Toward precision measurement of neutrinos

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Content

- **A brief review of ν oscillation**
- **The present and the future**
- **Earth matter effect in ν oscillation**
- **Low energy ν oscillation in the Earth**
- **High energy ν oscillation in the Earth**
- **Conclusions**

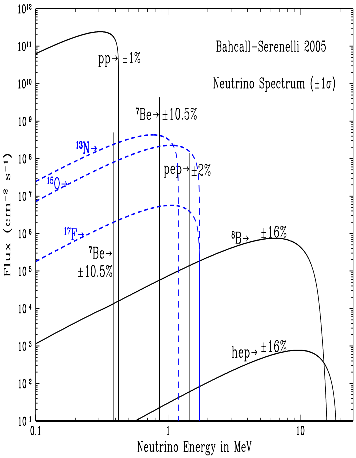
The solar neutrino problem:

ν produced in Sun:
pp ν: $p + p \to ^2 \mathrm{H} + e^+ + \nu_e$
Boron ν: $^8\mathrm{B} \to ^8 \mathrm{Be}^* + e^+ + \nu_e$
etc. $\cdots$

At low energy part of the spectrum
about $\frac{2}{3}$ ν_e flux detected

At high energy part of the spectrum
about $\frac{1}{3}$ ν_e flux detected

Missing of atmospheric ν_μ also discovered



ν oscillation

When flavor eigenstates are not the mass eigenstates (ν_1,ν_2),

$$i\frac{d}{dx}\begin{pmatrix}\nu_e\\ \nu'_\mu\end{pmatrix}=\frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\theta & \Delta m^2\sin 2\theta\\ \Delta m^2\sin 2\theta & \Delta m^2\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu'_\mu\end{pmatrix};$$

$$\begin{pmatrix}\nu_e\\ \nu'_\mu\end{pmatrix}=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}.$$

At distance x one obtains

$$|\nu_e(x)\rangle=\cos\theta e^{-iE_1x}|\nu_1\rangle+\sin\theta e^{-iE_2x}|\nu_2\rangle;$$

$$|\langle\nu_e|\nu_e(x)\rangle|^2=1-\sin^2 2\theta\sin^2\frac{\Delta m^2x}{4E}.$$

Averaging over phases or coherence lost(happened for $\nu_\odot$)

$$P_{ee}=1-\frac{1}{2}\sin^2 2\theta=\frac{1}{2}(1+\cos^2 2\theta).$$

$P_{ee}\geq 1/2$ **in vacuum oscillation.**

ν oscillation

L. Wolfenstein 1978, Mikheyev and Smirnov 1986.

Coherent forward scattering by medium modifies dispersion relation:

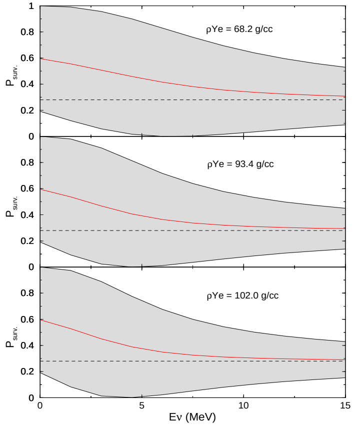
$$E = E_k + V,$$

E_k, **the kinetic energy;** V, **the potential energy. Examples include**

- **the optics : the case of electromagnetism;**
- **neutron optics: the case of strong interaction;**
- **MSW: the case of weak interaction,** $V_e = \sqrt{2} G_F N_e$**:**

$$i \frac{d}{dx} \begin{pmatrix} \nu_e \\ \nu'_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + V_e & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu'_\mu \end{pmatrix};$$

$$\begin{pmatrix} \nu_e \\ \nu'_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix} \begin{pmatrix} \nu_{m1} \\ \nu_{m2} \end{pmatrix}.$$



LMA MSW solution:

$$\Delta m^2 = (3.8 - 10) \times 10^{-5}\mathbf{eV}^2,$$
$$\tan^2\theta = 0.32 - 0.47$$

