$	$0.2249\rho^{1/2}$	$0.2253\rho^{1/2}$	$0.1163\rho^{1/2}$	$0.1177\rho^{1/2}$
$\kappa=0.10$	$0.2246\rho^{1/2}$	$0.2250\rho^{1/2}$	$0.1141\rho^{1/2}$	$0.1156\rho^{1/2}$
$\kappa=0.15$	$0.2241\rho^{1/2}$	$0.2245\rho^{1/2}$	$0.1106\rho^{1/2}$	$0.1121\rho^{1/2}$
$\kappa=0.20$	$0.2235\rho^{1/2}$	$0.2239\rho^{1/2}$	$0.1057\rho^{1/2}$	$0.1074\rho^{1/2}$
$\kappa=0.25$	$0.2226\rho^{1/2}$	$0.2230\rho^{1/2}$	$0.0998\rho^{1/2}$	$0.1017\rho^{1/2}$
$\kappa=0.30$	$0.2216\rho^{1/2}$	$0.2220\rho^{1/2}$	$0.0929\rho^{1/2}$	$0.0951\rho^{1/2}$

The critical temperature T_c obtained by the analytical S-L method (left column) and from numerical calculation (right column) with the chosen values of the backreaction parameter κ for the condensates of the scalar operators $<\mathcal{O}_1>$ and $<\mathcal{O}_2>$ in the case of 4-dimensional AdS black hole background. Here we fix the mass of the scalar field by $m^2L^2=-2$

The backreaction makes the critical temperature of the superconductor decrease.



$$m^2_{\rm eff} = m^2 + g^{tt}A_t^2 = -\frac{2}{L^2} - \frac{\phi^2}{ge^{-\chi(r)}}$$

The effective mass becomes more narrow and smaller in absolute value with the increase of the backreaction, and indicates that the higher backreaction in general make it harder for the scalar field to condense

2.3 Generalized holographic superconductor models with backreactions (s-wave)

Pan and Wang, *arxiv:1101.0222; PLB* 693, 159 (2010)

Motivation：

(a) Add the term psi^4 and more, and introduce a general class of gravity dual with backreactions to describe both first and second order phase transitions

(b) Construct numerically electrically charged black holes carrying scalar hair and obtain the influence of backreaction on the condensation and phase transition

(c) Comment on the so-called universal relation： $\omega_g / T_c \simeq 8$

The generalized action: $S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G_4}(R-2\Lambda)+\mathcal{L}_{matter}\right]$

The generalized Stuckelberg Lagrangian: **S. Franco et al. *PRD* 81, 041901(R) (2010)**

$$\mathcal{L}_{matter} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\psi\partial^\mu\psi - \frac{1}{2}m^2\psi^2 - \frac{1}{2}|\mathfrak{F}(\psi)|(\partial_\mu p - A_\mu)(\partial^\mu p - A^\mu)$$

The ansatz of the geometry of the 4-dimensional AdS black hole:

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2+dy^2)$$

The equations of motion:

$$\boxed{\mathfrak{F}(\psi) = \psi^2 + c_4\psi^4}$$

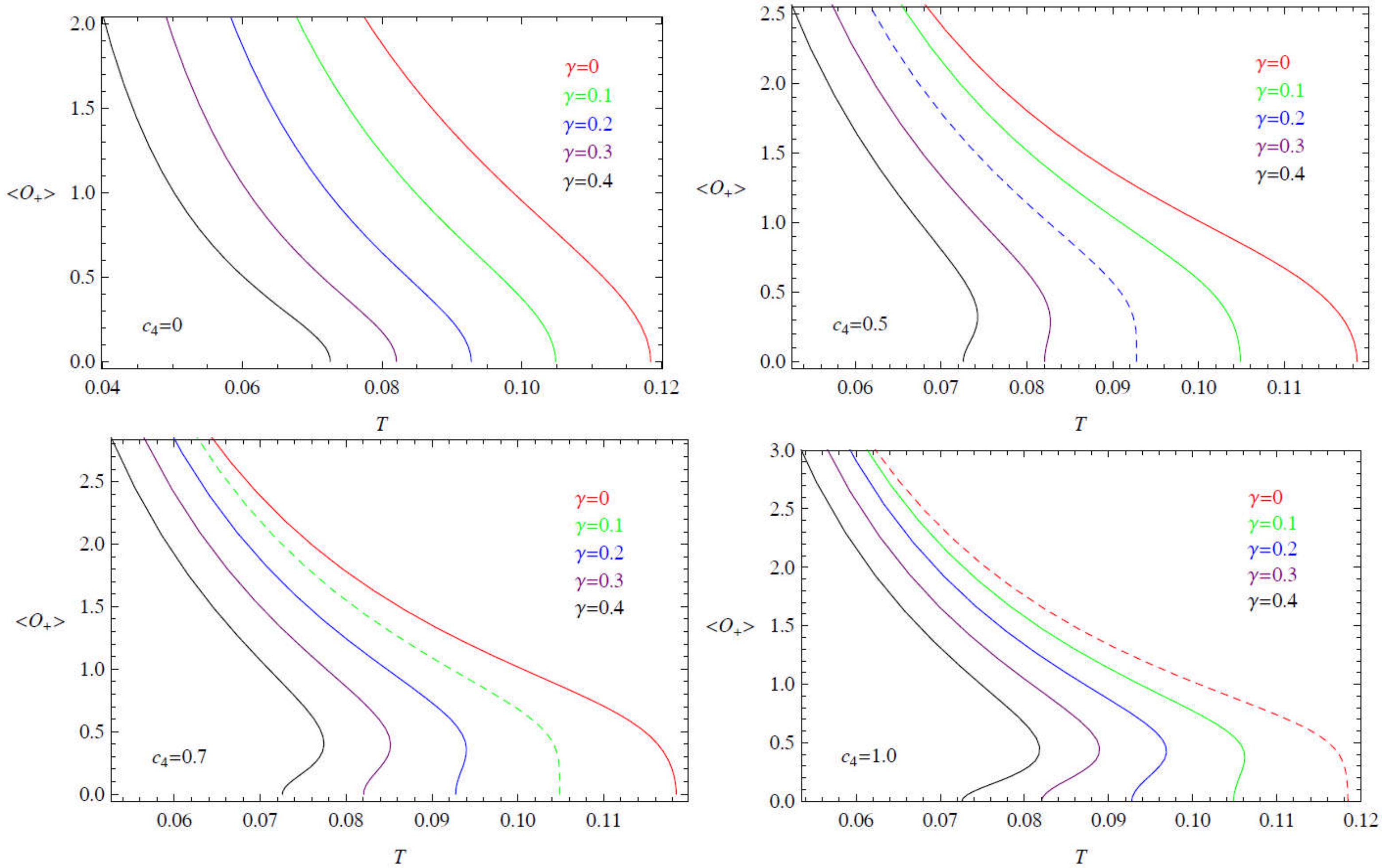
$$\chi' + \gamma\left[\frac{r}{2}\psi'^2 + \frac{r}{2g^2}e^\chi\phi^2\mathfrak{F}(\psi)\right] = 0,$$

$$g' - \left(\frac{3r}{L^2} - \frac{g}{r}\right) + \gamma r g\left[\frac{1}{4}\psi'^2 + \frac{1}{4g}e^\chi\phi'^2 + \frac{m^2}{4g}\psi^2 + \frac{1}{4g^2}e^\chi\phi^2\mathfrak{F}(\psi)\right] = 0,$$

$$\phi'' + \left(\frac{2}{r} + \frac{\chi'}{2}\right)\phi' - \frac{\mathfrak{F}(\psi)}{g}\phi = 0,$$

$$\psi'' + \left(\frac{2}{r} - \frac{\chi'}{2} + \frac{g'}{g}\right)\psi' - \frac{m^2}{g}\psi + \frac{1}{2g^2}e^\chi\phi^2\mathfrak{F}'(\psi) = 0,$$

A. Effects on the scalar condensation and phase transition



Dashed line in these panels corresponds to the case of the critical value of the backreaction which can separate the first- and second-order behavior.

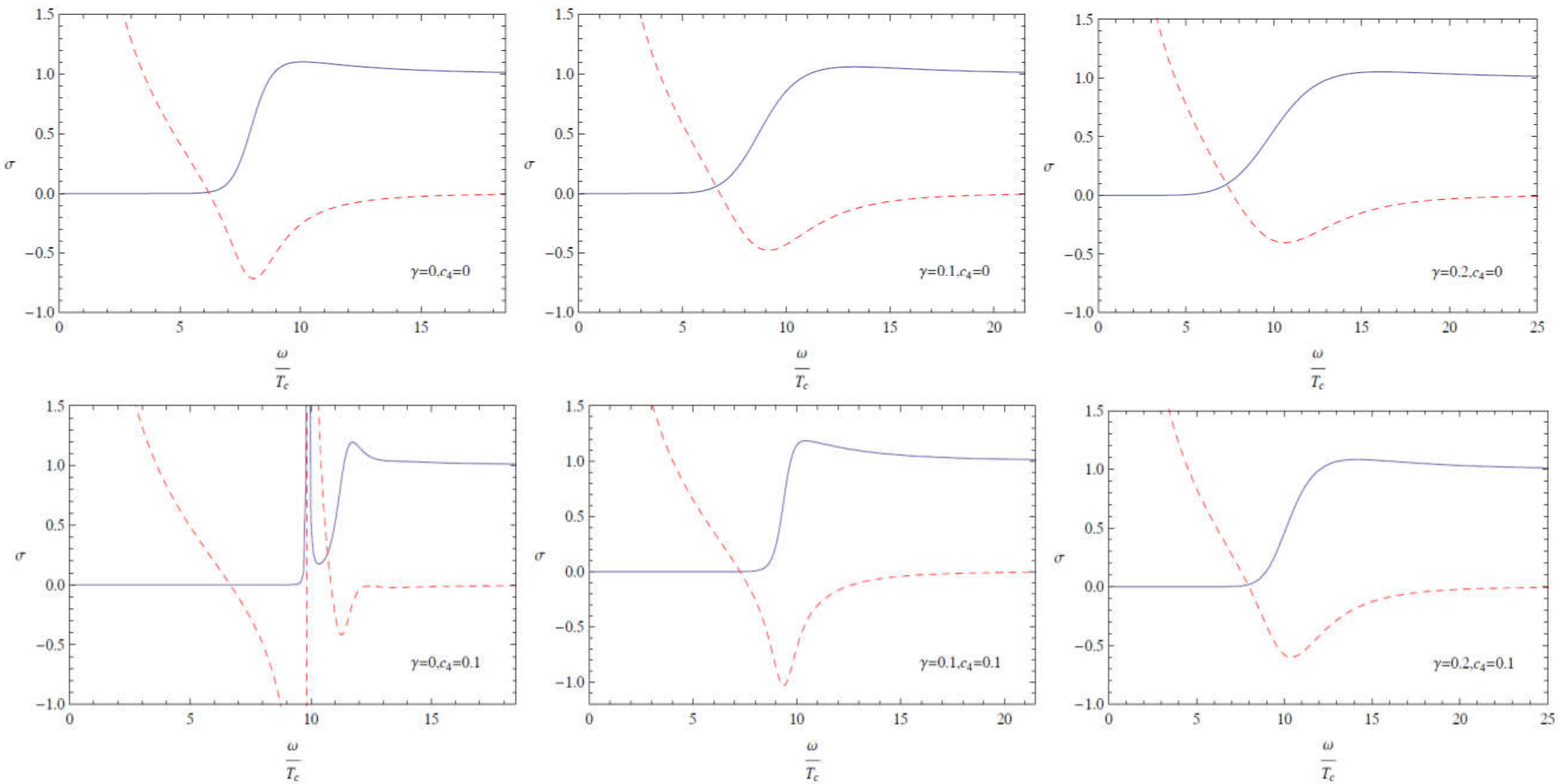
(a) For $c4>1.0$, the condensate does not drop to zero continuously at the critical temperature and the transition is first order for all values of the backreaction;

(b) For $0<c4<1.0$, not only $c4$ but also the backreaction can tune the order of the phase transition.

$$\gamma_c = 0.2 \text{ for } c_4 = 0.5,\ \gamma_c = 0.1 \text{ for } c_4 = 0.7 \text{ and } \gamma_c = 0 \text{ for } c_4 = 1$$

In addition to the model parameter $c4$, the spacetime backreaction can also bring richer descriptions in the phase transition. When the model parameter is larger, smaller backreaction can trigger the first order phase transition.

B. Effects on the conductivity



Gap ratio ω_g/T_c does change in the presence of the backreaction and model parameters.