- $L_{osc} \sim 200$ **km.**
- **For** $E \gtrsim 7$ **MeV,** $P_{ee} = \frac{1}{2}(1 - \cos 2\theta) \approx 0.3;$
- **for** $E \lesssim 1$ **MeV,** $P_{ee} = \frac{1}{2}(1 + \cos^2 2\theta) \approx 0.6.$
- $2EV_e/\Delta m^2_{21} \lesssim 0.08$ **in Earth; matter effect small in Earth.**

The present and the future

The present status

Oscillation of ν confirmed, LMA MSW established

- **Solar ν experiment(Homestake, Super-K, SNO, etc.), plus long baseline ν experiment(Kamland):**

$$\Delta m_{21}^2 \approx 0.8 \times 10^{-4} \ \mathbf{eV}^2, \ \ \tan^2\theta_{12} \approx 0.4$$

- **Atmospheric ν experiments(Super-K, SNO, etc.):**

$$|\Delta m_{32}^2| \approx 3 \times 10^{-3} \ \mathbf{eV}^2, \ \ \tan^2\theta_{23} \approx 1.0$$

- **Reactor ν experiment(Chooz, etc.):**

$$\sin^2 2\theta_{13} \lesssim 0.1$$

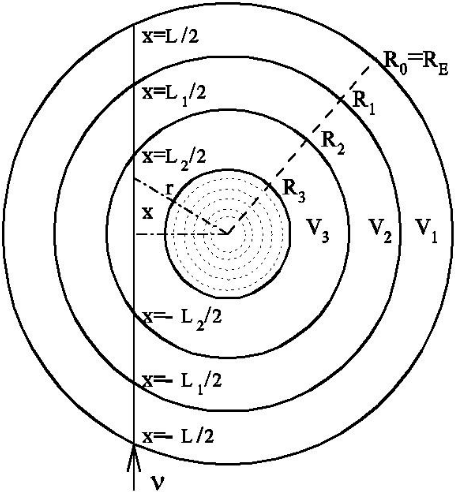
The present and the future

In the future we will measure

- **θ_{13} (reactor ν, long baseline exp)**
- **mass hierarchy(long baseline exp)**
- **CP violation in neutrinos(long baseline exp)**
- **absolute mass scale**
- **the Earth matter effect(long baseline, solar ν exp)**
- **nature of neutrino mass**
- **magnetic moment of neutrino, etc.**

Earth matter effect is important to most measurements

PReliminary Earth Model(PREM)



Matter profile very complicated

Earth matter density has many layers and changes

- **sharply between two layers**
- **slowly in a layer: density height**

$$h = \left(\frac{dV}{dx}\right)^{-1} \sim R_{Earth}$$

Earth matter effect is a challenge to future precision measurement

Quest to understand
the effect of the complex Earth matter
in neutrino oscillation

Earth matter effect

Oscillation length tells how fast ν oscillates

$$L_{21} = \frac{4\pi E}{\Delta m_{21}^2}, \quad L_{32} = \frac{4\pi E}{|\Delta m_{32}^2|}$$

Note that available measurements tell us

- **for $E \sim 10$ MeV (solar, supernova neutrinos)**

$$L_{21} \sim \textbf{hundreds km}, \quad L_{32} \sim \textbf{few km},$$

  $L_{21}, L_{32} \ll h$

- **for $E \gtrsim 500$ MeV**

$$L_{21} \gtrsim h$$

Earth matter effect

An important lesson:

oscillation well approximated by $1-2$ plus $2-3$ oscillation
i.e., oscillation in matter and in vacuum if θ_{13} small

$$H = \frac{1}{2E} U^* \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} U^\dagger + \begin{pmatrix} V_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Rotation in $2-3$ sector does not change the potential term

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Crucial to compare L_{21} and h to understand matter effect;
L_{31} not crucial when θ_{13} small

Low energy ν conversion in the Earth

For $E \lesssim 30$ MeV, $L_{21} \ll h$ (solar and supernova neutrinos)

ν_e **survival well reduced to $1-2$ oscillation**

$$i\frac{d}{dx}\begin{pmatrix}\psi_{1m}\\ \psi_{2m}\end{pmatrix} = \begin{pmatrix}-\frac{\Delta(x)}{4E} & -i\dot{\theta}_m(x)\\ i\dot{\theta}_m(x) & \frac{\Delta(x)}{4E}\end{pmatrix}\begin{pmatrix}\psi_{1m}\\ \psi_{2m}\end{pmatrix},$$