There is no universal relation for the gap ratio of the conductivity

s-wave: T. Nishioka, S. Ryu, and T. Takayanagi, ***JHEP*** 1003,131 (2010);
X.H. Ge, B. Wang, S.F. Wu, and G.H. Yang, ***JHEP*** 1008, 108 (2010);
J.P. Wu, Y. Cao, X.M. Kuang, and W.J. Li, ***Phys. Lett. B*** 697, 153 (2011).
Y. Ling, C. Niu, J. Wu, Z. Xian, and H. Zhang, ***Phys. Rev. Lett.*** 113, 091602 (2014).
L. Yin, D.F. Hou, and H.C. Ren, ***Phys. Rev. D*** 91, 026003 (2015);
Y.Q. Liu, Y.G. Gong, and B. Wang, ***JHEP*** 1602, 116 (2016);
C.J. Luo, X.M. Kuang, and F.W. Shu, ***Phys. Lett. B*** 759, 184 (2016).

p-wave, p+*i*p phase:

S.S. Gubser and S.S. Pufu, ***JHEP*** 0811, 033 (2008);
R.G. Cai, L. Li, L.F. Li, and R.Q. Yang, ***JHEP*** 1404, 016 (2014);
Y.B. Wu, J.W. Lu, *et al.*, ***Phys. Rev. D*** 90, 126006 (2014);
Z.Y. Nie, Q.Y. Pan, H.B. Zeng, and H. Zeng, ***EPJC*** 77, 69 (2017).

d-wave: J.W. Chen, Y.J. Kao, *et al.*, ***Phys. Rev. D*** 81, 106008 (2010);
F. Benini, C.P. Herzog, R. Rahman, and A. Yarom, ***JHEP*** 1011, 137 (2010);
H.B. Zeng, Y. Jiang, Z.Y. Fan, and H.S. Zong, ***Phys. Rev. D*** 82, 126014 (2010).

Competition and coexistence of superconducting order parameters:

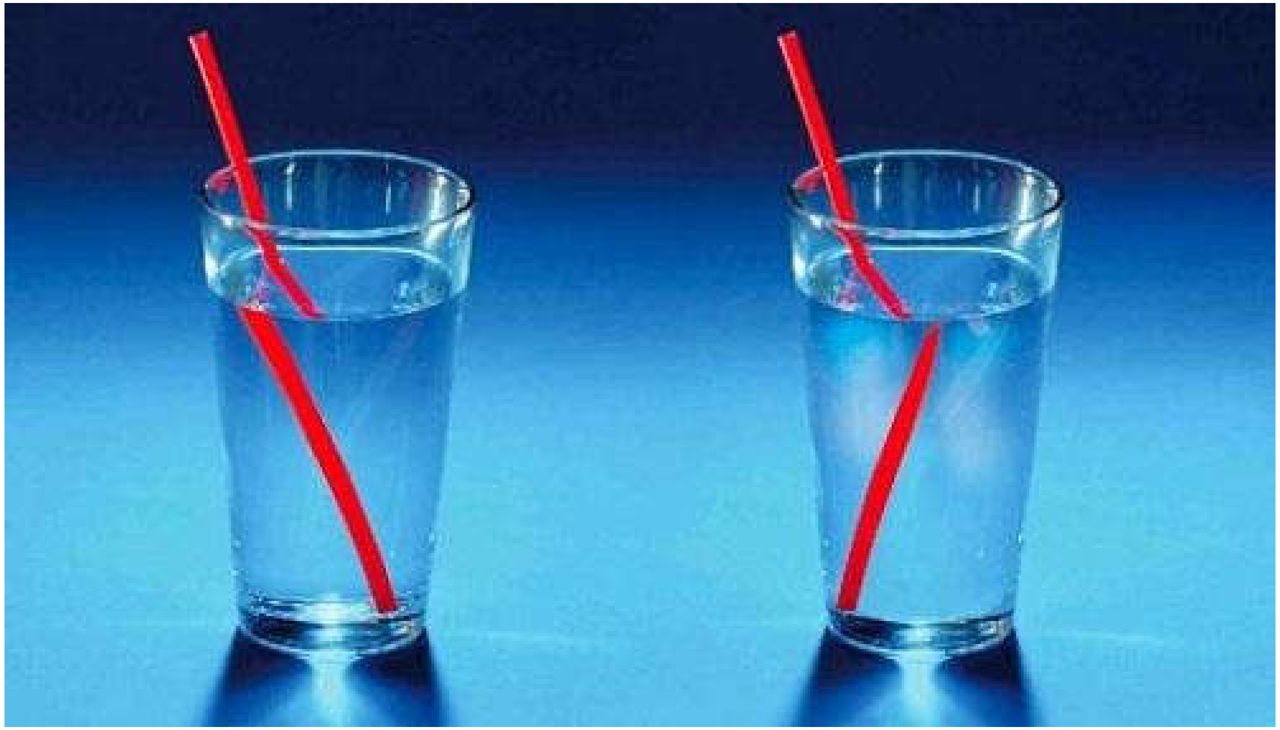
R.G. Cai, L. Li, L.F. Li, and R.Q. Yang, ***SCPMA*** 58, 060401 (2015); arXiv:1502.00437.
R. Li, Y. Tian, H.B. Zhang, and J.K. Zhao, ***Phys. Rev. D*** 94, 046003 (2016).

Josephson junction,……

G.T. Horowitz, J.E. Santos, and B. Way, ***Phys. Rev. Lett.*** 106, 221601 (2011);
Y.Q. Wang, Y.X. Liu, R.G. Cai, S. Takeuchi, and H.Q. Zhang, ***JHEP*** 1209, 058 (2012);
H.F. Li, L. Li, Y.Q. Wang, and H.Q. Zhang, ***JHEP*** 1412, 099 (2014);
Y.P. Hu, H.F. Li, H.B. Zeng, and H.Q. Zhang, ***Phys. Rev. D*** 93, 104009 (2016).

3、 Negative refraction in the holographic superconductors

The articial materials exhibit the exotic electromagnetic phenomena——negative refraction, i.e., the energy flux of the electromagnetic wave flows in the opposite direction with respect to the phase velocity



- V.G. Veselago, The electrodynamics of substances with simultaneously negative values of ε and μ, ***Sov. Phys. Usp.*** **10, 509 (1968).**

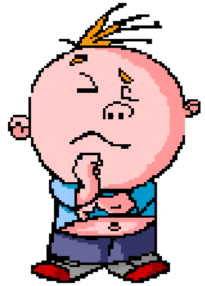
- D.R. Smith, W.J. Padilla, D.C. Vier, S.C. Nemat-Nasser, and S. Schultz, Composite Medium with Simultaneously Negative Permeability and Permittivity, ***Phys. Rev. Lett.*** **84, 4184 (2000).**

- J.B. Pendry, Negative Refraction Makes a Perfect Lens, ***Phys. Rev. Lett.*** **85, 3966 (2000).**

- R.A. Shelby, D.R. Smith, and S. Schultz, Experimental verification of a negative index of refraction, **Science 292, 77 (2001).**

奇异的负折射现象是人们研究的热点课题，人工制备的负折射率"超材料"不断涌现，相关成果在2003年度和2006年度两次被美国《科学》杂志评为全球十大科学进展之一。

An interesting proposal of high-temperature superconductors as negative index of refraction materials or metamaterials ???