$(\psi_{m1}, \psi_{m2})^T$, **neutrino mass state in matter.**

The survival probability of $\nu_\odot$ on the Earth

$$P_{ee} = \frac{1}{2}(1+\cos 2\theta_m(x_0)\cos 2\theta_{12}) - \cos 2\theta_m(x_0) f_{reg}\,.$$

$f_{reg} = P(\nu_2 \to \nu_e) - \sin^2\theta_{12}$, **the regeneration by the Earth**

Low energy ν conversion in the Earth

Adiabatic perturbation theory
(de Holanda, Liao, Smirnov, 2004)

Search for the solution of the following form

$$\begin{pmatrix} \psi_{1m}(x) \\ \psi_{2m}(x) \end{pmatrix} = \begin{pmatrix} e^{i\Phi(x)} & c(x)e^{-i\Phi(x)} \\ -c^*(x)e^{i\Phi(x)} & e^{-i\Phi(x)} \end{pmatrix} \begin{pmatrix} \psi_{1m}(x_0) \\ \psi_{2m}(x_0) \end{pmatrix}$$

$$\Phi(x) = \frac{1}{4E}\int_{x_0}^{x} dx'\Delta(x').$$

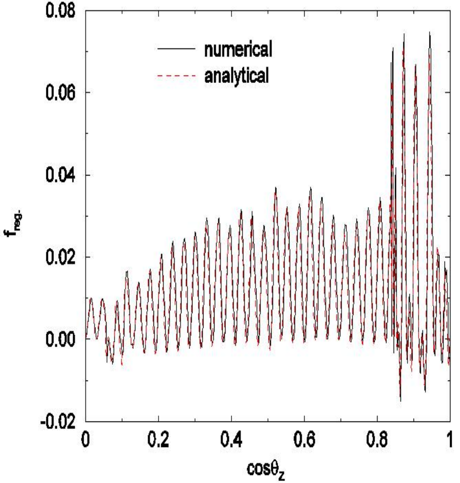
where $|c(x)| \ll 1$ is supposed to hold(adiabatic perturbation)

We get

$$c(x) = -\int_{x_0}^{x} dx' \frac{d\theta_m(x')}{dx'} \exp\left[-i\int_{x}^{x'} dx'' \frac{\Delta(x'')}{2E}\right].$$

Low energy ν conversion in the Earth

ν regeneration in the Earth

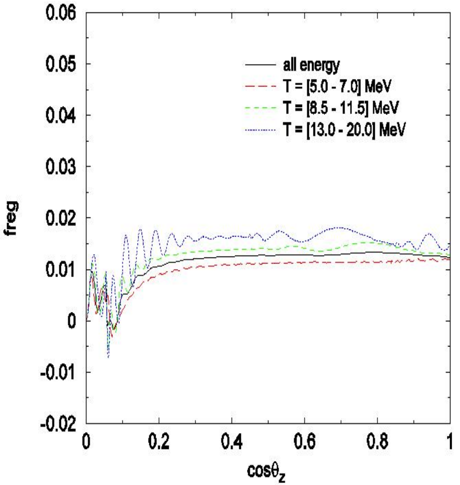


$$f_{reg} = \frac{2E\sin^2 2\theta}{\Delta m^2} \sin\Phi_0 \sum \Delta V_i \sin\Phi_i,$$
$$\Phi_i = \int_{-L_i/2}^{L_i/2} dx \frac{\Delta(x)}{4E}.$$

- **Leading contributions are from potential jumps between layers**
- **Analytic and numerical computations perfectly agree**
- **Oscillatory pattern is well understood using the adiabatic perturbation theory**