- A. Amariti, D. Forcella, A. Mariotti, and G. Policastro, Holographic optics and negative refractive index, ***JHEP*** **1104, 036 (2011); arXiv:1006.5714 [hep-th].**

- X. Gao and H.B. Zhang, Refractive index in holographic superconductors, ***JHEP*** **1008, 075 (2010); arXiv:1008.0720 [hep-th].**

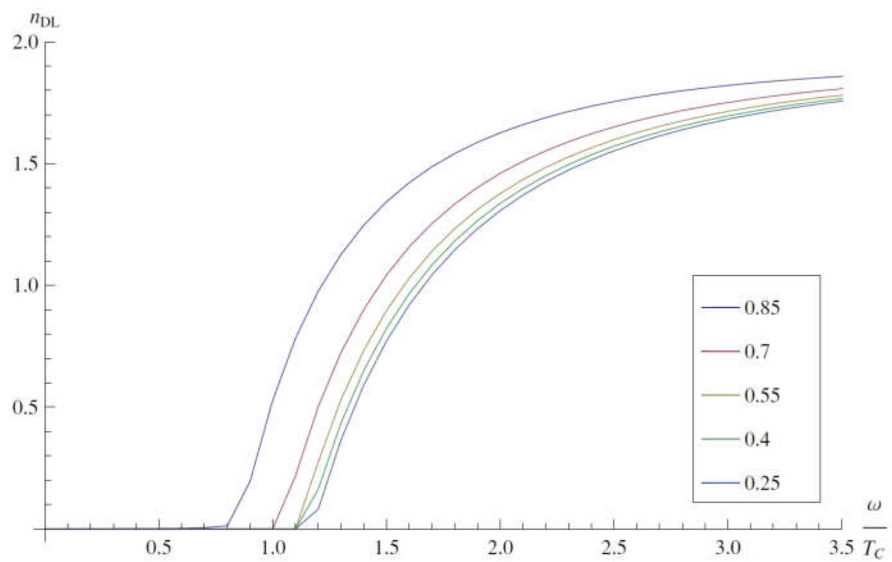
- X.H. Ge, K. Jo, and S.J. Sin, Hydrodynamics of RN AdS_4 black hole and holographic optics, ***JHEP*** **1103, 104 (2011).**

- S. Mahapatra, P. Phukon, and T. Sarkar, Generalized Superconductors and Holographic Optics, ***JHEP*** **1401, 135 (2014).**

The corresponding bulk action **_JHEP_ 08, 075 (2010)——first attempt**

$$S_{\text{bulk}} = \int d^5x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R+\frac{12}{L^2}\right)-\frac{1}{q^2}\left(\frac{1}{4}F_{ab}F^{ab}+|D_a\Phi|^2+m^2|\Phi|^2\right)\right]$$

Depine-Lakhtakia index as a function of frequency with various temperatures



The negative phase velocity does not appear in the holographic superconductors under such a situation

Motivation

1. Construct the holographic superconducting system with Born–Infeld electrodynamics which can have a negative refractive index；

2. Use Born–Infeld electrodynamics to lower the dissipative effects and study the effect of the Born–Infeld electrodynamics on the optical properties of strongly coupled field theories;

3. Examine the influence of the Born–Infeld correction to the Maxwell field on the holographic superconductor phase transition, and understand the influences of the 1/N or $1/\lambda$ (λ is the ’t Hooft coupling) corrections.

2.1 Holographic setup

Cheng, Pan, Yu, and Jing, to appear

Working in the probe limit, we consider a planar SAdS black hole background

$$ds^2 = -f(r)\,dt^2 + \frac{1}{f(r)}dr^2 + r^2\left(dx^2 + dy^2\right)$$

with $$f(r) = r^2\left(1 - \frac{r_h^3}{r^3}\right)$$

The matter Lagrangian is

$$L_{matter} = \frac{1}{b}\left(1 - \sqrt{1 + \frac{bF^2}{2}}\right) - \frac{(\partial_\mu \Psi)^2}{2} - \frac{m^2\Psi^2}{2} - |\mathrm{G}(\Psi)|(\partial\alpha - A)^2$$

Consider the following ansatz

$$\Psi = \Psi(r), \qquad A = \Phi(r)dt$$

The equations of motion reduce to

$$\Psi'' + \left(\frac{2}{r} + \frac{f'}{f}\right)\Psi' - \frac{m^2}{f}\Psi + \frac{\Phi^2}{f^2}\frac{dG(\Psi)}{d\Psi} = 0,$$

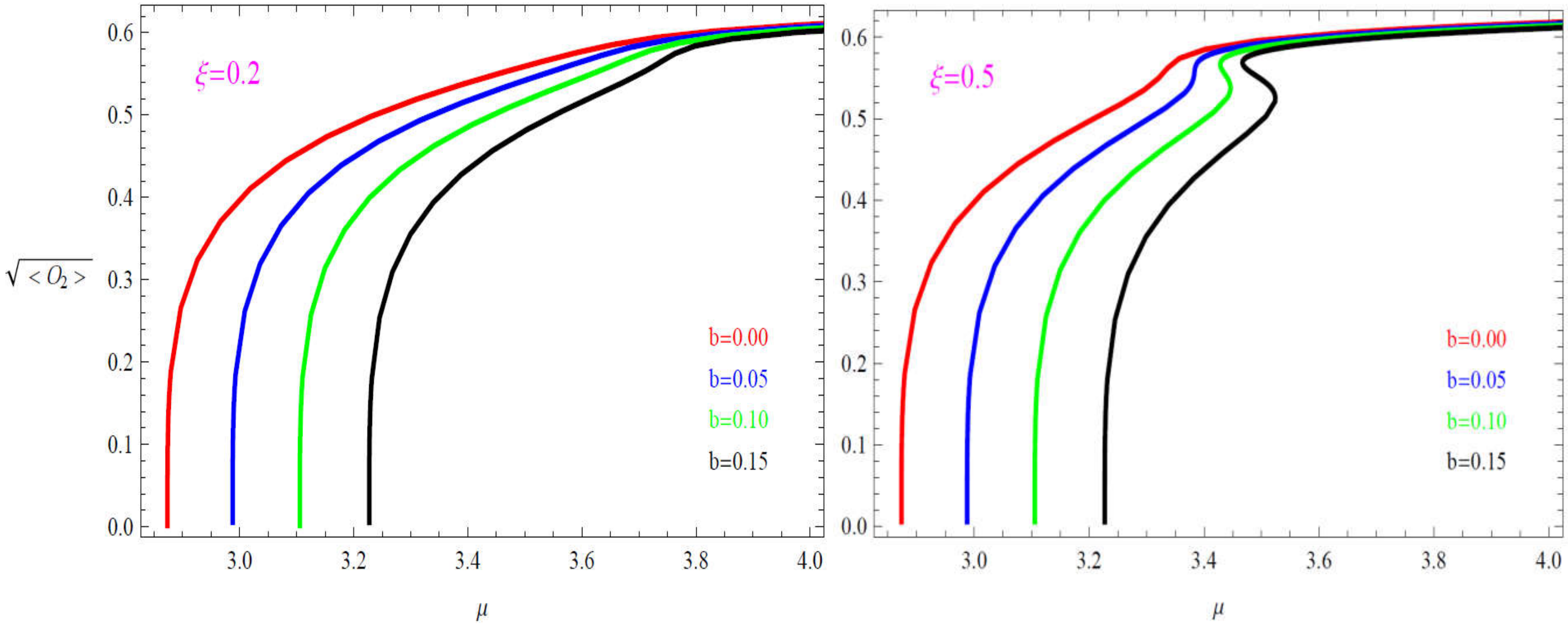
$$\Phi'' + \frac{2}{r}(1 - b\Phi'^2)\Phi' - \frac{2G(\Psi)}{f}(1 - b\Phi'^2)^{3/2}\Phi = 0$$