Low energy ν conversion in the Earth

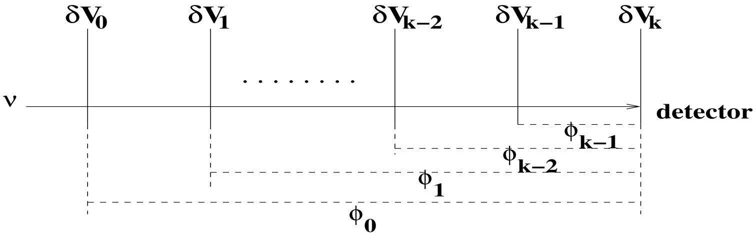
ν regeneration in the Earth



Properties:

- **Averaging over energy tremendously simplifies the oscillation pattern**
- **Neutrinos of horizontal direction still give complicated oscillation pattern in Earth regeneration**

Low energy ν conversion in the Earth



Extension to asymmetric matter profile

$$f_{reg} = -\frac{E \sin^2 2\theta_{12}}{\Delta m^2_{21}} \sum_{i=0}^{k} \delta V_i \cos 2\phi_i, \ \ \phi_i = \int_{x_i}^{x_k} dx \frac{\Delta(x)}{4E}, \ \ i = 0, \cdots, k$$

Main uncertainties in the Earth matter effects are from contributions of small structures close to the detectors.

High energy ν conversion in the Earth

For $E \gtrsim 0.5$ GeV,
$L_{21} \gtrsim h$, $L_{31} >$ or $< h$

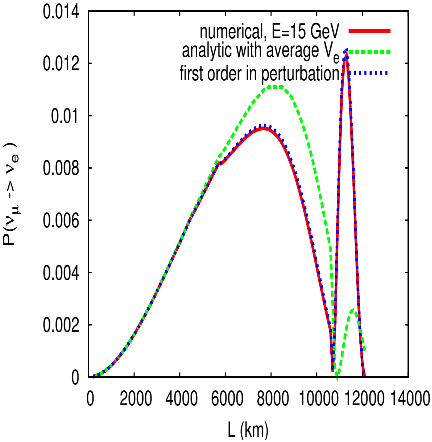
ν oscillation seems very complicated.

It turns out that
1) we can have a perturbation theory which perfectly describes the oscillation pattern

2) neutrino oscillation can be substantially simplified in the interested energy range

High energy ν conversion in the Earth

$\nu_\mu \to \nu_e$ **conversion vs. L**



A nice example:

For $E > 10$ **GeV** $(L_{21}, L_{31} > h)$**, neutrinos**

- **can not see the structure of the Earth very well**
- **see baseline dependent average potential in Earth**

Earth matter effect is well described by a formulation using the baseline dependent average potential

High energy ν conversion in the Earth

Liao, 2008; Liao, 2008

A baseline dependent perturbation theory

$$H = \bar{H} + \delta H, \quad \bar{H} = H_0 + \bar{V}$$

$$\delta H = \delta V = V(x) - \bar{V}, \quad \bar{V} = \frac{1}{L}\int_0^L dx \; V(x)$$

$\bar{V}$ depends on baseline

The transition matrix is found

$$M(L) = \bar{U}_m e^{-i\frac{\Delta}{2E}L}(1 - iC)\bar{U}_m^\dagger$$

$$C = \int_0^L dx \; e^{i\frac{\Delta}{2E}L}\bar{U}_m^\dagger \delta V(x)\bar{U}_m e^{-i\frac{\Delta}{2E}L}$$

This is a perturbation expanded using δV around the baseline dependent average potential $\bar{V}$

High energy ν conversion in the Earth

So

$$C_{jj} = \int_0^L dx (\bar{U}_m^\dagger \; \delta V(x) \; \bar{U}_m)_{jj} = 0,$$

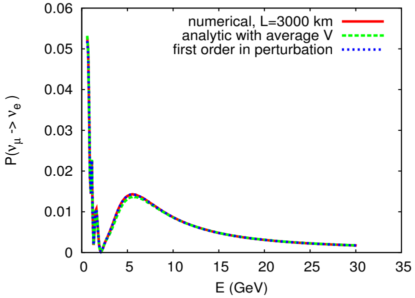
$$C_{jk} = \int_0^L dx \; e^{i\frac{\Delta_j - \Delta_k}{2E}} (\bar{U}_m^\dagger \; \delta V(x) \; \bar{U}_m)_{jk}, \; j \neq k$$

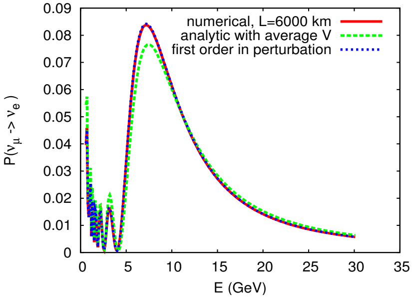
$|C_{jk}| \ll 1$ **needed as a good perturbation approximation**

C_{jk} **suppressed by**
1) $\delta V_e / \bar{V}_e \lesssim 0.3$
2) small $\Delta m_{21}^2 / (4E\bar{V}_e)$ **and** $\sin\theta_{13}$

High energy ν conversion in the Earth

$\nu_\mu \to \nu_e$ **conversion vs. E**



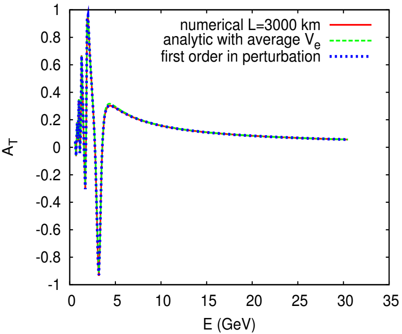


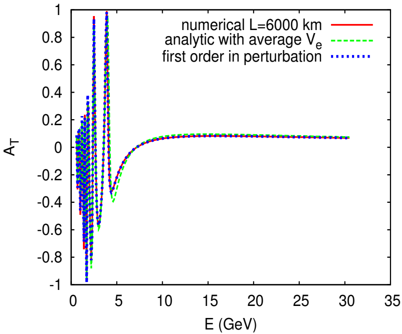
For $L \lesssim 6000$ km, the Earth matter effect is very well described by the baseline dependent average potential, the only parameter for a fixed baseline

Plus 1st order correction, the theory perfectly describes the oscillation of high energy ν in the Earth

High energy ν conversion in the Earth

Time reversal asymmetry vs. E

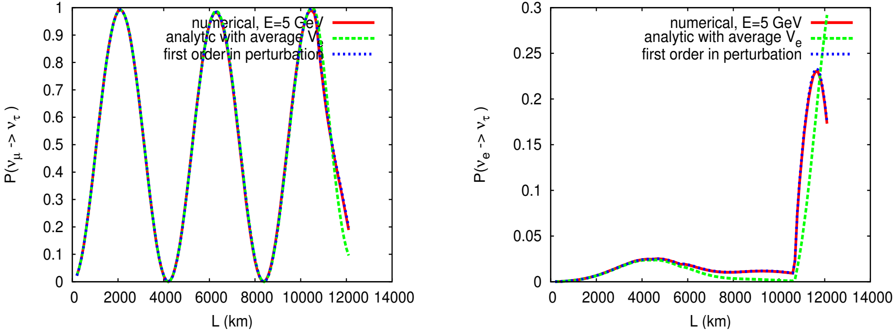




The CP violating effect in Earth matter is very well described by this formulation of ν oscillation

High energy ν conversion in the Earth

Left, $\nu_\mu \to \nu_\tau$ vs. L; right, $\nu_e \to \nu_\tau$ vs. L



$\nu_\mu - \nu_\tau$ is mainly vacuum oscillation, as can be seen

$\nu_e - \nu_\tau$ oscillation is very well described by the perturbation theory

Conclusions

- We found two nice theories which perfectly describe neutrino oscillation of interested energy range in Earth matter.
- Earth matter effect of interested energy range is well understood in these theories.
- For $E \lesssim 30$ MeV, the adiabatic perturbation theory says that main contributions are from potential jumps between layers of the Earth matter and averaging over energy can tremendously simplify the oscillation pattern.

Conclusions

- For $E \gtrsim 500$ MeV, expanding the potential around the baseline dependent average potential gives a perturbation theory
- This perturbation theory perfectly describes the probability and CP violating effect of oscillation of high energy neutrinos
- Ambiguities of Earth matter effect in measuring neutrino parameters in future experiments, in particular in measuring δ_{CP}, are properly addressed in the framework of this perturbation theory.