where we can use the gauge symmetry to fix the phase $\alpha = 0.$

The asymptotic expressions near the boundary

$$\Phi = \mu - \frac{\rho}{r}, \qquad \Psi = \frac{\Psi_{\lambda_-}}{r^{\lambda_-}} + \frac{\Psi_{\lambda_+}}{r^{\lambda_+}} \qquad \boxed{\lambda_\pm = \frac{3 \pm \sqrt{9+4m^2}}{2}}$$

Considering $m^2 = -2$ $\lambda_\mp = 1, 2$ $\langle O_2 \rangle \sim \Psi_{(2)}$ $G(\Psi) = \Psi^2 + \xi\Psi^8$



$\sqrt{< O_2 >}$ as a function of chemical potential

A. Higher Born–Infeld corrections make it harder for the condensation to form;

agrees with Jing and Chen, *PLB* 686, 68 (2010)

B. Higher Born–Infeld corrections make it easier for the emergence of the first-order phase transition.

agrees with Jing, Wang, Pan, and Chen, *PRD* 83, 066010 (2011)

2.2 Refractive index in generalized superconductors with Born-Infeld electrodynamics

Cheng, Pan, Yu, and Jing, to appear

The relevant quantity that is used to establish negative refractive index in a medium is called the Depine-Lakhtakia index [R.A. Depine and A. Lakhtakia, *MOTL* 41, 315 (2004)]

$$\eta_{DL} = Re[\epsilon(\omega)]|\mu(\omega)| + Re[\mu(\omega)]|\epsilon(\omega)|$$

with negativity of the DL index indicating that the phase velocity in the medium is opposite to the direction of energy flow.

Computation of the DL index involves a number of steps:

D.T. Son and A.O. Starinets, *JHEP* 09, 042 (2002); hep-th/0205051.

M. Dressel and G. Gruner, *Electrodynamics of Solids*, Cambridge University Press, Cambridge U.K. (2002)

The momentum dependent correlators:

$$G_T(\omega, K) = G_T^0(\omega) + K^2 G_T^2(\omega) + \cdots$$

The permittivity and the effective permeability

$$\epsilon(\omega) = 1 + \frac{4\pi}{\omega^2} C_{em}^2 G_T^0(\omega)$$
$$\mu(\omega) = \frac{1}{1 - 4\pi C_{em}^2 G_T^2(\omega)}$$

EM coupling constant (set to unity for numerical computations)

Consider a perturbation of the Maxwell field

$$A_x = A_x(r) e^{-i\omega t + iKy}$$

For the case where the bulk theory is an SAdS black hole

$$A_x'' + \left(\frac{f'}{f} + \frac{b\Phi'\Phi''}{1 - b\Phi'^2}\right) A_x' + \left[\frac{\omega^2}{f^2} - \frac{K^2}{r^2 f} - \frac{2\sqrt{1 - b\Phi'^2}\mathrm{G}(\Psi)}{f}\right] A_x = 0$$

With appropriate boundary conditions at the horizon:

$$A_x \propto f^{-\frac{i\omega}{3}}$$

Asymptotically $$A_x = A_x^{(0)} + \frac{A_x^{(1)}}{r} + \ldots$$

From the AdS/CFT dictionary

$$G_T = \frac{A_x^{(1)}}{A_x^{(0)}}$$

In order to obtain the DL index and other optical quantities in the boundary theory, we first calculate G_T^0 and G_T^2.

Expanding A_x in powers of K in same way

$$A_x = A_{x0} + K^2 A_{x2} + \ldots$$

Differential equations for A_{x0} and A_{x2} as

$$A''_{x0}+\left(\frac{f'}{f}+\frac{b\Phi'\Phi''}{1-b\Phi'^2}\right)A'_{x0}+\left[\frac{\omega^2}{f^2}-\frac{2\sqrt{1-b\Phi'^2}\mathrm{G}(\Psi)}{f}\right]A_{x0}=0,$$

$$A''_{x2}+\left(\frac{f'}{f}+\frac{b\Phi'\Phi''}{1-b\Phi'^2}\right)A'_{x2}+\left[\frac{\omega^2}{f^2}-\frac{2\sqrt{1-b\Phi'^2}\mathrm{G}(\Psi)}{f}\right]A_{x2}-\frac{A_{x0}}{r^2 f}=0$$

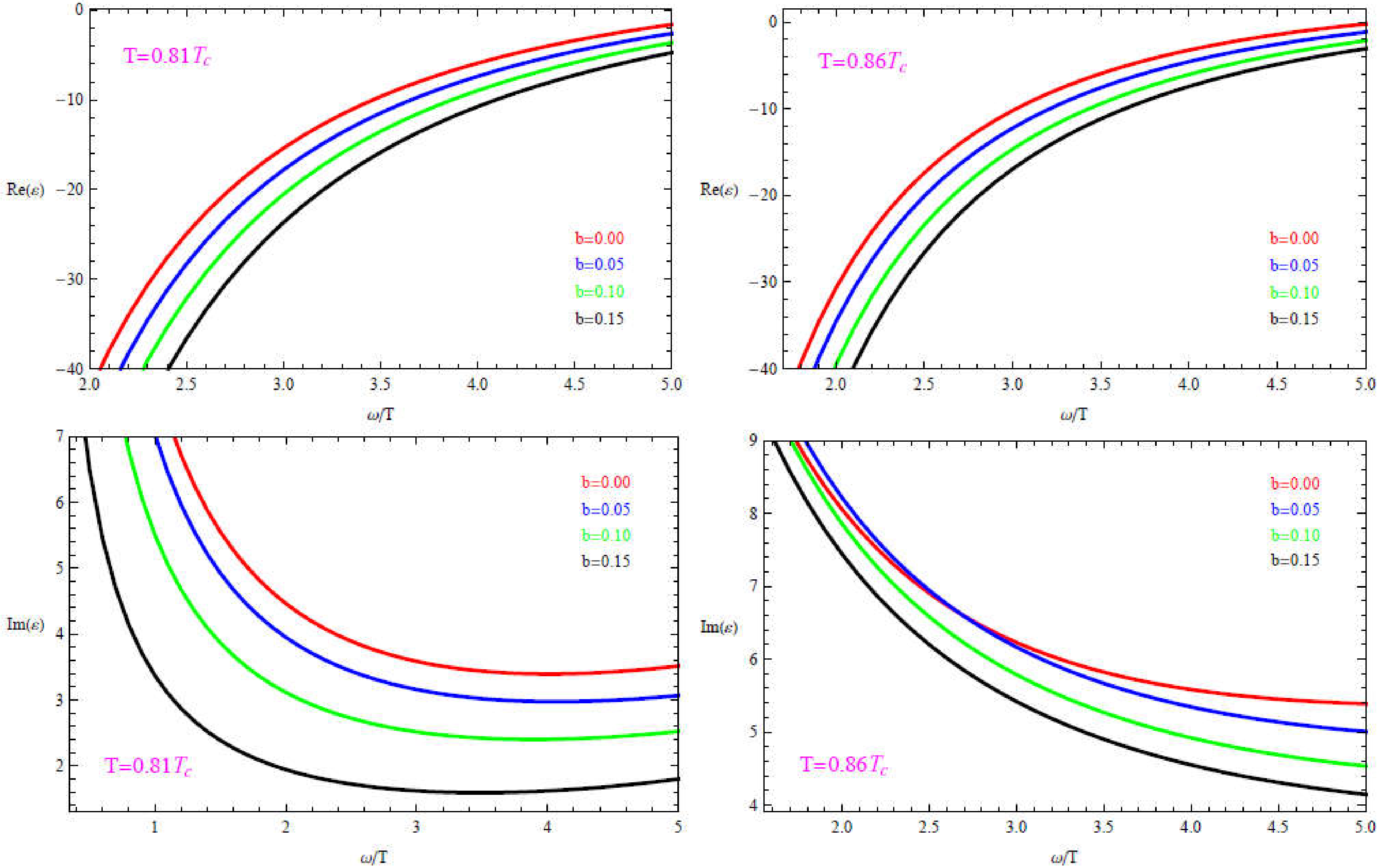
asymptotic forms

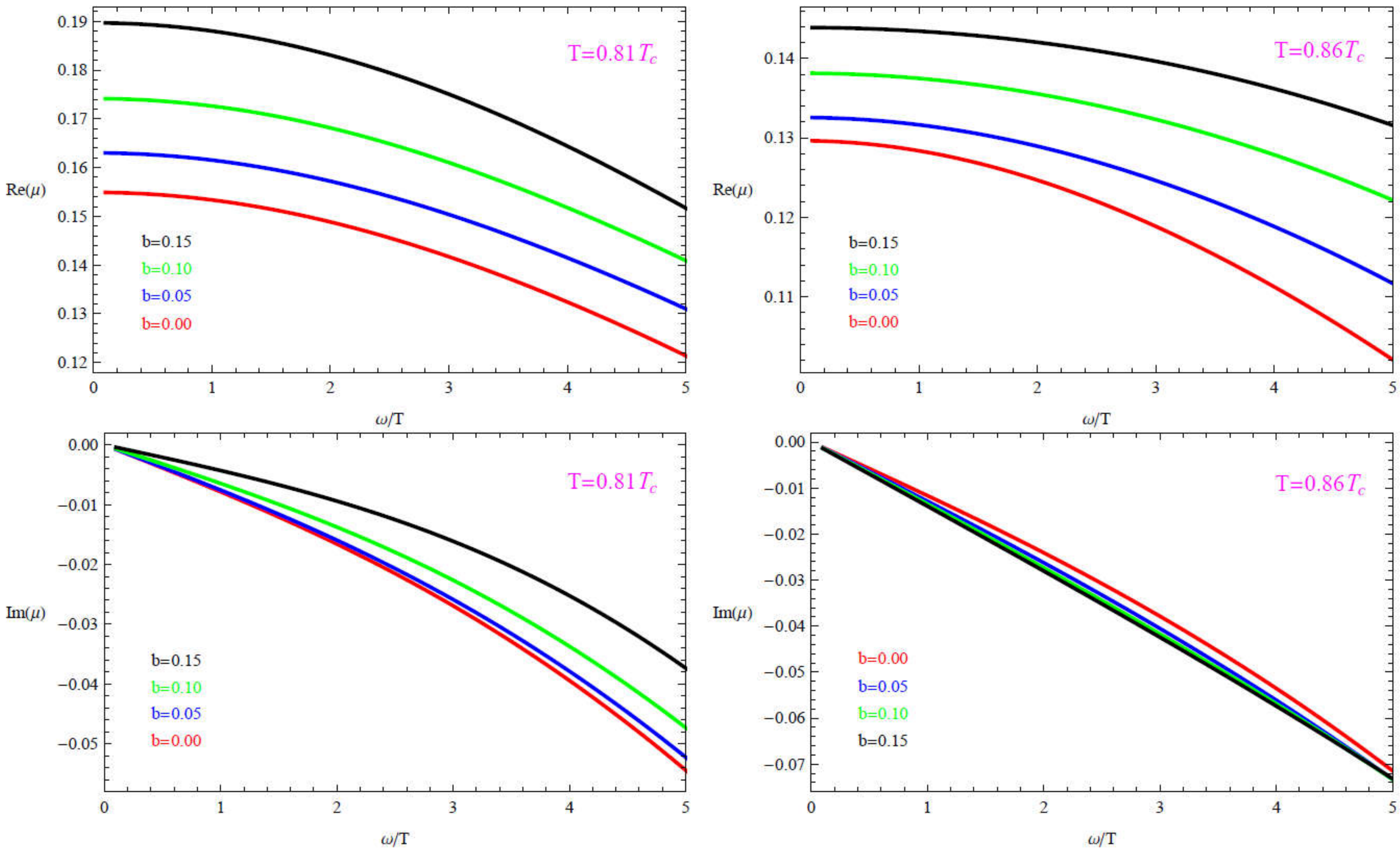
$$A_{x0}=A_{x0}^{(0)}+\frac{A_{x0}^{(1)}}{r}+\dots,\qquad A_{x2}=A_{x2}^{(0)}+\frac{A_{x2}^{(1)}}{r}+\dots$$

Therefore

$$\boxed{G_T^0=\frac{A_{x0}^{(1)}}{A_{x0}^{(0)}},\qquad G_T^2=\frac{A_{x0}^{(1)}}{A_{x0}^{(0)}}\left(\frac{A_{x2}^{(1)}}{A_{x0}^{(1)}}-\frac{A_{x2}^{(0)}}{A_{x0}^{(0)}}\right)}$$

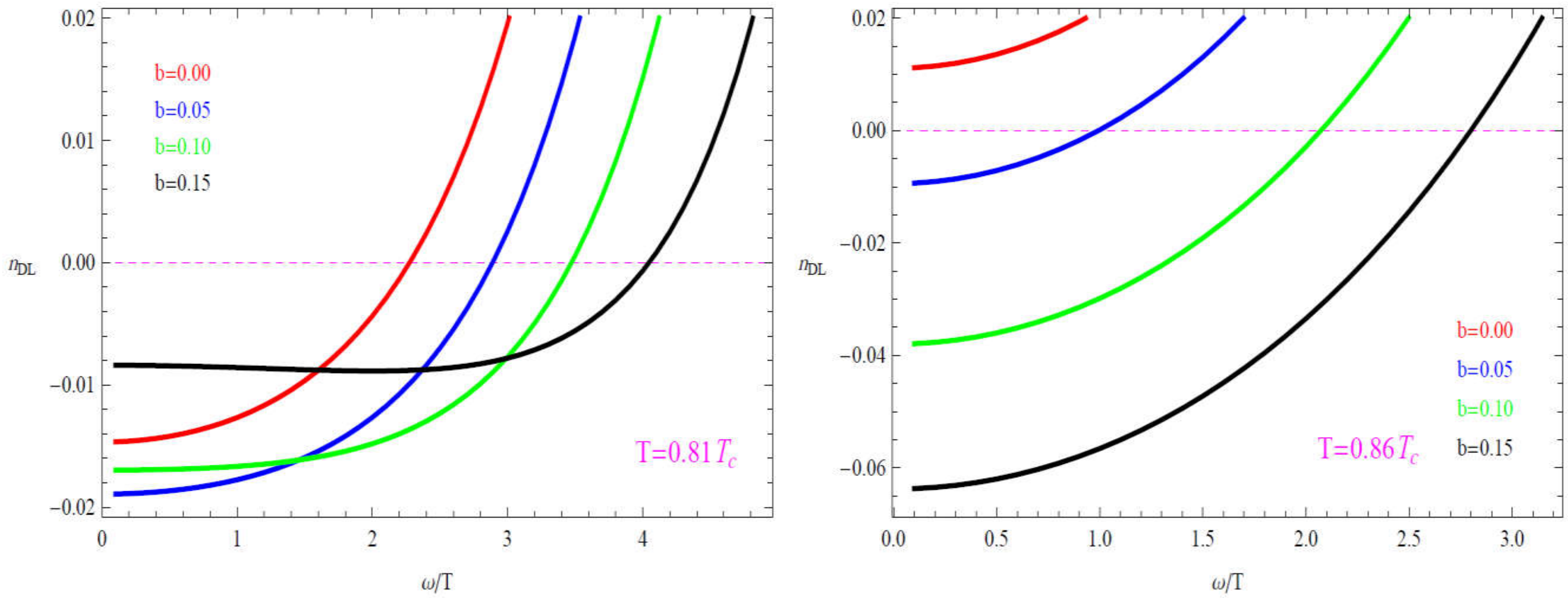
Numerical results




Re(μ)
ω/T
T=0.81 T_c
b=0.15
b=0.10
b=0.05
b=0.00
T=0.86 T_c
Im(μ)

Main results

For low frequencies, one can see the emergence of negative DL index, below a certain value of ω/T.



Higher Born–Infeld corrections make the range of frequencies or temperatures larger for which negative refraction is allowed

Check the validity of the results

The retarded current-current correlator is written down in a compact form

$$G_{xx}^{R} = G_T^0(\omega) + K^2 G_T^2(\omega) + \cdots$$

The series is restricted to the first two terms, provided,

$$\left|\frac{G_T^2(\omega) K^2}{G_T^0(\omega)}\right| \ll 1$$

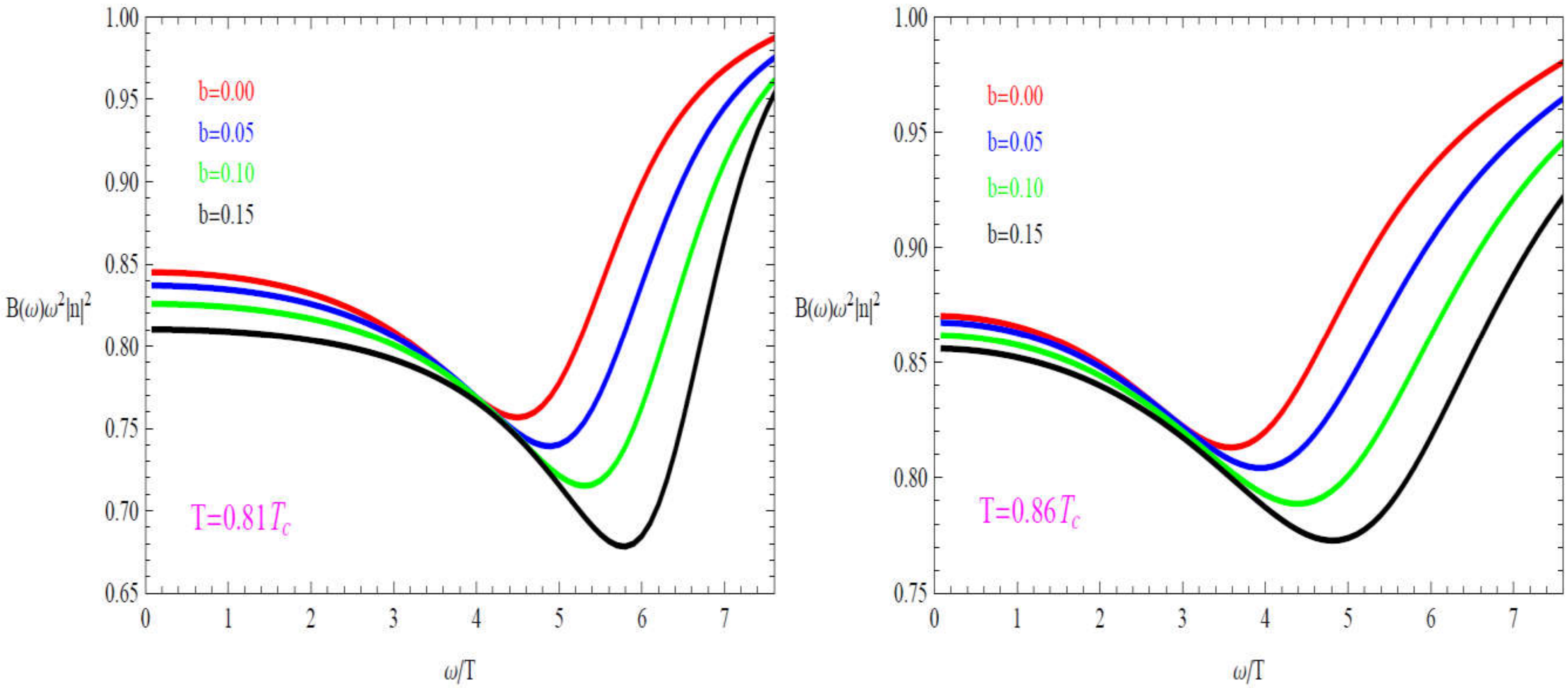
Or equivalently, one can write

$$\left|\frac{G_T^2(\omega) n^2}{G_T^0(\omega)}\right| \omega^2 \ll 1$$

Consequently, the $\epsilon-\mu$ analysis is valid only for those frequencies where the above constraint is not violated.

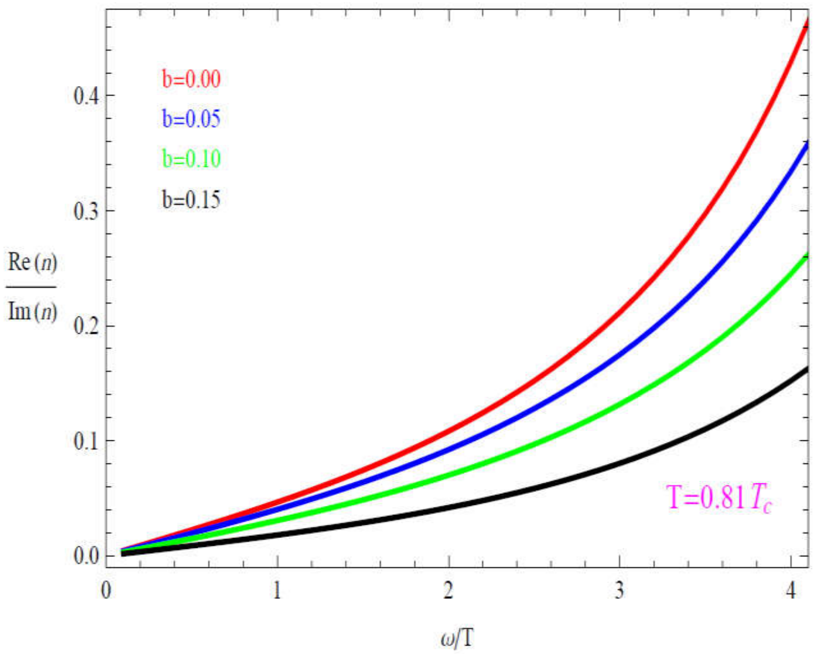
Within the plotted frequency range, $|\frac{G_T^2(\omega)n^2}{G_T^0(\omega)}|\omega^2$ is less than unity for ranges of frequencies in which the DL index is negative.

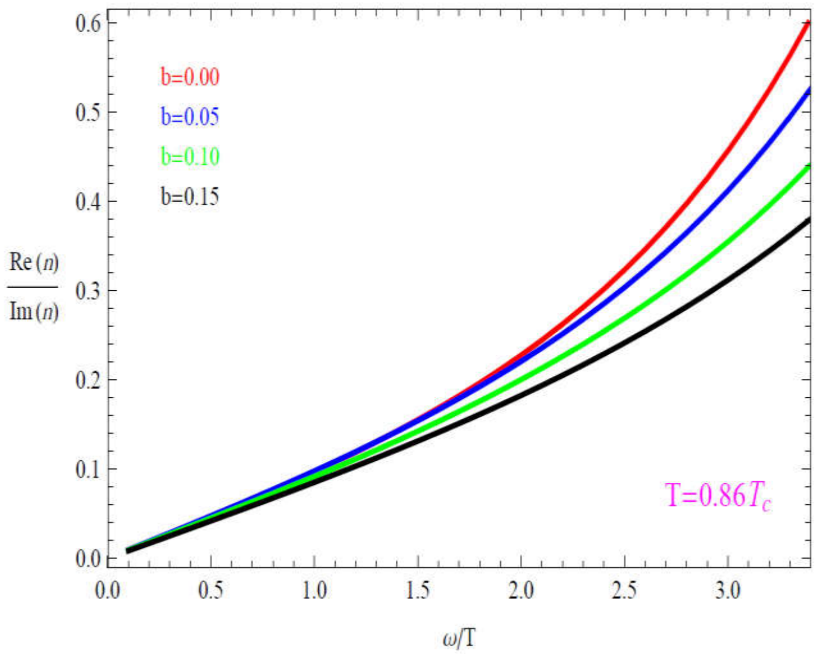
$$\boxed{|\frac{G_T^2(\omega)}{G_T^0(\omega)}| = B}$$



The dissipative effects

The propagation to dissipation ratio unfortunately tends to zero at very low frequencies, implying that in part of the domain of negative refraction, propagation of electromagnetic waves is virtually absent.





Born–Infeld electrodynamics can be used to lower the dissipative effects

4、 Conclusions and discussions

- **Negative refraction may appear at low frequencies in holographic superconducting systems,** for certain frequencies and temperature ranges, for specific values of the Born–Infeld parameter.

- **Higher Born–Infeld corrections make the range of frequencies or temperatures larger** for which negative refraction is allowed, and **can be used to lower the dissipative effects**.

- It would be of interest to generalize the study to different holographic models, for example **p-wave order** (the vector field), **d-wave order** (the tensor field), **competition and coexistence of order parameters**……

Thanks for your attention!